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Universidade Federal de Minas Gerais
Escola de Engenharia
Programa de Pós-Graduação em Engenharia Elétrica

PPGEE

Danny Augusto Vieira Tonidandel

The Ground Zero of Electrical Engineering:

the Flow Analogy for Electricity and Magnetism, from Antiquity to Telegraphy

Doctoral Thesis

Belo Horizonte
2021

Danny Augusto Vieira Tonidandel

The ground zero of electrical engineering: the flow analogy for electricity and magnetism, from antiquity to telegraphy

Thesis presented to the Graduate Program in Electrical Engineering at the Federal University of Minas Gerais, as part of the necessary requirements to obtain the title of Doctor in Electrical Engineering.

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Escola de Engenharia

Programa de Pós-Graduação em Engenharia Elétrica

Supervisor: Wallace do Couto Boaventura, PhD.
Co-supervisor: Antônio Emílio Angueth de Araújo, PhD.

Belo Horizonte
2021

Tonidandel, Danny Augusto Vieira.
T665g The ground zero of electrical engineering: the flow analogy for eletricity and magnetism, from antiquity to telegraphy [recurso eletrônico] / Danny Augusto Vieira Tonidandel. - 2021.
1 recurso online (231 f. : il., color.) : pdf.

Orientador: Wallace do Couto Boaventura.
Coorientador: Antônio Emílio Angueth de Araújo.

Tese (doutorado) - Universidade Federal de Minas Gerais, Escola de Engenharia.

Apêndices: f. 202-231
Bibliografia: f.174-201.

Exigências do sistema: Adobe Acrobat Reader.

1. Engenharia elétrica - Teses. 2. Epistemologia – Teses. 3. Linhas telegráficas - Teses. I. Boaventura, Wallace do Couto. II. Araújo, Antônio Emílio Angueth de. III. Universidade Federal de Minas Gerais. Escola de Engenharia. IV. Título.

CDU: 621.3(043)

Ficha catalográfica elaborada pela Bibliotecária Letícia Alves Vieira - CRB-6/2337
Biblioteca Prof. Mário Werneck - Escola de Engenharia da UFMG

"The Ground Zero of Electrical Engineering: The Flow Analogy for Electricity and Magnetism, from Antiquity to Telegraphy"

Danny Augusto Vieira Tonidandel

Tese de Doutorado submetida à Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação em Engenharia Elétrica da Escola de Engenharia da Universidade Federal de Minas Gerais, como requisito para obtenção do grau de Doutor em Engenharia Elétrica.

Aprovada em 06 de julho de 2021.

Por:

Prof. Dr. Wallace do Couto Boaventura
DEE (UFMG) - Orientador

Prof. Dr. Antônio Emílio Angueth de Araújo
DEE (UFMG)

André Koch Torres de Assis

Prof. Dr. André Koch Torres de Assis
Instituto de Física (UNICAMP)

Prof. Dr. Marcelo Azevedo Neves
Departamento de Física (UFRRJ)

Prof. Dr. Hani Camille Yehia
DELT (UFMG)

Davies William de Lima Monteiro

Prof. Dr. Davies William de Lima Monteiro
DEE (UFMG)

To the memory of my beloved parents, Maria Amélia and Ailton.

Acknowledgements

To the Supreme Source of life, first cause of all things, with gratitude!

To professor Wallace do Couto Boaventura, for his noble and wise guidance. To professor Antônio Emílio Angueth de Araújo, for all the shared wisdom, since the master's, from which the main theme of this research originated, a decade before. To both, for the memorable meetings! To the Graduate Program in Electrical Engineering (PPGEE) and the agency CAPES, for the institutional and material support. To prof. André K. T. Assis (Unicamp), for the relevant considerations, recommended readings and indications concerning the history of telegraphy, and for the kindness in accepting to be a member of the evaluation board, also formed by the professors Marcelo A. Neves (UFRRJ), Davies W. L. Monteiro (UFMG) and Hani C. Yehia (UFMG), to whom I am also grateful for the kindness and considerations.

To the Graduate Program in Sociology (PPGS), for the introductory course on sociology of knowledge, the Graduate Program in Physics, at UFOP, for the course on Mathematical Methods, and to the class colleagues, Luiz, Paulo and Ramon. To the lab colleagues (LAPIS): Geraldo, Samira, Mozzar, João Paulo, Rodrigo, Guilherme, Valcimar and Raphael, for the companionship. To the Control Engineering & Automation department, at UFOP, for the possibility of leaving for the doctorate, specially the colleagues Paulo, Adrielle, João Carlos and Regiane, for all support. To Celso Taveira and Andréa Lisly (DEHIS-UFOP), for the recommended readings, and to Mamede Dias, for the help with the Latin version of the abstract, thought as a silent homage to Ørsted's "obvious" discovery, which confirmed, two hundred years ago, the long felt relationship between electricity and magnetism.

To my family, brothers, aunts and uncles, nephews and nieces, with love. A warm hug to those friends who probably will not read the thesis, but supported me while I was writing it: Cristiano, Alba, Natiele, Kelly, Rubens, Bráulio and Gilson, for the pre-pandemic coffees and affection of always, and Luiz Barbieri, for the enlightening conversations. To my wife and life partner, Tati, for all moments, way beyond the conversations on history and literature; and, finally, to our dear son Francisco, who during his early years courageously endured a father and a mother in graduate school!

"—Imagine um cachorro basset, tão comprido, que a cabeça está no Rio e a ponta do rabo em Minas. Se se belisca a ponta do rabo, em Minas, a cabeça, no Rio, pega a latir...
—E é isso um telégrafo-sem-fio?
—Não. Isso é o telégrafo com fio. O sem-fio é a mesma coisa... mas sem o corpo do cachorro."

(João Guimarães Rosa, *Tutameia.*)

Abstract

Electricity has been thought and spoken of, throughout human history, as belonging to an almost transcendental realm, something simultaneously beyond space, time and any conception. The advent of the electrical science, in all its specificity, helped to lay the foundations for the techno-scientific revolution that would take place in the second half of the 19th century, a period in which the still non-existent branch of electrical engineering was not dissociated from Natural Philosophy. Way before the emergence of electrical telegraphy as a symbolic landmark for this applied branch of science, analogies played a fundamental role in the formation of some of its basic concepts, starting from the ancestral idea of flow and continuity, to the absorption of mathematical language in its framework.

Keywords: Electrical Engineering, History of Physics, Telegraphy, Analogy, Education, Epistemology.

Resumo

A Eletricidade foi, ao longo da história humana, pensada e falada como se pertencesse a um domínio quase transcendental, algo simultaneamente além do espaço, do tempo e de qualquer concepção. O advento da ciência elétrica, em toda sua especificidade, ajudou a lançar as bases da revolução tecno-científica que se daria na segunda metade do século dezenove, período em que o ainda não existente ramo da engenharia elétrica não estava dissociado da filosofia natural. Bem antes do aparecimento da telegrafia elétrica como um marco simbólico para esse ramo aplicado da ciência, as analogias físicas desempenharam um fundamental papel na formação de alguns de seus conceitos básicos, desde a ideia ancestral de fluxo e continuidade, até a absorção da linguagem matemática em seu arcabouço.

Palavras-chave: Engenharia Elétrica, História da Física, Telegrafia, Analogia, Educação, Epistemologia.

Summa

Electricitatis pertinere ad historiam humanitatem quasi transcendentalis locis intelliguntur erat, quam quod eodem tempore et simul spatio. Aduentus electrica scientia in suam proprietatem adiuvisti in eo monasterii fundamenta iacere ad scietiam revolutionem qui futurus erat secundum dimidium saeculo XIX ad tempus, in quibus nulla genere ex electrica ingeniarii non dissociantur de naturalibus. Longa ante specie electrica telegraphiae utpote significantem quos fixerunt priores in quo applicata genere scientiam, corporis similitudinibus praecipuum munus in formatione alicuius de sua conceptus de paterna agunt ideam de fluxus et continuitatem ad effusio de mathematical lingua compage vobis.

Significantis verbis: Electrica Scientia, Historia physica, Telegraphiae, Analogiae, Educationem, Epistemologiae.

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1 Prologue

The advent of electricity-based technology, in all its specificity, helped to lay the foundations for the techno-scientific revolution that would take place in the second half of the 19th century, a period in which the still non-existent branch of Electrical Engineering, with a fundamentally empirical character, was not dissociated from Natural Philosophy. Would it be possible to delimit a moment – or an event – through which the branch is structured as applied science, assuming the prominent role that it would enjoy in the following centuries? The hypothesis we intend to demonstrate is that the emergence of electric telegraphy can be considered as this symbolic landmark – "the ground zero" – having the physical analogies as fundamental elements in the formation of some of its basic concepts, from the ancestral idea of flow and continuity to the absorption of mathematical language in its theoretical framework. In this way, the collective conceptions of scientists involved with electrical telegraphy, in its theoretical and experimental aspects, would form one of the pillars of this landmark, from the first proposals, which allowed the laying of the first transatlantic telegraph cable, to the deduction of the telegrapher's equation, which could faithfully describe the spread of a telegraph signal along a wire.

Although it is not intended to write an exhaustive study, it is interesting to investigate certain points that relate the history of analogies to the idea of electrical flow, seeking to avoid, when possible, the existent philosophical labyrinths in the formation of such concepts. In addition, it is intended to draw a dialogue between history of theoretical models such as action at a distance, continuous action or field theory and their epistemological status, the extent to which analogies played a role in the advent of telegraphy and, in particular, of submarine telegraphy, such as the launch of the first atlantic cable for telegraph communications, one of the greatest technological events of the 19th century. That is, how and in which context were the physical analogies used, and with what implications? As a product of this research, it is intended to produce a text with a kind of history of electricity from the perspective of *physical analogy*, as being one of the bases upon which the Electrical Engineering itself was based: a study on analogies and its role in ground zero of Electrical Engineering, or the role of analogy in science and in the establishment of contemporary Electrical Engineering.

It is argued that analogies are part of the development of science and, in particular, are present in the history of electricity, as present as the deductive formalism is. The relationship between science and analogy – or rather, the use of analogy in science – is not recent, but in the nineteenth century it was used in an entirely new way compared to previous historical periods, since it was not only intended to provide explanations for the enigmas of life and the world, but also the development of electrical technology based on what was already known from the classic laws of mechanics. This movement would play an important role in the industrialization of the European and American economies. In fact, the success of the telegrsphic endeavours would change forever the conceptions of science and those of human relations, allowing, for the first time, a rapid communication over long distances, having a strong impact on the world economy and the globalization process.

Although under different names, the deductive method establishes that science is built from facts just as a house is made of stones. However, as Poincaré has one stated,[1] a collection of facts does not form a science, just as a pile of stones does not form a house. Much less history. The act of "collecting facts", at different times, is part of the motivations that led the pioneers of electrical science to interpret their own context; facing, not infrequently, enchanting technologies that, in many aspects, would divert them from the initial objectives that. Although a romantic vision leads us to think that the initial objective for science is for truth and knowledge, it is clear that the creation of scientifically based technology, as well as the elaboration of scientific principles adjusted to a certain technological panorama, has long been a delightful distraction.

Marc Bloch (1886-1944), in full awareness of the terror of a Nazi concentration camp,[2] already warned that the main aspect of history is not to be seen as a source of curiosity or chronology, but as a problem. This "history-problem" gives the scientific character to the history of science, and allows an adequate time frame to be made in order to answer a specific question, and not just a factual accumulation. If viewed in this way, the historiography would have the potential to produce a true transformation in the self-image of science, giving a new meaning to its role in the evolution of humanity. After all, the facts do not speak for themselves: "... documents and testimonies only speak when we know how to interrogate them."[3]

Thus, it is desirable to make explicit a historical and philosophical interpretation of science, as it will affect the criteria for the selection of historical material and the question to be answered, including in the resulting comments. This is an acceptable procedure, scientifically speaking: to establish a view on the scientific nature – such as the importance of inductive methods in its development – which will be tested against some historical situations and, if necessary, modify the theory in the light of new "experiments". In the words of Mary B. Hesse, "there is no pure data",[4] no facts without interpretation. All facts,

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[1] Poincaré 1890.

[2] Bloch was a French medievalist historian, one of the founders of the School of Annales – one of the most prominent historiographical movements of the 20th century – imprisoned in a concentration camp and shot by the Gestapo in 1944. He is considered by many to be one of the greatest historians of all time, having written, although incompletely, one of his most influential works, *Apology of History or the Historian's Craft*. This and other works were collected and published posthumously by his son, Etienne Bloch. Bloch 2001.

[3] Febvre 2011, p. 82.

[4] Hesse 1974.

whether experimental or historical, are interpreted in the light of a given theory, although implicitly.

Gaps inherent in the nonlinear character of history allow the researcher, at times, to conjecture about events that are still unclear, as soon as someone identifies them, or even, if one has scientific training, it is possible to suggest "insertions" in the course of transformation of certain technique, or in a scientific problem, for example, such as using a mathematical model or measurement artifact, in an attempt to understand the intention of those who developed them, or even to contribute significantly to that field of study. Such insertions must here be thought in their metaphorical sense and can be understood as an archaeological work, in which the researcher conjectures about historical gaps, having as a starting point only fragmented artifacts. These elements could, in theory, allow a connection between temporally separated realities starting from a conceptual union.

Take, for example, a well-known historiographical book on the formation of classical field theory, *The Origins of Field Theory*,[5] by Leslie Pearce Williams. In his book Williams argued that it was due to Faraday and not Maxwell the central role in the formulation of classical field theory, having inherited concepts from the *Naturphilosophie* of Newton's followers, in particular the point atom analogy of the Jesuit priest Ruggiero Boscovich (1711-1787), who had demonstrated, in the previous century, some problems with newtonian atomism. The author suggested that such a theoretical component was one of the main ingredients of Faraday's researches, contrary to the common sense at the time, that Faraday had conceived and gained confidence about the real existence of the lines of force as a result of his empirical and qualitative work on the "magnetic images".

A risk certainly faced by the author, and common in the development of this type of historical narrative, was to present evidences that might not be very convincing, as when he conjectured that Faraday's relationship with his first intellectual mentor, Sir Humphry Davy, was decisive in his adherence to Boscovich's ideas,[6] given Davy's well-known militancy within the *Naturphilosophie.*[7] In this case, the training for interpretation of historical events is, almost always, insufficient without an instrumental understanding on the area of knowledge that one seeks to study, which will inevitably make more difficult one in-depth reading of its development, essential for assessing the meaning of that science as a whole. Williams was a chemical engineer, as well as a professor in the history department at Cornell University. His most influential work, a biography of Michael Faraday prior to the *Origins of Field Theory*, helped, in some aspects, to create the *aura* around Michael Faraday's genius, as one of the most inventive empiricists of all times.[8]

This case is rich in two notable aspects. First of all, it is clear that the historian

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[5] Williams 1980.

[6] *Theoria Philosophiae Naturalis* (1758). See in Boscovich 1763.

[7] A contextualization of this problem can be seen in Heimann 1971.

[8] Williams 1965.

of formation will hardly be able to write about the history of a scientific concept without privileging, which is natural, inherent aspects of his field of work, that is, sociological, political, anthropological or cultural aspects, relegating the fundamental principles of those sciences to a secondary role. Another aspect, this one more encouraging, concerns to a relative lack of historiographical works with an initially untrained perspective. It serves as an incentive for non-professional historians if they dare to the inglorious task of research and historical reflection. It is said to be inglorious due to the high degree of strangeness that this type of work usually causes in its own environment, and which resides within the scope of transdisciplinarity, currently so commented on, but little carried out. Normally, research committees and funding agencies, as they are made up of elements drawn from the academic community itself, portray, naturally, the thoughtlessness and conceptions of their own environment, which results in the still low insertion and incentive to research in this field, like in Brazil especially.

Still in this respect, another problem must be commented. A superficial understanding, which implies a hasty reading of the history of science, can lead the unwary to hasty judgments about its progress, considering it as a rational, slow (and incredibly tedious!) march to a certain destination, as a collection of curiosities typical of the utilitarianism,[9] or even to the phenomenon of *negationism* that humanity has been witnessing, in the first quarter of the 21st century.[10] In this aspect, the importance of carrying out works in which the processes by which knowledge is produced is evidenced. A scientific work is not the result of personal convictions or common sense, nor a collection of self-evident concepts even if the information is "filtered" by the author. It is initially the result of an intense effort of reading, writing and rewriting, which involves planning, hypothesis making, project making, discussions in disciplines, submission of articles in specialized journals and evaluation by research committees, perhaps resulting from careful reading in hundreds books, articles and other sources, sometimes in different languages or even "dead" languages.

On the other hand, if it is considered that a scientific theory is the only way to acquire knowledge, it is recommended to avoid inconsistencies. For example, it would be strange to use a set of mathematical principles assuming that matter consists in a certain form and, just ahead, take another hypothesis for the formation of the same matter. This is, perhaps, the idea that made the most thirsty appetite student at all times ignore initial hypotheses, considering only the reality under his eyes and, consequently, his practice, as just a collection of experimental laws. However, the two ways of seeing are, in fact, useful tools for research, as long as they are not mixed, as Poincaré once said:[11] "... *Deux théories contradictoires peuvent en effect, pourvu qu'on ne les mêle pas, et qu'on n'y cherche pas le fond des*

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[9] Hesse 1962, chap. 1.

[10] See, for example, the interview given by the French philosopher Bruno Latour, one of the founders of the school of (social) studies of science and technology (S.T.S. - science and technology studies), for the The New York Times journal. Kofman 2018.

[11] Poincaré 1890.

choses, être toutes deux d'utiles instruments de recherches". In this respect, the thought meets Bachelard's "scientific spirit", when he affirmed that science is essentially built from constant rectifications of knowledge itself, widening the island's borders of human knowledge with the unknown, and not just an accumulation of facts.

It is essential for the scientist or engineer not only get to know more of his own training area, as well as perform a self-reflection and writing exercise. An experimentalist who knows little of the great questions of his/her field of work, which is essentially historical, may become an easy prey for naive generalizations that, by its very nature, crystallize beliefs that have only the appearance of knowledge. If we ask any educated person to answer what electricity is, it is common to receive sometimes vague or strict answers, which themselves repeat unconsciously older theories, that do not move too far away from the common sense.

And if for the layman, as well as for the educated, it seems difficult to imagine, in the the 21st century, why something like *electricity* needs explanation, because it is taken for granted, a common fact, the phenomenon of analogy in a broader sense suffers from the same evil:[12] if it is part of a constant and involuntary flow of thought, which pops out and soon disappears, why bother? Well, something other than the mythical apple must have fallen on Newton's head to arouse his interest in gravitation. Likewise, to study the role of analogies in the history of electrical science seems to be something very close to what it is to be human, or to human thought, something so familiar and present that it became invisible, having its attractive potential there. Thus, in the manner of the blind man called to observe the past to dilute his own ignorance, using other "eyes to see",[13] it is intended to take a look at important analogies present in the History of Electricity, within the frontier between science, history and philosophy, without having, however, the pretentious intention of exhausting any of the fields of observation.

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An underlying issue, which somehow guides the structuring of this work is, in essence: what type of imaginative conceptions had the Victorian scientists, at disposal, to build the science of electricity and magnetism? What traditions have they benefited from? Thus, the thesis is a reflection on the epistemic role of analogies in the structuring of scientific knowledge, intersected by historical questions on the formation of the idea of flow, in which the elements *electricity* and *magnetism*, *heat* and *light* will be associated. The analogies will be the mirror so that phenomena and theories somehow related to the history

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[12] Hofstader and Sander 2013.

[13] Mt 13.

of electricity can be portrayed, from the most remote conception to the establishment of the Electrical Engineering, a landmark that it is proposed to find.

The first structural reference is the work of Mary B. Hesse (1924-2016), especially for her study of analogies in the book *Forces and Fields*,[14] in which she addresses the concept of action at a distance, in addition to the book *The Structure of Scientific Inference*,[15] which represents a kind of synthesis of her entire work. In the first book the author presents the inseparability between different fields of knowledge, which are necessary to understand a concept as simple and, at the same time, as complex as this "phantasmagorical" principle, in the words of Albert Einstein. A second important structural reference is the work of Pierre Duhem (1861-1916), mainly for his studies on the physical models and structure of a scientific theory, as in the book *The aim and structure of physical theory.*[16]

An important stylistic reference is the work of author Paul J. Nahin, in particular the books *The Science of Radio*[17], *Dr. Euler's Fabulous Formula*,[18] *Oliver Heaviside*,[19] among others, in which the author proposes light and humorous ways to present complex scientific themes such as the factors that made radio possible, the discovery of the famous relation $e^{i\pi}+1=0$, or the biography of an "electrical genius", without setting aside the importance of a well based physical concept or a detailed mathematical demonstration. In some of his books the historical aspects play a preponderant role. In others, mathematics is the main focus, but permeated with historical narratives. Therefore, in an effort to contextualize the development of the idea of electric flow, it is sought to achieve a way of writing that seeks to bring the same elements to the reader.

Bearing in mind an event of great relevance, historical and scientific, which was the advent of telegraphy through Weber and Gauss, and of major projects such as the launch of the first transatlantic telegraphic cable, captained by one of the first mathematical theories of electrical propagation,[20] and considering further that the proposed solution was based on a physical analogy for the "flow of electricity", such events gain great importance, given that was early found that a more in-depth study of analogies would be necessary to support one of the main conjectures of this work: establishing an initial and symbolic framework for electrical engineering. In a way, the works of Hesse, Duhem and Nahin will be present throughout the text, like true beacons guiding a ship initially aimless. Therefore, the structuring of this thesis is evidenced as a historical, epistemological and cognitive reflection on the role of analogies in understanding and transformation of the scientific practice itself, even considering the considerably large time frame chosen.

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[14] Hesse 1962.
[15] Hesse 1974.
[16] Duhem 1954.
[17] Nahin 2001.
[18] Nahin 2006.
[19] Nahin 2002.
[20] Tonidandel, Boaventura, et al. 2018.

At the end of the work, there is a more historical approach, adopting as a starting point the beginning of telegraph technology, and the first large-scale project involving electrical telegraphy. The idea is therefore to write shorter chapters addressing different historical periods, highlighting, where possible, the discussion about the physical analogies that maintain some correlation with the mathematical theory of electricity, as its historical period, in particular, is characterized by a mixture of standards applied by many scholars, from different schools of thought as the mechanicism, positivism, cartesianism, empiricism and classicism, each one with their particular epistemic perspectives.

2 Analogy and Analogies

> *Passing from the most universal of all analogies to a very partial one, we find the same resemblance in mathematical form between two different phenomena giving rise to a physical theory...*
>
> (James Clerk Maxwell, *On Faraday's Lines of Force, 1855.*)

In his *Meditations on a Hobby Horse*, the art Historian Ernst Gombrich (1909-1936) narrates an old music hall joke that described a drunkard who used to lift his hat, in a gentle way, to every lamp-post he passed by, raising the question "... should we say that the liquor has so increased his power of abstraction that he is now able to isolate the formal quality of uprightness from both lamp-post and the human figure?"[1] Indeed, the own idea of abstraction as a mental act can take us on curious and strangely absurd paths, notably when tracing the role of scientific models, analogies and metaphors, keeping in mind that modeling is a clever way that humans conceive to supply an absent sense, like the sculptor who "abstracts" the shape under the chisel.

This is also true if the entity under the loupe resembles something elusive, like the slippering sand or the lightning cutting the night sky. What would be a good model for the electricity? If the advances in 20th century physics put this issue in check or not is still an open issue, but the famous statement of Hertz,[2] "... Maxwell's theory is Maxwell's system of equations" initiated a movement that would make physics extremely successful in a large spectrum. Before that, however, in a period prior to the eventual acceptance of the equations of electromagnetism as a good model for physical reality,[3] the Victorian science drank from a source that goes back to a long tradition that the new physics would sought to forget, the use of a highly hybrid, heterogeneous and reflexive way of problem solving, a mixture of non-formal (and apparently disconnected) methods as parts of the methodology of knowledge construction, and that will be, henceforth, the main search of this work.

"... In order therefore to appreciate the requirements of the (electrical) science, the student must make himself familiar with a considerable body of most intricate mathematics, the mere retention of which in memory materially interferes with further progress. The first process therefore in the effectual study of the science, must be one of simplifica-

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[1] Gombrich 1985.

[2] Hertz 1893, p. 46.

[3] James Clerk Maxwell published his book on electromagnetism, *A Treatise on Electricity and Magnetism*, in 1873 and died in 1879. In that period, his theory was not understood or even widely accepted. Above all, it was left to three British scientists (and a German), named by Hunt as "the Maxwellians", the task of developing and establishing what we now know as "Maxwell's theory", represented by the four equations of the electromagnetic field. See Hunt 1991, 2015.

tion and reduction of the results of previous investigation to a form in wich the mind can grasp them. The results of this simplification may take the form os a purely mathematical formula or of a physical hypothesis. In the first case, we entirely lose sight of the phenomena to be explained; and though we may trace the consequences of given laws, we can never obtain more extended views of the connexions of the subject. If on the other hand, we adopt a physical hypothesis, we see the phenomena only through a medium, and are liable to that blindness to facts and rashness in assumption which a partial explanation encourages. We must therefore discover some method of investigation which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on the physical science from which that conception is borrowed, so that it is neither drawn aside from object in pursuit of analytical subtleties, nor carried beyond the truth by a favorite hypothesis.

In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies...".

These are part of the first words of James Clerk Maxwell (1831-1879), in a paper published in 1855, the first of a series on the so called *electrical science*, in which he presented his view on analogy, together with some physical analogies he had identified: the laws of numbers, the similarity between corpuscular and wave theories of light.[4] Along de decades of 1840-60, a new heuristic for the electric and magnetic phenomena was in construction, when ambiguities in the orthodox view on the idea of *action at a distance* (AAD) were about to be questioned.[5] A reinterpretation of AAD as a field mathematics was underway, starting with an article by William Thomson, from the previous year (1854), in which he presented his flow analogy for electricity in a theory for the Atlantic telegraph cable project,[6] inspired in Fourier's theory of heat flow,[7] that would be the first to connect electrically the old (Europe) and the new World (America).

" ...Passing from the most universal of all analogies to a very partial one",[8] analogies would play, as "substitute realities",[9] an important role to Victorian's ideal of science. If analogical counterparts, in sense of differential equations, were a desirable way to validate a theory based on facts, according to the continental school (French, German),[10] on the other side of the English Channel, we had the "true factory" of Pierre Duhem's metaphor,[11] with pulleys, tubes, jams, strings and springs as valuable epistemic tools, with no fear of the *invisible*. In a context inherited from the long tradition of western philo-

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[4] Maxwell 1855.

[5] Frické 1982.

[6] Thomson 1854b.

[7] Fourier 1822.

[8] Maxwell 1855.

[9] Gombrich 1985.

[10] Kargon 1969; Wise 1981.

[11] Duhem 1954.

sophical thinking, the ethereal element could take on different formulations, on demand.

Maxwell believed that his method avoided adherence to a “concept of reality”, and the distant source fluids could be “quietly forgotten” to give rise to differential equations as the expression of Michael Faraday’s conceptions. However, different from what has been suggested,[12] this “forgetting process” was not intentional.[13] Indeed, drawing up a physical analogy meant becoming familiar with certain field of knowledge, but only to the point of being able to describe it through its own language, as in Hertz’s statement, but it is important to remember that, together with the system of equations of Victorian Physics, there was also a Victorian system of analogies.

In this thesis, it is sought the role of analogies in the history of physics and in the establishment of electrical engineering, and is argued that they constitute a relevant part, playing a role as important as the logical system in the structuring knowledge about electrical phenomena. Considering that significant achievements in the history of electricity and magnetism occurred not just in the last three centuries, it is possible that the dynamical nature of many discoveries is still lost in time, reason that makes us imagine if the undoubtedly brilliant minds behind the “ground zero” of electrical engineering, with the 19th century’s telegraphic technology, had their mirrors turned to a more distant past, now quietly forgotten.[14]

Analogies are part of what was conventionally called inductive inference, being much more than mere associations between words or concepts between different domains. They were present in the first models of ancient philosophy, but also in the first mathematical models and large-scale technological applications for electricity, in the 19th century. The attempt will be to show that physics and related sciences cannot be isolated with regard to the epistemic construction that is at the basis of Electrical Engineering, as a discipline, and it is believed that a real appreciation of the difficulty of this problem may change the conception of the role of each science in describing and modeling natural phenomenona. If 96% of the universe is still unknown, intellectual humility is a beacon signalizing that the current tools we have are not enough to fully understand any phenomenon. But if these limitations are recognized, paraphrasing Thomas Nagel,[15] it will be possible, eventually, to discover (or rediscover) new forms of scientific understanding; not the limits of what is really known, but of what can or cannot be known according to certain existing methods.

Lets us then break the first question into some parts: before the firsts physical or mathematical models for what would be called the electrical flow of current, in 19th century, it is necessary to glimpse what comes to be a analogical model *per se* and under what circumstances its basic categories can be elaborated. If understood as a projection of

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[12] Hon and Goldstein 2012.

[13] Tyler 2011.

[14] To the *longue durée* method of historical research, see Braudel and Mathews 1982.

[15] See Nagel 2012.

one domain over another, a model is something that reduces, in some way, the dimension of the entity under the magnifying glass, like a hologram,a special type of "photograph" that generates the perception of a 3D image when illuminated in a certain angle, a way of compressing information so that it can be stored and transmitted. In a way, this thought brings us to the shadow that entertain the cave dweller in the famous allegory of Plato, when he elaborates his theory of knowledge.

Plato, who would have lived between the 5th and 4th centuries before the Christian era, narrates that a certain group of people were captive to a recess of the earth, a cave, in which an opening let in a simple beam of light from a fire hidden behind a hill.[16] They were chained, staring at the nearby wall, unable to move, as they were totally immobilized. The only thing they could see was shadows cast on the wall ahead. As time went by, everyone got used to that situation, concluding that the shadows had a life of their own and should therefore be the objective reality of the world. Their belief was reinforced by personal and sensory experience, when they saw, every day, some shadows moving silently, others making strange sounds, others still standing, without ever being aware of a world beyond that.

Even if a member of the unfortunate group was suddenly elevated to the condition of a free man, walking on the surface, which would add a spatial dimension to his sight, the sculpted mental conditioning for several years would never allow him to fully understand what was happening. Stunned, the newly freed man would continue to look for patterns in the shadows projected by passers-by on the ground, clinging to the possibility that they were still the best way to answer the myriad questions that would follow, the best way to describe the world. Would his measuring instrument be his own capacity for understanding? If he did not assume an attitude of complete denial about what he saw, he would have difficulties in expressing, to the companions who stayed, what the reality is like beyond. In another dialogue, Socrates is confronted by a young sophist, who said "... the one who knows feels what he knows ...", as if to say that only sensations would be instruments for knowing reality. The master argues that, although the feeling of something that "exists"[17] is a way of *approaching the desired thing*, there is a risk of welcoming the sensation as the "thing" itself. So it is necessary to know how to make understandable something apparently out of control. How could anyone know anything at all?

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[16] In this excerpt from *The Republic*, Plato reports the dialogue between Socrates ($\approx 470 - 399\,BC$), who had been Plato's master, Glauco and Adimanto, who were his older brothers. Two other Socrates disciples are also known who have achieved a certain notoriety in history: Xenophon and Aristophanes. Although it is a well-known narrative, it is interesting to analyze it from this keyword, that a model is, in essence, a projection of one domain over another. See in Plato 2006, p. 210.

[17] Plato 2011b, Book vii.

2.1 The epistemic perspective

Every knowledge constitutes, simultaneously, a translation and a reconstruction, using symbolic tools in the form of representations, models, ideas, theories, essays.[18] For the ancient philosopher, a possible starting point to learn anything would be the attributes of the entity of interest, as many as he could identify, such as the suction capacity of a fluid, the power of attraction of fire, the emanated virtue from the magnet, the good or evil emitted by the look. In short, categorization, comparison, inference. However, as more attention is usually paid to words than things, as Descartes would say,[19] it is customary to accept terms not very well understood, considering the facts given itself, as the notion of natural phenomenon expressed through certain *categories*. An attempt will be made to show how the implicit or inattentive notion of categorizing a phenomenon can convey a limited notion of the reality sought, and this is directly linked to the "epistemic pillar" on which a theory is built. For this, it is necessary to make an abstraction of concepts received in a more or less thoughtless way, until they are examined again in the light of the renewed idea.

Consider, for example, a musical performance. When someone starts listening intently to an orchestra, he or she may have the clear notion that certain high or low notes, compasses with a slower or more vibrant tone cause different impressions and emotions in the audience. The performance of the piece can be considered as a physical phenomenon – in which the propagation of sound waves is the main element – and the epistemological matrix that seeks to understand what is between the orchestra and the audience can be called a physical theory. From what is produced, it is usually considered aspects of actuation, such as the movement of sound waves, changes in their amplitude or frequency, or what information was transmitted. All of this in the form of a theoretical representation expressed in a certain language, such as Mathematics. But how is it possible to objectively locate a phenomenon like this?

Unless the human element is considered a simple machine devoid of real intelligence, as an ultra-sophisticated diaphragm, other elements must be part of the involved phenomenology. After all, the spectator not only hears the sounds, but also sees the orchestra and is touched by its performance. Psychoacoustics could be a another attempt to model the problem, by describing some variables present and, in a way, also be explanatory with regard to the different emotions felt by the audience, in their psychological aspect, which are certainly part of the equation. However, even though it deserves the greatest consideration and respect, there must be a limit in its focus, because, although the cognitive aspects are not disregarded, the approach is essentially physiological, mechanical. And in a mechanical reality a phenomenon will be nothing but a mechanism to

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[18] See, e.g., the epistemic theory of complexity of Edgar Morin, in Morin 2018.

[19] Descartes 1983, p. 56.

be decoded.

The scientific world-view is based on premises that mostly rely on classical physics, where *nature* and *matter* are almost synonymous.[20] This correlates with the belief that complex phenomena can be reduced to their fundamental interactions, such as subatomic particles, or basic units like a neuron or gene, that engender more complex mechanisms. Such conceptions were highly successful in establishing not only the current point of human knowledge about nature, but also the great technological advances in contemporary society. Modern evolutionary theory aims to provide an overview of how the existence and development of life could be a consequence of particle physics equations, which could ultimately include an explanation for the functioning of the brain and cognitive abilities that enable human beings to make their discoveries.[21]

But today's scientists, as remarked by James Gleick, may not need particles so much, as physics itself is dominated by another intellectual model.[22] Although Claude Shannon was not thinking about fundamental physics when he elaborated his mathematical theory of communication, after him science wonders if the *bit* would not be the fundamental unit of the universe, a particle of another kind, without substance, more elementary than the matter itself.[23] In this light, the laws of physics would be no more than mere algorithms, based on this primordial "matter", the *information*. A relatively recent example of a theory based on these premises, in physics, can be found in the early 1970s, when Wheeler and Christodoulou (then Wheeler's graduate student) demonstrated that the total event horizon area of a black hole (the point of no return) did not decrease, which led Jacob Bekkenstein, another of Wheeler's students, to conjecture that the maximum entropy of any system, including a black hole, could be proportional to the event horizon area, a generalization of the second law of thermodynamics, which allowed him to calculate the maximum entropy of any system.

In the 1980s, Gerardus't Hooft tried to solve Stephen Hawking's *information paradox*, which emerged after his second major discovery, that black holes "evaporated", i.e., emitted radiation. But where did the entropy that entered the event horizon of a black hole go, since it could not get out of it because it required a supraluminal speed? Hooft suggested that the problem could be described from a physics with a smaller dimension, which led Leonard Susskind, in the early nineties, to demonstrate that the maximum amount of entropy in a region of space was proportional to the area of that region, instead of its volume, as one would expect.[24] This limit was called *holographic frontier*. In

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[20] For "problems" with classical physics in this context, see, for example, Schwartz et al. 2005, p. 1314.

[21] Nagel 2012, p. 20.

[22] Gleick 2013.

[23] Shannon 1948, 1938.

[24] Hooft won the Nobel Prize in physics in 1999, with Martinus JG Veltman, for his work with the quantum structure of electroweak interactions. In the 1980s, Hooft tried to solve Hawking's *information paradox*, where he had proposed that quantum fields near a black hole could be described from a theory with

other words, Susskind argued that an analogy for this “visual magic” could,[25] if true, be applied to the complete physical description of any system that occupies a 3D region, as if the visible universe could be described as a large hologram, “painted” on a vast and distant boundary.

This idea led Bekkenstein, in the 2000s, to propose another hypothesis, based on an analogy between entropy and information, to demonstrate that it would be possible not only to describe information in a given region, but also calculate its quantity in *bits*. To explain this, he used another physical analogy: if, for example, someone wants to know how much *information* exists in a bathtub containing water – i.e., the position and speed of each atom individually – it would be possible to leave the bathtub empty and gradually add drop by drop of water. Naturally, it is concluded that the total information is proportional to the volume of water in the bathtub. Now replace the word “bathtub” by “black hole”. In this analogy, a drop of water is equivalent to an elementary particle, such as a photon or any other particle, and entropy would be measured in separate units, such as particles or *bits* of information. Considering someone “filling the bathtub”, i.e., increasing the volume of a black hole by gradually adding such particles, one would still expect the same result, that the total information is proportional to the volume.

What he demonstrated, however, was that the natural expectation is not satisfied, which is to say the maximum amount of bits of “hidden” information depends not on its volume, but on its surface area, which is equal to the area of the event horizon – the so called “point of no return” – which is approximately one unit of Planck.[26,27] This and other analogies helped him, in some respect, to elaborate the conjecture: “... The amount of information in a region of space is proportional to the area of that region”. These analogies form an illustrative example on how the epistemic perspective is important in formulation of a physical theory, even today.

This link or epistemic operator can be seen in other formats in different disciplines, as an analogy between *digital computer* and *brain machinery*, in which the possible physical laws that governing the mental behaviour are no more than result of a complex computation, as in Dawkins’ *Selfish Gene*, which presents the human being as a clumsy and

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a lower dimension. See in Stephens et al. 1994; This led Leonard Susskind to propose a conjectural solution in 1995, which forms part of his *Holographic Principle* theory. See at Susskind 1995.

[25] Susskind 1995, p. 3.

[26] Bekenstein 2007, p. 71.

[27] The natural or Planck units form a system created in 1899 by Max Planck, which consists of measuring the diverse fundamental magnitudes of nature through a series of universal “constants”. That is, time, length, mass, electrical charge, temperature, etc. are expressed from the speed of light c (with dimension L/T), from the gravitation constant G (with dimension L^3/T^2M), from the constant of Coulomb force $\frac{1}{4\pi\epsilon_0}$ (with dimension ML^3/Q^2T^2), of the Boltzman constant k (with dimension ML^3/T^2K) and, finally, the reduced Planck constant $\hbar = \frac{h}{2\pi}$ (with dimension ML^2/T). The system is defined by making these five quantities to assume unitary values. Thus, the area of the event horizon is something close to $1A = \frac{G}{hc^3} \approx 10^{-66} cm^2$, which is a very small area.

genetically programmed robot.[28]On the opposite side, upon another epistemic pillar, the cerebral machinery could be less a processor with artificial intelligence than some kind of TV receiver, i.e., something that could function as a vehicle for the expression of thought, a kind of *transducer* for the mind, or a model on which cerebral biology could organize itself.[29] In light of this perspective, the mind could not necessarily be limited to the skull,[30] with a myriad of new analogies, with which another group of scientists claim to be part of a wider spectrum of life.[31],[32],[33]

So far in the above examples, even under the same general analogy or metaphor – *brain-computer* or *matter-information* – there are more conflicts between different conceptions instead of a common language that could unite useful knowledge in both perspectives. The first essentially sees the brain (or matter) as a mechanism that generates thought (or information) *versus* the second, which sees the matter as an organ of expression of thought, just as the violin is for the violinist or a software running on a computer. Raising the level of complexity, an even more fundamental question can be present. In the first perspective, the aim is to use fundamental elements in the categorization of a phenomenon, thus structuring a theory as a general mechanism, as a *bottom-up* approach. Hence the analogy of the brain as a computer, and the mind as a *software* running on this *machine*, or the 3D space as an hologram. The second perspective intends to use psychobiophysical or even cultural elements in order to elucidate some of the same problems, but starting from a *top-down* or holistic approach. What do the apparent contradictions suggest?

When an explanation is accepted from the same set of evidence to the detriment of another, as it is simpler and with less need to multiply entities, it is not always an aesthetic preference, but it is thought that the explanation that provides a better understanding is

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[28] In his book, *The Selfish Gene*, Richard Dawkins presents the human being as a sophisticated envelope created by genes in order to provide their own replication. See in Dawkins 1976; For a discussion of Dawkins' work, see de Franco 2014.

[29] The first and maybe the most notorious in-depth scientific study, carried out over two decades, which defies the purely physiological explanation for conscious experiences can be seen in van Lommel et al. 2001.

[30] Some authors argue that, at the moment when the brainstem reflexes disappear, conscious experiences could not be felt through the regular channels or would be in so deep layers that they would be ineffective, which defies the traditional hypothesis of *cerebral anoxia* as an explanation. They therefore propose that conscious experiences can occur due to a non-local nature of the mind, as can be seen in Trent-Von Haesler and Beauregard 2013.

[31] See, e.g., Beauregard et al. 2018.

[32] As the neurobiological basis of the brain is still unknown, and is currently the subject of intense debate, bolder proposals are still viewed with caution, due to their controversial potential. See, e.g., Martial et al. 2019.

[33] The idea of thinking "out of the head" has historical roots (van Eck and de Jong 2016, p. 14). For example, before the eyeball was considered a passive element in phenomena of sight, with the introduction of the "darkroom" analogy by Johannes Kepler (Kepler 1604) the prevailing idea was that the *optical pneuma* (a mixture of fire and air in some philosophical traditions) was somehow communicated from the seat of consciousness (*psyche*), passing through the optic nerve and reaching the eyes, being then "emanated". In treatises of medical philosophy, in Epicurean emanation doctrines (Carus 1985) as well as in Plato's writings (Plato 2013), is also possible to observe variations of the same notions.

more likely to be right.[34] The philosophical principle behind this was proposed in order to describe non-demonstrative inferences, i.e., in cases where evidence and theory are not deductively linked.[35] The point is that often the choice that provides a better understanding is not always the one that best fits or is the most "probable" to be true, but sometimes a reflection of the expectations of the formulator.

The organization of knowledge comprises, to a larger extent, operations of connection (union, inclusion, induction) and separation (comparison, opposition, selection, exclusion), in a continuous and recursive process.[36] As it is well known, in history of science, separation and analysis have been privileged over connection and synthesis. If there is a mismatch between different branches in human knowledge, notably separated, fragmented, compartmentalized in disciplines, there are also, at the other end of the spectrum, increasingly multidimensional and multidisciplinary problems that have been challenging the way of connecting local knowledge.[37] The disciplinary developments in the sciences, despite its countless benefits, have also brought all the inconvenient attachments, or collateral effects of overspecialization, as a kind of confinement of knowledge itself.

The case of the orchestra as a phenomenon to be studied would gain several potential contours. That is, the phenomenon experienced by the audience could be better understood by adding the listeners' complex perception, in all its magnitude: sight, hearing, social interactions,[38] cultural and spiritual,[39] taking into account a mix of physical, biological, psycho-social, anthropological phenomena ... all part of what each listener will understand as the totality of the phenomenon, ones "particular reality" of the cognitive process. If there were scientists in that environment,[40] the future models that they could develop would also be affected by their epistemic perspective.

What, then, would be a "good model" for electricity? In our real problem, returning to the initial question, it is possible that to create a physical model, perhaps only physics will not be sufficient. Despite the popular perception of science and scientists, as hermits inside a laboratory, with a blackboard with intricate equations in the background, as if science were a separate department from nature, exclusively logical, deductive, the reality

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[34] Nagel 2012, p. 17.

[35] Roughly speaking, the inference to the best explanation (IBE) is the distinction made between the most "pleasant" explanation – with potential to provide a "greater understanding" – and that which would be the explanation most "likely". The idea is a particular case of the concept of principle of sufficient reason, formulated by Leibniz, that all things are intelligible at some level, and if something appears to be occasional or arbitrary, it is because it is not yet known enough, which would explain why things are not arbitrary. See Lyons 2006.

[36] Morin 1992.

[37] According to Edgar Morin, the specialization closed in itself, which does not allow integration into a global problematic or conception of the whole of the object in which it considers only one aspect or a part, prevents not only that the global (which has been fragmented) from being seen, but also the essential (which has been diluted). See Morin 2018.

[38] See, for example, Krueger 2011.

[39] The word "spiritual" here refers to an existential human search. See, for example, Gallagher 2013.

[40] See, for example, Fuller 2016.

may be slightly different. If the history of science is not considered as a linear method to find the "correct answer", it will be possible, at times, to find situations in which theories were built not only on the solid bases of deduction and logic, but from other connections, even if fluid and subtle ones, between different domains, indicating a type of similarity that often enbles a better understanding of a phenomenon.

What do a cosmic web, a magnetic field line and a computer brain have in common? What is the relationship between heat flow and electric current, or between an electron and a wave, or between a harmonic series and quantum matrices? "... As our way of knowledge disengages objects from each other," as Edgar Morin would say,[41] we need to glimpse what unites the objects themselves. Regardless of how (or why) the human mind accomplishes this feat, this similarity is the act of learning in a deeper way, discovering through an *Analogy*.

2.2 Analogy and categorization

The spontaneous categorization made by the human mind is quite different and more complex than simply placing each thing in a specific "shelf". Alfred Noth Whitehead,[42] in his *The concept of Nature* (1919),[43] proposed that the relationship between matter (brain) and thought (mind) could be temporal in nature, in which the mind would be correlated to the brain by through a past-future relationship, as if these were two poles of the same magnet. The human mind would exist as a possibility in the brain, just as the future exists only as a possibility before a die is cast. Whitehead opposed cartesian dualism and conceived the reality of mind as a process, which evolves over time, a concept very close to the idea of a traveling wave.

It is interesting to note that some linguistics studies point to a similar notion to Whitehead's view.[44] In their *Surfaces and essences: analogy as the fuel and fire of thinking*, Hofstader and Sander claim that a category is, precisely, a dynamic structure that evolves over time, that allows, under certain circumstances, to be accessed.[45] When the word "category" is used, therefore, reference is made to a category "in someone's mind" – whether outside or inside the skull – which is opposed to the simple act of mechanically labeling any entity, as is normally done in hierarchies of taxonomies of botany (or astronomy) – with its kingdoms, classes, families and species, or as mere rules for mathematical operations. In other words, categorizing is different from classifying. If categorizations are considered to

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[41] Morin 2018, p. 24.

[42] Whitehead wrote with his student Bertrand Russel one of the greatest works of mathematical logic of the 20th century, *Principia Mathematica*. Whitehead and Russel 1963.

[43] Whitehead 1919; See also Whitehead 1929.

[44] See, for example Fuller 2016.

[45] See Hofstader and Sander 2013, pp. 13-15.

be a central element in human cognition, in all its fluidity, as their central hypothesis, the analogy is certainly its vehicle.[46]

But the very act of making analogies also suffers from the same simplistic stereotypes. According to the same study, the first stereotype is the understanding of "analogy" as a name for a particular class of sentences or mathematical associations. For example, the sentence: "day is for blue as night is for black" is one of them. In a stranger, quasi-formal notation, the sentence could be written as

$$\text{Day} : \text{Blue} \sim \text{Night} : \text{Black}\,, \tag{2.1}$$

or even as a pseudo-code:

$$\begin{aligned} &\textit{if}\quad (\text{Day})\ \textit{then}\ (\text{Blue})\ \textit{otherwise} \\ &\textit{if}\quad (\text{Night})\ \textit{then}\ (\text{Black})\ . \end{aligned} \tag{2.2}$$

Such expressions can be called *proportional analogies* (2.1) or *conditional* (2.2), a term equally based on an analogy between words and numbers, or the idea of that a numeric pair must have the same ratio as another pair, such as

$$\frac{D}{B_1} = \frac{N}{B_2}\,, \tag{2.3}$$

besides the assumption that conceptual entities can be transported to the numerical domain. In broader terms, it would then be possible to generalize a proportional analogy such as:

$$\text{Proportion} : \text{Quantities} \sim \text{Analogies} : \text{Concepts}\,. \tag{2.4}$$

The model, formed by a structure of four expressions or lexical entities,[47] is still taken seriously and has a close relationship with the syllogisms of ancient Greeks, such as Plato and Aristotle.[48] The structure of a proportional analogy like this would be formalized by Aristotle – although it was already used well before him – and is in the same domain as the classical rules of formal or logical reasoning, in particular deduction and induction. Thus, the fact that one usually thinks of *analogy* – the word itself – in this strict sense, has its origins back in time. However, this narrow view regarding the ability to make analogies leads to the almost inevitable conclusion that "analogical reasoning" is a type of narrow and linear mental activity, like a *Sherlock Holmes'* work, the immortalized character by Arthur Conan Doyle.[49] The impression created is that this supposed reasoning is a useful method, but in a very limited number of cases, as in a "test of intelligence".

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[46] See also Hofstader and Sander 2013, p. 20.

[47] Becker 1975; See also Hofstader and Sander 2013, p. 15.

[48] See an example formulated by Plato on page 50. This subject will be explored in section 3.2, which deals with numerical atomism of the Pythagoreans.

[49] See, for example, Nersessian and Chandrasekharan 2009.

However, the analogy considered as a natural and central element of human thought is not limited to these cases. In a syllogism, for example, the analogy is designed to provide *one, and only one* correct answer, as in the ramblings of a child from the interior of Minas Gerais state, in Brazil, about his morning snack: "... Hot milk is for cold days; today is cold, so I must drink hot milk, *uai*!" As much as these types of analogies are vital to our daily lives, there will be hardly a right answer for each one of them. It is like saying that the expression "climate of the Brazilian London" could lead to answers such as "Ouro Preto" or "São Paulo". That is, what is considered to be a striking feature of the city of "London" – in this case, the (bad) wheather – is clearly dependent on the cultural panorama. Although beautiful, the images provided by proportional analogies can lead the incautious to a misleading view regarding the process of elaborating analogies as a cognitive phenomenon, especially in the investigation of the so-called scientific innovations or in the evolution of human thought.

Another connotation for the act of making analogies consists in a type os "quasi-inspiration", a highly sophisticated form of reasoning, wich somehow has the ability to connect distant domains of knowledge. This kind of vision may create the idea of analogy as the product of a genial mind, as in those television documentaries such as "Newton Decoded". This conclusion can be both subtle and also tempting.[50] The idea of a good analogy only as a result of an unusual genius is supported, in fact, by several occurrences throughout the history of science, some of them presented along the scope of this research, as an elastic line to represent the action of a force, the sensory analogy in formulation of the principle of action at a distance, or in the equation between heat flow and electricity in a telegraphic engineering application. Starting from a perspective like this would be to admit that making analogies is an activity "for a few", extremely creative souls, responsible for probing the mysteries between universes never dreamed of and presenting them to us, mere mortals.

If this type of analogy is considered common to human thought,[51] even though, at first, the task seems to be in charge of a small class of privileged thinkers, the idea of analogy as a deep and sophisticated reasoning will be also applicable to situations closer to everyday life, as well as a method of scientific investigation.[52] For example, teaching activities are universally accepted as a fertile ground for analogies. Any engineering student will quickly recall the vision of an electrical circuit as the flow of a fluid, and will be able to model a thermal or a linear control system from a specific symbology, with the same electrical elements, by the way, but now with a higher level of abstraction than just the flow analogy. In general, these analogies use symbolic representations through "real" com-

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[50] Hofstader and Sander 2013, For this, see the preface of.

[51] As in Hosfstader's central thesis. In this regard, it is worth checking out a master class at Stanford University given by Leonard Hofstader in 2009, in Hofstader 2009.

[52] See, for example, Fuller 2016.

ponents, such as capacitors, resistors, or even “imaginary”, such as the so-called complex impedance method, common in the area of control engineering.

It is not necessary to explain each of these terms, but to realize that such mnemonic resources appear to connect domains in a more “technical” guise, but they will not be a target on this thesis. Furthermore, it is not intended to make reference to those analogies established simply *by definition*, that go back to an almost teleological (or even theological) conception of a static and immutable universe, such as “electricity is a property of gauge symmetry” or “energy is a property of the universe, constant and independent of the process”. Also disregarding the *naive analogies*, such as the spontaneous generation “theories” of medieval age,[53] proportional analogies, although considered sometimes, are so formally restricted that will not be widely explored. The second type, however, that will be called here *physical analogies*, like the one that William Thomson would do with his view on electricity and heat, in the problem of submarine telegraphy, forms a richer class for exploration, a profound method of investigation that allows to shed light on new things, belonging to other domains.

Inter-domain Physical analogies are, therefore, a path of interest. Understanding such a cognitive process, especially from a historical and philosophical point of view, may even allow an integration between areas of research today treated separately, such as epistemology, cognition, linguistics, mental imagery and conceptual change, philosophy of mind, theoretical physics, education and engineering.[54] For now, it is worth saying that the fundamental language of an analogy is almost always metaphorical, i.e., as if saying that a is b, without having to know whether a and b can be treated as equivalent. Even though it is sometimes confused with a hypothesis, an analogy is not intended to be one. For this reason, it can never be *proven* or *falsified*, having the usual conception of scientific knowledge as proven knowledge, demonstrated from a certain criterion, or not falsified, in which ineffectiveness has not yet been proven.

As in a given language, an analogy is used so that the assimilation of information is carried out at a deeper level of consciousness. That is, the structuring of an idea and its mirroring takes place via analogy.[55] Furthermore, the continued use of analogies with the aim of making practitioners more familiar with a certain language – such as a “grammar” – causes their own fading, due to their own dynamic nature, as the theory gains consistency. For example, in the 19th century it was understood that geometric counterparts would be the best way to attest to a theory based on empirical observation, such as Fourier’s “mathematization” of heat, or William Thomson’s equation for electricity and heat in an electrical telegraphy problem, which defined the so-called electric and magnetic fields through the continuity equation for flow systems, obtaining a geometric description for

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[53] See, for example, Menocchio’s picturesque theory on page 59.

[54] See, for example, Nersessian and Chandrasekharan 2009, p. 179.

[55] For this, see Bar 2007.

the motion of an electric current.

With the naturalization or appropriation of a certain language, an atom loses its meaning of “impenetrable ball”, to gradually assume a deeper concept, such as “singularity”. A “Faraday tube” may give way to “lines of force or field”, or lose its primitive association with the idea of fluid, to assume an even more tenuous, or even abstract, nature. This does not mean, however, that models supported in certain categories are abandoned because they present contradictions with the “previous ones”. The way of looking at the mode of action at a distance, for example, from the concept of central force, coexists “peacefully” with field theories, from classical electromagnetism to the standard model of particles. Contradictory theories and analogies are recurrently used by the scientist in his reasoning, whether intentionally or not, even for convenience.

In a broader spectrum, however, ways of looking at the same set of phenomena can be, and often are, disregarded, as when one sees an electromagnetic effect as purely dominant in its “particle” or “wave” character.[56] For example, when the wave theory of light “had to be abandoned” until it was reworked later, it had left strong impressions on a very special type of mathematical language that was, until then, used, and which goes back to the equations of the continuous wave motion in fluids, the hydrodynamics. This language carried, at its core, the relations of the physical scenario of that time, which would be revived by field theories. And as it is proposed to study the physical analogies in the history of electricity, it is desirable to take a more extended look at the formation of these concepts.

2.3 The aim of a physical theory

Physically, the notion expressed in the allegory of the cave may symbolize that the observation of phenomena does not put the observer in direct contact with the reality hidden under the veil of a “sensitive appearance”, but only with the appearance itself, according to the concept of Pierre Duhem, who was one of the leading thinkers of the twentieth century to address the issue.[57] That is, the measurable part of a given entity is nothing more than a small shadow cast on the wall. Although it is allowed to envision such appearances in a concrete way, a physical theory to describe it, although coherent, would not be complete or sufficient, since it leaves aside conditions that may be essential for the understanding of that phenomenon in its totality. A drop of dew on a leaf, if illuminated by sunlight, may look like a small diamond. If it falls on a pile of dust, it will form no more than a dark drop of mud. How to disregard the whole variety of colors, shapes, smells and nuances that escape the more attentive and systematic look of the “experimental control”?

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[56] But, in a way, it is always desirable (but not mandatory) that the models have a mathematical form, such as a differential equation or geometric counterpart, with testable consequences.

[57] Duhem 1954.

What is considered an important variable in the description of a given phenomenon may not be sufficient to elucidate it. Although the notion of physical theory can be seen as a pragmatic expression of intellectual elaboration, something close to Ernst Mach's (1838-1916)[58] "economy of thought" – in which Ockham's razor is a valuable assist in choosing simpler paths – it is noticeable that the universe will never fit into a set of mathematical equations, empirical laws or philosophical essay. To an orchestra, would it be possible to define an expression for the affection of each listener in relation to a certain movement in the musical piece?

This does not invalidate, however, the search for an economy of thought in the practical understanding of a phenomenon. The word *phenomenon* was used in 1786 by the german philosopher Immanuel Kant (1724-1804) in a way not far from etymological expression, as something that "strikes the eye" and is felt before any intellectual elaboration.[59] In his early writings, Kant made an attempt to establish a metaphysical basis for Isaac Newton's Theory of Universal Gravitation (1642-1727), and which, at least in the beginning, he seemed to accept without question. Roughly speaking, it can be said that Kant sought, here and there, to avoid initial hypotheses for the enigma of the creation of the universe, but he defined his cosmogony with principles derived from the great English scientist, especially his concepts of attractive and repulsive forces, which he believed geometric principles, derived from indisputable observations.

In later and more successful works, Kant distinguishes from what he considered the limits between natural sciences and metaphysics, merely because of the empirical aspect that he relegated to natural science. That is, tries to distinguish empirical knowledge or *a posteriori* – which represents what can be obtained from an experiment and cannot be separated from the impression of the senses – from knowledge that is universal, necessary, that does not requires the senses, what he called pure or *a priori*.[60] His metaphysics would then consider *a priori* principles such as space, time, substance and causality, without which it would be impossible to obtain any knowledge, being absolute and independent of any experiment. The rest would be the object of natural science.

However, certain elements were left out of the "pure side" of the scale, especially the idea of *matter*, a platform that would constitute one of the elementary principles of physics. For Kant, mobility was, in itself, a property of matter, which due to the its dynamic aspect could suffer the action of external forces, being continuous (i.e. it fills the entire space) and impenetrable in its essence.[61] The repulsion force itself was nothing less than matter itself, in its dynamic aspect, and the very three-dimensionality of space would have been caused by this inherent characteristic. In this, moreover, a fragrance of Ionian

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[58] Mach 1882.

[59] Kant 2017.

[60] Kant 1999, p. 7.

[61] Some of these elements were already exposed in his first works. See, for example, Kant 2017, §480.

pythagoreanism can be noted, due to the importance given to geometric relations in the formation of elements and substances.

The classical Greek tradition is certainly at the basis of the idea of cyclical time, regeneration, behavioral or habit-making law. It formed an implicit basis for contemporary theories, such as the evolutionary theory of species, being also present in the consequences of quantum theory, when the influence of observation on the result of an experiment is presented at some level. However, the idea of *natural law* in Kant's writings immediately takes us back to the Jewish tradition of eternal and immutable laws, which defined not only the *time arrow* of classical physics, but also the almost irresistible tendency of science, from different eras, to regard the universe as something static. Even though considered in its legal connotation, establishing a *law* should not be seen as the search for something literally "written" in a circumscribed region of the universe. But that was exactly what later traditions sought to do: finding phenomenal laws – despite their purpose – was one of the great pillars of Augusto Comte's french *positivism*, which continues to exert great influence on western scientific thought.

Considered a kind of epistemological realism based on the hypothetical-deductive method, positivism flourished in the midst of the nineteenth-century industrial atmosphere, prevailing in almost all branches of scientific activity and lasting until contemporary times, although under different denominations, such as Karl Popper's *falsificationism* :[62] a strong philosophical current that nurtured true dread of any metaphysical conception to understand the world. According to this current, any statement that could not be rigorously falsified should be dismissed as pseudoscience, superstition or psychologism, and the progress of humanity would depend exclusively on scientific knowledge, becoming one of the foundations of the so-called *scientism*. Popper never saw himself as a positivist,[63] although his theory is closer to positivism than to the deductive tradition of philosophy.

A problem usually attributed to contemporary falsificationism, by its very nature of demarcation, is precisely the propensity to inhibit and restrict the free questioning that is at the core of the scientific spirit. Evidently, controversies aside, not all knowledge that can be pursued should be pursued, and not everything that worth studying will fit into a "hypothesis test", or any other logical system of inference. A common criticism of the method, especially, is its non-recursive character, i.e., it cannot be applied to itself. Since it cannot be falsified, it should, in theory, be located in the field of pseudosciences. To this criticism Popper justified that the principle is not recursive because it is a *metatheory*, that is, a "theory of theories", which does not require justification. In any case, this opens a gap in the principle of falsifiability, and there may be circumstances in which it is not applicable, not being, therefore, universal, according to the theorems of the incompleteness of Gödel.

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[62] Popper 2005.

[63] See, e.g., Popper 1970.

Jean-Baptiste Joseph Fourier, in the preface to his 1822 *Théorie Analytique de la Chaleur*,[64] makes clear the point, stipulating that his intention was not to seek causes, nor to formulate hypotheses about the nature of heat – as their predecessors (Laplace, Poisson, Cauchy etc.) did, having as their basic motto for any dynamic theory the hypothesis of the existence of forces exchanged by molecular configurations – but laws, obtained from the observation of "facts" and experimentation: "... primary causes are unknown to us, but they are subject to simple and constant laws, which must be discovered by observation, being objects of study of natural philosophy".[65]

Fourier's concern was less based on discovering greater "truths" than make efforts to find "... the temperature of the skies, the planets, the infinite spaces ".[66] It did not matter whether the transmission was a fluid or any other way in which the movement was communicated. His model of reality consisted basically of reducing a problem to a mathematical equation and solving it "dryly", without hypothesis. The argument was that only macroscopic properties, and therefore measurable ones, should be considered. This was what the supporters of the analytical current understood as *Mathematical Theory*, whether for the heat or another entity. Comte himself would say that "... Mr. Fourier's beautiful series of research on the theory of heat offers us the most sensible verification of the preceding general observations ....",[67] showing the perfect integration of the theory of heat with the *positive* character of his philosophy.

Nonetheless, the systematization of physical laws has been useful in enabling the manipulation of sensible appearances, which, although exist in a domain not directly accessible, they allow, in the last degree, to form a theoretical set that the science of each time understands as a model. Because it is obtained from a well-defined process, such as the mechanics of a particle or by the vibration of a string in a musical instrument, a model will provide a context in which a particular theory can be tested, even though no theory can be tested in isolation. What is being tested is, according to the idea, a "set of beliefs",[68] designed "to save the phenomenon".[69] If one considers, for example, the mechanical models of Victorian science, it is possible to see that they were not necessarily intended to be literal descriptions of reality, but had a clear intention of making a theory understandable and verifiable. For a Victorian scientist, a pictorial resource had more to do with a mnemonic aspect than an intention to reveal a given phenomenon in depth. This

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[64] Fourier 1822.

[65] "... Les Causes primordiales ne nous sont point conues; mais elles sont assujetties à des lois simples et constantes, que l'on peut découvrir par l'observation, et dont l'étude est l'object de la philosophie neturelle". See in Fourier 1822.

[66] Fourier 1878.

[67] "... la belle série de recherches de M. Fourier sur la théorie de la chaleur. Elle nous offre la vérification très-sensible des remarques générales précédentes. En effet, dans ce travail, dont le càractère philosophique est si éminemment positif ...". See in Comte 1830, p. 17.

[68] Moles 1995.

[69] Duhem 1954.

tradition was, in a way, opposed to the continental school, particularly French, which had a greater appetite for strictly well-defined principles, typical of the Cartesian tradition.[70]

The participants of the French “mechanistic-molecular school”[71] would say that the superiority of their method relied on its anti-hypothetical caracter, erected from the mere observation of facts, which would, in principle, make it possible to derive the fundamental equations of a *general dynamics*, one of the sacred chalices of science at that time. Such approaches, due to their positive character, should be accepted even by “believers” of different doctrines, such as those who saw in the phenomenon of heat radiation a movement of a *caloric fluid* or vibrations of an elastic *ether*. But how to consider something that flows, that streams, without the subliminal hypothesis that if something flows, it is fluid? Even a mathematical formulation can carry an underlying hypothesis, since the hypothesis formulation is a characteristic of the expression of thought, which interprets as it sees, or as Duhem would say, “... an experiment is not simply an observation of a phenomenon; it is, moreover, the theoretical interpretation of that phenomenon”.[72] So, in a way, the basic categories are always *a priori*. This principle also highlights the existence of a mutual dependence between observation and the resulting model or theory.

As representations or mnemonic resources, models can enable a greater connection with the phenomenon. In this sense, the concept of analogy would undergo a certain formulation, and could be applied to the supposed relationship between the model and its correspondents, i.e., in the underlying reality that the model sought to mimetize, both in the sense of the variety of predicted experimental measurements, and later on, in the conception that the analogy, in the persona of a set of mathematical equations, could be seen as the very materialization of that reality, as in the analogies/equations of Maxwell. He was one of the few influential thinkers to provide an explicit, although subtle, analysis of the scientific method, particularly in relation to his theory of electricity and magnetism. Such a theory has been extensively commented over the past 150 years as support for deductive mathematical models, or inductive methods of scientific construction, but it is not often noticed that Maxwell himself claimed that his method was the authentic Newtonian way of “deducting from phenomena”, without the aid of unproven hypotheses.

Newton’s classic and influential idea of inference, which according to Mary Hesse had been a false move to solve what she called the transitivity paradox,[73] is what is now known as the “problem of induction”. For example, if a study concludes that all parrots in a given region are green, because “only green parrots were observed in a period p”,

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[70] Pierre Duhem himself was part of that tradition and would like to emphasize this aspect in the comparison between schools of scientific thought. See in Duhem 1954, p. 64.

[71] Kargon 1969.

[72] Duhem 1954, p. 180.

[73] The problem of transitivity seeks to determine how much a relationship can be expected to be transitive. That is, should it satisfy the logical condition “if f confirms h, and h confirms g, then f confirms g? See in Hesse 1974.

which was taken for granted, there is no guarantee that the next one to be observed could not be blue.[74] Newton, as will be seen in the following chapters, was not willing to consider his law of universal gravitation as a hypothesis, in the sense that the gravitational force was a concept postulated as a phenomenon. In a way, Newton hoped not only that gravitation would be seen as a mere description of the observed movements, so that the law of gravitation would be interpreted in the light of Kepler's mechanical laws of motion and the laws of planetary motion, but he also hoped to make predictions about comets and satellites. And the problem is precisely in using induction to justify induction itself.

The question of the relationship between physical and mathematical models was then undergoing profound changes, with intense debate about the ambiguities existing between the mode of action of forces, which could be by impact, as in the elastic collisions, at a distance, as in the *electro-magnetic* theories aligned with the Newtonian theory, or by a continuous medium, described by some *field theory*. Each one of them with its mathematical counterpart, in which more or less general phenomena could be explained. In his 1855 paper,[75] Maxwell presented examples of his view on analogy: the laws of numbers, the similarity between corpuscular and wave theories of light. A year earlier, when modelling the transatlantic telegraph cable,[76] William Thomson had presented his flow analogy, which related electromagnetic attraction to the heat and continuity equations. Based on this and Michael Faraday's pictorial representation in terms of the lines of force, Maxwell would develop his own analogy, connecting the theories of electric and magnetic actions at a distance, providing a mathematical representation of the lines of force.[77]

But Maxwell's insistence on a "formal" analogy between entities represented by equations did not directly imply an identity of physical processes, as when the fluid flow was compared to the movement of heat, current or electrical induction, without requiring that these processes involve fluids in motion. This may suggest that the considered entities in Maxwell's methodology of modeling were essentially cognitive, since his definition of *physical analogies* verged almost on the *ponderability*. According to Kargon, for example, the analogical method was *inductively preferable* for Maxwell to the logical (hypothetical-deductive), provided by mathematics.[78] With this perspective, analogies would be a "lesser" form of deduction from experiments, when these were applicable, which differs from the interpretation now defended, that of a stronger link between analogy and deductive methods.

As in the *linking operator* of the theory of complexity, analogies could be elements for connection and synthesis that still remain underdeveloped, or underestimated, in nat-

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[74] See Chalmers 1993, p. 31.

[75] Maxwell 1855.

[76] Thomson 1854b.

[77] A complete collection of Maxwell's papers can be found at Maxwell 1890.

[78] Kargon's theses can be seen in Kargon 1969.

ural sciences: “... As our way of knowing disables the objects themselves, we need to conceive what unites them. As it isolates objects from their natural context and from the whole of which they are part, it is a cognitive necessity to insert a particular knowledge in its context and place it together.”[79] Under this perspective, analogies gain importance in the historical studies on electricity.

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[79] Morin 2018, p. 24; See also Thompson Klein 2004.

3 Organic analogies

... the motion of the Earth awakens the internal breath of the globe.

(William Gilbert, *De Magnete*, 1600.)

See with the aid of light, feel the touch of the wind and hear its melody when in contact with the leaves; to remain silent before the lightning that cuts through the night and to receive, in the midst of the ocean of air, the warm embrace of the Sun. Such perceptions have, since ancient times, suggested the presence of a powerful and transcendent, impersonal and non-local unit, watching over the human destinies, which, until now, is the only species known to be capable of questioning its own existence. In addition to being responsible for the establishment of objective reality itself, such unity would be the primary cause of a mysterious *virtue*, which subsists and penetrates all beings in the universe, or whatever comes into existence: "... because in Him we live, move and exist,"[1] according to the biblical passage.

3.1 The shamanic web

Long before the studies of classical western antiquity, it is possible to *read between the lines* of ancient tribal societies traditions, that the first systems of knowledge representation produced by humanity were of spiritual origin: "... There is no religion that is not, at the same time, cosmology and speculation about the divine. If philosophy and sciences were born out of religion, it was because religion itself, in the beginning, played the part of science and philosophy",[2] according to the statement of the French anthropologist Émile Durkheim (1858-1917), which studied Australian and Amerindian populations. In these traditions it was possible to perceive the presence of a curious cosmogony or myth of universal creation, in which the symbolic representation of things recognized as sacred assumed the figure of a *totem*, represented by an animal that gave the clans a name and identity. The *totem*, an element considered alive in the daily lives of those communities, brought together all the spiritual qualities that inspired fear and reverence, security and balance, life and movement.

Although sacred things in their religious character assumed equal importance, the principle common to all of them was seen only as an impersonal virtue that was found in each of the beings, far beyond attribute or property of things themselves. Death could

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[1] The passage is found in the New Testament writings in Acts 17:28.

[2] Durkheim 1960.

arrive sneakily, but such a virtue would be present, alive and constant, like a great field, uniting all creation and all beings to a single universal web. For an Australian or American Indian, for example, it would be very difficult to conceptualize what a "horse" would be in the empirical sense of the word, which sounds strange to the contemporary Westerner. For the forestry, the *fundamental parcel* existing in the animal belonging to the "class of horses" was what made him, precisely, a horse. This principle subsists on the very physical existence of the little animal, like an avatar, a figure that has been borrowed, being returned to the earth after its existence, and that virtue or that vital and intelligent principle that animated it remained intact.

A Totem was not necessarily seen as a deity in the sense of the word. Even the gods (with a small "g") of the Israeli tribes, who would later be elements of the Judeo-Christian tradition, and even the anthropomorphic entities of the Greek or Hindu pantheon had a broader meaning: "... you are gods".[3] In fact, although they had similar characteristics to representations of forces of nature, the *mana* that animated the totemic spirit, as Durkheim called it, was understood more as a generating force, a fundamental part that was found inside and outside of things, which could emanate, radiate, heal or destroy, even at a distance.[4]

The notion is different from later conceptualizations about *pantheism*, a conception that interprets what is created as part of a whole, of the created being as part of the creative element, that is, a deity. In a way, both those who have a religious attitude, although subjective, in the face of the enigma of the external universe, and those who perceive in the world an "intelligent something" behind the events, cannot be immediately characterized as pantheistic, since for those societies the phenomena themselves were considered part of nature. At this time, there is no definition of "nature" and "being" as separate elements. Thus, the universe itself could not be considered, for those societies, as a deity.

A shaman, when forced to invoke a certain magic ritual in an imminent war situation, could, according to certain beliefs, have a real effect on his opponent from another tribe, perhaps miles away, even causing him physical death. This effect could be achieved through the manipulation of that substance, which was accessible from the knowledge of the wisest members of the tribe. At other times, objects could have strange properties. Reports from members of those communities stated that certain individuals, when coming into contact with certain objects, received from them a real shudder or *shock*, which could be compared to the action of lightning in a storm.[5]

For shamanic traditions, which are still alive in some parts of the globe, nature is considered alive, reproductive and autonomous. The mode of action of these forces,

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[3] The passage is found in the book of salms 82 : 6, from the Hebrew Bible, and also in the New Testament at (Jo 10 : 34) – "Isn't it written in your law, I said, you are gods?"

[4] Campbell 2004.

[5] See Durkheim 1960, p. 170.

however, can be framed, in the modern sense, or as a kind of analogy of attraction, either as a means of interaction in pairs, acting at a distance, or as continuous action, such as a phenomenon of fluid field, which could flow, propagate through space, acting wherever it went, even in an indefinite time. Based on this hypothesis, *mana* would be an ancestral understanding of humanity regarding the physical phenomena of mechanical action, in the first case, or field effect, in the second.

Obviously, ancestral humanity did not create, as far as is known, any objective system of knowledge, nor did it carry *a priori* a scientific concern in the strict sense of the term or, as Bachelard would say, "... human spirit does not start as a physics class...", but, contemplating the river "... as a sleeping shepherd sees the water running",[6] the remote man sought to provide explanations for the moral cycles of life, love and death. Such elements are present in several mythological narratives, alternating, naturally, characters and scenarios, maintaining relatively the same background. And for the real value of a symbolic or mythological expression to be accounted for, differently from what is thought, it is not enough to simply consider it a mere expression of human creativity, as a genre of fantastic narrative. Although not advocating the existence of a single interpretation for symbolic expression, as each symbol needs a keyword to give its meaning, the possible link between the *totemic mana* and the idea of force, omnipresent field or energy, in the conceptualization of contemporary science, can be seen as a controlled and intentional expression of certain universal principles that persist throughout human history.

For Joseph Campbell, who was one of the most influential scholars of comparative mythology in twentieth century, having such principles a spiritual nature, they would be present in a transfigured way as the unfolding of an elementary unity, a "... ubiquitous force from which all things and beings, a force that sustains and fills them throughout the period of their manifestation, to which they must return when their final dissolution."[7] In Campbell's monomyth, all mythological narratives symbolically tell the same story: the story of mankind, and the idea of *force* emerges as one of those fundamental elements. His work became known in the last quarter of the twentieth century for having influenced the cinematographic work of George Lucas – *Star Wars* – which had great impact on popular Western culture as a practically literal adaptation of Campbell's research, in which it seeks to highlight the thesis of *monomyth* as the journey of the human hero.

Each society is, at each time, impregnated with the *breath* of its own *mental clichés*, which reflect the natural anxiety of collectives in finding elements of their own context their religions, laws, cultural and knowledge systems. In the remote past, perhaps contemplating the external nature, men and women sought to understand themselves as individuals, such as "pieces of self-reflective universe". Today's humanity builds radio telescopes and

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[6] Bachelard 1992, p. 85.

[7] See at Campbell 2004, p. 271.

contemplates the outside universe, seeking to understand it, arriving sometimes at findings very similar to the ancestral ideas. This bias in the interpretation of reality can result in a very particular way of seeing and understanding each historical moment.

In a communication published on the prestigious pages of *Nature* magazine (2018),[8] it is possible to find a report in which researchers would have found evidence of a certain type of matter that was "lost" at the ends of the universe. This, according to estimates, would be part of a *filamentary structure* that possibly *connects* all parts of the universe, a true *cosmic web*.[9] In addition to the analogy of the cosmic web, which in some ways resembles the universal shamanic web, other analogies that are part of recent concepts can be listed, as in the theory of *dark fluid* and *dark energy*,[10] or even scientifically based novels, which transform the nineteenth-century submarine telegraph cable series into a *Victorian Internet*, based on what one author considers it to be the same attribute shared by both, the *connection*.[11]

Although the somewhat anachronistic approach of looking in the past towards the present, based on "social facts", natural in his time, the sociological method that had Durkheim as a pioneer became one of the universally accepted alternatives for understanding life and the customs of prehistoric societies. The thinking was based on the principle that these facts, by themselves, had objective reality, which can be treated using the indirect experimental method, or comparison. The degree of isolation in which those societies were immersed allowed them, in a way, to maintain customs closer to those of their ancestors. Durkheim noted that religion was not limited to enriching the history of ideas, but indeed contributed to the formation of scientific concepts common to the natural sciences, contrary to the thesis that it were a gradual and progressive evolution, with major contribution from philosophical thought. The proposition was that of a social origin for scientific thought.

In this respect it corroborates the thesis Cornford, who was a translator and professor of ancient philosophy at the University of Cambridge. In his book on the origins of classical thought, it is possible to see a certain indissolubility between ancestral ideas and the rational and artistic aspects present in the writings of Ionian thinkers.[12] In this Greece before Socrates, in parallel with the figure of the Shaman, who brought together the fig-

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[8] See in Nicastro et al. 2018.

[9] For a journalistic information on this subject, see Fang 2018.

[10] Farnes 2018.

[11] See Standage 2007.

[12] Francis M. Cornford (1874-1943) explores, in his research, the paths taken by the thinkers of antiquity, evidencing the inseparability between the figures of the sage, poet and prophet in the birthplace of Western philosophical thought, opposing the notion that sciences and religions were opposed by virtue of the nature of philosophical thought. Cornford was married to one of Charles Darwin's granddaughters, Francis Crofs Cornford (1866-1960), who was a prominent poet. Because of the similar names and because both were known figures at the time, the two were often referred to as *FMC* and *FCC*. See in Cornford 1989.

ures of the doctor, poet, seer, who “crossed” space-time through spiritual attributes,[13] the figure of the sage, who in a way used the intuition of the Shaman and the analytical ability of the doctor, is seen rising, however, differentiating himself by the search for a logical and aesthetically coherent unity, typical of rational thought.[14] In a way, the first Western philosophers brought together an awareness of their ancestral heritage, with the presence of religion and morality in a way mixed with mathematics, astronomy and arts.[15] ***Mana*** would give way to the virtues of the mind (*ψυχή*, ***psi-qui***) and fire (*φλγα*, ***flo-ga***). These, in turn, composing the elements of Empedocles (490-430 B.C.), earth, fire, water and air,[16] would be the agents that promote life and movement.

Obviously, Cornford’s thought is not unanimous, since he understood “Ionian science” more as a mythical construction than a scientific theory, which is firmly based on experimentation, as stressed Jean-Pierre Vernant (1914-2007). Ionian thought, according to Vernant, was not similar to the scientific thought developed later, both in methods and in inspiration.[17] In any case, among thinkers of remote Ionia, perhaps the best known is Tales of Miletus (or Milesia), who lived in a period between (624-546 B.C.), approximately. As is well known, the Ionian region, which was on the eastern side of the Aegean Sea (east of the Mediterranean Sea, currently Turkey), witnessed the flourishing of Greek civilization and would house thinkers such as Anaximander, Anaxymen, Anaxagoras, among others. Tales, which was made known from the writings of Aristotle,[18] believed that the soul was “... mixed with the totality of the universe...”, which would cause all things, for they, in turn, had also a soul, “... existing in the air or in the fire...”[19] elements that were part of what makes up animals and man; an entanglement between soul and matter, with different characteristics, although complementary.

Not only Tales, but Anaximander, realizing the perishable state of visible things, like the sand sliping through a mesh of a sieve, concluded that this was, precisely, a primordial characteristic. For the Ionian thinkers there was no clear distinction between what was matter or spirit, since “everything was contained in everything”. For this reason, it is difficult to conjecture whether Tales considered any causal relationship between soul and matter or vice versa, but it is certain that he flirted with the idea of soul as something very close to material reality, perhaps a kind of subtle matter, with mechanical characteristics, such as the known descriptions on the effect of the loadstone on iron. This idea was later related to experiments with amber. (*ήλεκτρο*, *i-lÃłctro*) and attached to the figure of Tales by Diogenes Laertios – who has had his reliability as a historian questioned on many

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[13] Cornford 1989, p. 140.

[14] Hesse 1962.

[15] Sparshott 1954.

[16] Aristotle 2010, 414b.

[17] Vernant 2005.

[18] Aristotle 2010, 411a.

[19] Aristotle 2010, 411a.

occasions.[20]

Although Diogenes' account on the figure of Tales can be disregarded,[21] the oldest known story comes from the dialogues attributed to Socrates, especially in the *Timaeus* by Plato, his work with a more "scientific" character. In this work it is possible to observe one of the first systematizations of the analogy between fluid flow and the notion of electric charge transfer. Subjected to the friction with objects of a different nature, amber changed its behavior and assumed properties similar to the magnet, being natural to argue that it yielded or received certain virtues, which seemed to flow from one body to another, provided that a path was established: "... To the same cause is due the flowing of water, the falling of the thunderbolt, and the force of attraction exercised by amber and the loadstone"[22]

In another dialogue, Socrates and the young Teetetum mention the figure of previous thinkers and poets, such as Protagoras, Heraclitus and Homer, who would bear the idea that "... all things originate from flow and movement."[23] Plato himself would have learned in his youth from Cratylus, the Heraclitean philosopher,[24] that "... a world of sensory perceptions is a world of flux."[25] However, as long as the wonderful properties of amber remained associated for a long time with prominently static events of electrification by friction and attraction, it was heat, personified in the figure of fire, the entity present in the primitive associations to the dynamic characteristic of the flow, imported from the geometric tradition of the Pythagoreans.

3.2 The atom of fire

A new phenomenon is hardly considered ideally in its objective character, even by those with a trained eye for it. When the human ancestor thought of fire, even giving it a name, how did this happen? One couldn't name it, except from what the fire did: consume, illuminate, disintegrate. Socrates said that we could not say that fire is this, but "... what, in certain circumstances, is like this".[26] Therefore, it is expected an objective explanation, in which the visible characteristics were more determinant to designate the new phenomenon. But no: it was mainly the rapid movements of fire that impressed the ancestral man, hence his name as "... the living, the agile, ag-nis, ig-nis".[27] If its notable feature was, first, the agility, it was necessary, therefore, agility to tame it.

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[20] See, for example, the discussion in Benjamin 1898.
[21] See also discussion at Assis 2010.
[22] The loadstone was also called by Plato as the *stone of Heracles*. See Plato 2013, 80c.
[23] Plato 2011b, sec. viii.
[24] Copleston 1993.
[25] Aristotle, *Metaphysics*, §A6, 987a, 32-5.
[26] Plato 2011b, 49e.
[27] Bachelard 1992.

In the mythological sphere, fire is always stolen. In a Brazilian indigenous legend of the *Kaiapós* tribe, a certain jaguar saves a man trapped in the crown of a tree, taking him to his shelter and offering roasted meat, as he did not know the fire. Over time, the female tries to devour the man, who ends up killing her and running away soon after. After returning to his village and making the fact known to the companions of the tribe, an expedition is then organized with the aim of stealing the fire from the jaguar's house, who would thereafter be condemned to eat raw meat.[28] It is curious to note that the narrative differs in words, but not in essence, in relation to the Greek myth of fire creation. Their traditions say that the Prometheus and Epimetheus brothers had been given the task of creating men and animals. Epimetheus starts the work but *consumes* all the raw material in the creation of the animals. Prometheus then assumed the post determined to create man from mud, but he was also unable to provide life to the newly modeled inert statue. He then decided to steal the fire from heavenly heights, to insert into the interior of man what would differentiate him from all creation.

Not far from the mythological sphere, Plato tells that Timaeus, having been presented by Critias to Socrates[29] as the one who had most endeavored to know nature and *kosmos* , he would be invited to speak about creation. He then tells that the Demiurg[30] contemplates the elements present in a universal pre-cosmic chaos and,[31] seeking to impose a certain order through from geometry[32] and mathematics,[33] gives rise to the element of fire. As a kind of divine artisan responsible for creation,[34] he shapes the world and its constituent elements from the *Archetype* (*αρξέτνπ*) or primordial model, which represents the set of possible ideas to be fully achieved.

The figure of the Demiurge in Plato's work is undoubtedly intriguing, having been subject of intense speculation throughout history. Sometimes interpreted as an esoteric or mythological reference to the Creator – or co-creator, since he only creates from something pre-existing – the Demiurge appears in Timaeus as a high commissioner or in charge, being also described as an educator, an example to be followed,[35] the highest model to be mirrored from the intellect, starting from rationality (*nous*) to true wisdom (*σοφία*). Kant, for example, categorized him as an element that generates order in the world. It is curious to note that, in some aspects, the narrative resembles certain interpretations of the Jewish tradition about the figure of *Elohym*, which appears in the *first* verse of the first Book of Genesis, which can be transliterated as *"Bereshit Bara Elohym et Hashamaim*

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[28] Mindlin 2002.

[29] Plato 2013, 27a-b.

[30] Plato 2013, 29a.

[31] Plato 2013, 30a3-5.

[32] Plato 2013, 69b.

[33] Plato 2013, 53b-c.

[34] Plato 2013, 35c, 55a5-6, 4c6.

[35] Plato 2011b, 176b.

Veet Haarets ...".[36] Another fascinating detail is that the expression *Elohym* means precisely Gods, plural of *El*, which is God. Thus, a possible translation of the first verse of Genesis would be "In the beginning of (the act of) creating, *created Elohym* the Heavens and the Earth." In this respect, Mckenzie states that "... Elohym here (can be read) not precisely as God, but as that level of being that is proper to God: the superhuman and the divine."[37]

Although the Jewish tradition used the idea of a "majestic plural" to justify the occurrence of *Elohym*, which appears only in the first verse of Genesis, instead of the singular *El*, one of the greatest medieval commentators on the Hebrew Bible, Rabbi Shlomo Yitzhaki, or simply Rashi (1040-1105), as he was also known, interpreted Elohym in the literal form: "... a council of angels."[38] That is, a group of beings that the Creator had at his disposal and that would have helped him in the creation. In this sense it is possible to notice a fine line between the first verse of Genesis and the Platonic narrative of the Demiurge. But unfortunately there is not known evidence of oriental influence on classical Greek culture, since Socrates himself, Plato's mentor, was a contemporary of Malachi, around 500 BC. Malachi was one of the last prophets of *Torah* or *Hebrew bible* – known in Christianity as *Old Testament* – which indicates that the sacred text for Jews and Christians was still "being written", so to speak, at the same time Socrates and his disciples were walking through Greece.

There is also a clear influence of Pythagoreanism in the Platonic narrative, not only because of the rituals begining at each new dialogue,[39] but especially for the geometric and kinetic shape with which it is attributed to the formation of things. What is certain is that Plato traveled to southern Italy and Sicily, to meet and talk with the remaining members of the Pythagorean school, when he was around 40 years old. On his return to the city-state of Athens, Plato founded his Academy, a name derived from its location, next to the sanctuary dedicated to the hero *Academus*,[40] who taught mathematics, among other sciences. Anyway, the character Timaeus, if he really existed, was a member of the extinct school of Pythagoras, and deduced the shape of the elements by assigning a corresponding

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[36] Transliterate means writing or transcribing a text in a particular language according to some system of characters. In this case, the Hebrew verse for the English alphabet is presented, so that it reproduces the same sound. The verses of Genesis, in Judaism, were all sung and passed on via oral tradition, thus justifying transliteration, in an attempt to preserve, as much as possible, the wealth of information.

[37] See in McKenzie 1948, p. 173.

[38] For more information on Rashi's interpretation, see Touitou 1990.

[39] One of these rituals was the invocation of the "muses". Plato 2013, 27c.

[40] Unlike modern universities that prioritize the "transmission" of information, similar to the sophist style of teaching through "lessons", Plato's academy can be considered asthe first university, where mathematics, astronomy, biology , botany and physical sciences were taught, beyond philosophy. The first academies used the Socratic method of *Maieutics*, which sought to help the apprentice to bring out the truth that resided within him. In teaching, seemingly naive questions were used, with which the apprentice gradually approached wisdom, from the motto "Know yourself" (*γνῶθι σεαυτόν*, gnōthi seauton). Achieving wisdom through the screening of truth, beauty (or goodness) and necessity was the end they were destined for. The method is explained by the character Socrates himself: "... they are labor pains, my dear Teetetum. You are not empty; something in your soul wants to come to light." See at Plato 2011b, p. 13.

figure to each one, like the cube for the earth,[41] the octahedron for air,[42] the icosahedron for water[43] and the pyramid for fire,[44] according to his moving characteristics.

The character starts, as it were, from the basic forms common to the Pythagorean tradition (triangles) for a three-dimensional formation, giving form to the shapes given by thought, with special attention to the element of fire, which is created from an existing material.[45] This material is a mathematical configuration or a geometric modeling from a cosmic matrix present in the universal archetype:[46] "... we selected two triangles from which fire and other elements were created: one is the isosceles, and as for the other, its longest side is always a square $(\delta\acute{\nu}\nu\alpha\mu\eta)$." The term in brackets, which is pronounced *dinami*, initially expresses the notion of mathematical power (x^2), and would later give rise to the word *dynamics*, being associated with a physical power, close to the contemporary formulation.[47]

From the elucidation of Timaeus emanates the idea that shapes, figures and numbers, by themselves, could have an objective existence. And the link between the created thing and the invisible cause was, precisely, to affirm that the world would, literally, be a mirror of something or *"...kosmos eikona tinos..."*.[48] In order to have an image, a mirror is also necessary, a bond that determines the similitude relationship between the model and the real, a *mirror analogy*. In his words: "... it is not possible for only two things to be beautifully composed without a third, as it is necessary to generate a bond between them that unites them. The most beautiful bond will be the one that makes the best union between itself and what it bonds to, which is, by nature, achieved in the most beautiful way through proportion."

There is no doubt that the elements constituted, for the Pythagorean school, "bodies",[49] having, in addition to a palpable reality, objective dimensions, such as depth, proportion or "harmony" , expressed from the numbers. This school was probably founded by Pythagoras in 600 BC and lasted until it was ironically destroyed by fire, spreading the idea that harmony, especially music, was strictly related to numbers, being the element present throughout the universe. The soul was the causative element of morphogenesis, being able to inhabit numerous bodies over time. They were, therefore, reincarnationists, a tradition perhaps inherited from Orphic culture, which believed in the successive purification of souls by means of successive lives.

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[41] Plato 2013, 55d; In this respect, it seems that Plato was right. It has recently been demonstrated that the best statistical model for the fragmentation of rocks is, precisely, the cube! See in Domokos et al. 2020.

[42] Plato 2013, 55a-56a.

[43] Plato 2013, 55b,56c.

[44] Plato 2013, 55d.

[45] Plato 2013, 31c.

[46] Plato 2013, 54d.

[47] Plato 2013, 54b; See also Plato 2011b, pp. 11-13,

[48] Plato 2013, 29b-c.

[49] Plato 2013, 53c.

The nature of numbers was studied by the Pythagoreans in order to have a more straight and pure life, according to their traditions. Because they were, in a way, ascetics, the numbers had for the Pythagoreans a *status* similar to the atom for the atomists, which should never be confused with the contemporary notion of atom.[50] It is not difficult to follow the syllogisms they used to justify the mechanism engendered by thought, something like:

- Elements are bodies;
- Bodies have depth and proportion;
- Proportion implies, in turn, on some surface, formed by triangles.
- Therefore, the elements are formed by triangles.

The question of "powers of the mind" was one of the criticisms that Aristotle made to Pythagorean thought, which suggested immaterial causes for natural phenomena, especially in the structuring of elements, an *intelligent design* in the contemporary sense. For Aristotle, the notion was not completely useless, since it was essential to suggest an initial trigger for movements, as a mechanical agent. According to the founder of the peripatetic school, even pre-Socratic thinkers thought as he did, i.e., "... Anaxagoras, in the very constitution of the universe, uses intelligence as a *deus ex machina* (machine god), and evokes Intelligence (with a capital I) only when he is in difficulty to give reason for something."[51]

In the apparent naivety of the Pythagorean syllogisms is embedded a principle that would be one of the greatest contributions to future scientific thought, that had a great influence on Plato's thought: that the property of things could be reduced, or explained, from of the properties of numbers, in which "... matter is not a number, but participates in numbers".[52] That is, there was no intention of seeing the number as something physical, but an attempt to break with the materialism of milesian cosmologists. In Pythagorean thought the entire universe was ordered according to sovereign principles of harmony and beauty, in which mathematics was the bond between the human mentality and this hidden reality, present in the field of Ideas: "... Nothing that resembles what is incomplete can become beautiful."[53]

Although there is no need for defenders to have his place guaranteed in history, historians' predilection for Aristotle's contributions in natural sciences is notorious. But

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[50] As strange as it may seem, a similar idea can be found, in which an equivalence is established between "thing" and "information". Some studies that suggest just that, as in Bekenstein's theory, which presents the *bit* of information as a "primordial element" of the universe. See in Bekenstein 2007.

[51] Aristotle, *Metaphysics*, 985a-20.

[52] Plato 2013, 990c5-991b4.

[53] Plato 2013, 30d.

the influence of Platonism and, consequently, of Pythagoreanism, in later scientific theories is still present, especially in the adoption of mathematics as a fundamental tool for science, in addition to keeping alive the idea of non-mechanical causes for physical phenomena. In this sense, Plato is closer to contemporary physics and engineering than Aristotle himself.

3.3 From organism to mechanism

In the ancient world, the first knowledge systems were established from the idea that the entire universe was “alive”, sharing the same principles that should govern the lives of beings. After all, animals and plants were organisms, just like man himself. Organisms support themselves, grow, multiply, are creative, have regenerative potential and maintain their own ends or goals. It is natural, therefore, that the first descriptions of natural phenomena should be based on analogies corresponding to organisms, although the debate regarding what “life”may be has not ceased over time. And the understanding of what an organism could be is, in a way, related to the idea of fluid, which has never been completely abandoned from philosophical-scientific conceptions, having, in truth, been transformed throughout history.

Anaxímenes tried to explain how the visible matter had arisen from the air, introducing notions of condensation and rarefaction.[54] Other events, some of them associated to medical practices, such as evaporation, breathing and circulation , would be governed by a certain power of attraction caused by fire. Heat had already been observed expanding air in a reservoir, which contracted when cooled. It should be able to drain some kind of invisible fluid to itself, which would cause the reservoir to expand. Upon cooling, such fluid would be expelled. It was observed that the Sun had the capacity of “drying” the water of a swamp[55] and the air, being expelled by the lungs in the act of breathing, pushing the air from the outside, causing a circular movement.[56] In a context where world conceptions were based on the geocentric model, with the Earth fixed at the center of the universe, the circularity alluded the supposed movement of the celestial spheres in comparison to the fixed stars. The mechanism, however, would be caused by some source of *internal fire*, which, in turn, would collide with the air when when it entered the body. Likewise, as in the waters of a river, each one would flow to its “commom place”, always in order to avoid emptiness.[57]

The world should really nourish horror by the void.[58] It was not conceivable that

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[54] Copleston 1993, p.26.

[55] Aristotle, *História dos animais*.

[56] Plato 2013, 79b.

[57] Plato 2013, 79d.

[58] Although the issue has not been an obstacle to the flourishing of science in various sectors, the problem has never been completely overcome. A few years ago, the physicist Stephen Hawking demonstrated in his radiation theory that even near a black hole, where there should be nothing, there is creation and

something was headed for “nothing”. Perhaps, the very horror that the human mind has always cultivated by the possibility of annihilation itself was responsible for the analog in Greek science. It was then necessary to reject the concept of absolute vacuum: even the empty space should be filled by something, having an objective dimension in a given domain, even if invisible or imponderable, since “emptiness” represented “nothing at all”. And to consider the existence of the void was as irrelevant as seeking the trivial solution to an algebraic equation. Or rather, the vacuum was only relevant in the sense of opposing its existence.

When observing a projectile in motion, it was concluded that the air pushed to the sides should create a vacuum, which would, in turn, be filled and annulled by the air itself, propelling the projectile forward and maintaining its movement.[59] It was a seductive idea, as all generalizations tend to be.[60] Any apparent phenomenon of attraction could be understood as a way to maintain coherence in universal laws, including the impossibility of a vacuum. Even the soul that came out through the pores of amber or the loadstone, and even the fall of the rays obeyed the “principle of the push”,[61] called by Aristotle ***antiperistasis*** (αντι - περίωσις, ***anti-periosis***), a term adapted from the Platonic concept of ***periosis*** (περίωσις), that designated something like “intimacy”, but that loses some of the original information when translated.[62] The term was naturalized in the medical literature, in which it is used to characterize, for example, the phenomenon of food swallowing and its processing inside the esophagus and intestine, hence the name “peristaltic movement”: supposedly involuntary movements, which push the food bolus along the digestive tract.

Although there are historical controversies regarding the concept of ***antiperistasis*** and its “power of attraction”, the idea represented a principle from which the vacuum was an impossibility of nature, related to the flow of matter in some process, in an organism. The metaphor in most of the explanations, however, remembered the functioning of a machine, which comes on the counter-intuitive side of organism and was closer to the idea of “thing”. Yes, an organism could even be considered a machine, as long as it was able to self-manage, based on a project that remains more or less invariable from generation to generation, but the opposite was no longer possible. A “thing”, by itself, has no purpose, does not grow or breathe; neither does it reproduce. Continuous movement and even attraction could then be possible due to a change in the configuration of these little things, called *atoms*, which would be the smallest portion of matter possible, so small that they

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distribution of matter, so that, so far, there is no strong evidence that there is really empty space in the universe.

[59] Plato 2013, 80a.

[60] Interesting to note that even today, in textbooks we can perceive a fragrance of these statements. David J. Griffiths, for example, in his famous book on electrodynamics, uses a mnemonic resource to better understand the phenomenon of induction, “... Nature abhors flow variations”, as expressed in Griffiths 2011, 210.

[61] Plato 2013, 80a.

[62] See Plato 2013, 79c.

could only be seen with the eyes of reason. In the sense of Democritus, who was the first (admittedly) atomist at the school founded by Leucipo,[63] atoms would be indivisible, impenetrable and eternal,[64] idea that remained cohesive in science until the 20th century, with the discovery of subatomic particles, especially the electron.

Atomism was the first mechanical interpretation for reality, the conception of a world in which bodies in motion would be part of a large machine, which in theory could be subdivided into its fundamental parts, in which "... all bodies are made up of all simple bodies".[65] Any and all experiences should be reduced, in some instance, to atomic configurations, matter and movement would be the only artificers of reality and matter would itself be generated from movement.[66] According to this view, the ability to be moved was, in fact, one of the properties of matter. The driving agent, however, would be in another category:[67] in Empedocles' doctrine, the *love* that unites or the *discord* that separates appear as agents of natural movements of aggregation and disintegration, a remote conception of the idea of force, which would even give meaning to the concept of time flow, as the world was nothing more than a big clock. Thus, if the sand in the universal hourglass were moved backwards, hypothetically, the first and immovable motor would be found, the cause of all movement.[68]

One of the old conceptions about the soul of an organism is closely linked to the idea of *free will* or *intelligent principle*, in which the "... the essence of the soul is a number moving itself."[69] Thus, a watch had almost all of these characteristics, in which the flow was relatively independent, almost without pushing. It had no soul and could not move on its own, but it was amenable to construction and replication. A skilled craftsman could easily build others. It was an attractive idea: a machine world would be possible for practical purposes, a simplification of the analogy of organism, which would survive in one way or another in subsequent conceptions. It would not be difficult, however, to reconcile the idea of attraction as an explanation for movements, and apply it to the most evident phenomena of life, given the proximity found between an organism and its mechanical abstraction.

In almost all elements, the doctrine of flow presented itself as a bridge between the mechanical and organic universe, and the thought that "it is not possible to enter the same river more than once", possibly pronounced by Heraclitus, has been readapted throughout history. An excerpt exposed in *Timaeus* illustrates the idea, in which Plato makes an association between fire and light in optical phenomena, with light being the

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[63] Copleston 1993, p.77.
[64] Aristotle 2009, 325a.
[65] Aristotle 2009, 335a-20.
[66] Aristotle 2009, 335b-15.
[67] Aristotle 2009, 335b-30.
[68] Aristotle, *Metaphysics*, 985a-25.
[69] Plutarch 1878; See also Plato 2011a, 245c.

flow of a fire that does not burn as part of the mechanism of vision:[70] "...The pure fire inside us, brother of the other, it was put to run through our eyes smoothly and continuously, so that it compressed the center of the eye as much as possible, in such a way that it would support other thicker species, in its totality, and filter only this pure species. In this way, when daylight surrounds the flow of vision, the like falls on the like, they become compact, uniting and reconciling in a single body along the axis of vision; what happens wherever that fire that comes out of the interior comes into contact with that which comes from outside. Thus, a homogeneity of impressions is generated, since the whole is very similar; if this whole touches something or if something touches it, it distributes its movements throughout the body to the soul, and produces the sensation that we call 'seeing'."[71]

It is worth noting that the ideas around fire, ether and soul have been a device used many times, in history, as a tautological response to a phenomenon difficult to explain or, as in the saying, "it's six of one and half a dozen of the other". On some occasions, these entities were considered similar. In others, they had different nature. For example, Hesiod's Theogony ($\approx$ 750-650 a.C.)[72] describes with great beauty that from *chaos* elements linked to the underworld and the upper world were born. From *Erebos*, a dark region connected to the underworld of the dead, night was born. From *Ether*, region of splendid luminosity of the sky, daylight emanated.[73] The Stoics, in turn, about 500 years later, followed the idea of Aristotle's ether and saw it as a kind of fire, although different from the four elements of Empedocles, that is, a fifth body, an essence, or quintessence.[74] Thus considered, as a fire coming from the first celestial sphere, light took part in the vision process in a complementary way to that flame that was spilled from the viewer's eyes, as expressed in Timaeus, " ... it is necessary to take into account that there are other types of fire, such as the flame and what emanates from the flame, which does not burn but provides the eyes with light, and what, when the flame is extinguished, subsists from it in the inflamed bodies."[75]

The phenomenon of vision and image formation was certainly a great enigma for ancient philosophers. Perhaps it was difficult to imagine that the image of a bird's flight was formed by the light that moved from the object to the eyes, taking shape "inside the head", instead of where it seemed to be, which is perfectly reasonable. Take the following thought experiment: if a particular object is shown to a child or lay adult and they are asked to report where they see the image, it seems illogical for them to point their finger in the direction other than the outside. How would it be possible to see what was outside

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[70] Plato 2013, 58d.

[71] Plato 2013, 45c-d.

[72] Hesiod, in addition to Homer, is one of the oldest Greek poets known, whose works have survived through the ages. His work is studied as a document of Greek culture and thought.

[73] Hesiod 1988, p. 91.

[74] Laêrtios 1987, VII-137.

[75] Plato 2013, 58d.

without something being transported from the eyes to the object?

Although some writers dismissed the materiality of light,[76] the consensus on the perception of vision was that the image was felt by some sensitive organ through the surrounding air, or as a disturbance that spread, like waves in the water, an anticipation of modern wave theory. For many ancients the vision was a non-local phenomenon, not restricted to the brain, similar to the iron that "feels" the influence of the magnet, and the magnet that "feel", for the same reason, the presence of iron. Almost like a proto-idea of a field phenomenon, the image would be like a flow of light that was directed into the eyes, while a fire would flow from the eyes towards the object. This is, in essence, one of the first theories of emanation, also called *extramission* of vision.

It is a human characteristic to give credit to one's own sensory experiences in the perception of what one thinks to be happening around, which is why the first descriptions of physical phenomena were based on perception, which is inseparable from the individual aspect of being. The auditory phenomenon, for example, if it were analyzed from the same perspective, would be similar to the formation of images, but now as an intrinsic phenomenon. That is, when hearing the sound of a lyre, the ancient philosopher or the layman of today would have the distinct sound sensation when the musical notes reached the hearing aid. If asked where in space they can hear the sound, their fingers would not point to the lyre, but towards the head.[77] Euclid, one of the great mathematicians of antiquity, tried to explain the idea of emanation of vision from geometry, and showed how *virtual images* projected from the eyes could explain the way one sees in a mirror, opposing the path traveled by mere luminous rays. According to Euclid, the light would be reflected by the mirrors, but virtual projections could pass through them, hence the expression "virtual image".[78] Euclid's ideas were adapted by Newton in 1704,[79] 2000 years later.

3.4 Rays and Emanations

The essential element in ancient science is the supremacy of the idea of movement in mechanical explanation of reality, that would be recovered in the high middle ages, in which the scope of Aristotelianism gave, in a way, a basis for a future theory for dynamics. It is obvious, however, to realize that this is a long step out from this "dialectical" dynamic towards an objective theory on matter and, in this respect, the influence of thinkers like Plato and Aristotle is exceedingly uncertain. In order to have a more precise notion of the transition between the metaphor of organism to a mechanical conception of nature, Aristotle could not have been an inspiration for anyone who wished to take the metaphor

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[76] As in Aristotle 2010, 418b.

[77] See, for example PereiraJr et al. 2018.

[78] See Haskins and Burton 1943.

[79] The digitized version of Newton's first edition of *Optiks* can be found at Newton 1704; A primeira tradução completa para o Português da mesma obra pode ser vista em Newton 2017c.

of mechanism to its "limits", as Kepler, Descartes, Galileo would do, and Newton later on.[80] Even taking the risk of confusing the boundaries between organism and mechanism, it was necessary a temporal retreat to a "purest" atmosphere of Ionian atomism and to abandon part of the Aristotelian ideal on matter in motion as a philosophical panorama, which dominated almost all medieval period.

The post-Aristotle philosophy would gain new airs from the third century before the Christian era. At this point one could no longer think of the great Greek city-states, especially after the rise of Alexander of Macedonia, who would have even been a student of Aristotle himself in the Lyceum.[81] After Alexander, the called the Hellenistic-Roman period sees a series of schools devoted to knowledge and the beginning of the cultivation of specific sciences flourished over almost 1500 years. In the so-called first period, between 300 BC and the first century after Christ, it is possible to observe the emergence of Stoic and Epicurean traditions, which were contemporary schools.

The Stoic tradition rejected the Platonic idea of transcendence but also Aristotle's universal concrete,[82] becoming one of the schools of greatest influence in Western thought, alongside aristotelianism. It was a doctrine that aimed at a proud and courageous behavior in face of life's challenges, in a practical way,[83] including illustrious members in his retinue, such as the emperor Marcus Aurelius, who lived possibly between 121-180 AD. Stoic cosmology often recurred to the figure of *logos* and to the Heraclitus' fire as substances, and also to borrowed elements from other schools, such as the *pneuma*, a kind of "cheering breath" or vital force. For Heraclitus *logos* consisted of a unity in universal order and harmony, which in history has been interpreted in different ways: the materialistic interpretation brings the figure of *logos* closer to some fundamental physical entity, such as fire. Another interpretation, of epistemological basis, brings the idea of a cognitive and necessary process to know the hidden nature of things. A third interpretation still considered it as a divine creative principle.[84]

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[80] See discussion in Hesse 1962, p. 73.
[81] Copleston 1993, 383.
[82] Copleston 1993, p. 386.
[83] See, for example, Epictetus 1994.
[84] Vieira 2018; Other interpretations can be seen in Rocha 2004.
[85] Copleston 1993, p. 386.
[86] See, for example, Epictetus 1994.

"cheering breath" or vital force. For Heraclitus, *logos* consisted of a unity in universal order and harmony, which in history has been interpreted in different ways: the materialistic interpretation brings the figure of *logos* closer to some fundamental physical entity, such as the fire. Another interpretation, of an epistemological basis, brings the idea of a cognitive and necessary process to know the hidden nature of things. A third interpretation still considered it as a divine creative principle.[87]

In stoicism, in general, everything was corporeal, although not limited, and all spiritual activity was a embodiment of this subtle subtle matter: soul, mind, emotions, attitudes and even God were considered as substances immersed in that fire or primitive breath, or as subtle ethereal currents that permeated the bins of matter. Some authors, however, started to use the supposed action of *pneuma* in mechanical terms. Discussions about the displacement of *pneuma* in a continuous medium - from the comparison with waves in an ocean - would suggest a spherical propagation of sound in the air, which would be used later by Descartes. On the other hand, the writings of Heron of Alexandria, a pioneer of engineering of the first century, would reinforce the idea of mechanical manipulation of the air from the atomist view, besides being one of the first philosophers to present the empirical character to validate a determined theory.

In his *Pneumatica*,[88] Heron (≈ 10-80 AD) presents a series of mechanical and automatic devices where ot possible to see his intention to show the convertibility between the elements air, fire, water and earth, and the material nature of these entities, as an anticipation of the principle of mass conservation. The best known of these devices is credited as one of the first steam engines in history. The air, according to the vision, would be formed by small portions, and the empty spaces between them could be reduced with the application of a force: "...The air particles are in contact with each other, such that they are not found everywhere, but empty spaces are left between them, as in beach sand: the grains of sand sould be thought of as corresponding to the air particles, and the air between the sand grains to the empty spaces between the particles of air."[89] Likewise, a force would appear whenever a vacuum was present, a vacuum that could be created artificially,[90] from that elastic property: the air was compressible. Even light could not pass through certain substances, such as glass, if matter were continuous and that space did not exist.[91]

The vision would even give rise to the interpretation that atoms could follow certain directions, and even light should be composed of atoms, spreading in the form of rays. One of the propellants of this idea was Titus Lucretius Carus (99 B.C. - A.D. 55), an Epicurean philosopher, who expressed an attempt to abolish the mysticism inherent in

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[87] Vieira 2018; Other interpretations can be seen in Rocha 2004.
[88] Woodcroft 1851.
[89] Woodcroft 1851, p. 02.
[90] Woodcroft 1851, p. 10.
[91] Woodcroft 1851, p. 09.

earlier conceptions. In his famous poem *Of the Nature of Things* (*De Rerum Natura*),[92] Lucretius presents his idea of emanation , attesting that not only light, but all things perceptible, should be expressions of a pre-existing nature. For the phenomenon of vision, in particular, the theory was based on the hypothesis that a material would be visually perceived by having detached some part from itself, such as a shell or replica of its external shape, that is, "... it must from all the bodies we see a perpetual flow, an emission, an emanation of elements that impress our eyes and move them to vision. The smells of certain bodies flow perpetually, as the cold leaves the rivers, the heat of the sun, and the waves of the sea, the wave that erodes the coastal dikes...".[93]

Epicureanism was an essentially materialistic doctrine that was based on the ideas of Epicurus of Samos, philosopher of the 4th century BC. Because it did not allocate space for any immortalist or spiritual digression, the idea of perception as movement of something detached from the whole was a condition that would avoid any less empirical explanation. In general, the concept of reality in epicureanism was limited to the five human senses. Although the idea was taken up on different occasions, and this certainly happened until the middle ages, at least, it is necessary to differentiate between the objectives of the philosophical endeavour then and current scientific practice. Philosophical traditions, which went from antiquity to the medieval age, had as their main objective the understanding of human reality and its relationship with the cycles of life, different from the contemporary ideal of nature as something objective, as in modern scientific reasoning. Although it could be observed from a more or less empirical approach, a physical phenomenon was considered only as an elucidation of metaphysical concepts, still distant from the concept of "research object". In the emanation theories In principle, taking inspiration from natural phenomena, it was thought of the emanation of reason, of providence and destiny, in order, in the end, to arrive at the radiation of fire and the flow of winds.

In the Neoplatonic schools of Alexandria ($\approx 300 - 600AD$), which had the most illustrious members Plotinus (204-270 A.D.) and St. Augustine (354-430 A.D.),[94] the term comes up to designate the action that moves from a higher spiritual level to a lower one. There, we see the metaphor of irradiation being used as a picture of relevant aspects of what was wanted to be achieved,[95] as the spiritual diffusion from the Divinity expressed in the confessions of St. Augustine, in a parallel between the movement of light in air and spiritual activity, when he asserts that "... as well as mass of air, this air that is above the earth, does not prevent the sunlight from passing through it, penetrating it, without breaking or tearing it, but filling it completely, so I thought that not only the body from the sky, and from the air, and from the sea, but also from the earth, they were accessible and

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[92] Carus 1985, 2016.

[93] Carus 1985, livro VI, p. 270.

[94] Also known as Augustine of Hippo.

[95] The idea of representation is studied in greater depth from the essays on art theory. See Gombrich 1985.

penetrable to You from all parts, from the largest to the smallest, to receive Your presence that, with invisible inspiration, governs, inside and out, all the things You created."[96]

Although loaded with emotionality characteristic of the religious aspect, it is evident that the illustration of metaphysical principles has served as an inspiration, in all ages, for a more detailed study of the science involved in the phenomenon. More than 900 years after Augustine, Thomas of Aquinas (1225-1274) states that fire could heat the similar from the transmission of what was inerent to it, stating that "the next end of every agent is to introduce in another the resemblance of its own form".[97] The fire, therefore, possessing heat, would transmit its image and similarity ahead, but only to objects that had, in themselves, something of that same fire, in an almost geometric process: each body divided into parts, each part repeating exactly the same pattern of the whole, like a recursive probability principle, as if it were possible to be, at the same time, the lung and the very air that one breathes. Immersed in an apparently chaotic whole, the small part could be born from this universal whole, repeating it. In the 20th century, a similar notion would support the discovery of the fractal geometry of Benoît Mandelbrot (1924-2010), but in the Middle Ages this view would be part of fantastic cogitations.

In the abundant medieval literature it is possible to observe that the ideas of generation, emanation and radiation were already beginning to affect the popular imagination, especially after the popularization of books and reading. In fact, books were one of the greatest technological innovations of that time, after the Guttemberg press, as one of the first effects of the Reformation, an omen of a new time. At this particular historical moment, alchemy, astrology and magic were almost universal beliefs, having lived on an equal footing with existing scientific theories. The ideas of spontaneous generation or "growth" from a seed impregnated with vital breath, pneuma or fire, could live peacefully with theories of a more pragmatic character.

The Historian Carlo Ginzburg when reported in 1939 the story of a miller named Menocchio (1532-1599) – who was condemned by the inquisition for his heretical ideas – pictured this transition period, when the "pre-scientific" thinking[98] would move slowly towards a rationalist mentality, which would flourish in just over a century. In a given statement, defendant Menocchio, a simple peasant who had little access to books, claims that according to his thought and belief "... everything was chaos, that is, earth, air, water and fire together ... and (I think) that in the beginning this world was nothing, that the sea water was beaten like foam and curdled like cheese, from which an infinity of worms was born; (and) these worms became men".[99]

Menocchio's speech synthesizes, in a way, a current of thought that was taking hold

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[96] Augustin 2008, p. 18, book vii.
[97] See Thomas Aquinas 2013, art. 7.
[98] According to Bachelard 1992, p. 101.
[99] See Ginzburg 2006.

at the end of the medieval period, in which, paradoxically, was embedded in Lucretius' idea of creation *ex nihilo*, "*...nil posse creari de nihilo*",[100] that is, nothing could arise from nothing, from chance. And since chance is nothing more than the ignored cause of an unknown effect, as Voltaire would say, astonishment at the enigma can even lead the unwary to the temptation of a superficial explanation; if it resists, however, it will be possible to glimpse the threshold of new knowledge. And it was precisely the astonishment at the idea of emanation of something invisible, but which should have an objective existence in some domain, which propelled the timid beginning of the science of magnetism. The organic analogy of emanation – of something that comes out of the bodies and fills the surrounding air, acting elsewhere – would be resumed. Going back to old concepts about the idea of flow, it would become one of the bases of the theories of propagation of light, heat and static phenomena of electrical and magnetic attraction of the 17th century, especially with the work of William Gilbert, still from the framework inherited from Greek science, having permeated scientific thought before the modern period.

3.5 Magno Magnete

At the height of medieval Europe, specially in newborn universities such as *Università di Bologna* (1088), *University of Oxford* (1167) and *Universidad de Salamanca* (1134), Aristotle had been the name of greatest influence.[101] However, the aristotelian intellectualism of macro explanations began to retreat after 1270, with a philosophy that was becoming more open to free discussion and to experimentation.[102] The *experimental philosophy* of magnetism had already been initially designed in the first century, when Lucretius developed a first model on magnetic action. He had made, in the form of poetry, an analogy between the action of the loadstone and the natural phenomena of life, such as the body sweat, the phenomenon of evaporation, the growth of body hair or the liquid that flows through the stalactites, in addition to reporting experiments carried out with iron filings, which "... seemed impatient to run away from the stone...".[103] In his explanation, this mixture of pioneer poet and experimental physicist suggests that the atoms of the magnet, for some reason, would collide the air that was between them and the attracted body, creating a vacuum that would be occupied by this same air, propelling the object ahead.

The feelings, since then, were of astonishment and admiration at the phenomenon, followed by a beautiful and harmonious explanation, in which it is possible to foresee the prodromes of the experimental analysis: "... men admire this stone; indeed, a chain is often seen forming with several rings that hang from it. It is possible to see five and even more, suspended from each other, swinging in the slight auras, each other passing the strength

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[100] Carus 2016, p.17.

[101] Untill the year of 1500, about 39 universitiies wold be founded in medieval Europe. See Le Goff 2007.

[102] Le Goff 2007, chap. 5.

[103] Carus 1985, 273.

and connections of the stone, so that this force is applied without interruption."[104] And although the most varied speculations about the real mechanism of action of the magnet, the elements that would mark the next advances in the area would be expressed in a millennium,[105] in medieval scholasticism, especially with the epistle of Pierre de Maricourt (*Petrus Peregrinus*) on the magnet, written in 1269.[106]

In a message addressed to a friend,[107] Maricourt says that while providing services on a military expedition, he conceived a mechanism capable of keeping an armillary sphere in motion, which was an astronomical instrument applied in navigations as a reduced model of the Ptolemaic cosmos, i.e., the great outer globe represented the celestial dome and the small central ball, the Earth. In this theoretical mechanism, a wheel encrusted with pieces of iron would be moved by the magnetic force, a kind of rudimentary magnetic motor, of incessant movement (*perpetui motus*), which was never able to build. More specifically, in the first part of the epistle, Peregrinus was concerned with explaining his discoveries about the magnet and, in the second, with the magnetic machine and the perpetual motion itself.

Before that, however, Peregrinus reports practical experiences. After specific considerations on how to machine the stone so that it had a spherical shape (Figs. 1), it indicates defined positions for what he calls *poles*, establishing the first bases of magnetic science: "... With an instrument with which crystals and other stones are rounded, let a loadstone be made into a globe and then polished. A needle or an elongated piece of iron is then placed on top of the loadstone and a line is drawn in the direction of the needle or iron, thus dividing the stone into two equal parts."[108] Additionally, he demonstrates that a fragment of a magnetic needle is still magnetic. Then, he makes practical indications for determining the poles, and to check which ones repel or attract. He also shows that a pole of greater strength can not only neutralize a weaker one with the same name, but also reverse its polarity.

Peregrinus' reputation would reach the other side of the English channel, when Friar Roger Bacon (1214-1294), of the then young university of Oxford, did not hide his admiration for the mathematical knowledge and technical skills of his Gallic colleague. In one of his most influential works, *Opus Tertium*,[109] Bacon presents the peculiar philosophy

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[104] Carus 1985, 269.

[105] It is even possible to see similarities between Lucretius' description and Descartes' vortex theory, in 16th century, and even more, the mention of iron filings leaves the question of whether Lucretius would have faced the magnetic images that would have great importance in Michael Faraday's conceptions of field theory, almost twenty centuries ahead. For a more detailed discussion on this topic, see Tonidandel, Araújo, and Boaventura 2018.

[106] Peregrinus 1904; A comentary on the epistle of Peregrinus can be found at Martins 2017.

[107] The letter was titled *Epistola ad Sigerus*. A 1558 edition of the letter addressed to *Sigerus* had the title: *Petri Peregrini Maricvrtensis: De Magnete, his Rota perpetui motus, libellus*. Peregrinus 1904.

[108] Peregrinus 1904, p.7.

[109] R. Bacon 1912.

of Pierre de Maricourt as the result of a great experimentalist, who used the technique instead of the usual "arsenal of words". The beautiful tribute, however, although offered by an eminent member of the scientific brotherhood, was not enough to grant Maricourt the necessary recognition. This is possibly due to the influence of Aristotelian thought in the middle ages, which should face an experimental approach with suspicion. In fact, the validation of knowledge from empirical data would not find many supporters before the 17th century, given the belief that this form would not be able to provide a satisfactory, or even valid, scientific explanation, given its heretical potential. Considering also the difficulty of propagating manuscripts before the press, Maricourt's discoveries fell into relative oblivion, being ignored in almost all of europe, at least until the year of 1600, when the letter was quoted in William Gilbert's book (1540-1603) *De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure*,[110] or "On the Magnet, Magnetick Bodies also, and on the Great Magnet the Earth".[111,112]

The letter of Peregrinus is currently considered a milestone in both experimental physics and electrical engineering, and his work shows that the experimental method was already being masterfully used in medieval Europe, contrary to what is commonly thought, two centuries before its pretended creator, Francis Bacon. It is no coincidence, moreover, that Francis Bacon had a public antipathy for the work of William Gilbert,[113] who was his contemporary and promoter of the epistle of Peregrinus, as a "... extravagant speculation based on insufficient data". In it, Gilbert traced the persistent influence of classical analogies on the attraction and repulsion of bodies and would enter for history as a great experimentalist, having also made important theoretical considerations, not restricting himself only to magnetism. And since the genesis of a concept is often different from its later formalizations, perhaps to make the reader unaware of the original paths followed by the pioneers, it is important to emphasize the erroneous view that an "outdated" theory must be forgotten.

Gilbert begins by doing a historical survey, comparing previous theories to his own reflections and weaving a clear differentiation between what would be a magnetic phenomenon around the loadstone and an electrical phenomenon, historically related to

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[110] Gilbert 1600.

[111] Gilbert's work would again be popularized at the end of the 19th century, with the publication of a pair of translations that were the subject of a divergence between Silvanus P. Thompson, who would publish a commemorative translation in the *De Magnete* tricentennial, in 1900, and the publisher Willey & Sons, which had published its own version a few years earlier (1983), translated by the american P. Fleury Motellay. Thompson had publicly disclosed his intention to translate Gilbert's work from Latin and claimed to have been harmed by the earlier publication by publisher Willey. The controversy can be followed in the pages of Science magazine, which presented a favorable position to Mottelay, written by an "hidden author" F. 1893; and in the British Nature, which contained the manifesto of Thompson 1902.

[112] Silvanus Thompson's edition of the book *De Magnete* can be found at Gilbert 1900; Mottelay's edition, in turn, can be found at Gilbert 1893.

[113] F. Bacon 1889.

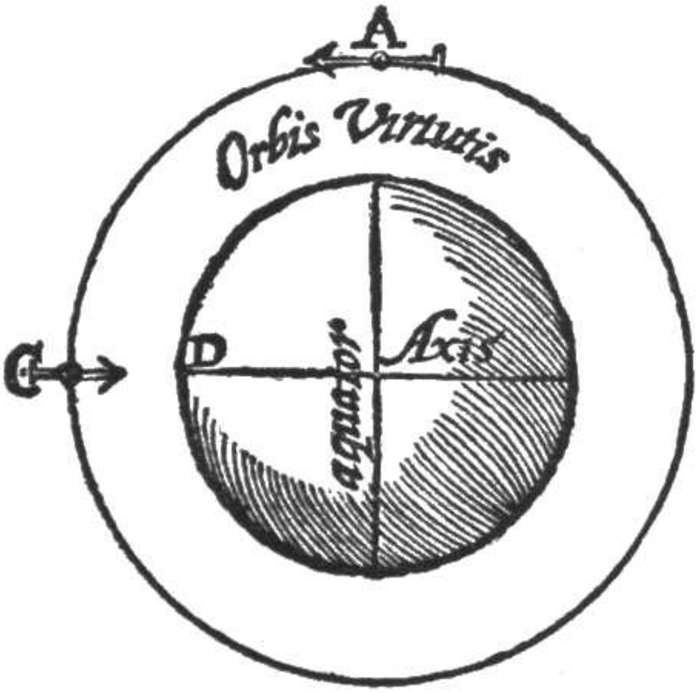

Figure 1 – The *terrella*, a model idealized by Maricourt and replicated by Gilbert for the study of magnetism. Source: Gilbert 1600, p. 97.

the figure of amber. Immediately, it refutes the old idea of the supposed power of attraction exerted by fire, defended by the writers of the *Materea Medica* like Galen,[114] who believed that an object could be attracted for three reasons: its essential or substance quality, i.e., if something has internal heat it is attracted by heat; by the vacuum suction mechanism, as found in medical instruments such as cupping glass;[115] and, finally, materials that would be attracted by verisimilitude, such as the *homeopathic* principle of "like attracts like".

In short, a stone was not attracted by stone, just as meat did not attract meat. Even certain materials, when heated, lost their power of attraction, as in the case of amber.[116] Even sunlight, concentrated in the legendary "burning mirror" of Archimedes,[117,118] was

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[114] Gilbert 1900, p. 50.

[115] According to the free dictionary, it is a glass vessel, from which the air has been exhausted by heat or a special suction apparatus, formerly applied to the skin in order to draw blood or other liquids to the surface.

[116] Gilbert, even proposes a challenge for those who wanted to prove the statement and build a versorium for themselves, to test the theories of attraction. Several experiments carried out by Gilbert had the support of a Versorium, which was one of the oldest laboratory instruments then created. It consisted of a mobile horizontal needle on a fixed base that could rotate freely, and was used to detect small variations in strength, such as the attraction of a piece of fabric by means of amber or a piece of iron by the magnet. Gilbert describes its construction in detail in Gilbert 1900, p. 48; interestingly, the term appears in Mottelay's translation with the name "electroscope": Gilbert 1893, p. 150.

[117] The burning mirror is a kind of concave mirror used at the time as a solar concentrator. The legend tells us that Archimedes built it in the second century BC to protect the city of Syracuse from Roman invasions. Some of Archimedes' achievements are also known from Plutarch's writings. Plutarch 1917, p. 241; A portuguese translation of Archimedes' Methods can be found at Magnaghi and Assis 2019.

[118] The story also appears in Cicero's work (106 - 43 B.C.). See at Cicero 2019, p. 21; Another portuguese

able to dissipate the electric effluvium from amber,[119] defended by writers like Plutarch (*Quaetionibus Platonicis*), about the existence of a thin layer of substance that, in some way, would make the air around the object more rarefied, which would cause air to flow from the densest to the least dense point.

However, Gilbert does not reject the existence of the *electrical effluvium*, nor does he completely rejects the possibility of attraction between "related" materials. Although he refutes Plutarch's theory of magnetic vacuum suction, he does not reject that the magnet's "soul" could somehow grant properties to iron.[120] However, he assignes the mechanism to some imponderable medium, a phenomenon that not necessarily depended on material interaction, unlike an electrical phenomenon. One reason for this belief was to have demonstrated that a flame could eliminate the powers of electrical attraction of some bodies.[121] Magnetized iron also lost its magnetism when heated to the point of blush – what today would be called *Curie temperature* – but this phenomenon gave a different explanation: as he observed that magnetism could act even between thick barriers of dense matter, Gilbert suggests that something went from the magnet to iron, and the loss of magnetization in the case of heating should be due to a deformation in the intimate structure of the iron.

The best explanation for this is the animistic view of some materials, which Gilbert did not want to hide. The loadstone was "... a wonderful thing in very many experiments, and like a living creature. And one of its remarkable virtues is that which the ancients considered to be a living soul in the sky, in the globes and in the stars, in the sun and in the moon."[122] Although Aristotle endorsed the soul only to celestial bodies and not to the fixed Earth, Gilbert opposes this position, suggesting that the terrestrial globe with its emanations, for the same reason, could not be "... condemned to exile":[123] "... We, however, consider that the whole universe is animated, and that all the globes, all the stars, and also the noble earth have been governed since the beginning by their own appointed souls and have the motives of self-conservation."[124]

In relation to the movement of celestial bodies, Gilbert presents his defense on the terrestrial movement, leaving between the lines his adherence to the heliocentric theory of Nicolau Copernicus (*De revolutionibus orbium coelestium*, 1543). He shows inconsistencies present in the aristotelian idea of a fixed world to explain the precession of the equinoxes[125] the movement of the tides and the tendency of bodies to be attracted to the center, an idea that would be developed by Newton from the conception of central force. The explicit

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translation can be found at Cicero, "De Republica (in Portuguese)", p. 285.

[119] Gilbert 1900, pp. 48-49.

[120] Gilbert 1900, p. 229.

[121] Gilbert 1900, pp. 52-53.

[122] Gilbert 1900, p. 208.

[123] Gilbert 1900, p. 209.

[124] Gilbert 1900, p. 209; In fact, the metaphor of "soul of the Earth" with its own conservation motives has been rekindled in contemporary debates about environmentalism. See, e.g., Lovelock 2011.

[125] Gilbert 1900, p. 227.

non-adherence to the Copernican model was understandable,[126] given its highly heretical character, which would be the engine of the accusations against Galileo Galilei, some years ahead, between the decades of (1610-1630).[127] From this point on, Gilbert deals with the distinction between what would be, by nature, "electric" and what was not, by saying that non-electric bodies do not emit a certain type of effluvium. The electrical properties of attraction, according to the idea, only happened when effluvia of the same nature "touched" themselves, so to speak.[128] This principle would have a great influence for more than a century, which can be seen in treatises on the strange subtlety of Boyle's effluvia,[129] or on Benjamin Franklin's theory of electric fluid.

Just as the origin of magnetic activity was, in his opinion, the center of the Earth, the activity of the magnet originated in what he called *terrella* (Fig. 1), which was the center of a magnet stone previously prepared in spherical shape, like Petrus Peregrinus had done 300 years before: "... The centre of magnetick virtues in the earth is the centre of de earth; and in a terrela is the centre of the stone."[130] This is a relevant point, given that the method of preparing the magnetic samples, after a machining process, facilitated the identification of the poles, as Peregrinus had stressed (Fig. 2). This may have suggested to Gilbert, in addition to his own convictions and experiments, what would be his greatest discovery: the analogy of the terrestrial planet as a great magnet, *Magno Magnete Tellure* or The Great Magnet the Earth.

The motor of magnetic activity was, for Gilbert, the movement of the Earth, that "... awakens the internal breath of the globe."[131] The movement of a conductive nucleus in the center of the Earth, possibly iron, was responsible for the generation and maintenance of the (today called) magnetosphere, an anticipation of the dynamo's geomagnetic theory.[132] Thus, the rays of the "magnetic virtue" would spread in all directions of the orb, showing that the action of the ancient stone of Heracles had a certain region of influence,[133] although different from the amber's electric effluvium, which appeared when it was rubbed. Orb is a synonym of planet or spherical object, and Gilbert uses the same term to refer to both Earth and other celestial bodies, at times, as to the spherical-shaped

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[126] A closer look at Gilbert makes his position clear, when he shows, for example, how ridiculous it would be to imagine an entire universe revolving around the Earth, instead of the other way around. Gilbert 1900, p. 220.

[127] The full adherence to the Copernican model would appear in a second work by Gilbert, *De Mundo Nostro Sublunari*, which would be published in 1628, after his death. See in Gilbert 1628.

[128] Gilbert 1900, p. 56.

[129] Boyle 1673, v.3.

[130] Gilbert 1900, p. 96.

[131] Gilbert 1900, p. 226.

[132] In a recent article in Nature magazine it was shown that not only the crust, but also the Earth's mantle contributes to the formation of the magnetosphere, showing that Gilbert's conjecture was correct, when he presented Earth's magnetism as an internal virtue generated through its rotation. Recent research has shown that even materials in extreme conditions, some between 410 and 660 km below the crust, participate in the generation of the magnetosphere. See in Kupenko et al. 2019.

[133] Gilbert 1900, p. 206.

loadstone or the region of influence of a magnet. Magnetic action, for him, could not exist in the air or any medium that was not, in turn, magnetic. Thus, iron would only become magnetic within the sphere of influence of the loadstone and the magnetic medium would have no existence of its own.[134] This dismisses the suggestion that Gilbert could have conceived a "field theory" for magnetism, but opens up a loophole for what would become the conception of action at a distance for a force.

Figure 2 – A blacksmith forging a piece of cast iron for experiments with the magnet. The words *avster* and *septentrio* indicate the southern (= south) and northern (= north) polarities. Source: Gilbert 1600, p. 139.

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[134] The observation is based on Gilbert's statement that the supposed sphere of influence of the magnet would not have its own existence "... or the forms are only effused and really subsist when magnetick substances are there. ... for the magnetick force does not pass through the whole medium or really exist as in a continuous body; so the orbes are magnetick, and yet not real orbes nor existent by themselves." Gilbert 1900, p. 205.

4 Mechanical analogies

> *... my goal here is to say that the Celestial Machine is not like a divine or animated being, but like a clock – and this clock, which is believed to be animated, depicts the glory of the Artisan.*
>
> (Johannes Kepler, in a *letter to the Bavarian Chancellor*, 1605.)

Science in the beginning of modernity had witnessed important movements. The occupation of the new world, the gradual acceptance of the heliocentric system, the press, the telescope, the universities. Movements that had disgust for"hidden" or"esoteric" causes would give rise to a process of valorization of reason, marking the transition from mechanics as a craft to a genuine science, opening space for adoption of mathematics as a scientific language.[1] The first action against the"old" philosophy then begin with the adoption of experimental control in evaluation of ancient certainties, forming the prodromes of a new philosophy of nature, as a gradual construction upon successive invalidation of previous theories. Until 17th century, what was missing for the science student, then called Natural Philosophy, was, in general, a methodological foundation on which one could prove the veracity and precision of facts. In fact, since Francis Bacon, the issue of empiricism was being drawn from a confirmation bias, that is, from the fact proved positively through the measurement between what was expected and what was obtained by a certain metric, which would then make possible generalize and predict new phenomena. From here, the word"fact" can be easily replaced by "data", considering the notion of datum as a facsimile of purity, not associated with any preconceived idea.

4.1 New old panorama

After Galileo Galilei (1564-1642), the universe gains the dimension of a book, a great book of nature,"... that one cannot understand until understand the language and know the characters with which it is written" .[2] Now, what is real must be factually controlled, measurable and concrete, and it is important to accept a "mathematical fact" and start from it to adapt a theory, if necessary, so that it fits to data.[3] It was necessary not only to confirm the truth of a theory, but to ensure that it could be challenged. It is not absurd to say that the great turbulence of the period, which would later be called *Scientific*

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[1] Le Goff 2007; See also Long 1997.

[2] Galilei 1983, p. 130.

[3] This does not mean, however, that the past ideas of medieval science were not controlled by the facts. Clemency for simplicity, born in medieval basements with Reverend Okham, would be adapted as a criterion to define the limits of scientific knowledge or, at least, to create a clearer separation between science and religion. See, for example, Collingwood 1945, part ii.

Revolution, is due less to the innovative character of the discoveries in the various fields of knowledge than to this change in the structural panorama of natural philosophy, an innovation methodological as a criterion for a theory.[4]

From now on, theories that use concepts considered obscure should be reviewed, but not totally abandoned, because, in general, it is not appropriate to think that the "modern" scientific panorama, in the sense of its participants, was characterized by a totally *a posteriori* attitude, or "without hypotheses" – as is usually thought, as hypotheses continued to be used. However, there is a clear opposition to the abuse of *ad hoc* hypotheses, i.e., those that could hardly be proven. They would come to be considered the fruit of a period rich in elements that go beyond the fantastic, and for repeated reasons, classified as legends that explained almost everything and almost nothing, at the same time.

A classic and widely debated example concerns the finding that certain astrological phenomena would not find support within the new program, starting (only) a split between astrology and the new astronomical science, as well as in other pre-modern sciences, such as alchemy and chemistry. And it is not an unknown fact that, throughout the 17th and 18th centuries, great natural philosophers such as Galileo, Kepler and Newton made regular use of them. The "post" of *Mathematicus*, held by Kepler at the court of the king of Bohemia, explicitly required astrological practice among his duties.[5] Galileo made astrological charts on demand. Newton secretly devoted many years of his life to studies on Alchemy, which at that time was considered fraudulent and forbidden, even liable to be condemned to hanging.[6] This, being considered an abject practice , as a remnant of a time when magic rituals were feared, usually had its experiences reported by the alchemist in a highly encrypted language, to be kept secret.[7] If an alchemist discovered any of these secrets, he qualified himself for a greater understanding of Nature.[8]

Although presumably supported by facts, theories of this nature would no longer find support within the new conjuncture and needed to be reformulated, with the difference that, from now on, an acceptable theory would gain a new aspect, being able to perpetuate itself as long as it was not refuted by the sieve of experimentation.[9] To say, however, that they rejected such conceptions just because of the new character of science is not entirely correct. Although with a renewed attitude, eighteenth-century science followed more or

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[4] In recent terminology, this innovation encompasses the principle of falsifiability. See chapter 2.

[5] See Field 1984, p. 190.

[6] See, for example, Dobbs 1982.

[7] For example, a synthesis of a certain biphasic metallic alloy was described as "conception of a hermaphrodite creature". The search for the mythical philosopher's stone did not aim to obtain, through shady processes, the vile metal, as is commonly believed, but was part of a search for divine attributes, being "God" understood as the essence of Nature (with"capital N) itself, as in Espinoza's conception, *Natura Naturata*.

[8] Recent research has been conducted to decode Newton's alchemical experiments, remaking his experiments, in where it is argued that even Newton's fundamental discoveries, such as the separation of colors, had great influence of his alchemical research. See in Newman 2019.

[9] Popper and Eccles 1977.

less the same rite that has been perpetuated, in one way or another, until these days: basic analogies being replaced by new ones, but still grounded on one or another metaphysical foundation. In this respect, a scientific theory is hardly completely dissociated from its *a priori* character.

The basic categories present in Greek conceptions were gradually being replaced by counterparts more suited to the new program. If an naive analogy was used to describe the digestion process, such as that fire that used to “appropriate itself of food” as in Platonic writings,[10] this automatically referred to an ancestral conception that extrapolated the idea of substantialization, even reaching a personification of fire.[11] This type of construction should be abandoned, not because of its fanciful character, but because comparisons with animal and human life were generally incompatible with a descriptive language such as geometry, making difficult the validatation through empirical data. In this context, it is clear that Gilbert’s studies on electricity and magnetism seem to be inextricably linked to the“pre-scientific” period, especially for his magnetic conception as something alive, possessing a soul, or for his electrical effluvium.

The trend of the new times was to think of the universe no longer as an organism, but as a large autonomous machine, put into operation by a wise artisan.[12] Thus, it would be more comfortable to consider the soul as having some counterpart, an equivalent to that“something indestructible” present in those virtues sung in prose and verse by the ancients. Concepts of antiperistasis, the horror vacui, the attraction power of *logos* and fire, of an autonomous soul, pneuma or animating breath and of attraction for similitude were being replaced by fundamental elements that would encompass the physics of following centuries: movement of waves in space, movements of corpuscules of light or heat, the attraction of a force.[13]

As side effects, such requirements would largely reduce the“search tree” of physics, a relief when thinking about a controllable universe with fewer variables, easier to be confronted with observation and measurement. This also shows that the formulation of theories has, as a subjective criterion, the filtering of what is considered most important by its proponents, which is why Gilbert will be best remembered for his analogy with a great terrestrial magnet, Newton will be remembered by his mechanics instead of his alchemy, or Descartes will be better known for metaphysics instead of his writings on physics.

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[10] See Plato 2013, §79d.

[11] This strategy is still used in works of high contemporary success, as in Richard Dawkins’ *selfish gene*, in which the author seeks to make a “reversal” in concept of life which, according to him, consists of a stratagem of the gene to replicate itself. See in Dawkins 1976.

[12] See section 4.2, p. 70.

[13] The search for a mechanical equivalent of soul would be at the heart of the formulation of the princicple of conservation of energy, in 19th century, as we can see at Mach 1854; But in earlier formulations can also be found, as in Kepler’s *species motus* Kepler 1609, p. 173; similarly, the formulation of living force or *vis viva* by Leibniz and, later on, by the Marquise du Châtelet, shared similar goals. See in Musielak 2014.

4.2 The solar virtue

The phenomena associated with magnetism or falling bodies found difficulties to be explained based on new ways of structuring knowledge. In case of tides, observed since ancient times, it was believed that the moon should exert some kind of influence by its brightness or even its radiated heat, the same way the "suction power" of solar heat would probably cause the phenomenon of evaporation. This thought was extended by medieval writers, such as St. Thomas of Aquinas,[14] stating that, yes, "... the flow and reflux of the sea do not result from the substantial form of the water, but they happen by virtue (of the movement) of the moon."[15] The causation of the tides, however, should be in a certain"hidden species", which with the movement of the Moon would print an action in the ocean, causing its elevation.

Still according to Aquinas, "... (the) natural bodies participate in certain hidden virtues, resulting from the *species*, by impression of the celestial bodies."[16] For one of the fathers of the Roman Church, at least, this species was similar to the magnetic action and would be emitted by the moon,[17] a concept that would become popular in other medieval theories, such as in the "multiplication of species" by friar Roger Bacon. While studying possible causes for radiation of light and heat, Bacon proposed the idea of *Species Magnetica*,[18] having been one of the first to make a clearer distinction between transmission (of the *magnetic species*) and bodily emanation, as in the atomist theories of Lucretius and Francis Bacon.

This idea flirts with the thinking of astrological traditions and would find in Johannes Kepler (1571-1630), more than in the prominent school led by Galileo, an element that could connect possible gravitational effects to external action. Gilbert had defended an idea of attraction through an internal action, of magnetic attraction as a virtue originating in the center of the material, or from the contact of the object with the region of influence of other "related" material, as in the pair formed between iron and loadstone. Kepler, on the other hand, who had a favorable, though unpopular, attitude to astrology,[19] combined with a strong intuition about the mathematical possibilities of his new astron-

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[14] Tomas Aquinas 2017.

[15] Tomas Aquinas 2017, Art. 3, p. 867.

[16] (Emphasis added). See in Tomas Aquinas 2017, Art. 2, p. 2347.

[17] It is worth mentioning that in the same section, Aquino attributes magnetic powers to the Diamond. If such information was not mistakenly replicated from the Greek to Latin translation of Augustine's work, which the author then commented on, it may have its origin in a misreading of the ancient philosophers, who were also the object of Augustine's analysis. It is notable that Plato made a reference in Timaeus about a legendary stone named *Adamas*, which was translated as *Adamanta* or diamond. It is very likely that adamanta is, in fact, hematite, the mineral that makes up the magnet stone. Cf. e.g., Plato 2013, S 59b.

[18] *De multiplicatione specierum.* Cf., R. Bacon 1912, Ch. vi, p. 34.

[19] Westman 2001.

omy,[20] does not follow exactly the Gilbert approach for the gravitational case.[21] Although he saw in the case of the tides a magnetic influence on the interaction between distant bodies, the root of his hypothesis for external action lies, in addition to his own personal and mystical convictions, in an analogy with the figure of the Sun as the source of a certain virtue, the physical cause of phenomena, and of the universe as a big clock. It is worth remembering that the window to the"universe" at that time it was much smaller, which would be today, with some restrictions, the solar system.

Kepler is not enough to generalize that all matter could exert mutual influence, but believed that this was the case between the Earth and moon, which had direct influence on the phenomenon of the tides, and the interaction between the sun and the five planets known in his time. In his youth, Kepler (1596) had suggested a geometric analogy between the proportions of the orbital distances of the five known planets, in addition to Earth – Saturn, Jupiter, Mars, Venus and Mercury, the main astronomical parameters of that time – and the famous platonic solids.[22] Kepler had been influenced by reading the ancient classics, especially the Pythagorean dialogues expressed in Plato's *Timaeus*,[23] in addition to the revolutionary work of Nicolau Copernicus, who proposed an alternative to the old system of Ptolemy, placing the Sun at the center of the cosmos. Guided by the possibilities, Kepler elaborates the iconic system of regular solids sequentially inscribed inside one another (cube, tetrahedron, dodecahedron, icosahedron and octahedron), which represented the proportion between the average distances between the orbits, with the Sun at the center (Fig. 3). From the same system, he establishes a relationship between them and the four elements of Empedocles (Earth, fire, water and air), in addition to the quintessence, the ether.

After having access to the records of Tycho Brahe – who had invited Kepler to work with him – Kepler realizes that the planetary orbits were not circular, as in Copernicus' theory (Fig. 4), but ellipses , with the Sun in one of the foci. Brahe had died early one year after Kepler's arrival, who would come to assume his position, with full possession of the data collected by the master. These data would have great influence on Kepler's entire life: Tycho Brahe had been the great astronomer of his time, in a period prior to the invention of the telescope, having collected observational data for almost three decades. However, it may be unreasonable to claim that the mere detection of an elliptical orbit, Kepler's first major discovery, in all its complexity, should have suggested the hypothesis of forces acting at a distance between Earth and the Sun.

Since the medieval age, astronomy had been considered more as a branch of ge-

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[20] Kepler 1609.

[21] Gilbert 1900, p. 226.

[22] Kepler's proportional analogy led to the following relationships between the planetary orbits and the regular solids: Saturn ≡ cube, Jupiter ≡ tetrahedron, Mars ≡ dodecahedron, Venus ≡ icosahedron and Mercury ≡ octahedron. See in Kepler 1596, p. 24.

[23] Plato 2013.

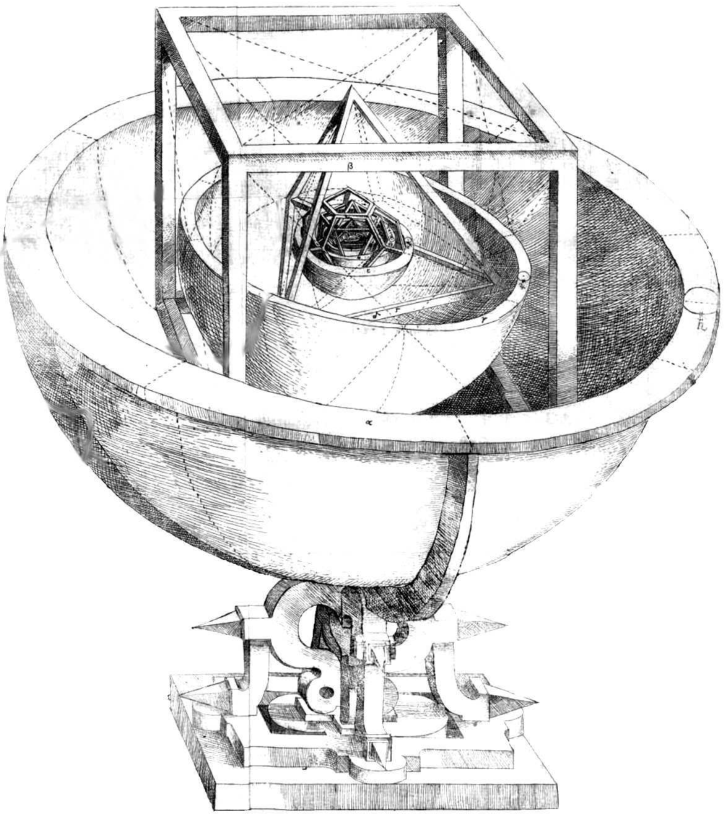

Figure 3 – Kepler's geometric analogy: a first model of the cosmos where proportions in the interplanetary space are presented as Platonic solids. Source: Kepler 1596, p. 24.

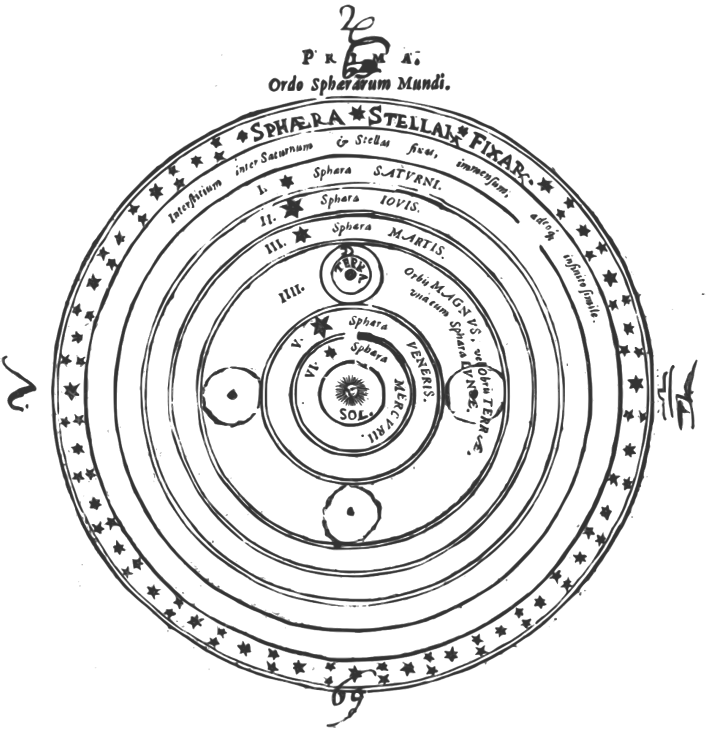

Figure 4 – The Copernican cosmos in the view of young Kepler. Source: Kepler 1596, p. 117.

ometry than an established science. Mathematics did not yet enjoy the *status* of universal language that it has in contemporary scientific thought, but had in geometry a preferable character, compared to other branches of mathematics, such as arithmetic, for example. Even an algebraic problem – like solving an equation – was thought in pictorial terms, where solutions were sought through the representation of a geometric structure. An emerging belief was that what had become true in geometrical terms should also be true for the celestial case.[24] And if Astronomy was a *celestial geometry*, as such it would act (in the relativism of the time) in the new conception of the cosmos. That is, why not think of the celestial movement as a geometric problem, where changing the reference point can simplify its resolution?

"Removing" the Earth from the center of the cosmos, shifting the focus of attention to the Sun, meant not only an enormous simplification in the geometric problem. In fact, it eliminated once and for all the intricate epicycles system of Ptolemy for planetary movements, but it also represented a whole change of aesthetic and methodological character, inspired by the reborn ideals of classical harmony and beauty, expressed in the work of Kepler and other thinkers and artists from that historical moment, as a return to the valorization of human reason and dignity. It is not surprising, moreover, that the Copernican model (Fig. 4) referred to the figure of a clock, an artifact that was already well known then.[25] In fact, in a letter written in 1605, in which he presented his New Astronomy to the Bavarian Chancellor, Johannes Kepler (1605) made clear this objective, by stating that, " ... my objective here is to say that the Celestial Machine is not like a divine or animated being, but like a clock (and this clock, which is believed to be animated, depicts the glory of the Artisan)."[26]

A year earlier (1604), in his *Ad Vitellionem Paralipomena*,[27] the first of the two books in which he studies the transmission and refraction of light,[28] Kepler hypothesizes that light would be nothing but light itself in motion.[29] Although its purely speculative character, he proposed that the intensity of the light emanated by Star King would remain constant in each ray,[30] decreasing in one spherical surface in proportion to its distance from the light

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[24] See Burtt 1923.

[25] One of the first mentions in Europe to a mechanical clock dates from 1300, when Dante Alighieri portrayed it in one verse from *The Divine Comedy.* The knowledge about manufacturing of mechanical clocks not based on water flow as a driving force is believed to have occurred around the 13th century, when trips to China began to be reported. Interestingly, such stories coincide with descriptions on magnetism and perpetual motion. Cf. de Solla Price 1964, p. 17.

[26] (free trans. of) "... *Scopus meus hic est, ut Caelestem machinam dicam non esse instar divinj animalis, sed instar horologij (.qui horologium credit esse animatum, is gloriam artificis tribuit operj.)*". See in Kepler et al. 1951, p. 146.

[27] Kepler 1604.

[28] Kepler's second book on this subject can be found at Kepler 1611.

[29] Kepler 1604, prop. vii, p. 9.

[30] "... *Sicut & habent spherica superficies, quibus origo lucis pro centro est, amplior ad angustiorem: ita se habet fortitudo seu densitas lucis radiorum in angustiori, ad illam in laxiori spherica superficie, hoc est, conuersim. ...* " See Kepler 1604, prop. ix, p. 10.

source, a principle that has been recognized since antiquity. From geometric considerations, it was possible for Kepler to state that the luminous intensity would decrease in the inverse squared proportion to the distance from the Sun.

While maintaining the central reference in the figure of the Sun, Kepler abandons the Copernican conception of natural and circular movement for the planets, and adopts, in his *Astronomia Nova* of 1609,[31] the analogy of *species motus*[32] as an emanating magnetic virtue,[33] that departs from the Sun and reaches the planets : "... the Solar matter is magnetic and, in turn, the surrounding space..."[34] In other words, the luminous intensity could be obtained by dividing the intensity of the source by the sphere area, considering the light diffused from a single point,[35] symbolically expressed by the relation $I \propto 1/d^2$. Interestingly, when Kepler discovered that the planetary orbits were not circular, he did not suggest that the (also called by him) *virtutis solaris*,[36] or "solar virtue", his conception of motive force, acted on an elliptical manner: no, it acted spherically, as well as light, although it had a magnetic nature (Fig. 5).

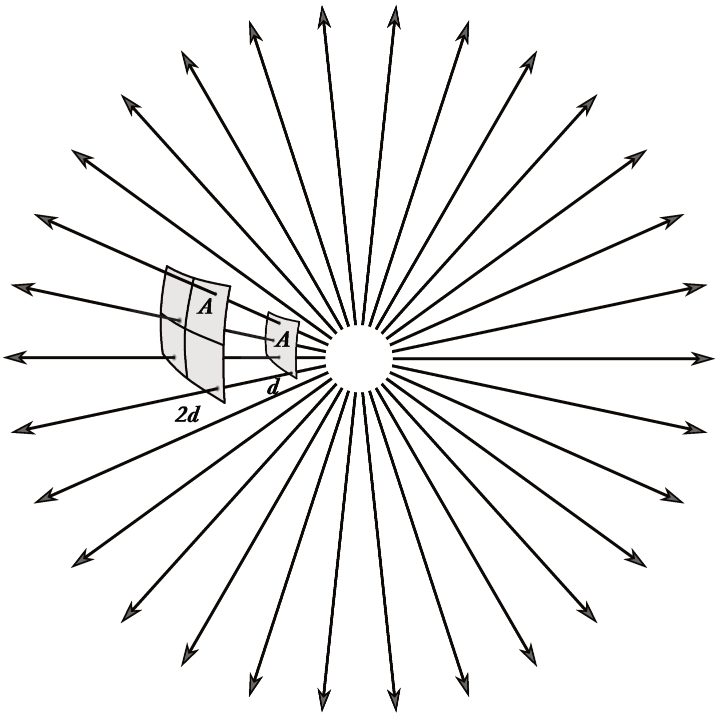

Figure 5 – A representation of the law of the inverse square of the distance, in which the light intensity of the source (Sun) decays with $1/d^2$.

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[31] Kepler 1609.

[32] Kepler 1609, p. 173, p. 240, p. 388.

[33] Kepler 1609, p. 308.

[34] Kepler 1609, p. 172.

[35] Currently this notion can be understood within the concept of energy *flow*, in that the intensity I can be considered as energy flow through an area A over time t: $I = \frac{E}{A \times t}$,, for any type of energy (heat, light, etc) emitted by a source.

[36] Kepler 1609, p. 35, p. 175.

Adopting the Aristotelian notion that every movement requires a *first motor*,[37] Kepler considers that the first component of the *solar virtue* was due to the movement of the Sun around its own axis, similar to what had already been proposed by Gilbert, in his analogy of the great magnet. Gilbert had conjectured that a *terrella*, which was his analogue to Earth, i.e., a perfectly machined magnet in spherical shape, would be able to rotate on its own axis in a period of 24 hours.[38] The rotation of the Sun, being tangential to the planetary movement, could be responsible for maintaining the orbits. Furthermore, the *species motus* acted, in some way, only in the plane of the orbits, without being given a more consistent explanation. Having initially admitted that this action obeyed the principle of the inverse square of distance, his observations would be consistent with the new discovery: the relationship between the distances from the planets to the Sun, their speeds and translation times, later called Kepler's second and third laws.[39]

Astronomia Nova[40] is considered Kepler's most important book and differs significantly from the former, such as *Mysterium Cosmographicum*,[41] especially after contact with the observations of Tycho Brahe, who had given Kepler data regarding his observations of planet Mars. Kepler's analogy of the *solar virtue*, unlike previous conceptions, is clearly a more geometric entity. For this and other reasons, the work is considered as one of the pillars of modern astrophysics. However, although he values the strength of the observations in his theory, and especially in the conception of the driving force that acted according to the inverse-square law, Kepler defends the production of knowledge from an *a priori* view, placing it in the same level as the metaphysical conceptions of his youth, in which the figure of the Sun appears in the"center of the world" as the divine matrix, an undisputed source of harmony and beauty: "... In addition, if the same thing that I have already demonstrated a posteriori (with observations), I say a priori (in line with the dignity and prestige of the Sun) showing that it is the source of life and the light of the world (which is seen in the movement of the stars), the beauty of the machine from all over the world, as well as the heat, from which everything flourishes, I think I also deserve to be heard."[42]

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[37] Aristotle 2009, §335b; Also in Aristotle, *Metaphysics*, §985a-25, See also sec. 3.3, p. 51.

[38] Gilbert's "tought experiment" comes from a similar suggestion made by Pierre de Maricourt at the end of his letter. See Peregrinus 1904; In the words of Gilbert, "*... terrellam super polos suos in meridiano suspensam, moveri circulariter integrâ revolutione 24 horis: Quod tamen nobis adhuc videre non contingit; de quo motu etiam dubitamus.*" See Gilbert 1600, p. 210, p. 220, p. 225; The same passage in the English translation can be found at Gilbert 1900, pp. 223–224.

[39] Currently the three laws are stated as follows: 1) "A planet orbits the Sun describing an ellipse, with the Sun occupying one of the foci."; 2)"The line connecting a planet to the Sun sweeps equal areas at equal times"; and 3) "The squares of the translation periods of the planets are proportional to the cubes of the major semi-axes of their orbits, or simply, $T^2 \propto D^3$".

[40] Kepler 1609.

[41] Kepler 1596.

[42] (Free. trans. of) "*... Sane si hoc ipsum quod jam a posteriori (ex observationibus) per longiusculam deductionem demonstravi, si hoc inquam a priori (ex dignitate & praestantia Solis) demonstrandum suscepissem, ut idem sit fons vitae mundi (quae vita in motu siderum spectatur.) qui est & lucis, quo totius machinae constat ornatus, qui itidem & caloris, quo omnia vegetantur; puto me aequis auribus audiri meruisse*". See in Kepler 1609, p. 169.

The translation of the excerpt shows the influence of Kepler's religious conceptions in his explanations, even though the *Astronomia Nova* was presented as a defense of the rebirth of empiricism, contrary to what appears in the *Mysterium Cosmographicum*. Kepler, who was a Lutheran, delegating to the Sun the role of the source of life and light in the world –*fons vitae mundi qui est & lucis*) – alludes to a New Testament passage (Jo 8:12), where it is said "... I am the light of the world". The mention of this detail is not accidental and is crucial to understand the role of analogies in Kepler's thinking and theories.[43]

4.3 Subtle matter

With the maturation of the new scientific proceding, universe would become "less organic" and mechanical analogies started to gain more space in elaboration of theories, which gained in "precision" but lost in "purpose". Until then, the drawn world from Greek considerations was separated into different "organs", with the Earth orb, which was located at the center, surrounded by water, then air, fire and, finally, the quintessence or cosmic ether. After the copernican revolution, followed by Kepler's theories, the withdrawal of the terrestrial planet from the center of the universe would also reverberate on what was beyond the lunar sphere. The stars would now be sympathetic when it comes to their own composition, making Earth "... similar to our skies".[44] The movement of bodies would no longer be the tendency to accomplish a certain fixed purpose: they were just movements, produced by the action of other pre-existing bodies, even if this action was caused by impact or some attraction or repulsion force.

The primary qualities gain in the pen of René Descartes (1596-1650) the nickname of simple appearances. From his conception of the absence of emptiness, Descartes is driven to a direct identification with the substantiality of space, which would necessarily be composed of bodies. Body, in his conception, would be nothing less than its own spatial extension, since this quality would be perceptible in itself, being, therefore, a truth. In his enjoyable and didactic *Discourse on Method* (1637),[45] which serves as basis for reflections on the nature of light in *Dioptric* and *Meteors*,[46] Descartes presents his first physical analogy as a mnemonic resource, whit which he aims to represent the phenomenon of refraction, keeping in mind the relationship between the angle of incidence and that of refraction. Whether Descartes' discovery of this principle was absorbed from Snell's observations (1591-1626),[47] matters less (here) than his attempt to explain luminous effects: light was an illusion.

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[43] The same passage was translated into relatively recent analyzes of Kepler's work, where this information is not considered. See, e.g., Itokazu 2006, p. 71.

[44] (Free trans. of) Descartes 1637, pp. 44.

[45] Descartes 1637.

[46] Descartes 2010.

[47] Which happened around 1621.

Kepler's view of the influence of the Sun acting (at a distance) through space on the planets was not acceptable to Descartes. And since the empty space did not exist in his conception, the light should be more like a disturbance transmitted in a subtle matter, an appearance sensed through an incompressible fluid, transmitted instantly, no matter the distance.[48] Like a blind man walking with a cane, perceiving his surroundings by tapping the floor,[49] there would be no need for something to be transmitted from the tip of the cane to the hand. Or, like wine coming out of a porous barrel, filled with grapes, where the top grapes would put pressure on the bottom ones (Fig. 6), it is only "...necessary that these pores be filled with some very subtle, and very fluid matter, which extends without interruption from the stars to us."[50] Thus, a force would not be transmitted except by direct contact, pressure or impact: light was pushed by the Sun, which was transmitted instantly, influencing the movement of the planets that naturally floated in this tenuous transparent substance, the vehicle of opaque matter and solar matter.

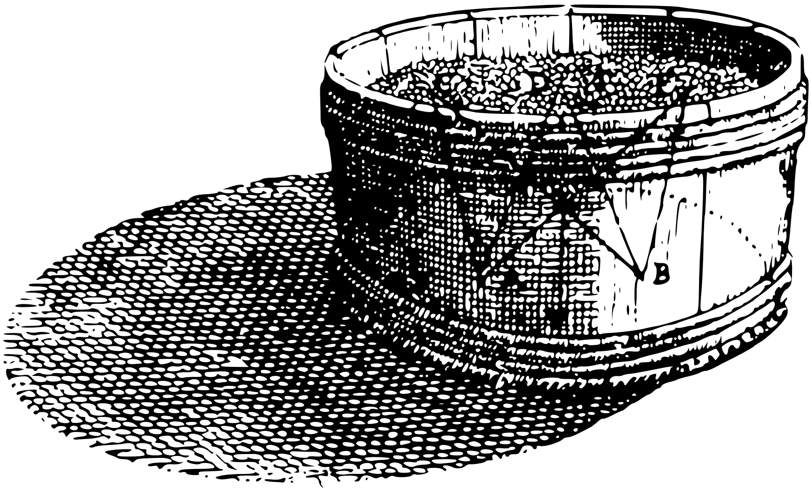

Figure 6 – A subtle matter like wine. In this analogy by Descartes, the points A and B where the wine flows represent"eyes open" in the line of action of light, that is, the act of "seeing". Source: Descartes 2010, p. 454.

The Cartesian universe was a complete set with three types of matter: luminous, which forms the Sun and stars, the opaque matter that forms Earth, planets and comets and, among them, the *Matière Subtile*, tenuous and transparent fluid, equivalent to the ether of other physical theories, composed of something like the grapes, small spheres of different sizes.[51] Therefore, it is natural to imagine that the movement of any of these spheres results in a tangle of small bodies moving together. The result, just like in Plato's circle of *periosis*,[52] where the flow occurs through the "principle of push", it becomes a joint

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[48] It is possible that Descartes had some familiarity with (what would become) the idea of conservation of *momentum*, not yet systematized.

[49] Descartes 2010, p. 453.

[50] Descartes 2010, p. 454.

[51] Descartes 2010, p. 5.

[52] See sec. 3.3, p. 51. See also Plato 2013, §79b.

and circular movement, generating a whirling mechanism or, simply, a *vortex* (Fig. 7). The dynamics of the vortexes would be at the very heart of the general movement of celestial spheres, the planets and also comets under the intricate swirling mechanism. Light would be a mere consequence (Fig. 8). Similarly, it follows that the heat would be produced by the radius of action of light,[53] and this would cause the "... continuous agitation" of the celestial particles,[54] as a kind of resistance to movement, like the heat observed in a material subjected to friction. In this respect, Descartes compares the perception of agitation to the sense of touch,[55] weaving a connection between heat and the idea of vibration.

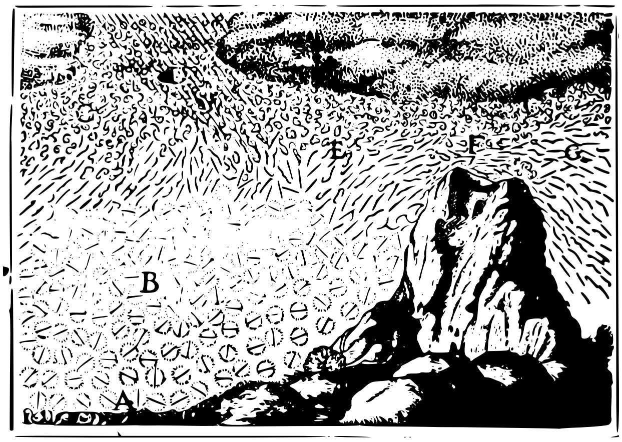

Figure 7 – Vortexes in Descartes' "liquid" sky. Source: Descartes 2010, p. 168.

In a similar fashion, Descartes applies the principle in explaining chemical phenomena and magnetic attractions. The magnetic action, which is also under the omnipresent yoke of vortexes, is explained by small particles that make up the subtle matter that pass through certain porous channels of the magnet and iron, with little or almost no resistance, and are also present around the planet, in the "sublunar layer".[56] On the outer part of the loadstone (Fig. 9), they form a kind of cloud, which surrounds its volume in an image similar to the magnetic lines of force showed by Faraday in 19th century. The direction of particle movement would be the cause of the orientation of a magnetic needle placed nearby, as well as the repulsion of equal poles and attraction of different poles. The given reason for the scarcity of magnetic materials was, precisely, the supposed absence of these pores, as those that should exist in ferrous materials. For the inventive application of his theory, Descartes tried to account for all magnetic phenomena, without postulating any other form of action besides the vortex mechanism.

Now Electric attraction, on the other hand, takes on a more confused meaning. Initially, admitting the almost impossibility and a certain lack of interest in throwing some

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[53] Descartes 1986, part iv.
[54] Descartes 1986, p. 91.
[55] Descartes 1986, p. 187.
[56] Descartes 1986, p. 234.

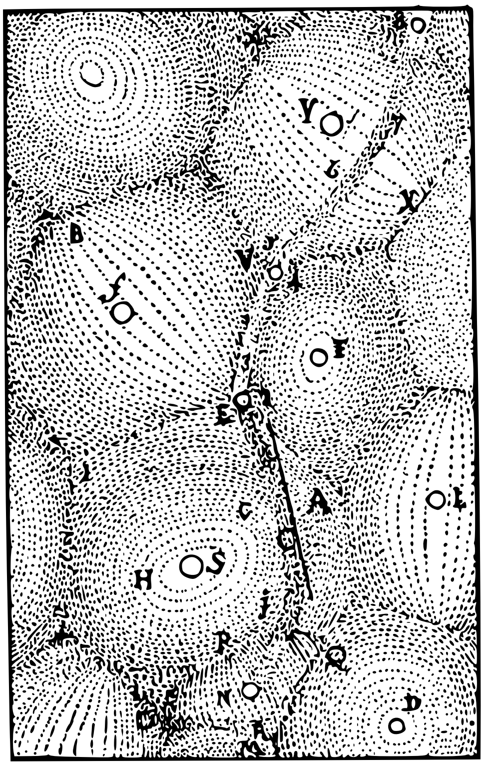

Figure 8 – The trajectory of a comet in the vortex mechanism, along the line NCEVB, passing through the Sun S and neighboring vortexes. Source: Descartes 1637, v. 3, p. 168.

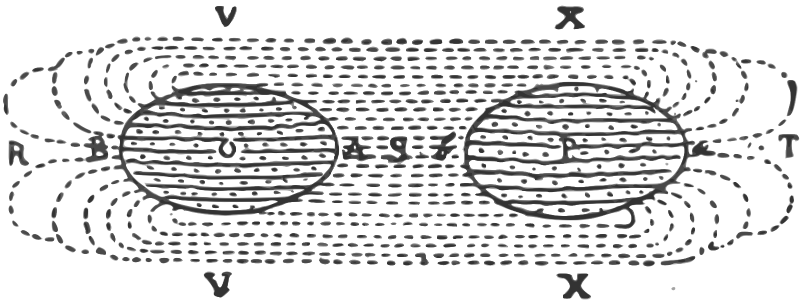

Figure 9 – Descartes's magnetic action. The traces represent the particles of matter passing through the poles of a magnet. Source: Descartes 1986, p. 247.

more light on the subject, Descartes proposes that materials such as amber, wax or the *jade* stone acquire properties similar to glass when rubbed.[57] Particles are removed from these"greasy" materials, which tend to clump together as dust near the original bodies, in the same way as a small drop of water in a laminar flow, in modern terminology, which does not seem to want to detach itself from its "companions". Making an analogy to the manufacturing process of the glasses, which take the form of the vessel where they are poured before solidifying, Descartes concludes that the electrical phenomenon consists of some type of fluidic movement. However, finding difficulties in explaining the attraction

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[57] Descartes 1986, p. 262.

process, he links the the heating generated by friction to phenomenon and states that the continuum occured from what was less fluid to more fluid – perhaps an allusion to specific mass of the substance –as a glass to air and air to a higher degree of finesse: “... It is reasonable to believe that while the most fluid in its matter passes continuously from the air to the glass and from the glass to the air, the less fluid particles found in the glass remain in the cracks that do not correspond to the air pores and there they join one another forming these strips, and hereby quickly acquire shapes so stable that they are not easily alterable.”[58]

Although it seems that Descartes’ analysis followed the same steps as Aristotle’s syllogisms, it is necessary to consider that, at this point, electrical phenomena were not of great interest as optical, magnetic or gravitational attraction, as presented in his version of the principle of inertia. For him, the luminous matter moved away from the Sun or a star due to its rotation, by a centrifugal force, an idea that was accepted for some time by its more intuitive character. But why wasn’t terrestrial matter ejected by the same principle? Simple: the non-existence of emptiness in planet’s sourrounding space prevented the substances of opaque matter from being ejected. For Descartes, the skies were liquid,[59] that is, formed from a non-elastic, “heavy”, incompressible fluid.

From the various analogies presented, Descartes’ attempt to connect his observations with his principle of rectilinear refraction of light, altough frustrated, was clear. Having published his law of refraction at 1637, he would end up “slipping” into corpuscular theory – which he did not believe – in his second discourse on Dioptrics.[60] In explaining the law of refraction, he considers light similar to a tennis ball that finds a denser medium, which would be equivalent to another “racket”. This means that its tangential speed does not change and, assuming the hypothesis (without giving a good reason) that the speed of the light beam would be different depending on the medium in which it was found, he manages to arrive at the correct form of the law of refraction.[61]

As the acceptance of a theory is directly coupled to the coherence it maintains with an entire theoretical network,[62] the physics of Descartes would fall into disrepute, although the validity of his mechanical analogies, especially when Røemer (1644-1710), around 1675,[63] found that light took time to propagate. The popular and pleasant appeal of Descartes writtings, with easy-to-see analogies, explains in part the success of his postulates in subsequent years, even after the great success of newtonian philosophy. In his own defense, Descartes would say that: “... I just tried to expose quite widely what I conceived

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[58] Descartes 1986, p. 263.

[59] Descartes 1986, p. 100.

[60] Descartes 2010, p. 457.

[61] For details, see note 26 in Newton 2017c, p. 85.

[62] See Hesse’s network theory at Hesse 1974.

[63] An analisys of Rømer’s experiments is made by Huygens in its *Traité de la lumiére*. See Huygens 1690, chap. 1.

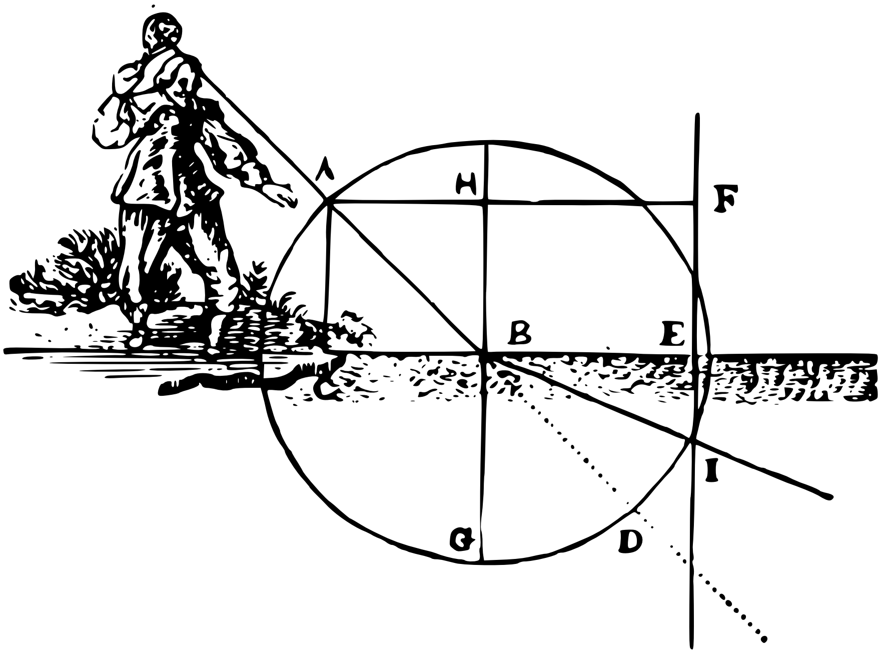

Figure 10 – Template for Descartes' law of refraction, which considers corpuscular emission. Source: Descartes 2010, p. 460.

of light; then, and at the right time, add something to the sun and the fixed stars, because light comes from almost all of them; of the heavens, because they transmit it; of planets, comets and Earth, because they reflect it; and, in particular, of all existing bodies on Earth, because they are either colored, or transparent, or shiny."[64]

However, the new perspective demanded a more concrete level regarding to experimentation and, due to this lack of connection between explanation and representation, cartesian physics would no longer find resonance, despite its influence in the rationalist movement. And even though a large part of his physical propositions were relegated to oblivion, some of his views on the mechanism of gravitation would endure, as well as his elementary concepts of geometric optics, which are still taught in textbooks. As Descartes' physics was being replaced, under the pretext of conceptual or empirical inconsistency, it would be a bridge to another view that was being developing at the same time, a corpuscular theory of light.

4.4 Elastic Effluvia

The flow analogy, for fire, heat, light or electricity would still be a universal principle of explanation until the 17th century, but now it took on a different dimension. Cartesian matter, confused with extension itself in Euclid's space, as a vast and incompressible fluid, absolutely homogeneous; portions of this universal substance could be animated by the constant whirling of vortexes. A vacuum between hard and impenetrable atoms would be

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[64] Descartes 1637, p. 43.

a mere illusion, but in the eyes of the atomist it would resemble small individual bodies. The intermediate fluid, which supposedly transmitted the force from one vortex to another, begins to assume, through a different optics, the configuration of tiny bodies, especially after the influential experiment of Torricelli.

Since ancient Greece and Rome, the knowledge on properties of fluids was being built along with the development of an “engineering”, as in the ancient construction techniques of clepsydras or water clocks and pumping of liquids, dated from the third century B.C.. Clepsydras were one of the first technological artifacts used not necessarily to measure time, but as part of aesthetic rituals to “simulate” the skies, although, eventually, they served to verify the passage of hours. The clepsydras of ancient Egypt, for example, were nothing more than vessels with a small hole in their bottom, which provided a constant water flow and made it possible to mark time.[65] Since then, the rate of water flow from an orifice was directly proportional to the height of the respective liquid column,[66] a mistake that went unnoticed even by Leonardo da Vinci (1452-1519), until it passed through the sieve of Galileo Galilei (1564-1642), in his *Dialogues concerning Two New Sciences*.[67]

In the 1630s, workers in the Italian mines faced a serious problem when realizing that the water pumps reached little more than 10.3 meters in height. Galileo had tried to explain the phenomenon, saying that a pump was able to suck air from the pipeline up to this limit, which made water take the place previously occupied by air, according to the old aristotelian aphorism of horror vacui.[68] In 1643, Evangelista Torricelli (1608-1647) sets up a similar scheme, but so that his research did not generate much fanfare, under the claim of “magical practices”, he imagined that a tube with mercury could represent the same problem, without occupying so much space, and creates a procedure that is widely discussed in elementary engineering books: place a glass tube buffered with mercury in another container that contained the same substance. After opening the tube that was partially submerged, he observed that the liquid went down to a relative height, 760 mm Hg, which is analogous to 10.3 m of water, taking into account the respective fluid densities.

Torricelli had been tutored by Galileo and continued the work of the master of Florence for the specific study of fluids. Galileo carried out important experiments on falling

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[65] See, e.g., de Solla Price 1964, p. 13.

[66] See Mills 1982.

[67] This is the best known work of Galileo, which came to appear in the *Index Librorum Prohibitorum*, the index of works prohibited by the *Court of the Holy Office* of the Catholic Church, when he was accused of heresy. The book presents a pleasant and thought-provoking dialogue, in the Socratic style (the maieutics), between three fictional characters: *Simplicio*, *Sagredo* and *Salviati*. These characters represented, in Galileo’s pen, the three great schools of thought at the time, and between the lines it was possible to observe that the first of them, Simplicio, represented the peripatetic school, i.e., aristotelian thought, which was then dominant. The second, Sagredo, represented Galileo’s contemporary philosophies, such as Descartes’ and, finally, Salviati, the proponent of the Two New Sciences, the study of *resistance* and *movement*, which represented the thought of the author. See in Galilei 1914; This book is also available in a Portuguese translation, in Galilei 1985.

[68] Galilei 1914, p. 11.

bodies, noting, for example, that the speed varied proportionally to the square root of height h, i.e., $v \propto \sqrt{h}$, where v represents the speed after the fall. But Torricelli and Vicenzo Viviano extended the theory, determining, among other things, the proportionality constant $v^2 = Kh$ (and that after Newton would be rewritten as $v^2 = 2gh$, with g due to the acceleration of gravity), later known as *Torricelli's theorem*. All of this about two years after Galileo's death.[69] The mercury barometer experiment, as it also became known, is currently used as a mere example of hydrostatic problems or demonstration in measurement of atmospheric pressure,[70] but back then it would throw a new light on the thinking then in vogue. Torricelli's experiment provided new elements and directed thoughts to a new epistemic question on the transmission of light, heat and magnetism: why did mercury stop at a specific height? What would have in that space left by the liquid column? Was that an absolute vacuum?

The experiment did not bring new elements from a theoretical point of view, as Torricelli's vacuum presented only a repository "completely empty of air",[71] but not exempt from any matter. Most thinkers believed that a medium, perhaps a rarefied matter, a mercury vapour, or even the ether, was more likely. There was no contradiction in postulating a *"superfluid"* hundreds of times lighter than air. Galileo himself (1623) had approached the subject of movement in regions "devoid of resistance", where an absolute vacuum would be impractical.[72] At another occasion, however, he demonstrates a closer view of Cartesianism, which advocated the transmission of pressure in a "liquid" sky,[73] and where only the tendency to movement could be detected. "... (Sensations) have no other existence than in us, (and I am) led to believe that heat is a phenomenon of this type, and that those materials that produce and make us perceive the heat in us, matter that we call with general name of fire, are a multitude of tiny bodies, with certain shapes, moving at enormous speed."[74] In short, heat was not the cause of movement, movement was the cause of heat.

It was convenient to adopt a similar posture in the interpretation of experiments, as many philosophers thought that influences such as heat, light and magnetism were something like hand-to-hand communication, felt through the modification of a subtle

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[69] The *Opera Geometrica* is divided into three parts, in which the second contains the *De Motu Gravium Naturaliter Descendentium e Projectorum*, where he extends Galileos research. See in Torricelli 1644, p. 95.

[70] At this subject, Ernst Mach used remark the value of historical studies in teaching in order to avoid the problem of superficiality: "... Quite analogous difficulties lie in wait for us when we go to school and take up more advanced studies, when propositions which have often cost several thousand years' labour of tought are represented to us as self evident...". For him, or we grow up accustomed to the puzzles and they trouble us no more, or we learn to understand them by the help of history and to consider them calmly from that pont of view. See in Mach 1911, p. 16.

[71] Huygens 1986, p. 8.

[72] Galileo cautiously called it "a medium completely devoid of air and other substances". See Galilei 1914, p. 72.

[73] Which is a entirelly opposite idea, since a liquid is a fluid of high density, that is, incompressible. See, e.g., Galilei 1983, p. 242.

[74] Galilei 1983, p. 241.

medium, instead of simple emission of bodies through the void from atomist theories. For eminent thinkers like Galileo, flame was able to dissociate matter, since "... we can take wood and see it go up in fire and light, but we do not see them recombine to form wood".[75] That is, heat was responsible for separating the luminous particles – which spread to the eyes through the movement of particles – in another type of *thing*, which he assumes not to be material, but would be able to fill the entire space. Like this,"... light is created by means of movement or, we mean, instantaneous expansion and diffusion, and potent by its own, I do not know if it should be called subtlety, lightness, immateriality or another condition different from all of these and still without a name, able , I say, to fill the imense spaces."[76]

But the scale started to change sides. Until then heat and light were part of the same sort of occurrences"... fluid as water",[77] but some philosophers would gain confidence in believing that, perhaps, there was no reason to deny the existence of the void between small bodies, as Robert Boyle (1627-1691) stated,[78] foreshadowing a possible inflection point in understanding the mechanism of action of forces, heat and light. Boyle uses results from some of his experiments to refute the supposed power of attraction of vacuum,[79] in addition to looking for mechanical causes for heat,[80] the antiperistasis,[81] magnetism[82] and electricity.[83] In most occasions, he uses the pneumatic machine of Otto Von Guericke (1602-1686), a Prussian engineer who had created a suction pump, around 1650.[84]

Much of Boyle's voluminous writings are an attempt to demonstrate that the corpuscular movement caused the various physical phenomena,[85] and there is a clear inclination towards a philosophy on which mechanical explanations could be based, inspired by the Baconian tradition,[86] being considered one of the drivers of the mechanistic program. Although not standing in favor or against the existence of the void, Boyle reports an experience that, according to which, the Epicurean hypothesis and that of other atomists emerges naturally. A simple perfume that, as soon as opened in a certain room, "... seems to fill the whole environment...".[87] In analogy, he states that those who made light a "... bodily effluvium of lucid bodies" are right to indicate that, like the perfume that occupies the

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[75] Galilei 1914, p. 60.
[76] Galilei 1983, p. 242.
[77] Galilei 1914, p. 242.
[78] Boyle 1772a, v.1, p. 137.
[79] Boyle 1772a, v.4, p. 128.
[80] Boyle 1772a, v. 4, p. 236.
[81] Boyle 1772a, v. 2, p. 659.
[82] Boyle 1772a, v. 4, p. 340.
[83] Boyle 1772a, v. 4, p. 345.
[84] von Guericke's studies – which would also be known for the invention of an electrostatic generator in 1663 – had shown that sound could not pass through Torricelli's vacuum in his 1672 *Experimenta Nova (ut vocantur) Magdeburgica de Vacuo Spatio.*
[85] Boyle 1772a, v. 1-2.
[86] See, for example, Boyle 1772e, v. 2, p. 5.
[87] Boyle 1772a, v. 1, p. 136.

whole environment, so the light did not leave any part without illumination in Torricelli's vacuum. He then suggests that, contrary to what the "plenists" thought ,[88] efforts should be concentrated on the "... trajectory of atoms through diaphanous bodies".[89] In other words, the propagation of impulses occured through an *elastic fluid.*

For Hobbes, one of plenists in Boyle's conception, matter should be infinitely divisible, even air, and space should be completely occupied by something. This allowed him, for example, to provide his version on the Torricelli's tube, saying that the column of mercury descended to give passage (from below) to even finer particles of air, which, in turn, went up the walls of the tube or pores of the glass, which would demonstrate the inadequacy of Boyle's elastic model for air.[90] In relation to light, heat and sound, Hobbes' explanation had the same characteristics as the Cartesian analogies, but more "sloppy" regarding to the possibility of experimental proof. In its conception, the rotation movement of the Sun would be the cause of light and heat, generated by changes in the pressures on the ether, such as contractions and expansions similar to the heartbeat. In this case, the pressure transmission would provide the sensation of light and heat. Electrical and magnetic attractions would be on the same level, being described by circular movements around the bodies that exercised the attraction. A certain movement of "vibration" was transmitted through the air by a process of emphresonance with attracted bodies, being the iron for the magnet, and the straw for the case of amber, for example.

The idea of resonance and especially of vibration gained a lot of strength in the second half of the century. Robert Hooke (1635-1703), in his *Micrographia*,[91] had also associated heat with some kind of vibratory process, dealing with what he called "congruence" or "incongruity" of bodies, associating heat with nothing more than a "pulse"[92] or "degree of agitation",[93] all of them within the category of fluids or "chemical spirits".[94] Likewise, light should be transmitted in a medium congruent to it, an analogy that Hooke also uses to account for electrical and magnetic phenomena.

Franciscus Linus (1595-1675),[95] on the other hand, claimed that the phenomenon of Torricelli's vacuum would be better explained by the action of "funicles" existing on the mercury's surface, which would act as membranes to increase rarefaction, in the Aristotelian sense,[96] inside the tube. It was from the attempt to refute Linus's hypothesis

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[88] By a "plenist", in Boyle's sense, is meant a reference to Cartesian thought, as in Hobbes, Linus, among others, even though they were not followers of Descartes' philosophy.

[89] Boyle 1772a, v. 1, p. 136.

[90] Boyle 1772a, v. 1, p. 186.

[91] Hooke 1665.

[92] Hooke 1665, p. 12.

[93] Hooke 1665, p. 15.

[94] This expression can currently be translated as "essence" or "essential oil", which consists of a process of obtaining oil from the distillation of a chemical compound.

[95] Franciscus Linus or *Francis Line* was a British Jesuit scientist and priest, known as the inventor of the magnetic clock. He was also known as a contemporary critic of the works of Boyle and Newton.

[96] Boyle 1772a, v. 1, p. 178.

that Boyle claimed to have found his mathematical relationship between volume and air weight for a gas confined at a constant temperature, where the pressure P and volume V are inversely proportional.[97] Boyle hoped to show that, just as he had used his vacuum experiments to refute attraction by suction,[98] electrical and magnetic phenomena would be some kind of impulse chaining, but he was unable to abandon Descartes' magnetic matter that circulated the Earth,[99] neither the electrical effluvium of previous theories.[100] In this respect, Boyle realized that the electrical attraction was not due to the movement of air, thus becoming favorable to the analogy of the electrical effluvium of Gilbert and Gassendi.[101] In Boyle's conception, heat, like Descartes, caused of the rotation movement of the air, causing it to expand, even though he observed that, in some cases, the air could expand when not heated.[102] The ether itself would also be a fluid medium of contiguous particles, a vehicle for light.

From Torricelli's experiment, the question of whether the ethereal fluid consisted of a continuum of infinitely divisible particles, or whether it was an ocean of corpuscles separated by a vacuum, began to lose strength as natural philosophers began to lose interest in the theme, which would be transferred to the metaphysical side of history due to the absolute inability to be tested empirically.[103] In any case, until the end of 17th century, although the corpuscular philosophy favored transmission theories dependent on a means of propagation or the emission of corpuscles, whether for light, heat, electricity or magnetism, most of them considered the action of contiguous way, i.e., by contact, since the issue of action at a distance was still seen as a hidden or esoteric cause, relegated to the same reward of the analogies of organism, propagated until the end of the middle ages.

In this respect, the analogies of corpuscular philosophy were not far from antiquity. If before the water flow suggested a world of matter in motion, now, heat generated by friction or flame's ability to dissociate matter suggested an effluvium of particles as agents that triggered physical phenomena. And if there is smoke, there is fire, as in the popular saying, where there was fire, there was also light. But if the mind was reluctant to admit that this fire acted at a distance, without the existence of a conductive medium, at least it was admitted that the light moved, and under any perspective it was necessary to admit some kind of movement. It was also clear that sound propagated through the air, and that

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[97] The relationship would later be called "Boyle's Law", $PV = k$. See, e.g., Boyle 1772b, v. 1, p. 137; In the eighteenth century, Daniel Bernoulli, would obtain the same relationship, guiving path for the kinetic theory of gases of the 19th century. See Bernoulli 1738.

[98] Boyle 1772a, v. 4, p. 128.

[99] Boyle 1772d, v. 4, p. 340.

[100] Boyle 1673, v. 3, p. 659.

[101] Boyle 1772a, v. 4, p. 345.

[102] Boyle 1772c, v. 3, p. 496, 1772a, v.1, pp. 1–54.

[103] On this, Mach used to say that "... we are acustomed to call concepts methaphysical, if we have forgotten how we reached them". Mach 1911.

electro-magnetic effects[104] could be associated with the movement of friction, mechanically induced or attenuated by heating. At the ends of the seventeenth century, the attempt to generalize it in terms of movement and impact becomes evident, where the aim is to include most universal effects in this program. But that was about to take a new turn with Newtonian Philosophy.

4.5 The divine sensorium

A beautiful landscape is not just an image. It suggests to the mind the absorption of the patterns of order and beauty existing in universe. For Isaac Newton (1642-1727), at least, the elements that formed the beautiful image of his theories were inspired not only by cartesianism, but also aristotelian physics, neoplatonic analogies, Kepler's philosophical and geometric conceptions, as well as the new science of Galileo Galilei. For all its complexity, Newton's work has been the subject of innumerable studies, under the most diverse approaches, impossible to be cited.[105] If it is clear to history that Newton tried to remove hypothetical elements from his laws of movement and theory of universal gravitation, which would influence generations ahead, it is not so evident that he had a real aversion to controversies and, perhaps because of that, reversed the traditional path in the publication of his first studies, or even suppressed essential elements used in his developments, in order to not compromise himself with his peers in academia.

Some of the basic categories that served as an initial motivation for Newton lie in the explorations of his youth, especially through the years 1664-1666, a period in which he obtained the most expressive results in mathematics and also in physics. Having started his higher studies at Cambridge University in 1661, he finds, at the time of his graduation, in 1665, a plague that took over England: like the great epidemic of the medieval period, which had wiped out much of Europe, the university closed its doors between 1665-1667. Seeing himself isolated in his family's home in the county of Linconshire, as is well known, the young philosopher experiences his most inventive period, in which he builds the bases of his greatest discoveries and inventions, such as the method of fluxions,[106] which is at the origin of differential and integral calculus, the principle of action and reaction, studies on the refraction of light and the phenomenon of color separation,[107] that would be subject of his first article,[108] and his latest book.[109] Naturally facing barriers in relation to his first findings,[110] Newton committed himself to publish his first book, *Philosophiae Naturalis*

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[104] The term is spelled that way for historical reasons, since the term "electromagnetic" would be coined only in nineteenth century. The two areas, although correlated, were, until that moment, viewed separately.

[105] A great part of Newton's extensive work can now be found online.

[106] Newton 1666.

[107] Newton 2020.

[108] Newton 1671; This paper is also available in a Portuguese translation, in Silva and Martins 1996.

[109] Newton 1704.

[110] Robert Hooke, for example, strongly criticized Newton on his early explorations in the field of optics.

Principia Mathematica, in 1686,[111] which would mark the adoption of mathematics as the universal language of science as never before, and would be immortalized as one of the most important publications in all scientific history.

Galileo had recognized that the uniform movement was a manifestation of inertia and that force was its cause. He had also noticed that an object launched in straight line would decay in form of a parabola, caused by the force of gravity, which pointed towards the center of Earth. Newton saw a fundamental principle there,[112] that would even explain the orbit of the planets (Fig. 11). Despite the premise of absolute rest,[113] Newton enunciates his first law as an empirical generalization, with which it is possible to recognize situations in which a force is acting: "... Every body remains in a state of rest or uniform movement in a straight line, unless it is forced to change that state by means of printed forces."[114] In the second axiom, he asserts that "... The change in movement is proportional to the printed driving force, and happens along the straight line where that force is printed".[115,116] As stated, the second law makes it possible to interpret the action of a force as being its own direction and measure, in which the constant of proportionality is the inertial mass of the body. The third Law, on the other hand, is conditioned to the interpretation adopted in each case, about which bodies are "in contact", either by pressure, traction on a rope or a collision between two objects. In his words, "... An equal reaction is always opposite for each action: or the mutual actions between two bodies are always the same and in opposite directions".[117]

Hooke's theory proposed the existence of two main colors, those that, with the exception of color blue, are part of the "extremes" of the visible spectrum (the range of the visible to the human eye is formed by the colors, red, orange, yellow , green, blue, indigo and violet). Hooke accused Newton of making an apology for the materiality of light, as himself believed that light should be part of some vibratory process, like pulses transmitted in a congruent environment. The colors, in turn, should be generated by changes in these pulses when crossing the walls of a prism. A letter in response to Hooke's accusations can be seen at: Newton 1672, p. 5086; The controversy also involved Cristiaan Huygens, to a lesser extent. See in Newton 1673.

[111] Newton 1686.

[112] Cf. Newton 1686, p. 20; See also Newton 2018, p. 61.

[113] There are still controversies on the subject today. The problem, still open, is to know if there really is any object at rest in universe or just in relation to a coordinate system.

[114] Free trans. of: "... *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*" Newton 1686, p. 12.

[115] Free trans. of: "... *Mutationem motis proportionalem esse vi motrici impressae, et fieri secundum lineam retam qua vis illa imprimitur.*" Newton 1686, p. 12.

[116] The term"change in motion" refers (in current terminology) to a change in momentum (= mv) and is linked to a static definition of mass, called by Newton the"quantity of matter" and which leads to the well-known cliché $F = m\dot{v} = m\ddot{x} = ma$. The force, in this conception, is the (rate of) variation of that quantity, or "fluent quantity". See, for example, Newton 1666; The notion of force attached to a static mass property, although valid for small speeds, fell out of use for velocities close to lightspeed. One of the postulates of relativistic physics, somewhat anticipated in 1892 by Heaviside (1850-1925), states that a "spherical" electric charge in motion can have its mass increased with increasing speed. See, for example, Heaviside 1892, p. 446.

[117] Free trans. of: "... *Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum in se mutuo sempre esse aequales et in partes contrarias dirigi.*" Newton 1686, p. 13; See also Newton's explanation shortly after presenting the three laws of motion (in the Portuguese translation) at: Newton 2018, p. 54.

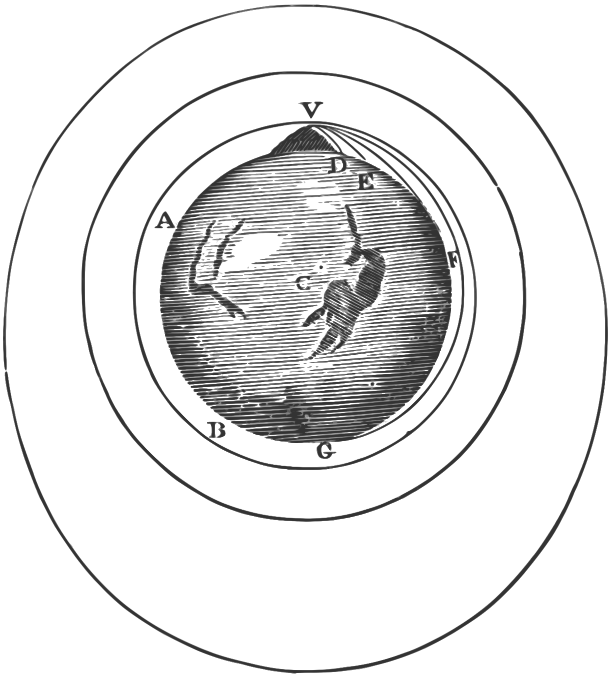

Figure 11 – Action of a centripetal force in Newton's view: a launched object is displaced from its rectilinear path by means of a force. If the initial speed is large enough, it will even be possible to place it in orbit. The illustration appears for the first time in a separate book entitled *A Treatise on the System of the World*, in 1728, 42 years after the first edition of the *Principia*, which would be a preliminary version of book III of *Principia*, in 1729. See Newton 1728, p. 6; Changing his mind, however, Newton decided to rewrite book III with a more detailed mathematics. Thus, when the next edition was printed, *System of the World (Treated Mathematically)* appears as book III, in which the figure is removed. In the most recent editions, however, the two texts appear in full, as well as the figure. See, for example, Newton 2017a, p. 181, p. 333.

The most appropriate attitude to face a work of the nature of the *Principia* is, precisely, that an independent and absolute concept of force is not necessary, but it is important to highlight the difference that Newton makes between mathematical properties of his laws of movement and its causes, "... *Mathematicus saltem est hic conceptus*",[118] which form a verifiable logical set. In the logical interpretation, it is enough to apply the second law in "reverse" sense: if someone perceives a body accelerating, without observing something to push it, how to explain the fact? If a body accelerates and there is "nobody" pushing, there is, according to the idea, the action of a coercive and invisible force, even if unreachable by sight. In other words, *action at a distance* emerges as a beautiful resource, through which it is possible to provide an explanation without, however, penetrate into its essence, diverting the viewer's gaze to something that can be measured. Would it be correct, then, to think that the principle of remote action sought to eliminate any reference

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[118] "... This is only a mathematical concept." Trans. of Newton 1686, p. 4; "... I intend here only to offer a mathematical notion of those forces, without considering their causes and physical positions." Newton 2018, p. 44.

to Nature's finalism?

To find out whether Newton sought to convey an absolute and independent concept for strength, or to eliminate forever Nature's finalism, there is no need to go much further than the enunciation of the laws of motion. This is even a recommendation he made in his *Rules of Reasoning in Philosophy*,[119] emphasizing that the filling of philosophical Scholium would not be necessary for the mathematical conception of force. Indeed, several elements of his work can be understood within the hypothetical-deductive scheme, from which Kepler's Laws, the principle of falling bodies, the orbit of comets and the phenomenon of tides are obtained. But, even if this were possible, Newton enjoyed prestige to establish a direct relationship between phenomenon and hypothesis as well. And it is precisely at this starting point, the phenomenon, that we must seek the root of his statements. This may even provide a plausible reason as to why the theory of universal gravitation has been viewed by adherents of operationalism and,[120] later on, of the more radical positivism, as an ideal form of theory, which certainly abstracted its fundamental principles between the lines of the *Principia*.

Although the initial objectives of his closest followers were based on the occasions when Newton oposed himself against to the creation of hypotheses, [121] a superficial interpretation may have contributed to a mistaken understanding of his thinking, such as seeing action at a distance as instantaneous action, or in the direct deduction of the inverse square law directly from the phenomena.[122] Like other historical figures, it will be a difficult task to find Newtonian thinking in Newton's figure. His riskier conjectures could not have influenced the philosophical community of his time, since much of this more open thinking was expressed through letters, restricted to a more intimate circle of friends.[123]

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[119] In his words, "... It is sufficient to read the definitions, the laws of movement and the first three sections of the first book." Newton 2017b, p. 181.

[120] Operationalism is an idea that presupposes the construction of knowledge as a series of successive, gradual, mechanical (and, why not say, boring!) operations, such as observation → experimentation → observation . . . (in physics) or → abstraction → deduction . . . (in mathematics). This topic is explored in Hesse 1962, p. 149.

[121] Newton, like any other scientist, elaborated numerous hypotheses throughout his life. The question that became famous is due more to specific cases in which he positioned himself contrary, as in the case of the causes of gravity

[122] The first edition of Principia (1686) was presented by Edmund Haley and prefaced by Newton himself. See in Newton 1686; Twenty-six years later (1713), Roger Cotes (1682-1716) was invited to preface the second Latin edition and, although without guiving names, had a clear objective of combating Descartes' vortex theory. In a passionate defense of what he considered to be the principles of Newtonian philosophy, Cotes labeled those sympathetic to Cartesianism as "... adepts of chimeras who spend their time uselessly, fixing a ridiculous fantasy", as if reads in the Brazilian translation in Newton 2018, p. 29; On another occasion, talking about the then "doctrine of action at a distance", Cotes says that"... we should not search for these laws out of uncertain conjectures." See translators' comments at Newton 2018, p. 308, note 8.

[123] Much of it would not be published until years later, and only recently have been the subject of research. Even the famous general scholium of *Principia*, which has been a source for countless debates over the years, was written only in 1713, when Newton had already exceeded 70 years and enjoyed immense prestige. By this time, Newton had already been elected president of the Royal Society of London (in 1703). See, e.g., Newton 1729, p. 387.

In the case of the theory of gravitation, for example, it is important to understand this context, in order to observe that there is a certain limit to how much a theory can be considered as a direct deduction from phenomena.

In presenting of the theory of gravitation, Newton starts to demonstrate Kepler's laws by accepting them as *phenomena*,[124] although they were called *hypotheses* in the first edition.[125] But what he does, in reality, is not a blind acceptance of Kepler's principles as a fait accompli. He first considers circular orbits and, later, proposes a correction that will shape elliptical orbits, known as Kepler's first law, in the form of a mathematical proposition. From then on, he states that, knowing the principles on which these phenomena depend, it is possible to follow a logical route, that of assuming phenomena as first approximations and inferring their consequences. That is, the argument is deductive, in the strict sense that deduction is understood by science, but only if certain categories are considered to be phenomena, which here assume the status of facts. In addition, other categories must be defined operationally, such as forces in the fixed star system, conservation of *momentum*, in addition to the rules of reasoning, defined as axioms. This is one of the methodological pillars by which Newton builds his reasoning, and is at the heart of his statement about deducing laws from phenomena.

Although stating that "... we must not admit more causes for natural things than those that are true and sufficient to explain appearances...",[126] Newton does not advocate a cause opposite to natural finalism. Since forces in Newton's conception ceased to be mere definitions to become real facts, i.e., forces viewed as attractions at a distance in absolute space, the theory of gravitation would be no less hypothetical than Kepler's first cogitations or even the Aristotelian conception of natural place. This is an aspect that can be considered, if one considers the theory of gravitation not as a mathematical law, but as a (meta)physical hypothesis.[127]

Finalism had been introduced by Aristotle as a basic necessity for life and was also a reality for Newton, who used the term "final cause" in the same sense: every end has a cause; every intelligent end has, for the same reason, an intelligent cause, and in the universe,"...it is not to be conceived that mere mechanical causes could give birth to so many regular motions...This most beautiful System of the Sun, Planets and Comets, could

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[124] Newton 2017b, p. 190.

[125] See Newton 1686, p. 401; For comparison, see Newton 2017b, prop. XIII, p. 210.

[126] See Newton 2017b, book iii, p. 185; The principles of reasoning in Philosophy do not appear in first edition of *Principia* as rules, but also as hypotheses. This even raises the question of whether Newton really made a clear distinction between these concepts, or was gradually gaining confidence over time, by stating that a hypothesis was all that not directly derived from a phenomenon. See Newton 1686, p. 402.

[127] In the sense of Pierre Duhem, any theory that aims to unveil absolute forms or elements is a metaphysical theory and, for this reason, must be discarded, being an attraction by a magnet, gravity, the chemical conception of an element, or any other explanatory theory. For the eminent philosopher, it was not conceivable to subordinate physics to metaphysics, as this would cause a disagreement between different "cosmological schools". See Duhem 1954, pp. 9–10, p. 189, pp. 47–48 e pp. 339–340.

only proceed from the counsel and dominion of an intelligent and powerful being...”.[128] This type of statement does not emerge from the common sense that is made about Newton’s thought, and would hardly be attributed to him. However, when it was conjectured that the illustration of metaphysical principles has served sometimes as inspiration, or as a motto for studying the science involved in phenomenon,[129] especially from the physical analogies, this could not be more present in the idea of acting (at a distance or not) of a force. The influence of spiritualist conceptions in Newton’s scientific work is still little explored, but even in his scientific treatises it is possible to see that the fascination about the “invisible” had some influence on his physical theories, even if they were discreetly reported.[130] “... What is there in places almost empty of matter, & whence is it that the Sun & Planets gravitate towards one another without dense matter between them? ...How do the motions of the body follow from the will?...”[131]

The questions that Newton was preparing would be tools of an even more subtle proposition, according to the famous *queries* of optics, which would appear in his new book. Always put in a negative way, they could be answered (by him or by the reader) with an emphatic *yes*:[132]“...Does it not appear from phænomena that there is a Being incorporeal living intelligent omnipresent, who in infinite space as it were in his sensory, sees the things themselves intimately, & throughly perceives them, & comprehends them wholy by their immediate presence to himself; of which things the images only carried through the organs of sense to itself, that which in us perceives & thinks, sees & beholds in its little sensorium (?). And tho every true step made in this Philosophy brings us not immediately to the knowledge of the first cause, yet it brings us nearer to it, & on that account is to be highly valued.”[133]

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[128] See Newton 1729, p. 388; See also Newton 2017b, prop. xlii, p. 328.

[129] See sec. 3.4, p. 55.

[130] Newton would keep such studies private for all his life, which justifies the relative ignorance of these writings even today. Even some of the his theological writings, such as a book on Daniel’s prophecies, did not exhibit the strong oppositions he had against religious orthodoxy, which shows a distance between his personal belief and his more technical theological writings. Even these, published posthumously, would go down in history more as a result of the ramblings of an elderly man, close to death, in comparison to the genius he demonstrated in exact sciences. In a period of conflict between the Anglican Church and Catholic tradition, Newton would develop a radically view against secular dogmas. For him, for example, Jesus was divine, but he was not God, and most the religions of his day were, in his sense, corrupt and idolatrous. Virtually all of Newton’s religious writings are now available online. See, for example, Newton 2008.

[131] The questions appear first in a “draft” version of the Queries, just before the Latin edition of 1706, in English. See Newton n.d., 247r; The same questions were already translated into latin in the second edition – *Optice* – which would be slightly changed in the following editions: “... *Unde est quod Sol & Planetae ad se invicem gravitent, sine Materia intersecta? ... Qui fit, ut Motus Corporis obsequantur Imperio Voluntatis? ...*”. See in Newton 1706, Qu. 20, p. 315.

[132] The first book on optics was published in English by Newton in 1704, where one can already see a first version of the *Queries*. The Latin edition *Optice* (1706) comes as a translation of the first, revised by the author, with some modified questions. See in Newton 1706; Other editions were published in 1717 and 1710 (in Latin), 1721 and 1730, the last being published posthumously. The first complete translation into Portuguese dates back to 2017. See Newton 2017c.

[133] Trans. of “... *Annon ex phaenomenis constat, esse Entem Incorporeum, Viventem, Intelligentem, Omnipraesentem,*

In other words, aren't the eyes a means by which the sensorium captures visual information? In infinite space, wouldn't there also be a sensorium through which the Creator himself, being omnipresent, would have access in "real time" to information, and through it, would act in the universe? In the analogy of the divine sensorium, even though he admitted the impossibility of finding the first cause, Newton assumes that Natural Philosophy can at least get closer to it. Exactly the same methodological criteria of the mathematical propositions, as in the case of Kepler's laws. And although some have seen only the didacticism of an "analog speech scheme",[134] Newton goes further and seeks a parallel of the image and similarity relationship between creature and Creator as a platform for a hypothesis on action at a distance.

The space, filled by the sensorium, would be for Divinity just as the mind was for the human observer. It could act at a distance, just as the mind moves a member by will. He does not see directly, feels through the sensory. In men, the eyes are vehicles of perception, but for Divinity, being incorporeal, this would not be necessary. This point even led to a great controversy with the German philosopher Gottfried Wilhelm Leibniz (1646-1716), who accused Newton of promoting an anthropomorphic and material vision of Divinity. Although he was not overtly Cartesian, as he did not identify matter with extension, Leibniz maintained a view on gravitation similar to the theory of Huygens and Descartes, having been one of the great opponents of Newton's principle action at a distance.

One of the controversies occurred after Leibniz wrote to Princess Caroline of Wales in November 1715, in which he claimed that Newton was paying a disservice to England's religious belief: " ... Natural Religion it self, seems to decay (in England) very much. Many will have Human Souls to be material: Others make God himself a corporeal Being. ... Sir Isaac Newton says, that Space is an Organ, which God makes use of to perceive Things by. But if God stands in need of any Organ to perceive Things by, it will follow, that they do not depend altogether upon him, nor were produced by him".[135] But this Being had no body for Newton, as he would stress, and neither was the sensory material. But it was certainly *something*. Newton's positioning was a high risk to be taken, which could even trouble his professional transit, even at that time. Leibniz was not willing to believe that the acting took place in any other way than by pressure or impact, according to the cartesian tradition, which he had embraced in this particular case. Newtonian action at a

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qui in Spatio infinito, tanquam Sensoriosuo, res Ipsas intime cernat, penitusque perspiciat, totasque intra se praesens praesentes complectatur; quarum quidem rerum Id quod in nobis sentit & cogitat, Imagines tantum ad se per Organa Sensuum delatas, in Sensoriolo suo percipit & contuetur? Utique si verus omnis in hac Philosophia factus progressus, non quidem statim nos ducit ad Causae primae cognitionem; at certe propius propiusque nos ad eam perpetuo adducit, eaque re permagni est aestimandus." Cf., Newton 1706, Liber iii, Qu. 20, p. 315; In the "draft" version of the Queries, when prepared the first Latin edition (1706), Newton expressed himself the same way, but in his mother language. See in Newton n.d., 247v.

[134] As in Hamou 2014.

[135] (Author's emphasis). See in Leibniz 2006b.

distance would require a perpetual miracle to happen.

Newton always had an aversion to controversy, as is known, but Samuel Clarke (1675-1729), who was an eminent English philosopher as well as pastor of the Anglican Church, came out in Newton's defense, fueling one controversy that lasted until Leibniz's death, in following year (1716): " ... that there are some in England, as well as in other Countries, who deny or very much corrupt even Natural Religion it self, is very true, and much to be lamented. But (next to the vitious Affections of Men) this is to be principally ascribed to the false Philosophy of the Materialists, to which the Mathematick Principles of Philosophy are the most directly repugnant...".[136] Later, he argues that Newton "... illustrates it by a *Similitude*: that as the Mind of Man, by its immediate presence to the pictures or images of things, form'd in the brain by the means of the organs of sensation, sees those pictures as if they were the things themselves; so God sees all things...". Thus, the sensorium would not be an organ like the brain, which possibly forms an image of external things, but the medium by which the mind is expressed: just as the brain was the vehicle of the mind, space was the vehicle of the incorporeal mind of God.

Despite the apparent teleological nature, if the characters of the story are changed in the analogy of the sensory, something curious happens: if the Sun acts on Earth, through space, in the same way that Earth acts on the Moon or in ourselves, how could it "know", at a distance, the force he must apply on Earth, since it is not present there? After all, as Leibniz thought,"... Nothing can any more act, or be Acted upon, where it is not present".[137] Knowing how the body's movements could obey the will, in analogy to what could happen between the Sun and the planets, [138] is like asking how a force could be transmitted from the Sun to Earth. That is, how did the action take place? Interactions between invisible bodies, a gravity field, transmission through some continuous medium? If a body cannot act where it is not, how can one define its position, or say where its action is? Can it be defined only from the sensory perception of touch?

Two centuries later, not wishing to reduce gravitation to what he believed to be unintelligible, the German physicist and philosopher Ernst Mach (1838-1916) would raise some of these questions, proposing a reversal in Leibniz's aphorism: " ... a body is where it acts".[139] But even this reversal would have difficulties in dismissing the idea that the Sun itself could "touch", in a way, the object at the moment of action.[140] Currently it would not be unreasonable to think that the Sun is, in a certain way, present on Earth, and "know" what happens there in terms of electromagnetic interactions, considering that the planet is immersed in the *heliosphere* . Action at a distance, like some kind of gravitational

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[136] See in Clarke 2006a.

[137] Clarke 2006b, p. 43.

[138] See note 131.

[139] Mach 1911, p. 56.

[140] Which is, in fact, a similar ideia to the emanation theories of the ancients. See views on the vision phenomenon in sec. 3.3, p. 55.

interaction, could also be understood with similar reasoning, as a constant presence or "omnipresence" of the Sun, as in the analogy of the divine sensorium. It is still possible to understand the critics received by Newton from Leibniz and others in his day.

The nature of the necessary "miracle" for action over great distances – as well as for atomic cohesion – was purely metaphysical, and was based on Leibniz's conception of continuity and his *principle of sufficient reason*, which rejected the possibility of attractive forces. If all matter was made up of finite atoms, immersed in a space devoid of matter, of what would space itself be formed? If perfection was inherent in creation, there would be no sufficient reason for a given proportion of matter instead of a vacuum. Now, a filled space seemed more sensible than a cluster of points surrounded by nothing, reason that made Leibniz discredit that the Otto von Guericke's pump,[141] as well as Torricelli's experience of the previous century, had produced an absolute vacuum: "... the effluvia of the load-Stone, and other very thin fluids may go through. ... If the Space (is) void of all Bodies... what is it then full of? Is it full of extended Spirits perhaps..."[142] If atoms were perfectly rigid units, all movement would be lost in a shock, which would make impossible *elastic collisions*. If they had elasticity, on the other hand, the material would be deformable and, therefore, would not be formed by discrete parts.

The same *corpus* of Leibniz's arguments would be adopted by some eminent Newtonians, as in Boscovich's mathematical analogy for the force between point atoms, which in no way could be abstracted from Leibniz's or Newton's philosophy. It would be a first attempt to diminish the importance of matter in comparison to the idea of invisible force, which required every particle to be connected to any other, at a distance. The maturation of the concept of action at a distance would close up a tradition to be explored with great success in the following decades of the 1700's and integrated the scientific imaginary of the 1800's, when the science of electricity and magnetism would be built, upon new constructs.

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[141] See note 84, p. 84.

[142] See in Leibniz 2006a, §7–10.

5 From elasticity to electricity

> *... So a straight spring when forcibly bent, must, to restore itself, contract that side which in the bending was extended, and extend that which was contracted... But the spring is not said to be charged with elasticity when bent, and discharged when unbent; its quantity of elasticity is always the same.*
>
> (Benjamin Franklin, in a Letter to Peter Collinson, 1748)

The eighteen century physics saw advances in the issue on how an action could be transmitted from one point to another in space, with some admissible alternatives. Between them was action by impact, in which elasticity was considered a fundamental property of bodies, as in the theory of collisions or in the theory of electrical and magnetic attractions. Other theories would consider the transmission of momemtum by deformations in a elastic medium or interactions with a fluid field, in which microscopic effects could be neglected. In the last case, the propagation demanded a temporal element where speed relied upon mechanical properties of an elastic solid, like a musical string, a jelly or a spring.

5.1 Attraction and repulsion

Newton's *Opticks* had opened a window to a new domain, with bolder hypotheses on the nature of light and the role of ether in its manifestation: could not light be made up of small particles emitted by a luminous body? By inertia, they would follow a straight line through space. But light also made curves, and the most basic optical effect of reflection could not be explained only by the ballistic approach. The smoothness of a mirror was deceptive under a microscope,[1] and the corpuscles "... would be scattered as much by the most polished Glass as by the roughest." As Newton conjectured, maybe reflection was caused by "... some power of the Body which is evenly diffused all over its surface, and by which it acts upon the ray without immediate contact."[2]

This average power was obviously a repulsive, or elastic, force, and the this idea was used to account for the phenomenon of refraction, but the repulsive aspect would not be sufficient. Newton's idea of elasticity meant "... particles mutually flying from each other ... (with) density as the compression",[3] in which the centrifugal forces were reciprocally proportional to the distances of their centers. Inversely, particles flying from each other with forces reciprocally proportional to the distances necessarily composed an elastic medium,

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[1] Hooke 1665.

[2] Newton 1704, prop. viii, p. 68.

[3] Newton 1729, book ii, prop xxiii, p. 77–79.

i.e., obeyed Boyle’s Law.[4] So, “... Doth not the Refraction of Light proceed from the different density of this Aetherial Medium in different places, the Light receding always from the denser parts of the Medium?”[5] But the issue would prove to be more complicated in the case of refraction. Depending on the angle of incidence, the particles could somehow overtake the repulsion and meet an attractive force, similar to the magnetic action. So, perhaps, light could have “sides” (polarities).[6]

The analogy was also attached to the algebraic operations with negative numbers, which only gained acceptance in Newton’s time: “... as in Algebra, where an affirmative Quantity vanish and cease, there negative ones begin; So in Mechanicks, where Attraction ceases, there a repulsive Virtue ought to succeed. And that there is such a Virtue, seems to follow from the Reflexions and Inflexions of the Rays of Light.”[7,8] Presented in this way, the analogy seemed to need no further explanation, as it apparently did not involve any embedded physical hypothesis. But the success with refraction did not go too far,[9] since the phenomena of rings, first observed by Hooke, would find difficulties to be adjusted to the laws of mechanics,[10] making Newton reconsider the role of an elastic fluid, “... hundreds of thousand times” more subtle than air.[11]

A similar strategy was implemented by Daniel Bernoulli (1700-1782), in his *De affectionibus atque motibus fluidorum elasticorum, praecipue autem aeris* (1738),[12] in which a portion of elastic particles put to collide randomly in a medium also behaved according to Boyle’s Law, coming to the conclusion that the average impact of particles gave the perception of a pressure in a closed container. Interestingly, both Newton’s and Bernoulli’s arguments considered two apparently irreconcilable modes for the action of a force: action at a distance or, more precisely, repulsion at a distance and action by impact. The same principle would later be used as an interpretation for the sensation of heat, i.e., as a measure of temperature, with the elaboration of the kinetic theory of gases in the nineteenth century.[13]

The idea of attraction and repulsion of light served also a philosophical justification for action at short distances, and became the favorite alternative for some eminent New-

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[4] Cf. sec. 4.2.

[5] Newton 1730, Qu. 19, p. 324.

[6] This question was addressed when Newton studied the problem of the double refraction in the Iceland crystal, identified primarly by Huygens. See Huygens 1690, p. 15; See also Newton 1730, p. 347.

[7] Newton 1730, Qu. 31, p. 370.

[8] *Inflexion* is the phenomenon now known as difraction. See Newton 1730, Qu. 29, p. 347.

[9] Newton 1730, pp. 170 – 187.

[10] The phenomenon of “rings” consisted on interference patterns, in the form of colored lines in thin films, such as in a soap bubble, or when light was reflected on two surfaces separated by a small distance. See, e.g., Williams 1980, p. 11.

[11] See, e.g., Newton 1730, Qu. 19, p. 324, Qu. 22, p. 327.

[12] Bernoulli 1738, p. 200.

[13] See Hesse 1962, p. 180.

tonians, such as the Jesuit friar Ruggiero Boscovich (1711-1787),[14] who having identified problems with the corpuscular philosophy, used the class of Leibniz arguments on action by contact and continuity to arrive at a whole new idea. Promoting a kind of fusion between Newtonian physics and Leibniz's metaphysical principles in his *Theoria Philosophiae Naturalis*(1763),[15] Boscovich suggested that matter consisted of points without dimension, which had no properties other than inertia and the ability to exert forces on each other. The nature of this force, as in Newton's thought, was its own measure, but now it was also a function of the distance between these dimensionless points (Fig. 12).

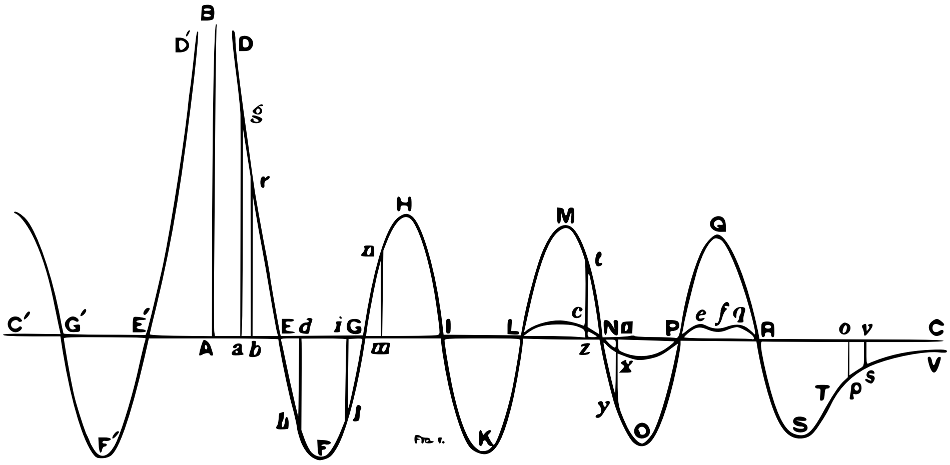

Figure 12 – Boscovich's elastic analogy: force between pairs of dimensionless material points. Source: Boscovich 1763, p. 133.

The force function curve approaches to a hyperbola for short distances. For longer distances, the curve has an asymptotic behaviour and, since the points have specifically no mass, the function $F(x)$ does not directly represent force, but an acceleration "felt" by the points as they approach. The analogy now is that of an elastic solid: if two ends of the "spring" approach each other in an act of compression, they acquire the propensity to move away, which increases as the distance decreases with compression. If the distance increases, on the other hand, the repulsion force decreases until it stops at a certain point. If the distance is increased again, a new force appears, now prone to attraction.

In general, Boscovich's bold attempt sought not only to support action at a distance, but to embrace in a single stroke the gravitational attraction to finite distances, the elastic collisions through repulsion, which prevented a "real" impact, the cohesion that maintained the stability of matter and the electrical and magnetic forces, proposing

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[14] Ruer Josip Bokovic, Roger Joseph Boscovich or Ruggiero Giuseppe Boscovich, was born in a territory that today belongs to Croatia and lived a large part of his life in Italy and France, which is why he preferred to write his name in Italian.

[15] The second edition of the book features a side-by-side translation of the original Latin text, with a preface by James Clerk Maxwell. See Boscovich 1763.

an unification of the Newtonian principles of gravitation, cohesion and fermentation as fundamental laws of nature. All of them under the tutelage of single a function![16]

The *adhoc* character of Boscovich's theory came from the fact that it was built on the suggested hypothesis of the force curve. Hence the idea that an object would be considered as if it had all its mass concentrated in its center of gravity, which was then treated as a point particle, instead of an infinite regression of elastic parts: "... therefore indeed I do not admit the idea of vacuum interspersed amongst matter, but I consider thar matter is interspersed in a vacuum & floats in it."[17] Any similarity with contemporary theories of the fundamental forces of nature or with the idea of singularity, as has already been considered,[18] remains a mere speculation, although the methods for obtaining the force curve are not far from more recent strategies in studies of behaviour of materials, especially for short distances on an atomic scale.

For longer distances, the Boscovich function closely resembles the binding potential energy curve or *potential well* curve, well-known in the field of materials science, which represents the potential energy in atomic systems with a defined position in space, as in a crystal. The curve is used to "predict" some properties of materials, such as cohesion, electrical conductivity, elasticity, plasticity, mechanical resistance or even behaviour in chemical reactions. The repulsion between two atoms that come together in a material is today related to the so-called *principle of Pauli*: when electronic clouds that surround the atoms start to overlap, the energy of the system abruptly increases.[19] Anyway, it seems pretty clear that Boscovich's fundamental idea of point particles suggested a panorama in which the interaction by means of forces had a more fundamental character than matter itself.

Boscovich's theory was an extension of Leibniz's theory of *monads*, which considered them as some sort of "living forces" or *vis viva*, concept that would later play an important role in the principle of conservation of energy, in the 19th century.[20] The replacement of the mass by a numerical abstraction goes back to the Pythagorean tradition,[21] giving meaning to matter only under the action of a force. A body was the combination of these dimensionless dots, transposing the argument of continuity as a fundamental character of matter, attributed to the continuity of a force governed by numbers, acting at a distance without intermediaries. As the units were mere euclidean points, at first it would not be a problem to occupy the same region of space. But the analogy did not allow two points to be "contiguous" from a mathematical point of view.[22] Furthermore, the idea of impene-

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[16] Boscovich 1763, p. 35.
[17] Boscovich 1763, p. 39.
[18] Anderton and Stoiljkovic 2018.
[19] Padmavathi 2011; Brenner 2000.
[20] Mach 1911, chap. 3.
[21] See chap. 3, sec. 3.2.
[22] Boscovich 1763, p. 19.

trability of matter was perfectly explained through the action of his *Law of Force*,[23] instead of direct contact, which directly linked the principle of material cohesion to the idea of elasticity.

5.2 The miraculous bottle of Leiden

The concepts of attractive and repulsive forces from the corpuscular tradition would unfold in some branches of physics, such as chemistry and the electrical science. Mechanical causes for the phenomena of electrical and magnetic attractions, cohesion and elasticity, had been the object of several studies. Evidence seemed to indicate that the attraction between particles was responsible for the phenomenon of the rise of liquids in a capillary tube, instead of the differential pressure between different environments (inside and outside the tube), or even for the gravitational attraction. Until then, however, there were few reasons to associate the principles of elasticity and cohesion with the phenomena of electrical attraction. The scenario started to change with a new group of theories that considered the principles of attraction and repulsion as fundamental manifestations of electricity, in which the idea of a new "elastic fluid", instead of a mere "emanated effluvium" of earlier theories, could now easily flow in conductors.

The idea of attraction and repulsion related to algebraic operations had influence on the work of Benjamin Franklin, at the moment he faced a new way of "storing" electrity inside the "... miraculous bottle!" of Leiden.[24] In fact, the Leiden phial would be a must-have device in any laboratory devoted to the study of electricity, in a time when the majority of instruments operated under the principle of electrification by friction. In 1660 Otto von Guericke (1602-1686) had built a machine that could generate a great amount of electric charges from friction with a sulfur sphere, allowing for the first time the use of high-voltage (static) electricity in experiments, considering that up to 1750, one of the few electrical apparatus at disposal to the "electricians" would be the *versorium* of Gilbert,[25] which was basically a non-magnetic needle used detect small variations in strength as, for example, the attraction of a piece of fabric by means of amber or a piece of iron by a magnet.[26]

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[23] Boscovich 1763, p. 15.

[24] In 1746, the eminent republican visited a family in the American city of Boston, when he was introduced to the invention of Pieter (or Petrus) van Musschenbroek (1692-1761), from the city of Leiden, in 1745. Amazed, Franklin wrote a letter on July 28 of the following year to the Royal Society of London, of which he was an honorary member, reporting on several experiments he had done with Leiden jars. He would performd experiments with electricity almost during a decade, reporting major discoveries in this field. See Franklin 1751, p. 3; See also a letter written by Franklin and addressed to John Mitchel, in a Portuguese translation, in which Franklin explains the formation of thunderstorms with lightning and thunder. Moura and Bonfim 2017.

[25] See note 116, p. 63.

[26] The versorium of Gilbert is not the oldest electric instrument invented by mankind, once its construction was apparently inspired by another apparatus created in 1546 by the Veronese poet, physician and philosopher Girolamo Fracastoro (1478-1553), called later *perpendiculum* of Fracastoro, in his homage. To further information, see Assis 2010, p. 35.

Until then, very few saw electricity as manifestation of a new force of Nature, but the invention of the Leiden jar was about to be an inflection point,[27] a sort of thing that could only be achieved, as it were, by "true believers" in the elastic nature of electricity, a device which, according to Thomas Kuhn "... might never have been discovered by a man exploring nature casually",[28] attracting attention of both philosophers and empiricists around the world and, therefore, becoming a central piece on the electrical research in the second half of the 18th century (Fig. 13).

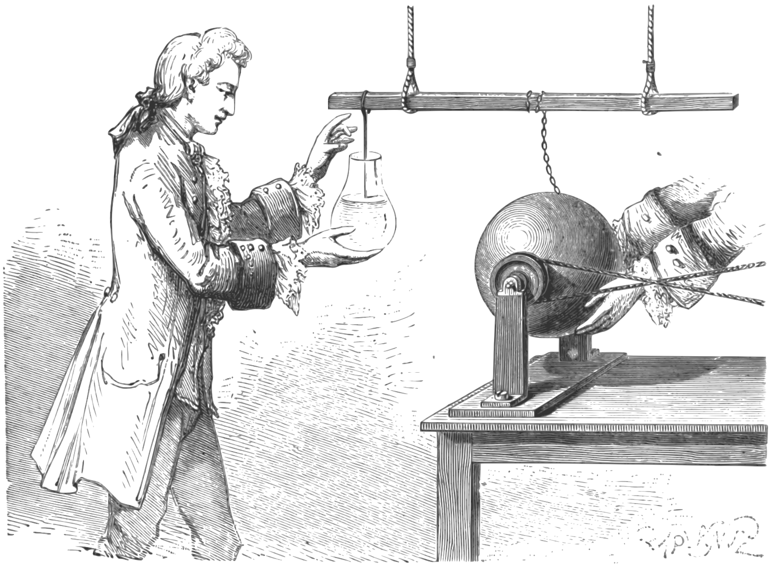

Figure 13 – A phial for the electric fluid. The picture represents a moment in which one perceives that electricity could, somehow, be stored in a jar filled with water, using metallic conductors to connect a frictional machine to a hand-held glass phial. The discovery had been made independently by the German cleric Ewald Georg von Kleist (1700-1748), in 1745. However, the illustration makes reference to the experiment that supposedly preceded the invention of the Leiden phial, credited to the Dutch professor Petrus Van Masschenbrock (1692-1761), from the university of Leiden, who certainly popularized the apparatus after 1746.

Commenting on the possible collisions between the electrical particles, Franklin used the idea of attraction and repulsion to suggest the existence of one single fluid according to Newton's idea of elasticity, i.e., as (positive and) mutually repulsive particles, "flying from each other". The negative and positive terms corresponded to a lack or excess of the singular fluid. An excess would be equivalent to a positively charged body, which were accumulated on the surface and surrounded the object, thus creating a kind of "repellent atmosphere"[29] to the other particles.[30] If a body had a lesser quantity than the neutral (or natural) state, it would be negatively charged. But this fluid analogy did not

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[27] Potamian and Walsh 1909, p. 86; Whittaker 1910, p. 41.

[28] Kuhn 1970, p. 17.

[29] Franklin 1747a, § 1.

[30] See also Franklin 1750, p. 2.

mean literally a flow of electricity as in a water flow, as one might think, but what Enrst Mach would later give the nickname of "mechanical equivalent" for water,[31] a bit more mathematical entity.

Franklin's experiments on electricity had a lot of publicity in history, often told as feats of a modern Prometheus, full of wonder and surrounded by mysticism.[32] One of them is of particular interest from the perspective of analogies, occasion in which the phenomenon of electric discharging, together with the idea of elasticity, could have suggested him the idea of wave propagation. Evidently this is only a curious historical conjecture, given that Franklin did not arrive at a concept that would be developed more almost a century later. In 1751, reading a friend's account of the effects of lightning on his ship, where it was noted that the magnetic compasses "... lost the virtue of the load-stone, or the poles were reversed", Franklin claimed to have "... given polarity to needles," reversing it at his pleasure,[33] in a replication of the lightning effect in his personal lab, using the charge of four Leiden phials connected in cascade. Could he then have anticipated the phenomenon of electrical oscillations?

To "give polarities" to compass needles at his wish mean that Franklin developed an "analogous" or modular phial, in which both dielectric and metallic parts could be easily removed, where, in his words, "... the tail of one serves to load the other", sometimes greatly increasing the charge. More specifically, in the experiment, the phials were arranged so that the inner coating of one connected to the outside of another (Fig. 14), and so on, until the last was grounded, being the first charged with a friction (von Guericke's) machine.[34] After "removing" the human influence, which was the part that received the "eletrical kiss",[35] he was able not only to demonstrate that the electric fluid was somehow stored inside – or within – the glass itself, but also that the capacity to store electricity varied with the way the Leiden bottles were connected.

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[31] According to Mach, Franklin's electrical fluid was equivalent to the weight of water, multiplied by the distance. That is, if a Leiden jar was discharged, the electricity did not magically disappear as the work was carried out, but it was assumed that different "electricities" moved to oposite locations. In other words, the mechanical equivalency for water would be the electrical energy, a concept to be formalized in the 19th century, by the "electrical mathematicians". See Mach 1911, p. 44.

[32] One of them is the "wonderful effect of points", or the "kite experiment" and many others can be found at Franklin 1751; Also in Franklin 2017.

[33] Franklin 1750.

[34] Franklin 1748, § 10.

[35] Reading Benjamin Franklin's work in the original is a good way to capture the panorama of experimental research until the 18th century, when the human body was an important element in experiments with electricity. In this case, in particular, Franklin created modular jars so that they could be removed and he could show where the electric fluid was "stored". But removing the human element was a more difficult task, since the lips were often used to feel electric shocks. This "Electrical Kiss" was even a theme for another experiment conducted by him, in which voluntary couples were asked to hold charged jars and, after that, they were invited to kiss(!), in order to demonstrate the electrization levels and the effect of an electrical discharge: "... let A and B stand on wax; or A on wax and B on the floor; give one of them the electrised phial in hand; let the other take hold of the wire; there will be a small spark; but when their lips approach, they will be struck and shock'd." See Franklin 1747b.

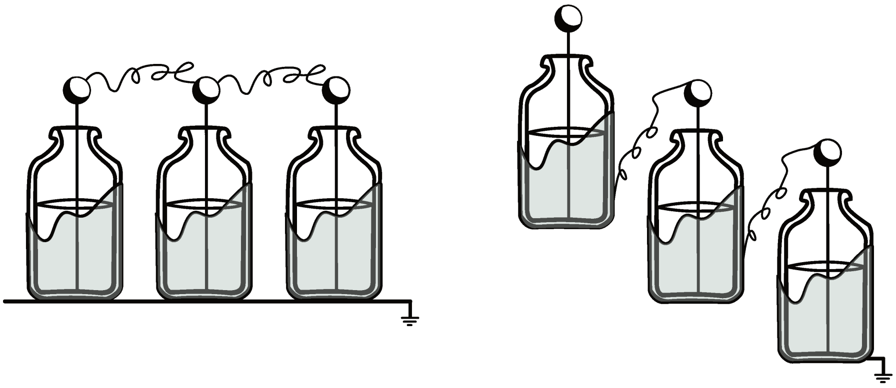

Figure 14 – Leiden phials connected in parallel (left) or in cascade (right).

As an outcome, the eminent republican noticed that a large amount of “electrical fire” could be transmitted with “... unconceivable quickness and violence”, but could not predict the polarization of the needles after discharge, that is, he could not indicate which end of the needle would be the north or the south pole, once every time different results were obtained. As it is today taught in electrostatics classes, the explanation is related to the oscillatory nature of the electrical current during the discharge process, a transitory phenomenon, but Franklin did not know that. Curiously, two years before, he made a remarkable analogy between the effect of the electric discharge of a Leiden phial and, amazingly, an elastic device such as a bent rod or a stretched spring, where he claims that “... a straight spring ...when forcibly bent, must, to restore itself, contract that side which in the bending was extended, and extend that which was contracted”.[36]

Franklin knew that a bent rod would oscillate a few times before turning to the state of rest, but with this analogy he was, in fact, being driven by the belief that the polarity of the needles was given by the (mechanical) force of the “electric fire”, stored in the glass, and not by the possibly elastic nature of the discharge, as he points out, in the same passage, “... the spring is not said to be *charged* with elasticity when bent, and discharged when unbent; its quantity of elasticity is always the same.”[37] He even stress that the comparison “... does not agree in every part”, and does not accept a similarity between the lighting phenomenon, the magnetization of compass needles and his own experiments on Leiden phials, and thus could not make the “correct analogy”: *charged jar and a stretched spring*. Obviously this would be a great step to be taken, and perhaps only could be achieved after the “official” recognition on the relation between electricity and magnetism, which apparently Franklin did not believe.[38]

The missing piece in the puzzle would be discovered only in 1826, when the French physicist Felix Savary discovered the phenomenon of oscillatory currents in the discharge of a Leiden phial. Guided by Øersted’s discovery (1820) of the influence of magnetism in

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[36] Franklin 1748, § 11-12.

[37] Franklin 1748, § 13; A remark on this analogy was made first by Potamian and Walsh 1909, p. 91.

[38] Franklin 1773.

galvanic circuits, Savary noted the same magnetic periodicity in fine steel needles – where Franklin only saw the work of chance or mechanical strength of the electric fire – when they were subjected to an electrical discharge, but arriving at the "correct" analogy. Later on, the often called "Faraday of America", Joseph Henry (1842), would develop his theory on the nature of discharge currents:[39] "... The discharge, whatever may be its nature, is not correctly represented (employing for simplicity the theory of Franklin) an imponderable fluid from one single transfer of side of the jar to the other; the phenomenon requires us to admit *the existence of a principal discharge in one direction and then several reflex actions backward and forward, each more feeble than the preceding, until equilibrium is attained.*"[40] This operation would be considered, by the nineteenth century electricians, analogous to the act of plucking a violin string in a medium with elastic characteristics.

5.3 Elastic solids and the wave equation

If we follow Franklin's track, it will be possible to see some attempts to treat electricity as a mathematical subject in a deeper level. One- and two-fluid theories for electricity and magnetism coexisted along the 18th century, and a two-fluid choice was, for many reasons, preferable, once the relationship between electrical and magnetic phenomena had been "felt" for a long time, but with no proofs. Breaking the loadstone meant two new magnets, and it was reasonable to consider a analogous behaviour for electricity, as Coulomb (1736-1806) put it, "*... le fluide magnétique, ainsi que le fluide életrique, agissent, soit par répulsion, soit par attraction*".[41] With Coulomb the two-fluid theory assumed a closer shape to the modern views on magnetism,[42] once he considered the smallest portion of magnetic substances as magnets themselves, acting in pairs, so that the magnetization process would be simply the orientation of the tiny magnets in one same direction.

Using the same terminology common to the fluid theories, Coulomb developed an experiment in which the measurement device acted under the elastic properties of a stem, called by him *balance életrique.* The balance was based on the property that filaments of metal could produce a reactive force in torsion, which were proportional to the angle of twist. With the torsion balance, Coulomb was able to arrive at the remarkable expression for the force between two kinds of electric charges (or magnetic poles), an analogue expression to Newton's gravitational law of 1687: *"... La force répulsive de deux petits globes életrisés de la même nature d'eletricité, est en raison inverse du carré de la distance du centre des deux globes."*[43]

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[39] For the history of discovery of electric oscilations, see Gluckman 1990.

[40] Henry's emphasis. See in Henry 1886.

[41] "... the magnetic fluid, as well as the electric fluid, act, either by repulsion or by attraction". See Coulomb 1785b, p. 578.

[42] Hesse 1962, p. 183.

[43] "... The repulsive force of two small globes electrified with the same kind of electricity, is inversely

The constant of proportionality was determined experimentally by Coulomb, once he believed that it depended exclusively on the nature of the metal used. The modern notation,

$$\vec{F}_{ji} = \frac{q_i q_j}{4\pi\epsilon_0}\frac{\hat{r}_{ij}}{{r^2}_{ij}}\,, \tag{5.1}$$

clearly depicts the action at a distance relation, where $\vec{F}_{ji}$ acts along the line between the two globes, $\hat{r}_{ij}$ is the unit vector that points from the charge j to i, r_{ij} is the distance between them, $\vec{r}_i$ (or $\vec{r}_j$) is the vector that points from the origin of the coordinate system to the body i (or j) and the constant ϵ_0 is the so-called vacuum permittivity.[44]

In Coulomb's epistemic pillar on action between two bodies, there must be some "... kind of spiritual affinity between them. The force which each of the two exerts is bound up with the presence of the other body. In order that force should be present at all, there must be at least two bodies present. In some way a magnet only obtains its force when another magnet is brought into its neighborhood." These are Heinrinch Hertz's words (1893),[45] picturing the purest state of Coulomb's view. Similarly to the formulation for the Newtonian gravitational force, Coulomb considered the physical greatness between two kinds of electricity as nothing more than the totality of the effects produced by it, associating the deflection of a pointer on a previously defined scale on his torsion balance. Only in the following centuries the question would start to be unveiled, with a clearer understanding on how a measure could actually be taken from a physical phenomenon.[46]

Part of the formalism that most engineers take now for granted was then scarcely understood, and the unclear perspective on the connection between physical phenomena and mathematical description was still a source of controversies, even after Newton's *Principia*. Mathematical descriptions related to elasticity would become more evident in study of vibrating strings, which would be fundamental to the concept of traveling waves, serv-

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proportional to the square of the distance between the centers of the two globes."See in Coulomb 1785a, p. 572.

[44] This is a modern notation of the so-called electrostatic (or Coulomb's) force, which appears only as a statement in Coulomb's original memoir. The formulation shown in (5.1) was extracted from Assis 1992, p. 18.

[45] For continuous action, as in the case of field theory, Hertz compares the action between two bodies as a " ... spiritual influence" instead of affinity. That is, "... altough we admit that we can only notice this action when we have at least two bodies, we further assume that each of the acting bodies continually strives to excite at all surrounding points attractions of definite magnitude and direction, even if no other similar bodies happen to be in the neighborhood." See in Hertz 1893, p. 22.

[46] In a given measurement instrument, when a physical quantity F is represented by a function $f(t)$, the possible dependency on $f(t)$ with other variables is ignored, and no one doubts that $f(t)$ "does not exist", even if one does not have an instrument capable of accurately measuring it. For example, if F is a force exerted between two charges, it is admissible for the magnitude $f(t)$ shown on a measurement instrument to represent it, even considering its intrinsic imperfection. This view requires the representation of F as an "ordinary" function, but only under a classical conception, once this is a slightly naive way of representing a "real" physical quantity F, if it is not considered as a distribution, which is a concept that would appear in physics only in the 20th century. In this context, a physical variable will be nothing more than the totality of the effects it produces, for example, the deflection D of a needle in a meter. See, e.g., Papoulis 1962.

ing as basis for the main analogies of Victorian science, in which electrostatic phenomena would start to be considered as a manifestation of the ethereal elasticity. If a mathematician is asked what the term wave is, the answer is likely to come in the form of a doubly periodic function or, in other words, a periodic disturbance in both space and time. For a string, a function $y(x,t)$ that satisfies some differential equation, a demonstration that worth to be followed.[47]

Consider an elastic string, fixed at its ends, like a tuned violin, and stretched, in order to be played by the violinist. The *elastic* feature means here that the string is not able to support shearing ou bending (perpendicular forces), being able to support only forces along its own length, i.e., a tension. The *stretched* feature means that the string at rest has a initial tension different of zero. When a force $f(x)$ is applied in order to put the string into action, how will it behave? Many tried to answer the question and the first answers started to emerge with the efforts of the Englishman Brook Taylor (1685-1731), and from a controversy between the French Jean Le Rond d'Alembert (1717-1783) and the Swiss mathematicians Daniel Bernoulli (1700-1782) and Leonhard Euler (1707-1783).[48] But it was d'Alembert who actually obtained the first equation of a vibrant string, publishing a study on its movement in 1747,[49] what later would be called the *one-dimensional wave equation*, one of the most important of physics and one of the most useful for engineers.

To this problem, consider also an even distribution of mass along its length, which means a constant mass per unit of length ρ. Now, consider that, at a time $t = 0$, the string is given a slight deflection as shown in (Fig. 15), in which a tiny piece of the string $x + \Delta(x)$ is considered. The curve describing the form of the string is $y(x,t)$, i.e., a function of two independent variables, where y is the deflection with respect to the horizontal axis for arbitrary values of x and t. Thus, the initial deflection is $y(x,0) = f(x)$. Another initial condition is that the string will be released from the rest, i.e., $\partial y/\partial t|_{t=0} = 0$ for all values of x. The requirement for fixed ends are $y(0,t) = 0$ and $y(l,t) = 0$, for all instants of time. d'Alembert's objective, at this point, was to obtain $y(x,t)$ to any value of x inside the interval 0 to t, for positive values of t.

Disregarding the gravitational force, if one looks closely at the string element in (Fig. 15), there will be only two forces acting. The angle $\theta(x,t)$ is the angle that the string

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[47] The procedure for obtaining this equation is widely known, and can be found in the best books on differential equations, as in Churchill 1963. It will be presented mostly the Euler's solution, once he obtained the most general form of the wave equation. The different approaches can be seen in Wheeler and Crummett 1987. The present demonstration, specifically, is an adaptation of Paul Nahin's biography of Leonhard Euler (Nahin 2006, p. 115), in addition to d'Alembert's own article (1747) and the analysis of Wheeler and Crummett of the vibrating string controversy. It will be very instructive in consonance with the objectives of this study, since the wave equation is one of the most important results of all physics and engineering. A small caveat must be made regarding Nahin's analysis, which presents an adaptation of the d'Alembert-Euler solution, but with some wrong calculations (or typing errors) along the text, specially in finding the general solution (Cf., e.g., Nahin 2006, p. 121).

[48] Wheeler and Crummett 1987.

[49] d'Alembert 1747.

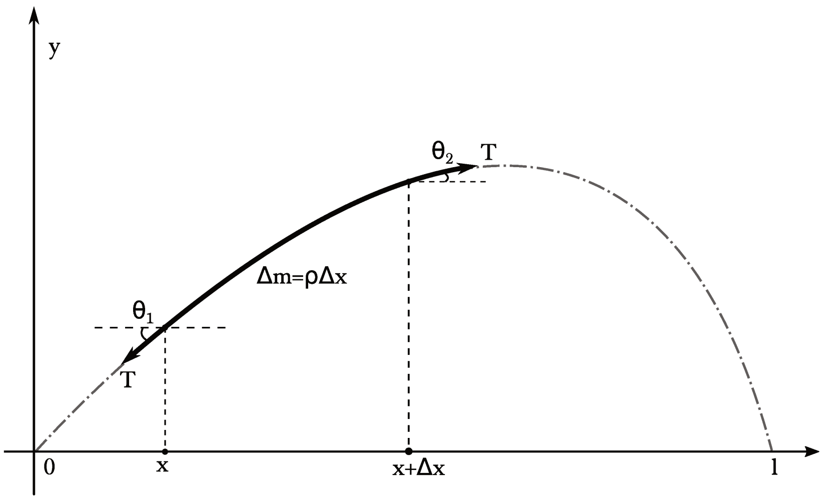

Figure 15 – A piece of an elastic string, with angles and forces represented.

element makes with the horizontal axis, with θ_1 and θ_2 being the elements of the left and right ends of the string, in which it is indicated the tension forces T. Considering small variations in the vibration of the string, the length of the element of string can be considered close to Δx and, for this reason, its mass will be $\Delta m = \rho\Delta x$. The vibration itself, that is, the vertical motion of the string can be obtained by Newton's second law *F=ma*, where the force will be the vertical component of the tension acting on the string, $T\sin(\theta_2) - T\sin(\theta_1)$, given by the mass and the vertical acceleration $\frac{\partial^2 y}{\partial x^2}$. Thus,

$$\rho\Delta x\frac{\partial^2 y}{\partial x^2} = T\sin(\theta_2) - T\sin(\theta_1)\,. \tag{5.2}$$

Because of the small amplitudes considered, for the sake of simplicity, $\sin(\theta) \approx \tan(\theta)$. So,

$$\rho\Delta x\frac{\partial^2 y}{\partial x^2} = T\tan(\theta_2) - T\tan(\theta_1)\,, \tag{5.3}$$

Or, dividing by $\rho\Delta x$,

$$\frac{\partial^2 y}{\partial x^2} = \frac{T}{\rho}\left[\frac{\tan(\theta_2) - \tan(\theta_1)}{\Delta x}\right]\,. \tag{5.4}$$

Imagining that the size of the rope starts to shrink, i.e., $\Delta x \to 0$, and looking closely at the right member of the previous equation, we will have precisely the definition of the partial derivative of $\tan(\theta)$ with respect to x,

$$\frac{\partial^2 y}{\partial x^2} = \frac{T}{\rho}\left[\frac{\partial}{\partial x}\tan(\theta)\right]\,. \tag{5.5}$$

In addition, from the curve it is noticeable that

$$\tan(\theta) = \frac{\partial y}{\partial x}\,,$$

evidencing that the result of the vibrating string problem will take the form of a second order partial differential equation,

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\rho}\frac{\partial^2 y}{\partial x^2} \,. \tag{5.6}$$

The factor T/ρ is especially noticeable in a physical context, and an engineer could quickly realize that, from a dimensional analysis (using the SI system of units),

$$\left[\frac{T}{\rho}\right] = \frac{kg \times m/s^2}{kg/m} = (m/s)^2 \,. \tag{5.7}$$

Admitting $T/\rho = c^2$, we will have a variable c with dimension of speed, but *speed* of what, precisely? Rewriting the equation in terms of c, the expression can be written as seen in modern textbooks:

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2} \,. \tag{5.8}$$

in which $y \equiv y(x,t)$ represents the displacement of the string in relation to the x axis and over a time t. To find Euler's solution, lets consider a change of variables, as it is assumed that $y(x,t)$ is twice differentiable by x and t. Thus,

$$v = ct - x \,, \tag{5.9}$$

$$w = ct + x \,. \tag{5.10}$$

It should be noted that v and w are independent variables, just as x and t, and so,

$$x = \frac{w-v}{2} \,, \tag{5.11}$$

$$t = \frac{w+v}{2c} \,. \tag{5.12}$$

By the chain rule, it is possible to write

$$\frac{\partial y}{\partial w} = \frac{\partial y}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial y}{\partial t}\frac{\partial t}{\partial w} = \frac{\partial y}{\partial x}\left(\frac{1}{2}\right) + \frac{\partial y}{\partial t}\left(\frac{1}{2c}\right) \,. \tag{5.13}$$

Differentiating again, now with respect to v, we will have[50]

$$\begin{aligned}
\frac{\partial}{\partial v}\left(\frac{\partial y}{\partial w}\right) &= \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial w}\right)\frac{\partial x}{\partial v} + \frac{\partial}{\partial t}\left(\frac{\partial y}{\partial w}\right)\frac{\partial t}{\partial v} \,,\\
&= \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial w}\right)\left(-\frac{1}{2}\right) + \frac{\partial}{\partial t}\left(\frac{\partial y}{\partial w}\right)\left(\frac{1}{2c}\right) \,,\\
&= \frac{\partial}{\partial x}\left[\frac{\partial y}{\partial x}\left(\frac{1}{2}\right) + \frac{\partial y}{\partial t}\left(\frac{1}{2c}\right)\right]\left(-\frac{1}{2}\right) + \frac{\partial}{\partial t}\left[\frac{\partial y}{\partial x}\left(\frac{1}{2}\right) + \frac{\partial y}{\partial t}\left(\frac{1}{2c}\right)\right]\left(\frac{1}{2c}\right) \,,\\
&= \frac{\partial^2 y}{\partial x^2}\left(-\frac{1}{4}\right) + \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial t}\right)\left(-\frac{1}{4c}\right) + \frac{\partial}{\partial t}\left(\frac{\partial y}{\partial x}\right)\left(\frac{1}{4c}\right) + \frac{\partial^2 y}{\partial t^2}\left(-\frac{1}{4c^2}\right) \,,\\
&= -\frac{1}{4}\left[\frac{\partial^2 y}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial x^2}\right] \,.
\end{aligned} \tag{5.14}$$

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[50] It is assumed here that $\partial^2 y/\partial t\partial x = \partial^2 y/\partial x\partial t$.

The expression inside the braces of 5.14 will be zero, once $y(x,t)$ satisfies the equation (5.8). So, for this reason,

$$\frac{\partial}{\partial v}\left(\frac{\partial y}{\partial w}\right) = 0\,. \tag{5.15}$$

And then, the general solution can be written:[51]

$$\begin{aligned} y(v,w) &= \Theta(v) + \Omega(w)\,, \\ y(x,t) &= \Theta(ct+x) + \Omega(ct-x)\,. \end{aligned} \tag{5.16}$$

From where one can see not only what is the physical meaning of the term c, which has the dimension of speed, but also the idea of *waves*. If someone has have a function $f(x-a)$, with $a > 0$, this means that $f(x-a)$ will be simply the function $f(x)$ shifted to the right by a. Thus, if one has have a function $\Omega(x)$, which is the function $\Omega(ct-x)$ at a time $t = 0$, with x positive, this means that $\Omega(ct-x)$ also represents the function $\Omega(x)$ shifted to the right – or to the left in the case of $\Theta(ct+x)$. It means that $\Omega(x)$ has traveled through a distance ct in a time t, at a speed c.

In the original problem, d'Alembert was trying to solve the boundary value problem:

$$y(0,t) = 0 \tag{5.17}$$

$$y(l,t) = 0 \tag{5.18}$$

$$\partial y/\partial t|_{t=0} = 0 \tag{5.19}$$

$$y(x,0) = f(x). \tag{5.20}$$

Applying the condition 5.17, we have $\Theta(ct) = -\Omega(ct)$. That is, the information $\Theta = -\Omega$ can lead the general solution to the form

$$y(x,t) = -\Omega(ct+x) + \Omega(ct-x)\,, \tag{5.21}$$

$$= \Omega(ct-x) - \Omega(ct+x)\,. \tag{5.22}$$

If we now apply the condition 5.18, we arrive at $\Omega(ct+l) - \Omega(ct-l) = 0$, or

$$\Omega(ct+l) = \Omega(ct-l)\,, \tag{5.23}$$

which means that the solution must be a periodic function of time. It is precisely at this point we can place the root of the vibrating string controversy, which involved three of the great physical mathematicians of all time. At that historical moment, the concept of function was not yet fully understood, and some eminent mathematicians were looking into the matter. d'Alembert believed that a function should be somewhat conditioned to

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[51] With the difference that d'Alembert used the variable s instead of x, $c = 1$, and the notation Ψ and Φ for the arbitrary functions. See in d'Alembert 1747, p. 216.

the ordinary methods of algebra and calculus. He was the “most mathematician” of the controversy, and sought to get away as much as possible from the physical panorama. His only imposition was that the solution should be an odd function and differentiable everywhere. His first presentation of the wave equation considered, for example, $c = 1$, and a solution (eq. 5.22) would be reduced to[52]

$$y(x,t) = \Omega(t-x) - \Omega(t+x)\,. \tag{5.24}$$

Euler, in turn, would propose a more “physical mathematical” approach. With a seemingly simpler view, he believed that a function could be defined simply if it were possible to draw the curve of $f(x) \times x$, in the same way as it would be done with a pen that slides over a piece of paper. The apparently simplistic view of Euler indicated that the function should be continuous and smooth, which is now understood as a “well-behaved” function, having a tangent and, therefore, being differentiable everywhere,[53] as he had obviously in mind the “plucked string” (Fig. 15). No one imagined that the solution only need to be valid for the interval $0 \leq x \leq l$. Since the expression presented in the general solution (eq. 5.16) also makes room for periodic functions with discontinuities (or even continuous but not smooth),[54] the controversy was then established.

Most of us who, in a way, have a solid and educated conception on the superposition principle – which was unknown at the time – would find d’Alembert’s vision strange, since he did not saw a vibrating string as a composition of distinct modes, but imagined each vibrating condition associated with a single one frequency. Daniel Bernoulli’s approach, in contrast, was closer to that of a modern physicist and was deeply engraved in physical terms. He suggested, for example, that one should listen to the sound of the string in order to understand it![55] From Bernoulli him came the first suggestion that a solution could emerge from a trigonometric series expansion, starting with the fundamental amplitude of the string, observed firsthand by Brook Taylor

$$A(x) = a\sin\left(\pi x/L\right)\ .$$

Bernoulli then argued that the general solution must be a sum of the string’s fundamental and higher harmonics:

$$\begin{aligned} y(x,t) &= b_1 \sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi c t}{l}\right) + b_2 \sin\left(\frac{2\pi x}{l}\right)\cos\left(\frac{2\pi c t}{l}\right) + \ldots\,, && (5.25)\\ &= \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi}{l}x\right)\cos\left(\frac{k\pi c}{l}t\right)\,, && (5.26)\end{aligned}$$

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[52] d’Alembert 1747.

[53] See in Wheeler and Crummett 1987.

[54] See, e.g., the sum of Weistrass in Tonidandel and Araújo 2015, p. 4.

[55] Wheeler and Crummett 1987, p. 34.

but he did not provide any mathematical justification for his arguments, and neither presented a method for calculation of the coefficients b_k. A method for this would be presented by Euler in 1777,[56] but the whole problem would be completely justified only by Fourier, in 1822, when he studied another elastic entity, now a fluid one, the heat.[57] Fourier would find out what the two friends Euler and Bernoulli missed, that when applying the last boundary condition (5.20), which represents the initial displacement of the string, $y(x,0) = f(x)$, it would be possible to find an impressive result, in which (if the coefficients b_k were known) they could perform an infinite sum of arbitrary functions to obtain any other![58] That is,

$$f(x) = \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi}{l}x\right) . \tag{5.27}$$

The one-dimensional wave equation was obtained within the physical panorama of the vibrating strings, and it is also an example of the complexity to conciliate mathematical models with physical hypothesis. Euler himself, in his early writings on the theory of sound,[59] had difficulties to harmonize his physical assumptions with the mathematical formalism. Despite d'Alembert's suggestion that the reasoning (of discontinuous functions) could be applied to sound propagation, since it was then believed that sound propagated in discontinuous pulses, as in Huygens light/sound analogy (Fig. 16), Euler argued that a function to represent such pulses would not be a solution to the wave equation. In a later work, however, he tried to justify the use of discontinuities on the sound propagation, with aid of the same wave equation,[60] using formalism expressed in the equation (5.16).

After d'Alembert, Euler and Bernoulli[61], mathematical models began to be more effectively considered as alternatives to physical hypotheses. With hydrodynamic models, Euler was able to describe the transmission of a disturbance through a homogeneous fluid medium, but leaving aside the issue of propagation through a solid medium, i.e., a medium where shear forces were present, which would even have a connection with the problem of vibrating strings: for that he considered a string made by n particles, and then made $n \to \infty$.[62] This question would be raised again in Young and Fresnel's theories for light, in 19th century.[63] Newton's corpuscular theory of light had been preferable throughout the century, but Young would reintroduce a wave theory, suggesting that a phenomena

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[56] Euler's method for calculating the coefficients of a trigonometric series is now widely used in Fourier series, but Fourier himself would not be aware of this method until years later, although Euler developed it when Fourier was only 9 years old! For the Euler and Fourier methods, see Tonidandel and Araújo 2017, p. 46.

[57] Fourier 1822.

[58] Actually, the issue if a little bit more complicated that that. To make this assertion true, the function $f(x)$ must obey the so called Dirichlet criteria. To more, see note 77, p. 120.

[59] L. Euler 1727.

[60] L. Euler 1766.

[61] Bernoulli 1738.

[62] L. Euler 1736a,b.

[63] Pedersen 2008.

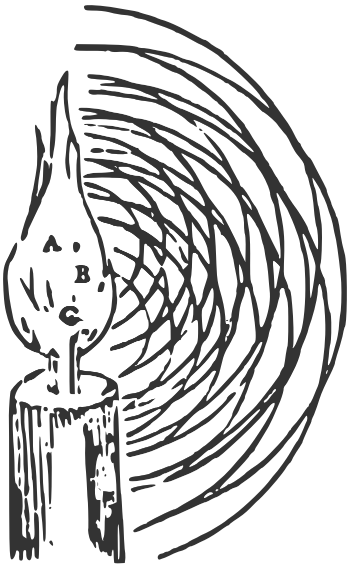

Figure 16 – Huygens view of light as a wave that propagates in independent pulses, but without the idea of periodicity. Source: Huygens 1690, p. 15.

hitherto hardly explained, like polarization, could be understood from the propagation of transverse waves, contrary to the common sense in which the ether was thought, in analogy to the longitudinal waves of sound. This movement would start the search, in the 19th century, for a mechanical model for the ether as an elastic solid, like the propagation of a wave on a vibrating string.

Other attempts to apply mathematical formalism to electrical phenomena in relation to elasticity were made in the same period and worth to be commented, as, for example, the proposal of the physicist Johann A. Euler (1734-1800), son of the great Leonhard Euler. Following the general guidelines proposed by Franklin, Johann Euler's electrical fluid in his *Recherches Sur la Cause Physique de l'Electricité* would be nothing less than the ether contained in subtances, and where in Franklin's the lack or excess of the electrical fluid corresponded to a positive or negative charge, in J. Euler's theory it could be replaced by the the ethereal elasticity, "*... de sorte que la cause de ces phénomenes doit être attribuiée à l'inégalite de ressort de l'éther, qui se irouve renfermé dans les corps*".[64]

Johann Euler often used the french terms *ressort* and *élasticité* to denote similar concepts of ethereal elasticity, and the first word approximated to the Boylean concept of air elasticity,[65] a more tangible and mechanical entity than the second one.[66] Possibly under the influence of his father, J. Euler took a further step towards a mathematical model for the electrical fluid, in which the flow of electricity would be described at a macroscopic level with differential equations for the velocity components of the fluid, as a function of

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[64] "... so that the cause of these phenomena must be attributed to the elastic inequality of the ether, which is enclosed inside the bodies". See J. A. Euler 1759, p. 126.

[65] See Boyle 1772b; see also chap. 4.

[66] Nardi and Silva 2021, p. 9.

time. To describe electrostatic phenomena with hydrodynamic concepts would evidently be an advance towards a field theory for electricity, although J. Euler's attempts did not gain much attention at that moment.

5.4 Elastic fluids and the heat equation

The eighteenth century had seen a rapidly growing number of theories to be attached to the category of elastic fluids, specially considering the nature of heat. In all times of mankind, the majestic presence of the sun was considered a divine matrix of life, an inexhaustible source of light and heat. But if, according to the corpuscular philosophy, light should be composed of matter, even if imponderable, and heat just a simple form of movement, why no changes in the sun's dimensions had been detected? Bacon and Descartes suggested that heat was the result of *intestine* movements of molecules within the cosmic ether. After all, there were evidences to corroborate the view, once it was possible to observe daily cases of movement producing heat: the phenomenon of boiling, the metal that was worked with hammer blows, the phenomenon of conduction in solid bodies.

Newton often explained heat in terms of vibrations of small ethereal particles, as appears in the pages of *Optics*, but he had also spoken of heat radiation through the ether as an elastic fluid, and it was an easy transition for his followers to postulate heat as a substance: the result of the movement of a very special one, partly reviving the old theories of elements, which saw the action of one of these elements in fire, a *comburent* fluid called *phlogiston*, φλογιστν (*flogisto*, wich means "burning up") and φλγα (*flo-ga*, which means fire).[67] The phlogiston theory of Johann Joachim Becher and Georg Ernst Stahlhad was one of the three elements that came to replace the traditional four, in which it was believed that such a combustible exuded from metals on ignition. It had some success in 17th century, mostly explaining chemical reactions,[68] but it also had a rival theory, which could explain an even wider range of phenomena in chemistry, a heat-fluid theory known as *caloric theory*.

The term caloric had been coined by Antoine Laurent Lavoisier (1743-1794), and literally represented an "atmosphere of heat", repelent particles around molecules of chemical compounds. The caloric theory was intended to be superior to phlogiston's, given that the phlogiston, on some occasions, could not explain why some chemical reactions resulted in a product that had a greater weight than the original components. Thus, the phlogiston should, in some cases, have a negative mass. Despite this, in other circumstances, the concepts of heat and caloric came to be treated as the same entity. The caloric was often seen as an elastic fluid similar to electricity, with mutual repellent parts, but that were attracted by ordinary matter. This hypothesis explained some success in explaining effects such as

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[67] See section 3.1, p. 41.

[68] Newman 2019.

the dilation in hot bodies, due to mutual repulsion of the excess caloric parts inside the body. In addition, the theory did well for bodies at low temperatures, mostly after Joseph Black had shown some evidence on this respect.[69]

A fluid that generated heat was a very reasonable concept and would explain a great number of phenomena: the idea of something very subtle, that could pass from one body to another, being responsible for the phenomenon of conduction and causing variations in temperature, being preserved throughout process. However, a fluidic theory for heat began to lose its strength as a result of Rumford's experiments on heat generated from friction.[70] After all, these kind of elastic fluid theories were highly qualitative, and, in a way, tautologically based. That is, they explained almost everything and almost nothing, simultaneously. Anyhow, a change of epistemological character was already going on, since the early days of Newton and Euler. Although it was realized that the caloric did not need, necessarily, to be associated with a substance, but only with some *unknown cause* that could keep the constituent parts of matter apart, its effects would start to be considered mathematically, from the first theories of continuous action.

In 19th century France, above all, it would emerge works able to provide credibility to the dynamic theory of heat, which aimed to provide it with a mathematical drapery or, more specifically, to the process of conduction, similar to what Newton had done with the universal gravitation and optics: "... Les Causes primordiales ne nous sont point conues; mais elles sont assujetties à des lois simples et constantes, que l'on peut découvrir par l'observation, et dont l'étude est l'object de la philosophie neturelle. Le chaleur, comme la gravité, toutes les substances de l'univers, ses rayons occupent toute les parties de l'espace. Le but de notre ouvrage est d'exposer les lois mathématiques que suit cet élément. Cette théorie formera désormais une des branches les plus importantes de la physique générale."

These are the opening words of Jean-Baptiste Joseph Fourier, in the preface to his 1822 *Théorie Analytique de la Chaleur*,[71] where he makes clear the intention of not pursuing causes, nor to formulate hypotheses on the nature of heat – as his predecessors (Laplace, Poisson, Cauchy etc.) had done, but laws obtained from the observation of "facts" and experimentation. Fourier's concern was less based on discovering greater "truths", concentrating on determining "... the temperature of the skies, the planets, the infinite spaces ...". It did not matter whether the transmission was a fluid or any other way in which the movement was communicated. His reality model basically consisted of reducing a problem to a mathematical equation and solving it "dryly", without hypothesis, as it has

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[69] Mach 1911.

[70] Brown 1949.

[71] See Fourier 1822; The same passage can be found in the english translation: "... Primary causes are unknown to us; but are subject to simple and constant laws, which may be the object of natural philosophy. Heat, like gravity, penetrates every substance of the universe, its rays occupy all parts of space. The object of our work is to set forth the mathematical laws in which this element obeys. The theory of heat will hereafter form one of the most important branches of general physics." See in Fourier 1878.

already been discussed.[72]

The argument was that only macroscopic properties, and therefore measurable ones, should be considered, that is, Fourier states that the rate of heat flow, independently of the unknown causes, would be proportional to gradient of temperature between to mediums subjected to different sources of heat. This was what the supporters of the analytical current understood as a *Mathematical Theory*, whether for the heat or another entity. But how to consider something that flows, that streams, without the subliminal hypothesis that if something flows, it is fluid? Based on the analytical principle, it was then possible to obtain the mathematical model of the problem, in terms of a partial differential equation that would undoubtedly be a great landmark for the physics and engineering: the heat equation.

It is of interest to follow the steps of Fourier in demonstrating the heat equation, which would be the first step for his great discovery, the now called Fourier series. Lets consider two cross sections of a solid represented by two planes at temperatures u (left) and $u + \Delta u$ (right) respectively. As the right plane is at a higher temperature, the gradient "points" towards the unit vector $\hat{n}$, that is, the gradient is a vector that points in the direction of the growth of a function ($\frac{\partial u}{\partial n}\hat{n}$ is also called a directional derivative of u in the direction of $\hat{n}$). The heat flow ϕ , responsible for balancing temperatures, will be similarly proportional to the temperature gradient, but in the opposite direction. Thus,[73]

$$\phi = -K\frac{\partial u}{\partial n}\hat{n}\,. \tag{5.28}$$

in which K is a proportionality constant related to the material. If the flow through an area S in a volume V is considered through the plans x, y and z, we have:

$$\phi_x = -K\frac{\partial u}{\partial x}\hat{\imath}\,; \phi_y = -K\frac{\partial u}{\partial y}\hat{\jmath}\,; \phi_z = -K\frac{\partial u}{\partial z}\hat{k}\,; \tag{5.29}$$

and, for this reason,

$$\begin{aligned} \phi &= -K\left\{\frac{\partial u}{\partial x}\hat{\imath} + \frac{\partial u}{\partial y}\hat{\jmath} + \frac{\partial u}{\partial z}\hat{k}\right\} \\ &= -K\vec{\nabla}u\,. \end{aligned} \tag{5.30}$$

Also, consider the small volume element $\Delta V = \Delta x\Delta y\Delta z$ of a volume solid V, the constant of proportionality K, the specific heat c and specific mass (density) μ (Fig. 17). The flow per unit area that goes through EFGH in the volume element will be $-K\frac{\partial u}{\partial x}\big|_x$, where u represents the temperature. Since the EFGH face area is $\Delta y\Delta z$, the heat transferred in a time Δt will be:

$$-K\frac{\partial u}{\partial x}\bigg|_x \Delta y\Delta z\Delta t\,. \tag{5.31}$$

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[72] See section 2.3, p 34.

[73] See Fourier 1878, p. 109.

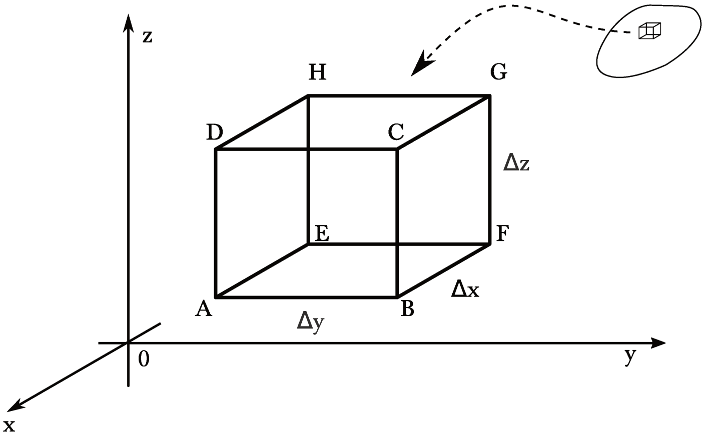

Figure 17 – Heat flow in an element of volume.

Similarly, the flow through the ABCD face will therefore be:

$$-K\frac{\partial u}{\partial x}\bigg|_{x+\Delta x}\Delta y\Delta z\Delta t\,. \tag{5.32}$$

The heat flow that passes through the volume ΔV in the direction x will, in turn, be the flow that enters minus what comes out in the time interval Δt:

$$\phi_{yz}=\left\{K\frac{\partial u}{\partial x}\bigg|_{x+\Delta x}-K\frac{\partial u}{\partial x}\bigg|_{x}\right\}\Delta y\Delta z\Delta t\,. \tag{5.33}$$

What happens in a similar way for y and z directions:

$$\phi_{xz}=\left\{K\frac{\partial u}{\partial y}\bigg|_{y+\Delta y}-K\frac{\partial u}{\partial y}\bigg|_{y}\right\}\Delta x\Delta z\Delta t\,; \tag{5.34}$$

$$\phi_{xy}=\left\{K\frac{\partial u}{\partial z}\bigg|_{z+\Delta z}-K\frac{\partial u}{\partial z}\bigg|_{z}\right\}\Delta x\Delta y\Delta t\,. \tag{5.35}$$

In short, the flow through the volume ΔV will be the sum of these three equations, which represents the heat needed to increase the temperature of a volume ΔV by an amount Δu.

On the other hand, it is known that the heat needed to raise the temperature of a quantity Δu a body of mass m is $m \times c \times \Delta u$, in which $m = \mu\Delta V = \mu\Delta x\Delta y\Delta z$. Therefore, the sum of the heat flow equations will result in:

$$\phi_{yz}+\phi_{xz}+\phi_{xy}=\mu\Delta x\Delta y\Delta z\times c\times\Delta u\,. \tag{5.36}$$

Substituting this result in the sum of the flow equations, dividing the two members by ΔV, we get:

$$\frac{K\frac{\partial u}{\partial x}\big|_{x+\Delta x}-K\frac{\partial u}{\partial x}\big|_{x}}{\Delta x}+\frac{K\frac{\partial u}{\partial y}\big|_{y+\Delta y}-K\frac{\partial u}{\partial y}\big|_{y}}{\Delta y}+$$

$$+\frac{K\frac{\partial u}{\partial z}\big|_{z+\Delta z}-K\frac{\partial u}{\partial z}\big|_{z}}{\Delta z}=\mu c\frac{\Delta u}{\Delta t}\,. \tag{5.37}$$

This equation has a very peculiar format, known to every student of calculus. If the limit is taken by making Δt and ΔV tend to zero, *voilà*:

$$\frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z}\right)=\mu c\frac{\partial u}{\partial t}\,.$$

And as K is a constant in isotropic materials,

$$K\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=\mu c\frac{\partial u}{\partial t}\,,$$

and can be rewritten as:

$$\frac{\partial u}{\partial t}=k\nabla^2 u\,. \tag{5.38}$$

In which $u \equiv u(x,y,z,t)$ is the temperature distribution over time and $k=\frac{K}{c\mu}$ is the so-called diffusivity of the material. This equation is called three-dimensional heat equation.

In the case of the diffusion of a substance in a porous medium, the function u denotes *concentration* instead of *temperature* – i.e., $u(x,y,z,t)$ will represent the mass of the substance that diffuses per unit volume of the solid – and the constant k will be called the *diffusion constant*. Under these conditions, the heat equation could be renamed to the diffusion equation. This clearly demonstrates the fluid analogy for heat transfer. The natural step, therefore, would be to find a solution to the equation.

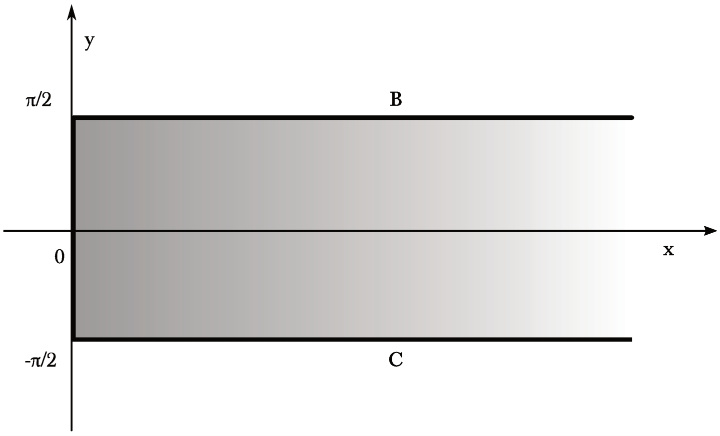

Figure 18 – Fourier's heat conductor solid.

In section *First example of the use of trigonometric series in the theory of heat*, of the chapter *Propagation of Heat in an Infinite Rectangular Solid*,[74] Fourier sought to find the profile of permanent temperatures in a solid conductor, which he called a "prism"(Fig. 18). Since he was interested in the stationary regime of heat conduction, i.e., the "final" temperature values, u does not depend on time, and therefore $\frac{\partial u}{\partial t}=0$. Thus, the heat equation is transformed into the Laplace equation, $\nabla^2 u=0$. Disregarding the thickness of the solid,

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0\,, \tag{5.39}$$

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[74] Fourier 1822, sec. 2, chap. 3.

forming the boundary value problem subject to conditions:[75]

$$\begin{aligned} \left|u(x,y)\right| &< M \quad \text{[limited temperature]}\,; && (5.40)\\ u(0,l) &= 0\,; && (5.41)\\ u(x,\pm\frac{\pi}{2}) &= 0\,; && (5.42)\\ u(0,y) &= f(y)\,; && (5.43) \end{aligned}$$

in which $l = \frac{\pi}{2}$. Basically, Fourier considered $u(x,y)$ to be a periodic function with period $2l = \pi$.

Working on this reduced form, Fourier was able to find what would represent the final temperature distribution, after establishing what would be known as the method of separation of variables – i.e., making $u(x,y) = \alpha(x)\beta(y)$ – and the principle of superposition, which is nothing less than considering the general solution as a sum of all particular solutions. Thus, assuming that the function u of the independent variables x and y is "separable", we have

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} &= \beta\frac{\partial^2 \alpha}{\partial x^2}\,, && (5.44)\\ \frac{\partial^2 u}{\partial y^2} &= \alpha\frac{\partial^2 \beta}{\partial y^2}\,. && (5.45) \end{aligned}$$

And substituting this expression in Laplace's equation, we arrive at

$$\frac{1}{\alpha}\frac{\partial^2 \alpha}{\partial x^2} + \frac{1}{\beta}\frac{\partial^2 \beta}{\partial y^2} = 0\,. \tag{5.46}$$

And since each member of the separated equation is a function of just one variable, the two sides will be equal for every x and y if each one is equal to the same arbitrary "constant" m. Thus,

$$\begin{aligned} \frac{1}{\alpha}\frac{\partial^2 \alpha}{\partial x^2} &= m^2\,, && (5.47)\\ \frac{1}{\beta}\frac{\partial^2 \beta}{\partial y^2} &= -m^2\,. && (5.48) \end{aligned}$$

Assuming an exponential-type solution $\alpha(x) = Ce^{\omega x}$, replacing it in the equation (5.47), we have

$$\omega^2 Ce^{\omega x} - m^2 Ce^{\omega x} = 0 \Rightarrow \omega = \pm m\,. \tag{5.49}$$

Then, $\alpha(x) = A_1 e^{-mx} + A_2 e^{mx}$. Analogously, in the equation (5.48), $\omega = \pm mj$, in which $j = \sqrt{-1}$. Then, by the Euler's formula,

$$\begin{aligned} \beta(y) &= A_3 e^{jmy} + A_4 e^{-jmy}\,, && (5.50)\\ &= A_3\left[\cos(my) + j\sin(my)\right] + A_4\left[\cos(my) - j\sin(my)\right]\,, && (5.51)\\ &= [A_2 + A_3]\cos(my) + j\,[A_2 - A_3]\sin(my)\,. && (5.52)\\ &= B\cos(my) + jC\sin(my)\,, && (5.53) \end{aligned}$$

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[75] Fourier also considers the plans B and C subject to a constant null temperature.

if we incorporate the coefficients within the same constants B and C.

From the first boundary condition (5.40), Fourier observed that since the temperature cannot increase indefinitely as $x \longrightarrow \infty$ (assuming $m > 0$), the positive exponential term can be disregarded. Then $\alpha(x) = Ae^{-mx}$. He also considered that the function $u(x,y)$, which represented the steady state of temperatures in the solid, should be "extremely small" when x takes on large values.[76] And as $u(x,y)$ is a function of the "real world", it is possible to significantly simplify the problem, disregarding the imaginary term of the equation. Thus, $\beta(y) = B\cos(my)$.

Bringing together the previously separated function, it follows that the function $u(x,y)$ represents, so far, a smoothed cosine. To visualize this function, imagine a surfer on the crest of a wave. If the variable y represented the time (which does not happen in this case) the surfer would not notice any oscillation, that is, he would not observe the sinusoidal behaviour of the wave, only the exponential decay along x, feeling a gradually decreasing amplitude. Since y is, like x, a spatial dimension, what we would see would be a "snapshot" of the wave along x. Then,

$$u(x,y) = \alpha(x)\beta(y) = Ae^{-mx}B\cos(my)\,. \tag{5.54}$$

According to the second boundary condition (5.41), it is possible to see that $\cos ml = 0$. This means that, except for the trivial solution, $ml = (2n+1)\frac{\pi}{2}$. That is, the expression $m = (2n+1)\frac{\pi}{2l}$, in which n is an integer, shows that the "arbitrary constant" m is not that arbitrary! However, as $l = \pi/2 \Rightarrow m = (2n+1)$, m assumes only odd values. Since AB will be different for every value of m (or n), we can incorporate the constants of the "gathered" equation, making $AB = c_m$. Thus, $u_m(x,y) = c_m e^{-mx}\cos(my)$, which represents a particular solution of the equation. The general solution can be found by the sum of all particular solutions, which establishes the *principle of superposition*:

$$\begin{aligned} u(x,y) &= \sum_{\substack{m=2n+1\\ n=0}}^{\infty} c_m e^{-mx}\cos my \\ &= c_1e^{-x}\cos y + c_3e^{-3x}\cos 3y + c_5e^{-5x}\cos 5y + \dots\,. \end{aligned} \tag{5.55}$$

When applying the initial condition (5.43), i.e., $u(0,y) = f(y)$, Fourier obtained his famous trigonometric series, which represented the temperature profile of the solid, in the form of an infinite sum

$$\begin{aligned} f(y) &= \sum_{m=1}^{\infty} c_m\cos my \\ &= c_1\cos y + c_3\cos 3y + c_5\cos 5y + \dots\,. \end{aligned} \tag{5.56}$$

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[76] Fourier 1822, p. 134.

An impressive result, showing that it was possible, if the coefficients c_m were known, to perform an infinite sum of cosine functions to approximate "any" function![77]

Throughout his work, Fourier used sinusoidal functions in order to solve various particular cases, in steady state. His analogy in considering the movement of the supposed caloric fluid, even labeling it as an unknown cause, adjusting the heat problem to the Newtonian paradigm – especially after its great discoveries, the infinite series and transform – caused a sensation throughout Europe, finding a fertile cradle especially in England, in first quarter of the 19th century. The dynamical theory of heat did not completely surpass the caloric fluid theory until an equivalence between mechanical and calorific energy was established, in the middle of the nineteen, based on the suggestions of Clausius, Helmholtz and Thomson. The question of the repulsion of the supposed particles of caloric has then become somewhat obsolete, being replaced by the statistical theory of heat. Throughout the century, still, a large number of theories demanded some (or several) type of elastic fluids. Sometimes a mixture of fire, light, heat, ether. Other times the phlogiston or the electric fluid. Oliver Lodge, for example, one of the pioneers of electromagnetism used to say that he enjoyed "pondering the imponderable."[78]

That is, what all these entities shared with each other was precisely the character that made them their greatest weakness: volatility, subtlety, elasticity. They were examples of a period in which the principles of convertibility between different forms of energy were not known. With the revival of this *mathematical positivism*,[79] born between the lines of Newton's Principia or by the writing of Roger Cotes, and which now had one of its most eminent members in Fourier, there is a tendency to replace previous analog structures, such as theories of elastic matter or models of particles by equivalent mathematical abstractions, still aware that, as Mach would say, "... matter is an abstraction of the same type, as good or bad as it can be. We know as much about the soul as we know about matter."[80]

In the new scientific movement it was necessary to *matematize* the phenomenon, in order to cover it with meaning. Following Fourier's example, it follows George Ohm's

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[77] In fact, "any" is a too optimistic expression, once the function $f(y)$ must obey some criteria, called the Dirichlet criteria. Roughly speaking, a Dirichlet condition is one that specifies the value of a function in a boundary, which ensures that it can be expressed as a Fourier series. Specifically, the criteria termed after the German mathematician define *suficient* but not *necessary* conditions to the convergence of a Fourier series. To this, if a function is periodic, then its partial sums will converge to $f(y)$ for all y if, beyond being absolutely integrable (i.e., having a finite integral over a period), it also has, necessarily

1. A finite number of discontinuities in a period, and,
2. A finite number of extreme points (maxima and minima) along a period.

[78] Lodge 1909.
[79] Duhem 1954.
[80] Mach 1911, p. 48.

(1789-1854) attempts,[81] with electrical conduction in straight metallic conductors, Stokes and Green, with the theory of elasticity, and, finally, William Thomson (1824-1907) – the most devoted disciple of Fourier – with the “trilogy” in which he would expose his analogy between electricity and heat: the movement of heat in connection with the mathematical theory of electricity,[82] the theory of equilibrium electricity,[83] and, finally, the theory of the electric telegraph.[84] The latter, after its adherence to the technological issues of underwater telegraphy, which would culminate in the launch of the first cable for telegraph communications between the old and the new World: the transatlantic cable. The door to fast communications over long distances was about to be opened.

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[81] Ohm 1827.

[82] Thomson 1842.

[83] Thomson 1854a.

[84] Thomson 1854b.

6 The electric telegraph

... I believe electro-magnetic telegraphy could be brought to a state of perfection, and made to assume such proportions as almost to startle the imagination.
(Carl Friedrich Gauss, in a letter to a friend, 1833.)

... Faraday no doubt was perfectly familiar with the electrical appetite of the water wire as compared with the [earth] air wire. ...The mental translation of propagation of electricity into propagation of heat which your equations shown to be permissible puts the mode of transfer in a clear light.
(George Gabriel Stokes, in a letter to William Thomson, 1854.)

... We may ring a bell at a distance in other ways, as by forcing air into a long tube, at the other end of which is a cylinder with a piston which is made to fly out and strike the bell. We may also use a wire; but instead of pulling it, we may connect it at one end with a voltaic battery, and at the other with an electro-magnet, and thus ring the bell by electricity.
(James Clerk Maxwell, 1873.)

... A mental representation of many of the phenomena connected with electrical oscillations is also very simply got by the fluid analogy.
(Oliver Heaviside, in his Eletrical Papers, 1894.)

At the end of the eighteenth century, a new trend sought to highlight the technological progress of individual countries and their insertion on the world stage, specially from those new products coming out of manufactures, in an evident process of mechanization. Inspired by British economic and expansionist liberalism, the need for developing industrial fields and stimulate foreign trade was promoting a real cultural transformation in the Old World. On the continental side of the English Channel, while the first popular uprisings based on the enlightenment ideas broke out, exhibitions of manufactured products are organized, where inventions and ideas are presented as a celebration of a new era, aiming to educate and inform the masses for the new markets.[1]

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[1] Briggs and Burke 2009.

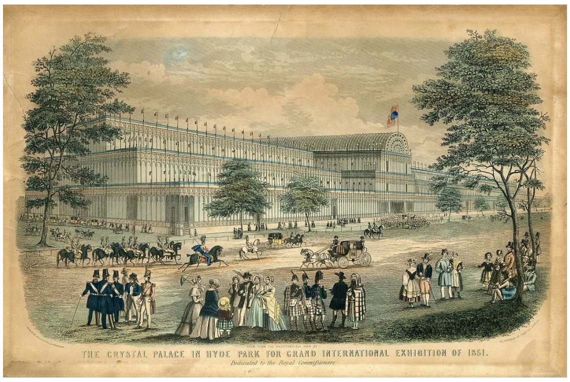

Figure 19 – The crystal palace, stage of the Universal Exhibition of All Nations, 1851. Source: Read & Co. Engravers & Printers (PD-US), 1851.

6.1 Early telegraphy

Between 1797 and 1798, the first exhibitions were organized in Paris, and in the first jubilee after the turn of the century (1849), more than 4500 fairs would take place. Around 1834, under French initiative, the internationalization process started and, in 1851, the *Great Exhibition of the Works of Industry of All Nations* opened its doors in the magnificent Crystal Palace of Hyde Park, an exuberant structure in steel and glass, designed by a self-taught gardener, Joseph Paxton (1803-1865), a symbol of the modern London (Fig. 19). The first conveyed message was evidently a political one, in which the host country, in this case the British empire, could export not only the products of a growing and powerful industry, but also its traditions and culture. Additionally, the *universal exhibition of 1851* promised to revolutionize communications through the newest and most promising invention, which fed on a mysterious and invisible fluid: the electric telegraph.

Other novelties would be presented at the same exposition,[2] such as the vulcanized rubber and the fax machine, but the telegraph would be, evidently, the great star. Electric telegraphy is one of the culminations of a long history of communications, and has one of its roots in the optical or semaphoric system of Claude Chappe, from 1792.[3] Used mainly for military purposes, but also with institutional appeal from organizations and governments, the optical telegraph of Chappe used a mechanical system located in a signaling tower, in which blades were positioned according to a certain symbolic coding.[4] The visual coding was coordinated by human operators located at the top of each tower, who observed the signal with the help of telescopes (Fig. 20), at stations that were up to 10 kilometers apart from each other. One operator sent a sequence of symbols, spaced regularly, while the other repeated the same operation confirming the integrity of the information. Such towers spread all over France, which is a relatively small country, with

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[2] Yapp and Ellis 1851.

[3] Murray 1905.

[4] Chappe's symbolic encoding was something like:

Figure 20 – The Optical telegraph of Claude Chappe, represented by a 19th century painting. (Unknown author, public domain-US).

area around 550 km^2.

From 1790, more than 500 stations were covering the french territory, forming a corridor of almost 5000 km. The speed of transmission was also impressive: a good operator could transmit about three symbols per minute, that is, a short sentence could be transmitted every hour.[5] It does not seem very much, but considering the involved distances, a single symbol, in the same terms, could reach a distance of up to 900 km in one single hour![6] This was impressive, since information such as letters, price lists for negotiations or even a national security alert could take weeks to arrive on horseback at its destination. The telegraphed messages had also a double layer of encryption, as a kind of point-to-point encryption, in which the operators were unaware of its content. They received the coded messages before sending, which could provide for the capture of information by an unauthorized person.[7] Chappe's optical telegraph facilities were used in France until the second half of the 19th century and inspired similar systems all over Europe, but would prove impractical on a larger scale.

In the previous centuries, the first devices to equip the laboratories dedicated to

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[5] The transmission rate of symbols, considering that the receiver had to confirm the integrity of the information by sending the same sequence to the transmitter, can be estimated by, approximately, $\frac{3\,symb}{2\,min} \times \frac{60\,min}{h} = 90\frac{symb}{h} \approx 10\frac{words}{h}$, considering a sentence of approximately 10 or 90 symbols.

[6] This means that the "speed"of each symbol is approximately $\frac{3}{2}\frac{symb}{min} \times \frac{1\,min}{60\,s} = \frac{1}{40}\frac{symb}{s}$, considering a distance of 10km per tower. So, in one our we have the average "speed" per symbol: $\frac{1\,symb}{40\,s} \times \frac{10\,km}{symb} = \frac{1}{4}\frac{km}{s} \times \frac{3600\,s}{1\,h} = 900\frac{km}{h}$.

[7] Obviously, the anachronistic expression *point-to-point* serves only as a figure of speech, but resembles a similar idea. In this regard, Standage informs that even so the messages could be intercepted or "hacked", another anachronism that the author alowed himself to do. See Standage 2007, p. 105.

electricity had been developed, such as the versorium of Gilbert (1600), the electrostatic generators, the Leiden phials and the battery of Alessandro Volta (1745-1827), in 1800. Since it was believed possible to, literally, store electricity in a bottle, it was realized that it could be used for communications. One of the first attempts to use static electricity in sending messages was conducted inside the Carthusian convent of Paris, an experiment conducted by the Abbot Jean Antoine Nollet (1700-1770), which ended up involving the entire community of the monastery and even the French royal guard. By that time, some electrical properties of metals were already known, such as their conductive capacity, and thus, in Nollet's experiment, about 180 soldiers suspended a long iron wire of approximately 1800 m in the air, holding it in pairs. When connecting one end to a Leiden jar loaded with the von Guericke's machine,[8] the result, more than expected, was an immense shock, "instantly" felt by the soldiers in front of the King of France!

However, only after Øersted's discovery (1820),[9] that the idea of using electricity as a mean of "transmitting intelligence" begins to take shape. The simple experiment reported by Hans Christian Øersted (1777-1851) confirmed the long felt relation between electricity and magnetism, demonstrating a deflective magnetic needle in presence of a galvanic current nearby. At that same year, the remarkable experiments of André-Marie Ampère (between the short years of 1820-1826) demonstrated the interaction between electrical current elements as an action at a distance law. Ampère's main hypothesis from the Øersted experiment was that the electromagnetic interactions arose, fundamentally, from interactions between electrical elements. For him, such elements should exist both on earth and on magnets and, therefore, would be responsible for all magnetic interactions. That is, all magnetic properties would be, fundamentally, a result of electrical interactions. Ampère's simple but powerful conjectures helped him to elaborate a whole formalism for the electrodynamics, but had also, as a result, the development of important measurement devices, such as the Galvanometer.[10]

With the discovery of the electrical induction by Michael Faraday, in 1831,[11] the pathway for the electrical telegraph started to become a viable reality, once it enabled the construction of the first electromagnets and magnetic switches. Although the first electromagnet was proposed by William Sturgeon, in 1825,[12] the impact of Faraday's achievements boosted the first complete telegraphy systems, starting with the Gauss-Weber telegraph of 1833, which connected the Institute of Physics at the University of Göttingen, where Wilhelm Eduard Weber (1804-1891) was director, to the Astronomical Observatory

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[8] Lardner and Bright 1867, p. 40.

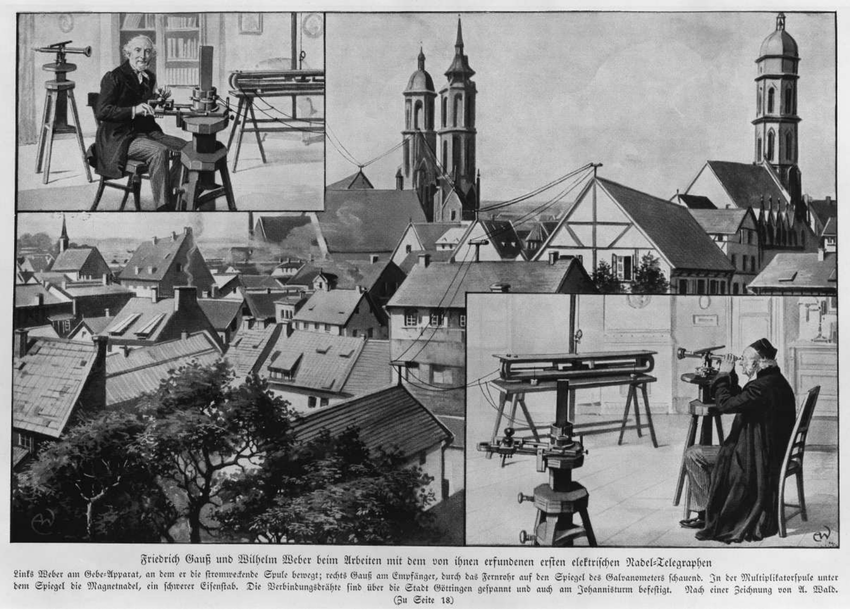
[9] Øersted 1820; A portuguese translation of this paper can be found at Øersted 1986.

[10] In addition to the expression *electrodynamics*, Ampére coined as well the expressions *electromagnetism*, *electromagnetics* and *electrostatics*. As for the the construction of the Galvanometer, it was realized under the suggestion of Ampère by Pouillet, in 1837. For this and for the life and work of Ampère, see Assis and Chaib 2011, p. 59,p. 291–297.

[11] Faraday 1855; See also, in a Portuguese translation, Faraday 2011.

[12] Morus 1992.

directed by Carl Friedrich Gauss (1777-1785), covering up a distance of approximately 2.5 km (Fig. 21). The transmission line consisted of two wires, in a time when the earth had not yet been considered as a possible return path for the electrical signals.



Friedrich Gauß und Wilhelm Weber beim Arbeiten mit dem von ihnen erfundenen erſten elektriſchen Nadel-Telegraphen

Links Weber am Gebe-Apparat, an dem er die ſtromweckende Spule bewegt; rechts Gauß am Empfänger, durch das Fernrohr auf den Spiegel des Galvanometers ſchauend. In der Multiplikatorſpule unter dem Spiegel die Magnetnadel, ein ſchwerer Eiſenſtab. Die Verbindungsdrähte ſind über die Stadt Göttingen geſpannt und auch am Johannisturm befeſtigt. Nach einer Zeichnung von A. Wald.
(Zu Seite 18)

Figure 21 – The Gauss-Weber telegraph built in 1833. In the image it is possible to observe Gauss on the right, at the receiver, and Weber on the left.

The system was constructed and installed by Weber, as it can be seen in a letter written by Gauss on 1833,[13] in which he describes not only the apparatus, "... it consists of a galvanic circuit conducted through wires stretched through the air", but also its basic principles, "... both ends of the wire are connected with a multiplicator, the one at my end consisting of one hundred and seventy, that in Weber's laboratory of fifty coils of wire, each wound around a one-pound magnet suspended according to a method which I have devised. By a simple contrivance – which I have named a commutator – I can reverse the current instantaneously. Carefully operating my voltaic pile, I can cause so violent a motion of the needle in the laboratory to take place that it strikes a bell, [...] We have already made use of this apparatus for telegraphic experiments, which have resulted successfully in the transmission of entire words and small phrases." A representation of the Gauss-Weber telegraph can be seen in (Fig. 22).

The influence of the earth on electrical conductivity had been considered by Desaguilers,[14] and its influence on the transmission or storage of electricity was a key ele-

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[13] Gauss 1889.

[14] John Theophilus Desaguliers (1683-1744) was a member of the Royal Society of London, serving as an experimental assistant to Isaac Newton. Today he would be called an Engineer in contemporary terminology, once e acted in the popularization of Newton's mechanics, especially in practical applications involving electricity: he became well known for these experiments after publishing a dissertation on the subject. Desaguilers used to promote small public lectures to popularize his ideas, just like the modern *TED* talks. This type of approach was common and lasted between the 18th and 19th centuries. Fara-

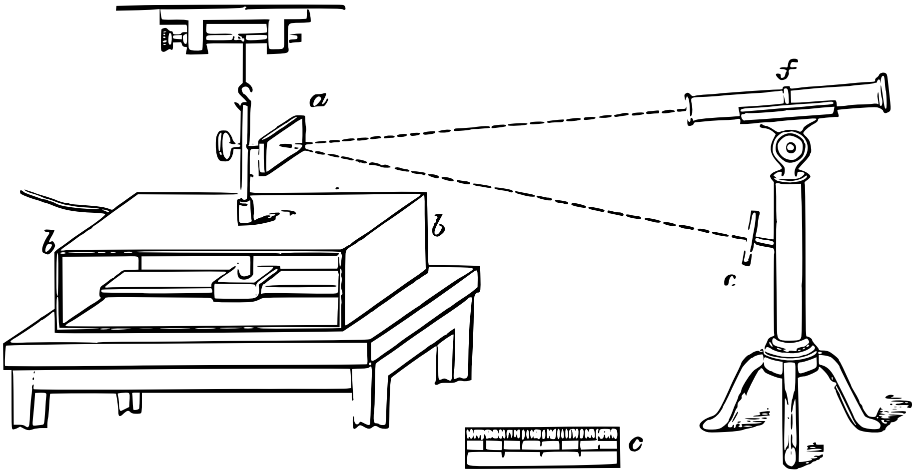

Figure 22 – Gauss and Weber's apparatus for the first telegraph, built in 1833, which shows the mirror galvanometer invented to "amplify" the transmitted signal. The movement of a magnetic needle inside a eletromagnet bb moves a, a small mirror with a counter-balance attached to the axle of the magnet suspended by a thread. The scale image c could be seen by means of the reflection of a mirror, with the aid of the telescope at f. Source: Lardner and Bright 1867, p. 42

ment in the 18th century's experiments with Leiden phials.[15] However, although Ampère was probably the first to propose the existence of electrical currents inside the terrestrial globe,[16] the electrical conductivity of the earth only started to be considered, for practical purposes, along the years of 1836-1837, when Carl August Steinheil, a Bavarian professor of Physics – who had been a student of Weber and Gauss in 1823 – realized that the telegraphic lines could be grounded to serve precisely as a return path for signals. In 1835, Gauss suggested that the Nürnberg-Fürth railroad tracks could be used as telegraphic conductors. Following the tip, Steinheil found that the tracks were not sufficiently insulated, even when the chair seats were well insulated. This led him to consider that telegraph signals could escape through the earth, and, therefore, could be used as a possible return route.

Steinheil's discovery (made between 1837-1838) was, for electrical telegraphy, of great importance, and since he did not want to register a patent, the innovation was quickly absorbed.[17] Subsequently, a confluence of technologies inspired by the Gauss-Weber system can be seen, especially after the Cooke and Wheatstone's patents in England (1837) and Samuel Morse's in the USA (1840), in which the electric telegraph takes, among var-

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day, for example, was well known for promoting similar lectures, even with children in the audience, at Christmas celebrations, following the example of his first tutor, Humphry Davy. Desaguliers 1742.

[15] Franklin 2017.

[16] Assis and Chaib 2011, p. 60.

[17] Schneider 1999.

ious configurations, the form of a mechanical switch[18] connected to a battery,[19] with the other end connected by a metallic wire to an electromagnet or relay. The relay had the role of attracting a metallic part, acting as a transducer and converting the electrical signal into a mechanical one, to be printed on a ribbon of coiled paper (Fig. 23).

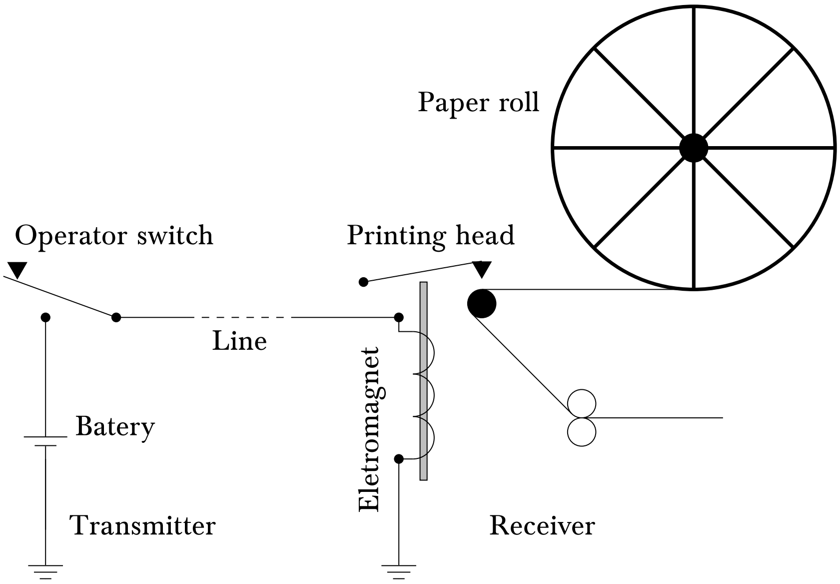

Figure 23 – Printing apparatus of Samuel Morse's telegraph. Source: Wikimedia commons.

In addition to Steinheil's contributions, one last aspect to be considered in the popularization of electric telegraphy has a close relationship with the railway expansion itself. Over the 1830s, the aereal lines for terrestrial telegraphy had already spread along Europe and North America, but it would not gain popular support until a notorious case, which illustrates that peculiar historical moment: "The murder of Sarah Hart by John Tawell". Tawell (1804-1907) was a commercial representative, widowed by his wife Mary Tawell in December 1838, died of tuberculosis. During the last cares for his wife, Tawell hires the nurse Sarah Hart, with whom he ends up developing a love affair. After his wife's decease, John and Sarah carry on their relationship, at the time she starts to provide services as a maid, resulting in two children, from 1840 to 1841. Two years after the birth of their second child, Tawell meets Mrs. Sarah Cuttforth, also a widow, asking her to marry him, thus ending the relationship with Sarah Hart. In the following years, however,

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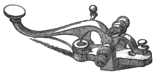
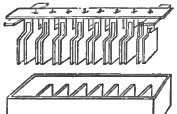
[18] A type of mechanical switch: 

[19] The "voltaic" battery had already been perfected at this point. One of its common configurations consisted of alternating copper and zinc plates, in a wooden box, dipped in an acidic solution, similar to contemporary automotive batteries: 

he would go into a deep financial crisis and, as he regularly paid two alimony payments to his ex-lover, he decided to plan a murder. On the January 1st, 1845, John, on one of his visits, poisons Sarah with prussic acid and flees to the train station, taking the train from Salt Hill to London.

Witnesses told the police of having seen the suspect go to the aforementioned location and the police force promptly used the new technology to notify London's Paddington station of the ongoing escape. A telegraphic message is then sent and, in a few minutes, arrives at the next station. The trip from Salt Hill to London via locomotive lasted no more than 50 minutes, and as soon as the steam reaches the desired destination, the event that would change forever the popular imagination in relation to telegraphy takes place: John is promptly received by the police force, while disembarking. The arrest of John Tawell in 1845, thanks to the new telegraphic technology, which existed less than 15 years (Fig. 24) would direct everyone's eyes to the telegraph, including in relation to its profit potential. The markets would not be the same and telegraphy would play an important role in the ongoing industrial revolution.

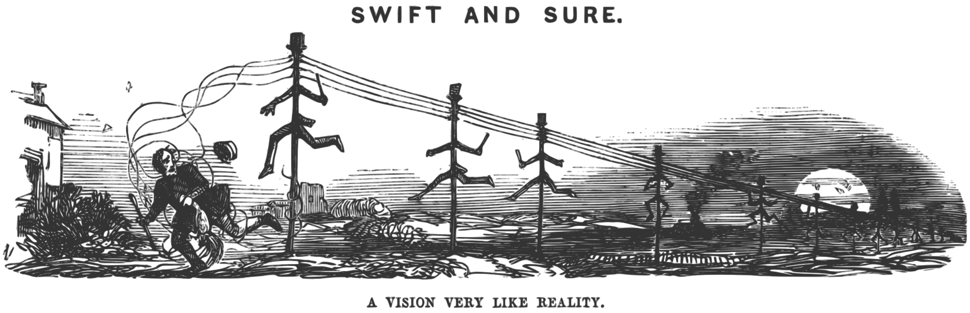


Figure 24 – The first arrest with the help of the electric telegraph and the repercussion in the popular imagination, according to a current newspaper. Source: Punch Magazine (1849).

In the second half of the 19th century, much of Europe and the United States would be interconnected, separately, by terrestrial lines. Communication at this point was already a routine task (Fig. 25), while the dream of connecting the old and the new world by means of a telegraph cable starts to take shape. The prospects for new markets would fill the eyes of many visionaries, eager for the possibilities of progress and prosperity.[20] Until then only ships and railways could transport, over long distances, not only people and belongings, but also letters, an indispensable way for local, national and transnational communication, without which any business on a larger scale would get off the ground. Ships would also be essential for carrying out ambitious projects to launch telegraph lines across the ocean, in order to interconnect distant regions and expand the possibilities of communication.

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[20] Cookson 2006.

Figure 25 – A Typical view from inside a telegraph office.

The attempts to launch a communications cable that linked the two continents would culminate in one of the greatest episodes in the history of technology, certainly constituting the largest and most daring enterprise, in terms of engineering, of that century. The enterprise would be, in magnitude of efforts and financial investment, proportionally higher than the Apollo XI mission, which would take mankind to the lunar soil, in 1969.[21] In addition, it would be an immensely risky investment, which took place between 1855, when the first unsuccessful attempts took place, until the successful launch in 1866 (Fig. 26).[22]

More than just an engineering venture, the successful launch of a transatlantic cable for telegraphy would have a profound economic and social impact on humanity, as a result of a joint effort of hundreds of researchers, engineers and enthusiasts of the new electrical science, having propitiated innumerable technological and scientific advances, in almost all areas of the human knowledge, from the exact sciences to humanities. In fact, the newborn telegraph (and later electrical) engineering itself would gain maturity when facing the problems of underwater telegraphy. The world would never be the same after the transatlantic cable and it really was not. The bottom of the atlantic ocean, hitherto unexplored, was the first challenge, but not the greatest. Before the project, it was difficult to have a clear understanding of the risks involved. Until then, submarine cables had

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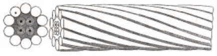
[21] It is estimated that the total cost of launching the atlantic telegraph cable in the 1850-1860 decade was around 210 million pounds, which would be equivalent to around 700 billion, today. That's right: in today's currency, the investiment would cost approximatelly $ 1,000,000,000,000 (a trillion dollars)! See, e.g., Murray 1905, p. 211.

[22] Communication via submarine cables started a few years earlier, with an experimental cable being launched in the Dover Strait (the narrowest part of the English Channel, with approximately 33 km), between France and England, in 1850, having failed shortly after the launch. A year later, a new cable, which had about 7 tons and was coated with the vegetable resin called *gutta-percha*, was successfully launched (see figure next) in the same strait: 

been launched over relatively short distances and it did not take too long before technical difficulties put into question the real possibility of reaching the long-dreamed objective.

A colossal investment was being made to the atlantic cable project of 1858. Is is important to remember that, in the economical side, the role of the "financial genius" behind the initiative, the American industrialist Cyrus West Field (1819-1892). At the age of 34 (1854), Field had already retired for becoming one of the first millionaires worldwide (the word, incidentally, had been coined just then), working with the cellulose industry. For years, Field would be obsessed with the idea of launching a cable that could unite the two continents, and would become one of the founders of the *New York, Newfoundland and London Telegraph Company*, created to take the project ahead, having been present, on board of the "American side" (the ship USS Niagara), in all launch attempts, from the first failure in 1858 to the successful launch in 1866.

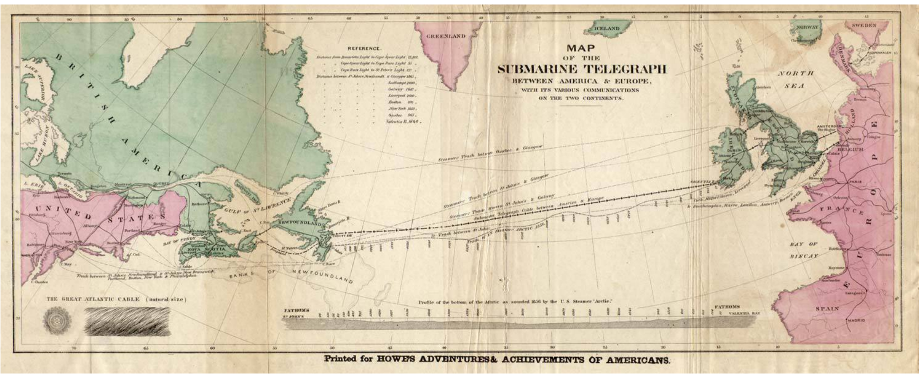


Figure 26 – The first operational transatlantic cable, launched in 1866, between *Valentia Bay*, UK, and *Trinity Bay, Newfoundland*, in the former British colonies of North America, currently Canada.

6.2 The electric puzzle

Submarine telegraphy appeared to be, indeed, a new field of work. From the first days, seemingly overwhelming technical barriers appeared. Problems of mechanical nature were innumerable: how to lay a cable without breaking it, after countless failures? How to transport them over long distances, considering their colossal masses? Is there a ship capable of such a feat? And does the conductive material, its manufacturing process, its purity, have a real influence on the quality of communications?

In addition to the known problems back then, such as the cables insulation, another mysterious phenomenon would prove to be really disheartening for the pioneers: the harmonic distortion, first demonstrated in 1853 by the English *electrician* Latimer Clark (1822-1898), on the recently landed Anglo-Danish cable,[23] which was several hundred kilometers long. Clark was a railroad builder who learned the trade by watching his brother,

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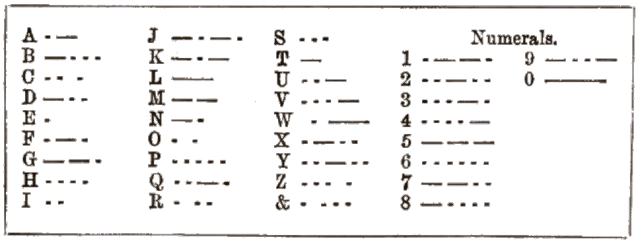
[23] In June of 1852 Clark had already observed the phenomenon in terrestrial lines that covered the cities



A	·—	J	—·—·	S	···		Numerals.		
B	—···	K	—·—	T	—	1	·——·	9	—··—
C	·· ·	L	⸺	U	··—	2	··—··	0	⸻
D	—··	M	——	V	···—	3	···—·		
E	·	N	—·	W	·——	4	····—		
F	·—·	O	· ·	X	·—··	5	———		
G	——·	P	·····	Y	··—··	6	······		
H	····	Q	··—·	Z	··· ·	7	——··		
I	··	R	· ··	&	· ···	8	—····		

Figure 27 – Convention for the transmission of telegraph signals, a sequence of dots and dashes known as the *Morse code.*

Edwin Clark, a Structure Engineer. In fact, the terrestrial telegraphy itself had initially used the railway lines to launch the transmission lines. In 1850, at the invitation of the president of the newly formed *Electric Telegraph Company*, Latimer and his brother joined the ETC team and were introduced to the branch of electrical matters.[24] Some of the first electrical engineers would migrate from the construction industry in the middle of the century.

But the term *distortion* is an anachronistic denomination. In submarine cables, the phenomenon then named *retardation* caused the conveyed signals (Fig. 27) from one end of the cable to arrive, in a way, "mixed", i.e., slightly delayed and elongated at the other end, making it difficult to read and interpret. After observing the phenomenon, Latimer Clark conducted a series of tests on underground lines (in which it was also possible to perceive the distortion) and, at the invitation of prof. (*Sir*) George Biddell Airy (1801-1892) – the "royal astronomer" – introduced them to Michael Faraday (1791-1867).[25]

After hearing from Øersted that "... the electric conflict acts in a revolving manner",[26] Faraday conducted a series of experiments in 1821 that led him to the phenomenon of electromagnetic rotations – the principle of the electric motor. He imagined that the action of a galvanic current on the magnet was not a push-and-pull force, but a spinning force instead. By 1831 Faraday made his most celebrated discovery, that magnetism could "produce" electrical currents, what became known as *electric induction*,[27] ideas that apparently opposed to the principle of action at a distance. Faraday's ideas on the condition of the

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of London, Leeds and Liverpool. But the first public demonstration to be given by him, in presence of Faraday, Airy and some directors of the *Electric Telegraph Company*, would be made only in October 4, 1853. See Weaver 1909, p. 4.

[24] Society 1889.

[25] The legend has it that, having worked as a bookbinder in a bookstore, Faraday had the opportunity to read scientific books and encyclopedias. Knowing his interest in scientific matters, a client of the establishment would have offered Faraday a ticket to attend a series of four conferences, given by the famous chemist Humphry Davy, of the Royal Institute of London. Impressed by the series of lectures, Faraday would have take some notes, writting a letter to the great scientist, asking for a job as his laboratory assistant. Being accepted in this role for almost 10 years, Faraday conducted independent studies in chemistry and electricity. The most famous biography of Michael Faraday can be seen at Williams 1965.

[26] Øersted 1820; See Maxwell 1873, p. 341.

[27] Faraday 1855.

medium between two bodies under some electromagnetic interaction, would profoundly influence Maxwell, especially Faraday's work on magnetic images, once he apparently had few mathematical skills, having developed his theory from a series of visual analogies, such as the concept of *lines of force*. In contrast to the continental school of physics and the Newtonian tradition, Faraday saw in the phenomenon of retardation a way of reinforcing his own theories – and philosophical choices – on the true nature of the electromagnetic interactions: his concept of field.

Most of the scientists of the time, especially French such as Laplace, Cauchy, Poisson and Germans, such as Navier and Weber, followed the line of Ampère and saw in electricity mechanisms of interaction between central forces. For this mechanical-molecular school, in Kargon's aception,[28] any inherent properties behind the transfer of heat or electricity should be discovered first by assuming the existence of particles and configurations of forces, which would, in principle, allow them to deduce verifiable consequences for such configurations. Others, members of the analytical tradition – such as George Simon Ohm[29] and Fourier[30] – sought literally to *matematize* the phenomenon. Starting from anti-hypothetical methodologies, they submitted physics to mathematical analysis, with geometric descriptions of reality in terms of differential equations.

Faraday, in contrast, believed that the flow of electricity in a conductor was resultant of a process of continuous breaking, from an accumulated tension, like an elastic solid immersed in a dieletric medium. As soon as Clark introduced him to the phenomenon of distortion, Faraday suggested that the problem was due to the fact that the cable behaved like a huge Leiden jar, which should be responsible for the delays.[31] Since 1838, while commenting on Wheatstone's experiments on the "velocity of electricity", Faraday conjectured that a delay would occur on account of the electrostatic capacity of the circuit. In 1850, Werner Siemens, of Berlin, observed the same effect in a paper on testing faults in telegraph lines. However, in aerial lines, the phenomenon would be difficult to visualize given its low capacitance, and would "charge up" too fast to be perceived, and it was not until 1853 that the *retardation* was experimentally demonstrated by Clark, as already mentioned.[32] On January 20 of the following year (1854), Faraday reports the fact to the Royal Institute of London, where he was director, publishing his observations.

The main reason, according to Faraday, was that, when one end of the cable came into contact with the positive side of the battery, a current of positive electricity flowed out of the cable, which was in contact with the gutta-percha coating, leaving a negatively charged interior. This would cause, by induction, a second current of electricity to flow to

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[28] Kargon 1969.
[29] Ohm 1827.
[30] Fourier 1822.
[31] Faraday 1855, p. 508.
[32] Weaver 1909, p. 4.

the other end of the cable, which was grounded, considered now as a common reservoir, capable of supplying the "deficiency" of positive electricity, until reaching a static equilibrium. In short, the voltage would be electrically induced, and it was, to Faraday, the cause of the conduction. Later on, Maxwell would summarize that specific idea, in presenting an elastic analogy – *tension of ropes and the pressure of rods* – to understand what Faraday saw as *tension*, from the abstract conception of geometrical lines of force to physical lines: "... the motion which the magnetic or electric force tends to produce is invariably such as to shorten the lines of force and to allow them to spread out laterally from each other." The medium would posses, therefore, "... a state of stress, consisting of a *tension*, like that of a rope, in the direction of the lines of force, combined with a pressure in all directions at right angles to them."[33]

Curiously, the analogy to justify the mediated action was not far from the common sense of the time, regarding to telegraph's functioning: "... when we ring a bell by means of a wire, the successive parts of the wire are first tightened and then moved, till at last the bell is rung at a distance by a process in which all the intermediate particles of the wire have taken part one after the other. We may ring a bell at a distance in other ways, as by forcing air into a long tube, at the other end of which is a cylinder with a piston which is made to fly out and strike the bell. We may also use a wire; but instead of pulling it, we may connect it at one end with a voltaic battery, and at the other with an electro-magnet, and thus ring the bell by electricity."[34] Even though the electrical fluid pushed by Volta's battery (Fig. 31) took time to propagate, the anomaly would not be a real problem except for long distances. But for submarine telegraphy, the fact could undermine the dream of fast and intercontinental communications and, therefore, make the transatlantic cable a great and useless *white elephant*. The possibility of overcoming this challenge would attract an influential character in the history of science and engineering, who would become the leading mentality in the cable landing venture.

That same year of 1854, a young Scottish professor was hurriedly leaving a meeting of the British Association, of Liverpool, when approached by a young student about an *electric puzzle*, but not paying immediate attention. The student was son of the famous Irish mathematician William Hamilton but, as he feared losing the train for the city of Glasgow, where he lived, William Thomson (1824-1907) addresses the question to a friend walking beside him, George Gabriel Stokes (1819 -1903),[35] claiming that Stokes was as or more competent to answer to the student's request. The content of the question involved the

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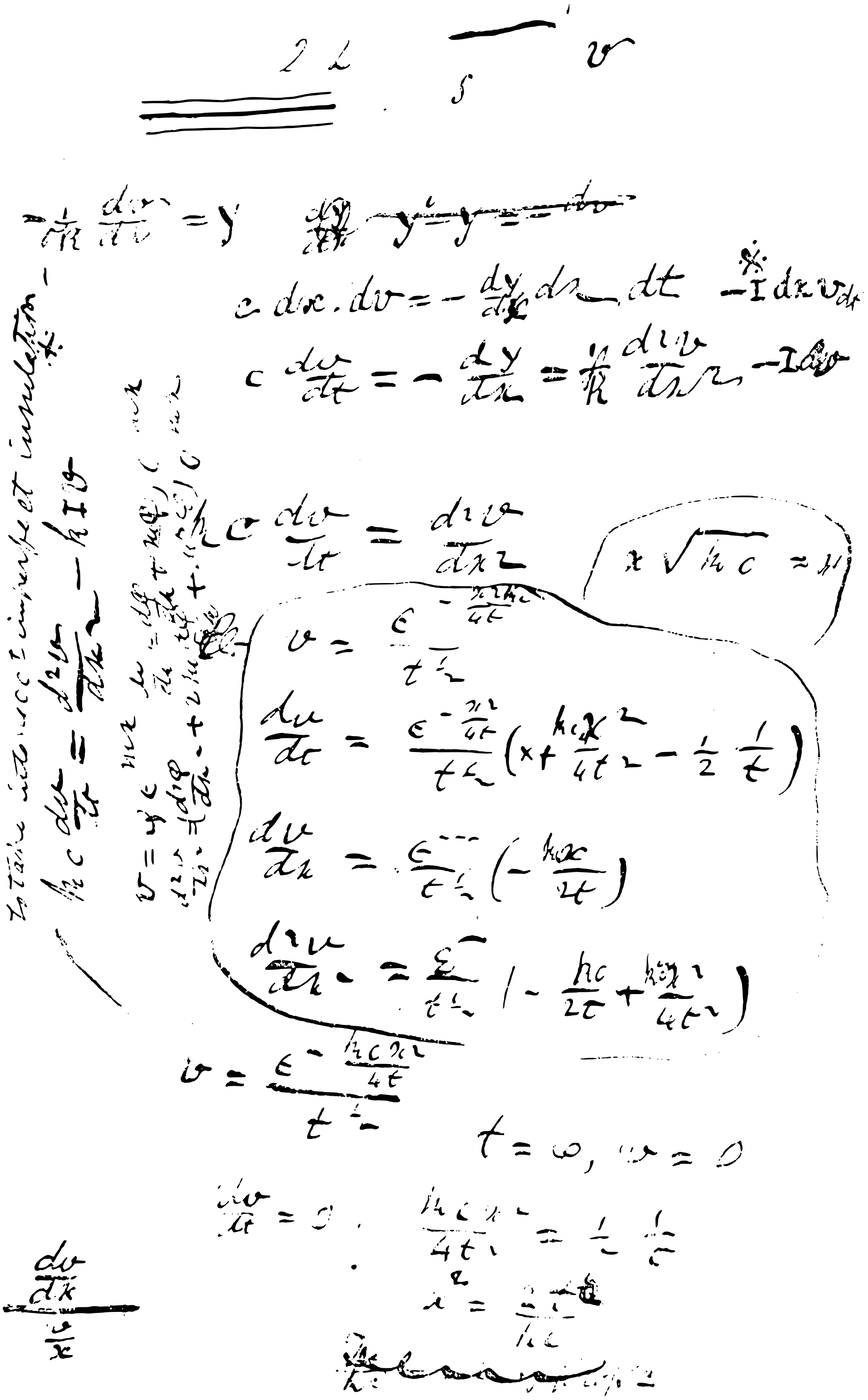
[33] See the second paper of Maxwell on electricity, at Maxwell 1873, p. 311; A critical analysis of this paper can be seen at Tort et al. 2004.

[34] Maxwell 1873, p. 312.

[35] George Gabriel Stokes (1819-1903) at this point, was a highly respected professor of mathematics at Cambridge University – the same chair that was occupied by Isaac Newton and Stephen Hawking, more recently – having been elected secretary of the Royal Society of London that year, a position he would hold until 1885, when elected president of the entity.

distortion problem observed by Clark and reported by Faraday, immediately catching up Stokes' attention.[36]



$c\,dx.dv = -\frac{dy}{dx}dx\,dt$

$c\frac{dv}{dt} = -\frac{dy}{dx} = \frac{1}{k}\frac{d^2v}{dx^2}$

To take into acc^t imperfect insulation

$kc\frac{dv}{dt} = \frac{d^2v}{dx^2} - kIv$

$kc\frac{dv}{dt} = \frac{d^2v}{dx^2}$

$x\sqrt{kc} = \mu$

$v = \frac{\epsilon^{-\frac{x^2kc}{4t}}}{t^{\frac{1}{2}}}$

$\frac{dv}{dt} = \frac{\epsilon^{-\frac{x^2}{4t}}}{t^{\frac{1}{2}}}\left(x + \frac{kx^2}{4t^2} - \frac{1}{2}\frac{1}{t}\right)$

$\frac{dv}{dx} = \frac{\epsilon^{\cdots}}{t^{\frac{1}{2}}}\left(-\frac{kx}{2t}\right)$

$\frac{d^2v}{dx^2} = \frac{\epsilon^{-}}{t^{\frac{1}{2}}}\left(-\frac{kc}{2t} + \frac{k^2x^2}{4t^2}\right)$

$v = \frac{\epsilon^{-\frac{kcx^2}{4t}}}{t^{\frac{1}{2}}}$

$t = \infty,\ v = 0$

$\frac{dv}{dt} = 0.\quad \frac{kcx^2}{4t^2} = \frac{1}{2}\frac{1}{t}$

$x^2 = \frac{2t}{kc}$

Figure 28 – On the back of the letter sent by Stokes, in Oct. 16, 1854, Thomson seeks a quick solution to the "electric puzzle". Source: Stokes 1990c.

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[36] Thomson 1889.

On October 16, Stokes writes a letter to his friend reporting the open problem and Thomson immediately starts working on the "new" theory (Fig. 28).[37] It was, possibly, the opportunity he had been waiting for a long time.[38] Facing the intriguing problem, the pair Stokes and Thomson would exchange a series of letters that would change the history of telegraphy, in a joint effort for the atlantic cable project. One of these letters,[39] published partially under the title *On the theory of the electric telegraph*,[40] sought to provide a operational solution, although incomplete, to the problem. The approach would be, in fact, the first well-founded mathematical theory for the electric telegraphy, and the recommendations would go beyond just mathematics: not only "explanations" for the mysterious phenomenon of distortion, which could put an end to the ambitious project, but also indications for practical applications of the theory, ranging from cable estimation and fabrication, to new forms of measuring the present variables. In addition to the suggestions, Thomson was personally involved in all stages, from the confection to the landing of the cables.

Thomson's first attempt to find a solution came less than two weeks after receiving Stokes' letter, on October 28. He started modeling the cable as a cylindrical, straight, very long and perfectly insulated conductor, what a modern engineer would call *leakage free* conductor (Fig. 29). In those days, any electrical conductor, as well as an underwater cable, should be isolated by a vegetable resin called *gutta-percha*, a resin obtained from the *Palaquium tree*, which grew in the rain forest and was widely used in dental filling, golf balls and, of course, electrical isolation. Its industrial use declined with the development of plastics and synthetic resins, derived from petroleum, although it had superior insulation properties than rubber, for example. When landed into the ocean, a submarine cable

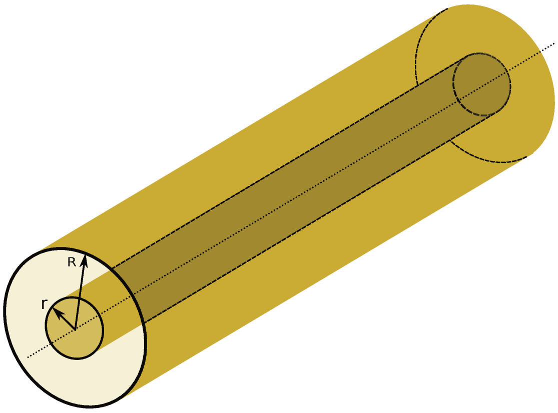

Figure 29 – Geometry of the leakage-free submarine cable, imagined by Thomson. The configuration is similar to what would be called today a *coaxial cable*.

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[37] Stokes 1990c.

[38] To know why, see the appendix B. See also See Lloyd 2007.

[39] Thomson 1990a.

[40] Thomson 1854b.

covered with gutta-percha would form a system of two conductors: the internal, which would be the telegraph wire itself, and the external, formed by the interface with the sea water, which is conductive. Thus, the geometry of the cable assumed a configuration similar to what would be called today as a *coaxial cable*. Like Faraday, Thomson realized that this configuration was electrically similar to an old one known to electricians back then: the cable was a colossal Leiden jar,[41] a device that preceded the modern capacitor.

Thomson also suggests that the state of the "uniform motion of heat", of his previous works,[42] enabled him with great ease to investigate the *capacity* of a Leiden jar. His approach, by the way, would become a classic example in electrical engineering and physics courses at universities, which is worth presenting.[43] For a Leiden phial (Fig. 30), consider an infinitesimal thickness of the dielectric (insulator) formed by the gutta-percha resin, so that the distance between the external A' and internal A conductors, at any point, is a small portion of the radii of curvature. At this point, one of Thomson's main ideas was

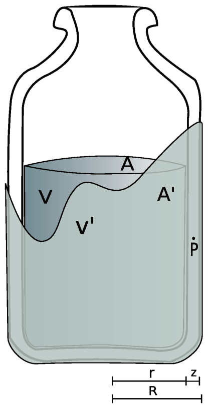

Figure 30 – A cable as a huge Leiden jar: a phial for the electrical fluid, with both outer and inner conductive surfaces explicit.

the mathematical analogy between heat and electricity, in which two key questions would possibly be simmering in his mind: assuming that two points are on different electrical potentials V and V', what would happen if the two equipotential surfaces were subject to different temperatures? What if the variables *temperature* and *electrical potential* could be, somewhat, analogous?[44]

This was a somewhat similar approach to Georg Simon Ohm's (1789-1854), of 1827.[45] Ohm's attempt was based on a variable volumetric density of charge along the length of the wire, in analogy with the analytical treatment of Fourier for the "motion" of heat, which was based on a temperature gradient of a body. That is, considering a wire

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[41] See sec. 5.2, p. 100.

[42] Thomson 1854a,b.

[43] Thomson 1855.

[44] To a discussion on this topic, see Wise 1979; and also Wise 1981.

[45] Ohm 1827.

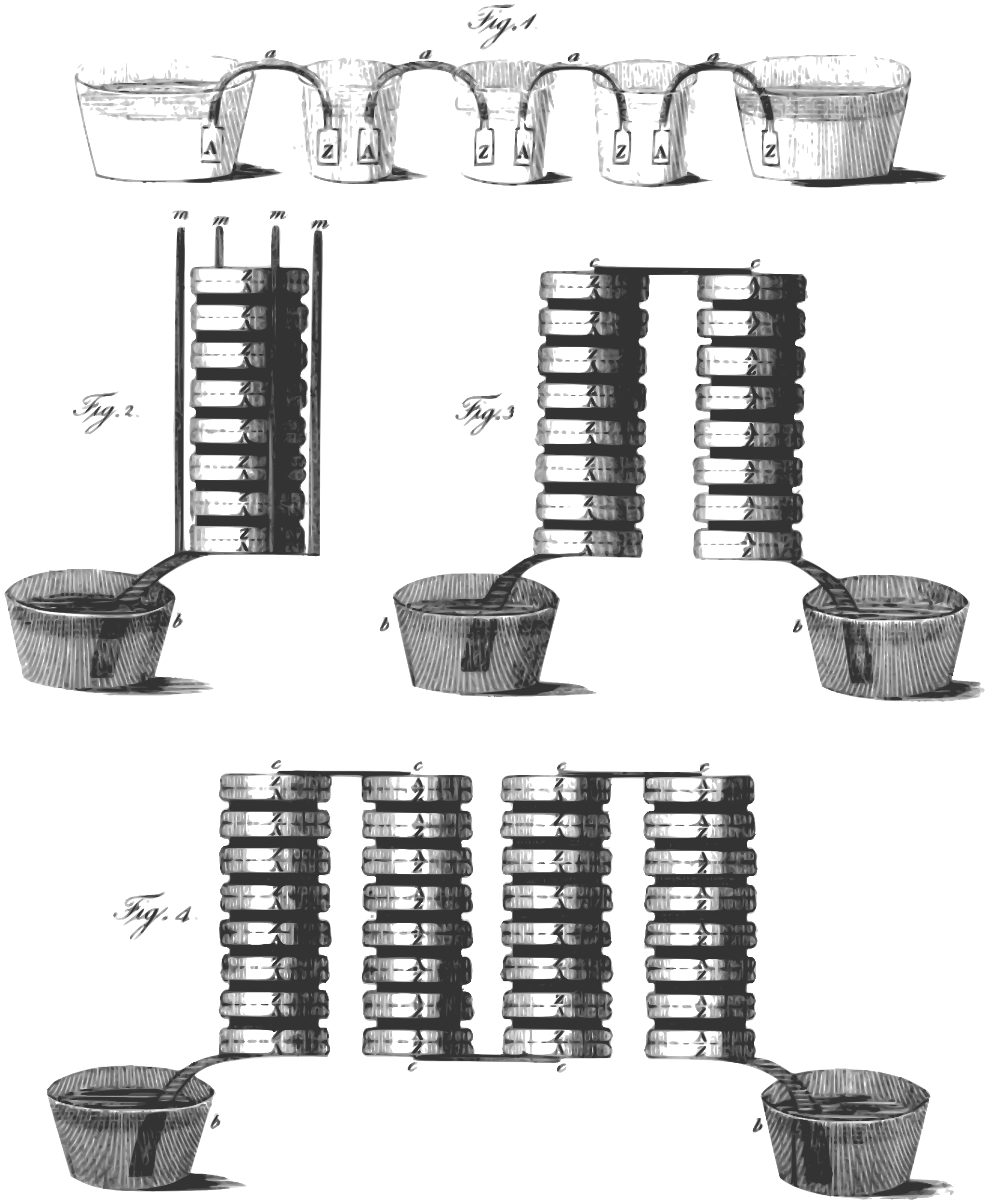

Figure 31 – Volta's chemical-physical analogue to a frog's leg (1800). The voltaic pile was the first electrical device to continuously provide an electric current. It was a response to his friend Luigi Galvani's hypothesis on *animal electricity* (1780), after having been shown by him that an electric current was capable of moving a frog's leg: "*… je me vis obligé de combattre la prétendue électricité animale de Galvani.*" Volta created then an analogy in which animal parts could be exchanged by a set of of different metals in a brine-soaked cloth, to promote the electrical conduction. When the extremities were connected by a metalilc wire, apparently the same *Galvanic current* flowed through the *appareil electro-moteur*. Source: Volta 1800, p. 423, p. 431.

carrying a *galvanic current* in a region where there is no action of an electromotive force,[46] the pushing force "felt" by the electric charges (see Fig. 31) – in the opposite direction to that of the resistance of the wire – would be due to a gradient in the volumetric density of charges.[47] This vision would be, incidentally, a point of divergence raised by Gustav Robert Kirchhoff (1824-1887), who stated that the only constant quantity inside a conductor in electrostatic equilibrium would be the electric potential, and not the volumetric density of charge, as Ohm stated. The electromotive force inside an conducting wire, with a constant current, would be proportional to the variation of this potential in relation to the longitudinal coordinate, i.e., along the length of the wire. The potential itself would have its origin, according to Kirchhoff, in the free charges spread across the surface of the conductor.[48]

Thomson, in turn, instead of Ohm's *flowing pipe analogy*, although choosing a *Leiden phial geometry*, followed, in a way, Faraday's recommendations, but placed himself in the "intermediate path" between Ohm and Kirchhoff's views. Henceforth, the electric potentials of the two surfaces, along a radially disposed line between them, would be in "arithmetic progression". But what does it mean? With the statement, a not very clear definition is made of the present electric field (Fig. 32). That is, the "arithmetic progression" indicated that the rate of change of the potential, through the point P, would be equal to

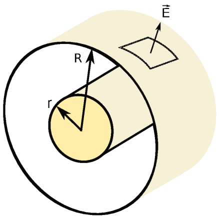

Figure 32 – Electric field.

$$\frac{V - V'}{z}, \tag{6.1}$$

which could be translated, as:[49]

$$\vec{E} = -\nabla V, \tag{6.2}$$

---

[46] The expression *electro-moteu* (electromotive), credited to Alessandro Volta, indicated some sort of a "pushing force" on the electrical charges, inside a battery, or *appareil electro-moteu.* See Volta 1800, p. 421.

[47] See Ohm 1827, pp. 402-418; In this respect Weber would argue that Ohm was mistaken because, while for heat transfer the temperature gradient was a necessary condition for local heat transfer, the same was not necessary true for electrical charges, which would be in contradiction with the laws of electrostatics, namely, the one that prohibited the existence of charges inside a conductor, but only on the surface. See in Assis and Hernandes 2009, pp. 205-206.

[48] See Kirchhoff 1850; See also Assis and Hernandes 2009, p. 223.

[49] It is interesting to observe, by the way, how a well mathematically trained mentallity like Thomson's could overcome, by far, the lack of more adequate tools in dealing with difficult problems, as it was the case of vectorial calculus, which had not yet been "invented".

in which ∇V corresponds, in modern notation, to the gradient of the potencial and $\vec{E}$ the "implicitly defined" electric field.[50]

The establishment of this relationship ties, even, the concept of "difference of electric potential" to the idea of central force – i.e., a force as the gradient of a scalar field, as obtained by Lagrange in 1777. In other words,

$$\mathscr{V} = V'_+ - V'_- = -\int_{(-)}^{(+)} E \cdot dl\,. \tag{6.3}$$

This also shows that the potential considered in isolation had no physical meaning for Thomson, but only the difference of potential, regardless the reference frame. For example, if someone is asked about the altitude of the city of Ouro Preto (in the state of Minas Gerais, Brazil), this person could answer considering the sea level, which is a traditionally accepted convention. If one considers the measurement in relation to the *pico da Neblina* (in the state of Amazonas), e.g., it would only be necessary to add (or subtract) some constant value to the previous one, but a different answer would not alter the "real world". The same analogy applies to the case of the potential, in his view.

The name *electric potential* would be a slightly misleading attribution if carried over to the present day, as it suggests the tendency to associate it with potential energy. This can lead to a slight confusion regarding the understanding of electrical phenomena, once there is a connection between *potential* and *potential energy*, as in the gravitational case, for example. Likewise, the word *potential*, as used by Thomson, designated an equipotential surface, in analogy to the two surfaces subjected to different temperatures (isotherms), that is, between which there would be a gradient of temperature, which allows a transfer of heat.

Thomson's analogy of heat to understand electromagnetic phenomena arises, therefore, from electrostatics, in which Newton's Laws of mechanics were the main language. In the first article of a series,[51] he proposes a connection between the concept of electric potential, after reading one of Green's works (1828),[52] and temperature (isotherms and equipotentials), considering both electricity and heat as, in his words, "... very thin fluids", governed by central forces. For two surfaces considered *geometrically analogous*, a heat conductor and an electrified one, the heat flow and the force of attraction in an "... electrical point" (a *proof charge*) would be proportional. Therefore, the direction of the heat flow would be in the line of action between the two charged surfaces. In short, the direction

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[50] The advantage of vector notation for the electric potential – using the operator ∇ – is really quite interesting, since $\vec{E}$ is a *3D* vector quantity, whereas V is a scalar. A vector function $\vec{E}$ has information on the three independent functions in a Euclidean space, because they are independent of each other and are related by the condition $\nabla \times \vec{E} = 0$, which means that the field lines "come out ", diverge, that is, the electric field (whatever it is) is not rotational.

[51] Thomson 1842.

[52] Green 1852.

of the heat flow would be the same as the electrical force for the surfaces considered geometrically analogous. This path is also of interest of this investigation, and the first result would be a procedure that today is widely taught in basic electromagnetism courses.

Keeping in mind the analogy *heat as viscous fluid*, lets consider a "... homogeneous distribution of heat" along an ellipsoid of revolution,[53] in which isotherms and equipotential surfaces are considered analogous. For the solid subjected to the action of a heat source, imagine now that the temperature in steady state will depend on the position r, that is, an entity defined in a field. Thomson also argues that to find the potential (or temperature) v produced by a single source, at a distance r – which would be the same for points located at the same distance from the source – it could be mounted an expression which indicates that its spatial variation obeys the inverse square law,

$$r^2 \frac{dv}{dr} = A \Rightarrow \frac{dv}{dr} = \frac{A}{r^2}\,, \tag{6.4}$$

which indicates the presence of a central force, acting along a straight line, at a distance, being the promoter of the heat flow. Thus, integrating the equation and incorporating the negative sign in a constant, we have:

$$v(r) = \frac{A}{r} + \mathscr{C}\,. \tag{6.5}$$

Also applying the condition $\lim_{r\to\infty} v(r) = 0$, we have $\mathscr{C} = 0$. Thus,

$$v(r) = \frac{A}{r}\,. \tag{6.6}$$

Additionally, considering the contribution to temperature from other heat sources in a element $d\omega^2$, we have:

$$v(r_1) = \frac{\rho_1 d\omega_1^2}{r_1}\,, \tag{6.7}$$

in which ρ_1 it is a constant that represents the contribution of sources coming from other parts of the surface. The *testing* point is at a distance r_1 from the sources, which are located inside the considered element of area. From the divergence theorem (although an explicit mention has not been made), he realizes that the temperature of the isotherm v_1 is the same on the surface and inside:

$$v_1 = \int\int \frac{\rho_1 d\omega_1^2}{r_1}\,, \tag{6.8}$$

arriving at the formulation of Green's function, which gives the force of attraction, towards the x axis, as the gradient of a certain "potential function":[54]

$$F = -\frac{d}{dx}\int\int \frac{\rho_1 d\omega_1^2}{r_1}\,, \tag{6.9}$$

---

[53] Thomson 1842, p. 4.

[54] Always considering an equivalence between an equipotential surface and an isotherm.

considering a surface "... covered by an attractive medium", which would currently be called an electrically charged surface with *density of charge* ρ_1, given by the condition $v = v_1$. Thomson frequently used the terms "homogeneous" and "distribution ", among others, while referring to heat as a *viscous fluid*. He also used to use the term *electrical density* in reference to Coulomb's works. Before that, however, it is common to see in his writings the expression "... thickness of the stratum", with the same connotation.[55]

"Mixing" once again the concepts of electricity and heat, the author shows that for any isotherm, the force also pointed, for the same reason, to the opposite direction to the temperature gradient, giving shape to an expression for the flow of heat,

$$\phi = -\frac{dv}{dn}\hat{n}\,, \tag{6.10}$$

in which $\hat{n}$ is a vector normal to the heat conducting surface.[56]Thomson has now enough elements for the mathematical model of the Atlantic cable as a large Leiden jar. Considering a cable of length L and radius S, and using Thomson's definition of *amount of electricity* Q,[57] – the electric charge – and considering an element of area $\vec{da}$ (Fig. 32), it is possible to follow more or less the same steps as his in demonstration of the capacitance formula:

$$\int \vec{E}\cdot\vec{da} = E2\pi SL = \frac{1}{\epsilon_0}Q\,, \tag{6.11}$$

which is similar to what would become known as Gauss's Law,

$$\oint \vec{E}\cdot\vec{da} = \frac{1}{\epsilon_0}Q\,, \tag{6.12}$$

with Q being the charge closed in a surface, which leads to

$$E = \frac{Q}{2\pi\epsilon_0 L}\frac{1}{S}\hat{s}\,. \tag{6.13}$$

So the potential between the two conductors of a Leiden phial (Fig. 30) or the two cylinders of underwater cable (Fig. 32) will be

$$\begin{aligned} V = V(R) - V(r) = -\int_r^R E\cdot dl \quad &= \\ &= -\frac{Q}{2\pi\epsilon_0 L}\frac{1}{S}\int_r^R \frac{1}{S}ds \\ &= -\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{R}{r}\right)\,. \end{aligned} \tag{6.14}$$

Thus, as $Q = CV$,

$$Q = C\left[-\frac{Q}{2\pi\epsilon_0 L}\right]\ln\left(\frac{R}{r}\right)\,. \tag{6.15}$$

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[55] Thomson 1872, p. 48.

[56] See Thomson 1842, p. 4.

[57] Thomson 1855.

Therefore, the *electrostatic capacity* of the cable, or capacitance per unit length $c = C/L$, will be:[58]

$$c = \frac{C}{L} = \frac{2\pi\epsilon_0}{\ln\left(\frac{R}{r}\right)}\,, \tag{6.16}$$

in which $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}\, F/m$. The electrostatic capacity or capacitance of the submarine cable is then obtained by what Thomson called a “coaxial model”, and its value was **huge**! Making a comparison with a “real” Leiden phial, using the actual data from the cable that would be landed under the waters of the Atlantic,[59] which had an internal diameter of the gutta-percha insulation of approximately $1.58\, mm$, Thomson calculates that the cable would have a capacitance equivalent to a Leiden jar with a surface area of $771 m^2$ ($8300\,\mathrm{ft}^2$), resulting in a capacitance of approximately $0.12\, \mu F/m$, which was very large compared to the aerial telegraph lines of the time.

Now it is possible to establish the first boundary conditions for the problem, considering as main parameters the resistance r,[60] and the capacitance or electrostatic capacity c, all per unit length x. Initially, Thomson refuses that the problem was of a mere electrostatic induction, as Faraday had proposed, and suggests that the electrical resistance should be the main parameter to hinder the flow of electricity. Disregarding the self-inductance of the cable, the greater the resistance, the longer the cable charging time would be. Thomson further argues that there would be a voltage drop along the length of the cable, as a consequence of the resistance to the passage of a (current of) electricity $i(x,t)$,[61] which was the already famous Ohm’s technical *cliché*.[62] That is, the distribution of electric potential would be

$$\frac{\partial v}{\partial x} = -r i(x,t)\,. \tag{6.17}$$

Likewise, the *distribution* of current in relation to the variation of potential would be determined by the cable capacitance, that is, the higher the capacitance, the greater the “time constant”, in an anachronistic definition, of the large Leiden phial,

$$\frac{\partial i}{\partial x} = -c\frac{\partial v}{\partial t}\,. \tag{6.18}$$

Eliminating the element of current $i(x,t)$, replacing the second relation in the first and deriving it with respect to x, we have that

$$\frac{\partial^2 v}{\partial x^2} = -r\frac{\partial i}{\partial x}\,. \tag{6.19}$$

---

[58] In practice, Thomson obtained the value $c = \frac{C}{L} = \frac{1}{2\log\left(\frac{R}{r}\right)}$, which is obtained by dividing the result by $4\pi\epsilon_0$, which provides the value in the Gaussian system, instead of the international system.

[59] Thomson 1872, p. 38.

[60] The notation used for the resistance, at the time, was the letter k.

[61] I the original notation, Thomson used $\gamma(x,t)$, instead of $i(x,t)$ to designate the current. This was probably because he used the notation $i = \sqrt{-1}$ to denote the imaginary number.

[62] The Ohm’s law, $V = RI$ or $\vec{J} = \sigma\vec{E}$, in modern notation, is an example of a constitutive law, in which the equation is an empirical description of the properties of some material, and does not depend on the theory used, or the geometry of the piece of material.

Substituting in the second relation, it comes:

$$\frac{\partial^2 v}{\partial x^2} = r\left(-c\frac{\partial v}{\partial t}\right) = -rc\frac{\partial v}{\partial t}\,. \tag{6.20}$$

Therefore, arises the equation:

$$rc\frac{dv}{dt} = \frac{d^2 v}{dx^2}\,, \tag{6.21}$$

which he termed as the "... equation of electrical excitation in a submarine telegraph-wire, perfectly insulated by its gutta percha covering."

The first model of the Atlantic telegraph cable using the equation of heat diffusion would be called, by a modern electrical engineer, as the "non-inductive and leakage-free transmission line". That is, considering the resistance and capacitance of the cable as fundamental parameters, the model highlights an aspect very familiar to Thomson's spirit. He had already faced a very similar equation, years ago, in his youthful days. It was identical, in terms, to the expression of the linear motion of heat, obtained by Fourier, and would be perfectly adaptable to the practical issues of a telegraph wire. In fact, if the electric puzzle could be correctly solved, all investment in the design of the atlantic cable would be feasible. Never, in all the ages of mankind, would there be so much expectation (and money!) around the solution of a differential equation.

6.3 The law of squares

Thomson investigated diverse particular solutions of the heat equation adapted to telegraph signals transmission. In a letter sent to Stokes, in October 30 (1854), he stressed that "... an application of the theory of the transmission of electricity along a submarine telegraph wire... shows how the question raised by Faraday as to the practicability of sending distinct signals along such a length as the 2000 or 3000 miles of wire that would be required for America, may be answered. The general investigation will show how exactly how much the sharpness of the signals will be worn down, and will show what maximum strength of current through the apparatus in America, would be produced by a specified battery action on the end in England..."[63]

In the first case, Thomson considered Fourier's Integral theorem (the inverse transform), once the problem consisted on a "semi-infinite" cable, as in the case of the linear solid conductor of Fourier, which was "... perfectly adaptable for answering the practical questions regarding the use of the telegraph wire."[64] The first strategy was to imagine an

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[63] See in Thomson 1990b, p. 176.

[64] Thomson 1854b, p. 384; See also, in the Portuguese translation, Tonidandel, Boaventura, et al. 2018, p. 271.

impulsive entry that represented a switch being quickly actuated (Fig. 33),

$$
\begin{aligned}
v(z,t) &= V\,,\ \text{for } 0 \le t < T\,, && (6.22)\\
v(z,t) &= 0\,,\ \text{for } t < 0 \text{ or } t > T\,, && (6.23)\\
v(0,t) &= 0\,. && (6.24)
\end{aligned}
$$

in which V is a finite value of electrical potential. The first adaptation was to consider

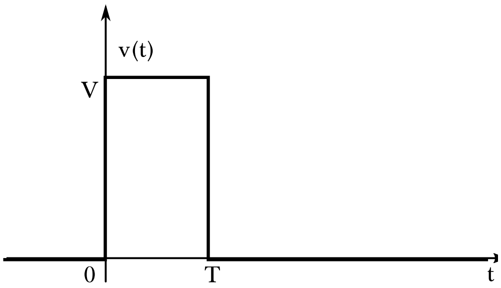

Figure 33 – A rectangular pulse.

a change of variables $z = x\sqrt{rc}$ in the cable equation (6.21). This supposition was fruit of Fourier's cleverness and it was, in fact, a trick used also by Thomson to eliminate the term rc. That is, by the *chain rule*,

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial z}\frac{\partial z}{\partial x} \Rightarrow \frac{\partial v}{\partial x} = \frac{\partial v}{\partial z}\sqrt{rc}\,. \tag{6.25}$$

Thus,

$$
\begin{aligned}
\frac{\partial^2 v}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial z}\sqrt{rc}\right)\,,\\
&= \sqrt{rc}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial z}\,,\right)\\
&= \sqrt{rc}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x}\,,\right)\\
&= \sqrt{rc}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\sqrt{rc}\right)\\
&= rc\frac{\partial^2 v}{\partial z^2}\,. && (6.26)
\end{aligned}
$$

Substituting in the heat equation, we have,

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial z^2} = 0\,, \tag{6.27}$$

which was the same equation worked by Fourier. Under these conditions, Thomson tried to show that the equation

$$v(x) = \frac{2V}{\pi}T\int_0^\infty e^{-x\sqrt{rc\omega}}\cos\left(2\omega\theta - x\sqrt{rc\omega}\right)d\omega\,, \tag{6.28}$$

which also was adapted from Fourier's integral theorem,[65]

$$f(x) = \frac{2}{\pi} \int_0^\infty \cos(qx)\,dq \int_0^\infty f(x) \cos(qx)\,dx\,. \tag{6.29}$$

would be a solution representing the distribution of potential along the cable, but he was not as successful.[66] In that case, Thomson hoped to find a *primary solution* that could be analogous to Fourier's heat problem, which consisted in finding the temperature profile in a "... thin and infinite solid", or a line heat conductor heat[67], an extension of the two-dimensional problem. When applying one of the boundary conditions, Fourier arrived at the result in form of an *integral equation*, which is now called the *inverse Fourier transform.* Similarly, to the "finite" case, the initial temperature profile in a planar solid with *spatial periodicity* resulted in a sum of cosines, the very first of the *Fourier series.* In that particular case, the steady state regime led him, by simplification, to Laplace's equation and, consequently, to the series. In the one-dimensional problem, the semi-infinite line represented, mathematically, an "infinite period."[68] His first analysis would appear very clumsy if compared with the most basic modern tools in treatment of linear systems, and will not be detailed.

It was Stokes that, in fact, came to the most general solution, in analogy to Fourier's heat problem. A few days later (Nov. 4), he writes a letter to his friend Thomson, presenting his version of the electrical puzzle of Hamilton's son: "... Faraday no doubt was perfectly familiar with the electrical appetite of the water wire as compared with the [earth] air wire. ... I wrote simply for my own information, taking for granted that the subject was all ABC to you. I merely endeavored to put the question clearly as it occurred to my own mind. ... *The mental translation of propagation of electricity into propagation of heat* which your equations shown to be permissible puts the mode of transfer in a clear light."[69] Following his own narrative, having worked in "... various forms of the equation $\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$ under the conditions $v(x,0) = 0$ and $v(0,t) = f(t)$," Stokes found that the Fourier transformation could really be used to achieve the same effect as the method of separation of variables, regarding to solving a partial differential equation. So, he could write the solution with a

---

[65] Fourier arrived primarily in the trigonometric and half-spectrum form, i.e., $0 < x < \infty$. The exponential form, $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$, in which $\lim_{T\to\infty} c_n = \frac{1}{2\pi} F(\omega) d\omega$ is a natural consequence of the first.

[66] Thomson 1854b, p.384; See the analysis of Thomoson's first attempts in the Portuguese transation of his 1854 paper at Tonidandel, Boaventura, et al. 2018.

[67] Fourier 1878.

[68] To know how he came to his remarkable equation, see Tonidandel and Araújo 2017, p. 54.

[69] My emphasis. See in Stokes 1990b, p.179.

single integral equation of the form[70] $v(\alpha,t) = \int\limits_0^{\infty} \omega(t,\alpha)\sin\alpha x dx$, to arrive at[71]

$$v(x,t) = \frac{x}{2\pi^{\frac{1}{2}}} \int\limits_0^{t} (t-t')^{-\frac{3}{2}} e^{\frac{x^2}{4(t-t')}} f(t') dt' \,. \tag{6.30}$$

This equation represented what would today be called the *impulsive response* of the system, once it permitted to know the behavior of the potential after an arbitrary signal was applied at one end of the cable (e.g., when a telegraph key was suddenly applied). Making a change of variables $u^2 = \frac{x^2}{4(t-t')}$ for any values of k and c, respectively resistance and capacitance per unit length, the Stokes integral becomes,

$$v(x,t) = \frac{2}{\sqrt{\pi}} \int_{\frac{x}{2}\left(\frac{rc}{t}\right)^{1/2}}^{\infty} e^{-u^2} du \,, \tag{6.31}$$

with $t \geq 0$ and $x \geq 0$.[72] This is a well known integral, called the *complementary error function*, *erfc*, in which erfc(b) = $1 - \operatorname{erf}(b) = 1 - \int\limits_0^{b} e^{-u^2} du$. So

$$\begin{aligned} v(x,t) &= \operatorname{erfc}\left[\frac{x}{2}\left(\frac{rc}{t}\right)^{1/2}\right] , && (6.32) \\ &= 1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2}\left(\frac{rc}{t}\right)^{1/2}} e^{-u^2} du \,. && (6.33) \end{aligned}$$

Which gives the most general solution to the distribution of potential along the cable. As the formulation (6.33) is more easily manipulable, we have, from equation 6.18, the response in terms of current can be found by calculating the derivative of equation (6.33) with respect to x. Thus,

$$\begin{aligned} \frac{\partial v(x,t)}{\partial x} &= -\frac{2}{\sqrt{\pi}} \left\{ \frac{\partial}{\partial x}\left(e^{-u^2}\right) du + e^{-\left[\frac{x}{2}\left(\frac{rc}{t}\right)^{1/2}\right]^2} \frac{\partial}{\partial x}\left[\frac{x}{2}\left(\frac{rc}{t}\right)^{1/2}\right] - e^0 \frac{\partial}{\partial x}(0) \right\} , && (6.34) \\ &= -\frac{2}{\sqrt{\pi}} \left\{ 0 + e^{-\frac{x^2 rc}{4t}} \frac{1}{2}\left(\frac{rc}{t}\right)^{1/2} \right\} , && (6.35) \\ &= -\left(\frac{rc}{\pi t}\right)^{1/2} e^{-\frac{x^2 rc}{4t}} \,. && (6.36) \end{aligned}$$

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[70] Stokes' objective at this point consisted in using the Fourier integral transformation to transform a partial differential equation (with partial derivatives) into an ordinary one. Applying the transformation with respect to the variable x on the two members of the heat equation (using the notation $\mathcal{F}\{\}$ to denote the operation), he could arrive at something like $\frac{d}{dt}\mathcal{F}\{v\} - \kappa\omega^2\mathcal{F}\{v\} = 0$ (in the homogeneous equation), which was evidently easier to work out. In fact, he wrote in a most general form, already considering the initial condition $v(0,t) = f(t)$. So, he arrived at $\frac{\partial\omega}{\partial t} + \alpha^2\omega - \frac{2}{\pi}\alpha f(t)$, in which $\mathcal{F}\{v(x,t)\} = \omega(\alpha,t)$. See in Thomson 1854b, p. 390; For a similar problem, see Tonidandel and Araújo 2017, p. 60.

[71] He did this (see note 70) differentiating $v(\alpha,t)$ (calculating $\frac{\partial v}{\partial x}$ and $\frac{\partial^2 v}{\partial x^2}$) and substituting into the heat equation (6.27).

[72] Try, for example, making $x = 0$. From the probabilities formula of De Moivre, we have $v(0,t) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du = \frac{2}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2} = 1$. The formula $\int_0^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$ is a very important integral, once it "appears" in the study of various natural phenomena. It was obtained for the first time in 1733, by Abraham De Moivre (1667-1754).

Thus, given that $\frac{\partial v}{\partial x} = ri(x,t) \Rightarrow i(x,t) = -\frac{1}{k}\frac{\partial v}{\partial x}$ (equation 6.19), the distribution of current $i(x,t)$ along the cable, maintaining the original notation according to Stokes analysis (and where he omitted the calculations, by the way) will be:

$$i(x,t) = \left(\frac{c}{\pi r t}\right)^{1/2} e^{-\frac{x^2 rc}{4t}} \,. \tag{6.37}$$

And, finally, the value of t_{max} can be found, in each case, making $i(x,t)$ or $v(x,t)$ a maximum point (i.e, making $\frac{\partial i(x,t)}{\partial t} = 0$ or $\frac{\partial v(x,t)}{\partial t} = 0$). For example, in the equation 6.37, calculating the time derivative and equating to zero, we have

$$t = \frac{x^2 rc}{2} \,. \tag{6.38}$$

This value was called "time of the maximum electrodynamic effect"[73] and the first conclusion was that "... the retardations of signals are proportional to the squares of the distances, and not to the distances simply". Although the result in the equation 6.38 expressed some sort of relation in which the delay of the maximum signal depended on x^2 instead of x, as anyone would expect, in fact, the cause of another type of delay: if the "law of squares" was really correct, it could make long distance telegraphy impracticable. A delay in the propagation of signals was expected and had already been observed, but the fact of being proportional to x^2 was something really strange. However, before analyzing the consequences of this "law", we have to investigate another particular result, now of greater interest. This one had to do specifically with the phase distortion phenomenon (a term Stokes and Thomson did not used): two separated pulses sent by a telegraph operator could easily end up overlapping each other unless T was sufficiently large and also the time between subsequent symbols. Or else, a sent pulse would disperse along the length of the cable, being attenuated to the point of almost disappearing when it reached the other end. The strategy, now imagined by Thomson, was to admit a periodic solution of the equation (6.27) as an entry signal:

$$v(0,t) = \mathrm{Re}\left\{e^{j\omega t}\right\} = \cos(\omega t) \,, \tag{6.39}$$

in which $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$. Although Thomson did not give any demonstration of this, it is not difficult to arrive at his first relevant result.[74] Admitting, at first, that the temporal variation of $v(x,t)$ happens along the cable in the form $e^{j\omega t}$ and considering that the spatial and temporal variables are separable, we have,

$$v(x,t) = \mathrm{Re}\left\{A(x)e^{j\omega t}\right\} \,, \tag{6.40}$$

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[73] Thomson 1854b, p. 386; Tonidandel, Boaventura, et al. 2018, p. 272.

[74] A discussion on this topic can be seen at Nahin 2002, p. 39.

with $A(0) = 1$. Substituting in the cable equation

$$\begin{aligned}
\frac{\partial^2 v}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right), \\
&= \frac{\partial}{\partial x}\left(\frac{\partial A}{\partial x} e^{j\omega t}\right), \\
&= e^{j\omega t}\frac{\partial^2 A}{\partial x^2}. \\
rc\frac{\partial v}{\partial t} &= rc\frac{\partial}{\partial t}\left[A e^{j\omega t}\right], \\
&= rcj\omega A(x) e^{j\omega t}. \qquad (6.41)
\end{aligned}$$

And so,

$$\frac{\partial^2 A(x)}{\partial x^2} = j\omega rcA(x). \qquad (6.42)$$

As the booklet says, when the derivative of a solution is proportional to the function itself, there is an exponential solution of the type $A(x) = e^{px}$. So, substituting in the equation (6.42), we have

$$p^2 = j\omega rc \Rightarrow p = \pm\sqrt{j\omega rc}. \qquad (6.43)$$

But, we know that,

$$p^2 = j\omega rc = \left[\left(\frac{1}{2}\omega rc\right)^{1/2} + j\left(\frac{1}{2}\omega rc\right)^{1/2}\right]^2, \qquad (6.44)$$

and thus,

$$p = \pm(j\omega rc)^{1/2} = \pm\left[\left(\frac{1}{2}\omega rc\right)^{1/2} + j\left(\frac{1}{2}\omega rc\right)^{1/2}\right]. \qquad (6.45)$$

Additionally, only the negative roots make physical sense in a semi-infinite cable ($p > 0$ would led to $v \longrightarrow \infty$). Then

$$A(x) = e^{-\left[\left(\frac{1}{2}\omega rc\right)^{1/2} + j\left(\frac{1}{2}\omega rc\right)^{1/2}\right]x}. \qquad (6.46)$$

Henceforth, writing the complete solution, choosing only the real part considering Euler's relation $e^{ja} = \cos(a) + j\sin(a)$:

$$\begin{aligned}
v(x,t) &= \mathrm{Re}\left\{A(x)e^{j\omega t}\right\}, \\
&= \mathrm{Re}\left\{e^{-\left[\left(\frac{1}{2}\omega rc\right)^{1/2} + j\left(\frac{1}{2}\omega rc\right)^{1/2}\right]x} e^{j\omega t}\right\}, \\
&= \mathrm{Re}\left\{e^{-\left(\frac{1}{2}\omega rc\right)^{1/2}x} e^{j\left[\omega t - \left(\frac{1}{2}\omega rc\right)^{1/2}x\right]}\right\}, \\
&= e^{-\left(\frac{1}{2}\omega rc\right)^{1/2}x}\cos\left[\omega t - \left(\frac{1}{2}\omega rc\right)^{1/2}x\right]. \qquad (6.47)
\end{aligned}$$

in which $t > 0$ and $x > 0$.[75] This solution implies an oscillatory response with a monotonically decreasing amplitude in length (the distance form the input end), in which higher frequencies would decay more rapidly. To visualize this, imagine a boat surfing on the crest of a big wave. A sailor on board is unable to notice the oscillations, that is, the sinusoidal behavior, but feels the wave amplitude smoothly decreasing over time. This condition requires the cosine factor (or the phase angle) to remain constant, and defines what Thomson called "the diffusion speed" of a signal with angular frequency ω, i.e.,

$$\omega t - \left(\frac{1}{2}\omega rc\right)^{1/2} x = \mathcal{C}\,. \tag{6.48}$$

Taking the time derivative,

$$\begin{aligned} \omega - \left(\frac{1}{2}\omega rc\right)^{1/2} \frac{dx}{dt} &= 0\,, && (6.49) \\ \frac{dx}{dt} &= \left(\frac{2\omega}{rc}\right)^{1/2}\,. && (6.50) \end{aligned}$$

The ultimate consequence of this result showed that a huge cable would apart and stretch out the input signals, which means that it would not be sufficient to simply raise the voltage amplitude to overcome the attenuation. That is a direct consequence of Thomson's analysis, and the effect explains the shape of the "arrival curves" for the current signals observed after applying various steps in one end of the cabe (Fig. 34).

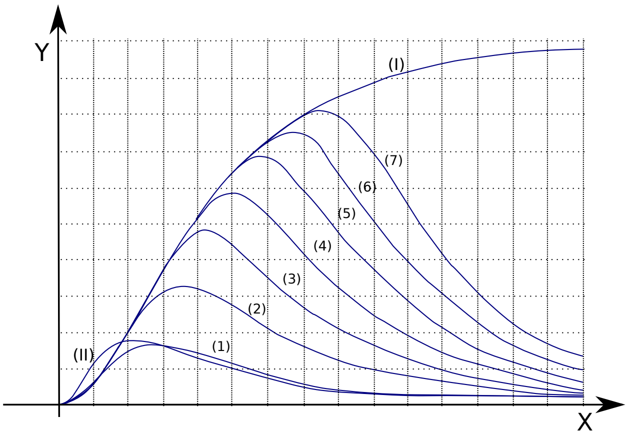

Figure 34 – Arrival curves from the input signals.

To understand the magnitude of this problem, we only need a little bit of arithmetic, while observing the monotonic exponential decay of the equation 6.47. That is, a signal of the type e^{-ax} will have its amplitude attenuated by a factor $1/e$ ($\approx$ 37%) at a distance

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[75] Or, in terms of z and T, $z = x\sqrt{rc}$ e $\omega = 2\pi/T$, using the notation of the original analysis:$v(z,t) = \cos\left(\frac{2\pi}{T}t - z\sqrt{\frac{\pi}{T}}\right)e^{-\sqrt{\frac{\pi}{T}}z}$. At this point Thomson was wrong in his calculations or presentation, once he had come up with $\cos\left(\frac{2\pi jt}{T} - z\sqrt{\frac{\pi j}{T}}\right)e^{-z\sqrt{\frac{\pi j}{T}}}$.

$x = 1/a$. A typical value of the product rc for a telegraph cable of that nature was around $rc \approx 2 \times 10^{-16}$, in the Gaussian system (cgs).[76] Thus, considering the factor $e^{-\left(\frac{1}{2}\omega rc\right)^{1/2} x}$, with $a = \left(\frac{1}{2}\omega rc\right)^{1/2}$, we have

$$\begin{aligned}
x \approx \frac{1}{a} &= \frac{1}{\left(\frac{1}{2}\omega rc\right)^{1/2}}\,, && (6.51)\\
&= \left(\frac{2}{\omega rc}\right)^{1/2}\,, && (6.52)\\
&= \frac{1}{\sqrt{\omega}}\left(\frac{2}{2 \times 10^{-16}}\right)^{1/2}\,, && (6.53)\\
&= \frac{10^{3}}{\sqrt{\omega}}\,\mathrm{cm} \times \frac{10^{-5}\,\mathrm{km}}{\mathrm{cm}}\,, && (6.54)\\
&\approx \frac{398}{\sqrt{f}}\,\mathrm{km}\,, && (6.55)
\end{aligned}$$

in which $\omega = 2\pi f$. This means that a signal with frequency 100 Hz, for example, would be 37% attenuated at every 39.8 km, approximately. For a 3200 km ($\approx$ 2000 miles) cable, a sign would be almost unreadable.

The previous analysis, which was roughly done by Thomson and Stokes from the law of squares, shows that, for high frequencies, the diffusion "speed" could exceed the velocity of light, i.e., an almost and instantaneous propagation of signals, and ultimately, instantaneous action at a distance. The analysis also highlights the limitations of the diffusion equation when dealing with the problem of propagation of electrical signals as a heat flow, or in other words, as a fluid flow, when disregarding the self-inductance of the Atlantic cable. In fact, at that point it was not a great problem, since the missing connection between electrical and optical phenomena was about to be made, a few years later. However, it suggested that some kind of modification should be made in the diffusion equation. But before that, it is necessary to take a step ahead, to the moment when the first mission left the bay of Valentia, Ireland, towards the deep waters of the Atlantic, in 1857.

6.4 The Atlantic cable [1857-1866]

The idea of launching a telegraph cable over the Atlantic Ocean bed was beginning to take shape, but in the 1850s, the proposal raised a cloud of disbelief.[77] Most people, even

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[76] We know this estimative for the constant rc from Lord Rayleigh's *The theory of sound* (1890). See Strutt 1894, v. 1, p. 466; A similar analysis was made by P.J. Nahin in his book on Heaviside life and work; however, he used apparently incorrect values for the constant rc, while making reference to Rayleigh's data. He used $rc = 5 \times 10^{-17}$, while the actual value provided by Rayleigh was $rc = 2 \times 10^{-16}$. Nahin's commentary on the distortion phenomenon can be seen at Nahin 2002, p. 37.

[77] A more detailed version of this narrative can be found at Potamian and Walsh 1909, p. 373.

in specialized branches, considered the idea mere utopia. Cable manufacturers said that such an apparatus could not be built, as a cable of this nature, over 2000 km, was simply unthinkable. The engineers of the time were almost certain that it could not be landed into the ocean bed, given its enormous mass. But with the glimpse of potential financial gains, specially after Thomson and Stokes' careful analysis, making sure that the feat was achievable, the investors began to consider the idea.

Something similar had already been considered at the beginning of the decade, and the place chosen should be the one with the shortest distance between the European continent and North America, between Valentia bay, in Ireland, and the city of Newfoundland (currently Canada). In 1852, the *Newfoundland Telegraph Company* had been created for a similar purpose, to link the arctic circle to city of New York, so that the news from ships arriving in the northern British colonies could be telegraphed to the financial heart of the United States. The first attempts to laying a cable under the mysterious waters of the Atlantic took place in 1857, when the then newly created *Atlantic Telegraphic Company*, carrying out one of the first surveys – by the newly invented radar – on the ocean floor, demonstrating what they believed to be a more or less flat surface on the ocean floor, which became known as the *telegraphic-plateau*. The result of the first attempts was a series of failures, with huge losses. After just over 300 km the cable would be broken and forever lost in the immensity. A huge loss, which led some of the investors to ruin.

A second attempt, after a new fundraising, was carried out in 1858, when the American ship *USS Niagara* and the English frigate *HMS Agammenon* would meet halfway between the two continents. Each one would carry one half of the cable. After the first encounter, the halves would be welded and the large cable could be slowly landed into the sea bed,[78] with ships now sailing in opposite directions. William Thomson was at the British ship, hired as chief *Electrician*. Possibly, at that moment, his mind wandered until the backward days when, in his adolescence, he had been faced with Fourier's mathematical artwork and his equation of linear motion of heat. Now, with its new guise, it would be possible to "transmit intelligence", by electric means, through the silent and deep waters of the Atlantic. After a series of minor accidents, which forced the ships to return for leagues back, the *HMS Agamemnon* landed one end of the cable on the Irish side, in Valentia bay. The *USS Niagara*, on the other hand, did not have a smooth journey, facing severe storms, which almost led it to a shipwreck, managing to arrive safely at *Trinity Bay*, Newfoundland, on August 5, 1858, truly a remarkable date, in which, for the first time, the two continents would be electrically united, completing the first phase of the great engineering feat of the 19th century.

Although it was not completely understood by the group of investors who financed the project, the signals were very difficult to see, given its large capacitance, and it was up

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[78] See appendix A.

to William Thomson, from one of the recommendations of his 1854 study, to develop an ultra-sensitive device, so that the minimum signs of voltage could be detected and decoded messages. With this, the phenomenon of distortion could be contoured and it would not hinder the reading and interpretation of messages. The result expressed by the "law of squares" (eq. 6.38) suggested that the delay of the maximum signal of current depended on x^2 instead of x, as expected. The delay itself did not cause as much astonishment, but the dependence on the square of the distance caused much suspicion, and could put the viability of the project in jeopardy, once the time to transmit a short message could be so long that could make all investment unfeasible (almost 8 minutes per word).

Trying to work around this problem, Thomson developed his *optical galvanometer*, or *mirror galvanometer*, which was not new in essence, once it had already been used by Weber and Gauss. However, following the recommendations of Weber, Thomson built a slightly modified version: a small current carrying the telegraphic signal, crossing a tiny galvanometer coil, would, in turn, cause a small deflection of a needle. This deflection would be "amplified" by the light of a candle, projected on a screen, in which a graduated scale could show even very small variations of the signal (Fig. 35). Thomson was, in fact, one of the first engineers to realize that a decrease in instrument sizes could lead to an increase in measurement sensitivity.[79]

The use of the *mirror galvanometer*, one of the creations that would enable the practical use of the Atlantic cable, was not fully operational at this point, as it did not yet allow any form of recording. It was necessary to continuously watch the deflection of the magnetic needle and, simultaneously, take notes of the values associated to the messages, essentially the same proceeding of Gauss and Weber in the telegraph of 1833. But the experience with this rudimentary device from the first expedition would enable him to create, years later, the *Siphon recorder*, an automatic device that consisted of a capillary-sized siphon, which was directly connected to the galvanometer's movement mechanism, and which recorded the small deflections.

One of the most influential critics of Thomson and Stokes's physical-mathematical approach was the physician – and self-declared "electrical expert" – Edward Orange Wildman Whitehouse (1816-1890).[80] Claiming to have performed experiments on his own, and being a supporter of the class of "practical men", in his words, Whitehouse was one of those who were very suspicious of the presence of mathematics in the engineering place. Having been taken to the head of the *Atlantic Telegraph Company*, Wildman sustained that, for the transmission over long distances, it would be enough that the signals were transmitted from a high voltage source, of a few thousand volts. The belief, also shared by other members of the Engineering board, was also economically motivated, since it would be

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[79] Potamian and Walsh 1909, p. 375.
[80] Hunt 1996.

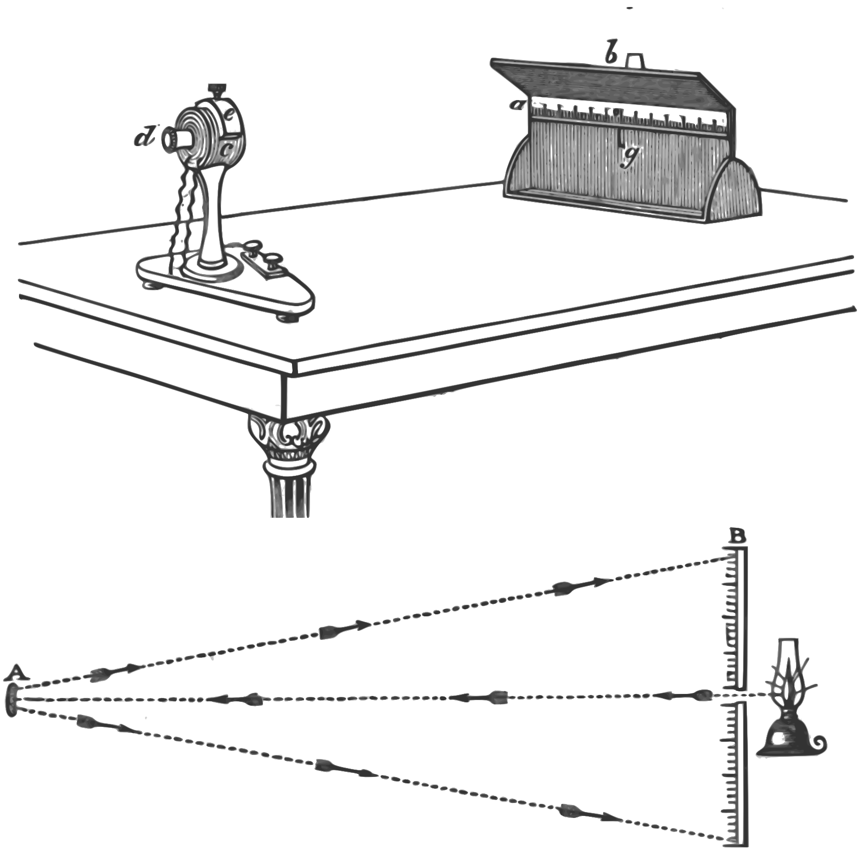

Figure 35 – Thomson's mirror galvanometer: developed to detect small variations of current in a telegraphic line. A small current crossed a tiny galvanometer coil, causing a small deflection of a magnetic needle. This deflection was "amplified" by the light of a candle, projected on a screen, in which a graduated scale could show even very small variations of the signal.

possible to build a cheaper wire, a cable with a small cross section area and, consequently, less metallic copper. There is no need to say that the project conception choice, taken by the company board of directors and made before the cable manufacturing, would result in the stupendous failure of the 1858 mission. The high voltage impressed by 380 Daniell's cells, used at the extremity of *Valentia* bay,[81] would break the gutta-percha isolation a few weeks later. During its short life, the first version of the great submarine cable would carry only 366 messages between the United States and England. As a result, Whitehouse would be fired from his position.

The negative publicity of the failure would have also a collateral effect, and the phenomenon would provoke an almost widespread belief that Thomson's *law of squares* was indeed a principle of Nature, which was far from being. In the sociological aspect, the social perception had been altered, and the most directly involved characters started

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[81] The Daniell cell was a type of electrochemical battery, invented by John Frederic Daniell in 1836, as an improvement of Volta's battery. The cell consisted of a copper pot filled with a copper sulfate solution, in which it was immersed in sulfuric acid which contained a zinc electrode. This battery would be at the heart of the definition of volt (V), years later, at the convention for the International System of Units, where the electromotive force of a Daniell cell would be considered $1\,V$.

to see in Thomson's figure someone important to decide on the subject, taking his view on the importance of mathematics to a higher level of awareness among the "population". The problem was that, in this respect, *Thomson was also wrong*, and the model, while mathematically consistent, was physically deficient. This misinterpretation would cause a delay in the practical application of submarine telegraphy for a long time, until the discovery that the solution could be "outside" the cable, so to speak, with Heaviside's work on the role of self-induction, in the late 1870s.

Figure 36 – The Great Eastern, *fishing* a broken cable in the 1865 mission.

In 1865, when it was made another attempt to launch a cable under the Atlantic ocean, Thomson, whose reputation had increased enormously, was conducted again as a scientific consultant. Now the cable would be transported on just one ship, then the largest ship in the world, the *Great Eastern*, created by engineer Isambard Kingdom Brunel (1806-1859), who would never had the chance to see it in operation. Leaving Valentia bay, the ship was now sliding slowly, gradually landing the newest cable. However, when almost 1700 km had already been layed, the cable broke and was lost, again, as in the 1857 mission. Attempts to fishing the cable were made (Fig. 36), but without success.[82] A year later, however, after a new financial collection,[83] now equipped with an improved and well manufactured cable, a paying-out machinery (Fig. 37)[84] and a whole staff of engineers, the landing was finally successful, after the American end arrived at Trinity bay, on 13th July 1866.

Finally, it is now necessary to make a brief comment on the end of this story, since the advent of electrical telegraphy, in all its extension, would bring a profound change in

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[82] The 1865 cable would be retrieved after 1866 and put into operation.

[83] About $ 4,000,000 at that time, almost $ 100,000 billion on today's currency.

[84] See appendix A.

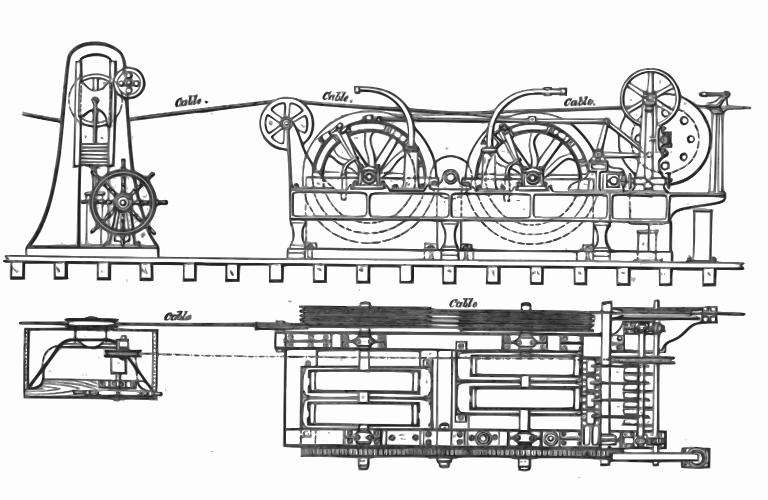

Figure 37 – The paying-out machine, used to gradually land the submarine cable into the ocean bed.

human relations, propitiated by the revolution in communications. And the last technical step was taken from the solution of the harmonic distortion problem.

6.5 The telegrapher's equation

It is said that when the great novelist Charles Dickens (1812-1870) moved to a new residence in the city of Kent in 1856, he asked them to install a hidden door in his office, amid the hundreds of books on the shelves. The door was camouflaged in the middle of the shelves,[85] containing a series of fictional titles that, if removed, would give way to a "secret" room. Among the three volumes of *Five minutes in China*, the nine volumes of *Cat's lives*, the single volume of *Architecture of Noah* and *The oysters of Shelley* – an allusion to the famous writer of *Frankenstein* – there was a singular title, *Heaviside's Conversations with Nobody*. The title of Dickens's fake book very well portrayed the stereotype of the lonely hermit scientist, with a peculiar sense of humor, but certainly with a inventive mind, Oliver Heaviside (1850-1925).[86]

Since Heaviside literally retired from his job as a telegraphic operator, he would have one of his most productive years in science. During this period he published a remarkable series of articles on telegraph theory. The series evidenced a great mathematical sophistication, but also a deep practical knowledge about real telegraphic problems, in which one objective was clear: to provide a rapid advance for the new needs of the new born Telegraphic Engineering and the telegraphic industry, that was growing rapidly, or, in Heaviside's words, "... modernize it in accordance with Maxwell's ideas".[87] During the

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[85] Although presented as an anecdote, the door to Dickens home study office is real, and so the book list. See Fadiman and Bernard 2000.

[86] See, e.g., the widelly known biogaphy of Heaviside, by Nahin 2002, p. 43.

[87] Heaviside 1894, p. 82.

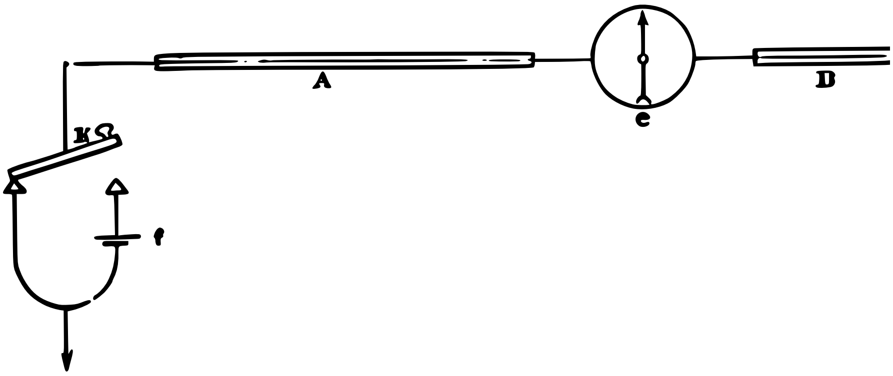

Figure 38 – Heaviside's representation for the transmission line (1876), as a starting point to his developments. This picture can be "translated", in modern terms, into Fig. 39.

years of 1876-1879, three important articles are published in the *Philosophical Magazine*.[88] In the first one, *On the extra current*,[89] the 26-year-old "nobody" showed his talent, obtaining a differential equation for the potential $v(x,t)$ along an extensive submarine cable. He provided only a brief explanation of how he arrived at the equation, but the starting point of Heaviside was certainly Thomson's derivation of the submarine cable equation, the equation of diffusion ef electricity, from 1854, but now without neglecting self-inductance of the system.[90]

Thomson himself had affirmed, years after his achievement,[91] to have considered the phenomenon of self-inductance initially, leaving the annotation in one of his research notebooks (July 1852),[92] but believing that it would play a minority role in the whole project, he considered it negligible. Heaviside, then, taking uniform values of resistance r, capacitance c and inductance l per unit of length throughout a cable of length ℓ, stated that "... to find an expression for the potential and the current at any point of a cable insulated at one end, at any time, the only way, as far as I am aware, is to follow the method given by Sir William Thomson in 1855,[93] making the necessary alterations to suit the changed conditions of the problem."[94] To achieve this, he used a first approximation of what would be known today as a *transmission line* (Fig. 38), which can be "translated" into the model expressed in (Fig. 39). The capacitance was formed, in the model, by the interface between the dielectric outside the wire and the water, which served in his mind as the return conductor.[95] This was his first step, as Heaviside believed, "... towards getting

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[88] Heaviside 1876, 1877, 1879; A collection of Heaviside's papers can be found at Heaviside 1892, 1893, 1894, 1899, 1912.

[89] Heaviside 1876.

[90] Heaviside 1874.

[91] Thomson 1882, p. 534.

[92] See Thomson n.d.; See also Jordan 1982.

[93] See Thomson 1855.

[94] Heaviside 1874, p. 48.

[95] See Heaviside 1894, p. 79, p. 81.

out of the wire into the dielectric".[96]

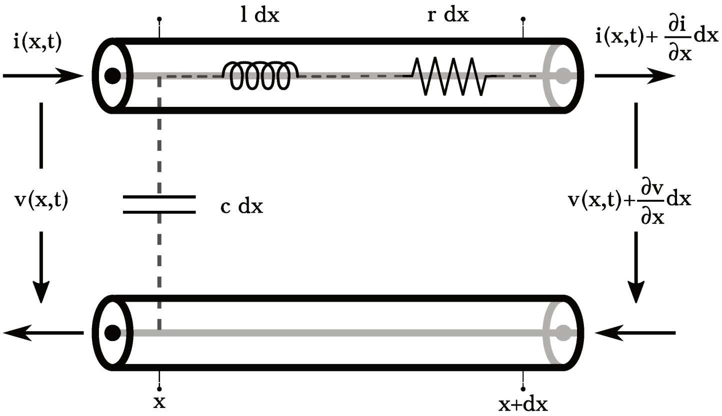

Figure 39 – Model of the telegraph transmission line of 1876, in which Heaviside develops the concept of distributed elements. The line is conducting a current $i(x,t)$ under a difference of potential $v(x,t)$. Even though he did not provide any clear representation of this, only a brief description, the line parameters are now *distributed* (not lumped) along the line, i.e., the capacitance c (F/m), resistance r $(\Omega/m)$and inductance l (H/m) are considered per unit of length m. The dashed lines represent here the parameters' non-locality condition of his first approximation in a line element of length dx.

The telegraph line is conducting a current $i(x,t)$ under a difference of potential $v(x,t)$, both functions of length and time. The line parameters are now distributed, i.e., the capacitance c (F/m), resistance r (Ω/m) and inductance l (H/m) are considered per unit of length m, in a line element of length dx, as a first approximation.[97] As his first model considered an infinitesimal length of cable, he equated the voltage drop and the electric potential,[98] arriving at

$$v(x,t) - l dx\frac{\partial i}{\partial t} - r i(x,t)dx - \left(v(x,t) + \frac{\partial v}{\partial x}dx\right) = 0\,, \tag{6.56}$$

and so,

$$\frac{\partial v}{\partial x} = -l\frac{\partial i}{\partial t} - r i(x,t)\,. \tag{6.57}$$

This equation had a clear physical meaning for Heaviside and, at this point, a slight comment can be made. The variation of the voltage along the transmission line was due

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[96] Heaviside 1894, p. 79; A discussion in Portuguese about a text by Heaviside that deals with action at a distance can be seen in Santos et al. 2020.

[97] He used the notation V for the potential drop, R for the resistance, C for the current (in wich $C = \frac{\partial q}{\partial t}$, with q the electric charge) and S for the capacitance (electrostatic capacity).

[98] Altough knowing it was a wrong approximation to make, specially in virtue of the presence of the self-inductance, Heaviside justified the assertion based on the capacitive dominance ($q = CV$) and the absent necessity, in his view, to restrict the configuration to round wires or simmetrically arranged returns. See in Heaviside 1894, p. 81.

to the voltage drop in its distributed resistance (Ohm's law) and inductance (Faraday's law),[99] which he called "... rate of increase of the momentum of that current",[100] or "... electric force of inertia",[101] an analogy that indicated the effect of opposition caused by the induced voltage, in order to establish a current value that was opposed to its own variation, as it usually happened when one tried to brake a flywheel, for example. **For those class of analogies, the inductance (l) was the dynamical equivalent to the mass of fluid per unit of length, the current (i) would be the velocity, (li) the momentum and $\left(l\frac{\partial i}{\partial t}\right)$ the force that must be applied to increase this "velocity". For the same reason, $-l\frac{\partial i}{\partial t}$ (i.e., the Lenz law) would be the force of reaction and the magnetic energy would have the same shape of the kinetic energy,[102] i.e., $\frac{1}{2}mv^2$ and $\frac{1}{2}li^2$.**

For the current, Heaviside used the continuity equation,[103] where the difference between the input and output currents are proportional to the tension, in a first place,[104] and secondly due to the capacitive effect, that is, charges that accumulate in (or disappear from) the section dx of the conductor. In other words, when there is a difference between the currents (i on the left and $i\frac{\partial i}{\partial x}dx$ on the right), the excess will accumulate in form of a rise of charge in the capacitor.[105] Here, if one intends to maintain the flow analogy,[106] that is, if the current is considered as a fluid flow, according to Heaviside, it must be an incompressible fluid, although it can accumulate on the surface of the wire. The effect describing the tendency for a current to flow on the surface of a cylindrical conductor would be formaly presented by Horace Lamb, in 1883, kown as *the skin effect*,[107] would also be investigated later by Heaviside, in another occasion. Then, the rate of variation of this charge will be

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial t}\left[cv(x,t)dx\right] = c\frac{\partial v}{\partial t}dx\,. \tag{6.58}$$

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[99] That is $v_l = N\frac{\partial \phi}{\partial t} = N\frac{\partial \phi}{\partial i}\frac{\partial i}{\partial t} = l\frac{\partial i}{\partial t}$, in which N is the number of spires and ϕ is the magnetic flux (Wb).

[100] Heaviside 1892, p. 54.

[101] He usually used different notations, writing sometimes $-L\dot{C}$ or $s\ddot{Q}$, in wich Q represented the charge, $\dot{Q} = C$ the current and $L = s$ the inductance. Cf. Heaviside 1894, p. 83.

[102] Heaviside 1894, p. 84.

[103] A modern notation for the continuity equation is $\nabla.\vec{J} = -\frac{\partial \rho}{\partial t}$. It establishes that the divergence of the current density J $\left(A/m^2\right)$ is equal to the negative rate of change in the density of charge ρ $\left(C/m^3\right)$, which is, essentially, a symbolic representation of a fluent quantity, in this case the electrical current.

[104] He would later use a more realistic approximation, considering a leakage term, wich was proportional to $v(x,t)gdx$, where g $\left(\Omega^{-1}/m\right)$ is called the conductance per unit of length. In this case, the equation 6.57 can be written as $i(x,t) + \frac{\partial i(x,t)}{\partial x}dx + v(x,t).g.dx - i(x,t) = -c\frac{\partial v}{\partial t}dx = 0$, as it is going to be considered further. See Heaviside 1894, pp. 81–82; See also Heaviside 1892, p. 53.

[105] Heaviside 1894, p. 79.

[106] Heaviside was at this point, a little bit controversial. Although he demonstrated an open willingness to forget the fluids of Victorian science, "... I am no believer in this fluid" (1894, p. 79), just ahead, as it was usual in a period when the *Maxwellians* were getting used to the new language of electromagnetism, he advocated favorably to it by saying: "... the water-pipe analogy is, however, simple enough" (1894, p. 83).

[107] Lamb 1883.

Thus, given that $i(x,t) = \frac{\partial q}{\partial t}$ and $q = cv(x,t)$,

$$i(x,t) + \frac{\partial i(x,t)}{\partial x}dx - i(x,t) = -c\frac{\partial v}{\partial t}dx\,. \tag{6.59}$$

"Dividing" by dx, we have the equation for electric charge on the surface of the wire between $x + dx$,

$$\frac{\partial i}{\partial x} = -c\frac{\partial v}{\partial t}\,. \tag{6.60}$$

Now, with the system formed by the equations for the voltage (6.57) and the current (6.60), Heaviside was able to mount the final expression. Taking the second derivative of 6.57 with respect to x,

$$\frac{\partial^2 v}{\partial x^2} = -r\frac{\partial i}{\partial x} - l\frac{\partial^2 i}{\partial t \partial x}\,, \tag{6.61}$$

Now, substituting the equation 6.60 in 6.61 results in,

$$\frac{\partial^2 v}{\partial x^2} = -r\left[-c\frac{\partial v}{\partial t}\right] - l\frac{\partial^2 i}{\partial t \partial x}\,. \tag{6.62}$$

Analogously, taking and the second derivative of equation 6.60 with respect to t,

$$\frac{\partial^2 i}{\partial x \partial t} = -c\frac{\partial^2 v}{\partial t^2}\,. \tag{6.63}$$

Finally, inserting the equation 6.63 in 6.62 leads to

$$\frac{\partial^2 v}{\partial x^2} = rc\frac{\partial v}{\partial t} + lc\frac{\partial^2 v}{\partial t^2}\,, \tag{6.64}$$

or

$$\boxed{\frac{1}{lc}\frac{\partial^2 v}{\partial x^2} = \frac{r}{l}\frac{\partial v}{\partial t} + \frac{\partial^2 v}{\partial t^2}\,,}$$

in which $\frac{1}{\sqrt{lc}}$, and $v(x,t)$ is the potential at every point x, in a time t.

This equation was termed the *Telegrapher's Equation* or *Telegraphy Equation*, and it indeed contained more information than Thomson's diffusion equation for the submarine telegraph cable. It was, in fact, the first version obtained by Heaviside from William Thomson's diffusion equation, in 1876.[108] It is important to reaffirm here that Heaviside did not give any demonstration of this, although he wrote the expressions for current and voltage, which govern the propagation of electromagnetic waves on single-phase lines. The equation 6.64 admitted some interesting special cases. The most evident are the conditions in which $l = 0$ and $r = 0$. For the first, $l = 0$, the self-inductance of the cable is being neglected and, in this case, the relation becomes the well-known *heat equation*,

$$\frac{\partial^2 v}{\partial x^2} = rc\frac{\partial v}{\partial t}\,. \tag{6.65}$$

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[108] See in Heaviside 1876, p. 54; See also Heaviside 1894, p. 82.

Now neglecting the resistance per unit of length, that is, for $r = 0$, we arrive at the famous equation of the vibrating string, the *wave equation* of section 5.3:

$$\frac{\partial^2 v}{\partial x^2} = lc\frac{\partial^2 v}{\partial t^2} \, . \tag{6.66}$$

That is, the special cases allowed, as a consequence, not only the possibility of wave propagation at finite velocities,[109] but also the solution of harmonic distortion problem, which hindered a really fast communication through the Atlantic telegraph cable. Understanding this solution would pave the way for the appearance of the telephone, at the end of the century.[110]

Although Heaviside was, apparently, unaware of the first consequence in 1876,[111] he used the particular solutions to infer the solution "by inspection", which would be solution of the charging problem.[112] If a cable of length ℓ is fed by a battery for some time, then the stationary current I and voltage V will be

$$\begin{aligned} I &= \frac{V}{r\ell} \, , & (6.67) \\ V &= V\left(1 - \frac{x}{\ell}\right) \, , & (6.68) \end{aligned}$$

with $0 \leq x \leq \ell$. Then, imagining that each end of the cable would be connected to a perfect ground, he realized that the effect of the new inductive term, at the cable equation, would be to induce oscillations in the voltage signal as it decreased during the discharge process. For that, he wrote the Fourier series for $i(x,t)$ and $v(x,t)$.

A second consequence, the solution of the distortion (but not the attenuation) problem would be given by Heaviside some years later – one more time without giving any formal demonstration. The "archaeological" work here is an important step, since it can lead us to instructive insights on the development of electrical engineering. In the transmission line, consider now a more "realistic" approximation, considering a leakage term represented by a shunt conductance g (Fig. 40).[113] The current at the leakage term will be proportional to $v(x,t)gdx$, in which g is simply the reciprocal *Ohmic* resistance per unit of length $\left(\Omega^{-1}/m\right)$. In this case, the equation of continuity for the current can be

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[109] It is worth remembering that the existence of electromagnetic waves would only be confirmed by Hertz in 1893. See Hertz 1893.

[110] Jordan 1982.

[111] About ten years later, Heaviside would afirm that "...c is a quantity occuring in Weber's hypothesis, the velocity with which two particles of electricity must move in order that the electrostatic repulsion and electrostatic attraction may balance". See in Heaviside 1894, p. 81; The constant c was first estabilished by Wilhelm Weber as a ratio of units of electric charge. These units were defined in terms of Coulomb's force (chapter 5) and the force between elements of currents of Ampère (sec. 6.1). For the story of c, see Mendelson 2006.

[112] He used, in fact, the same strategy of the vibrating string problem, arriving at a solution in form of a Fourier series. See in Heaviside 1876, p. 54.

[113] The basic theory of the transmission line can be seen in Araújo and Neves 2005, pp. 137–169.

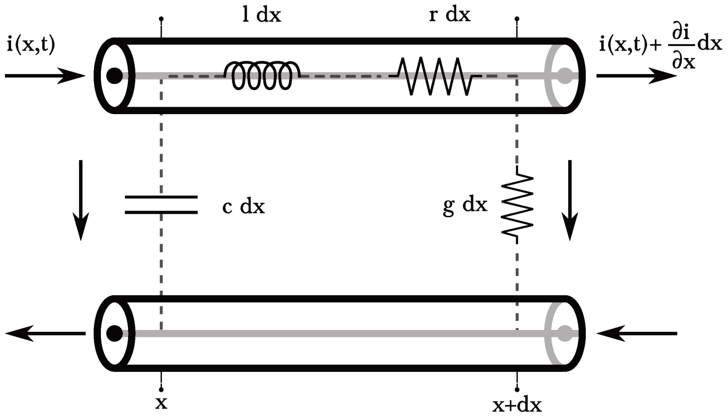

Figure 40 – Model of the telegraph transmission line to apply the continuity equation for the current. The difference between the input and output currents is proportional to the accumulated charges in section dx and to the voltage at the leakage conductance g (Ω^{-1}/m).

written as

$$i(x,t)+\frac{\partial i}{\partial x}dx+v(x,t).g.dx-i(x,t)=-c\frac{\partial v}{\partial t}dx\,, \tag{6.69}$$

$$\frac{\partial i}{\partial x}=-v(x,t).g-c\frac{\partial v}{\partial t}\,. \tag{6.70}$$

which means, according to the interpretation, that the spatial variation of the current along the telegraph line is due to the "leakage" of charges to the subsequent element of the line, and also to the accumulation of these charges on the conductor surface. Considering also the equation for the voltage,

$$\frac{\partial v}{\partial x}=-l\frac{\partial i}{\partial t}-ri(x,t)\,. \tag{6.71}$$

Adopting the same procedure as previously done, taking the derivative of 6.71 with respect to x, comes

$$\frac{\partial^2 v}{\partial x^2}=-l\frac{\partial^2 i}{\partial t\partial x}-r\frac{\partial i}{\partial x}\,. \tag{6.72}$$

Replacing 6.70 in 6.72, results in

$$\frac{\partial^2 v}{\partial x^2}=-r\left[-g.v(x,t)-c\frac{\partial v}{\partial t}\right]-l\frac{\partial^2 i}{\partial t\partial x}\,. \tag{6.73}$$

Now taking the second derivative of 6.70 with respect to t,

$$\frac{\partial^2 i}{\partial x\partial t}=-g\frac{\partial v}{\partial t}-c\frac{\partial^2 v}{\partial t^2}\,. \tag{6.74}$$

And, thus, replacing 6.74 in 6.73 leads to,

$$\frac{\partial^2 v}{\partial x^2}=-r\left[-g.v(x,t)-c\frac{\partial v}{\partial t}\right]-l\left[-g\frac{\partial v}{\partial t}-c\frac{\partial^2 v}{\partial t^2}\right]\,. \tag{6.75}$$

And, finally, rearranging the terms, we arrive at

$$\frac{1}{lc}\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} + \left[\frac{r}{l} + \frac{g}{c}\right]\frac{\partial v}{\partial t} + \frac{gr}{lc}v(x,t)\,. \tag{6.76}$$

The equation 6.76 is also called the telegrapher's equation, and it was probably used by Heaviside to arrive at one of his most famous achievements. His insights, then supported by Maxwell's theory, led him to a new conception of the practical problems of telegraphy, in a way reformulating the basic knowledge on the propagation of signals along a telegraph cable. The concept of distributed parameters would now yield excellent credit for the telegraph engineering, once Heaviside realized that the condition to solve the long known problem of distortion of signals could now be achieved if he imposed one single condition:

$$\boxed{\frac{r}{l} = \frac{g}{c}\,.}$$

"... what we do is to make the circuit distortionless... by the additional leakage to compensate the additional resistance of the wires."[114] For the simpler case (eq. 6.66), considering a null resistance, in addition to the signal not being distorted, it will also not be attenuated, which will represent a traveling wave, propagating with the speed of light along the cable.[115] He also noted that usually $r/g >> l/c$ and suggested that one should increase the inductance of the circuit by adding discrete inductors along the line to fulfill the equality. Additionally, to solve many of the problems involving practical telegraphy, Heaviside invented his own mathematics – what he called *Experimental Mathematics* – specially his *Operational Calculus*, which would be at the heart of the modern integral transform techniques, such as the *Laplace transform*.[116] The operational calculus was developed precisely to solve differential equations such as the telegrapher's equation and, from his various discoveries, a great amount of modern electrical engineering science would emerge. But, on the issue of harmonic distortion itself, which resulted in the equation that predicted the propagation of electromagnetic waves, was Heaviside really a pioneer? **No, he certainly was not!**[117]

The technical question on the distortion problem could have been overlapped decades before if Kirchhoff's approach, which was based on Weber's force, had been widely

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[114] See Heaviside 1894, p. 140.

[115] In fact, Heaviside arrived at the condition form the solution of the telegraphy equation, in which he used his Operational Calculus. In the solution he noticed that the now called *constant of propagation* $\gamma = \sqrt{rg - lc + (j\omega)^2 lc}$ would have a pure imaginary term if $r/l = g/c$, which means a propagation without distortion (the term in dv/dt in eq. 6.76 is null), but still with attenuation. This situation is easily visualized by the term in $v(x,t)$, which does not disappear, evidencing the differential equation in x, which will have an exponential solution. See Araújo and Neves 2005, p. 142.

[116] To see the history of the Operational Calculus and Integral Transforms, see Tonidandel and Araújo 2017, p. 6; See also Tonidandel and Araújo 2012b.

[117] Even recent review articles seem to ignore this issue, as can be seen in Christopher 2018.

considered. This, however, would depend on a philosophical choice that the English physicists like Heaviside were not willing to corroborate. In 1857, just three years after Thomson and Stokes's study on the transatlantic project, while the first mission for the transcontinental submarine cable left the port of Valentia, Gustav Kirchhoff (1824-1887) was preparing a set of impressive papers in which he obtained not only an expression geometrically identical to the telegrapher's equation,[118] but also demonstrated that, while starting from the epistemic pillar of action at a distance – without the concept of fields or any other swirling mechanism in the ether – he could arrive at an outstanding result: that electric disturbances could travel along wires of negligible resistance with the velocity of light![119]

Kirchhoff could demonstrate the validity of such waves using only the Newtonian laws of movement unfolded by Coulomb, Weber, Ampère and others, before the discovery of Maxwell's equations. Not only Kirchhoff, but also Weber would deduce the telegrapher's equation, independently, at that same year of 1857, while the first transatlantic mission was ongoing, by taking into account the notes of Faraday on the self-induction, and using premises similar to those of Kirchhoff.[120] Both of their premises started from the Weber's force relation, and would predict, particularly, the propagation of electric waves in a long wire. In the first paper, Kirchhoff derived the telegraphy equation for a signal that propagated along a thin conductor, with a circular cross-section, which could even have an arbitrary curvature.[121]

Writing the generalized version of Ohm's law, Kirchhoff considered free charges moving along the surface of a conductive wire.[122] To account for the effects of the self-induction in the three dimensional space, he considered the induction caused by the variation of the current in all parts of the wire, that is, in the form of an differential equation[123]

$$\vec{J} = \kappa \left(\nabla v + \frac{\partial \vec{A}}{\partial t} \right) , \tag{6.77}$$

in which $\vec{J} = (J_x, J_y, J_z)$ is the density of electrical current, κ is the conductivity of the wire, $v(x, y, z, t)$ is the electric potential and $\vec{A}$ is an analogue function to the modern vector magnetic potential, where $\frac{\partial \vec{A}}{\partial t}$ executes a similar function to the electric field $\vec{E}$,

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[118] Kirchhoff 1857a.

[119] Kirchhoff 1857b; A translation of this paper can be found at Graneau and Assis 1994.

[120] In a paper published in 1857, *Bemerkung zu dem Aufsatz des Herrn* (Poggendorff 2021), J. C. Poggendorff makes an observation on Kirchhoff's experiments related to his 1857 paper *On the motion of electricity in wires* (Kirchhoff 1857a) and to Weber's 1864 paper *Determinations of electrodynamic units, especially on electric oscillations* (Weber 1864, with English translation in Weber 2021). Poggendorff stressed the similarity of the two approaches and the remarkable achievements of Kirchhoff and Weber.

[121] Kirchhoff 1857a; See also Assis and Hernandes 2009, p. 223.

[122] See note 62, in p. 143.

[123] The main source of this demonstration can be found at Assis 1992; Assis and Hernandes 2005, 2009; Kirchhoff 1850, 1857a. In the first three references, the authors present Kirchhoff's demonstration based on Weber's electrodynamics, using the International System of units.

which Kirchhoff also deduced from Weber's force:[124]

$$\vec{F}_{ji} = \frac{q_i q_j}{4\pi\epsilon_0}\frac{\hat{r}^2_{ij}}{r^2_{ij}}\left(1 - \frac{\dot{r}^2_{ij}}{2c^2} + \frac{r_{ij}\ddot{r}_{ij}}{c^2}\right) , \tag{6.78}$$

which represents the impressed force made by a charge q_j in a charge q_i, where $\vec{F}_{ji}$ acts along the line between the two charges, $\hat{r}_{ij}$ is the unit vector that points from the charge j to i. Additionally, $r_{ij} = \vec{r}_i - \vec{r}_j$ is the distance between $\vec{r}_i$ and $\vec{r}_j$, where $\vec{r}_i$ (or $\vec{r}_j$) is the vector that points from the origin of the coordinate system to the body i (or j), the constant ϵ_0 is the so-called vacuum permittivity and $\dot{r}_{ij} \equiv \frac{d}{dt}r_{ij}$, $\ddot{r}_{ij} \equiv \frac{d^2}{dt^2}r_{ij} = \frac{d}{dt}\dot{r}_{ij}$.

The most important characteristic of Weber's formula is that it obeys Newton's law of action and reaction, whatever the state of motion of the charges, i.e. $\vec{F}_{ij} = -\vec{F}_{ji}$ for any referential frame, even non-inertial. The force of Weber is also directly related with Coulomb's force,[125] which is essentially a particular case of Weber's when the charges are at rest, relative to each other, i.e., when $\dot{r}_{ij} = \ddot{r}_{ij} = 0$.[126] The constant c is the ratio of the electrostatic and electromagnetic units of charge (which are equal to the speed of light $3.8{\times}10^8 m/s$).[127] Weber obtained this expression in 1846, from the Fechner's hypothesis.[128]

Next, Kirchhoff calculates v from equation 6.77 by integrating the effect due to all free superficial charges,

$$v(x,y,z,t) = \frac{1}{4\pi\epsilon_0}\int\int\frac{\sigma(x',y',z',t)}{|\vec{r} - \vec{r}'|}da' , \tag{6.79}$$

in which $\vec{r}$ is the point where the potential is being considered, and σ is the surface density of charge. When integrating this equation for a wire of length ℓ and radius α, without specifying σ, but imposing only that $\alpha \leq \ell$, in addition to neglecting the curvature effects of the wire, Kirchhoff gets to the impressive result for the electric potential,

$$v(s,t) = \frac{\alpha\sigma(s,t)}{\epsilon_0}\ln\left(\frac{\ell}{\alpha}\right) , \tag{6.80}$$

in which s is a variable related to the curvature of the wire (Fig. 41):

The vector potential $\vec{A}$ was also obtained from the equation 6.78, while integrating the component dependent on the acceleration of charges $-q\partial\vec{A}/\partial t$ along the volume of the wire,[129]

$$\vec{A}(s,t) = \frac{\mu_0}{2\pi}I(s,t)\left(\ln\frac{\ell}{\alpha}\right)\hat{s} , \tag{6.81}$$

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[124] See Assis 1992, p. 42.

[125] See eq. 5.1, p. 105, on chapter 5.

[126] The Weber's force is also related with Ampère's force, which can be obtained from the expression 6.78. See Assis and Chaib 2011.

[127] See Weber 2021.

[128] To the steps taken by Weber in the develpment of his equation, see Whittaker 1910, p. 200-211.

[129] The procedure is similar to what is done today, when we want to calculate the potential from the electric field $\vec{E} = -\nabla v$, with $\vec{F} = q_0\vec{E}$.

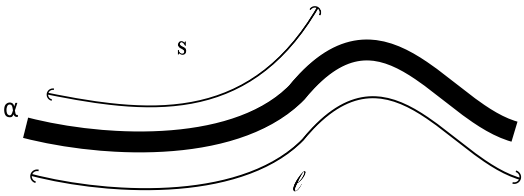

Figure 41 – Kirchhoff's curved telegraph wire (1857), with length l, radius of curvature α and a variable distance s. Adapted from Assis and Hernandes 2009, p. 224.

in which $I(s,t)$ is the current. Considering $I(s,t) = J\pi\alpha^2$ and $R = \frac{\ell}{\pi\kappa\alpha^2}$ as the electrical resistance of the conductor, he wrote the component of the Ohm's law along the length of the wire, as

$$\frac{\partial\sigma}{\partial s} + \frac{1}{2\pi\alpha}\frac{1}{c^2}\frac{\partial I}{\partial t} = -\frac{\epsilon_0 R}{\alpha\ell\ln(\ell/\alpha)}I\,. \tag{6.82}$$

To write a single expression relating the variables σ and I, Kirchhoff used the continuity equation, i.e., the equation of the conservation of charges,[130]

$$\frac{\partial I}{\partial s} = -2\pi\alpha\frac{\partial\sigma}{\partial t}\,. \tag{6.83}$$

Finally, using the equations 6.82 and 6.83, Kirchhoff was able to find:

$$\frac{\partial^2 u}{\partial s^2} = \frac{2\pi\epsilon_0 R}{\ell\ln(\ell/\alpha)}\frac{\partial u}{\partial t} + \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}\,, \tag{6.84}$$

which represents the displacement u of a positive particle in the point s. In other words, this relation is nothing less than the telegrapher's equation, geometrically identical to equation (6.64). Here, u can represent I, σ, v, or even the longitudinal component along the wire $\vec{A}$. If the resistance of the wire is neglected, as also remarked, we have the possibility of waves propagating with light speed in vacuum.[131]

Throughout the 1880s and 1890s Heaviside would end up being recognized,[132] after the efforts of William Thomson[133] – then sacred *Lord Kelvin* for his achievements with underwater telegraphy – as one of the main actors of the new role of mathematics in the place of Engineering, especially after Hertz's confirmation of the electric waves in 1893,[134] then interpreted as a confirmation on the validity of Maxwell's theory.[135] Although Heaviside claimed to be unaware of Weber's approach to this specific case until 1876,[136] it

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[130] See 103, in p. 103.

[131] A similar equation was obtained by Weber in the paper of 1864. See Weber 2021, p. 35; The conection with the heat equation, where Thomson's equation of the electric telegraph submarine is analyzed, can be found in Weber 2021, p. 76.

[132] Heaviside 1892, 1894.

[133] See Thomson 1889.

[134] Hertz 1893.

[135] Jordan 1982.

[136] See the coments by Heaviside in the footnote of the last page (p. 61) of his paper *On the extra current*, in Heaviside 1876; See also the comments on Weber's approach, in Heaviside 1894, p. 81.

is possible that Weber's and Kirchhoff's derivation, being supported by another epistemic pillar, was in part neglected because the Maxwellians – which had in Heaviside one of their prominent members – sought to break with this tradition after the reformulation of the principle of action at a distance (AAD) initiated by Faraday, Thomson,[137] and carried out by Maxwell.[138] Maxwell had occasion to claim that his equations were fully compatible with Weber's theory and probably would not have as much conviction in the supremacy of his methods as his followers,[139] but would not live long enough to see the unfolding of his efforts.[140]

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[137] The establishment of the classical field theory of electromagnetism by the hands of Maxwell's followers can be found in Hunt 2015.

[138] There is also a "geopolitical explanation", which is related to the economic and military role of the British empire at the world scenario in the 19th century, which dominated not only the seas but also the ocean floor after the submarine telegraphy. This would obviously reflex in the eventual acceptance of the Maxwellian field theory – or the theory of Maxwell by the Maxwellians – to the detriment of continental school of action at a distance, which would not be properly recognized with the pioneering of telegraphy, from the technical questions to the "theoretical choices" to be carried on. Naturally, a "British theory" would be preferable to most part of British scientists, regardless the quality of a "foreign" or "peripheral theory".

[139] See in Maxwell 1890.

[140] Maxwell died prematurely from cancer at age 48, in 1879, with the same type of illness and age as his mother, Frances Hodson Cay. See in Waterston and Macmillan Shearer 2006.

7 Epilogue

Ponderability of matter, ether, heat, electricity and magnetism were fundamental questions for most part of the 19th century natural philosophers. The general expectation was that, among such conceptions, a physics of flow could provide elements to address even greater problems, such as a general dynamics for the ether, where the principle of mutual conversion into different types of energy was rooted in the idea of continuity, "... a law that should permeate Nature",[1] as Stokes had the opportunity to stress. Historically, the tendency to provide explanations based on action-by-contact or medium-dependent theories, in opposition to the action at a distance heuristics, is often associated to a process of depersonalisation of nature, which reflected the desire to eliminate the organic analogies in favor to mechanical ones, once the major part of mechanical devices in 19th century physics acted by contact. However, the contradictory element between the ideas of mediated action and action at a distance, which continually alternate throughout the history of science, is only apparent: atomism and continuity, idealism and materialism, mechanism and vitalism, randomness and determinism, causality and randomness, matter and spirit often changed sides throughout history.

The British physics approach in the 19th century, unlike the continental school, differed mainly by the indiscriminate use of unobservable entities, such as elastic lines of force or any other cognitive resource. Hence the idea that the use of mechanical analogies would be mere suggested tools for the lack of concrete evidence. Indeed, we look at the scientific compendiums of that time, it is not uncommon to feel walking in a real *factory* of Duhem's metaphor: iron bars, wooden pulleys, inertia wheels, smoke rings, tubes filled with viscous fluids as jam or glycerine and elastic entities like springs, strings or gelatin. The etheric element in this context could assume different formulations and contradictory properties, on demand. Like a gelatin, it could support transverse waves of light. As an elastic fluid, it could allow planets to navigate smoothly through the sidereal ocean, as in Stokes words, "... it seems to me that we can attach no idea to the motion of heat without entering into some speculation as to the nature of heat",[2] an entity that explained almost everything, and almost nothing.

The classic tradition of philosophy, on which much of the Victorian science was based on, seemed to suggest that macroscopic models of nature carried with it conjectures about the imponderable nature of certain elements. However, in a similar fashion, the eighteenth-century models of attraction and repulsion, inherited from a particular reading of Newton's *Principia* and carried forward by part of the continental school, were not so

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[1] Stokes 1880, p. 78.
[2] Stokes 1990a, p. 151.

far from the speculative field, as well. This can be verified by a certain rupture carried out by the analytic school, which had in Fourier one of its most prominent members, and which sought to replace a physical problem by descriptive equations, in a radically anti-hypothetical methodology. Geometric analogies played an important role until then, as they were believed to be the best way to verify the veracity of a physical theory based on empirical observation, with the notion that differential equations could represent almost all physical phenomena. Previous theories, which focused strictly on direct observation of processes, subtly began to seek a legitimacy of geometric structures themselves, in order to correspond to the experimental results of a dreamed general dynamics. This movement occurred in British physics, but not in the continental school, which continued to search for general principles based on simple configurations of forces and distances.

In a previous research, a decade ago, we studied a certain class of mathematical tools that, at least in part, were an attempt to answer the question of "what electricity might be", in a broader sense of the term.[3] One of these tools, the *Operational Calculus* (1892), which is now part of the techniques of the *Laplace transform*, emerged as an attempt to solve a very special equation: first obtained by Kirchhoff and Weber in 1857 using the heuristics of action at a distance, and re-worked years later by Heaviside upon the ideas of Maxwell as a medium-dependent theory, the *Telegrapher's Equation* had the ability to describe with precision the propagation of electromagnetic signals in a telegraphic wire. Kirrchhoff and Weber took into account the discoveries of Øersted (1820) and Faraday (1831) to deduce the equation, considering that Weber had already developed the first operational telegraphic system, with Gauss, in 1833. Oliver Heaviside obtained his own version of the equation (1876), using as a starting point the previous work of William Thomson (1854), which, by his turn, adapted the *heat diffusion equation* of Fourier (1822) for the flow of electricity in the project of the transatlantic cable, to be launched in 1857.

The contradictions generated by the telegrapher's equation is an alternative that could be addressed in depth, in a future research. That is, could the ability to explain the physical reality from an equation be questioned, given that different epistemic conceptions gave rise to the same equation? That is, two antagonistic conceptions such as the models of action at a distance and continuous action could reach similar results, similar equations and predictions. What can these results tell us about the "thing" of interest? The formulation of the telegrapher's equation would contribute to the adoption of self-induction as one of the fundamental parameters of the transmission line,[4] predicted the existence of electromagnetic waves – so that a signal could travel without distortions with the speed of light – making possible the transmission of voice signals: the telephone would be the new dream to catch up.

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[3] See Tonidandel 2011; See also Tonidandel and Araújo 2017.

[4] To a good introduction to the subject, see Jordan 1982.

Heaviside was part of a very selected (or privileged?) group of scientists who believed that a certain group of equations could be a good answer to the nature of electricity and magnetism, an obvious path to the development of a new branch of knowledge: they were part of a movement that would make physics extremely successful, on a broad spectrum, forming the conceptual foundations of the contemporary Electrical Engineering. This would obviously reflex in the eventual acceptance of the Maxwellian theory to the detriment of continental school of action at a distance, which would not be properly recognized with the pioneering of telegraphy, from the technical questions to the theoretical "choices" to be carried on. Naturally, a "British theory" was preferable to the British scientists, some of them part of the all-mighty Royal Society, regardless the quality of a "foreign" or "peripheral" theory, like Weber and Kirchhoff's.

This is also related to the economic and military role of the British empire at the world stage in the 19th century, which dominated not only the seas but also the ocean floor, after the submarine telegraphy. In contrast to the first optical telegraph systems in France, the electrical telegraphy in Britain was not developed by the military, but by the private sector. Initially taking advantage of the railway infrastructure, electrical telegraphy would revolutionize the overland communications along the decades of 1830-1840. However, the development of submarine telegraphy would be crucial to the imperial point of view, and it was definitely be an imperial product,[5] which promoted the first cross-channel cable in 1851 and the transatlantic cable missions during 1857-1866. When the first mission parted from the Irish coast (see Fig. 42) to reach the American side, in 27 July 1866 aboard the *Great Eastern* of Brunel, the conveyed message made clear what power would dominate the age of the telegraph.

Figure 42 – European end of the 1866 cable at *Valentia Island*, Ireland. Source: Flanagan 2015.

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[5] Ferguson 2004, p. 168.

The previous question opens up a field for studying the controversies among scientists who sought to highlight their own views on the same subject. Although not locating Maxwell's thought in his historical figure, as Heaviside himself declared, "... Maxwell was only one-half a Maxwellian",[6] the clearly preference for the ideas of Maxwell and his predecessors, such as William Thomson, and the influence of these in the establishment of the electrical engineering was a motivation, in a way, to seek their "collective imaginary", in the sense of "collective conceptions". In his debut article on electricity, Maxwell presented his peculiar interpretation of science: "... In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies".[7] Maxwell used this rule of thumb as a kind of metalanguage to develop his own mathematical language, but the method was not his invention. In a period previous to Maxwell's theories, scientists benefited from a source that goes back to a long tradition that the new physics would seek to forget.

It took almost five centuries of recent history for the humanity to discover the "obvious" fact that the continents were once united, that the coast of Brazil fit into the African coast, a puzzle that has been part of children's games since maps of the New World began to be made. Similarly, the simple experiment reported by Ørsted, in 1820,[8] confirmed the long felt relationship between electricity and magnetism, demonstrating a vivid magnetic needle in presence of a galvanic current nearby. Now, to establish a symbolic ground for the not-so-obvious beginning of Electrical Engineering it is not ignored that, institutionally, the area is a somewhat consequence of the 19th century societies of telegraph engineers, both in Europe and America,[9] but these are more natural consequences of the professionalization of a segment, than a landmark for a new branch of knowledge, properly.

Some of the usual practices common to the area of Electrical Engineering already existed before telegraphy, but they were certainly intensified after its advent, in the 1830s. Electrical Engineering is a confluence of techniques and theories related to the management of electricity, as well as the professionalization of the science of electricity and magnetism themselves, which was not dissociated from Natural Philosophy until then. Professionals in this field would stop being mere electricians – i.e., hobbyists who had an interest in the subject or professionals from other areas, such as civil and mechanical engineers, who worked on some aspect of telegraph lines, whether aerial lines, highways, or submarine cables – to become authentic dedicated professionals. However, the area as a

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[6] Statement used by Hunt in his excelent book. See Hunt 1991, 2015.

[7] Maxwell 1855.

[8] Ørsted 1820; See the Portuguese translation in Ørsted 1986.

[9] For example, the British Society of Telegraph Engineers (STE), formed in 1872, would become the Institute of Electrical Engineers (IEE) in the late 1880's (See, e.g., Thomson 1889 On the other side of the Atlantic ocean, the American Institute of Electrical Engineers (AIEE, of 1884), merging with the 1912 Institute of Radio Engineers (IRE) would form, in 1963, the Institute of Electrical and Electronics Engineers (see IEEE 2021), currently the world's largest professional technical organization. However, the main subject addressed in this thesis is far from being a "union's issue".

whole professionalized with submarine telegraphy and, in particular, with the project of the transatlantic cable, which involved the cooperation of hundreds of professionals, from all areas.

Several engineering areas would gain maturity with telegraphy as a whole, and some of which worth to be mentioned: oceanography, which was created to probe the ocean floor, metallurgy, researching metallic materials such as copper alloys, which needed a high degree of purity, in a rapidly growing scale of production; mechanics, materials science, especially developing techniques of wrapping, covering and launching underwater or aerial lines; naval industry, given the need to build larger and larger ships, and many others. Along the 1850s, a significant number in submarine cables had been launched by several nations, which highlights the relevance of underwater telegraphy in the world economic scenario (Fig. 43). At the beginning of the twentieth-century, a vast network of underwater telegraph cables would connect the continents (Fig. 44), shortening distances and, in a way, shrinking the world. For these and other reasons, one cannot ignore the influence of electrical telegraphy in the world economy, a fact that reinforces the hypothesis that the advent of telegraphy can be seen as a metaphor for the establishment of Electrical Engineering.

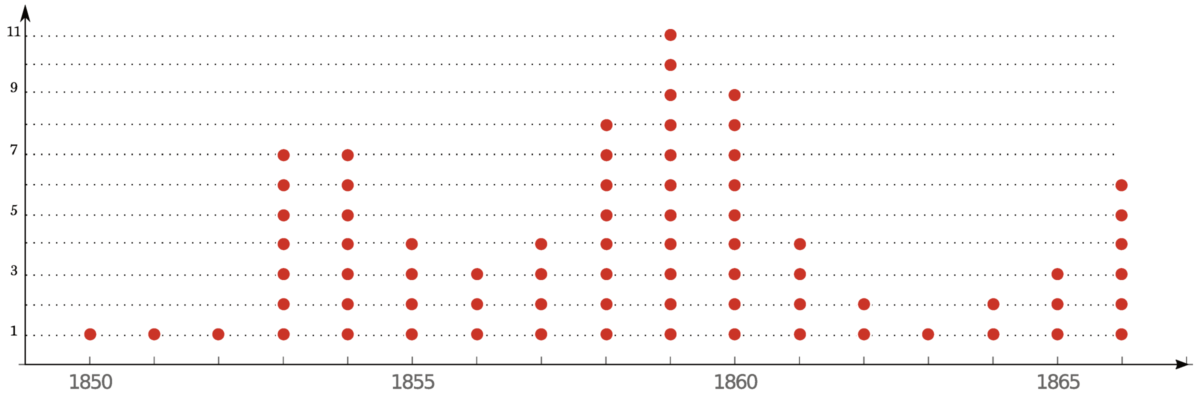

Figure 43 – Submarine cables launched by the Great Britain along the years 1850-1867. Source: Lardner and Bright 1867.

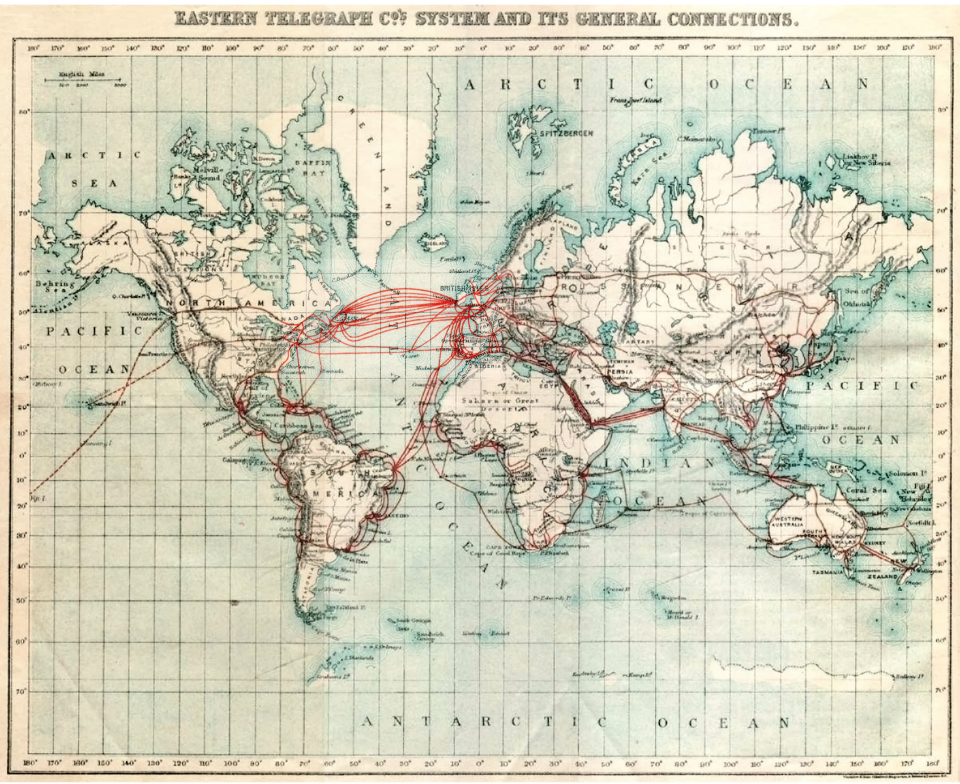

Figure 44 – The world wide web of telegraphy, in 1901. Source: IEEE History Center.

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Appendix

APPENDIX A – Telegraphic Machinery

This section presents a glimpse of the fundamental machinery and devices developed by the telegraph industry – that do not appear in the text – in vectorized images, in public domain.

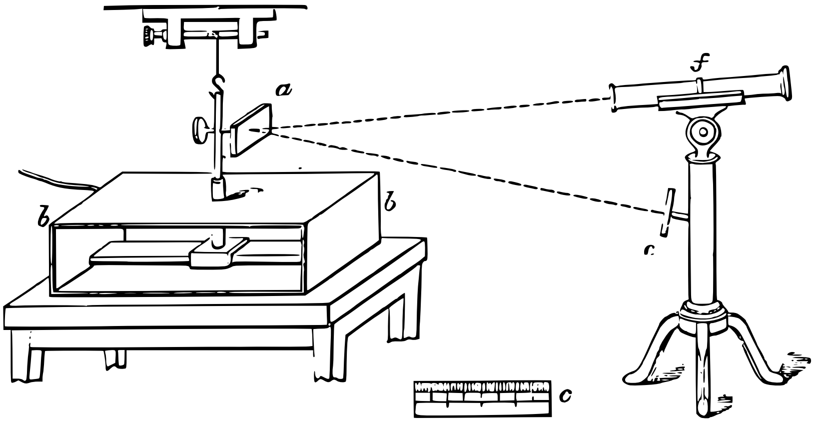


Figure 45 – The Gauss-Weber telegraph of 1833.

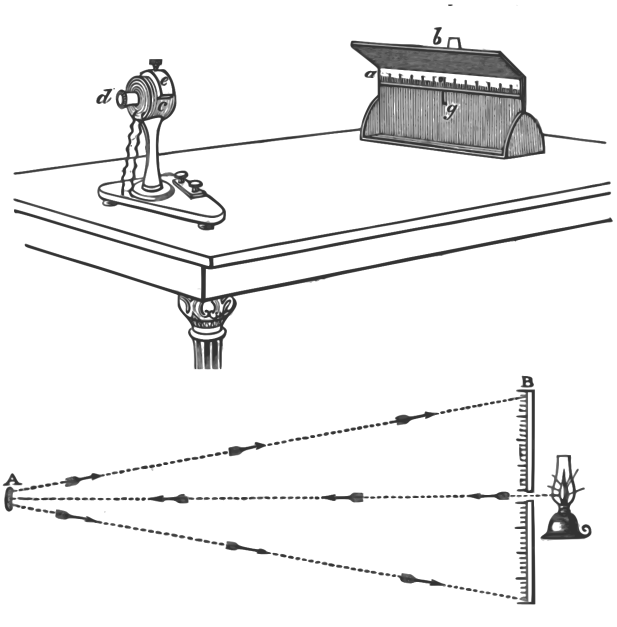
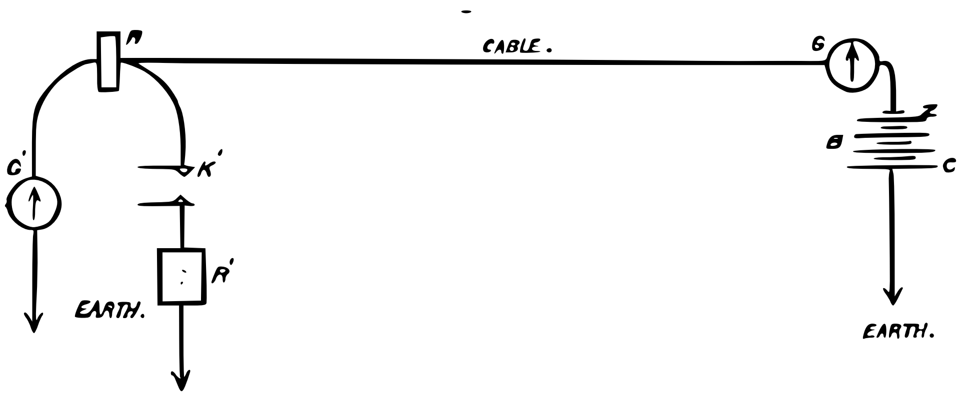


Figure 46 – Thomson's mirror galvanometer, developed to detect minute variations of current in a telegraph wire.



B, Battery on board ship.
G, Thomson's galvanometer, through which the battery is connected to the cable.
G', Another galvanometer on land in connection with resistance R.
K, A contact key by which the current from the cable can be turned through a comparatively low resistance R'.

Figure 47 – Scheme of Thomson's mirror galvanometer.

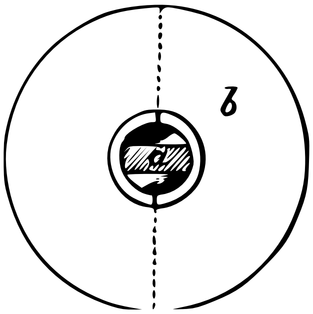


Figure 48 – The atlantic mirror galvanometer coil, with a mirror and magnetic needle.

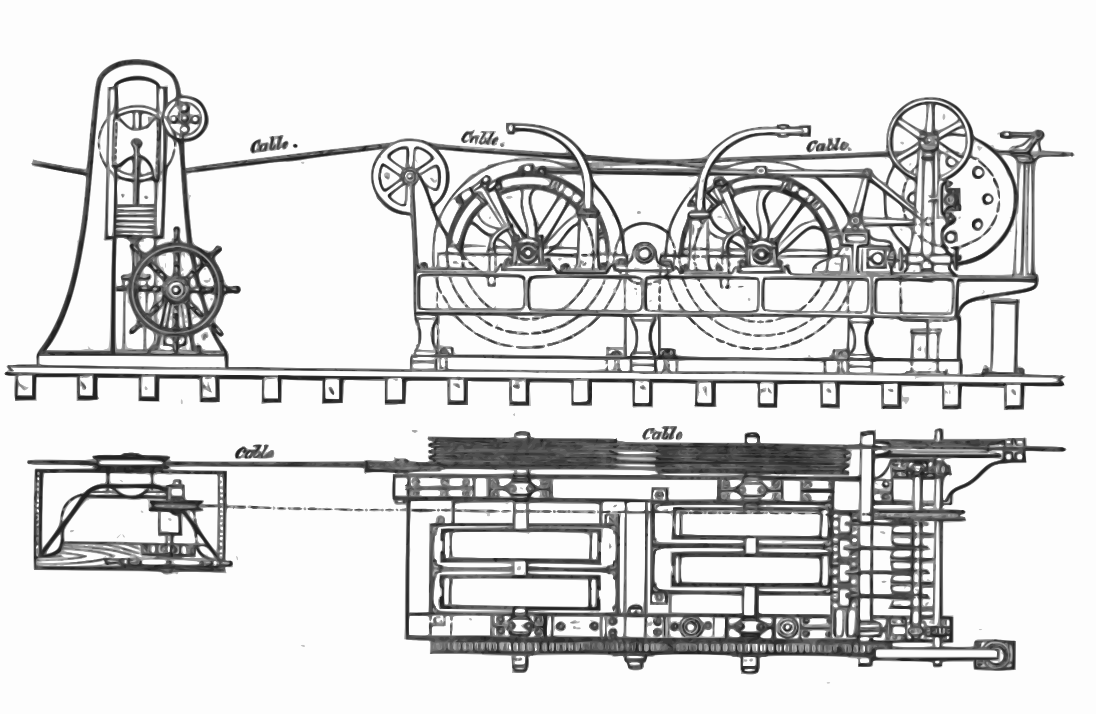


Figure 49 – The Submarine cable paying-out machine.

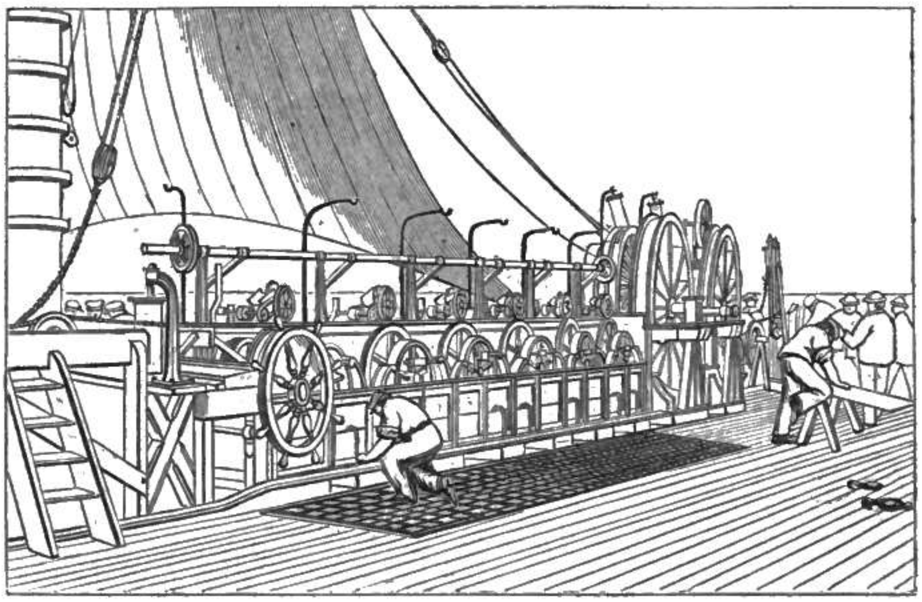


Figure 50 – The paying-out machine of the 1856 cable.

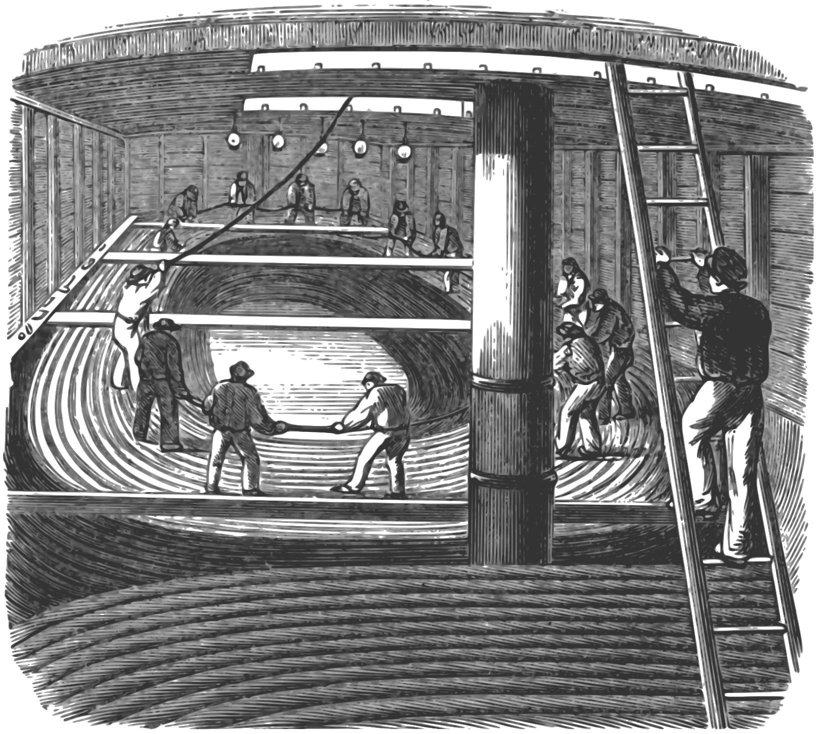


Figure 51 – Ancestral method of winding submarine cables on a ship.

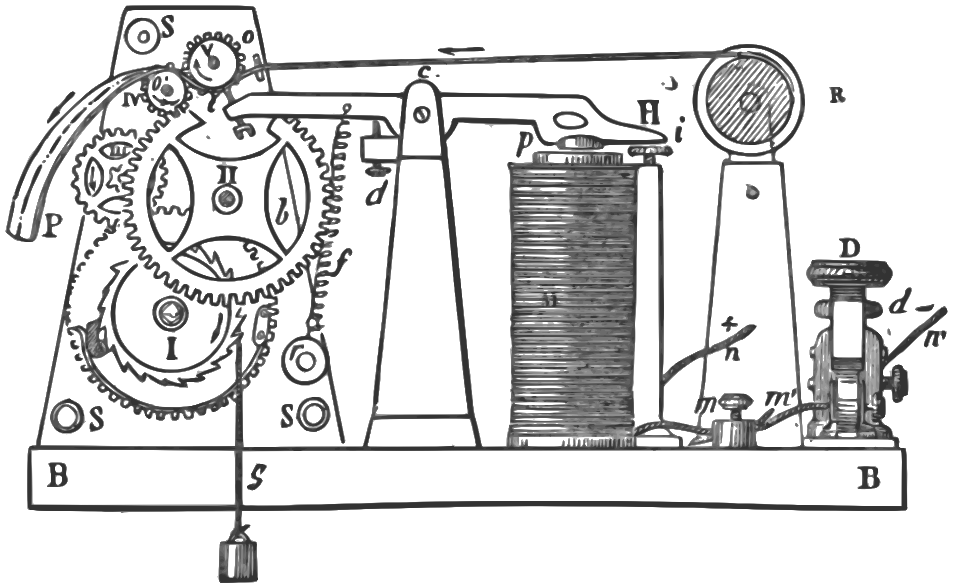


Figure 52 – Samuel Morse's recording apparatus.

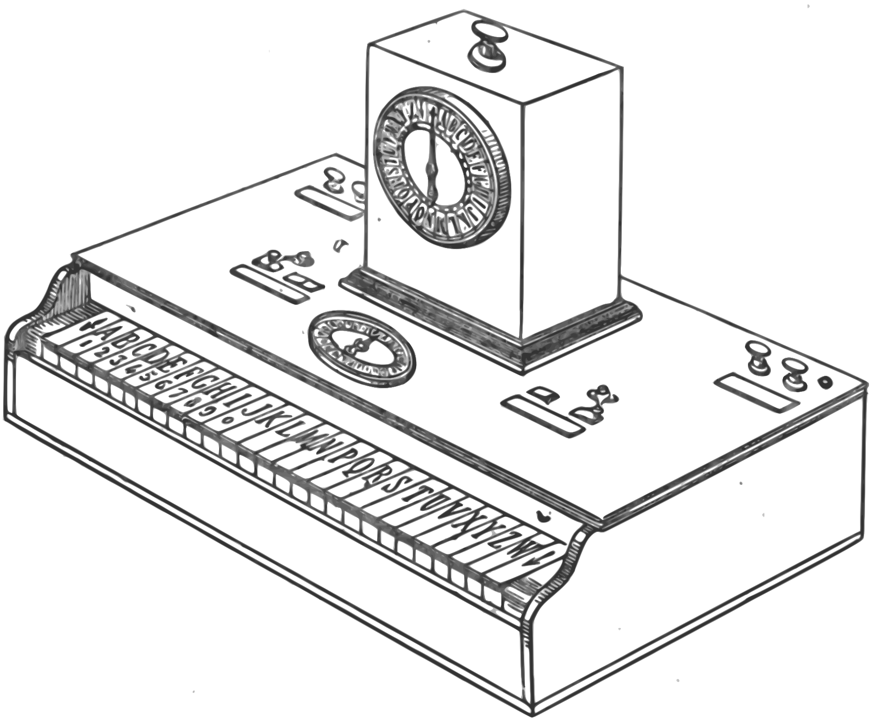


Figure 53 – Froment's alphabetical telegraph, with the keyboard in the shape of a piano.

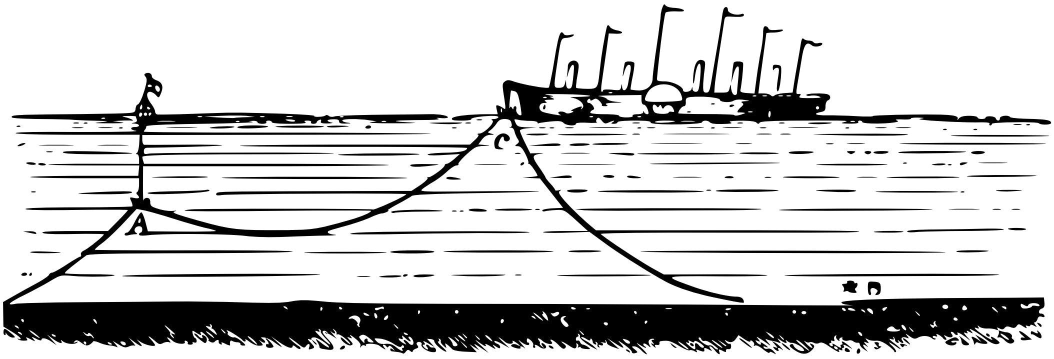


Figure 54 – Fishing the cable of 1865 with the Great Eastern.

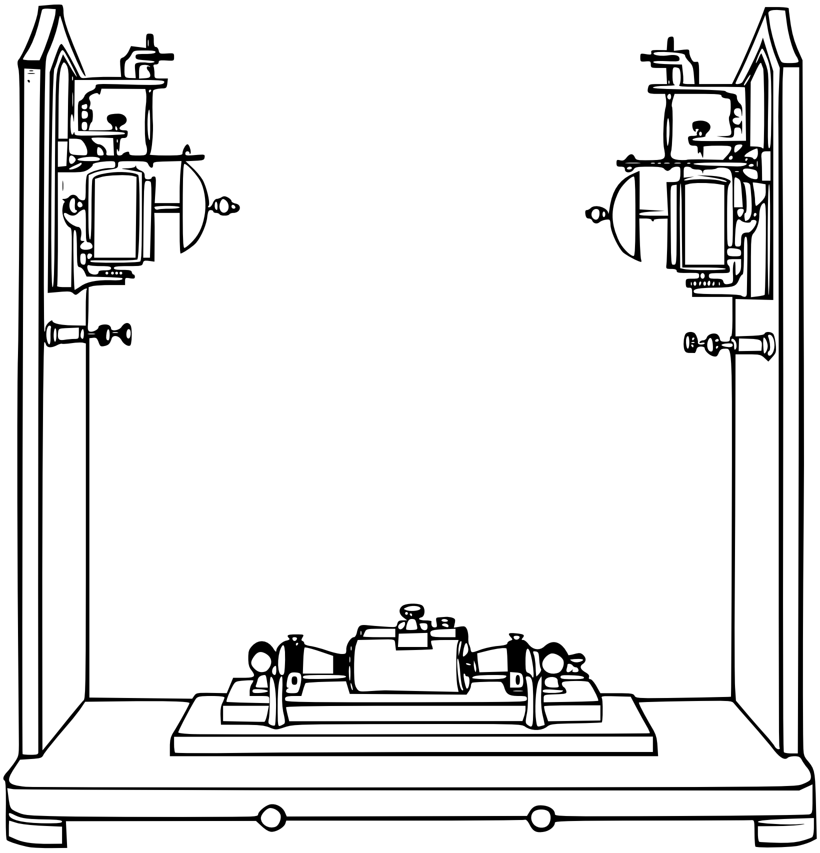


Figure 55 – Bright's acoustic telegraph.

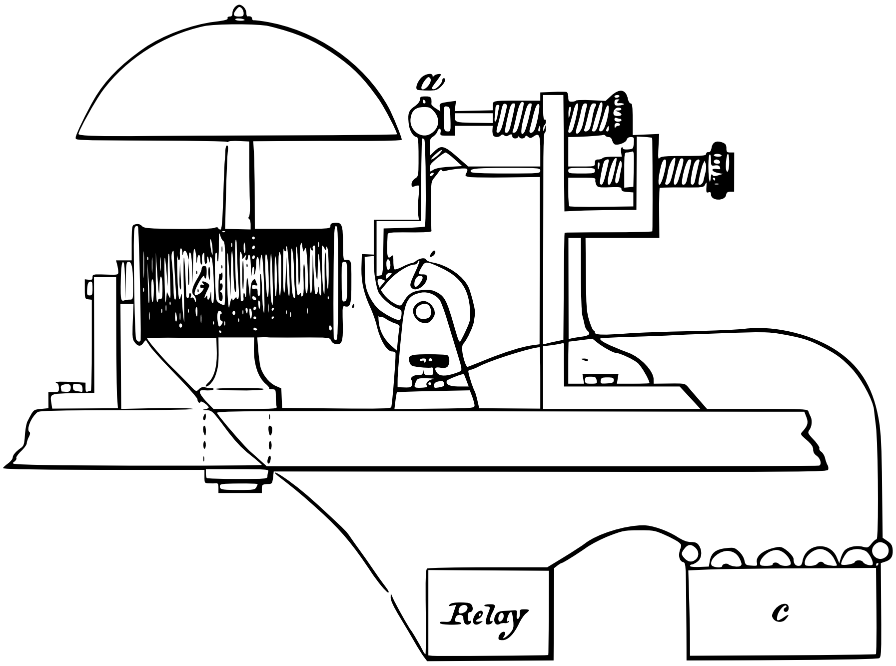


Figure 56 – Scheme of Bright's acoustic telegraph.

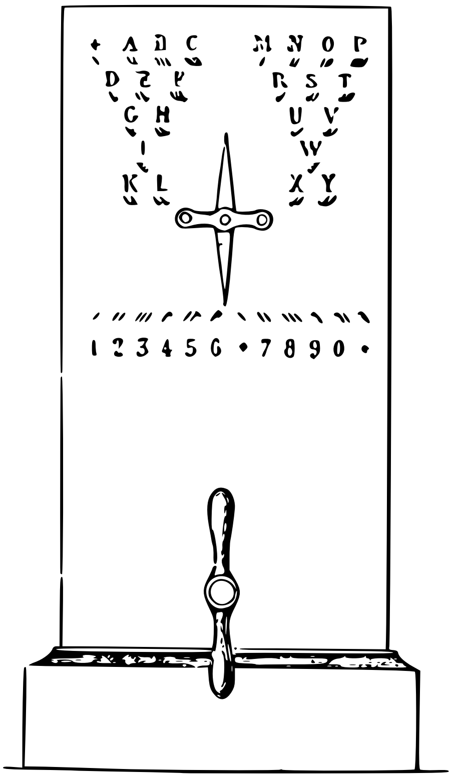


Figure 57 – Single needle telegraph of Cooke and Wheatstone.

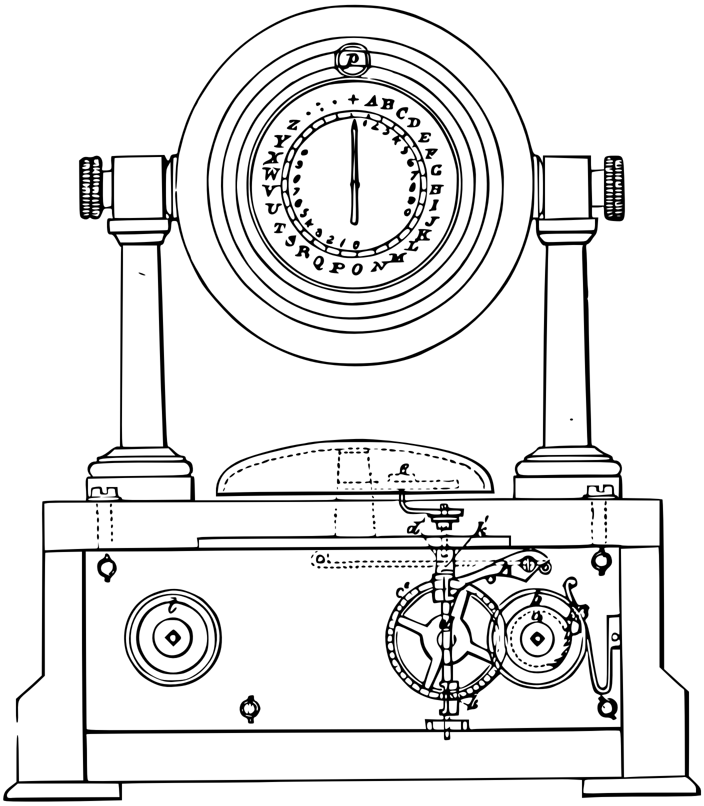


Figure 58 – Wheatstone’s receiving apparatus and alarm.

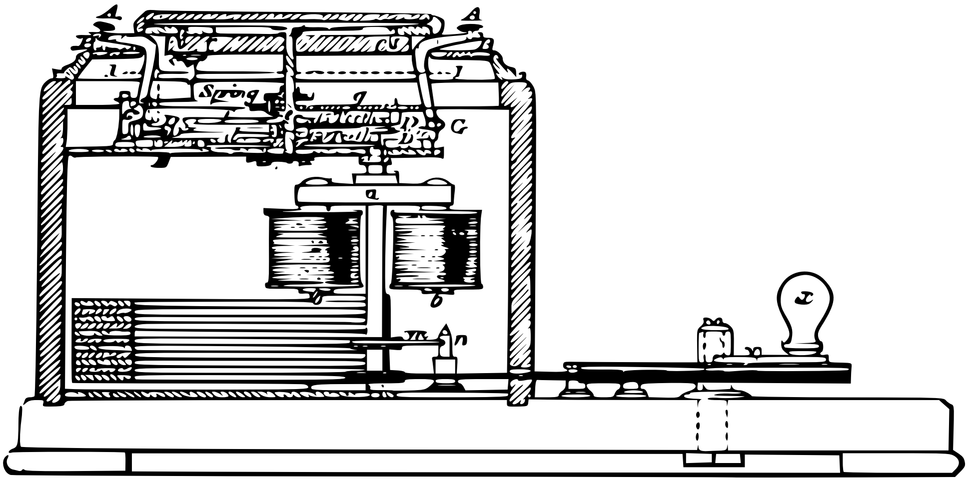


Figure 59 – Wheatstone’s sending apparatus.

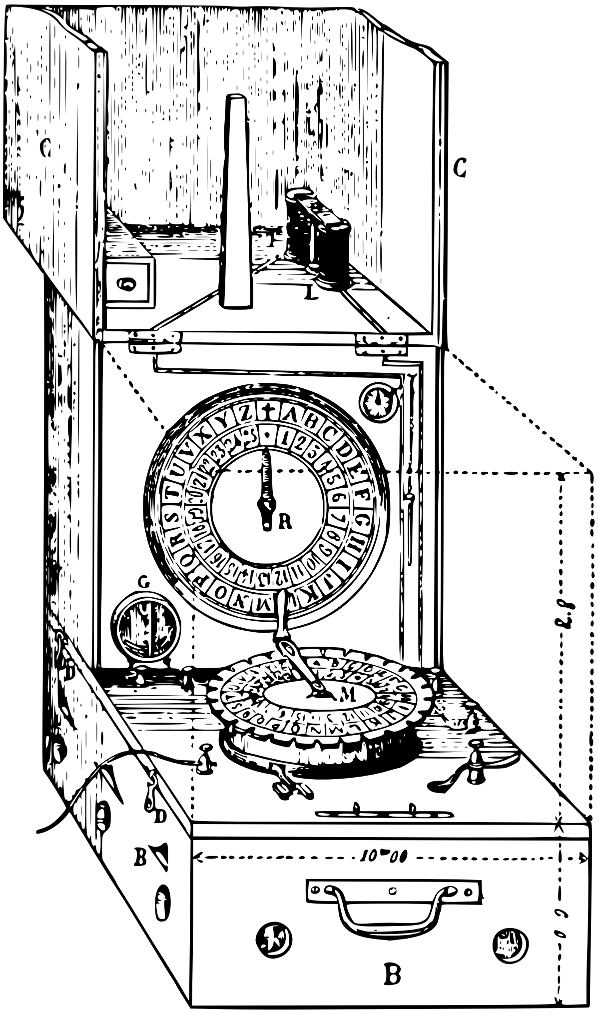


Figure 60 – Breguet’s portable alphabetical telegraph.

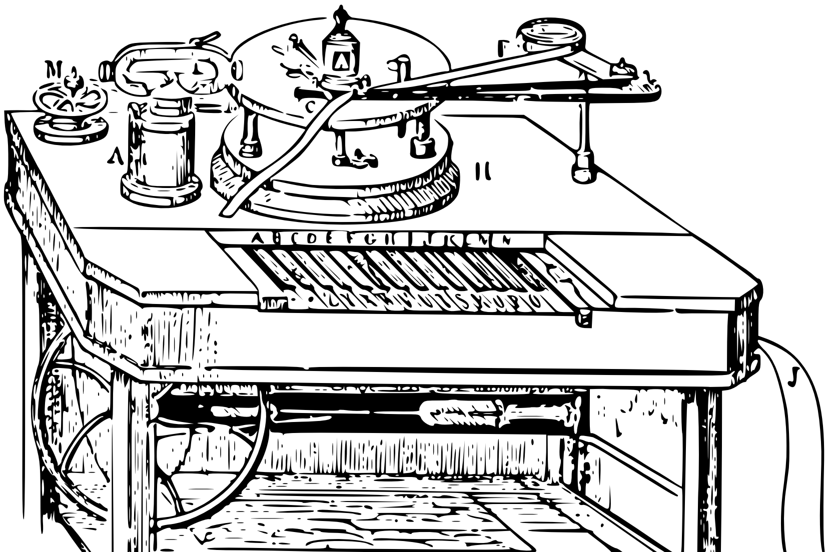


Figure 61 – House’s printing telegraph.

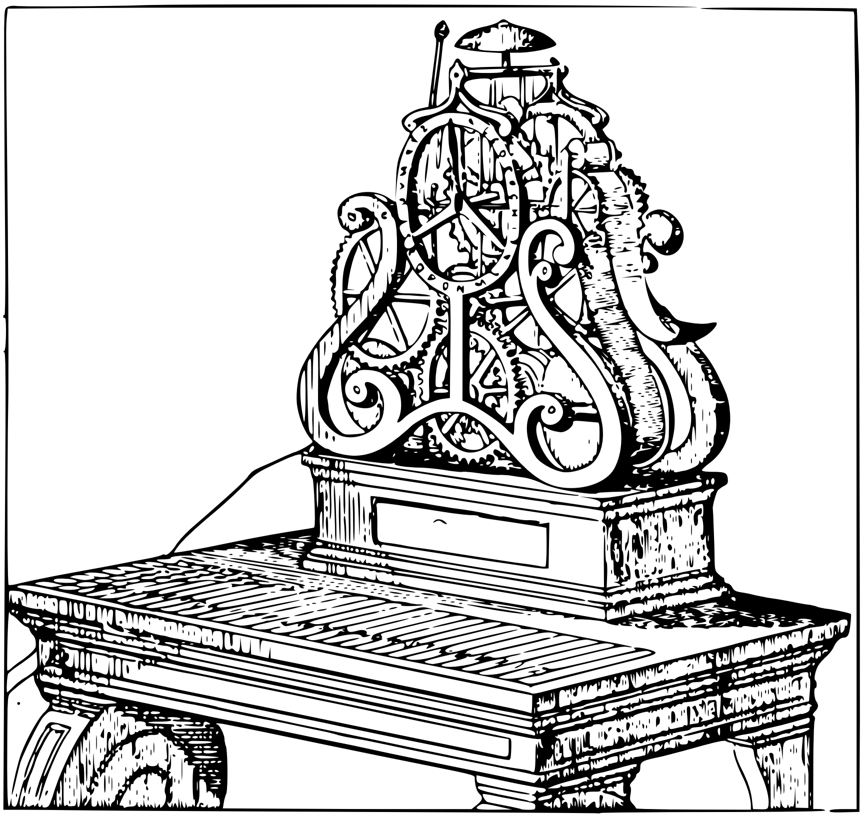


Figure 62 – House's and Brett's type printer.

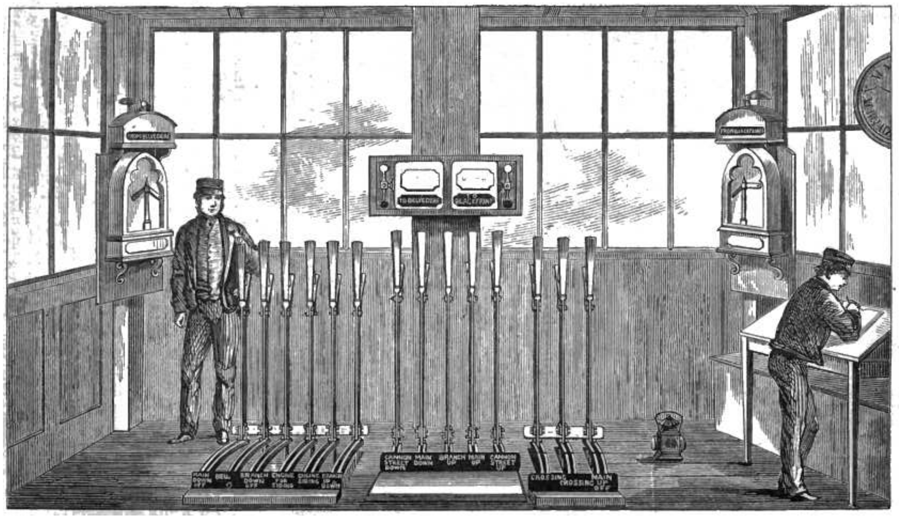


Figure 63 – A signal box.

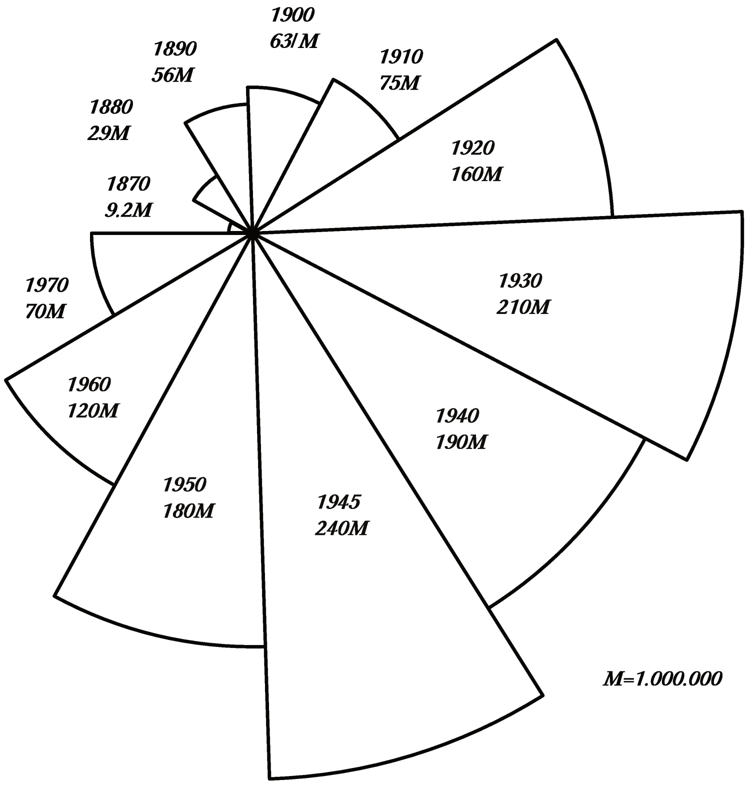


Figure 64 – Messages sent by the Western Telegraph Union between 1870-1970. It is possible to observe a rapid growth in communications at the end of the 19th and a decline from 1930 onwards, except for the peak during World War II, when radio and telephone became popular, returning to the levels of the previous century. The infographic's peculiar format is inspired by the work of Florence Nightingale, one of the pioneers of statistics, who during the Crimean War (1854) sought the causes of the high mortality rates among wounded combatants.

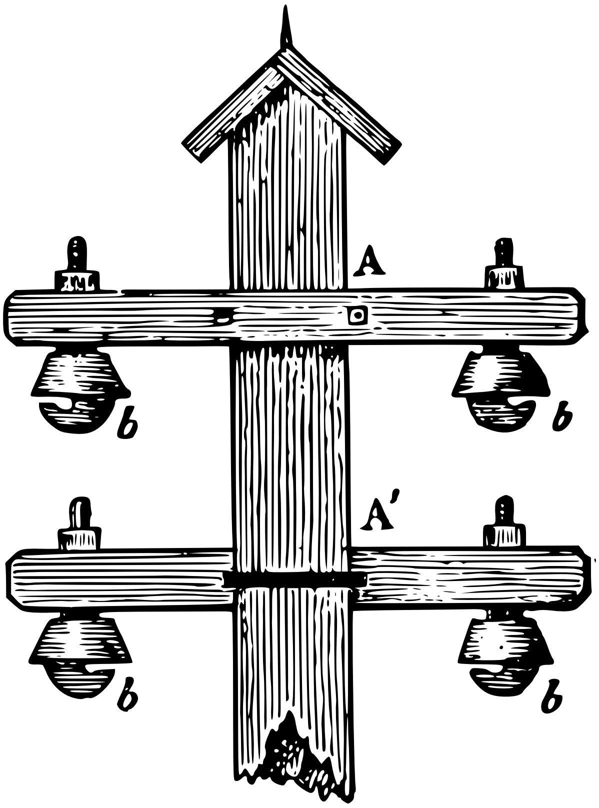


Figure 65 – An aerial telegraphic post.

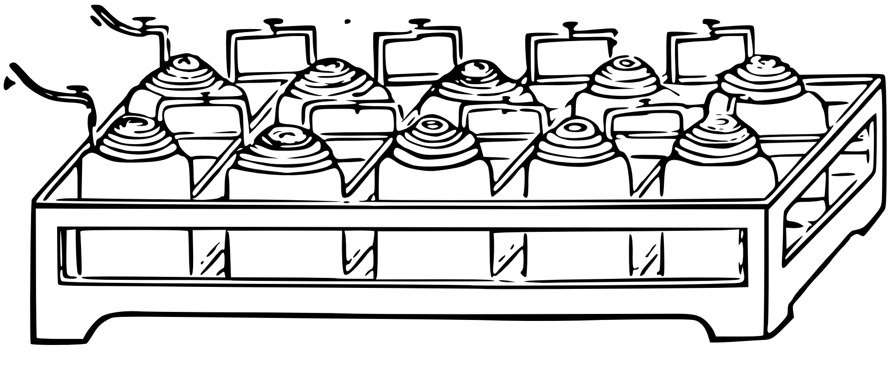


Figure 66 – The Daniel's battery.

Figure 67 – Cookes’ first telegraph.

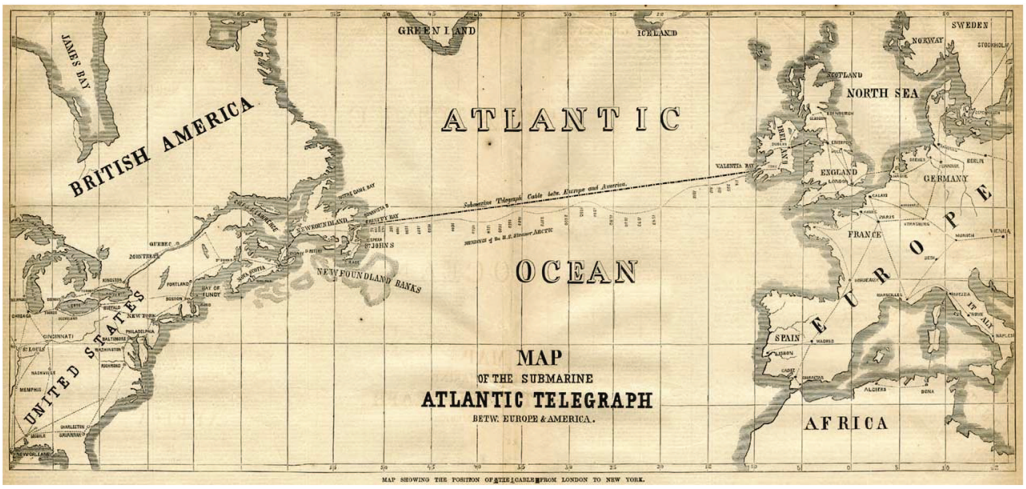


Figure 68 – Map of the transatlantic cable of 1865.

APPENDIX B – Lork Kelvin: the Victorian dynamo

Who was William Thomson?[1] Although his life and work have already been profoundly written in countless writings, it is opportune to establish a brief commentary on his life and education, which will provide elements to contextualize his avid adherence to the problem of underwater telegraphy.

From a very young age, William Thomson already had a great aptitude and interest for science subjects. Son of a respected mathematician, James Thomson, who had been appointed professor for the mathematics chair at the University of Glasgow in 1832, little William, then 10, with his brother James, aged 11, informally attends his father's classes. Before long, meeting the legal deadlines for admission to higher education centers, the brothers would formally enroll at the University of Glasgow,[2] acquiring conditions for graduation, and even winning the first prizes in mathematics, astronomy and natural philosophy of their lives. Like William, his Brother James would, in the future, become a professor of Engineering and also a member of the Royal Society of London.

At 16 William is officially admitted as a student at the University of Cambridge, transferring there even before receiving his degree in Glasgow, where he had spent the last 6 years. Before, however, the definitive change, he had the opportunity to know, under the guidance of the substitute professor of Natural Philosophy, J. P. Nichol, the *Thèorie Analytique de la Chaleur*', by Joseph Fourier, being absolutely fascinated! Years later, Thomson would claim that Nichol, in addition to his own father, had been a major influence in his life, especially since he introduced him, in addition to Fourier, to other members of the French school, such as Lagrange's *Mécanique Analytique* and Laplace's *Mécanique Celeste*. The French tradition stood out for the importance of differential and integral calculus in the solution of physical problems. He would even claim, perhaps with some exaggeration, that he had devoured Fourier's book in one night, on a trip. It is certain that the analytical theory of heat would have significant influence throughout the course of his career: Fourier was his idol![3]

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[1] To a summary on Thomson's life and work, see Lloyd 2007.

[2] By the way, this was not uncommon, once Scotland's third largest city, Glasgow, had a strong tradition in the educational field. By the end of the eighteenth century, more than half of the university's students were the children of local merchants or industrialists, which today would be called the "upper-middle class", compared to just under 10% of them, at Cambridge University. Thompson 1910.

[3] And not only William was the only fan of the Baron of Fourier. A few years later, his brother James, who became a professor of mechanical engineering, would create a mechanical device that would make a Fourier series! See Thomson 1882, p. 272.

In his final year as a student in Glasgow (1840-1841), Thomson published the first and bold article in the Cambridge Mathematics journal, under the pseudonym *PQR*, entitled *About Fourier expansions in trigonometric series*,[4] in which he defends the Fourier method as opposed to the claims of a Cambridge professor, Kelland, who had pointed out possible inaccuracies in Fourier's work. In the following year, at the age of 17, Thomson published his first original work, *On the uniform movement of heat in homogeneous solid bodies, and its connection with the mathematical theory of electricity*,[5] in which he adapts the theory from the Gallic master to the problem of electrostatics, based on the works of Green and Poison. This article, moreover, in which it suggests for the first time an analogy between heat and electricity. After a brief stint as a mathematics tutor at Cambridge, already graduated, William applied for the chair of Natural Philosophy at his home university, after the death of ex-professor Meikleham in 1846. He would return to teach alongside his father, prof. James Thomson, who continued to lead the Mathematics chair until his death in 1849. Thomson was only 22 years old. His presentation for the professorship vacancy was entitled*De motu caloris per Terra Corpus* or, *The movement of heat through the terrestrial body.* Thomson would hold the position until his retirement in 1899 at the age of 75.

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[4] Thomson 1882.

[5] Thomson 1842.

APPENDIX C – Publications

Below is a list of selected publications, placed entirely from 2021 until 2017, along the doctorate period, including the oldest, and from previous periods, dating back to 2010, selected for informational purposes. The main objective of this list is to provide an overview of the line of research developed over the past few years. There are also two papers that are in the process of submission, in 2021.

2020-2021

The present set of two papers, in process of submission, aims to answer the following question: what type of imaginative conceptions had the Victorian scientists, at disposal, to build the science of electricity and magnetism? What traditions have they benefited from? One possible answer is summarized in two parts:

- Maxwell's mirrors - part I: physical analogies in the age of organism
- Maxwell's mirrors - part II: physical analogies in the age of mechanism.

Maxwell, Thomson and other Victorian scientists often used the language of "physical analogies" as a tool for modeling and investigation, especially concerning the science of electricity and magnetism, considered by them as an important tool. That question made us imagine if Maxwell had his mirrors turned to a more distant past, silently forgotten

Maxwell's mirrors - part I: physical analogies in the age of organism

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Abstract

The advent of the electrical science, in all its specificity, helped to lay the foundations for the techno-scientific revolution that would have taken place in the second half of the 19th century, a period in which the still nonexistent branch of electrical engineering was not dissociated from Natural Philosophy. Way before the emergence of electrical telegraphy as a symbolic landmark for this applied branch of science, analogies played a fundamental role in formation of some basic concepts, starting from the ancestral idea of flow and continuity, to the absorption of mathematical language in its framework. Although Maxwell and other Victorian thinkers used the so called "physical analogies" as a shortcut to modeling and investigation, it is possible to found an early use of these methods, especially when we try to investigate the history of analogies related to the idea of electrical (or another type of) action. Although an exhaustive study is not the object of writing, we aim to take, in a couple of papers, a closer look at important analogies in history of electricity, in frontier between Science and Philosophy. In part I, specifically, the focus is on what we call "organic analogies", from antiquity to the dawn of modernity.

Keywords: History of Science, Philosophy of Science, Physics, Epistemology, Analogy, Model

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1. Introduction

In his *Meditations on a Hobby Horse (1963)*, the art Historian Ernst Gombrich (1909-1936) narrates an old music hall joke that described a drunkard who used to lift his hat, in a gentle way, to every lamp-post he passed by, raising the question "... should we say that the liquor has so increased his power of abstraction that he is now able to isolate the formal quality of uprightness from both lamp-post and the human figure?"(Gombrich, 1985) Indeed, the own idea of abstraction as a mental act can take us on curious and strangely absurd paths, notably when tracing the role of models in science, keeping in mind that modeling is a clever way that humans conceive to supply an absent sense, like the sculptor who "abstracts" the shape under the chisel.

This is also true if the entity under the loupe resembles something elusive, like the slippering sand or the lightning that cuts across the sky. What would be a good model for electricity? If we ask an educated person what electricity is, paraphrasing Bachelard,Bachelard (1992) it will be possible to receive vague or strict answers, repeating unconsciously older theories that do not move far from common sense. If the same is posed to a scientist, it will be no surprise to find the same sort of answers, at least initially, although science is often thought of as a separate department from Nature, a exclusively logical and deductive one, and the "scientist" as a hermit inside the laboratory, with intricate equations in the blackboard.

James Clerk Maxwell (1831 - 1879) tried to provide an answer for this question and, in fact, was one of the few influential thinkers to provide an explicit analysis of the scientific method. His ideas have been extensively commented over the past 150 years as support for deductive or inductive methods of scientific construction, under the most diverse approaches.(See, e.g. Cat, 2001; Hon & Goldstein, 2012; Humphreys, 2019; Kargon, 1969; Silva, 2006) Nevertheless, it is not often noticed that Maxwell himself claimed that his conceptions on "physical analogies" (i.e., the inductive approach) was the authentic Newtonian way of "deducting from phenomena", without the aid of unproven hypotheses,Maxwell (1855) as it has been stressed long ago.(Cf. Hesse, 1974, p. 259) To him, analogies were part of the whole development of electrical science.

Taking into account that significant achievements in history of electricity and magnetism occurred not just in the last three centuries, it is possible that the dynamical nature of many discoveries, which also pass through analogies, are still lost in time, reason that makes us imagine if Maxwell had his "mirrors" turned to a more distant past, silently

Maxwell's mirrors - part II: physical analogies in the age of mechanism

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Abstract

Some of the first theories and technological applications for electricity and magnetism in 19th century physics were based on a kind of idealism that remounts a long period, which extends from classical antiquity to modern ages. In this respect, the so called "physical analogies" formed a kind of metalanguage for scientists like Maxwell, Thomson and Stokes, which used them as a shortcut to modeling and investigation, and as a ground to found their theories with aid of mathematical language, especially in the reformulation of the Newtonian heuristics of action at a distance (AAD). Part I focus on the idea of a "living universe", where the first descriptions of physical phenomena were based on analogies corresponding to living organisms. But the Western World had witnessed important changes after Middle Ages and the analogies would change in the same fashion. With a "less organic" universe, "more mechanical" analogies gained space in elaboration of theories, which improved "in precision" but lost in "purpose", a process that would open a pathway for the Maxwellian's rhetoric in 19th century.

Keywords: History of Science, Philosophy of Science, Epistemology, Analogy, Electrical Engineering

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1. Introduction

If it is known that science was not a modern invention, it is not often thought that such a construction frequently does not go through formal methods, but with the clever and inventive way of making analogies. The first action against the "old" philosophy with the advent of "modern" science began with the adoption of experimental control in evaluation of ancient certainties, establishing the prodromes of a new science of nature. Since Francis Bacon, the methodological foundation for validation of facts was being drawn from a confirmation bias, and the basic categories of Greek science would gradually be replaced by counterparts more suited to the new born program, after 17th century.

A "naive analogy"[1] used to describe the digestion process, as the fire that "... appropriated itself of the food",[2, §79d] for instance, automatically reassembled an ancestral conception that extrapolated the idea of substantiation with the personification of fire,[1] but from now on this type of construction should be abandoned, not because its fanciful character, but because organism-based comparisons were no longer compatible with a descriptive language such as geometry.

In this context, openly organic analogues like Gilbert's models seemed to be inextricably linked to the past, but we aim to show that this can be a misleading path. Sentences like: "... we must not admit more causes for natural things than those that are sufficient to explain appearances..." and "... mechanical causes could (not) give birth to so many regular motions..." appear to be separated by two eras, but , in fact, were pronounced by the same source. The heuristic of AAD, for instance, main target to be reformulated by Victorian physicists like Maxwell and others, had the aid of many singular and, sometimes, picturesque, analogies. In the same spirit of part I, this article is organized as a look to the dynamics of analogies that would make part of Victorian physicists' imaginary of theoretical construction.

2. The Solar Virtue

The phenomena associated to magnetism and falling bodies found difficulties to be explained in the medieval conception of nature. As for the tidal phenomenon, it was believed that the moon should exert some kind of influence by its brightness or even radiated heat, the same way the "suction power" of the sun caused evaporation. The idea was extended by writers like Thomas Aquinas, who believed that a "hidden species" emitted by the moon,

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[1] Interestingly, the strategy still finds resonance in scientific writings of relative contemporary success.[See, e.g., 3]

2018

Here we present the two articles published in 2018. The first one presents a brief history of electricity and Magnetism from some literary, philosophical and scientific narratives throughout history: from antiquity to the Middle Ages.[1] The second presents a critical reading and an analysis of the paper *On the theory of the electric telegraph*.[2]

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[1] Tonidandel, Araújo, and Boaventura 2018.

[2] Tonidandel, Boaventura, et al. 2018.

Documento
Document

A primeira teoria matemática da propagação elétrica: uma leitura estendida do artigo "Sobre a teoria do telégrafo elétrico", de William Thomson

DANNY AUGUSTO VIEIRA TONIDANDEL
Universidade Federal de Ouro Preto | UFOP

WALLACE DO COUTRO BOAVENTURA
Universidade Federal de Minas Gerais | UFMG

ANTÔNIO EMÍLIO ANGUETH DE ARAÚJO
Universidade Federal de Minas Gerais | UFMG

RESUMO Este trabalho tem como focos principais um evento singular ocorrido em meados do século XIX – o lançamento do primeiro cabo para comunicações (elétricas) intercontinentais, o grande "cabo transatlântico" – e um artigo igualmente singular, em que a primeira teoria matematicamente consistente que visava "explicar" como a eletricidade poderia ser transmitida por um condutor foi elaborada por William Thomson (futuro Lorde Kelvin), intitulado "On the theory of the electric telegraph" [Sobre a teoria do telégrafo elétrico], de 1854. Nele foi apresentada uma analogia na qual considerava-se a eletricidade como um fluido que se difundia através do cabo, a exemplo do que havia feito Fourier, anos antes, no problema da condução de calor. Como forma de investigar mais a fundo as relações e analogias entre a formação dos conceitos de eletricidade e calor, apresenta-se uma tradução detalhadamente comentada do referido artigo, que os autores acreditam ter sido um dos motores da grande revolução tecnológica que seria vista no século seguinte.

Palavras-chave história da física – história da engenharia elétrica – William Thomson.

Introdução

Criar modelos daquilo que acredita perceber é a forma que o ser humano se utiliza para suprir sentidos que não possui. Ainda que perceba a existência de determinado objeto ou fenômeno, a capacidade de enxergar sua realidade íntima estará, possivelmente, relegada a um futuro infinitamente distante. Um modelo, se entendido como uma projeção de um domínio sobre o outro, tal qual a sombra que entretêm o morador da caverna na famosa alegoria de Platão,[1] é algo que reduz de alguma maneira a dimensão da entidade sob a lupa, fazendo com que parte da informação seja para sempre perdida.[2]

Por ser obtido a partir de um processo bem definido, como a dinâmica de uma partícula ou pela vibração da corda em um instrumento musical, um modelo (mecânico, pictórico ou matemático) fornecerá um contexto no qual determinada

Revista Brasileira de Ensino de Física, vol. 40, nº 4, e4602 (2018)
www.scielo.br/rbef
DOI: 10.1590/1806-9126-RBEF-2018-0046

História da Física e Ciências Afins

História da Eletricidade e do Magnetismo: da Antiguidade à Idade Média

The History of Electricity and Magnetism: from antiquity to middle ages

Danny Augusto Vieira Tonidandel*[1,3], Antônio Emílio Angueth de Araújo[2], Wallace do Couto Boaventura[2]

[1]Universidade Federal de Ouro Preto, Departamento de Engenharia de Controle e Automação, Campus Morro do Cruzeiro, s/n, Ouro Preto, MG, Brasil
[2]Universidade Federal de Minas Gerais, Departamento de Engenharia Elétrica, Belo Horizonte, MG, Brasil
[3]Universidade Federal de Minas Gerais, Programa de Pós-graduação em Engenharia Elétrica, Belo Horizonte, MG, Brasil

Recebido em 16 de Fevereiro, 2018. Revisado em 28 de Maio, 2018. Aceito em 28 de Maio, 2018.

Os conceitos de eletricidade e magnetismo, muito antes de figurarem como aspectos basilares da ciência e tecnologia contemporânea, tiveram suas virtudes cantadas em prosa e verso, analisadas em perquirições filosóficas, enaltecidas em tratados morais, aplicadas na medicina e até na literatura mitológica. A história de sua descoberta, que acreditamos ainda não ter sido concluída, se confunde com a própria história do conhecimento humano, desde épocas remotas. Considerando sua importância, buscar-se-á, portanto, tecer alguns fatos e narrativas a respeito da formação dos conceitos de eletricidade e magnetismo ao longo da história antiga até a Idade Média, visando contribuir com os estudos relativos a este recorte temporal no contexto da física.

Palavras-chave: História da Física, história da eletricidade e do magnetismo.

The concepts of electricity and magnetism, way before appearing as fundamental aspects of contemporary science and techonology, had their virtues sung in prose and verse, analyzed in philosophical studies, extolled in moral treatises, applied in medicine and, even in mythological literature. The history of its discovery, which we believe has not yet been completed, is confused with the very history of human knowledge, since remote times. Considering its importance, we will therefore try to weave some facts and narratives regarding to the formation of the concepts of electricity and magnetism throughout ancient history until the Middle Ages, in order to contribute with the studies related to this temporal cut in the context of physics.

Keywords: History of physics, History of electricity and magnetism.

1. Introdução

Prosa e poesia, ética e moral, medicina e mitologia se entrelaçam nos primeiros relatos conhecidos sobre eletricidade e magnetismo. Muito antes de se tornarem objeto de ciência, observadores da Antiguidade notavam que determinados materiais, ora submetidos ao atrito com materiais de natureza diferente, cediam ou recebiam determinadas “virtudes”, que pareciam transferir-se de um corpo ao outro, fazendo com que estes, ora inertes, ganhassem propriedades similares ao da “pedra ímã”. A “capacidade de transferência” sugeria que um entre material, dotado de massa, jornadeava livremente entre os corpos – desde que estabelecido um caminho – tornando-os concentrados ou “carregados”. Tais virtudes foram enaltecidas em diversas produções, que acabaram por formar, ao longo do tempo, o inventivo e genioso caminho de descoberta dos fenômenos eletromagnéticos. Neste contexto, buscamos apresentar fatos e narrativas acerca da história da eletricidade e do magnetismo em um recorte temporal que vai da Antiguidade até a Idade Medieval, culminando em trabalhos que sedimentaram um dos pilares fundamentais da física e tecnologia contemporâneas, o conceito de campo.

2. Do século XXVII a.C, na China, até os gregos antigos

Parece haver indícios de que o homem usava propriedades magnéticas de certas substâncias num passado longínquo. Registros históricos indicam que no ano de 2637 a.C., as tropas do imperador chinês Huang-ti se perderam na enevoada planície de Tchu-lu, quando perseguiam as tropas inimigas do príncipe rebelde Tchi-yu. Dadas as circunstâncias, Huang-ti construiu uma carruagem sobre a qual uma grande figura feminina, de braços abertos, girava livremente de modo a sempre indicar o sul, qualquer que fosse a direção tomada pela carruagem. Suspeita-se, com muita razão, que ligada a essa figura houvesse uma espécie de agulha magnética que lembrasse uma bússola [1]. A este registro tão antigo seguem-se outros, todos na China.

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*Endereço de correspondência: tonidandel@ufop.edu.br.

2017

The book *Invertendo Domínios: o conceito de transformada*, published in 2017,[3] verses on the history of integral transformations, or simply *Transforms*.

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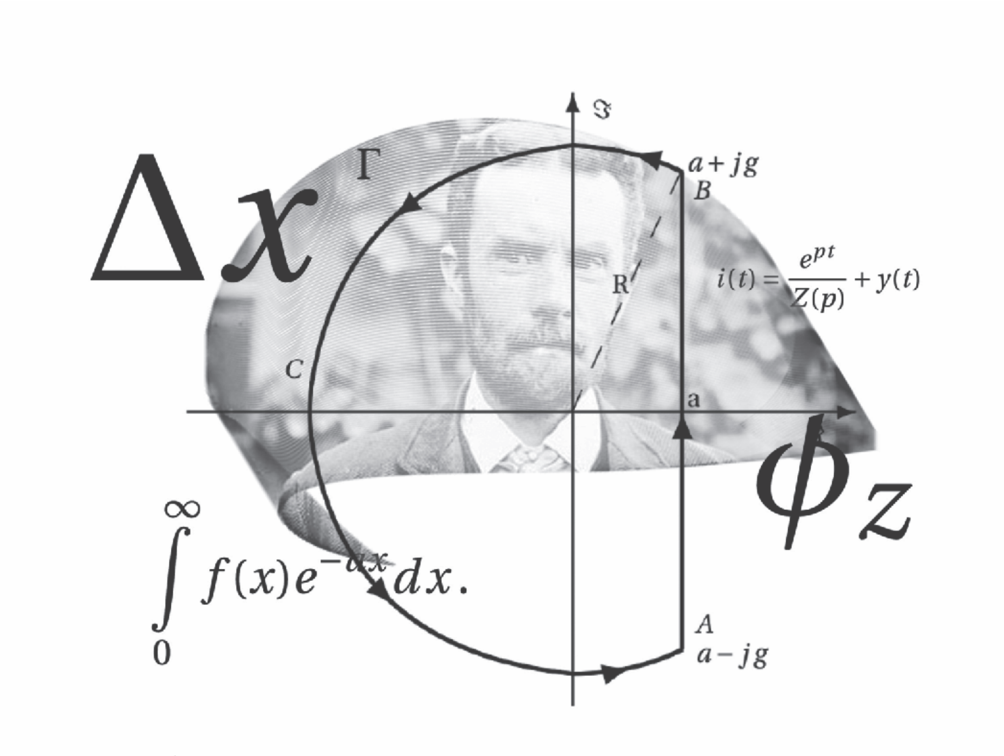
[3] Tonidandel and Araújo 2017.

DANNY AUGUSTO VIEIRA TONIDANDEL
ANTÔNIO EMÍLIO ANGUETH DE ARAÚJO



INVERTENDO DOMÍNIOS

O CONCEITO DE TRANSFORMADA

2010-2016

In this section we present the first page of each paper published from 2010 to 2016.[4,5,6,7,8,9]

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[4] Tonidandel 2016.
[5] Tonidandel and Araújo 2015.
[6] Araújo and Tonidandel 2013.
[7] Tonidandel and Araújo 2012a.
[8] Tonidandel and Araújo 2012b.
[9] Tonidandel 2010.

DOI: http://dx.doi.org/10.5007/2175-7941.2016v33n2p399

Eficácia sem razão [da matemática]+*[1]

Danny A. V. Tonidandel[2]
Departamento de Engenharia de Controle e Automação – Escola de Minas
Universidade Federal de Ouro Preto
Ouro Preto – MG

Resumo

Em 2003, o matemático Americano Alex Kasman (2003) escreveu um belo conto de "ficção matemática" intitulado "Unreasonable Effectiveness", uma das várias respostas existentes ao clássico artigo de Wigner (1960) "The unreasonable effectiveness of mathematics in the natural sciences". Na versão de Kasman, uma pesquisadora acaba, acidentalmente, descobrindo a resposta para a questão de como uma nova teoria encontra, em algum momento, uma utilidade prática na ciência. Isto é, como resultados abstratos, construídos sem quaisquer alicerces no "mundo real", acabam se tornando tão úteis, mesmo em áreas completamente diversas? Neste artigo é proposta uma tradução comentada deste delicioso ensaio, tanto como proposta não convencional de experiência didática quanto uma reflexão sobre os rumos do desenvolvimento científico, propiciados pela Matemática e Física. Como objetivo secundário, procura-se trabalhar a motivação do estudante na busca por soluções não triviais para problemas científicos e filosóficos.

Palavras-chave: *Ensino de Física e Matemática; Filosofia da Ciência; Pensamento criativo.*

+ The unreasonable effectiveness [of mathematics]

* *Recebido: outubro de 2015.*
Aceito: abril de 2016.

[2] E-mail: tonidandel@ufop.br

Revista Brasileira de Ensino de Física, v. 37, n. 3, 3306 (2015)
www.sbfisica.org.br
DOI: http://dx.doi.org/10.1590/S1806-11173731851

A função delta revisitada: De Heaviside a Dirac

(The delta function revisited: from Heaviside to Dirac)

D.A.V. Tonidandel[1], A.E.A. Araújo[2]

[1] *Universidade Federal de Ouro Preto, Ouro Preto, MG, Brasil*
[2] *Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brasil*

Recebido em 9/3/2015; Aceito em 25/3/2015; Publicado em 30/9/2015

A função delta (δ), nomeada após o trabalho pioneiro do físico inglês Paul Adrien Maurice Dirac (⋆1902, †1984) em 1927, tornou-se ferramenta primordial para a ciência e engenharia atuais, com aplicações que vão da teoria quântica até o controle de processos industriais. Ela tem a capacidade facilitar a obtenção de inúmeros resultados que, de outra forma, necessitariam de complicados argumentos. Não obstante, suas definições na literatura são, frequentemente, apresentadas com pouco significado, mesmo que tenham sido corretamente aplicadas na solução de problemas. Este artigo tentará mostrar o engenhoso e inventivo caminho de desenvolvimento desta extraordinária ferramenta, que, além de Dirac, teve a contribuição de outros nomes, como o matemático francês Laurent Moise Schwartz (⋆1915, † 2002), com a teoria das distribuições, e do excêntrico físico-matemático e autodidata inglês chamado Oliver Heaviside (⋆1850, † 1925).

Palavras-chave: função impulso, delta de Dirac, história da engenharia elétrica, teoria das distribuições.

The delta (δ) function, named after the English physicist Paul Adrien Maurice Dirac (⋆1902, †1984) in a 1927 pioneer work, became a fundamental tool to modern science and engineering, with applications covering areas from quantum theory to industrial process control. It facilitates the obtention of countless results that, in other ways, would require complicated arguments. Nonetheless, its definitions in literature are, frequently, presented with few or no meaning, even though they had been correctly applied in the solution of several problems. This paper intends to show the ingenious and inventive way of development of this extraordinary tool that, besides Dirac, had the contribution of others, such as the french mathematician Laurent Moise Schwartz (⋆1915, †2002), with the theory of distributions, as well as the eccentric and self-taught English mathematical physicist called Oliver Heaviside (⋆1850, †1925).

Keywords: impulse function, Dirac's delta, history of physics, history of electrical engineering, theory of distributions.

1. Introdução

Em 2003, o matemático americano Alex Kasman escreveu um belo conto de "ficção matemática" [1], em que uma pesquisadora acaba, acidentalmente, descobrindo a resposta para a questão de como uma nova teoria encontra, em algum momento, uma utilidade prática na ciência, i.e., como resultados em matemática abstrata, construídos sem quaisquer alicerces no "mundo real", acabam se tornando tão úteis, mesmo em áreas diversas. Naturalmente após Hilbert, Riemann e Einstein, a personagem Amanda Birnbaum refletia, ninguém mais acha estranho duas retas paralelas que se encontram no infinito, como afirma a geometria não-Euclidiana. Os números imaginários então, continuava, quando entraram em cena [2], tampouco eram considerados matemática "de verdade"! Atualmente, as funções de onda de uma partícula subatômica são consideradas entidades complexas em sua natureza... mas antes!

De fato, diversas ferramentas hoje "encaradas" com despudor, tomaram corpo e consistência em momentos em que o próprio ato de observar consistia sério entrave, como no surgimento da teoria quântica [3, p. 20]. Em verdade, ao longo da historiografia da ciência, pode-se observar que a matemática acabou sobrepondo-se a esta incapacidade de "ver" um sem número de vezes, e os instrumentos criados para "iluminar" os objetos de interesse eram puramente abstratos.[2] Esse era o contexto

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[1] E-mail: tonidandel@decat.em.ufop.br.

[2] Esta analogia é baseada num experimento proposto pelo físico Alemão Werner karl Heisenberg (⋆1901, †1976) para determinar a posição e o momento de um elétron, que não podem ser observados diretamente, pois têm dimensão menor que os comprimentos de onda da luz visível. São ideias que originaram o famoso "princípio da incerteza de Heisenberg".

J Control Autom Electr Syst (2013) 24:388–395
DOI 10.1007/s40313-013-0030-5

Steinmetz and the Concept of Phasor: A Forgotten Story

A. E. A. Araújo · D. A. V. Tonidandel

Received: 21 May 2012 / Revised: 16 July 2012 / Accepted: 6 September 2012 / Published online: 17 April 2013

Abstract The history of the concept of phasor is often neglected when it is introduced in textbooks on circuits. The presentation does not emphasize the historical aspects, which is natural. This paper intends to recover the original and creative way the concept of complex numbers, then almost unknown to engineers, was applied to electric circuits in sinusoidal steady-state. As usual in physics and engineering, the theory of phasor could have been anticipated by earlier researchers, if they had followed their original reasonings. Maxwell and Heaviside had proved the meal, but could not, or were not interested in writing the recipe.

Keywords Phasor · Steady-state · Circuit theory.

1 Introduction

Textbooks on circuit analysis, in general, treat the subject of the sinusoidal steady-state in a standard form, with two possible variations. The first approach is to consider a rotating vector, with angular velocity equal to the frequency of the sinusoidal wave it intends to represent, making the horizontal projection of this vector to correspond to the instantaneous value of this wave, at the instant of time at which the projection is taken (Johnson et al. 1990; Kerchner and Corcoran 1977; Sadiku 2008; valkenburg 1974).

Another way to introduce such concept, mathematically more formal, is presented in Nilsson and Riedel (2009), where the authors define a phasor transform, intended to transform one generalized sinusoid in a complex number.

Textbooks on physics which cover the subject do not innovate in what was described above (Feynman et al. 1963).

The concept of phasor was created through a different process, if compared with the pedagogical schema chosen by several authors dealing with the topic. This creation had two anticipations before taking its definitive shape. The first appears in a almost forgotten Maxwell's (1868) paper that presents, for the first time (Blanchard 1941), the solution of the current in a series RLC circuit, feeded by an alternating sinusoidal voltage source. The other one appears in a Oliver Heaviside's paper, published by the *Royal Society of London* (Heaviside 1893b).

The genesis of a concept is, many times, different from later formulations, so as to make the reader completely unaware of the original path followed by the pioneer. These pathways are going to be the object of the present paper. It is intended, at first, to describe the unique way in which Maxwell addresses the problem, showing how his genius took him to a solution very similar to the modern method. It is observed a possible anticipation of the idea of using complex number in the treatment of sinusoidal steady-state.

Three curiosities are presented. The first is the interesting story of an article in which appears for the first time the concept of phasor, then presented in the AIEE (American Institute of Electrical Engineering) meeting—IEEE grandmother's institution—and not published in the meeting proceedings, for lack of funds. The second is the apparent inexistence of a relationship between Steinmetz and Heaviside, two electrical engineering giants who, living in the same period—end of 19th, beginning of 20th century—, apparently did not know each other's work. The third is the

A. E. A. Araújo
Universidade Federal de Minas Gerais-UFMG,
Belo Horizonte, Minas Gerais, Brazil
e-mail: araujo@cpdee.ufmg.br

D. A. V. Tonidandel (✉)
Universidade Federal de Ouro Preto-UFOP,
Ouro Preto, Minas Gerais, Brazil
e-mail: tonidandel@decat.em.ufop.br

CONECTANDO TRANSFORMADAS: FOURIER E LAPLACE

D.A.V. Tonidandel*, A.E.A. Araújo†

*Faculdade Santa Rita - FaSaR
Conselheiro Lafaiete, Minas Gerais, Brasil

†Universidade Federal de Minas Gerais - UFMG
Belo Horizonte, Minas Gerais, Brasil

Emails: `tonidandel@ufmg.br, araujo@cpdee.ufmg.br`

Abstract— Transforms are essential tools in electrical engineering and related areas and it is extremely important that the future engineer has an firm concept of their utility. Based on that, it is intended to create a unified view of the Laplace and Fourier transforms, articulating their concept and combining the mathematical and the historic point of view, without forgetting the human aspects of the developments. Even though the final product is hardly measurable, the students' opinions and the experience of professors seem to indicate that this approach is of great relevance in creating a solid basis of learning.

Keywords— Laplace transform, Fourier transform, step function, higher education.

Resumo— As transformadas são ferramentas essenciais em engenharia elétrica e áreas afins, sendo de extrema importância que o futuro engenheiro tenha uma noção integral de sua utilidade. A partir dessa premissa, objetiva-se criar uma visão conjunta das transformadas de *Laplace* e *Fourier*, unificando seu conceito, com a fundamentação matemática acrescida do aspecto humano e histórico. E embora o produto final seja algo de difícil mensuração, a experiência de professores e a opinião de alunos parecem indicar que o uso desta abordagem é de grande relevância ao criar uma base sólida de aprendizado.

Palavras-chave— Transformada de Laplace, transformada de Fourier, função degrau unitário, educação superior.

1 Introdução

A transformada de *Fourier* – em conjunto com a transformada de *Laplace* – constitui, provavelmente, na principal ferrameta [matemática] do engenheiro eletricista. Juntamente com as séries de *Fourier*, desempenha importante papel em diversas áreas, desde comunicações, processamento de sinais, sistemas de controle, antenas, além de ser extremamente útil na resolução de problemas de valor de contorno[1], embora com um poderio relativamente menor que a transformada de *Laplace* em alguns aspectos. Por exemplo, para calcular a resposta de um circuito elétrico a um dado valor de entrada [tensão ou corrente], a transformada de *Laplace* fornece a resposta transitória e permanente. Já a transformada de *Fourier* limita-se ao regime permanente. Ela pode ser definida como uma soma [integral] ponderada de senóides complexas:

$$F(\omega) \triangleq \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \,. \tag{1}$$

A transformada inversa,

$$f(t) \triangleq \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \,, \tag{2}$$

converte a transformada de volta para $f(t)$. Isto porém não é novidade. O nome "transformada de *Fourier*" refere-se ao grande físico-matemático francês *Jean-Baptiste Joseph Fourier* ($\star$1768 † 1830), em razão de seu grande – em todos os aspectos – e aclamado livro *Théorie Analytique de la Chaleur*.

> Diz-se frequentemente que se imaginarmos a função $f(t)$ como um feixe de luz, então a transformada de *Fourier*, como um prisma, quebra a função em diversos componentes de frequência ω que a compõe, cada uma de intensidade $F(\omega)$. As várias frequências seriam chamadas *cores* e dessa forma, a transformada de *Fourier* forneceria o *espectro de cores* do sinal. Fazendo o caminho contrário, a transformada inversa de *Fourier* combina o espectro, ou seja, combina todas as cores, para retornar à função original.

De maneira similar, a transformada de *Laplace* é definida, em termos atuais, pela equação:

$$\mathcal{L}\{f(t)\} = F(s) \triangleq \int_{-\infty}^{\infty} e^{-st} f(t) dt \,, \tag{3}$$

com a transformada inversa expressa pela integral de inversão de *Bromwich*, também chamada de integral de *Mellin-Fourier* (Bromwich, 1916)[2]:

$$f(t) = \mathcal{L}^{-1}\{F(s)\} \triangleq \frac{1}{2\pi j} \oint_{\Gamma} F(s) e^{st} ds \,, \tag{4}$$

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[1] Equações Diferenciais Parciais - EDPs.

[2] A integral de inversão que fornece a transformada de *Laplace* inversa foi obtida, em 1916, pelo matemático da Universidade de Cambridge *Thomas Jonh I'Anson Bromwich* ($\star$1875 † 1929), que utilizou a forma complexa da integral [leia-se transformada] de Fourier $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} d\omega \int_{-\infty}^{\infty} f(u) e^{-j\omega u} du \,.$

Fechar

DECIFRANDO A TRANSFORMADA $\mathcal{Z}$

Danny Augusto Vieira Tonidandel*

**Programa de Pós-graduação em Engenharia Elétrica - PPGEE*
Universidade Federal de Minas Gerais - UFMG
Belo Horizonte, Minas Gerais, Brasil

Email: `tonidandel@ufmg.br`

Abstract— A widely used mathematical tool in the study of discrete-time systems is the $\mathcal{Z}$-transform. It can be understood as the discrete-equivalent to the *Laplace* transform, although an intimate connection between $\mathcal{Z}$, *Laplace* and *Fourier* transforms exists. Among its advantages, a large class of functions that does not have *Fourier* transform, for example, have $\mathcal{Z}$-transform. An attempt to make easy the integration of topics concerned with the transform techniques, a historic approach combined with a mathematical view of $\mathcal{Z}$-transform is presented. In addition, techniques of system discretization are described, such as the *Tustin* transform, and an example is presented.

Keywords— $\mathcal{Z}$-transform, Bilinear transform, System Discretization, Residues, Laurent Series, Tustin transform.

Resumo— Uma ferramenta matemática largamente utilizada no estudo de sistemas de tempo discreto é a transformada $\mathcal{Z}$. Ela pode ser entendida como o equivalente discreto à transformada de *Laplace* no caso contínuo, mas pode-se mostrar uma íntima relação entre ela e as transformadas de *Laplace* e *Fourier*. Entre suas vantagens está o fato de que uma grande classe de funções que não possuem transformada de *Fourier* por exemplo, possuem a transformada $\mathcal{Z}$. Na tentativa de auxiliar a integração de conhecimentos com relação às diversas técnicas de transformação, são apresentados alguns traços históricos juntamente com uma definição matemática rigorosa da transformada $\mathcal{Z}$. Em seguida, são mostradas algumas técnicas para discretização de sistemas contínuos, como a transformação de Tustin, além de exemplos de aplicação.

Palavras-chave— Transformada $\mathcal{Z}$, transformação Bilinear, Discretização de Sistemas, Resíduos, Séries de Laurent, transformação de Tustin.

1 Introdução

1.1 O conceito de transformada

Em engenharia, a ideia por trás do nome "transformada" consiste basicamente em uma operação matemática que tem por finalidade promover algum tipo de simplificação. Dessa forma, o logaritmo consiste, provavelmente, na ferramenta mais antiga de que se tem notícia cujo conceito se aproxima da ideia de transformada, uma vez que transforma multiplicações e divisões em somas e subtrações, além de ser útil na resolução de equações cujos expoentes são desconhecidos (Boyer, 1974). Em verdade, o conceito das transformadas vai muito além dos logaritmos no contexto da engenharia, em que desempenham papel importante. Entre as mais conhecidas (e com certeza mais utilizadas) figura a transformada $\mathcal{Z}$.

1.2 Por que $\mathcal{Z}$?

A transformada $\mathcal{Z}$ é um método operacional muito útil no tratamento de sistemas (de tempo) discretos. Seu nome já é em si incomum, por se tratar de uma letra do alfabeto e não o nome de algum cientista famoso. Sabe-se que a transformada de *Laplace* (*Pierre-Simon de Laplace* ($\star$1749 † 1827)) tem sido usada desde longa data na solução de equações diferenciais contínuas e invariantes no tempo. Entretanto, métodos para o tratamento de problemas de tempo discreto são relativamente recentes. De acordo com (Strum and Kirk, 1994), um método para a resolução de equações de diferenças lineares e invariantes no tempo foi apresentado por *Gardner* e *Barnes*, aos seus alunos de engenharia no ínicio da década de 1940. Eles aplicaram tal procedimento, que era baseado principalmente em "*jump functions*"[1], na resolução de linhas de transmissão e aplicações envolvendo funções de *Bessel*. Tal abordagem era bastante complexa, e, na tentativa de "dividir para simplificar", uma transformação de um sinal amostrado foi proposta em 1947 por *Witold Hurewicz* ($\star$1904 † 1956) (Kuperberg, 1996). Tal transformação era escrita como função da sequência amostrada f (no domínio do tempo) ao invés do número complexo z da notação moderna (definição 1.1):

$$\mathcal{T}\left\{f[kT]\right\} \triangleq \sum_{k=0}^{\infty} f(kT)\zeta^{-k}\,. \tag{1}$$

Em 1952, cinco anos após a tentativa de *Hurewicz*, a transformação foi batizada de "*transformada $\mathcal{Z}$*" pelo "*Sampled-data control group*", liderada por *John Ralph Raggazini* ($\star$1912 † 1988), com *Eliahu Ibrahim Jury* (que, na época, era aluno de doutorado de *Raggazini*, mas que acabou sendo um dos principais desenvolvedores da teoria), *Lotfi Zadeh* (famoso pela criação da lógica *Fuzzy*) e colaboradores da *Columbia University*,

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[1] funções usadas para representar uma sequência de dados amostrados.