Roozbeh Haghnazar Koochaksaraei

Qualitative analysis in many-objective optimization with visualization methods

Belo Horizonte - Minas Gerais

June, 2017

Roozbeh Haghnazar Koochaksaraei

Qualitative analysis in many-objective optimization with visualization methods

Final thesis presented to the Graduate Program in Electrical Engineering of the Federal University of Minas Gerais in partial fulfillment of the requirements for the degree of Doctor in Electrical Engineering.

Federal University of Minas Gerais - UFMG

Graduate Program in Electrical Engineering - PPGEE

Machine Intelligence and Data Science Laboratory - MINDS

Supervisor: Prof. Dr. Frederico Gadelha Guimarães

Co-supervisor: Dr. Vitor Nazário Coelho

Belo Horizonte - Minas Gerais

June, 2017

K82q

Haghnazar Koochaksaraei, Roozbeh.
Qualitative analysis in many-objective optimization with visualization methods [recurso eletrônico] / Roozbeh Haghnazar Koochaksaraei . - 2017.
1 recurso online (xvi, 80 f. : il., color.) : pdf.

Orientador: Frederico Gadelha Guimarães.
Coorientador: Vitor Nazário Coelho.

Tese (doutorado) - Universidade Federal de Minas Gerais, Escola de Engenharia.

Bibliografia: f. 73-80.
Exigências do sistema: Adobe Acrobat Reader.

1. Engenharia elétrica - Teses. 2. Visualização de dados - Teses. 3. Otimização - Teses. 4. Pesquisa qualitativa - Teses. 5. Processo decisorio - Teses. I. Guimarães, Frederico Gadelha. II. Coelho, Vitor Nazário. III. Universidade Federal de Minas Gerais. Escola de Engenharia. IV.Título

CDU: 621.3(043)

Ficha catalográfica elaborada pela Bibliotecária Roseli Alves de Oliveira CRB/6 2121
Biblioteca Prof. Mário Werneck, Escola de Engenharia da UFMG

UNIVERSIDADE FEDERAL DE MINAS GERAIS
ESCOLA DE ENGENHARIA
Programa de Pós-Graduação em Engenharia Elétrica

ATA DA 255ª DEFESA DE TESE DE DOUTORADO
DO PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA ELÉTRICA

ATA DE DEFESA DE TESE DE DOUTORADO do aluno **Roozbeh Haghnazar Koochaksaraei** - registro de matrícula de número 2013744638. Às 09:00 horas do dia 23 do mês de junho de 2017, reuniu-se na Escola de Engenharia da UFMG a Comissão Examinadora da TESE DE DOUTORADO para julgar, em exame final, o trabalho intitulado **"Qualitative Analysis in Many-objective Optimization with Visualization Methods"** da Área de Concentração em Sistemas de Computação e Telecomunicações. O Prof. Frederico Gadelha Guimarães, orientador do aluno, abriu a sessão apresentando os membros da Comissão e, dando continuidade aos trabalhos, informou aos presentes que, de acordo com o Regulamento do Programa no seu Art. 8.16, será considerado APROVADO na defesa da Tese de Doutorado o candidato que obtiver a aprovação unânime dos membros da Comissão Examinadora. Em seguida deu início à apresentação do trabalho pelo Candidato. Ao final da apresentação seguiu-se a arguição do candidato pelos examinadores. Logo após o término da arguição a Comissão Examinadora se reuniu, sem a presença do Candidato e do público, e elegeu o Prof ..Frederico....Gadelha..................... para presidir a fase de avaliação do trabalho, constituída de deliberação individual de APROVAÇÃO ou de REPROVAÇÃO e expedição do resultado final. As deliberações individuais de cada membro da Comissão Examinadora foram as seguintes:

Membro da Comissão Examinadora	Instituição de Origem	Deliberação	Assinatura
Prof. Dr. Frederico Gadelha Guimarães - Orientador	DEE (UFMG)	Aprovado	
Dr. Vitor Nazario Coelho - Coorientador	IC (UFF)		
Prof. Dr. Igor Coelho Machado	IME (UERJ)	APROVADO	
Prof. Dr. Rodrigo Tomás Nogueira Cardoso	DFM (CEFET/MG)	Aprovado	
Prof. Dr. Ricardo Hiroshi Caldeira Takahashi	DMAT (UFMG)	aprovado	
Prof. Dr. Hossein Javedani Sadaei	DEE (UFMG)	Aprovada	

Tendo como base as deliberações dos membros da Comissão Examinadora a Tese de Doutorado foi ......aprovada.................. O resultado final de .aprovado.......... foi comunicado publicamente ao Candidato pelo Presidente da Comissão, ressaltando que a obtenção do Grau de Doutor em ENGENHARIA ELÉTRICA fica condicionada à entrega do TEXTO FINAL da Tese de Doutorado.O Candidato terá um prazo máximo de 30 (trinta) dias, a partir desta data, para fazer as CORREÇÕES DE FORMA e entregar o texto final da Tese de Doutorado na secretaria do PPGEE/UFMG. As correções de forma exigidas pelos membros da Comissão Examinadora deverão ser registradas em um exemplar do texto da Tese de Doutorado, cuja verificação ficará sob a responsabilidade do Presidente da Banca Examinadora. Nada mais havendo a tratar o Presidente encerrou a reunião e lavrou a presente ATA, que será assinada pelo Presidente da Comissão Examinadora. Belo Horizonte, 23 de junho de 2017.

ASSINATURA DO PRESIDENTE DA COMISSÃO EXAMINADORA

Acknowledgements

After intense period of four years, today is the day: writing this thank you note is the finishing touch of my dissertation. I have to admit that it was an intense period of learning for me, not only in the scientific arena, but also on a personal level. Writing this thesis has had a big impact on me and I would like to reflect on the people who have supported and helped me so much throughout this period.

To start with, I am thankful to God, for giving me the strength to finish this job.

I would like to express my sincere gratitude to my advisor *Prof. Frederico Gadelha Guimaraes* who has been the ideal thesis supervisor for me. Dear professor, your sage advice, motivation, insightful criticisms, and patient encouragement aided me to write this thesis in innumerable ways. During these years you have been more than a professor to me and I am so thankful for that. Furthermore, you have been the most amazing professor that I have ever had. Thank you dear Fred. You have taught me how to learn.

Beside my advisor, I have to appreciate all my professors and friends here in Brazil and Iran for their supports and friendship that helped me to move on through this path easily. In addition, I appreciate all Brazilians, your amazing land, and your government for this given opportunity to me. Specially I gratefully acknowledge support from *CNPq, Brazil.*

I would also like to have a special appreciation to my dear parents and my only brother, Ramtin, for all of their help, unconditional love, and support. My dear and lovely family, I have to avouch to your main role in my progresses in different steps and levels of my life and educations during the past decades.

Finally, super special thanks to my friends. Nobody can deny your role in this success and finishing. I have to admit that your presence along this stormy way reasoned me to move on strongly. I sincere admire you for your patience, forbearance and all of your supports and help during this period. You have been always there for me.

Thank you very much, everyone.

Roozbeh Haghnazar Koochaksaraei

June 2017

Resumo

Problemas de otimização com muitos objetivos apresentam vários desafios para os métodos de otimização atuais. Dentre essas, a visualização de soluções é um obstáculo importante para a interpretação dos resultados. Ter a habilidade de visualizar resultados parciais ou finais, de um problema multi-objetivo com várias dimensões, fornece vantagens chave para o otimizador bem como para o tomador de decisões, com relação a compreensão do problema e interpretação de resultados. Neste estudo, propõe-se uma ferramenta de visualização multi-propósito a ser aplicada em um processo de design evolucionário. A ferramenta de visualização proposta, denominada Visualização e Mapeamento em Arcos (VMA), contém duas partes e utilizações diferentes. VMA fornece duas importantes categorias de informação qualitativa sobre espaços de várias dimensões. A primeira parte da ferramenta mapeia as soluções do espaço de alta dimensão para as formas 2D, com base na extração da relação entre os objetivos. Em seguida, a segunda parte, mapeia as soluções do espaço de objetivos de alta dimensionalidade em uma forma 2D de espalhamento, baseada na norma e informações de ângulo entre os objetivos. A abordagem preserva algumas características desejáveis do espaço de objetivos, como a forma da Fronteira Pareto, sua localização, relações entre os objetivos, etc. Com o apoio desta ferramenta o decisor pode obter informações sobre a forma da frente de Pareto, a área explorada pelos algoritmos, uma estimativa qualitativa do desempenho do algoritmo, relação entre os objetivos, localização das soluções e sua dispersão. Além disso, este aplicativo tem escalabilidade e flexibilidade em relação ao número de objetivos e tamanho da população. Adicionalmente, o VMA permite ao decisor identificar visualmente regiões pouco exploradas do espaço de objetivos e determinar vetores de peso para guiar a busca por uma região específica ou preferida. Finalmente, os resultados experimentais mostram que esta ferramenta pode desempenhar um papel de métrica de desempenho e auxiliar o processo evolucionário de busca por soluções.

Palavras-chave: Visualização de Dados, Otimização de Muitos Objetivos, Análise Qualitativa, Análise Quantitativa, Relacionamento entre Objetivos, Tomada de Decisão Multi-Critério, Distribuição Individual.

Abstract

Many-objective Optimization Problems present various challenges to the current optimization methods. Among these, the visualization gap is an important obstacle to the interpretation of results. Having the ability of visualizing partial or final results of a high-dimensional multi-objective problem provides key advantages to the optimizer and also to the decision-maker in terms of understanding the problem and interpreting results. In this study, a multi-purposed visualization tool is proposed to be applied in an evolutionary design process. The proposed visualization tool, named Visualization and Mapping on Arcs (VMA), contains two different parts and usabilities. VMA provides two important categories of qualitative information from high-dimensional spaces. The first part of tool maps the solutions from high-dimensional space into the 2D forms based to extract the relationship between objectives. Then, the second part, maps the solutions from high-dimensional objective space into a 2D form of scattering that is based on norm and angle information. Meanwhile, it preserves some desirable characteristics of objective space, such as the shape of the Pareto front, its location, relations between objectives etc. With the support of this tool the decision-maker can obtain information about the shape of the Pareto front, the range of explored area by the algorithms, qualitative estimation of algorithm performance, relation between objectives, location of solutions and their dispersion. Furthermore, this application has scalability and flexibility about the number of objectives and population size. Additionally, VMA allows the decision-maker to visually identify poorly explored regions of the objective space and determine weight vectors to guide the search to a specific or preferred region. Finally, experimental results show that this tool can play a role of performance metric and help the evolutionary solving process.

Keywords: Data Visualization, Many-Objective Optimization, Qualitative Analysis, Quantitative Analysis, Objective Relationship, Multi Criteria Decision Making, Individual Distribution.

List of Figures

List of Tables

List of Algorithms

Contents

Chapter 1

Introduction

1.1 Introduction

Real-world problems are very often characterized by multiple objectives, leading to the so-called Multi-Objective Optimization Problems (MOOP) Deb et al. [2016] and multi-criteria decision making problems Triantaphyllou [2000]. Multi-objective problems have been intensively investigated in the optimization community, however usually focusing on a low number of objectives. The conflict between the objectives lead to the existence of a set of trade-off solutions, called the Pareto-optimal solutions Mishra et al. [2002], and its corresponding image in the space of objectives is the Pareto front. Finding the complete and exact set of Pareto-optimal solutions is a hard problem in many practical and engineering problems, therefore approximations of this set are the usual outcome of multi-objective algorithms, specially those relying on heuristic methods such as evolutionary algorithms. MOOP with at least four objectives are known as many-objective optimization problems (MaOPs) Farina and Amato [2002], Li et al. [2015]. This border is somewhat informal but with practical meaning and based on empirical studies about the downgrading performance of most multi-objective algorithms when the number of objectives increase. Multi-objective Evolutionary Algorithms (MOEAs) have been successful and well-suited to low dimensional multi-objective problems, but their performance degrades with the increasing number of objectives, as discussed in many different studies Li et al. [2015], von Lücken et al. [2014], He and Yen [2016b].

Among the different challenges posed by MaOPs, the visualization gap is an important obstacle to the interpretation of the results, when handling with more than three objectives Walker et al. [2013]. An intuitive and high quality visualization can enable the decision-maker to recognize characteristics of the problem, realize the underlying trade-off, range of explored areas, among other features Tusar and Filipic [2015].

In low-dimensional problems, containing less than four objectives, solutions can be represented on 2D by using 2D or 3D scatter plots. Although it cannot provide appropriate details in three-dimensional spaces, those methods have been helpful to provide enough information for the decision makers Var [1998]. One can still use 3D plots to represent up to 5-dimensional problems, if resorting to other attributes such as size and color, but interpretation and understanding is hard to the decision makers. Beyond that, in higher dimensional spaces, other visualization methods should be used and some approaches are reviewed later in this paper (Section 2).

Most researchers use quantitative analysis strategies in order to asses and evaluate the results in high dimensional spaces. Quantitative analysis and statistical measures are a good policy in evaluating heuristics, although it came later in the MOEA community, specially to improve the qualitative analysis that was ubiquitous in the early days of the field. Recently, in MaOP studies, the qualitative analysis and the visualization of results has been abandoned, relying mostly on performance measures and indices Van Veldhuizen and Lamont [1998]. In order to figure out suitable solutions in a many-objective space, there are still several intuitive challenges to analyze the problem for decision makers, optimizers and end users. For instance, in Tran et al. [2015], Cai and Yuping [2015], Deb and Sundar [2006], Liu et al. [2017], Zhu et al. [2017], several proposed methods and algorithms were discussed, being applied on the high dimensional multi-objective problems. All those studies considered quantitative analysis for evaluating and reaching conclusions, such as comparing or illustrating the performance of different methods. In this sense, nobody can deny the fact that the results of solving strategies do not allow decision makers to have an intuitive overview of the obtained solutions. This gap is clear in different solving strategies for MaOPs Cai and Yuping [2015], Esquivel et al. [2002], Schneider and Krohling [2014] and also Haghnazar Koochaksaraei et al. [2016], Freitas et al. [2013], Marler and Arora [2004a], Saxena et al. [2013], Singh et al. [2011].

Therefore, this article tries to fill this gap by proposing and developing a novel visualization tool, named Visualization and Mapping on Arcs (VMA), which contains a complementary usage to enable decision makers to solve many-objective optimization problems through qualitative analysis schemes. In the proposed approach, there is an iterative strategy for solving a many-objective problem, which is discussed in Section 3. The proposed VMA tool has two aspects: (i) the visualization of the relation between objectives, which might be helpful for objective reduction; (ii) the visualization of high-dimensional solutions of a MaOP, which can be useful for checking the distribution of the solutions in objective space, the shape of the Pareto front, and for qualitative comparison overview on the performance of different MOEA on the problem. The proposed interactive tool is not only suitable for developers, but also can reach end users, providing a more didactic manner for understanding conflicts and relations in real-world problems.

Totally, among all strategies, this research considered two different goals as complementary tasks in order to figure out the solution to the MaOP. Objective reduction and finding the unexplored area are the two main strategies which are going to be explored along this manuscript. For this purpose, we introduce a tool with different abilities to provide several useful information. In fact, we developed initial ideas in Haghnazar Koochaksaraei et al. [2016], a brief manuscript with a visualization tool able to analyze the relation between objectives. The present paper is a large extension of that work with an additional visualization tool for solutions in many-objective spaces. By finding the relationship between objectives, the proposed framework can allow the optimizer to decide on the aggregation of those objectives that are in harmony, in order to reduce the problem size. It is known that objective reduction can improve the efficiency of the solving algorithms, particularly MOEA, for exploring all over the problem space Brockhoff and Zitzler [2006]. In spite of this strategy, there is no guarantee to have full exploration throughout the space. Thus, the decision maker or optimizer must be able to realize the area which has not been properly explored by the solving algorithms. This ability is provided as a huge and influential expansion that is presented in this text. In this way, the decision maker, through reference point based algorithms such as NSGA-III, MOEA/D, MOPSO families etc. Cai and Yuping [2015],Deb and Sundar [2006],Liu et al. [2017],Zhu et al. [2017] and the obtained information from the proposed tool, is able to explore neglected regions. In Section **??**, this aspect of the tool will be detailed and discussed.

Altogether, the proposed application covers two general purposes. The first one includes a more precise comprehension about the relation between objectives in several aspects, focusing on realizing the harmony and conflict among them. The proposed visualization method for this helps to reduce the number of objectives. The second one involves mapping the high dimensional space into a 2D scattered form. This mapping provides several abilities to the decision maker, which are produced by illustrating the distribution of individuals on the estimated fronts.

1.2 Objectives of thesis

According to the above text, the main objectives of this study is finding an efficient strategy to solve many objective problems with a desired performance, both related to speed and solutions quality. It covers objectives which are written in following:

1. To product a tool and method in order to have qualitative analysis about many objective problems.
2. To provide the qualitative strategy for solving the many-objective problems.
3. To realize the harmony and conflicts between objectives in order to possibly ignore objectives.

4. To map the solutions from the high-dimensional spaces into the 2D VMA coordinate.

5. To find the characteristics of the obtained solutions.

6. To compare several individuals data sets which can be obtained from different algorithms or one algorithm in the different properties setting.

7. To explore the unexplored region of Pareto-Front or cover the specified directions considering the goals and requirements.

1.3 Challenges

The mentioned items are the objectives of this study and work which face several challenges, such as:

1. The mathematical methods to reduce problem dimensions are not so user friendly for the decision makers. On the other hand, since this is the subjective procedure, the decision makers need to have comprehensible overview and information about the problem space. Therefor the new tool should be efficient and easy to understand.

2. The harmony and conflict in different areas of objectives have different meanings and the tool should be flexible in this terms.

3. The concept of optimal solutions as an answer of problem is not useful in practical problems in real situations and real world. It should select considering different criteria according to the decision maker preferences.

4. Local optimum regions is one of the huge challenge in optimization which avoid algorithms to explore allover the Pareto-Fronts.

5. Analyzing the behavior of the algorithms in optimization processes prevent decision makers to apply the best values of parameters. For instance they might apply large number of population and iteration for the algorithm.

6. New tool and algorithm must be flexible and adaptable with the number of objectives.

1.4 Claimed contributions

As it was mentioned before, the main goal of this study is applying data visualization on the high-dimensional space problems in order to assess the space and reach to the proper and optimum solutions. In this way, three different majors have been studied. Data-Visualization, Many Objective Optimization and Multi Criteria Decision Making are the three complementary areas which can be helpful to achieve the most proper set of solutions Walker et al. [2012]. As a summary, the most important contributions are:

1. Mapping the high-dimensional spaces on the 2D forms coordinator considering different usabilities.

2. Visualizing the relationships between objectives or criteria.

3. Assessing the pattern of the relationships between objectives.

4. Evaluating the algorithms during their processes, stating novel stopping criteria or directing the search.

5. Enabling decision makers to control the search done by optimization algorithms, based on their preferences.

6. Simulating the patterns of individuals distribution in the space.

7. Giving an overview of the problem space to the end users.

8. Enable the decision makers to rank the solutions and alternatives based on their desires, requirements and goals.

9. Decrease the complexity of the problems in order to increase the speed and convergence accuracy.

The above items are the main contributions of this study. It is clear that, each one of the items contains several branches and sub-items. In the following some of the important publications which have been obtained from this study or applied this tool are listed.

1. A New Visualization Tool in Many-Objective Optimization Problems, Roozbeh Haghnazar Koochaksaraei, Rasul Enayatifar, Frederico Gadelha Guimarães, Hybrid Artificial Intelligent Systems (HAIS 2016), pp 213-224.

2. A New Visualization Method in Many-Objective Optimization through Mapping Arcs, Roozbeh Haghnazar Koochaksaraei, Ivan Reinaldo Meneghini, Vitor Nazario Coelho, Frederico Gadelha Guimaraes, submitted paper in Knowledge-Based Systems.

3. A multi-objective green UAV routing problem", Bruno N. Coelho, Vitor N. Coelho, Igor M. Coelho, Luiz S. Ochi, Roozbeh Haghnazar, et al., Computers & Operations Research.
   This work applied the proposed tool for understanding properties of the obtained sets of non-dominated solutions, tackling a novel UAV routing problem with several objectives.

In conclusion, in spite of existing several challenges, this study presents a strategy which contains two different tandem methods which are possible to be helpful when used sequentially in a problem solving process. It means, through this strategy, at first, the decision maker, as a user, should analyze the problem and reduce the objectives as much as possible, after that, through assessing the obtained solutions and considering specific preferences. Thus, explore all the desired region of a Pareto Fronts can reach to the optimum solutions. Then, decision maker can rank the obtained alternatives by using MCDM methods.

Chapter 2

Literature Review

This section summarizes a set of most useful and popular visualization tools which have been applied on the many-objective optimization problems (MaOPs). After that, it makes the overview on the different and popular MCDM methods which can be applied at the final step of optimization procedures in order to rank the final solutions on the Pareto-Front based on the decision makers' preferences and needs.

2.1 Visualizing the high dimensional problem spaces

As it was mentioned before, visualizing results in MaOPs is an important hindrance to provide a qualitative information for several usages such as: analyzing the problem (understanding correlations, impacts and consequences); objective reduction; decision making; etc. Hence, each tool has been designed to drive the user to his/her goals.

There are different approaches to visualize solutions in many-objective spaces. For instance these papers Saxena et al. [2013], Freitas et al. [2013], Brockhoff and Zitzler [2009], Lygoe et al. [2010] have tried to reduce the number of objectives and visualize the problem spaces. Objective reduction has been advocated by some researchers as a way to improve the ability of solving the problem, by understanding relations (in terms of harmony and conflict) between objectives and reducing the MaOP to a version with a smaller number of objectives, which is usually easier to solve with the current MOEAs. For instance, in Brockhoff and Zitzler [2009], the authors have tried to detect and recognize the conflicts between pairs of objectives and merge the ones who have harmony together. In this way, they followed several goals such as visualization, decision making, computational cost, etc. This strategy decreases difficulties when the problem contains more than three objectives. In spite of this research it should be considered that objective reduction is not necessarily able to improve the visualization understanding. Freitas et al. Freitas et al. [2013] tried to use aggregation trees to find the amount of harmony between objectives and according to the conflict and harmony the decision maker can merge objectives together. Simultaneously, it used parallel coordinates as a suitable visualization tool to present the relations between pairs of objectives. The adjacency of objectives in parallel coordinates was defined by the sequence of aggregations suggested in the aggregation tree. Lygoe et al. Lygoe et al. [2010] presented a many-objective decision making process on a real world problem with six objectives. They applied a modular process that has been designed to cluster the Pareto front by applying a rule-based Principal Component Analysis framework, including preferences articulation for potential objective reduction. Saxena et al. Saxena et al. [2013] discussed about high computational cost and difficulties in visualizing the problem space. They considered both challenges simultaneously as effective criteria for solving and decision making issues. Hence, development of generic and robust objective reduction approaches becomes important. Therefore, they present a principal component analysis and maximum variance unfolding based framework for linear and nonlinear objective reduction algorithms, respectively. We presented a new visualization tool in Haghnazar Koochaksaraei et al. [2016] in order to satisfy the lacks of current tools. This tool provides the relation among objectives simultaneously. It guides decision makers to asses the relationship between one objective with all the others at the same time. Moreover, the relationship can be seen in a specific range of objective values. For instance, if the goal of optimization is minimizing, the decision makers or optimizers are able to asses the relationships of objectives on the lower boundaries. In this current study, the proposed strategy will show how the provided information of this tool can be usable and helpful in the solving process. One of the primary researches that had tried to visualize the population of multi-objective problems is given in Fonseca and Fleming [1993], which described a rank-based fitness assignment method for multi-objective genetic algorithm (MOGA). Fonseca and Fleming used the method of parallel coordinates in order to show the values of objectives in the solutions provided by MOGA. Pryke et al. Pryke et al. [2007] believed that visualization of high dimensional parameter spaces has been traditionally neglected. Hence, they did one of the researches which tried to visualize the high dimensional spaces. They proposed a visualization method based on heatmaps, for the simultaneous visualization of objective and parameter spaces. They illustrated its application on a simple 3D test function and also applied heatmaps to the analysis of real-world optimization problems. In Walker et al. [2010], a method for ordering a many-objective optimization individuals in order to simplify a more perceptible visualization, without any needs to objective reduction, was suggested. Walker also presented multiple tools method in Walker et al. [2013] involving RadViz and heatmap in order to visualize mutually the non-dominated solution sets. First, they used RadViz and exploited interpretations of barycentric coordinates for convex polygons and simplices to map a mutually non-dominated set to the interior of a regular convex polygon in the plane, providing an intuitive representation of the solutions and objectives. Secondly, they introduced a new measure of the similarity of solutions, called the dominance distance, which captures the order relations between solutions. This metric provides an embedding in Euclidean space, which is shown to yield coherent visualizations in two dimensions. Zhenan and Gray He and Yen [2016b] have done one of the most recent researches about visualization in many-objective problems. They presented multi purposed research in order to visualize the high dimensional spaces and performance metric of solving algorithms. The visualization tool is based on mapping the high dimensional space into the 2-D polar coordinate plot while preserving the important characteristics of objective spaces such as Pareto relationship, shape and location of the Pareto front and the distribution of individuals. According to the observed information from this tool, the performance metric, named polar-metric has been designed. This metric system, enables decision makers to have a comprehensive comparison among MOEAs.

In general, many researchers have proposed strategies based on diverse aspects in order to bring solutions on the papers or display screens. On the other hand there are several principles which guide the designers to develop a proper tool according to the needs and conditions. In fact, data visualization principles specify cons and pros of design methods in each situation. In order to promote the rise of generic and didactic approaches, the designer of the tool proposed in this current research has tried to respect those rules, which are discussed in Section **??**.

As mentioned, in a low-dimensional space, with two or three objectives, scatter plot plays the role of popular tool, because it can illustrate the important characteristics of individuals in problem space such as location, distribution, shape of approximate fronts, etc. In fact, in those low-dimensional spaces, each objective can be directly assigned to a specific axis. Through scatter plots, decision makers can easily make selections and pick up preferred solutions. Totally, there have been two major categories that brought high-dimensional spaces into the 2-D screens. The first way applies additional attributes such as color, size, shape, etc. to the existing tools. Secondly, mapping high-dimensional spaces on the 2-D spaces by developing the new versions of Data Visualization tools. For instance, Bubble charts Ashby [2000] or RadViz can be considered as extensions of scatter plots.

Since Fonseca and Fleming Fonseca and Fleming [1993], parallel coordinates have been one of the most popular tools that have been applied on the high-dimensional spaces Freitas et al. [2013], Inselberg and Dimsdale [1990], Inselberg [2009]. The main advantage is that this tool is flexible about the dimensional number. For N-dimensional spaces, parallel coordinates represent an axis for each objective. Hence, it illustrates the original value of each objective on each axis. In other words, there is no need of sophisticated mapping of spaces. Moreover, this method provides some information about the approximate Pareto front and relationships between objectives. Although this information could be useful it is only obtained just about the adjacent objectives. This limit causes some lacks on trade-off analyzing among all objectives. In order to figure out the trade-offs, each solution is represented polylined with its vertexes. The vertexes are located on the parallel axes, where each axis illustrates the objectives and the values of the solution on that objective locates the vertex on the axis.

Another popular tool, often used to represent high-dimensional spaces, are the Heatmaps Hettenhausen et al. [2010], von Lücken et al. [2014], Pryke et al. [2007]. In Data-Visualization principles, heat map is a 2-D tool which contains X and Y axis and it uses color base gradients as an attribute to show the changes among the data set. Hence, the combination heatmaps with parallel coordinates results in a tool which uses color gradient as an attribute. Those colors illustrate the form of the distribution of the solutions in the space, while the parallel coordinates apply polylines to represent them. Meanwhile, heatmaps create a continuous space based on colors which is not proper for the discrete data space such as solution information. In addition, it is normally not able to indicate trade-off between objectives. In spite of that, it does not require mapping the objective space to the 2-D space to show the data.

Another interesting category of tools involve the use of mapping techniques. Basically, these tools uses mapping techniques in order to provide 2-D information based on M-Dimensional information for each individual. Although all mapping methods try to preserve the original information of each individual and the original estimated Pareto front, they do not avoid loosing some information. On the other hand, they are scalable to many dimensions, which helps visualization tools to be flexible and scalable. Self Organized Map Flexer [2001], Kohonen [2013], Radar Graph (also known as Radar Graph or Polar Graph) Grinstein et al. [2001], Hoffman et al. [2002], Freitas et al. [2013], ISOmapVlachos et al. [2002] and Sammon mappingHe and Yen [2016a], Valdes and Barton [2007] could be considered in the category of mapping transmission and visualization. It means each individual in this category does not need to contain the original information in order to be visual. For instance, ISOMAP presents a meaningful transition in the lower dimension while it contains low number of data clusters Vlachos et al. [2002]. Meanwhile it preserves the shape and geometry information of solutions during the transition from high-dimensional to the 2-D space. As another example, Self-Organizing Map (SOM) is an automatic data-analysis method. It is usually applied for solving clustering problems, as well as data exploration in several different fields and majors. In addition, it is a derivative of Neural Network, with a low computational complexity. This projection can be visualized in several methods and graphical strategies to extract and reveal the properties of fundamental information about a given data set Polzlbauer et al. [2005]. Accordingly, cons and pros of these methods have a strong dependency on the method of visualization, based on the applied attributes of the tool. Polar charts or RadViz are another visualization tool which are widely applied in different high-dimensional data sets. A polar chart is a circular graph for plotting polar coordinates, in fact, a special type of the parallel coordinates. According to its usage, it can be used on the mapping methods, using the angle and radius as the input data or consider the original data from each dimension (as in the parallel coordinates). The next method which has enough usability for mapping is Sammon Mapping, which has in its core a feature extraction algorithm that has been widely used for pattern recognition and exploratory data analysis Kovács and Abonyi [2002]. The idea is to minimize the stress function in order to save the local distance based on iterative methods (such as gradient descent). Moreover, this method has been advocated as a proper method to preserve the distribution form of the solutions.

In view of the wide possibilities to combine and create visualization tools, the authors believe that categorizing the methods might not be a proper way to evaluate and encapsulate them. That is why one can find some tools, such as polar graph, which can be used with mapped or original data. Moreover, the applied attributes can affect on the classification of the tools and methodologies. For instance, if color were considered as an attribute, the designer could use it with scatters or gradient in order to support the discrete or continuous space.

Finally, since visualization is usually used to extract the features of the individuals, as well as comprehending the problem space itself, the chosen attributes and tools play a vital role on qualitative analysis.

2.2 Overview of MCDM

2.2.1 What is MCDM

During the past decades, beside of the other technologies, choosing the best options among the lots of different alternatives has had a dramatic progress in modeling of MCDM problems Stewart [1992] is a good survey to this progress. These kind of methods can be very useful in several challenges of management. That is why the researchers in the management scopes are trying to find, make and improve the methods as a powerful method in decision making scope.

Imagine that there are a lot of alternatives like different cars, and there are several criteria and you want to choose the best one considering the criteria that is specified even with different weights. What is the way for choosing the best alternatives? That's why the manager should use Decision Making methods. And when there are some criteria you can use Multi Criteria Decision Making (MCDM).

On the other word, MCDM refers to making decision in the situation that there are several criteria. In the essence, a MCDM problem is formed into hierarchy of four elements which are: goal, objectives, criteria, and the alternativesWatthayu [2009]. These elements can be formed into the matrix. Assume that, $A = \{a_i, ..., a_m\}$ is a set of alternatives and $C = \{c_j, ..., c_n\}$ is a set of criteria according to which desirability of an action is judged. A decision matrix is a relation between A and C which is defined like it : $D = A \times C$ and it is a $m \times n$ matrix, in which elementd_{ij} indicates that value of alternative a_i when it is assessing in terms of criteria c_j. Even it can be assumed that the decision maker has considered the weights of criteria as $W = \{w_j, .., w_n\}$. Finally the total points of each alternative is obtained by the following equation:

$$A_i = \Sigma_j w_j d_{ij}$$

The result of overall score calculation for all alternatives are the rank of them. The highest option is the chosen option for decision maker. In the following this text is assessing some of the basic MCDM methods with the different approach.

2.2.2 AHP

This method is created in 1970 by Dr.Saati. This method is based on the pairwise comparison that means all alternatives are compared with each other as a pair sets and according to the several criteria. In this method the complex problem is divided to the some smaller parts. In each new part priority of each criterion based on the goal and the value of each alternative in the specific criterion calculated by the pairwise comparison matrix and arithmetic mean. And finally based on the combination of these weights, the final weights of each alternative will be obtained.

2.2.3 WASPAS

The first method which is presented here is WASPAS. The WASPAS method is a new MCDM method based on the Weighted Sum Model (WSM) and Weighted Product Model (WPM). Zavadskas et al. (2012) have demonstrated that this aggregation makes WASPAS better than WSM and WPM models E.K.Zavaskas et al. [2012]. In the following the steps of this method has been shown: In the above algorithm it is obtained that this

Algorithm 1 WASPAS Algorithm

if *opt* is the maximum value **then** ▷ Check that the criterion should be max or min
$\bar{X} \leftarrow \bar{x}_{i,j} = \frac{x_{i,j}}{\underset{i}{opt} x_{i,j}}$
else
$\bar{X} \leftarrow \bar{x}_{i,j} = \frac{\underset{i}{opt} x_{i,j}}{x_{i,j}}$
end if$\bar{\bar{xs}}_{i,j} = \bar{x}_{i,j} \times q_j$
$\bar{\bar{xp}}_{i,j} = \bar{x}_{i,j}^{q_j}$
$WPS_i = 0.5 \sum_{j=1}^{n} \bar{\bar{xs}}_{i,j} + 0.5 \prod_{j=1}^{n} \bar{\bar{xp}}_{i,j}$

method can consider about the both of minimizing and maximizing in its process. In the first step, the algorithm starts to make the normalize matrix from the initial decision making matrix. After calculating $\bar{X}$ the normalized value for each alternative should be calculated in terms of its criterion's weight q_j in order to make the summarizing part of $WASPAS$ weighted and normalized decision matrix. And also, the multiplication part. Finally, it should make the overall score with the mixture of those both of WSM ans WPM methods.

2.2.4 COPRAS

Complex Proportional Assessment method which is called COPRAS is used When the overall efficiency of an alternative is required to be appraised, it is vital to identify the most important criteria, to evaluate alternatives and assess information regarding these criteria; methods are used for evaluating the criteria to meet the DMs' needs. Decision analysis is applied for a situation in which a DM has to choose among several alternatives considering a particular set, usually conflicting criteria. For this reason, Complex proportional assessment (COPRAS) method which was developed by Zavadskas and Kaklauskas (1996) can be applied for. In real situations, most of the criteria are used for alternative evaluation deals with vague future, and values of the criteria cannot be stated precisely. The procedure of applying the COPRAS method consists in the following steps:

Algorithm 2 COPRAS Algorithm

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix}$$ ▷ Constructing the Decision Making Matrix X

$q_j \leftarrow w$ ▷ Significance of criteria which are determined by decision maker

$\bar{X} = \bar{x_{ij}} \leftarrow \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}}$ ▷ Normalize the X

$\hat{X} \leftarrow \bar{x_{ij}} \times q_j$

$P_i = \sum_{j=1}^{k} \hat{x}_{ij}$

$R_i = \sum_{j=k+1}^{m} \hat{x}_{ij}$ ▷ $m - k$ is the number of criteria that must be minimized

$R_{min} = \min_i R_i$ ▷ Calculating the relative significance of each alternative

$$Q_i = P_i + \frac{R_{min} \sum_{i=1}^{n} R_i}{R_i \sum_{i=1}^{n} \frac{R_{min}}{R_i}}$$

Sort Q_i

Like the general situation there is matrix X as the decision making matrix that contains alternatives and their value for each criterion. and q_j shows the ratio and significance of each criterion. Normalizing is the first real step of process in order to equalize the criteria and their values. After normalizing, and considering their significances, the method should calculate the P for the criteria which should be maximized and R for the criteria which should be minimized. Because Copras is the method that is usable for the problems that have minimizing and maximizing in their criteria. Finally, Q can show the overall score of alternatives, and the highest one is the selected one.

2.2.5 VIKOR

VIKOR method or Multi-criteria Optimization and Compromise Solution is a compromise MADM method, presented by Opricovic for the first time Opricovic [1998] , Opricovic and Tzeng [2002]. The framework of VIKOR method is based on the compromise programming of MCDM by comparing the measure of closeness to the ideal alternative Wu et al. [2009]. In continue it is going to show the steps of VIKOR:

Algorithm 3 VIKOR Method

$\bar{X} \leftarrow \bar{x} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$ ▷ Normalizing the Decision Making Matrix X

for $NumberofCriteria$ **do**

  $f_j^* \leftarrow max f_{ij}$

  $f_j^- \leftarrow min f_{ij}$

end for

$S_i = \sum_{j=1}^{n} w_j \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)}$

$R_i = MAX[w_j \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)}]$

$Q_i = v[\frac{S_i - S^*}{S^- - S^*}] + (1 - v)[\frac{R_i - R^*}{R^- - R^*}]$

Sort the Q

Assuming that each alternative is evaluated by each criterion function, we can represent this as a matrix $[a_{ij}]$, with $i = 1, \ldots, n$, $j = 1, \ldots, m$, where there m alternatives evaluated with n criteria. a_{ij} represents the value of ith criterion for the alternative j. Like other decision-making methods, the first step of the algorithm is normalization to equalize the values in different kinds of criteria. The first step in the method is the normalization of the data, leading to the normalized decision matrix:

$$x_{ij} = \frac{a_{ij}}{\sqrt{\sum_{j=1}^{n} a_{ij}^2}} \tag{2.1}$$

Determine the best f_i^* and worst f_i^- values of all criterion functions:

$$f_i^* \longleftarrow \max_j x_{ij} \tag{2.2}$$

$$f_i^- \longleftarrow \min_j x_{ij} \tag{2.3}$$

Compute the values S_j and R_j for each alternative:

$$S_j = \sum_{i=1}^{n} \omega_i \frac{f_i^* - f_{ij}}{f_i^* - f_i^-} \tag{2.4}$$

$$R_j = \max_i \left[\omega_i \frac{f_i^* - f_{ij}}{f_i^* - f_i^-} \right] \tag{2.5}$$

where ω_i are the weights expressing the relative importance of the criteria. Compute the values Q_j for each alternative:

$$Q_j = \nu \left[\frac{S_j - S^*}{S^- - S^*} \right] + (1 - \nu) \left[\frac{R_j - R^*}{R^- - R^*} \right] \tag{2.6}$$

where:

$$S^* = \min_j S_j, \quad S^- = \max_j S_j$$

$$R^* = \min_j R_j, \quad R^- = \max_j R_j$$

The parameter ν is a compromise weight between the maximum group utility (majority rule) and the minimum individual regret (veto rule). Usually $\nu = 0.5$ is adopted. In other words, increasing ν enables the method to consider a majority vote point more than a minimum regret point (veto). In Section **??**, we will show the effects of the combination.

Once the S_j, R_j and Q_j values are obtained we can rank the alternatives. Rank the alternatives by sorting Q in ascending ordersuch that the best alternative has the minimum value of Q.

2.2.6 TOPSIS

This method has been presented by Yoon and Hwang [1981]. In this method, m alternatives are assessing by the n criteria. Each problem is considering as a geometric system which contains m point in the n dimensional space. In this method, the purpose is maximizing the distance from the ideal negative point and minimizing the same solution's distance from the positive ideal point.

As it mentioned before, this algorithm tries to select the closest alternative to the ideal and best feasible point in the space. The concept is very similar to the VIKOR and the difference is in the methods of calculation. That's why the results of TOPSIS and VIKOR are very similar to each other. In continue, it is going to present the proposed methods and tools.

Algorithm 4 TOPSIS Algorithm

$\bar{X} \leftarrow \bar{x_{ij}} = \frac{x_{ij}}{\sqrt{\Sigma x_{ij}^2}}$ ▷ Normalizing
$V = \bar{X} \times W_{n \times n}$ ▷ Weighted normalised decision matrix
for Number of Criteria **do**
    $A_i^+ \leftarrow max\ f_i$ ▷ Getting the positive ideal point
    $A_i^- \leftarrow min\ f_i$ ▷ Getting the Negative ideal point
end for
$d_i^+ = \sqrt{\sum_{j=1}^{n} (V_{ij} \times V_j^-)^2}$
$d_i^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^-)^2}$
$C_i = \frac{d_i}{(d_i + d_i^+)}$

Chapter 3

Multi-objective Evolutionary Optimization

Regarding the case study of the proposed tool, this chapter discusses some general definitions and considerations the Many-objective optimizations and presents three instants of the applicable algorithms which can accept our tool as the visualizer. The algorithms have been chosen based on their relevance in the current spectrum of research in Many-Objective Optimization and its adequacy to the proposal presented. On the other hand, according to our tool, reference point based algorithms category was our constraint. This chapter is divided into two parts: Section 2.1 gives a brief introduction to the problem of Many-Objective Optimization, presenting its general concepts while Section 2.2 deals with the technique used to solve the presented optimization problem. The end of the second section is devoted to exposing the MOEA/D, NSGA-III, and MOEA/DD.

3.1 Many-Objective Optimization

First of all, what is the Many-Objective Optimization? Loosely speaking, many-objective optimization problems are defined as problems with four or more objectives which should be solved simultaneously in spite of their conflicts. Two-objective and three-objective problems fall into a different class as the resulting Pareto-optimal front and, in most cases, can be comprehensively visualized by graphical means. Although a strict upper bound on the number of objectives for a many-objective optimization problem is not so clear, except for a few occasions. The optimization of each function can be given by the search of its greater value (problem of maximization), or the search of its lowest value (minimization problem) or a mixture of these. However, usually optimizers consider a MOO functions with the same goals. for instance, they should be minimized because the maximization of a function f is equivalent to the minimization of $-f$. In this way the objectives can always be presented as minimization problems Deb [2001], Gaspar-Cunha et al. [2012], Coello et al. [2006].

Definition 1 (Multi-objective optimization problem) *It is the problem of optimization given by:*

$$x^{\star} = \arg\min F(x) = (f_1(x), f_2(x), ..., f_m(x))$$
$$\textit{Subject to:} \begin{cases} g_i(x) \leq 0, \ 1 \leq i \leq p \\ h_j(x) = 0, \ 1 \leq j \leq q \\ x \in \mathcal{S} \end{cases} \tag{3.1}$$

$g_i(x)$ p are the inequality constraints and $h_j(x)$ q are the equality constraints.

Where m is the number of objective functions, p is the number of inequity constraint and q is the number of equity constraint. $x \in E^n$ is vector of *design variables*, where n is the number of independent variables x_i. $F(x) \in E^k$ is a vector of objective functions $F_i(x) : E^n \nRightarrow E^1$. $F(x)$ are also called *objective, criteria, payoff functions, cost functions,* or *value function* Marler and Arora [2004b]. The gradient of $F_i(x)$ with respect to x is written as $\nabla_x F_i(x) \in E^n$. x_i^* is the point that minimizes the objective function $F_i(x)$. The *feasible design space* Ω is defined as the set $\{x|g_i(x) \leq 0, i = 1, 2, .., p; and h_j(x) = 0, j = 1, 2, .., q\}$, where x must belong to Ω. If the only constraint of the MOP is $x \in S$ or $S = \Omega$, the problem is called unrestricted.

On the one hand, the ineffectiveness of Pareto dominance, aggravation of the conflict within convergence and diversity, and inefficiency of the recombination process, accompanied by rapid progression of time or space demand and parameter sensitivity, have been notable barriers to the form of many-objective search algorithms. On the other hand, the infeasibility of solutions' direct observation, the difficulty of the design of the trade-off surface, and the challenge of explaining the relationship among objectives and articulating preferences lead to severe difficulties in algorithm execution investigation, comparison, and decision-making method. All of these imply an important requirement for innovative methodologies in many-objective optimization. In the next Section, some of the Many-Objective Optimization Techniques will be reviewed.

3.2 Many-Objective Optimization Techniques

Evolutionary multi-objective optimization (EMO) methodologies have proven to be extremely successful in obtaining well-converged solutions for optimization problems with two and three objectives. Some of these auspicious methods incorporate Strength Pareto Evolutionary Algorithm (SPEA) Zitzler [1999], SPEA2 Zitzler et al. [2001], Non-dominated Sorting Genetic Algorithm NSGA-II Deb et al. [2002] and Pareto Archived Evolution Strategy (PAES) Knowles and Corne [1999].

While all these methodologies have shown achievement, it is essential to acknowledge that many real-world problems have more than three objectives. Scalability tests for these methods highlight several obstacles linking to convergence, diversity, and calculation time. In conclusion, it is necessary to come up with innovative methodologies or to enhance existing ones to be able to deal with a higher number of objectives. While many-objective optimization is a somewhat new field of study, it is necessary to take note that some effort on this had previously begun in the beginning 1990s. One of the most initial algorithms which have been implemented to many-objective problems is MOGA Fonseca et al. [1993]. MOGA was examined on the four objective Pegasus gas turbine engine optimization problems [11]. After then several researchers have tried to solve different real-world and simulated many-objective optimization problems. The majority of the activity in this field has grown in the last decade.

One of the most successful algorithms in literature is NSGAII Deb et al. [2002]. It is frequently employed as a baseline algorithm for connection with new algorithms. The NSGA-II is a computationally quick and elitist MOEA based on a non-dominated sorting procedure. It additionally employs a specific diversity-preserving mechanism to get a set of well-spread Pareto-optimal solutions. Several NSGA-II reforms have been introduced, recently, to get the algorithm further effective in handling a higher number of objectives. ϵ-NSGA-II Kollat and Reed [2006] connects NSGA-II with an ϵ-dominance archive, adaptive population sizing, and time continuation [32]. This algorithm has also been applied to several different real-world many-objective problems Kollat and Reed [2006, 2007].

Hadka and Reed Hadka and Reed [2012] introduced Borg MOEA which is an algorithm created for handling many-objective, multimodal problems utilizing an auto-adaptive multi-operator recombination operator that can better search in various problem areas. It employs multiple different recombination drivers such as the SBX, parent-centric crossover (PCX), simplex crossover (SPX), polynomial mutation (PM), and numerous others. This enables the algorithm to immediately adapt to the problem's local characteristics and modify as needed.

Bringmann et al. Bringmann et al. [2011] proposed an algorithm that operates with a conventional notion of approximation. The advanced algorithm, named Approximation-Guided Evolution (AGE), exceeded state-of-the-art methods in terms of the desired additive approximation and the covered hypervolume on standard benchmark functions (with many objectives) provided a fixed time budget. Notwithstanding the good execution of problems with many objectives, AGE was not so effective for problems with few objectives. An extra major issue with AGE was that it collected all non-dominated points seen so far in an archive that considerably changed its runtime.

Asafuddoula et al. Asafuddoula et al. [2014] introduced a decomposition-based evolutionary algorithm with adaptive epsilon comparison (DBEA-Eps). The algorithm is created using a steady state pattern and employs writing directions to guide the exploration. The equilibrium between diversity and convergence is controlled adopting an adaptive epsilon comparison.

Deb and Jain Deb and Jain [2013] lately proposed the NSGA-III which employs a reference point-based method for many-objective optimization. The amount of reference points is comparable to the population size guaranteeing that each population member is connected with a reference point. This secures diversity as the reference points are uniformly distributed over the normalized hyper-plane. The process, which has been produced especially for many-objective optimization, confirmed better performance in comparison to methods such as the MOEA/D Zhang and Li [2007] and NSGA-II Deb et al. [2002].

One of the important extensions is the U-NSGA-III Seada and Deb [2015] which is a combined evolutionary algorithm able of solving single/multi/ many-objective problems. The U-NSGA-III was capable to provide equivalent and sometimes better performance in comparison to a real-coded genetic algorithm, NSGA-II, and NSGA-III.

A knee point driven evolutionary method (KnEA) Zhang et al. [2014] has been introduced lately in which the knee points between the non-dominated solutions in the current population are provided decision for mating selection and environmental selection. The authors show that preference of knee points can practically be seen as a bias towards higher hypervolume which serves in obtaining good convergence and diversity. Lately, various review papers have been published in the literature, which concentrates on the use of multi-objective evolutionary algorithms to solve MaOPs also recently formed many-objective evolutionary algorithms to address MaOPs Bechikh et al. [2017].

The MaOEAs are grouped into various categories, viz. relaxed dominance-based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches. Furthermore, these sections include several frameworks or methods used to solve MaOPs. He and Yen in He and Yen [2017] presented and analyzed visualization methods used in many-objective optimization problems. They have categorized these methods into five different categories. The purpose of visualization approaches is to visualize the population in many-objective optimization problems to assess the performance of the algorithm and for decision making. There are some quality metrics observed in literature used for performance evaluation, viz. Hypervolume (HV), Epsilon, Generational distance (GD), and Inverted generational distance (IGD). Each quality metric's different varieties proposed Bezerra et al. [2017].

3.2.1 Exploring MOEA/D, NSGA-III, and MOEA/DD

This section aims to provide an overview on some famous and popular Many Objective Evolutionary Algorithms which have been using in most of the recent researches. There are many other algorithms such as MOPSO, MDMOEA, EMOA, NSGA-II, PCSEA, and etc. but in this research the author has used NSGA-III, MOEA/D and MOEA/DD as some instances to illustrate the proposed methodology and its capabilities. It is clear that this methodology can be utilized on the other Many Objective Evolutionary Algorithms. There are some reasons that the researcher team selected these algorithms that will be discussed in the following chapters. For example, MOEAD has a specific characteristic to reproduce the vector of browsing area and this capability helps the user to use the proposed visualization tool to create that vector and re-run the algorithm.
On the other hand some of the current evolutionary algorithms such as NSGA-II are not fully designed of a many objective optimization problem. In fact, NSGA-II is more proper for multi-objective problem spaces and also single objective problems. But there are some researches that introduced NSGA-II as a MoEAs. It should be noted that multi objective problem space means 2 and 3 objectives while more than that is known as many-objective problem spaces.
In spite of the above facts, the proposed methodology in this research can be utilized for all other algorithms and the end user can use it to improve the performance of the optimization process.

3.2.1.1 MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition

This method proposed a multiobjective evolutionary algorithm based on decomposition (MOEA/D) Zhang and Li [2007]. It actually decomposed a multiobjective optimization problem into a representation of scalar optimization subproblems and optimizes them concurrently. Each subproblem is optimized by only using information from its various neighboring subproblems, which gives MOEA/D have more moderate computational complexity at each generation than MOGLS and nondominated sorting genetic algorithm II (NSGA-II). The strength of MOEA/D with a small population, the scalability, and sensitivity of MOEA/D makes this method very applicable in a wide range of problems. Since the part of this thesis is related to this method, the algorithm is explained in algorithm 5.

Algorithm 5 MOEA/D

Input
$MOP \leftarrow input_problem$
$N \leftarrow Thenumberofsubproblemsconsideredin MOEAD/D$
$\lambda^1, \lambda^2, \ldots, \lambda^N$: uniformly distributed weight vectors; p_c: crossover rate; p_m: mutation rate.
$T \leftarrow The number of the weight vectors in the neighborhood of each \lambda$
gen_{max}
Output
EP;
procedure MOEAD/D
$EP \leftarrow \emptyset$
$gen \leftarrow 0$
$P_0 = \{x^0, x^1, x^2, ..., x^n, \}$ ▷ Initiate population
$FV^i \leftarrow F(x^i)$
$\lambda^{i(1)}, \ldots, \lambda^{i(W)}$, by Euclidean distance and set $\phi(i) = \{i(1), \ldots, i(W)\}$.
Generate an initial population $x_1, \ldots, x_N$ and evaluate $f_u(x_j)$ for each individual.

for $\lambda^i, i = 1..N$ **do**
$W \leftarrow$closest weight vectors
$\lambda^{i(1)}, \ldots, \lambda^{i(W)}, byEuclidean distance and set \phi(i) = \{i(1), \ldots, i(W)\}$.
Generate an initial population $x_1, \ldots, x_N$ and evaluate $f_u(x_j)$ for each individual.
Initialize $z = (z_1, \ldots, z_m)$.
end for
repeat ▷ Update phase
$gen = gen + 1$
for $i \leftarrow 1$ **to** N **do**
Reproduction: Generate a new solution y by two individuals x_u and x_l using crossover and mutation operators, where $u, l \in \phi(i)$.
Improvement: Improve y by using a problem-specific improvement repair operator, which is optional.
Update of z: For $j = 1, \ldots, m$, if $f_j(y) < z_j$, set $z_j = f_j(y)$.

Update of neighboring solutions: For each $k \in \phi(i)$, if $g(y|\lambda^k, z) \geq g(x_k|\lambda^k, z)$,
set $x_k = yandf_j(x_k) = f_j(y), j = 1, \ldots, m$.

Update of EP: Remove those solutions dominated by y from EP and add y to EP if it is not dominated by any member in EP.
end for
until $gen = gen_{max}$
return EP
end procedure

3.2.1.2 NSGA-III

In one important research work, Deb and Jain [2013] proposed a reference-point-based many-objective evolutionary algorithm following NSGA-II framework that indicated population members that are nondominated, but close to a set of supplied reference points. The suggested NSGA-III is utilized to a number of many-objective test problems with three to 15 objectives and compared with two versions of a recently suggested EMO algorithm (MOEA/D). While each of the two MOEA/D methods works properly on various classes of problems, the proposed NSGA-III is determined to present satisfying results on all problems studied in their paper. Without any doubt, NSGA-III is one of important algorithm and because we are using this algorithm in this thesis, it is essential to bring its algorithm here:

Algorithm 6 Generation t of NSGA-III procedure

```
    Input
    H structured reference points
    Z^s or supplied aspiration points Z^a
    parent population P_t
    Output
    P_{t+1}

  procedure NSGA-III
      S_t ← ∅, i = 1
      Q_t = Recombination+Mutation(P_t)
      R_t = P_t ∪ Q_t
      (F_1, F_2, ...) = Non-dominated-sort(R_t)
      repeat
          S_t = S_t ∪ F_i and i = i + 1
      until ‖S_t‖ ← N
      Last front to be included: F_l = F_i
      if ‖St‖ = N then
          P_{t+1} = S_t
      else
          P_{t+1} = ∪_{j=1}^{l-1} F_j
          Points to be chosen from F_l : K = N − ‖P_{t+1}‖
          Normalize objectives and create reference set Z^r: Normalize(f^n, S_t, Z^r, Z^s, Z^a)
          Associate each member s of S_t with a reference point:[π(s), d(s)] =
  Associate(S_t, Z^r)%π(s) : closest reference point, d : distance between s and π(s)
          Compute niche count of reference point j ∈ Z^r : ρ = Σ_{s∈S_t/F_1} ((π(s) = j)?1 : 0)
          Choose K members one at a time from F_l to construct P_{t+1} : Niching
  (K, ρ_j, π, d, Z^r, F_l, P_{t+1})
      end if
  end procedure
```

3.2.1.3 MOEA/DD

Although traditional MOEAs have gained prominent success in solving MOPs, they usually fail to solve MaOPs. The analyses after the failure can be connected to the tendency of pick pressure exerted by the criterion for analyzing solutions and the loss of population variety in the manner of evolution. The weakness of selection pressure decreases the convergence speed of an MOEA, and the loss of population diversity heads to a very poor distribution of the resulting population. Consequently, setting between convergence and diversity becomes a crucial issue for evolutionary algorithms to solve MaOPs. A lot of attempts have been made to deal with this issue, and some techniques are added into MOEAs to keep balance between convergence and diversity. One of the most strong evolutionary algorithms for solving MaOPs may be the evolutionary many-objective optimization algorithm based on dominance and decomposition (MOEA/DD) proposed in Li et al. [2014]. In MOEA/DD, each individual is connected with a subregion uniquely defined by a weight vector, and each weight vector (or subregion) is attached to a neighborhood. In an iterative step, mating parents is extracted from the neighboring subregions of the current weight vector with a given probability δ, or the whole population with a low probability $1 - \delta$. In case that no associated individual exists in the selected subregions, mating parents are randomly taken from the entire population. And then serval traditional genetic operators such as the simulated binary crossover (SBX) Deb et al. [1995] and the polynomial mutation Deb and Goyal [1996], etc., are implemented on the chosen parents to produce an offspring. Consequently, the offspring is applied to replace the worst solution within the current population defined by a hybrid method based on decomposition and dominance. All the solutions are arranged into N subregions, and split into various levels according to their dominance relationship. To discover the worst solution, the most crowded region is first distinguished from the subregions associated with the solutions in the last domination level, and the solution with the largest PBI value is chosen as the worst one. The following is General framework of of MOEA/DD.

Algorithm 7 General framework of MOEA/DD

```
Output
population P

procedure MOEA/DD
    [P, W, E] ← INITIALIZATION()
    P is the parent population
    W is the weight vector set
    E is the neighborhood index set
    while termination criterion is not fulfilled do
        for i ← 1 to N do
            P̄ ← MATINGSELECTION(E(i), P)
            S ← VARIATION(P̄)
        end for
        for each x^c ∈ S // x^c is an offspring do
            P ← UPDATEPOPULATION(P, x^c)
        end for
    end while
    return P
end procedure
```

Chapter 4

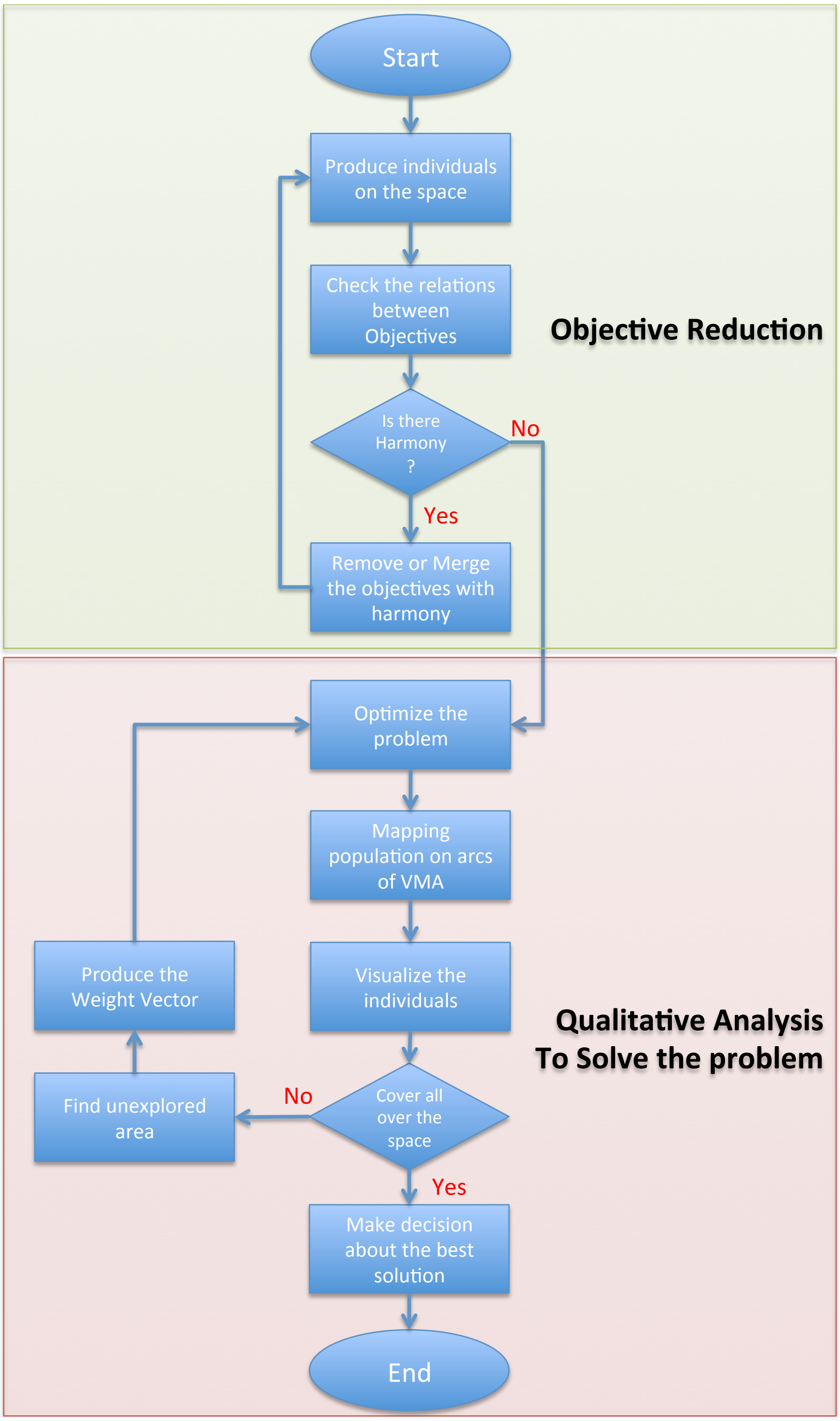
Proposed Method

4.1 Evolutionary solving approach

After presenting an overview of the popular tools and methods in the last section, this section is going to present a new composite methodology to solve MaOPS and evaluate MOEAs. This methodology contains iterative steps to solve the MaOPs, which are shown in the algorithm flowchart depicted in Figure 1. Comparisons of the performance and abilities of algorithms is another functionality of the proposed tool and method. This ability enables users to compare the obtained solutions from several algorithms simultaneously through the visualized individuals on the screen and extract diverse information from them. There are some variables in design language that help designers to devise a good tool based on their needs Ward et al. [2010], Aigner. These variables are position, size & length, shape, value, and color. Each one of those variables have their own characteristics and power to serve a purpose. Position is the strongest variable and shape is great to recognize the different classes, while colors can be useful for qualitative data. The order of effective impact can be, position & length, area & angle, ratio scale and finally we can say that 3D and Gradient are the least effective and can be the last option.

As already emphasized, this research introduces an innovative tool that provides users with useful information about: relations between objectives; distribution of individuals; approximate form of Pareto Front; poorly explored area of objective space; and the information about reference point or vector in the unexplored area. In this sense, it helps decision makers to understand the problem and, consequently, being able to reduce the number of objectives and push the solving algorithm to cover all over the space. Also, since decision makers can see the individuals on the 2D, therefore they are able to compare visually the performance of different algorithms.

Figure 1 – Flowchart of the solving process, describing both sequential and iterative parts.



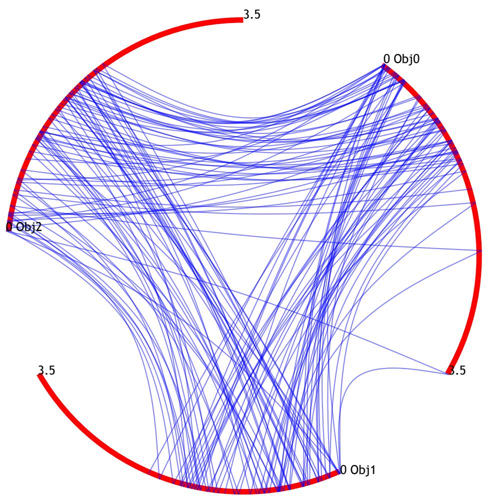
4.2 Objective Reduction through visualization

In the first part of the proposed tool, we adopt the visual information-seeking mantra, also known as *Shneiderman Mantra*, as the design concept Brown, Ben Shneiderman Catherine Plaisant [2010]. The Shneiderman Mantra summarizes visual design guidelines and provides a framework for designing information visualization tools, as described in Section 4.2.0.1. According to this concept, the first step of visualizing data should be the overview and after that zooming and filtering can help the user to have detailed information. In order to illustrate the details, linking and highlighting (detailed in Section 4.2.0.2) help users to see on demand the accurate information that is useful for them.

4.2.0.1 Overview Step

Figure 2 shows the overview of data to the user according to the mentioned points, generated considering 50 solutions from the DTLZ-2 problem with three objectives The range of each objective is represented along an arc, forming a circle. Solutions in the objective space are represented by individual lines connecting the corresponding values of the objectives for that solution. There are two kinds of overview in this frame. The first kind of view shows the links individually connecting one value of an objective to another value of a different objective for the same solution. For example, in Figure 2, there are two links in the end point of the *Obj0* that connect to the first points of *Obj1, Obj2*, showing that there is conflict between these objectives.

Figure 2 – Relation of three objectives in DTLZ-2 using 50 solutions.



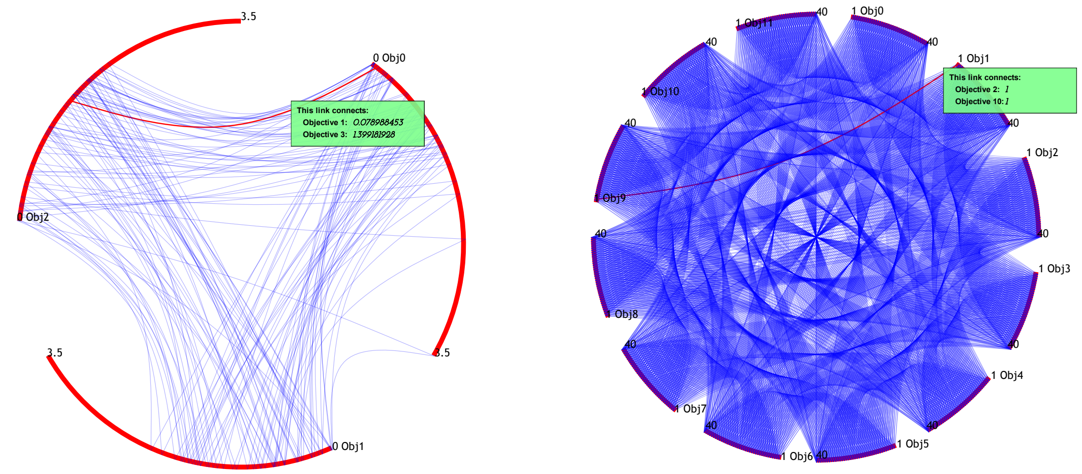
Another kind of general information which has been extracted from this overview is the overall pattern of conflict or harmony between objectives. Looking more closely, the links from the lower bound of the objectives are denser, since this is a minimization problem. The slightly denser pattern of links from the lower bound of *Obj0* to the lower band of *Obj1* and the another dense pattern from the lower bound of *Obj1* to the middle bound of *Obj2* show some line crossings, which demonstrates some moderate conflict between these objectives. In this sense, the nature (sometimes called as essence) of the problem, or objective space, can be extracted and identified through the chaos or discipline of links (the density among them). In this sense, the discipline of the connections between the objectives may create a visual pattern.

4.2.0.2 Detail Step

It is clear that the user needs more information than what the overview presents. Since having enough information is necessary to make the best decision, visualization tools should be able to produce and present enough and appropriate information for the end user based on the problem, needs and goals. According to *Shneiderman Mantra*, after presenting the overview, the tool should provide more details for supporting conclusions. In addition, *linking & highlighting* Ward et al. [2010] can be a good way for achieving those insights.

Figure 3 presents the tool in situations with 3 and 12 objectives, where many solutions have been produced on them. This amount of data makes a mess of links with different patterns. In this sense, the end user is not able to have clear information. Due to the huge amount of links available in the tool, highlighting emerges as a suitable strategy for improving the presentation condition. These cases motivates the designer to provide hovering for links in order to give information about a specific solution.

Figure 3 – Highlighting a specific link to get the detail information



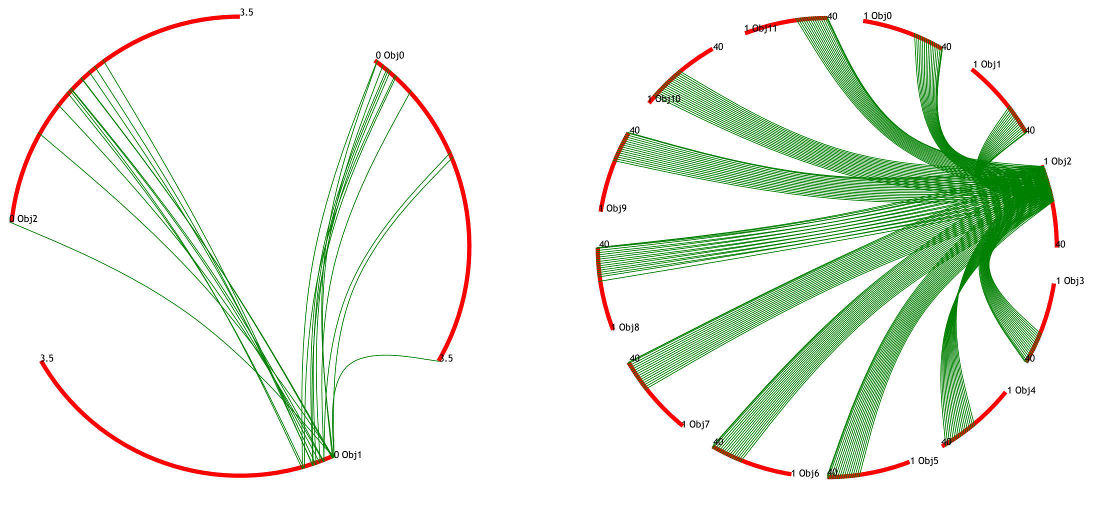
(a) Set of solutions with 3 objectives (b) Set of solutions with 12 objectives

For instance, in Figure 3a, one can see the link of a solution between *Obj0* and *Obj1*, which has the values of 0.07 and 1.39 for the in the first and third objectives, respectively. Therefore, the user can understand and realize that this highlighted link shows a little conflict between first and third objectives, because it has connected the lower bound of first objective to the middle range of third one. The highlighted link of Figure 3b connects *Obj1* and *Obj9*, both with the same value. For the last case, a high harmony, for this solution between these two objectives, can be observed.

It is recognizable that this level of details is not enough to realize the harmony and conflict of information. For this reason, we tried to provide a complementary way to have a total and details view in one window. In particular, trying to accomplish the goal of illustrating the relations among objectives for realizing and finding harmonies and conflicts between them. Based on the main goal, a strategy is needed that can help to have a flexible comparison among the different boundaries and intervals of objectives. Hence, we need to have information about the amount of harmony in intervals of an objective with other ones.

To support the above requisite, the designer has provided and designed the *frame focus* facility to have a flexible analysis. This feature is depicted in Figure 4, which presents the relationship among objectives for the special interval of one objective. For instance, in a three-objective space, the lower bound of *Obj1* has been selected and there are some links that connect this interval to the other objectives. As it is shown, this bound has links to the lower, middle and upper bounds of *Obj0* and also has connections to the lower and middle bounds of *Obj2*. This visual information shows the appropriate conflict between second objective and two others in lower bound. In addition, the same analysis for the 12-objective space demonstrates a more regular link pattern.

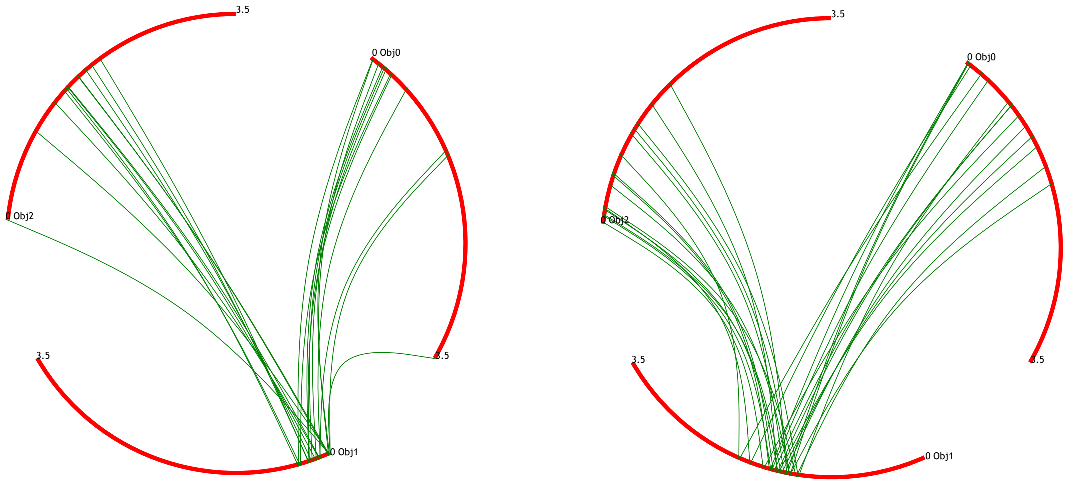
Figure 4 – Specify an interval to find relationships between objectives



(a) Highlighted solutions with 3 objectives (b) Highlighted solutions with 12 objectives

All of the extracted information is helpful in different comparison purposes. As it is shown in Figure 5, there is a comparison between different boundaries of *Obj1* with the other objectives in 3-objective space, which is useful in order to find the trade-off behavior in the objective space. For example, the selected region in the lower boundary of the second objective has connections to all the regions of the first objective and the first half of the third objective. Meanwhile, the middle boundary of second objective has connections to the lower half part of the first and lower and middle part of the third objectives. Therefore, the user can realize the different trade-offs in different regions of *Obj1* in this space. This comparison and analysis about the behavior of several intervals of an objective can be helpful when there is not full harmony or full conflict between objectives.

Figure 5 – Comparing the objectives in different regions

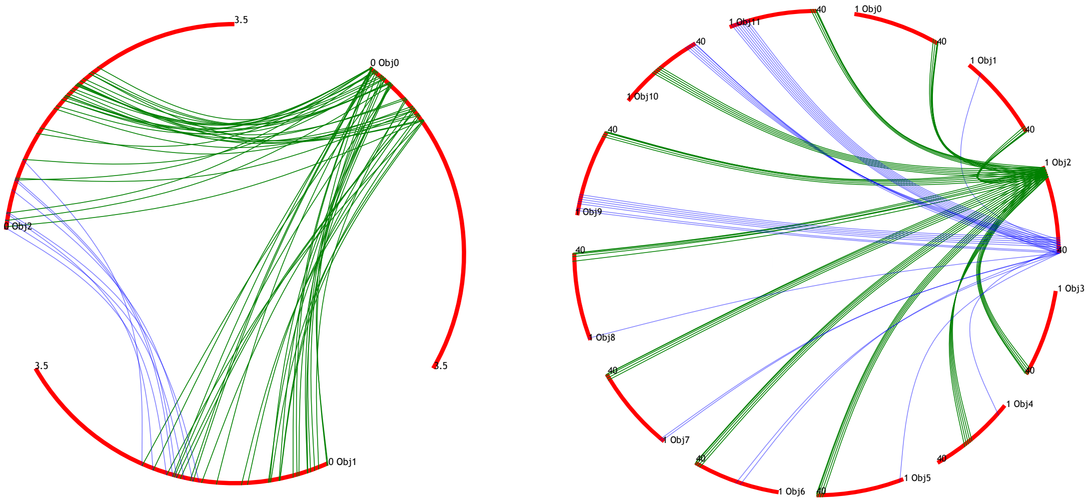


(a) Highlighted solutions with 3 objectives (b) Highlighted solutions with 12 objectives

A feature that can distinct a powerful visualization tool is related to its ability to accommodate a wide range of analyses. For example, the several forms of comparing the links and connection between objectives. The user should be able to extract different information from various viewpoints of the objective space and possible solutions. Here, the designer has tried to design and implement a useful tool to present the necessary information.

Another way to perform a useful comparison between different intervals of objectives is using multi-color links in separated frames in one view, as exemplified in Figure 6. Both examples show the behavior of different regions of objectives in terms of their relationships with the others. This feature enables the user to assess trade-offs of the problem in different regions of objectives at the same time.

Figure 6 – Comparing the objective trade-offs in different regions in different goals



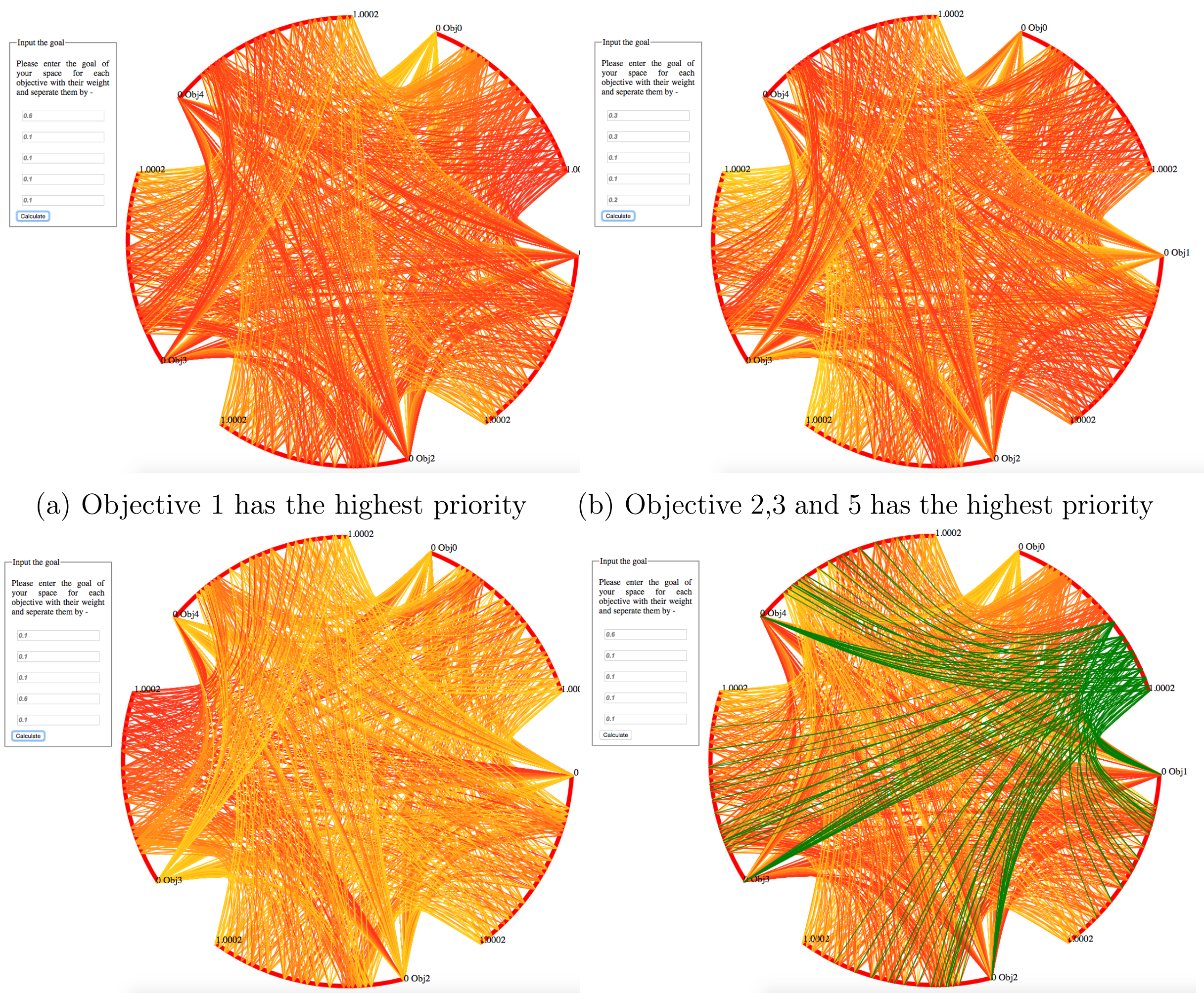
(a) Highlighted solutions with 3 objectives (b) Highlighted solutions with 12 objectives

In the results we are going to illustrate how the tool can be used for objective reduction in MaOPS.

4.2.0.3 Relationship based on criteria

In addition to the all of explained abilities, there can be different subsidiary usabilities. As it is clear, evaluating the relationship and objective reduction are subjective activities. It means, there is a deep dependency between them and the problem conditions and the preferences of the decision maker. Thus, visualizing the relationships according to the criteria can be a efficient ability for the decision makers.

Figure 7 – Applying MCDM method on the obtained Pareto-Front of DTLZ2 through MOEA/D



(a) Objective 1 has the highest priority (b) Objective 2,3 and 5 has the highest priority

(c) Objective 4 has the highest priority (d) Relationship on the goal region

Figure 7 demonstrates a sample of the applying the MCDM on the VMA tool. In this example, the Pareto-Front of 5-objective DTLZ2 has been obtained by MOEA/D algorithm. Then, the decision maker set the weight of 5 criteria for the embedded VIKOR algorithm in different situation and each criteria allocated to one objective. It should be noticed that in the algorithm we consider higher value as a better condition. For instance, in figure 7a the weight of first criteria is set as the highest value. Therefore, it can be seen that the solutions with the higher value of objective 1 have been highlighted, and the relationship of that area with the others are illustrated with the same color. This figure shows the conflict between them. On the other side, the priority is not set on just one objective. The priority on figure 7b has been divided between objective 1,2 and 5. Hence, the highlighted area is not concentrated on just one region. In opposite of figure7a&7c the highlighted solutions have been spread among the length of those three objectives.

Combination of this ability and frame drawing of the tool, enables the decision makers to check out the relationship with more detailed information based on their preferences. Figure7d illustrates the selected solution relationship by the decision maker based on the goals which have been shown in figure7a. In this figure the green lines show the clear relationship and conflict on the desired area of the decision maker.

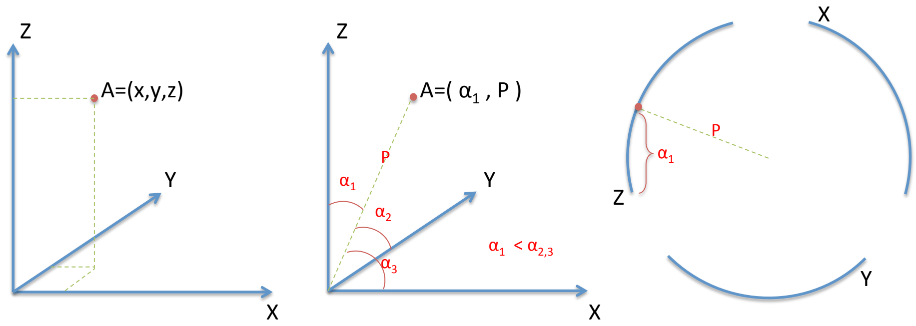
4.3 Visualizing the solutions distribution.

This section introduces a complementary extension of the proposed Visualization Tool (VMA). The extension plays the role of the second part of the Diagram illustrated in Figure 1. In addition, it opens the possibility of using the proposed framework for reducing the number of objectives. By allowing better visualization of the distribution of individuals in high-dimensional spaces, the proposal helps the end user to have an overview and detailed information about the status of the individuals in different conditions. Hence, the decision makers can extract diverse information through the qualitative analysis of the problem. In particular, the status of each step of an optimization process can be viewed, analyzed and interpreted, in the search for guiding the next step of the method.

Basically, the first ability of the extension is mapping the distribution of individuals in high-dimensional spaces to a 2D form. Each point in the N-dimensional space contains N properties to be illustrated and transformed. Meanwhile, in order to represent these points, on displays or papers, it seems smart to consider only two or three properties. Hence, the first step of visualization is mapping N-*Property* information into 3-*Property* information. Figure 8 demonstrates this concept on the 3D space.

As it can be noticed, Point A is in the 3D space and can be declared into two forms. The well-known Cartesian coordinates, exemplified in the first graph, contains 3 properties which are assigned to each axis. On the other hand, this point can be defined by three properties (ρ, α, i). In this set, ρ is the norm of point A or, in other words, the length of the defined vector from the center to the point A. The vector makes three angles with the three axes, among them, the smallest one is the second property for point A (namely α), and i is the axis in which the smallest angle occurs (closest axis in angular

Figure 8 – Mapping a point from 3-D Cartesian space to the 2-D space. Point A with 3 properties in Cartesian coordinator has been converted to the vector with 3 angles $\alpha_{1,2,3}$ which are the angle of the vector with Z,Y,X axis in order to be ready for illustrating on 2D displays.



distance). Following this strategy, each point from high dimensional space can be redefined by just these three properties. Equation 4.1 illustrates the method of this mapping.

$$\begin{aligned} A &= (x_1, x_2, ..., x_n) \\ \rho &= \|A\|_2 \\ \alpha_i &\leftarrow \arccos\left(\frac{x_i}{\rho}\right) \\ \theta &\leftarrow \min\left(\alpha_i\right) \end{aligned} \tag{4.1}$$

In fact, a point A, with n properties in Cartesian space, is the input of this mapping. Property x_i is the value of the ith coordinate of the point, which is allocated to the ith objective. Variable α_i represents the angle of axis i and the vector of A, which is obtained through the Euclidean norm and each property of the point. Following this transformation, there is a vector with specified length (ρ) and n angles. Thus, point A can be redefined by the smallest angle and the length of the vector. As it can be seen in Figure 8, this redefinition is proper for 2D displays. Therefore, each individual has the suitable property set to locate on VMA method. As it was mentioned before, each arc is allocated to one objective or axis. Hence, there are three arcs which are assigned to the axes X, Y, Z, which are the hosts of obtained solutions set.

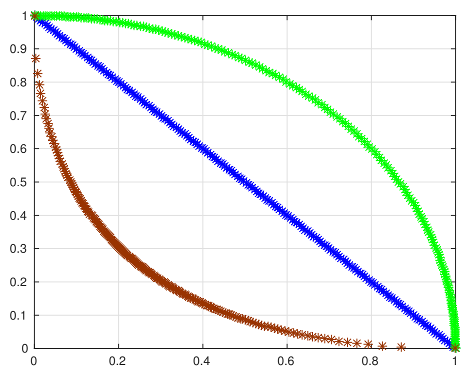
Based on this strategy method, each solution (or individual) is defined with 3 properties (θ, ρ, i), where i is the closest axis. Therefore, the radius of the arcs is the maximum ρ in the available set. Moreover, the length of each arc is the angle of hyper-diagonal of N-dimensional space which is calculated by Equation 4.2.

$$\arccos\left(\frac{1}{\sqrt{n}}\right) \tag{4.2}$$

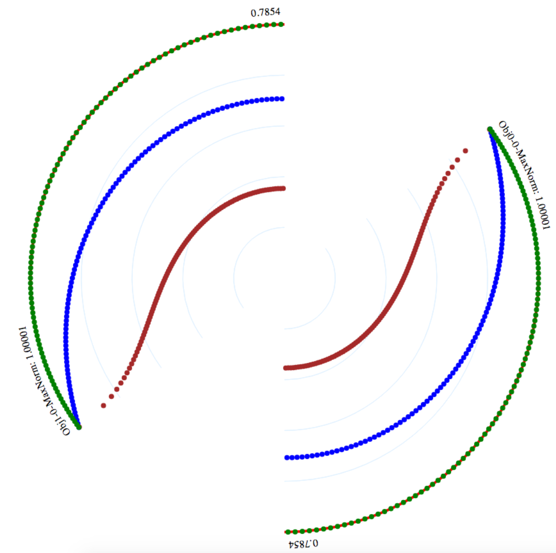
Now, the replacing algorithm uses the 3rd property of each solution (closest axis) to specify the target arc. Then, by the value of θ, find the place of it on the arc and finally, by the

value of ρ, drag the point on the radius of the arc. By iterating of this algorithm, all the solutions can be located on their target arcs (objectives).
Figure 9 illustrates the application of the concept of the VMA approach in three sets of points in two and three dimensional spaces, all of them with constant p-norm. The brown points have constant 0.5-norm, the blue points have constant 1-norm and the green points have constant 2-norm. Thus, the blue points in the 2 dimensional space are located in a straight line (or a plane, in the 3 dimensional space) and the green points in 2 dimensional space are located in a circle in the first quadrant (or a sphere in the first octant for the 3 dimensional space).

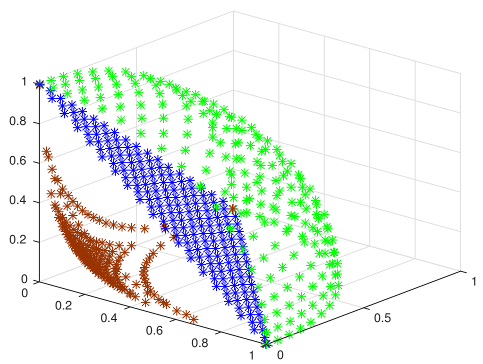
Figure 9 – Cartesian coordinate and VMA representation



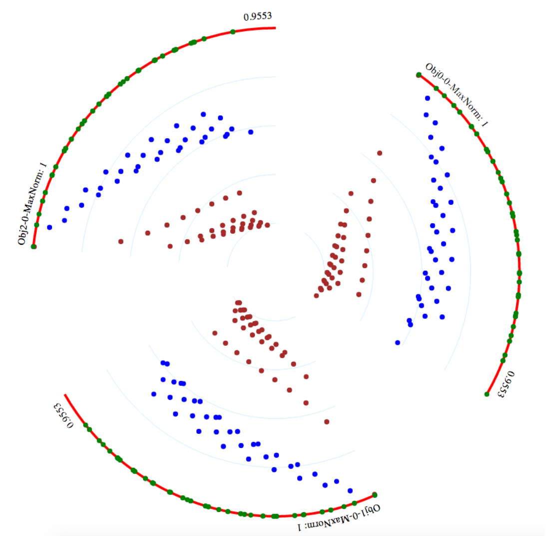
(a) Points with constant p-norm in the 2 dimensional space using rectangular coordinates.



(b) The corresponding distribution of points in 9a using VMA.



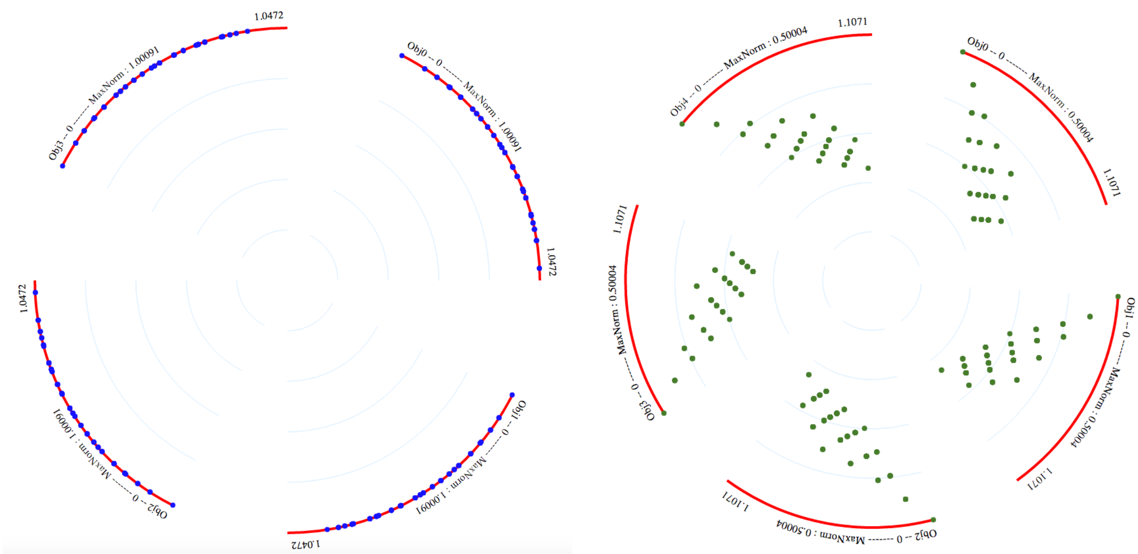
(c) Points with constant p-norm in the 3 dimensional space using rectangular coordinates.



(d) The corresponding distribution of points in 9c using VMA.

Considering the observer at the origin of the rectangular coordinate system (point $O = (0, \ldots, 0)$), figure 9 presents a convex (brown), a flat (blue) and a concave (green) Pareto front, in 2 and 3 dimensional spaces. The application of the VMA approach preserves the order in which these points appear in the rectangular coordinate system. The green points, which represent the concave data set in VMA, are completely located on the arc, and it means that every point on the space has the same distance from the center of the coordinate system. Meanwhile the brown points represent the shape of convex front in Cartesian coordinate. According to the discussed concept, the beginning of the arc represents the location of allocated axis and the end of it indicates the location of the hyper-diagonal in the high-dimensional spaces (point $D = (1, \ldots, 1)$). In the extremes, corresponding to small angles, the points in the three sets are closer in both representations. As they get closer to the hyper diagonal and the center, the angular distance from the axis is increasing until reaching the maximum value $\theta_{\max}$. Based on this, for greater angles on the arc the relative distances between the sets of points increase. Figures 9c and 9d illustrate the same concept for 3D space. Figure 10 shows the visualization of results of estimated Pareto fronts in VMA. In figure 10a, we show the approximated Pareto Front for DTLZ2 problem with 4 objectives, obtained with NSGA-III, and in figure 10a, we show the approximated Pareto front for DTLZ1 problem with 5 objectives, obtained with MOEA/D algorithm. Those problems have a well defined Pareto Front: for the DTLZ1 problem it is a flat surface and for the DTLZ2 problem it is a hyper sphere. Comparing figure 10 with figure 9, it is easy to recognize the shapes of the Pareto fronts in each problem.

Figure 10 – Distribution of obtained Pareto Front.



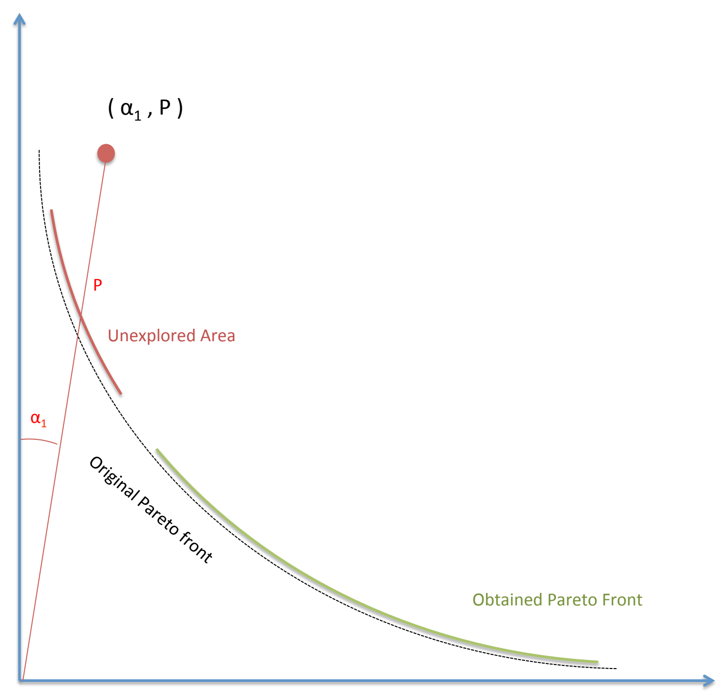
(a) NSGAIII on DTLZ2 with 4 objectives. (b) MOEAD on DTLZ1 with 5 objectives.

4.3.0.1 Exploiting unexplored areas

After presenting the overview of distribution of the individuals, the next important ability is figuring out unexplored area in the problem space. Trapping in the local optimums is a significant challenge that has been affecting the solving process of different algorithms. Hence, the authors believe that detecting the covered area of space can be an effective and efficient ability that enables the users to figure out the performance and efficiency of their methods. Considering the design principle of the arc, each arc can cover the region between the allocated axis and the hyper-diagonal. Therefore, the demonstrated distribution of the individuals can give the image of the covered area in the objective spaces. This image illustrates the distribution of the obtained individuals by an specific run of an algorithm.

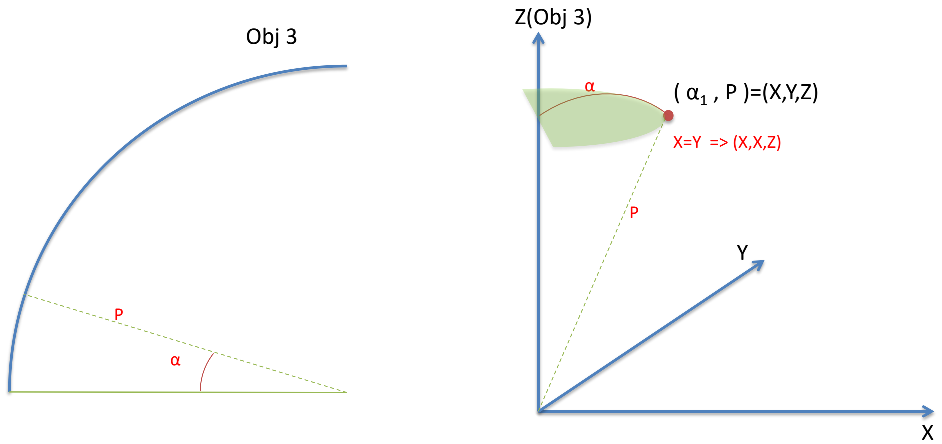
Taking advantage of these tools, the end user may be able to figure out the unexplored area in the problem space. After that, an additional step can guide decision maker choice. The optimization process can use the vector tool in VMA to declare the vector or reference point for the next iteration, guided by specific preference. In this sense, the algorithm tries to start its search with individuals around the defined vector. In this step, the decision maker can define the radius around the vector in order to control the amount of coverage on that area, as illustrated in Figure 11. As it is shown, the green part is the area that had been covered before, meanwhile the dashed vector refers to unexplored areas. Based on this vector and the radius around it, the algorithm can guide its search. Hence, by iterating this process, the decision maker ensures that the algorithm may cover different areas all over the feasible image region, in the objective space.

Figure 11 – Declare the vector in direction of unexplored area.



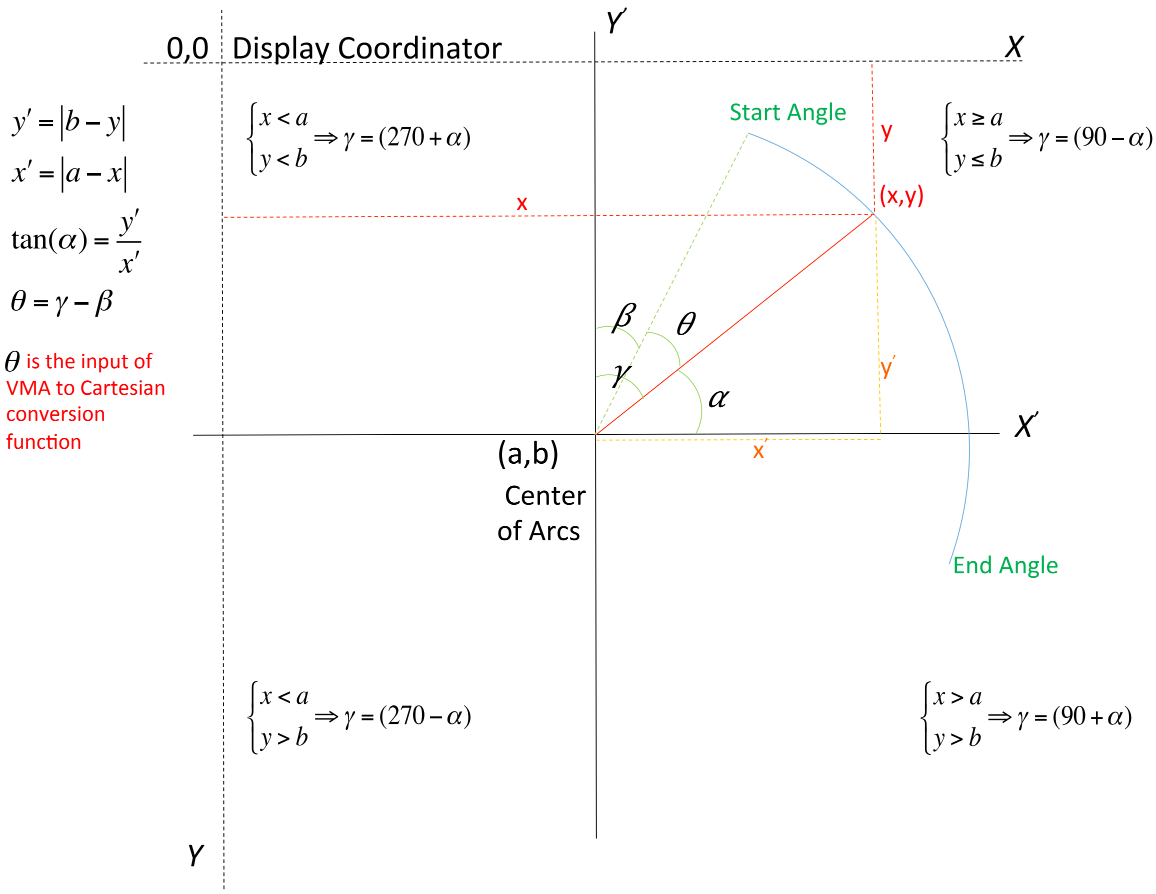
The definition of this vector brings a new geometric challenge. In n dimensional space, any vector can be identified by n angles of this vector with the axes. These angles are often called direction or directional angles. As only the smallest of them is used in VMA, some information is lost of course. Hence, in this step, the tool has to do a reverse process to convert a point in VMA coordinator into a region in the rectangular coordinate system. Figure 12 exemplifies the conversion concept. As it can be seen, the goal of this mapping is giving a point in the Cartesian coordinator.

Figure 12 – Mapping a point in VMA into a region in the rectangular coordinate.



In the following, the trigonometric equations illustrate the concept of this translation from VMA system to Cartesian system. In this part, it is necessary to calculate the angle θ when user clicks on the arc at a specific point. Figure 13 shown this situation. In this figure there are two coordinate systems. The system XY, whose origin $(0, 0)$ is located in the upper left corner of the display, and the system $X'Y'$, whose origin $(0', 0')$ is located at the center of the VMA representation, with coordinate point (a, b) of the XY coordinate system. Let θ be the angle, in the $X'Y'$ system, obtained by selecting a point on an arc of the VMA system, with (x, y) coordinates in the system XY. In order to find θ, the differences between the coordinates x and y of the point and a and b of the center of the VMA system, which is represented by $x^{'}$ and $y^{'}$, need to be calculated, that is, $x' = x - a$ and $y' = y - b$. Then, in the $X'Y'$ coordination system, it is possible to calculate the angle, say α, with the tangent formula: $\tan\alpha = \frac{y^{'}}{x^{'}}$. With this method, any angle between the line segment defined by the points (a, b) and (x, y) and the axis X' or Y' of the new coordination system can be found. Thus, θ is calculated by summation or subtraction of these angles.

Figure 13 – Finding the angle of vector (θ).



However, as illustrated in Figure 13, there is no difference between $\tan(45°)$ and $\tan(180° + 45°)$. Actually, the new coordinate system divides the screen into four different regions. This regions are started from Y' axis and it is clockwise and the standard region two is defined as region four here. For each region, there is a conditional statement which makes calculation of the possible angles. For example, in region one, which is defined by $x \geq a$ and $y \leq b$, γ is defined by $\gamma = (90° - \alpha)$, and in region four, which is defined by $x < a$ and $y < b$, γ is defined by $\gamma = (270° + \alpha)$. Therefore, with this method, it is possible to calculate θ and proceed to the next step.

It is important to calculate the preference vector $x = (x_1, \ldots, x_n)$ in the n dimensional objective space. Suppose that the directional angle θ determined previously is associated with the x_i coordinate of the preference vector. Due to loss of information in the VMA system, all other angles are set to the same value, say φ. We know that if $\kappa_1, \ldots \kappa_n$ are the directional angles of the vector $x = (x_1, \ldots, x_n)$, then $\cos^2 \kappa_1 + \ldots + \cos^2 \kappa_n = 1$.

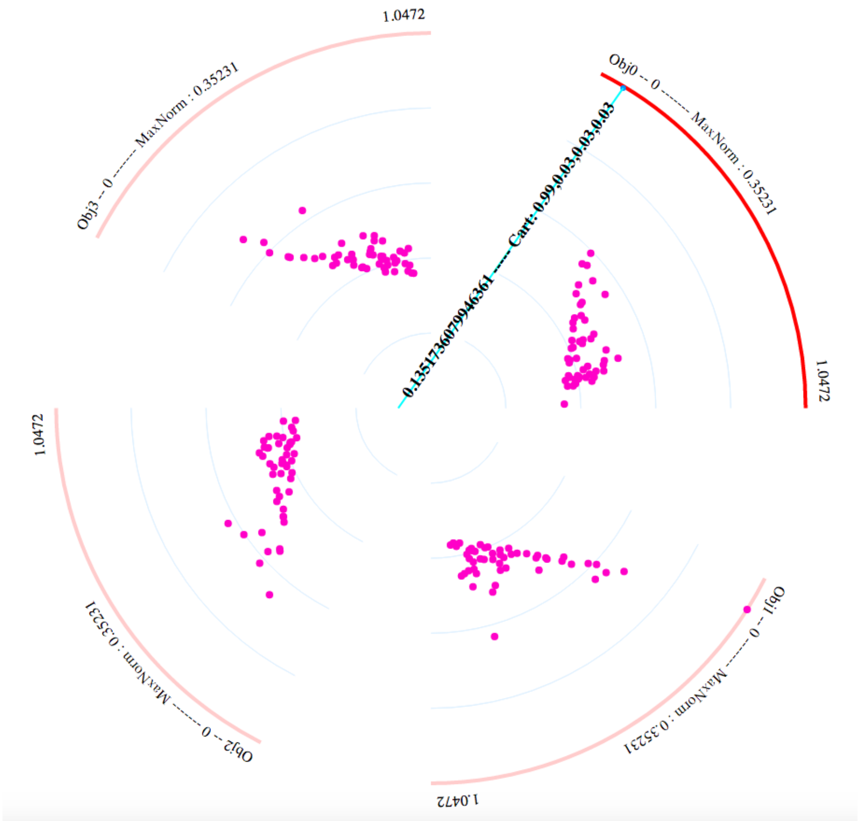
In this case we have:

$$\begin{aligned}
\cos^2\theta + \cos^2\varphi \ldots + \cos^2\varphi &= 1 \\
\cos^2\theta + (n-1)\cos^2\varphi &= 1 \\
\cos\varphi &= \sqrt{\frac{1-\cos^2\theta}{n-1}} \\
\varphi &= \arccos\left(\frac{\sin\theta}{\sqrt{n-1}}\right)
\end{aligned}$$

And the vector $p = (\cos\varphi,\ \ldots\ ,\cos\theta,\ \ldots\ ,\ \cos\varphi)$ is the preference vector obtained, with $||p|| = 1$.

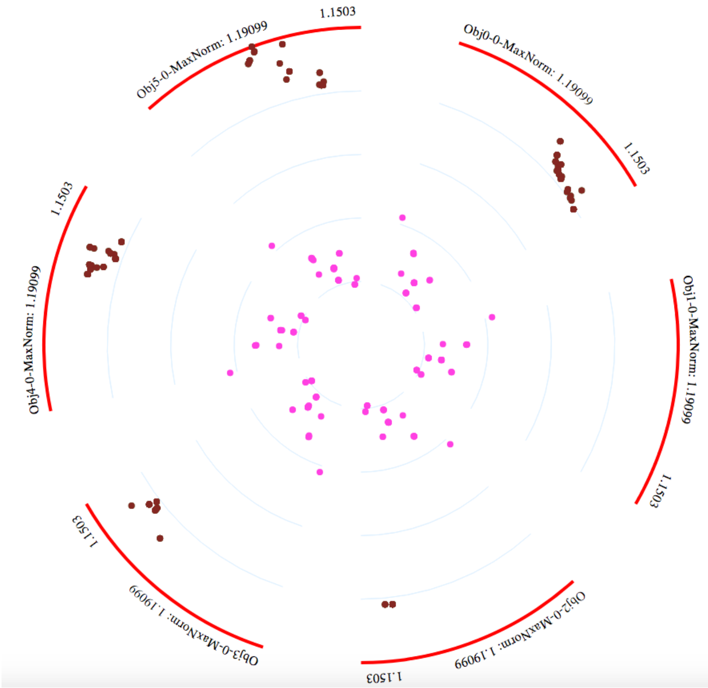
Figure 14 provides an example of these concepts in the visualization tool. As it can be seen, the MOEA/D algorithm tried to find the Pareto Front but, for this specific case, the decision maker has found a tiny area which does not contain any individual. Hence, the decision-maker clicked on the point on objective 0 in order to draw a vector. The proposed application provides the vector, according to the mentioned theory and equations, in order to present a point in 4-dimensional Cartesian system, in the objective space. Then, the optimization method can use this vector to search on that desired area, which will be illustrated and discussed at Section 5.

Figure 14 – The blue vector contains the Cartesian coordinator information that the vector addresses the unexplored area.



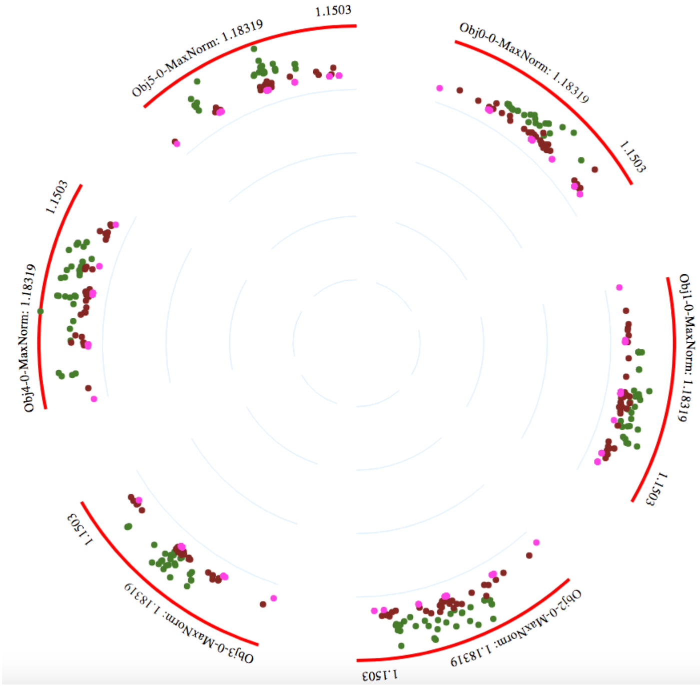
After assessing the exploration ability based on the desired search region, the final significant facility is the qualitative evaluation of the performance of different algorithms in the same problem space or the visualization of different sets of solutions in the high-dimensional objective space. To illustrate this part, consider problems DTLZ1 and DTLZ2 with 6 objectives. Fig. 15 shows the approximation of the Pareto front in DTLZ1 by using MOEA/D in two different moments of the optimization process: the estimate of the Pareto front after 50 generations (brown) and after 200 generations (pink). The Pareto front in DTLZ1 is a plane in the objective space, and this shape can be correctly identified in the tool. The color is the attribute that plays the role of categorizing the dataset into the three classes. In this case, VMA finds the maximum norm among all the individuals, setting the radius of arcs to the maximum norm. After that, it locates each point based on its information that has been mentioned before.

Figure 15 – Visualizing the evolution of the approximation of the Pareto front in DTLZ1 with 6 objectives.



In Figure 16, it is possible to visualize the evolution of the population along the generations for comparing different moments of the optimization in the same benchmark problem, in this case the DTLZ2 problem. The green data set is the approximation of the Pareto front after 10 generations of MOEA/D, brown data set is the approximation after 50 generations and the pink data set is the approximation of the Pareto front after 300 generations. One can clearly see the evolution of the population in the VMA method. The Pareto front in DTLZ2 is a spherical surface, this si why those points are distributed along the arcs. After 300 generations, one an clearly see that the solutions got aligned with the weight vectors. Moreover, the tool provides additional information such as an approximate shape of the Pareto Front; the amount of coverage in the objective space; the density and redundancy of individuals in the specific area(s); performance of algorithms; among others. This diverse category of information is an important assistance that the VMA is able to provide.

Figure 16 – Visualizing the evolution of the approximation of the Pareto front in DTLZ2 with 6 objectives.



In Figure 16, it is possible to visualize the evolution of the population along the generations for comparing different moments of the optimization in the same benchmark problem, in this case the DTLZ2 problem. The green data set is the approximation of the Pareto front after 10 generations of MOEA/D, brown data set is the approximation after 50 generations and the pink data set is the approximation of the Pareto front after 300 generations. One can clearly see the evolution of the population in the VMA method. The Pareto front in DTLZ2 is a spherical surface, this si why those points are distributed along the arcs. After 300 generations, one an clearly see that the solutions got aligned with the weight vectors. Moreover, the tool provides additional information such as an approximate shape of the Pareto Front; the amount of coverage in the objective space; the density and redundancy of individuals in the specific area(s); performance of algorithms; among others. This diverse category of information is an important assistance that the VMA is able to provide.

As a brief conclusion, the VMA consists into two general phases. The first one was discussed at the beginning of Section **??**, playing the role of figuring out the objective relationships. It enables the decision makers to extract enough reasons in order to ignore objectives or merge them together. They can repeat this process up to the point that there will not be any harmony between objectives. It is noteworthy that these iterations are completely subjective based on the requirements of the problems and goals of decision makers. The duty of the 2nd phase is visualizing the individuals distribution, mapping the high-dimensional problem space into the 2-D space. Furthermore, it presents several ideas and information which have been assessed in order to reach to the desired solutions. Qualitative analysis can be more useful than the quantitative analysis for the wide range of users, in particular, for those without specific mathematical knowledge expertise.

Chapter 5

Applications and experimental results

As a summary, this article follows two issues about solving strategy and qualitative performance analysis of MaOEAs. Solving strategy contains sequential and iterative process to reach the proper result. Although the quantitative indicators are widely used in the analysis of solutions of multiobjective optimization problems, some characteristics of the obtained solutions are more evident when analyzed through some graphic resource. Likewise, the decision maker can present his/her preferences using numerical and graphical parameters. We believe that the joint use of numerical and visual parameters is of utmost importance to guide the optimization process and analyze the results. In this sense, this section presents some possibilities for applying the proposed visualization method in order to enrich the analysis of the results, as well as improving the search for novel solutions. The following applications will be presented for the proposed visualization method:

1. Reduction of the number of objectives aiming at improving the quality of the solution found when approaching problems with many objectives;

2. Identification of regions with solutions of poor quality and/or quantity in the objective space. In this step, the decision maker is able to figure out the unexplored area and provide a reference point based on it.

3. Comparison of the performance of different algorithms in the same problem.

5.0.1 Objective reduction experiments

In general way, a multiobjective problem presents conflicting objectives, i.e. the improvement of any objective is accompanied by a deterioration in the quality of the other ones. Benchmark problems, like the DTLZ family, were designed for highlighting these characteristics. Furthermore, many objective algorithms need to face different challenges, such as escaping from several local maxima and minima and dealing with different characteristics such as non-convexity, discontinuity or symmetry of the Pareto Front.

Nevertheless, in real world applications, the absence of conflict between objectives can only be verified in the final solution obtained by any multi-objective algorithm, since, in general way, this multi-objective problem does not have an explicit analytical expression and the objective space presents a complex landscape. In practical problems it is highly possible that there are objectives in harmony or with low conflict. This is not the case in benchmark problems, since they are designed to generate scalable conflicting objectives. When it is possible, the reduction of the number of objectives may lead to an improvement in the quality of the final solution and the convergence of the algorithm used to solve it.

As introduced, the first step of presented approach is objective reduction, a subjective activity assisted by the proposed visualization method. In this sense, the decision maker should analyze the environment and based on the visual relation of objectives, decide on reducing the number of them. In order to decrease the number of objectives, there are different methods such as aggregation, Principal Component Analysis (PCA) or simply removing the objectives. On the other hand, based on specific problem characteristics, the amount of harmony and objectives can be effective on the decisions. For instance, the amount of harmony, the specific region of harmony, the goal of each objective, etc can affect on the selected set of objectives which should be removed or constrained or merged. According to this goal, we have designed a multi-objective problem with quadratic functions, as shown in Eq. 5.1, such that there is some intentional degree of harmony in groups of objectives.

$$
\begin{aligned}
\mathbf{x}^\star = \arg\min F(\mathbf{x}) &= (f_1(\mathbf{x}), \ldots, f_{10}(\mathbf{x})) \\
\text{where} & \\
f_i(\mathbf{x}) &= (\mathbf{x} - c_i)(\mathbf{x} - c_i)^t, \; 1 \leq i \leq 10 \\
\mathbf{x} &\in [0,1]^{20}
\end{aligned}
\tag{5.1}
$$

In the quadratic optimization problem (5.1), the c_i points (centres) of each f_i objective were chosen in order to form two clusters $C_a = \{c_1, c_2, c_3, c_4, c_5\}$ and $C_b = \{c_6, c_7, c_8, c_9, c_{10}\}$ of points in the objective space, making the problem strongly biased. The points c_i in each cluster are $c_i = c + \delta_i$, being: $c = (0.25, \ldots, 0.25)$ for the cluster C_a; $c = (0.75, \ldots, 0.75)$ for the cluster C_b; and $\delta_i = (\delta_{i,1}, \ldots \delta_{i,20})$ for both clusters. Variable $\delta_{i,j}$ was set as a random variable with normal distribution (mean $\mu = 0$ and variance $\sigma^2 = 0.08$). The spatial distribution of these clusters are shown using parallel coordinates (Figure 17) In all the experiments, the same set of centres was used. The main idea is to optimize the original problem and after the solving process, the obtained set of non-dominated solutions is analyzed on the VMA harmony tool. An expert person without knowledge about the positions of the centres tried to check the harmony and provided a set of objectives as candidates for aggregation. Based on the provided set, the objectives are merged by the average value of the objectives in harmony. This is how the process would be done with assistance of the VMA tool. Along with this qualitative analysis, for the purpose

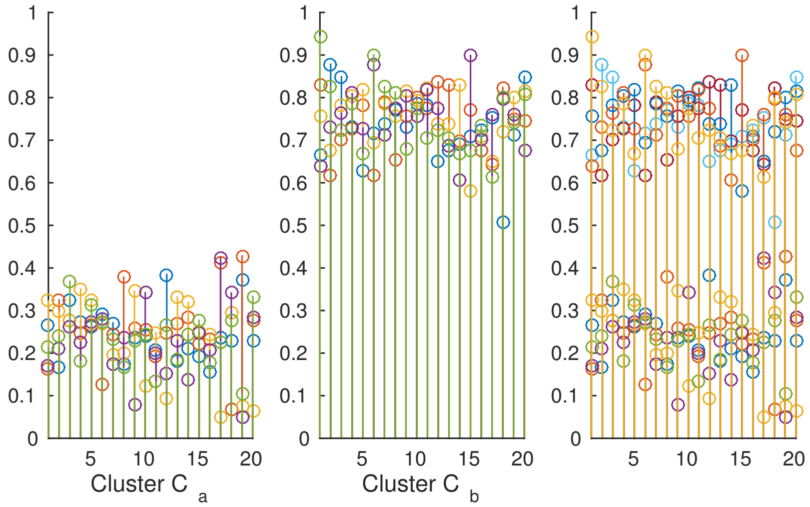


Figure 17 – Parallel coordinates of the centres of the quadratic function

of comparison and validation, we have tried to use a quantitative method with the same concept of harmony detection and objective reduction in order to confirm the validity of the qualitative method.

In this scenario, a mathematical procedure for dimensionality reduction of the problem is presented, inspired by the procedure of identification of harmony proposed by the VMA approach, with the goal of comparing the objective analysis with the subjective one. By this quantitative analysis, two or more objectives are merged according to their harmony, which is identified when two or more objectives have a high concentration of low values. This situation is illustrated in the Figure 18, that shows a hypothetical objective set in parallel coordinates, where the objectives 3, 4, 5 and 10 have a high concentration of their population in low values.

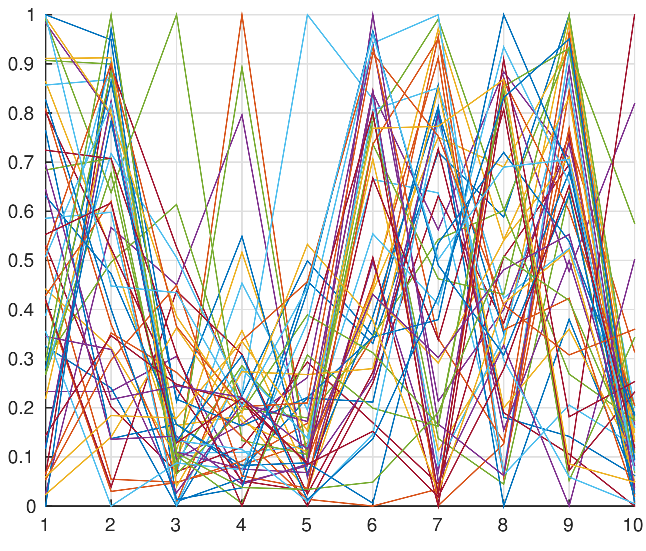


Figure 18 – Objectives in harmony

In order to compute this concentration, the numerical procedure for the determination of harmony is presented below:

1. Normalize each objective in the Pareto Front in $[0, 1]$ interval;

2. Set the parameter η that defines the harmony between the objectives. The η parameter defines the threshold where the minimum value of each objective is located. For instance, setting $\eta = 0.1$ we would look for solutions in $[0, 0.1] \subset [0, 1]$;

3. For each objective f_i, if at least $\phi\%$ of the population have their value of the objective i smaller than η, i.e. $f_i(x) \leq \eta$, then f_i would be a candidate to be in harmony with other objectives. For each individual in the subpopulation $P_i = \{x_{i1}, \ldots, x_{is}\}$ with $f_i(x_{ik}) \leq \eta$, evaluate:

   a) $\mu_1 = \frac{1}{s}\sum_{\ell=1}^{s} f_i(x_{i\ell})$, e.g. the mean of $f_i(x_{ik}),\ x_{ik} \in P_i$

   b) For each objective $f_j,\ j \neq i$, evaluate $\mu_2 = \frac{1}{s}\sum_{\ell=1}^{s} f_j(x_{s\ell})$, e.g. the mean of $f_j(x_{ik}),\ x_{ik} \in P_i$.

   c) Evaluate $\omega_{ij} = 1 - |\mu_1 - \mu_2|$. If $\omega_{ij} \geq \delta$ the objectives i and j are in harmony.

For each objective f_i, the procedure presented between items 3a to 3c returns a list $\lambda_i = \{j_1, \ldots, j_{ki}\}$ of kj objectives in harmony with objective f_i: objective f_{j_1} is in harmony with objective f_i, objective f_{j_2} is in harmony with objective f_i, ..., objective $f_{j_{ki}}$ is in harmony with objective f_i. At the end, we have a set of lists $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$. The parameters used in the experiments are listed in the Table 5.0.1.

Parameter	Value
η	0.5
ϕ	0.7
δ	0.7

Table 1 – Parameters for the numerical analysis of harmony

Using this procedure of identification of harmony between objectives, some tests were performed in the quadratic function presented in equation 5.1. In the initial step a MOEA is used to obtain an estimate of the Pareto front for the problem with 10 objectives. After obtaining an estimate of the Pareto front, the search for harmony between objectives is carried out. The objectives in harmony presented in each list Λ are grouped into a new goal, which is the simple average of these objectives. The MOEA is used again in the reduced problem, generating a new estimate. This procedure is repeated until at least two goals remain and the quality of the solution set found, measured with the hypervolume indicator, is calculated and saved in each step of the way and presented in Table 2.

Test	Objective selected to merge	number of objectives	Hypervolume
1	–	10	4.956×10^7
	[4, 5, 6, 8, 9, 10]	5	$\mathbf{5.935}\times10^7$
	[1, 3, 4, 5]	2	2.791×10^7
2	–	10	4.922×10^7
	[1, 2, 3, 4, 5, 10]	5	$\mathbf{5.324}\times10^7$
3	–	10	$4{,}988\times10^7$
	[2, 3, 4, 5, 6, 8, 9, 10]	3	$\mathbf{8.342}\times10^7$
4	–	10	4.841×10^7
	[3, 4, 5, 6, 9, 10]	5	$\mathbf{6.789}\times10^7$
	[2, 3, 5]	3	$\mathbf{6.287}\times10^7$
5	–	10	4.976×10^7
	[3, 4, 5, 6, 9, 10]	5	$\mathbf{6.775}\times10^7$
	[2, 3, 4, 5]	2	3.707×10^7

Table 2 – Summary test on objective reduction

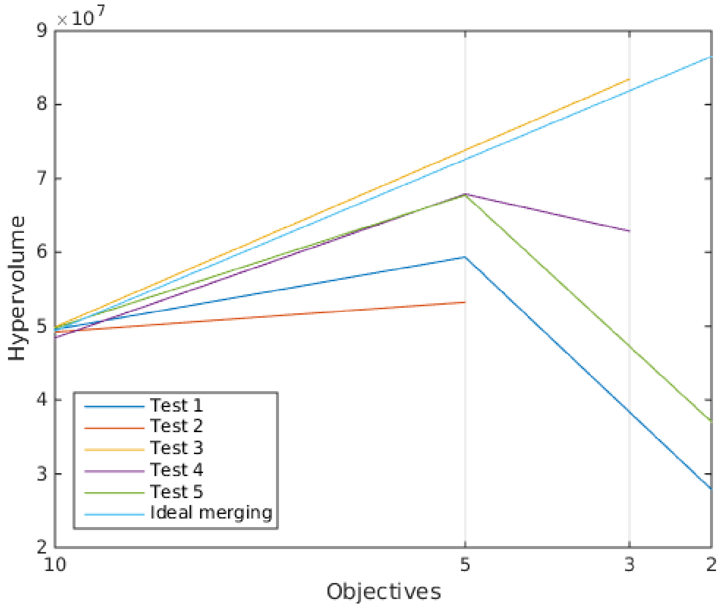


Figure 19 – Hypervolume variation

For the calculation of the hypervolume indicator the reference point was the point whose coordinates corresponded to the highest value of the objectives found in all the tests, namely the point $p = (6.5, 6.5, \ldots, 6.5)$. Therefore, higher values of hypervolume mean better quality of the approximation of the Pareto front.

To understand the results in Table 2, consider test 1 for instance. In the first step, optimizing the problem with 10 objectives produced an estimate whose hypervolume is 4.956×10^7. The automatic procedure identified groups of objectives in harmony, reducing the problem to one with 5 objectives. A new estimate of the Pareto front is obtained by optimizing the reduced problem. After evaluating this set in the original 10 objectives and computing the hypervolume, we get 5.935×10^7. This shows that solving the reduced problem with 5 objectives leads to a better estimate in terms of hypervolume than solving the original problem with 10 objectives. This results is not surprising, since the MOEA usually have a hard time solving MaOPs. In the last step, the automatic procedure reduced the number of objectives to two. Optimizing the bi-objective problem leads to an estimate with hypervolume (calculated in the original 10 objective space) of 2.791×10^7. Probably the reduction performed in the last step was wrong, degrading the hypervolume, because it was guided by the heuristic procedure described before, without any aid of visualization.

Next, in order to contrast with previous procedure, still for the Quadratic Function, the application of the VMA tool is displayed in the Figures 20 and 21. In these figures, the harmony between the objectives 1 to 5 (figure 20) and 6 to 10 (figure 20) are clearly observed. Merging these objectives into a new one, by the average values of each original values, we reduce the dimensionality of the problem from 10 objectives to a new multiobjective problem with only two objectives, each objective having a centre at an average of each cluster. The first objective is the average value of the objectives evaluated in the cluster C_a and the second objective evaluated by the average value of the objectives in the cluster C_b. After the optimization process in this reduced problem, the estimate of the Pareto obtained is evaluated on the original 10 objective problem in order to calculate the corresponding hypervolume. For the non dominated solution obtained, the value of the hypervolume is 8.653×10^7. The Figure 19 shows the values of the hypervolume of the qualitative and quantitative procedure for the objective reduction.

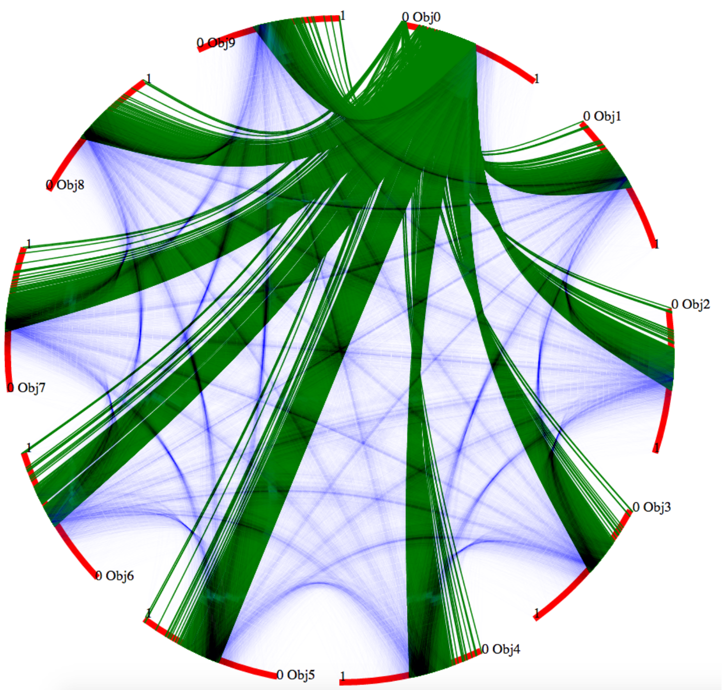


Figure 20 – Harmony between objectives 1 to 5 on the biased Quadratic Function

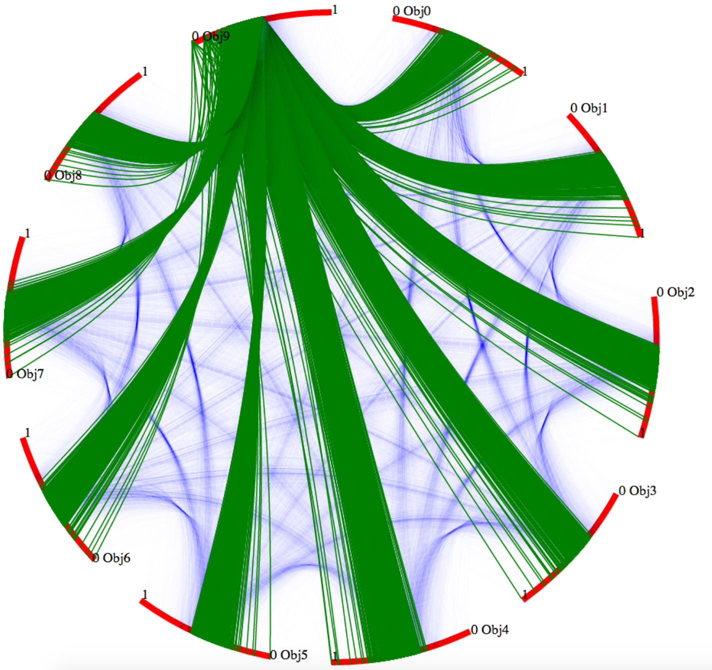


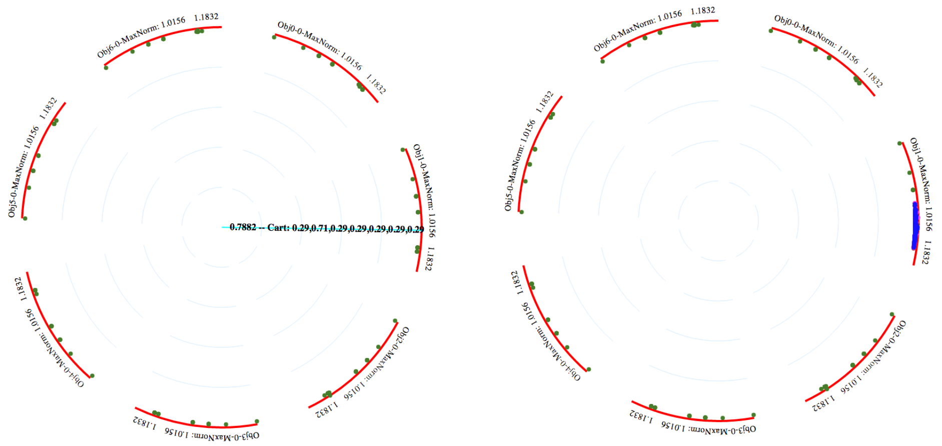
Figure 21 – Harmony between objectives 6 to 10 on the biased Quadratic Function

5.0.2 Covering the specific regions

Covering the space is one of the important challenges in Many Objective problem solving algorithm from two viewpoints. The first reason, is scalability, means by growing the size of problem, the needed population is increasing exponentially to cover allover the Pareto-Front. The second reason is searching the specific and desired direction and area. Therefore, through the iterative process, decision maker can push the algorithm to search the desired and unexplored directions to cover the Pareto front or cover the specific direction.

With the VMA procedure, it is possible to identify empty regions in the objective space. The Figure 22a shows the solution of the DTLZ2 problem obtained by the MOEA/D algorithm. In this figure, it is possible to identify empty regions and a vector defined by the Decision Maker indicating the region where he wants to perform a localized search. Using a cone of weight vectors, see Meneghini and Guimarães [2017], in the MOEA/D, with the same parameters of the original problem, it is possible to obtain new solutions in the region indicated by the Decision Maker, as shown in the Figure 22b.

Figure 22 – Produce the new population based on the decision maker preference



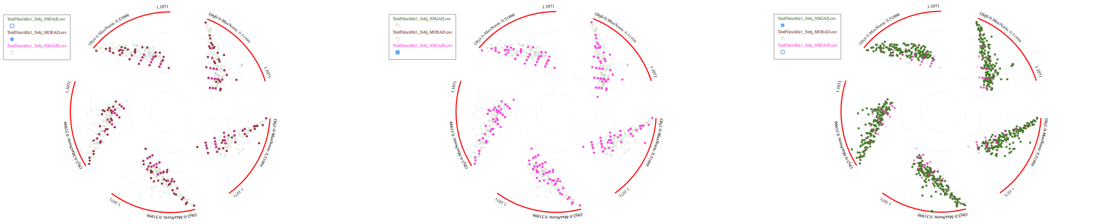
(a) Specify an unexplored region (b) Produced individuals (blue dots) around the vector

5.0.3 Analyzing the utilization of algorithms

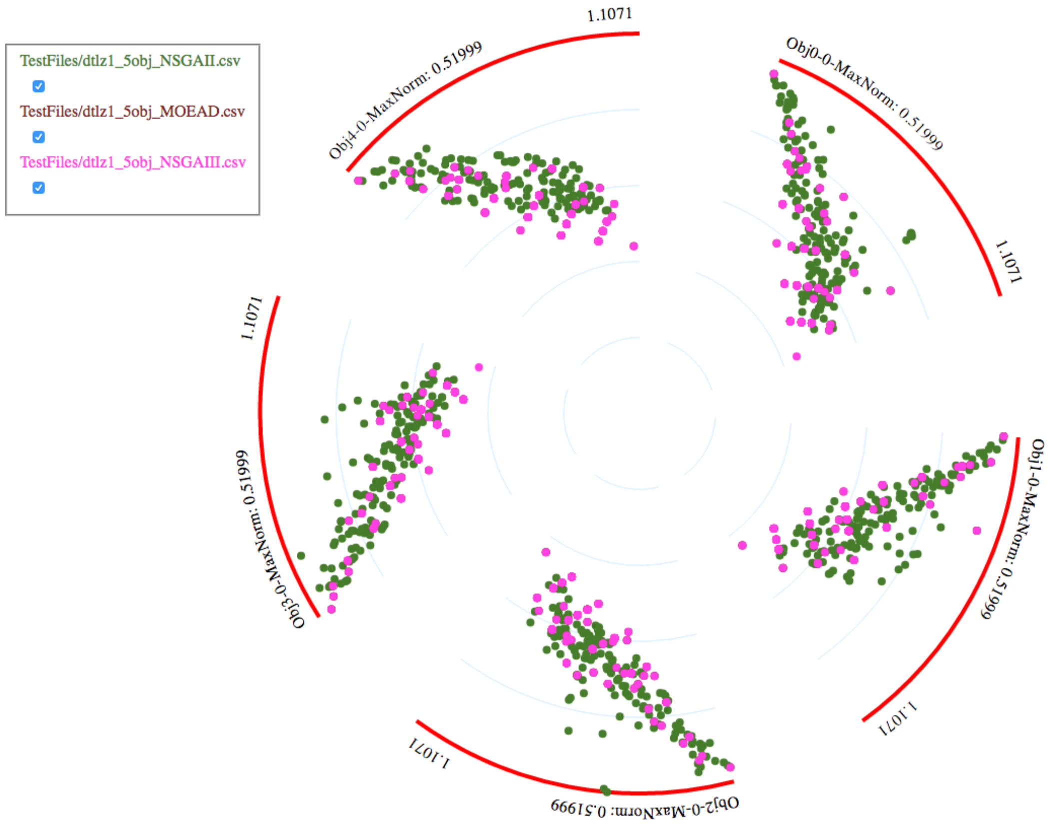
Another application of the tool is assessing the performance, efficiency and behavior of algorithms on different problems and compare them with each other. Hence, in this test, we have run MOEAD, NSGA III and NSGA II on DTLZ problems to evaluate their behavior and efficiency. All tests were performed on PLatEMO Tian et al. [2017], with 700 individuals in the population and 1000 generations.

For the first test, we ran the three algorithms on DTLZ1 problem which is demonstrated in Figure 23. Since we cannot have the live edition on the paper, the authors decided to keep turned on just one data set in order to keep highlight. In this case, the other data sets become transparent with the low opacity. Therefore, the user can see the selected data set clearly, whereas the other data sets have a shadow of colors to give the overview to the user. Hence, Figures 23a,23b,23c are giving the information about MOEA/D, NSGAIII and NSGAII, respectively. Figure 23d illustrates all datasets together. Moreover, the brown points are not visible in Figure 23d because they are covered by the pink points which are assigned to NSGAIII population. In spite of this, when we turned off NSGAIII and NSGAII, the MOEA/D population has been highlighted and appeared in Figure 23a. Therefore, you can compare it with Figure 23b to retrieve the differences. With more details, one of the important differences between MOEA/D and NSGAIII is the concentration and density of population. With more attention, it can be figured out the brown points have better spread whereas the pink points have more concentration in the center of their cluster. This kind of information is useful for different decision making situation and gives the good overview to the decision makers.

Figure 23 – Analyzing NSGAIII, NSGA II and MOEA/D on DTLZ 1 problem



(a) MOEA/D is turned on (b) NSGAIII is turned on (c) NSGAII is turned on



(d)

As it can be seen in Figure 23d, NSGAIII and MOEA/D have a similar distribution and convergence of the population and it is possible to see the alignment of the final solution with the weight vectors (pink points in Figure 24). In addition, these two algorithms could have better coverage than the NSGAII because they have points in different angles of objectives and it shows that they could cover them completely.

Another important characteristics that we can see is that the MOEA/D and NSGA-III have better convergence than the NSGA-II. It is clear when we analyze regions with same angle: In those regions, the solutions of MOEA/D and NSGA-III have a lower norm than the solution of the NSGA-II.

These characteristics are expected because NSGA-II is not designed for many objective problems. The align of the solutions with the weight vectors and solutions with same angle and different norms are shown in detail in Figure 24.

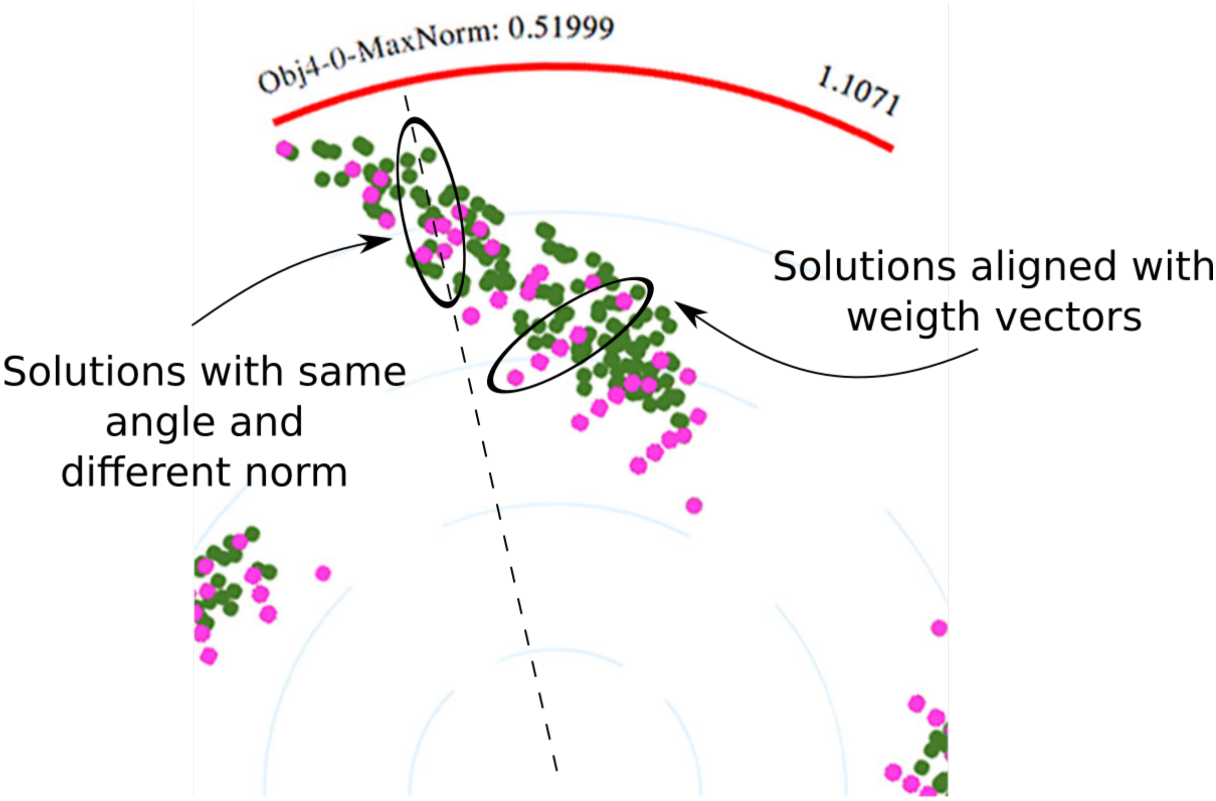
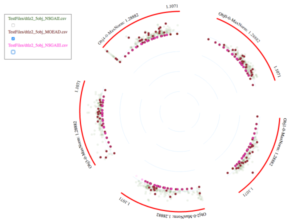


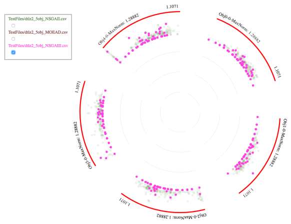
Figure 24 – Solutions from NSGA-II and NSGA-III with different characteristics on DTLZ1 problem.

As another experiment, figure 25 illustrates the obtained solutions from the same algorithms on DTLZ2 problem. Once again, it is possible to observe the best performance of algorithm MOEA/D and NSGA-III in comparison to algorithm NSGA-II. With a closer look at the result of NSGA II, the concentration of solutions in the beginning of the Obj 4 arch shows the population's density in the objective's axis which is not a proper behavior for a many objective optimization algorithm. On the other hand NSGA-III has had a better performance of MOEAD/D is some situations. For instance, NSGA-III could brows some areas closer to the axises (beginning of the arcs) which are not covered by MOEAD. Figure 26 shows in detail two unwanted situations. On the left there are many solutions on the border of the objective space. That area is a place where algorithms that use dominance relations, like NSGA-II, are not efficient to find new solutions, tending to generate weakly dominated solutions. The another unwanted situation is a poor diversity of the solutions in specific regions, where there are often empty spaces in the objective space. This situation was addressed in the previous section.

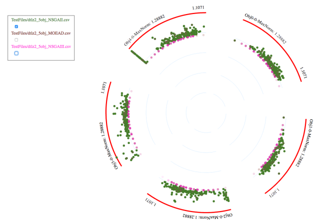
Figure 25 – Analyzing NSGAIII, NSGA II and MOEA/D on DTLZ 2 problem



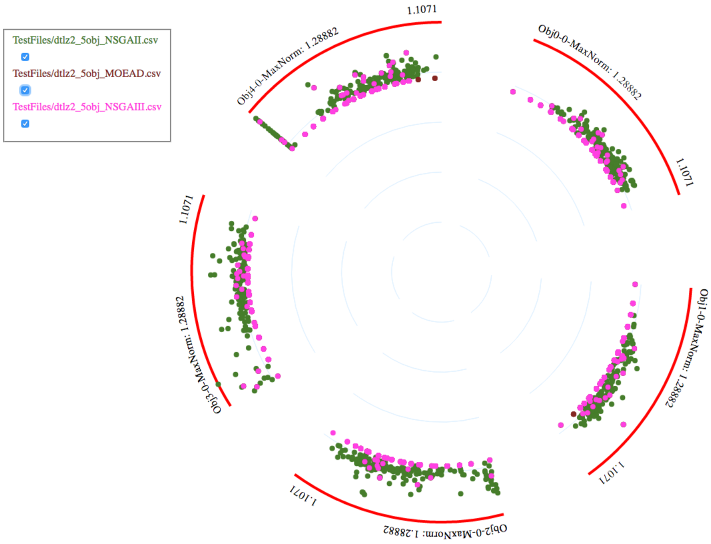
(a) MOEA/D is turned on



(b) NSGAIII is turned on



(c) NSGAII is turned on



(d)

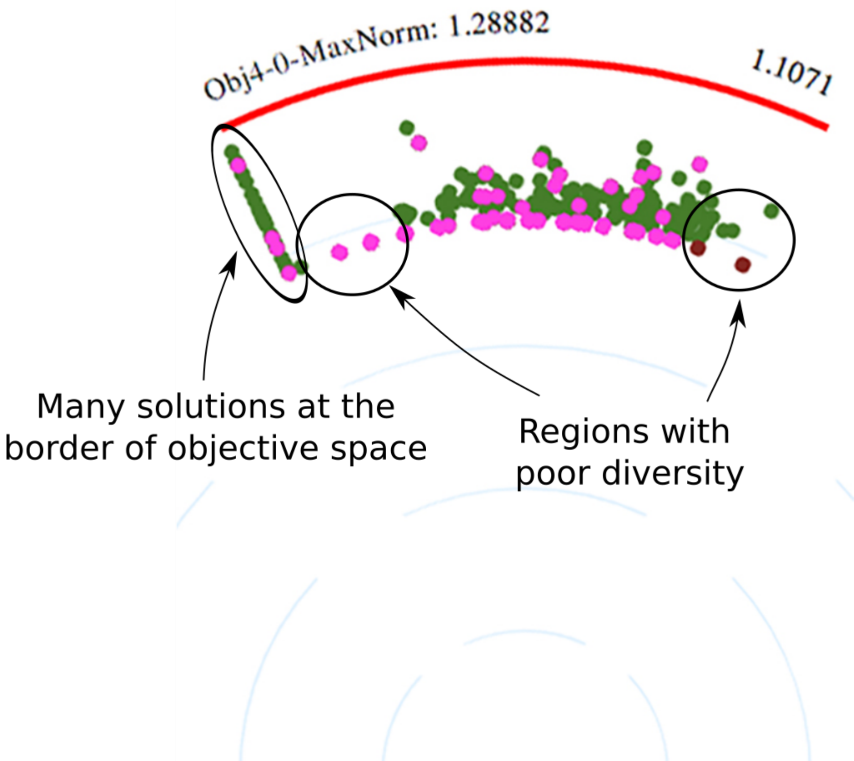


Figure 26 – Regions with poor diversity and unbalanced solutions on DTLZ2 problem.

Chapter 6

Conclusion

As a summary, this text followed two issues about solving strategy and qualitative performance analysis of MaOEAs. According to the mentioned facts, size of the many-objective problems, the form of the problem space, the preferences of the decision makers, etc. are the important challenges which prevent algorithms from reaching to the desired solutions. Moreover, the desire solution has different meaning and perception for each person based on his preferences, goals and needs. Therefore, satisfying the challenges are the requisites to reach to the optimum solutions. On the other hand, a common solving procedures contains two main steps. At first, decision makers tries to achieve the Pareto-Front of the problem. Then, through the multi criteria decision making methods tries to rank the obtained solutions considering the needs and weight of the criteria. In addition, the second step of the mentioned process is dependent to the firs part. It means if, the procedure can not obtain the proper and optimum solutions, ranking them can be an ineffectual work. For this reason, this study regarded the first step of this procedure which is called solving strategy in the text.

This solving strategy contains sequential and iterative process to reach the proper result. In this way, we introduced a tool that allows the decision maker to reduce the number of objectives through the first visualization tool of VMA.

As previously known, the MOEAs are more likely to perform better in problems with smaller number of objectives, therefore they would benefit better from smart/efficient objective reduction. Since, the objective reduction is a subjective process, the decision makers or the expert people should be able to see and evaluate the relations between objectives visually. On the other hand, in some cases, the visualized data-sets is more meaningful to the experts and decision makers practically. Moreover, objective reduction has a deep dependency to the several conditions of problems and preferences. For instance, according to the minimization, maximization or the criteria and prerequisites, the decision maker should be able to check the lower or upper boundary of objectives or comparing them together. After that, he can make decision about the objective reduction methods or the ones which should be reduced. Totally, this part attacks to the challenge of the size of problems.

After that, the second tool of proposed framework (VMA) is able to map the individuals from Cartesian coordinate system to the arcs in VMA tool. According to the methodology, this tool converts the N-property individuals from N-dimensional spaces to the 2-property entity in order to map and illustrate solutions on 2D displays. This mapping gives the ability of visualizing the high dimensional spaces in 2-D forms without any need to objective reduction. Based on this conversion and mapping, we expect that decision makers will have the visual distribution of individuals throughout the space while the characteristics of many-objective space are preserved. Thus, the decision maker is able to figure out the unexplored region and provide a preference point based on it or even generate more solutions in any specific region based on his goals and needs. A previously defined reference point or vector guides the search of the algorithms, and this can be done iteratively and with interaction with the decision-maker. On the other world, after each execution of algorithm, the decision maker can specify the unexplored area and execute the algorithm again to provide the solution on that region. By repeating the execution based on the new references points, the decision maker can visualize all the obtained data-set at the same moment and see all the obtained individuals on the Pareto Front.

On the other hand, the proposed tool also has some abilities to present qualitative information in order to compare or verify the performance of MaOEAs. Moreover, the proposed visualization method was able to provide diverse facilities and information for comparing several algorithms performance together simultaneously. In fact, performance metrics based on quantitative analysis are relevant and have been extensively used in MaOP, whereas the qualitative analysis has been barely used given the visualization gap in high dimensional spaces. Therefore, the author believes that this matter is a huge lack in MaOPs analyzing.

As it has been shown in the previous chapters, the objective reduction tool of VMA enables the decision makers to reduce the objectives considering the several matters such as the goal of optimization or the inherent criteria of the problems. In the experimental result, we have tried to use the simplest way of objective reduction, for both mathematically and visually approaches. Because, we wanted to evaluate the utilization of qualitative analysis. Hence, we had to be independent of the objective reduction methods. Then, as it has demonstrated, the result are acceptable and trustable. In addition, this tool has this ability to evaluate the relations based on the criteria. In this case, the decision maker is able to see the hottest point of each objective according to the weight of each criteria. After that, there is a problem with the optimum size. Thus, the algorithms can solve it as well as possible, and trough the mentioned abilities and given qualitative information, the decision maker can drive the algorithm to reach the desired solutions.

The experiments demonstrated that the defined objectives in this study has been satisfied. This study has introduced the visualization tool to provide the qualitative information. Based on that, it can produce the qualitative strategy to solve the MOPs. Figuring out the harmony and conflicts between objectives is the next important challenge. The second tool of VMA maps the obtained populations in MOPs in to the 2D and comprehensible space to find out the characteristics of the space and the found solutions data-sets. It enables the decision makers to compares the algorithms with each other or themselves. It means, decision maker can check the progress of a algorithms in the different iterations, or compare it with the others. And finally, the decision maker can explore the desired and specific area of the space.

Simple cases of study were carried out in order to highlight and illustrate the functionalities of the set of visualization procedures described along this paper. Hopefully, these examples showed the benefit and flexibility brought by the proposed tool in analyzing and interpreting results in many-objective optimization.

References

W. Aigner. Visual Information-Seeking Mantra Share. URL http://www.infovis-wiki.net/index.php/Visual{_}Information-Seeking{_}Mantra.

M. Asafuddoula, T. Ray, and R. Sarker. A decomposition-based evolutionary algorithm for many objective optimization. *IEEE Transactions on Evolutionary Computation*, 19 (3):445–460, 2014.

M. Ashby. Multi-objective optimization in material design and selection. *Acta Materialia*, 48(1):359–369, 2000. ISSN 13596454. doi: 10.1016/S1359-6454(99)00304-3.

S. Bechikh, M. Elarbi, and L. B. Said. Many-objective optimization using evolutionary algorithms: A survey. In *Recent advances in evolutionary multi-objective optimization*, pages 105–137. Springer, 2017.

M. C. S. J. Ben Shneiderman Catherine Plaisant. *Designing the User Interface: Strategies for Effective Human-Computer Interaction.* Addison-Wesley, 5 edition, 2010. ISBN 0321537351.

L. C. Bezerra, M. López-Ibáñez, and T. Stützle. An empirical assessment of the properties of inverted generational distance on multi-and many-objective optimization. In *International Conference on Evolutionary Multi-Criterion Optimization*, pages 31–45. Springer, 2017.

K. Bringmann, T. Friedrich, F. Neumann, and M. Wagner. Approximation-guided evolutionary multi-objective optimization. In *Twenty-Second International Joint Conference on Artificial Intelligence*, 2011.

D. Brockhoff and E. Zitzler. Are all objectives necessary? on dimensionality reduction in evolutionary multiobjective optimization. In *Parallel Problem Solving from Nature-PPSN IX*, pages 533–542. Springer, 2006.

D. Brockhoff and E. Zitzler. Objective Reduction in Evolutionary Multiobjective Optimization: Theory and Applications. *Evolutionary Computation*, 17(2):135–166, jun 2009. ISSN 1063-6560. doi: 10.1162/evco.2009.17.2.135. URL http://www.mitpressjournals.org/doi/10.1162/evco.2009.17.2.135.

S. Brown. Shneiderman's mantra. URL http://www.codingthearchitecture.com/2015/01/08/shneidermans{_}mantra.html.

D. Cai and W. Yuping. A new uniform evolutionary algorithm based on decomposition and CDAS for many-objective optimization. *Knowledge-Based Systems*, 85:131–142, 2015. ISSN 09507051. doi: 10.1016/j.knosys.2015.04.025.

C. A. C. Coello, G. B. Lamont, and D. A. V. Veldhuizen. *Evolutionary Algorithms for Solving Multi-Objective Problems (Genetic and Evolutionary Computation)*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2006. ISBN 0387332545.

K. Deb. *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Inc., New York, NY, USA, 2001. ISBN 047187339X.

K. Deb and M. Goyal. A combined genetic adaptive search (geneas) for engineering design. *Computer Science and informatics*, 26:30–45, 1996.

K. Deb and H. Jain. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part i: solving problems with box constraints. *IEEE transactions on evolutionary computation*, 18(4):577–601, 2013.

K. Deb and J. Sundar. Reference point based multi-objective optimization using evolutionary algorithms. In *Proceedings of the 8th annual conference on Genetic and evolutionary computation - GECCO '06*, page 635, New York, New York, USA, 2006. ACM Press. ISBN 1595931864. doi: 10.1145/1143997.1144112. URL http://portal.acm.org/citation.cfm?doid=1143997.1144112.

K. Deb, R. B. Agrawal, et al. Simulated binary crossover for continuous search space. *Complex systems*, 9(2):115–148, 1995.

K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: Nsga-ii. *IEEE transactions on evolutionary computation*, 6(2):182–197, 2002.

K. Deb, K. Sindhya, and J. Hakanen. Multi-objective optimization. In *Decision Sciences: Theory and Practice*, pages 145–184. CRC Press, 2016.

E.K.Zavaskas, Z.Turskis, J.Antucheviciene, and A.Zakarevicius. Optimizationation of Weighte ed Aggre egated Sum Pro duct Ass sessment t. *Elektronika IR Elektrotechnika*, pages 3–6, 2012.

S. Esquivel, S. Ferrero, R. Gallard, C. Salto, H. Alfonso, and M. Schütz. Enhanced evolutionary algorithms for single and multiobjective optimization in the job shop scheduling problem. *Knowledge-Based Systems*, 15(1):13–25, 2002. ISSN 09507051. doi: 10.1016/S0950-7051(01)00117-4.

M. Farina and P. Amato. On the optimal solution definition for many-criteria optimization problems. In *2002 Annual Meeting of the North American Fuzzy Information Processing Society Proceedings. NAFIPS-FLINT 2002 (Cat. No. 02TH8622)*, pages 233–238, 2002. doi: 10.1109/NAFIPS.2002.1018061.

A. Flexer. On the use of self-organizing maps for clustering and visualization. *Intelligent Data Analysis*, 5(5):373–384, 2001. ISSN 1088-467X.

C. M. Fonseca and P. J. Fleming. Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization. *5th Int. Conf. Genetic Algorithms*, pages 416–423, 1993.

C. M. Fonseca, P. J. Fleming, et al. Genetic algorithms for multiobjective optimization: Formulationdiscussion and generalization. In *Icga*, volume 93, pages 416–423. Citeseer, 1993.

A. R. Freitas, P. J. Fleming, and F. G. Guimaraes. A Non-Parametric Harmony-Based Objective Reduction Method for Many-Objective Optimization. *International Conference on Systems, Man, and Cybernetics (SMC)*, pages 651–656, 2013.

A. Gaspar-Cunha, R. Takahashi, and C. A. H. Antunes. *Manual de computação evolutiva e metaheurística*. Imprensa da Universidade de Coimbra/UFMG, 2012. ISBN 978-989-26-0583-8 (PDF). doi: http://dx.doi.org/10.14195/978-989-26-0583-8. URL https://digitalis.uc.pt/handle/10316.2/5655.

G. Grinstein, M. Trutschl, and U. Cvek. High-Dimensional Visualizations. In *Proceedings of the Visual Data Mining Workshop, KDD*, 2001.

D. Hadka and P. Reed. Diagnostic assessment of search controls and failure modes in many-objective evolutionary optimization. *Evolutionary computation*, 20(3):423–452, 2012.

R. Haghnazar Koochaksaraei, R. Enayatifar, and F. G. Guimarães. A New Visualization Tool in Many-Objective Optimization Problems. pages 213–224. Springer International Publishing, 2016. doi: 10.1007/978-3-319-32034-2_18. URL http://link.springer.com/10.1007/978-3-319-32034-2{_}18.

Z. He and G. G. Yen. An improved visualization approach in many-objective optimization. In *2016 IEEE Congress on Evolutionary Computation (CEC)*, pages 1618–1625. IEEE, jul 2016a. ISBN 978-1-5090-0623-6. doi: 10.1109/CEC.2016.7743982. URL http://ieeexplore.ieee.org/document/7743982/.

Z. He and G. G. Yen. Visualization and Performance Metric in Many-Objective Optimization. *IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION*, 20(3), 2016b. doi: 10.1109/TEVC.2015.2472283.

Z. He and G. G. Yen. Comparison of visualization approaches in many-objective optimization. In *2017 IEEE Congress on Evolutionary Computation (CEC)*, pages 357–363. IEEE, 2017.

J. Hettenhausen, A. Lewis, and S. Mostaghim. Interactive multi-objective particle swarm optimization with heatmap-visualization-based user interface. *Engineering Optimization*, 42(2):119–139, feb 2010. ISSN 0305-215X. doi: 10.1080/03052150903042632. URL http://www.tandfonline.com/doi/abs/10.1080/03052150903042632.

P. Hoffman, G. Grinstein, K. Marx, I. Grosse, and E. Stanley. DNA visual and analytic data mining. In *Proceedings. Visualization '97 (Cat. No. 97CB36155)*, pages 437–441,. IEEE, 2002. ISBN 0-8186-8262-0. doi: 10.1109/VISUAL.1997.663916. URL http://ieeexplore.ieee.org/document/663916/.

A. Inselberg. *Parallel Coordinates*. Springer New York, New York, NY, 2009. URL http://link.springer.com/10.1007/978-0-387-68628-8{_}6.

A. Inselberg and B. Dimsdale. Parallel coordinates: a tool for visualizing multi-dimensional geometry. *VIS '90 Proceedings of the 1st conference on Visualization '90*, pages 361–378, 1990.

J. Knowles and D. Corne. The pareto archived evolution strategy: A new baseline algorithm for pareto multiobjective optimisation. In *Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406)*, volume 1, pages 98–105. IEEE, 1999.

T. Kohonen. Essentials of the self-organizing map. *Neural Networks*, 37:52–65, 2013. ISSN 08936080. doi: 10.1016/j.neunet.2012.09.018.

J. B. Kollat and P. M. Reed. Comparing state-of-the-art evolutionary multi-objective algorithms for long-term groundwater monitoring design. *Advances in Water Resources*, 29(6):792–807, 2006.

J. B. Kollat and P. M. Reed. A computational scaling analysis of multiobjective evolutionary algorithms in long-term groundwater monitoring applications. *Advances in Water Resources*, 30(3):408–419, 2007.

A. Kovács and J. Abonyi. Visualization of Fuzzy Clustering Results by Modified Sammon Mapping. In *Proceedings of the 3rd international symposium of hungarian researchers on computational intelligence*, pages 177–188, 2002.

B. Li, J. Li, K. Tang, and X. Yao. Many-objective evolutionary algorithms: A survey. *ACM Comput. Surv.*, 48(1):13:1–13:35, Sept. 2015. ISSN 0360-0300. doi: 10.1145/2792984. URL http://doi.acm.org/10.1145/2792984.

K. Li, K. Deb, Q. Zhang, and S. Kwong. An evolutionary many-objective optimization algorithm based on dominance and decomposition. *IEEE Transactions on Evolutionary Computation*, 19(5):694–716, 2014.

Y. Liu, D. Gong, X. Sun, and Y. Zhang. Many-objective evolutionary optimization based on reference points. *Applied Soft Computing*, 50:344–355, 2017. ISSN 15684946. doi: 10.1016/j.asoc.2016.11.009.

R. J. Lygoe, M. Cary, and P. J. Fleming. A Many-Objective Optimisation Decision-Making Process Applied to Automotive Diesel Engine Calibration. pages 638–646. Springer Berlin Heidelberg, 2010. doi: 10.1007/978-3-642-17298-4_72. URL http://link.springer.com/10.1007/978-3-642-17298-4{_}72.

R. Marler and J. Arora. Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, 26(6):369–395, apr 2004a. ISSN 1615-147X. doi: 10.1007/s00158-003-0368-6. URL http://link.springer.com/10.1007/s00158-003-0368-6.

R. T. Marler and J. S. Arora. Survey of multi-objective optimization methods for engineering. *Structural and multidisciplinary optimization*, 26(6):369–395, 2004b.

I. R. Meneghini and F. G. Guimarães. Evolutionary method for weight vector generation in multi-objective algorithm based on decomposition and aggregation. In *2017 IEEE Congress on Evolutionary Computation (CEC)*, 2017.

S. K. Mishra, P. Ganapati, S. Meher, and R. Majhi. A fast multiobjective evolutionary algorithm for finding wellspread pareto-optimal solutions. In *In KanGAL Report No. 2003002, Indian Institute Of Technology Kanpur*. Citeseer, 2002.

S. Opricovic. Multicriteria optimization of civil engineering systems. *Faculty of Civil Engineering, Belgrade*, 2(1):5–21, 1998.

S. Opricovic and G.-H. Tzeng. Multicriteria planning of post-earthquake sustainable reconstruction. *Computer-Aided Civil and Infrastructure Engineering*, 17(3):211–220, 2002.

G. Polzlbauer, M. Dittenbach, and A. Rauber. A visualization technique for self-organizing maps with vector fields to obtain the cluster structure at desired levels of detail. In *Proceedings. 2005 IEEE International Joint Conference on Neural Networks, 2005.*, volume 3, pages 1558–1563. IEEE, 2005. ISBN 0-7803-9048-2. doi: 10.1109/IJCNN.2005.1556110. URL http://ieeexplore.ieee.org/document/1556110/.

A. Pryke, S. Mostaghim, and A. Nazemi. Heatmap Visualization of Population Based Multi Objective Algorithms. In *Evolutionary Multi-Criterion Optimization*, pages 361–375.

Springer Berlin Heidelberg, Berlin, Heidelberg, 2007. doi: 10.1007/978-3-540-70928-2_29. URL http://link.springer.com/10.1007/978-3-540-70928-2{_}29.

D. K. Saxena, J. A. Duro, A. Tiwari, K. Deb, and Q. Zhang. Objective Reduction in Many-Objective Optimization: Linear and Nonlinear Algorithms. *IEEE Transactions on Evolutionary Computation*, 17(1):77–99, feb 2013. ISSN 1089-778X. doi: 10.1109/TEVC.2012.2185847. URL http://ieeexplore.ieee.org/document/6151114/.

E. R. Schneider and R. A. Krohling. A hybrid approach using TOPSIS, Differential Evolution, and Tabu Search to find multiple solutions of constrained non-linear integer optimization problems. *Knowledge-Based Systems*, 62:47–56, 2014. ISSN 09507051. doi: 10.1016/j.knosys.2014.02.015.

H. Seada and K. Deb. U-nsga-iii: a unified evolutionary optimization procedure for single, multiple, and many objectives: proof-of-principle results. In *International conference on evolutionary multi-criterion optimization*, pages 34–49. Springer, 2015.

H. K. Singh, A. Isaacs, and T. Ray. A Pareto Corner Search Evolutionary Algorithm and Dimensionality Reduction in Many-Objective Optimization Problems. *IEEE Transactions on Evolutionary Computation*, 15(4):539–556, aug 2011. ISSN 1089-778X. doi: 10.1109/TEVC.2010.2093579. URL http://ieeexplore.ieee.org/document/5699918/.

T. J. Stewart. A critical survey on the status of multiple criteria decision making theory and practice. *Omega*, 20(5-6):569–586, Sept. 1992. URL http://www.sciencedirect.com/science/article/pii/030504839290003P.

Y. Tian, R. Cheng, X. Zhang, and Y. Jin. PlatEMO: A MATLAB Platform for Evolutionary Multi-Objective Optimization. *ArXiv e-prints*, Jan. 2017.

D.-H. Tran, M.-Y. Cheng, and D. Prayogo. A novel Multiple Objective Symbiotic Organisms Search (MOSOS) for time cost labor utilization tradeoff problem. *Knowledge-Based Systems*, 94:132–145, 2015. doi: 10.1016/j.knosys.2015.11.016.

E. Triantaphyllou. Multi-criteria decision making methods. In *Multi-criteria Decision Making Methods: A Comparative Study*, pages 5–21. Springer, 2000.

T. Tusar and B. Filipic. Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method. *IEEE Transactions on Evolutionary Computation*, 19(2):225–245, 2015. ISSN 1089778X. doi: 10.1109/TEVC.2014.2313407.

J. J. Valdes and A. J. Barton. Visualizing high dimensional objective spaces for multi-objective optimization: A virtual reality approach. In *2007 IEEE Congress on Evolutionary Computation*, pages 4199–4206. IEEE, sep 2007. ISBN 978-1-4244-1339-3. doi: 10.1109/CEC.2007.4425019. URL http://ieeexplore.ieee.org/document/4425019/.

D. A. Van Veldhuizen and G. B. Lamont. Multiobjective evolutionary algorithm research: A history and analysis. Technical report, Citeseer, 1998.

I. Var. Multivariate data analysis. *vectors*, 8(2):125–136, 1998.

M. Vlachos, C. Domeniconi, D. Gunopulos, G. Kollios, and N. Koudas. Non-linear dimensionality reduction techniques for classification and visualization. In *Proceedings of the eighth ACM SIGKDD international conference on Knowledge discovery and data mining - KDD '02*, page 645, New York, New York, USA, 2002. ACM Press. ISBN 158113567X. doi: 10.1145/775047.775143. URL http://portal.acm.org/citation.cfm?doid=775047.775143.

C. von Lücken, B. Barán, and C. Brizuela. A survey on multi-objective evolutionary algorithms for many-objective problems. *Computational Optimization and Applications*, 58(3):707–756, 2014. ISSN 1573-2894. doi: 10.1007/s10589-014-9644-1. URL http://dx.doi.org/10.1007/s10589-014-9644-1.

C. von Lücken, B. Barán, and C. Brizuela. A survey on multi-objective evolutionary algorithms for many-objective problems. *Computational Optimization and Applications*, 58(3):707–756, feb 2014. ISSN 0926-6003. doi: 10.1007/s10589-014-9644-1. URL http://link.springer.com/10.1007/s10589-014-9644-1.

D. J. Walker, R. M. Everson, and J. E. Fieldsend. Visualisation and ordering of many-objective populations. In *IEEE Congress on Evolutionary Computation*, pages 1–8. IEEE, jul 2010. ISBN 978-1-4244-6909-3. doi: 10.1109/CEC.2010.5586078. URL http://ieeexplore.ieee.org/document/5586078/.

D. J. Walker, R. Everson, and J. E. Fieldsend. Visualizing mutually nondominating solution sets in many-objective optimization. *IEEE Transactions on Evolutionary Computation*, 17(2):165–184, 2012.

D. J. Walker, R. M. Everson, and J. E. Fieldsend. Visualizing mutually nondominating solution sets in many-objective optimization. *IEEE Transactions on Evolutionary Computation*, 17(2):165–184, 2013. ISSN 1089778X. doi: 10.1109/TEVC.2012.2225064.

M. O. Ward, G. Grinstein, and D. Keim. *Interactive Data Visualization: Foundations, Techniques, and Applications*. CRC Press, 2010. ISBN 143986554X. URL https://books.google.com/books?hl=en{&}lr={&}id=Kk7NBQAAQBAJ{&}pgis=1.

W. Watthayu. Loopy Belief Propagation: Bayesian Networks for Multi-Criteria Decision Making (MCDM). *International Journal of Hybrid Information Technology*, 2, 2009.

C. Wu, D. Barnes, D. Rosenberg, and X. Luo. An analytic network process-mixed integer multi-objective programming model for partner selection in agile supply chains. *Production Planning and Control*, 20(3):254–275, 2009.

K. Yoon and C. L. Hwang. TOPSIS (technique for order preference by similarity to ideal solution)–a multiple attribute decision making, w: Multiple attribute decision making–methods and applications, a state-of-the-at survey, 1981.

Q. Zhang and H. Li. Moea/d: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on evolutionary computation*, 11(6):712–731, 2007.

X. Zhang, Y. Tian, and Y. Jin. A knee point-driven evolutionary algorithm for many-objective optimization. *IEEE Transactions on Evolutionary Computation*, 19(6):761–776, 2014.

Y. Zhu, J. Liang, J. Chen, and Z. Ming. An improved NSGA-III algorithm for feature selection used in intrusion detection. *Knowledge-Based Systems*, 116:74–85, 2017. ISSN 09507051. doi: 10.1016/j.knosys.2016.10.030.

E. Zitzler. *Evolutionary algorithms for multiobjective optimization: Methods and applications*, volume 63. Citeseer, 1999.

E. Zitzler, M. Laumanns, and L. Thiele. Spea2: Improving the strength pareto evolutionary algorithm. *TIK-report*, 103, 2001.