Fernando Fernandes

Population Aging and Retirement Age Policy: Period Balances, Age Rebalances and Generational Imbalances

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Supervisor: Prof. Cássio Maldonado Turra
Co-supervisor: Prof. Eduardo Luiz Gonçalves Rios Neto

Belo Horizonte, MG
Centro de Desenvolvimento e Planejamento Regional
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Never give in, never give in, never, never, never — in nothing great or small, large or petty — never give in except to convictions of honour and good sense.

Winston Churchill

Resumo

O envelhecimento populacional impacta sistemas de aposentadoria estruturados sobre equilíbrios financeiros de período, conhecidos como sistemas de repartição simples (PAYG), porque altera a razão entre as populações com 65 anos de idade e mais (i.e., potenciais beneficiários) e de 20 a 64 anos de idade (i.e., potenciais contribuintes). O envelhecimento populacional também impacta o retorno de sistemas PAYG às coortes de nascimento, porque mudanças nas razões de período entre beneficiários e contribuintes levam a contribuições e benefícios de ciclo de vida distintos entre coortes diferentes.

Nosso objetivo principal é examinar até que ponto o envelhecimento populacional impacta equilíbrios de período e desequilíbrios geracionais de sistemas PAYG. Nos concentramos em três objetivos específicos. Primeiro, examinamos a contribuição de nascimentos, mortes e migrações para o envelhecimento da população mundial. Segundo, analisamos o peso do envelhecimento populacional sobre sistemas PAYG sob uma perspectiva de período. Por último, investigamos os efeitos do envelhecimento populacional nos retornos de sistemas PAYG às coortes de nascimento.

Utilizamos dados da revisão 2017 das projeções e estimativas oficiais de população das Nações Unidas que cobre 150 anos de 1950 a 2100. Primeiro, aplicamos uma expressão matemática introduzida por Preston, Himes e Eggers (1989) que decompõe a taxa de mudança na idade média de uma população em efeitos rejuvenescedores de nascimentos, mortes e migrações. Segundo, aplicamos um método introduzido por Bayo e Faber (1981) que ajusta a idade de aposentadoria baseado em ganhos na idade média à morte. Por último, aplicamos uma expressão matemática introduzida por Samuelson (1958) e expandida por Aaron (1966) para medir o retorno de sistemas PAYG às coortes de nascimento.

Mostramos que transições demográficas diferem ao longo de um padrão geral entre o efeito rejuvenescedor de nascimentos e o efeito rejuvenescedor de mortes. Propomos uma categorização dos estágios da transição demográfica baseada em níveis e indicadores dos efeitos rejuvenescedores de nascimentos e mortes. Quanto ao peso do envelhecimento populacional, argumentamos que políticas que ajustam a idade de aposentadoria de sistemas PAYG baseadas em ganhos de longevidade são intrinsecamente ineficazes, e que quando a mortalidade de idosos declina, políticas baseadas em ganhos na idade modal à morte podem ser menos ineficazes. Também mostramos que quando populações envelhecem sistemas PAYG de benefícios fixos (DB) são preferíveis aos de contribuições fixas (DC) porque o primeiro provê retornos às coortes mais altos e menos variáveis. Ainda, elevar a razão entre benefícios per capita e salários líquidos per capita de sistemas PAYG de posição relativa fixa (FRP) pode levar a retornos às coortes mais baixos e com reduções mais rápidas. Afirmamos que uma idade de aposentadoria fixa mais alta tende a elevar os retornos de sistemas PAYG DB às coortes, e a reduzir os de sistemas PAYG DC. Em sistemas PAYG que ajustam a idade de aposentadoria baseados em ganhos de longevidade, uma idade de aposentadoria mais alta tende a reduzir os retornos de sistemas PAYG DB às coortes, e a elevar os de sistemas PAYG DC.

Palavras-chave: População. Envelhecimento. Política de Aposentadoria. Equidade Intergeracional. Repartição Simples. Estudos Multinacionais.

Abstract

Population aging impacts retirement systems structured on period financial balances, known as pay-as-you-go (PAYG) systems, because it changes the ratio of the population 65 years of age and older (i.e., potential beneficiaries) to the population 20 to 64 years of age (i.e., potential contributors). Population aging also impacts the return of PAYG systems to birth cohorts, because changes in period relations between beneficiaries and contributors lead to distinct life cycle contributions and benefits among different birth cohorts.

Our primary objective is to examine the extent to which population aging affects period balances and generational imbalances of PAYG systems. We concentrate on three specific objectives. First, we examine the contribution of births, deaths, and migrations to world population aging. Second, we analyze the burden of population aging in PAYG systems from a period perspective. Last, we investigate the effects of population aging on the returns of PAYG systems to birth cohorts.

We draw data from the 2017 revision of the official United Nations population estimates and projections that covers 150 years from 1950 to 2100. First, we apply a mathematical expression introduced by Preston, Himes, and Eggers (1989), which decomposes the rate of change in the mean age of a population into the rejuvenating effects of births, deaths, and migrations. Second, we apply a method introduced by Bayo and Faber (1981) that adjusts the retirement age based on gains in the mean age at death. Last, we apply a mathematical expression introduced by Samuelson (1958) and expanded by Aaron (1966) to measure the return of PAYG systems to birth cohorts.

We show that demographic transitions differ alongside a general concerted pattern between the rejuvenating effect of births and the rejuvenating effect of deaths. We propose a categorization of the stages of the demographic transition based on levels and indicators of the rejuvenating effects of births and deaths. Regarding the burden of population aging, we argue that policies that adjust the retirement age of PAYG systems based on gains in longevity are intrinsically ineffective, and when old-age mortality declines, policies based alternatively on gains in the modal age at death may be less ineffective. We also show that when populations age defined benefit (DB) PAYG systems are preferable to defined contribution (DC) PAYG systems because the former yields higher and less variable returns to birth cohorts. In addition, increasing the ratio of per capita benefits to per capita net wages of fixed relative position (FRP) PAYG systems may lead to lower and faster decreases of the returns to birth cohorts. We further claim that a higher fixed retirement age tends to increase the returns to birth cohorts of DB PAYG systems, and to decrease the returns to birth cohorts of DC PAYG systems. In PAYG systems that adjust the retirement age based on gains in longevity, a higher retirement age tends to decrease the returns to birth cohorts of DB PAYG systems, and to increase the returns to birth cohorts of DC PAYG systems.

Keywords: Population. Aging. Retirement Policy. Intergenerational Equity. Pay as You Go. Multicountry Studies.

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List of Abbreviations and Acronyms

2017 UN REVISION	2017 revision of the official United Nations population estimates and projections 34, 35, 37, 39, 46, 48, 49, 53, 72, 112, 185, 188, 189, 201, 202
DB	defined benefit 13, 63, 64, 72–74, 78, 79, 105, 107–114, 116, 118, 120, 121, 123, 132, 133, 136, 145, 146, 149, 150, 159, 163, 165, 166
DC	defined contribution 13, 63, 64, 72–74, 78, 79, 105, 107–114, 116, 118, 120, 121, 123, 132, 135, 136, 145, 148–150, 159, 163, 165, 166
EOADR	equivalent old age dependency ratio 73, 86, 88, 92, 103, 165
ERA	equivalent retirement age 65–68, 72, 73, 78, 79, 85, 86, 88, 92, 93, 97, 100, 102, 103, 105, 112, 132, 145, 146, 148–150, 163, 165–167
FRP	fixed relative position 13, 64, 72–74, 78–80, 103, 105, 107, 108, 112–114, 116, 118, 121, 123, 159, 163, 165, 166
HMD	Human Mortality Database 195
IRR	intrinsic rate of return 106–114, 116, 118, 120, 121, 123, 132, 133, 135, 136, 145, 146, 148–150, 159, 163, 165, 166
`MortalityLaws`	MortalityLaws `R` package . . . . 189, *see also* `Rstats`
NDC	notional defined contribution 61
OADR	old age dependency ratio 33, 43, 48, 61, 62, 68, 72, 73, 85, 86, 88, 92, 100, 103, 107, 109, 159, 165
OASDI	Old-Age, Survivors and Disability Insurance Program 110
PAA	Population Association of America 44
PAYG	pay-as-you-go 13, 33–36, 61–64, 72–74, 78–80, 85, 97, 103, 105–114, 116, 118, 120, 121, 123, 132, 133, 135, 136, 145, 146, 148–150, 159, 163, 165, 166
PHE I	first mathematical expression from Preston, Himes, and Eggers (1989) 45, 46, 48
PHE II	second mathematical expression from Preston, Himes, and Eggers (1989) 45, 46, 48

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Contents

1 *Introduction*

Discern of the coming on of years, and think not to do the same things still; for age will not be defied.

Francis Bacon, Of Regiment of Health

1.1 MOTIVATION

As the world goes through the demographic transition, it moves from times of high mortality and fertility, and population growth rates around zero to a contemporary era of low mortality and fertility, and minimum or negative population growth rates. Consequently, the age distribution of the world population has changed significantly from initial larger proportions in the younger age groups to intermediary higher proportions in the working or producing age groups to final increasing proportions in the older age groups (Lee, 2003; Dyson, 2010). The proportion of the world population with 65 years of age and older was 5.1% in 1950, 6.9% in 2000, and it is expected to be 15.8% in 2050, and 22.5% in 2100 (United Nations, 2017c). Population aging is ubiquitous: by 2050 the proportion of the population with 65 years of age and older will be 28% in Europe, between 18% and 20% in Asia, the Americas and Oceania, and 6% in Africa. By 2100, these numbers are expected to be about 30% in Europe and the Americas, 26% in Asia and Oceania, and 15% in Africa (United Nations, 2017c). Yet across the globe, the demographic transition has varied concerning the onset, pace, and scale of mortality and fertility declines (Reher, 2004, 2011), leading to different processes of population aging. Several scholars have looked at population aging in various contexts and from distinct angles, both theoretically and empirically. But the international literature lacks a systematic comparative analysis of the demographic determinants of population aging.

Population aging impacts social programs such as education, health care, and retirement primarily because it changes the relation between beneficiaries and contributors, who usually are of different age groups. In the case of retirement systems structured on period financial balances, known as pay-as-you-go (PAYG) systems, sustainability is directly affected by variations in the old age dependency ratio (OADR), that is, the ratio of the population 65 years of age and older (i.e., potential beneficiaries) to the population 20 to 64 years of age (i.e., potential contributors). The world's OADR has grown from 9.9% in 1950 to 12.8% in 2000, and is expected to be 28.3% in 2050, reaching 41.8% in 2100 (United Nations, 2017c). Many policies may buffer the burden of population aging in PAYG systems, such as varying contributions, benefits, or both, and changing the normal ages of contribution and retirement. Among the alternative policies, there is an increasing debate among actuaries, demographers, and economists about adjusting the retirement age based on gains in longevity. But since PAYG systems are established primarily on period financial balances, adjusting the retirement age based on gains in longevity, a life cycle characteristic, may not be effective in lessening the impact of population aging if the contribution of mortality to changes in the population age distribution is only moderate. Therefore, PAYG systems should contemplate the role of the demographic determinants of population aging in the definition of retirement age policies. Moreover, when old-age mortality declines, adjusting the retirement age based on gains in the mean age at death may be less effective than based on gains in the modal age at death, because increases in the retirement age will be slower than the gains in longevity for most beneficiaries of the PAYG system.

Population aging also impacts the return of PAYG systems to birth cohorts, because changes in period relations between beneficiaries and contributors lead to distinct life cycle contributions and life cycle benefits among different birth cohorts. That is, PAYG systems are built on intergenerational transfers that are intrinsically redistributive, and likely favor some birth cohorts to the detriment of others, leading to inequitable returns among generations. Ultimately, life cycle intergenerational transfers of PAYG systems relate to long-term equality among generations, and thus the long-term stability of PAYG systems hinge on their intergenerational equality as well (KEYFITZ, 1985). Therefore, to investigate the returns of PAYG systems to different birth cohorts, it is necessary to consider the role of alternative period policy designs, as well as the effects of retirement age policies and the demographic determinants of population aging.

1.2 OBJECTIVES

Our primary objective is to *examine the extent to which population aging affects period balances and generational imbalances of pay-as-you-go (*PAYG*) systems*. To address this broad research question, we use the 2017 revision of the official United Nations population estimates and projections (2017 UN REVISION) (UNITED NATIONS, 2017c,d) that covers 150 years from 1950 to 2100, and concentrate on three specific objectives.

First, we *examine the contribution of births, deaths, and migrations to world population aging*. Our analysis covers populations that are in distinct stages of the demographic transition, allowing us to analyze the role of the demographic determinants of population aging in diverse demographic contexts. We discuss and apply a mathematical expression introduced by Preston, Himes, and Eggers (1989) in a seminal article, which decomposes the rate of change in the mean age of a population into the rejuvenating effects of births, deaths, and migrations. To our understanding, this is the first time someone tests the authors' mathematical expression with such a long and comprehensive data set. Our results add to the earlier international studies that have tried to describe population aging concerning changes in the demographic variables.

Second, we *analyze the burden of population aging in* PAYG *systems from a period perspective*. We benefit from using data that represent all stages of the demographic transition and a variety of demographic trajectories, to elucidate the primacy of changes in period population age structures to the equilibrium of PAYG systems. We investigate to what degree the burden of population aging befalls contributors and beneficiaries in different period policy designs of PAYG systems. Also, we propose a framework to investigate the effectiveness of policies that adjust the retirement age based on gains in longevity to counterbalance the effects of population aging. We examine and apply a method introduced by Bayo and Faber (1981) that adjusts the retirement age based on gains in the mean age at death. We also compare the authors' initial adjustment with a new measurement for the age of retirement based on gains in the modal age at death.

As aforementioned, the consequences of population aging are not restricted to period balances and may affect birth cohorts distinctly, depending on the stage of the demographic transition and the type of adjustments adopted in the PAYG systems. Therefore, as our last goal, we *investigate the effects of population aging on the returns of* PAYG *systems to birth cohorts*. We analyze the returns of PAYG systems to birth cohorts under different period policy designs, different fixed retirement ages, and retirement ages based on gains in longevity. Also, we investigate the influence of the determinants of population aging on the returns of PAYG systems to birth cohorts. We discuss and apply a mathematical expression introduced by Samuelson (1958) and expanded by Aaron (1966) to measure the return of PAYG systems to birth cohorts.

1.3 ORGANIZATION OF THE DISSERTATION

This dissertation is organized into this introduction, four chapters, a conclusion, and five appendixes and one annex with supplemental materials to each chapter.

In chapter 2 (Materials), we present the data and outline the notation we use throughout this dissertation. This chapter has two main sections. In section 2.1, we present the data we draw from the 2017 UN REVISION (UNITED NATIONS, 2017c,d); specify the projection variant we use; detail its geographic coverage and the variables we work with; fine-tune the 2017 UN REVISION regional and subregional classification of countries and areas; and adjust the open-ended age groups of populations, deaths and life tables. Last, in section 2.2, we outline the notation we use in this dissertation. Appendix A and Annex A have supplemental material to chapter 2. In Appendix B (Model old-age mortality), which also supplements chapter 2, we model old-age mortality to obtain life tables with the same open-ended age groups as populations and deaths, and also equalize the first age group between life tables, and populations and deaths.

Chapter 3 (Population aging) has five main sections in addition to an introduction and a conclusion. In section 3.2, we compare some demographic measures of population aging. In section 3.3, we present approaches that demographers use to investigate the demographic determinants of population aging, including two mathematical expressions introduced by Preston, Himes, and Eggers (1989). In section 3.4, we detail our methods. In section 3.5, we decompose the rate of change in the mean age of a population. Last, in section 3.6, we propose a categorization of the stages of the demographic transition based on the demographic determinants of population aging. Appendix C has supplemental material to chapter 3.

In the next two chapters, we use a stylized demographic model to investigate the burden of population aging in PAYG systems in the world from 1950 to 2100, respectively from period and cohort perspectives.

Chapter 4 (Period balances, age rebalances) has seven main sections in addition to an introduction and a conclusion. In section 4.2, we present the main attributes of PAYG systems in three alternative period policy designs: two classic and a third proposed by Musgrave (1981). In section 4.3, we review different approaches for measuring the old-age threshold or adjusting the retirement age based on gains in longevity, including a method introduced by Bayo and Faber (1981); and present policies of a selected group of countries that adjust the normal retirement age or retirement pensions based on gains in longevity. In section 4.4, we detail our methods and assumptions. In section 4.5, we estimate the distribution of the burden of population aging between contributors and beneficiaries in different period policy designs of PAYG systems. In section 4.6, we assess the change in the retirement age based on gains in longevity, and how much it alleviates the burden of population aging on contributors and beneficiaries. In section 4.7, we propose a framework to investigate the effectiveness of policies that adjust the retirement age based on gains in longevity. Last, in section 4.8, we propose adjusting the retirement age based on gains in the modal age at death and evaluate its effectiveness. Appendix D has supplemental material to chapter 4.

Chapter 5 (Period balances, generational imbalances) has five main sections in addition to an introduction and a conclusion. In section 5.2, we present a mathematical expression introduced by Samuelson (1958) and expanded by Aaron (1966) to measure the return of PAYG systems to birth cohorts. In section 5.3, we detail our methods and assumptions. In section 5.4, we estimate the return of PAYG systems to birth cohorts under different period policy designs of PAYG systems. In section 5.5, we evaluate how different fixed retirement ages, and retirement ages based on gains in longevity influence the returns of different period policy designs of PAYG systems to birth cohorts. Last, in section 5.6, we analyze the influence of the determinants of population aging on the returns of PAYG systems to birth cohorts. Appendix E has supplemental material to chapter 5.

Last, in chapter 6 (Conclusion), we present our general concluding remarks.

2 *Materials*

In this chapter, we present the data and outline the notation we use throughout this dissertation.

2.1 DATA

2.1.1 *Introduction*

We draw data from the 2017 revision of the official United Nations population estimates and projections (2017 UN REVISION) (UNITED NATIONS, 2017c,d). It covers 150 years from 1950 to 2100, which are divided into two periods: 1950–2015 (estimates) and 2015–2100 (projections). First, we specify the projection variant we use and its fertility, mortality and migration assumptions. Second, we detail its geographic coverage, and the variables we use and their characteristics. Third, we fine-tune its regional and subregional classification of countries and areas. Last, we adjust the open-ended age groups of populations, deaths and life tables.

2.1.2 *Projection variant*

The 2017 UN REVISION has nine projection variants that are the combination of five fertility, two mortality and two international migration assumptions (Table 1). We use the medium fertility projection variant, which combines the medium fertility, normal mortality, and normal international migration assumptions. The medium fertility and normal mortality assumptions reflect the median trajectories of probabilistic projections of total fertility and life expectancy at birth. The normal international migration assumption results from past estimates and the policy of each country towards future flows (UNITED NATIONS, 2017d).[1]

2.1.3 *Geographic coverage and variables*

The 2017 UN REVISION covers a total of 233 countries and areas. It includes detailed data (e.g., population by five-year age groups) for the 201 countries and areas that had 90,000 or more inhabitants in 2017, and only total populations and growth rates for the remaining 32 (UNITED NATIONS, 2017d, p. 1). We include these 201 countries and areas, both sexes combined, and the variables presented in Table 2.

[1] The other fertility assumptions are: a) low-fertility, 0.5 births below the medium variant; b) high-fertility, 0.5 births above the medium variant; c) constant-fertility, constant at the level estimated for 2010–2015; and d) instant-replacement, the level necessary to ensure a net reproduction rate of 1.0 starting in 2015–2020.

Table 1 – Projection variants of the 2017 UN REVISION by fertility, mortality and international migration assumptions

Projection variant	Assumptions		
	Fertility	Mortality	International migration
Low fertility	Low	Normal	Normal
Medium fertility	Medium	Normal	Normal
High fertility	High	Normal	Normal
Constant fertility	Constant as of 2010–2015	Normal	Normal
Instant replacement fertility	Instant replacement as of 2015–2020	Normal	Normal
Momentum	Instant replacement as of 2015–2020	Constant as of 2010–2015	Zero as of 2015–2020
Constant mortality	Medium	Constant as of 2010–2015	Normal
No change	Constant as of 2010–2015	Constant as of 2010–2015	Normal
Zero migration	Medium	Normal	Zero as of 2015–2020

Source: United Nations (2017d, p. 31).

Table 2 – Data of the 2017 UN REVISION by variable, age group, and year or period

Variable	Age groups	Years / Periods (1)
List of locations with code, description, region and subregion (2)	…	…
Populations (3)	Five-year	Annually from 1950 to 2100
Deaths (4)	Five-year	Five-year periods from 1950–1955 to 2095–2100
Abridged life tables (5)	0–1, 1–4, and five-year	Five-year periods from 1950–1955 to 2095–2100
Demographic indicators (6)	Total	Five-year periods from 1950–1955 to 2095–2100

Source: United Nations (2017c).
(1): Annual data refer to 1 July of the year indicated. Data for five-year periods are from 1 July of the first year to 30 June of the final year.
(2): `https://esa.un.org/unpd/wpp/DVD/Files/4_OtherFiles/WPP2017_F01_locations.xlsx`
(3): `https://esa.un.org/unpd/wpp/DVD/Files/1_Indicators(Standard)/CSV_FILES/WPP2017_PopulationByAgeSex_Medium.csv`
(4): `https://esa.un.org/unpd/wpp/DVD/Files/1_Indicators(Standard)/EXCEL_FILES/3_Mortality/WPP2017_MORT_F04_1_deaths_by_age_both_sexes.xlsx`
(5): `https://esa.un.org/unpd/wpp/DVD/Files/1_Indicators(Standard)/CSV_FILES/WPP2017_LifeTable.csv`
(6): E.g., crude birth rate. See complete list in section A.1, Appendix A. `https://esa.un.org/unpd/wpp/DVD/Files/1_Indicators(Standard)/CSV_FILES/WPP2017_period_indicators_medium.csv`
…: Not applicable.

2.1.4 *Geographic classification by regions and subregions*

The 2017 UN REVISION (UNITED NATIONS, 2017b,c, 2017d, p. vii) follows the names and composition of geographic areas of the United Nations' *Standard Country or Area Codes for Statistical Use (M49)* (UNITED NATIONS, 2018), but with two differences. First, the 2017 UN REVISION groups its countries and areas into six regions: Africa, Asia, Europe, Latin America and the Caribbean, Northern America, and Oceania; whereas the United Nations (2018) adopts five geographic regions based on continental regions: Africa, Asia, Europe, Americas, and Oceania. Second, while the 2017 UN REVISION combines the Southern Asia and Central Asia subregions into South-Central Asia; the United Nations (2018) classifies Central Asia and Southern Asia as separate subregions since 2005.[2] Yet none of the 2017 UN REVISION's geographic classification criteria help us to either summarize or drill down its data. First, Northern America has no subregions, and only two countries with detailed data (i.e., Canada and United States of America); second, we risk loosing information when we combine subregions. Therefore, we fine-tune the 2017 UN REVISION's regional and subregional classification of countries and areas. First, we adopt the United Nations (2018)'s standard, specifically, five geographic regions, and Central Asia and Southern Asia as separate subregions. Second, we remove Latin America and the Caribbean as a subregion, but maintain its subregions under Americas; that is, we categorize Americas' subregions as the Caribbean, Central America, South America, and Northern America.

2.1.5 *Open-ended age group*

Since the open-ended age group has no determined age at its end, it has no specified length. The higher the initial age of the open-ended age group, the fewer people survive to it, the narrower its expected length, and thus the smaller the errors in methods that work with it. For example, methods that assume the life expectancy at the initial age of the open-ended age group as an estimate of the mean age of the population or the mean age at death of this age group, or methods that use the open-ended age group of periods as a proxy for the last age group of cohorts. Most of our methods work with the open-ended age group, and some also incorporate simultaneous use of distinct variables by age groups (e.g., populations or deaths multiplied by life table functions). Therefore, the initial age of the open-ended age group of populations, deaths and life tables must be the same and as high as possible.

We may assume observed or theoretical age distributions to increase the initial age of the open-ended age group of populations or deaths. But this may result in inconsistent and incomparable data, and is unfeasible for most periods, countries and areas. Still, we can consistently model old-age age-specific death rates to increase the initial age of the open-ended age group of life tables. Therefore, we make the following changes to obtain populations and deaths with the same open-ended age groups as life tables (Table 3): a) populations from 1990 to 2100: decrease open-ended age group to 95+ (add 95–99 and 100+); b) life tables from 1950–1955 to

[2] See Annex A for United Nations (2017b): 2017 UN REVISION, Classification of countries.

1985–1990: add open-ended age group 80+; and c) life tables from 1950–1955 to 2095–2100: increase open-ended age group to 95+.

Table 3 – Open-ended age groups of the 2017 UN REVISION by variable, year or period, and before and after adjustments

Variable	Years / Periods (1)	Open-ended age group	
		Before	After
Populations	1950 to 1989	80+	80+
Deaths	1950–1955 to 1985–1990	95+	95+
Abridged life tables	1950–1955 to 1985–1990	85+	80+ and 95+
Populations	1990 to 2100	100+	95+
Deaths	1990–1995 to 2095–2100	95+	95+
Abridged life tables	1990–1995 to 2095–2100	85+	95+

Source: Author's calculations, based on United Nations (2017c).
(1): Annual data refer to 1 July of the year indicated. Data for five-year periods are from 1 July of the first year to 30 June of the final year.

In Appendix B, we model old-age mortality to obtain life tables with the same open-ended age groups as populations and deaths. We calculate life table functions for the 80+, 85–89, 90–94, and 95+ age groups; and also equalize the first age group between life tables (0 and 1–4 years originally) and populations and deaths (0–4 years).

2.2 NOTATION

2.2.1 *Introduction*

We base our notation on the traditional symbols of demography (KEYFITZ, 1977; PRESTON; HEUVELINE; GUILLOT, 2001), and follow or adapt some notation on subscripts, superscripts and operators from Canudas-Romo (2003, chapter 2).

We represent a demographic function or variable u measured over a continuous dimension x as $u(x)$, but if the dimension x is discrete or a subgroup, we represent it as u_x. We always denote time as t, for example, $u(t)$, $u(x,t)$ or $u_x(t)$. If x is an interval between x and $x+n$, then we write $u_{x,n}(t)$; likewise, if we are measuring u over a period from t to $t+h$, we write down the function or variable as $u(x,t,t+h)$ or $u_x(t,t+h)$. We always denote age as a, for instance, $u(a)$ for continuous, u_a for single (exact) age, and $u_{a,n}$ for age group. When variable u is measured over more than one dimension, we use superscripts or additional subscripts. For example, if variable u is measured over dimensions x and z at time t, we denote it as $u(x,z,t)$, or $u_x^z(t)$; if u has dimensions x, z, i, j, at time t we represent it by $u(x,z,i,j,t)$ or ${}_i^j u_x^z(t)$. Thus, we may have: life expectancy at age a and time t as $\mathring{e}(a,t)$; age-specific death rate between ages a and $a+n$ at time t as $m_{a,n}(t)$; population between ages a and $a+n$ for area i at period t to $t+h$ as $N_{a,n}^i(t,t+h)$; or the mean age of population of area i at time t as $N_{\bar{a}}^i(t)$.

2.2.2 *Operators and measures*

We illustrate operators (e.g., differences, summations, averages) and measures (e.g., proportions) by the addition of a Greek letter or a math accent. If the operation is over the function or variable, we add the letter or the accent to the demographic function or variable. If the operation is over the dimension or subgroup, we add the letter or the accent to the respective subscript or superscript.

a) differences are represented by adding a capital Greek delta

$$\Delta u(t) = u(t+h) - u(t) \tag{2.1a}$$

$$u_{\Delta x}(t) = u_{xa}(t) - u_{xb}(t) \tag{2.1b}$$

b) relative differences are indicated by including a capital Greek theta

$$\Theta u(t) = \frac{\Delta u(t)}{u(t)} \tag{2.2a}$$

$$u_{\Theta x}(t) = \frac{u_{\Delta x}(t)}{u_{x_a}(t)} \tag{2.2b}$$

c) absolute differences are expressed by adding vertical bars around the capital Greek delta

$$|\Delta| u(t) = |\Delta u(t)| \tag{2.3a}$$

$$u_{|\Delta x|}(t) = \left|u_{\Delta x}(t)\right| \tag{2.3b}$$

d) absolute relative differences are represented by adding vertical bars around the capital Greek theta

$$|\Theta| u(t) = |\Theta u(t)| \tag{2.4a}$$

$$u_{|\Theta x|}(t) = \left|u_{\Theta x}(t)\right| \tag{2.4b}$$

e) summations are expressed by substituting a dot for the dimension symbol

$$u_{\cdot}(t) = \sum\nolimits_x u_x(t) \tag{2.5a}$$

$$u_{\cdot}^{z}(t) = \sum\nolimits_x u_x^{z}(t) \tag{2.5b}$$

f) averages are expressed by adding a bar over (e.g., for an arithmetic mean)

$$\overline{u}_{\cdot}(t) = \frac{u_{\cdot}(t)}{n} \tag{2.6a}$$

$$u_{\overline{x}}^{z}(t) = \frac{u_{\cdot}^{z}(t)}{n} \tag{2.6b}$$

g) maximums are indicated by a vee (∨)

$$\vee u_{\cdot}(t) = \max_x u_x(t) \tag{2.7a}$$

$$u_{\vee x}^{z}(t) = \max_x u_x^{z}(t) \tag{2.7b}$$

h) minimums are denoted by a wedge (∧)

$$\wedge u_{\cdot}(t) = \min_x u_x(t) \tag{2.8a}$$

$$u^z_{\wedge x}(t) = \min_x u^z_x(t) \tag{2.8b}$$

i) ratios or proportions are indicated by a tilde over

$$\tilde{u}(t) = \frac{u(t)}{s(t)} \tag{2.9a}$$

$$u^z_{\tilde{x}a}(t) = \frac{u^z_{xa}(t)}{u^z_{xb}(t)} \tag{2.9b}$$

3 *Population aging*

3.1 INTRODUCTION

As the world goes through the demographic transition, it moves from times of high mortality and fertility, and population growth rates around zero to a contemporary era of low mortality and fertility, and minimum or negative population growth rates. Consequently, the age distribution of the world population has changed significantly from initial larger proportions in the younger age groups to intermediary higher proportions in the working or producing age groups to final increasing proportions in the older age groups (LEE, 2003; DYSON, 2010). Yet across the globe, the demographic transition has varied concerning the onset, pace, and scale of mortality and fertility declines (REHER, 2004, 2011), leading to different processes of population aging. Several scholars have looked at population aging in various contexts and from distinct angles, both theoretically and empirically. But the international literature lacks a systematic comparative analysis of the demographic determinants of population aging.

We examine the contribution of births, deaths, and migrations to world population aging from 1950 to 2100. Our analysis covers populations that are in distinct stages of the demographic transition, allowing us to analyze the role of the demographic determinants of population aging in diverse demographic contexts. First, we compare some demographic measures of population aging. Second, we present approaches that demographers use to investigate the demographic determinants of population aging, including two mathematical expressions introduced by Preston, Himes, and Eggers (1989) to decompose the rate of change in the mean age of a population. Third, we detail our methods. Fourth, we decompose the rate of change in the mean age of a population. Last, we propose a categorization of the stages of the demographic transition based on the demographic determinants of population aging.

3.2 MEASURES OF POPULATION AGING

We can measure population aging based on summary indicators of the age structure that can be either proportions of age groups, ratios between age groups, or measures of central tendency. The age groups are commonly based on life cycle stages related to physical or economic conditions, including young, working or producing adults, and old or retired. The ratio of the old-age population to the working age population is known as the old age dependency ratio (OADR), and its inverse is known as the support ratio (GOLDSTEIN, 2009; HOBBS, 2004). Here, we adopt the age group categorization of the United Nations (2017a, p. 33–35, 87): zero to 19 years of age for the younger ages (0–19 years), 20 to 64 years of age for the working ages (20–64 years), and 65 years of age and older for the older ages (65+ years). Accordingly, the United Nations (2017a) OADR is the ratio of the population 65 years of age and older to the population 20 to 64 years of age.

Common measures of central tendency of the age structure are the mean, median, and mode ages of the population. Preston, Himes, and Eggers (1989) used the rate of change in the mean age of the population as an indicator of population aging. On the one hand, the disadvantages of using the mean age are the non-symmetric age structure of populations and the required assumptions about the population age distribution within the open-ended age group (GOLDSTEIN, 2009; HOBBS, 2004). On the other hand, the mean age is easier to understand (GOLDSTEIN, 2009), is the leading measure of central tendency used in the social sciences (PRESTON; HIMES; EGGERS, 1989; PRESTON; STOKES, 2012), is influenced by all values in the distribution and to its variations (HOBBS, 2004; MURPHY, 2017), gives more weight to values at the right tail of the age structure (i.e., the oldest ages), and is related to the covariance with age (PRESTON; HIMES; EGGERS, 1989; VAUPEL; CANUDAS-ROMO, 2002). Also, it is highly correlated to the proportion of the total population 65 years of age and older (PRESTON; HIMES; EGGERS, 1989; MURPHY, 2017).[1]

3.3 DEMOGRAPHIC DETERMINANTS OF POPULATION AGING

Traditionally, demographers use two approaches to investigate the demographic contexts that promote changes in the age structures of populations. The first is founded on the formal dynamics and comparative statics of the stable population model (COALE, 1957, 1972; KEYFITZ, 1968, 1977; LEE, 1994; LOTKA, 1922, 1939; PRESTON, 1974). The second is based on counterfactual population projections (GRIGSBY; OLSHANSKY, 1989; HERMALIN, 1966; HEUVELINE, 1999; LEE; ZHOU, 2017; MOREIRA, 1997; YU; HORIUCHI, 1987).

Although distinct, both approaches consistently reach the same conclusions that fertility is the main determinant of population aging. For example, at the Population Association of America (PAA) presidential address of 1980, Siegel (1980) reviewed stable population theory and population projections studies from demographers. He defined as an error of interpretation the belief that the decline of mortality is the primary factor of population aging, and associated this error to lay persons, many social scientists, and government officials. According to Siegel (1980), demographers have a responsibility to spread the correct message, namely, fertility is the primary determinant of population aging (SIEGEL, 1980, p. 346–347). Thirty years later, Dyson (2010, p. 20–21) indirectly endorsed Siegel (1980)'s view, by emphasizing that the causal relationship between fertility decline and population aging is "deterministic — the consequence of basic population dynamics [...]", and that "[m]any people incorrectly ascribe population ageing within the [demographic] transition to mortality decline" (DYSON, 2010, p. 231).

Indeed, this conclusion is "consistent with a stylized demographic transition model" (MURPHY, 2017, p. 257), as delineated by Dyson (2010, p. 20–23): pre-transitional populations are young because fertility rates are high; as mortality declines, first at childhood ages, populations become younger; later, mortality declines at all ages with negligible consequences for the age structure; but as fertility declines, the proportion of the population in the younger age groups falls, and

[1] Preston, Himes, and Eggers (1989) for 17 regions of the world in 1970 and 1980, Murphy (2017) for 11 European countries from 1850 to 2012.

populations age; ultimately, death and birth rates are low and balanced, and the growth of populations are around zero; post-transitional populations are old because fertility rates are low. Dyson (2010) used comparative statics of the pre-transitional and 2010 age structures of Sweden and Sri Lanka to support his assertions.

The stable population model, however, has limited applicability to access the demographic contexts responsible for population aging, since very few modern populations meet the condition of stability (LEE; ZHOU, 2017; PRESTON; HIMES; EGGERS, 1989; PRESTON; STOKES, 2012). Besides, counterfactual population projections assume unrealistic scenarios (i.e., constant mortality or fertility over very long periods); and are sensitive to the choice of the starting date, which may lead to conflicting conclusions (e.g., changes in fertility made the population older vis-à-vis younger) (MURPHY, 2017). Moreover, no population follows a simple demographic transition model, or observes long-term constant mortality and fertility (MURPHY, 2017). Therefore, both approaches have limitations to explain what we can observe in practice (MURPHY, 2017; PRESTON; HIMES; EGGERS, 1989; PRESTON; STOKES, 2012). Also, they do not quantify the influence of mortality and fertility to population aging, and consequently derive weak factual evidence that fertility is the primary determinant of population aging (MURPHY, 2017).

Nevertheless, the central point from the stable population model and its extensions to non-stable populations (BENNETT; HORIUCHI, 1981; PRESTON; COALE, 1982) stands. The age distribution of any population changes not because of mortality, fertility, or migration levels, but because mortality, fertility, or migration rates are changing or have changed in the recent past (HORIUCHI; PRESTON, 1988; PRESTON; HIMES; EGGERS, 1989). Usually, there is a confusion between levels of rates with changes in rates because people's minds "perform the wrong experiment" (PRESTON; STOKES, 2012, p.224) or employ the wrong verb tense. As an illustration, a population with low levels of fertility or old-age mortality is older than it *would be* if it had higher levels of fertility or old-age mortality, not necessarily older than it *was*.

A breakthrough came in 1989. Preston, Himes, and Eggers (1989) introduced two related expressions that quantify the demographic contexts responsible for changes in the age structure of any population at a moment in time. Specifically, they developed two mathematical expressions to decompose the rate of change in the mean age of a population into its demographic determinants. The first mathematical expression from Preston, Himes, and Eggers (1989) (PHE I) decomposes the rate of change in the mean age of a population into *rejuvenating effects* of births, deaths, in-migration, and out-migration. The second mathematical expression from Preston, Himes, and Eggers (1989) (PHE II) decomposes the rate of change in the mean age of a population into age-specific population growth rates, age-specific proportions in the total population, and age-specific differences to the mean age of the population.[2]

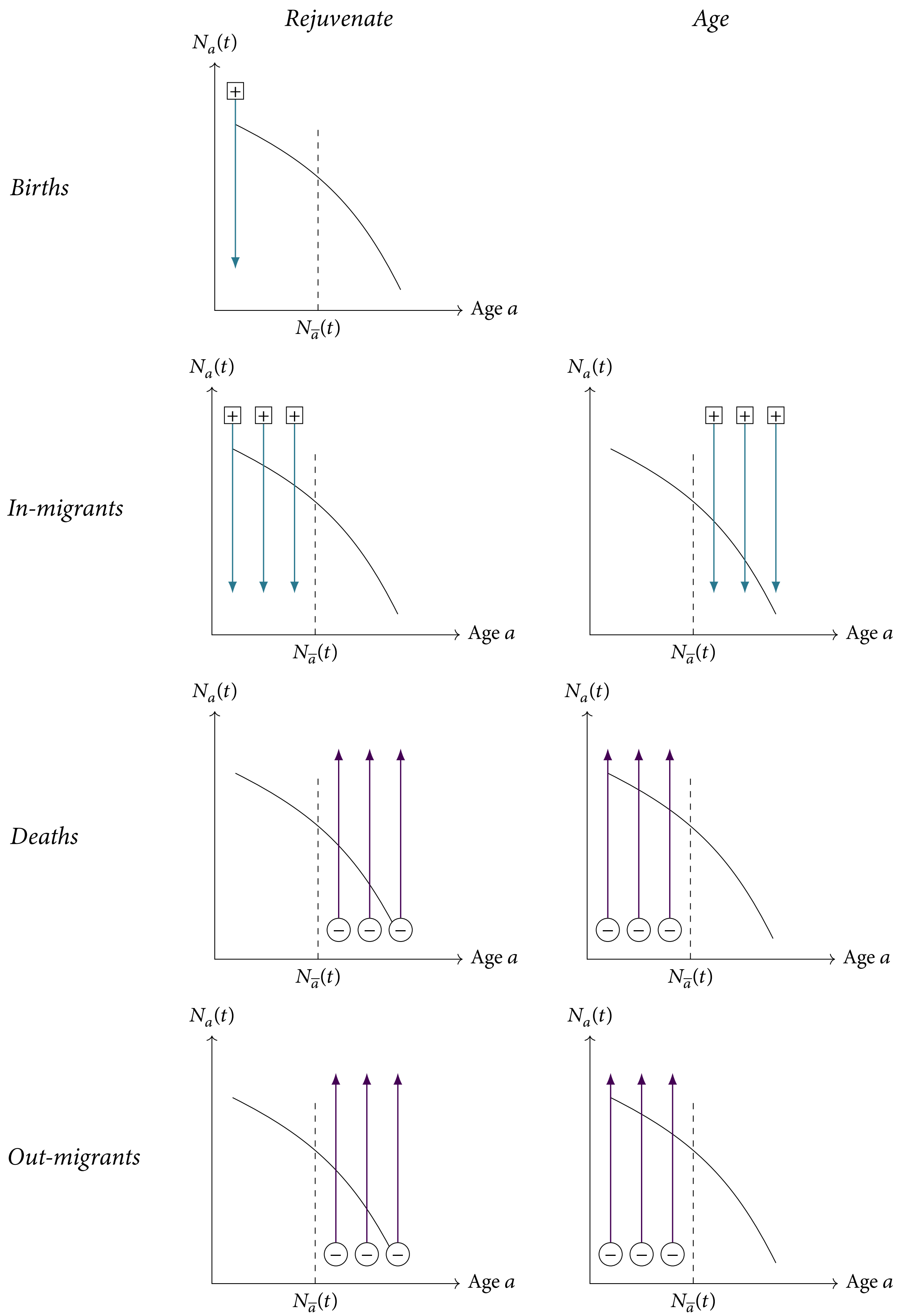
[2] Preston, Himes, and Eggers (1989) further mapped the age-specific population growth rates of the PHE II into the adjacent birth cohorts' rate of change in births, rate of change in the cumulative age-specific mortality rates, and rate of change in the cumulative age-specific net migration rates (HORIUCHI; PRESTON, 1988; PRESTON; COALE, 1982). Murphy (2017) extended the PHE II, by decomposing the birth cohort component into a fertility rate term and the corresponding population at risk, incorporating both the current direct effect of fertility and the indirect effect of historical fertility, mortality, and migration rates.

3.4 METHODS

The PHE II is preferable to the PHE I, since it decomposes aging into a fertility rate term and the corresponding population at risk (MURPHY, 2017), a mortality rate term, and a migration rate term, whereas the PHE I incorporates effects of the age structure when it decomposes aging into the rejuvenating effects of births, deaths, in-migration, and out-migration. The PHE II, however, demands a minimum of one hundred years of continuous data to calculate the first change of the mean ages, which may limit its applicability to recent periods and short-term comparison intervals, as in Preston, Himes, and Eggers (1989) for the United States and Sweden in 1980–1985, and Preston and Stokes (2012) for more developed and less developed countries in 2005–2010 (MURPHY, 2017). Since we do not want to limit the period scope of our analysis, we use the PHE I with the 2017 UN REVISION.

The PHE I builds upon one fundamental demographic truth: every person ages one year by each one calendar year. Therefore, any population has the natural tendency to age if there are no births, no deaths, and no migration. Births enter populations at age zero, below the mean age of the population; therefore, they rejuvenate populations. In-migrants also enter populations; if the mean age of in-migrants is below the mean age of the population, they rejuvenate populations. On the contrary, deaths and out-migrants exit populations; for both variables, if the mean age of occurrence is below the mean age of the population, they age populations (PRESTON; HIMES; EGGERS, 1989). Figure 1 uses stylized population age distributions to illustrate how births, in-migrants, deaths, and out-migrants rejuvenate or age populations when their mean ages are either below or above the mean age of the population ($N_{\overline{a}}$).

Figure 1 – How births, in-migrants, deaths, and out-migrants rejuvenate or age populations



Source: Author's creation, based on Preston, Himes, and Eggers (1989).
Note: ⊞ indicate entrances to the population. ⊖ indicate exits from the population.

Formally, these associations can be expressed in Equation 3.1 (PRESTON; HIMES; EGGERS, 1989, p. 695). Let N be population; a, age; t, time; I, in-migrants; D, deaths; O, out-migrants; b, crude birth rate; i, crude in-migration rate; d, crude death rate; o, crude out-migration rate; and $dN_{\overline{a}}(t)/dt$, the first derivative of the mean age of the population ($N_{\overline{a}}$) with respect to time:

$$\begin{aligned} \frac{dN_{\overline{a}}(t)}{dt} = 1 & \\ & - b(t) \cdot N_{\overline{a}}(t) \\ & - i(t) \cdot [N_{\overline{a}}(t) - I_{\overline{a}}(t)] \\ & - d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] \\ & - o(t) \cdot [O_{\overline{a}}(t) - N_{\overline{a}}(t)] \end{aligned} \tag{3.1}$$

In Equation 3.1, 1 is the population natural tendency to age one time unit ($dN_{\overline{a}}(t)$) by each one calendar time unit (dt), $b(t) \cdot N_{\overline{a}}(t)$ is the rejuvenating effect of births, $d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)]$ is the rejuvenating effect of deaths, $i(t) \cdot [N_{\overline{a}}(t) - I_{\overline{a}}(t)]$ is the rejuvenating effect of in-migration, and $o(t) \cdot [O_{\overline{a}}(t) - N_{\overline{a}}(t)]$ is the rejuvenating effect of out-migration. That is, the rejuvenating effects of births, deaths, in-migration, and out-migration are the products of their respective relative volumes (i.e., crude rates) and age selectivities (i.e., mean age differences to the mean age of the population).

Since the 2017 UN REVISION does not include migration age schedules, and is limited to net numbers of migrants ($I - O$) and net migration rates ($i - o$), it precludes the estimation of the mean age of migrations. Therefore, we adopt an approach similar to the one used elsewhere (PRESTON; HIMES; EGGERS, 1989; PRESTON; STOKES, 2012) for the PHE II, and compute the rejuvenating effect of net migration as a residual ($\epsilon_{\overline{a}}$), specifically,

$$\epsilon_{\overline{a}}(t) = i(t) \cdot [N_{\overline{a}}(t) - I_{\overline{a}}(t)] + o(t) \cdot [O_{\overline{a}}(t) - N_{\overline{a}}(t)] \tag{3.2}$$

$$\implies \frac{dN_{\overline{a}}(t)}{dt} = 1 - b(t) \cdot N_{\overline{a}}(t) - d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] - \epsilon_{\overline{a}}(t) \tag{3.3}$$

We analyze how population aging varies in the world from 1950 to 2100.[3] Specifically, we decompose the rate of change in the mean age of the population according to Equation 3.3.[4]

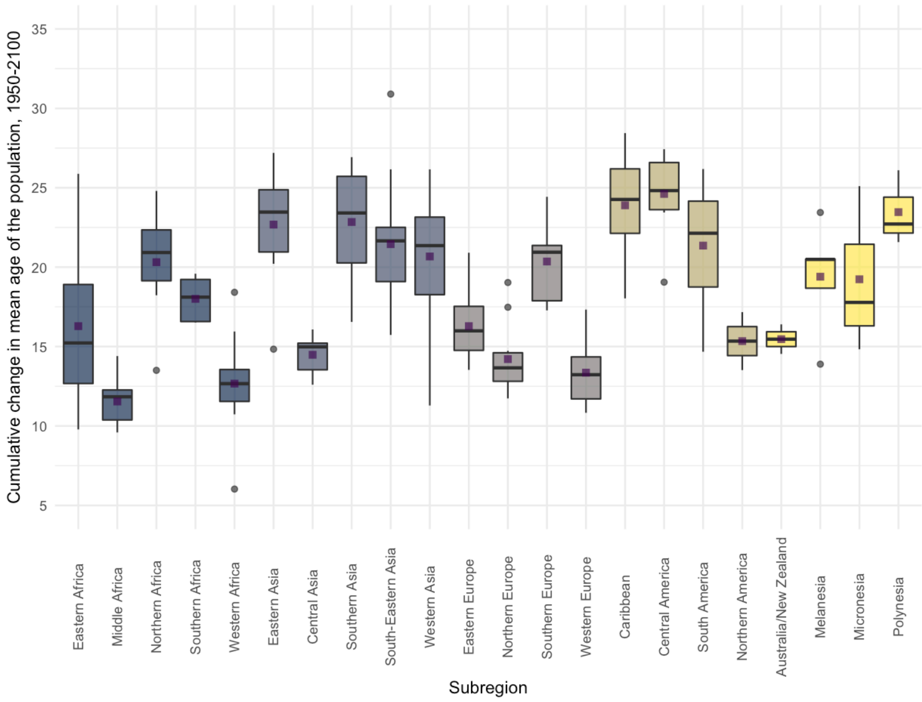
3 For a study that applies the PHE I to Brazil from 1950 to 2100, see Myrrha, Turra, and Wajnman (2017).

4 In the 2017 UN REVISION, for 6,030 observations from 201 countries and areas multiplied by 30 five-year periods, the $N_{\overline{a}}$ and the OADR are correlated at 0.928 (Pearson), and partially correlated at 0.928 (control for country/area), 0.857 (control for year), and 0.857 (control for year and country/area). Figures 118 and 119 in section C.1, Appendix C, plot the $N_{\overline{a}}$ by the OADR, whole population and subregions, confirming the high correlation between the two measures.

3.5 DEMOGRAPHIC DETERMINANTS OF WORLD POPULATION AGING

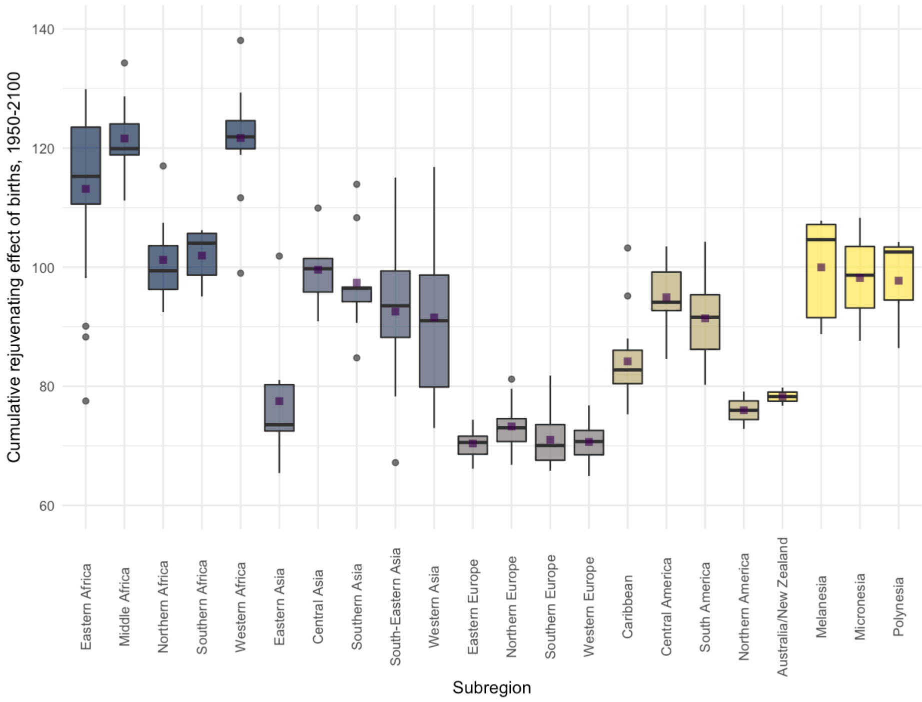
If there were no births, deaths or migration during the 150 years covered by the 2017 UN REVISION, the mean age of all populations would have increased by the same 150 years. Most populations, however, age between 10 and 25 years (Figure 2), with some aging as little as 6 years (Benin) and some aging as much as 31 years (Singapore). Although some subregions present similar levels of cumulative changes in the mean age of the population (e.g., Western Africa, Central Asia, and Northern Europe), the demographic determinants of aging behind these changes are quite distinct. This is what we observe in Figures 3 and 4 that present the cumulative rejuvenating effect of births and the cumulative rejuvenating effect of deaths from 1950 to 2100 by subregion. The subregions of Europe have the lowest cumulative rejuvenating effect of births (around 70 years), and the highest cumulative rejuvenating effect of deaths (around 60 years). Eastern, Middle, and Western Africa subregions present the highest cumulative rejuvenating effect of births (around 120 years), and the lowest cumulative rejuvenating effect of deaths (around 15 years). The demographic determinants of aging of Central Asia are intermediary to these, presenting cumulative rejuvenating effect of births around 100 years, and cumulative rejuvenating effect of deaths around 35 years. Cumulative rejuvenating effect of births is prominent in Niger (138 years), Angola (134 years), Somalia and Mali (around 130 years), Hungary and Greece (about 66 years), and Japan and Germany (about 65 years). Extreme cumulative rejuvenating effect of deaths are found in Niger (0.79 year), Angola (5.9 years), Mali (7.6 years), Somalia (10 years), Croatia (67 years), Hungary (68 years), and Bulgaria (69 years) and Latvia (69.5 years). The cumulative rejuvenating effect of migration (Figure 5) is small when compared with those from births or deaths. Nevertheless, migration ages the Caribbean and Polynesia, and rejuvenates Western Asia, Northern Europe, Western Europe, Northern America, and Australia/New Zealand. Most notably, migration cumulatively ages the United States Virgin Islands by 18 years, Grenada by 14 years, Jamaica, Martinique, Saint Vincent and the Grenadines, Barbados, and Guadeloupe between 10 and 12 years; and cumulatively rejuvenates Canada, Switzerland, Australia and Kuwait between 10 and 12 years, Luxembourg and Bahrain by 16 years, Macao by 19 years, and Qatar and the United Arab Emirates by 29 years.

Figure 2 – Cumulative change in mean age of the population from 1950 to 2100 by subregions



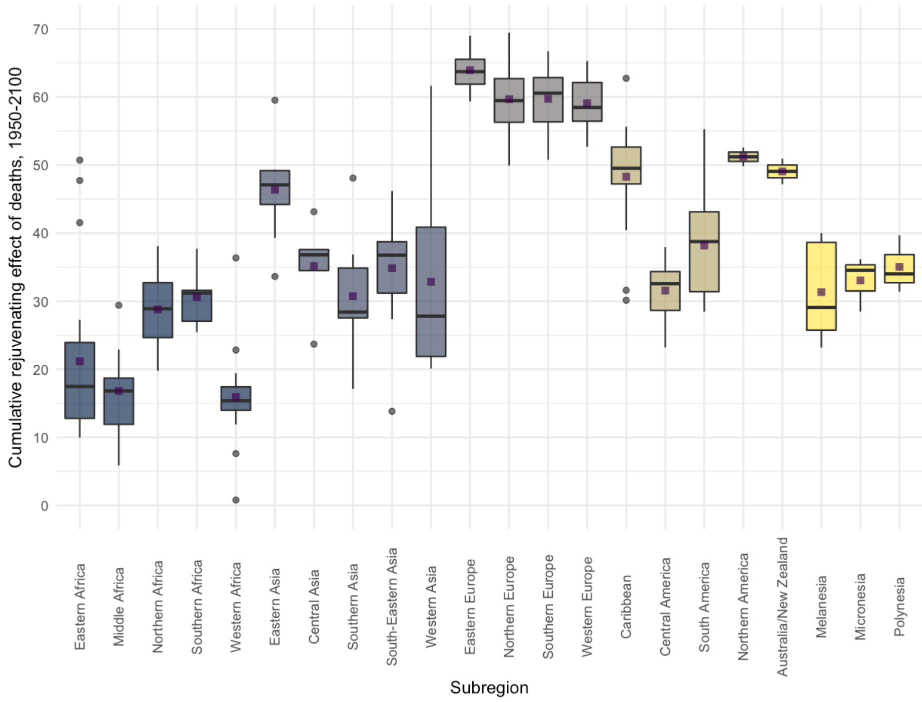
Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

Figure 3 – Cumulative rejuvenating effect of births from 1950 to 2100 by subregions



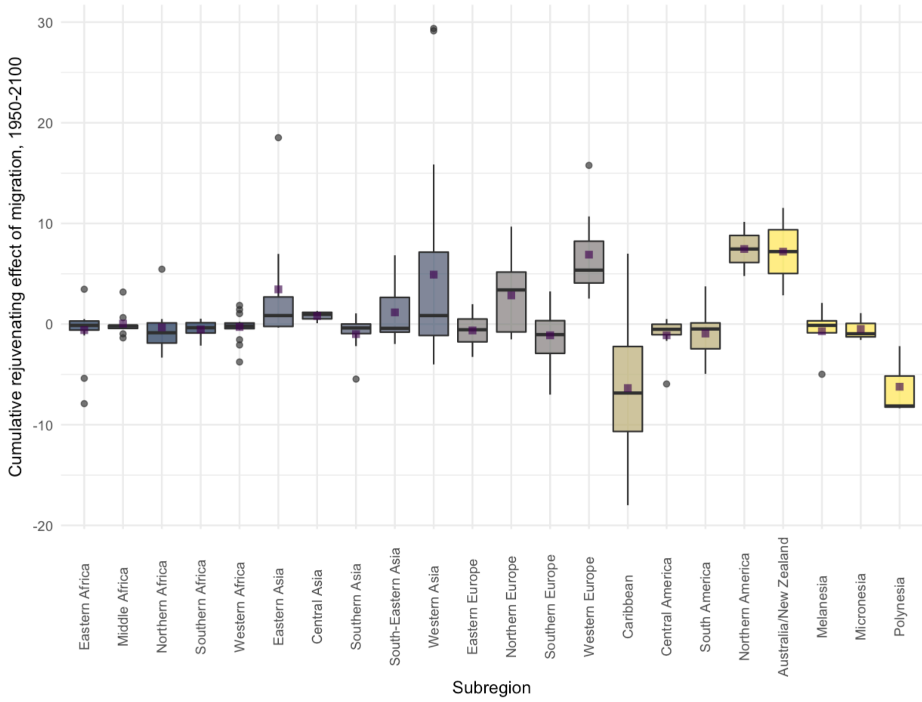
Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

Figure 4 – Cumulative rejuvenating effect of deaths from 1950 to 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

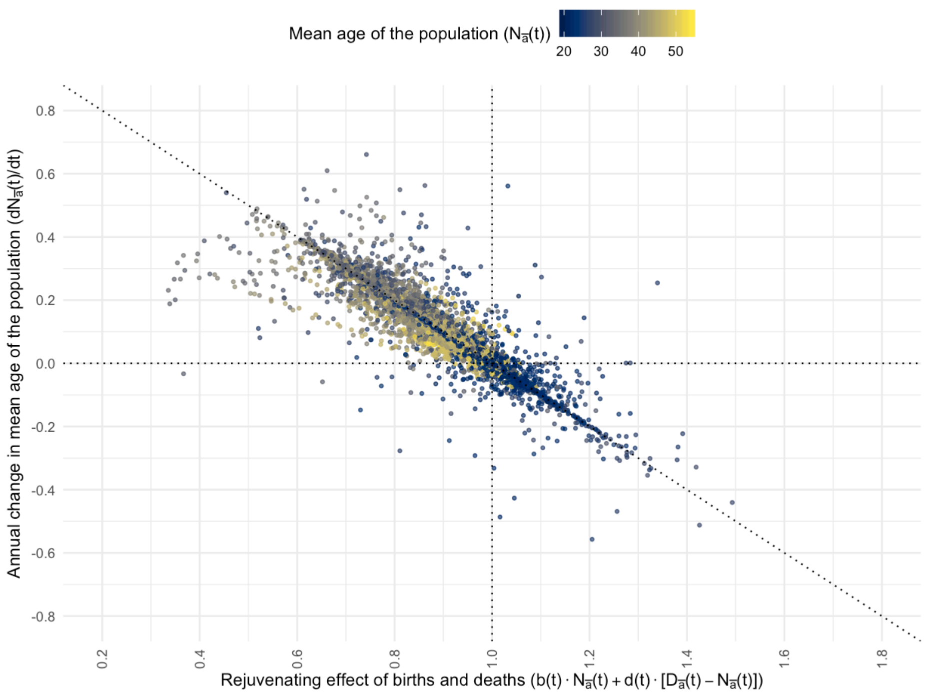
Figure 5 – Cumulative rejuvenating effect of migration from 1950 to 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

In Figure 6, we present the combined rejuvenating effect of births and deaths ($b(t) \cdot N_{\overline{a}}(t) + d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)]$) by the annual rate of change in the mean age of the population ($dN_{\overline{a}}(t)/dt$). The observations are displayed around a line that represents Equation 3.3 when the rejuvenating effect of net migration ($\epsilon_{\overline{a}}$) is equal to zero. Observations that depart from this line indicate the existing rejuvenating effects of net migration ($\epsilon_{\overline{a}}$). The combined rejuvenating effect of births and deaths varies from around 1.3 to 0.6 years per calendar year, while the change in the mean age of the population varies from rejuvenating 0.3 year per calendar year to aging 0.4 year per calendar year.[5] Figure 7 shows the rejuvenating effect of net migration ($\epsilon_{\overline{a}}$) by the annual rate of change in the mean age of the population ($dN_{\overline{a}}(t)/dt$).[6]

Figure 6 – Combined rejuvenating effect of births and deaths ($b(t) \cdot N_{\overline{a}}(t) + d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)]$) by annual rate of change in the mean age of the population ($dN_{\overline{a}}(t)/dt$)



Source: Author's calculations, based on United Nations (2017c).

5 Figures 120 and 121 in section C.2, Appendix C, detail these results by subregions.

6 Since the rejuvenating effect of net migration ($\epsilon_{\overline{a}}$) is computed as a residual, it incorporates any errors that are inherent to our estimates. Depending on the age selectivity between the mean age of the population ($N_{\overline{a}}$) and the mean age of net migration, $\epsilon_{\overline{a}}$ may be close to zero even if the net migration rates are high. Still, independently of the age selectivity between the mean age of the population ($N_{\overline{a}}$) and the mean age of net migration, $\epsilon_{\overline{a}}$ should always be close to zero if the net migration rates and the inherent errors in our estimates are very low. Our values of $\epsilon_{\overline{a}}$ are consistent with robust estimates, that is, they are close to zero when absolute net migration rates are less than 0.0001 (see Figures 122 and 123 in section C.2, Appendix C).

Figure 7 – Rejuvenating effect of net migration ($\epsilon_{\bar{a}}$) by annual rate of change in the mean age of the population ($dN_{\bar{a}}(t)/dt$)

Source: Author's calculations, based on United Nations (2017c).

3.6 DEMOGRAPHIC DETERMINANTS OF WORLD POPULATION AGING AND THE DEMOGRAPHIC TRANSITION

Based on the comparative statics of stable populations, Preston, Himes, and Eggers (1989) demonstrated that there is a pattern between the rejuvenating effect of births and the rejuvenating effect of deaths from scenarios of high mortality and high fertility to scenarios of low mortality and low fertility. The higher are mortality levels and fertility levels, the more of the combined rejuvenating effects of births and deaths originate from births, the less come from deaths. As mortality and then fertility decline, the more of the combined rejuvenating effects of births and deaths come from deaths, the less originate from births. We analyze how this *concerted pattern* unfolds in the diverse demographic scenario of the 2017 UN REVISION. We both examine whether there is a general concerted pattern between the rejuvenating effect of births and the rejuvenating effect of deaths along the demographic transition, and propose a categorization of the stages of the demographic transition based on the demographic determinants of population aging.

In Figure 8, we present the rejuvenating effect of deaths by the rejuvenating effect of births.[7] Let the rejuvenating effect of net migration ($\epsilon_{\bar{a}}$) be equal to zero. Populations that are in the *mean age stability line* have combined rejuvenating effects of births and deaths equal to one,

[7] Figure 124 in section C.3, Appendix C, details Figure 8 by subregions.

the mean ages of the populations are constant, populations are neither aging nor rejuvenating.[8] Populations that are above this line have combined rejuvenating effects of births and deaths greater than one, the mean ages of the populations are decreasing, populations are rejuvenating.[9] Populations that are below this line have combined rejuvenating effects of births and deaths less than one, the mean ages of the populations are increasing, populations are aging.[10] Figure 8 suggests that there is a general concerted pattern between the rejuvenating effect of births and the rejuvenating effect of deaths along the demographic transition, which we propose to categorize into seven stages. We summarize these stages by indicators of the rejuvenating effect of births, the rejuvenating effect of deaths, and the mean age of the population in Table 4.[11]

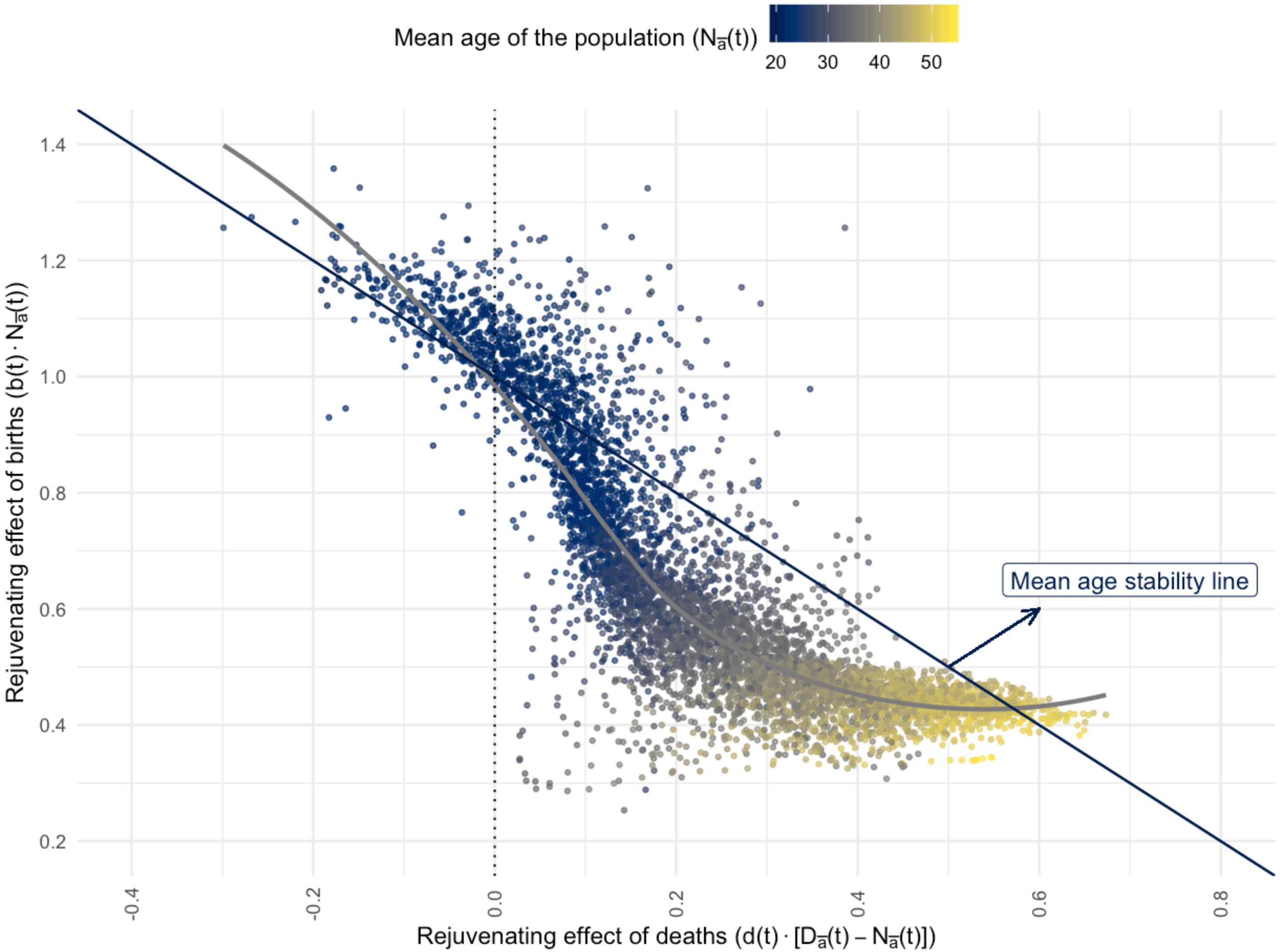
8 Equation 3.3 for $b(t) \cdot N_{\overline{a}}(t) + d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] = 1$ and $\epsilon_{\overline{a}}(t) = 0 \implies dN_{\overline{a}}(t)/dt = 0$.

9 $b(t) \cdot N_{\overline{a}}(t) + d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] > 1$ and $\epsilon_{\overline{a}}(t) = 0 \implies dN_{\overline{a}}(t)/dt < 0$.

10 $b(t) \cdot N_{\overline{a}}(t) + d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] < 1$ and $\epsilon_{\overline{a}}(t) = 0 \implies dN_{\overline{a}}(t)/dt > 0$.

11 Table 16 in section C.3, Appendix C, replicates Table 4 with additional indicators of the mean age of the population, the rejuvenating effect of births, and the rejuvenating effect of deaths.

Figure 8 – Rejuvenating effect of deaths ($d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)]$) by rejuvenating effect of births ($b(t) \cdot N_{\bar{a}}(t)$)



Source: Author's calculations, based on United Nations (2017c).

Table 4 – Indicators of the mean age of the population, rejuvenating effect of births, and rejuvenating effect of deaths by stage of the demographic transition

Stage	Mean age of the population		Rejuvenating effect of births $(b(t) \cdot N_{\bar{a}}(t))$	Rejuvenating effect of deaths $(d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)])$	Combined rejuvenating effect of births and rejuvenating effect of deaths	Example Countries
	$N_{\bar{a}}(t)$	$\frac{d}{dt} N_{\bar{a}}(t)$				
1	decrease	negative	1.0, 1.2	-0.2, 0.0	> 1	Turkey (1950–1955) Afghanistan (1980–1985) Somalia (1990–1995)
1A	minimum	zero	1.0	0.0	= 1	Peru (1965–1970) Pakistan (1970–1975) Chad (2005–2010)
2	increase	positive	1.0, 0.6	0.0, 0.2	< 1	Bulgaria (1950–1955) China (1955–1960) Angola (2010–2015)
3	increase	maximum	0.6	0.2	< 1	Japan (1970–1975) Philippines (2025–2030) Niger (2095–2100)
4	increase	positive	0.6, 0.4	0.2, 0.6	< 1	Austria (1950–1955) United States (1965–1970) Brazil (2025–2030)
4A	maximum	zero	0.4	0.6	= 1	Portugal (2060–2065) Spain (2060–2065) Jamaica (2080–2085)
5	decrease	negative	0.4	> 0.6	> 1	Poland (2070–2075) Albania (2085–2090) Puerto Rico (2090–2095)

Source: Author's creation and calculations, based on Preston, Himes, and Eggers (1989) and United Nations (2017c).

Initially, at *Stage 1*, mortality levels and fertility levels are high. Deaths are concentrated at infancy and childhood ages, and thus the age selectivity of deaths are negative (Figure 9). Consequently, populations observe negative rejuvenating effects of deaths between −0.2 and 0, and positive rejuvenating effects of births between 1.0 and 1.2. Despite the negative rejuvenating effects of deaths, the combined rejuvenating effects of births and deaths are greater than one, and thus populations rejuvenate. Then, mortality declines, largely at infancy and childhood ages, and thus the age distribution of deaths shifts to older ages and the age selectivity of deaths gradually increase to zero (Figure 9). The crude death rates (d) solely decline (Figure 10). Consequently, the rejuvenating effects of deaths increase exclusively from the rise of the age selectivity of deaths. Next, fertility declines, and thus the rejuvenating effects of births decrease. The rejuvenating effects of births decrease faster than the rejuvenating effects of deaths increase; consequently, the rates of change in the mean age of the population increase.[12] At the end of *Stage 1*, the combined rejuvenating effects of births and deaths cross the mean age stability line, and the mean ages of the populations ($N_{\overline{a}}$) reach a local minimum. We indicate this moment as *Stage 1A* at Table 4.

At *Stage 2*, populations observe positive rejuvenating effects of deaths between 0 and 0.2, and rejuvenating effects of births between 1.0 and 0.6. The combined rejuvenating effects of births and deaths are less than one, that is, populations age. Mortality and fertility continue to decline. Mortality declines at infancy and childhood ages accelerate, and thus the age selectivity of deaths steeply increase from 0 to 30 (Figure 9). The crude death rates (d) predominantly decline (Figure 10). Consequently, the rejuvenating effects of deaths still increase fundamentally from the rise of the age selectivity of deaths. The rejuvenating effects of births still decrease faster than the rejuvenating effects of deaths increase; consequently, the rates of change in the mean age of the population still increase.

At *Stage 3*, rejuvenating effects of deaths are around 0.2, and rejuvenating effects of births are just below 0.6. The combined rejuvenating effects of births and deaths are still less than one, and thus populations still age. Both rejuvenating effects arrive at an inflection point where the rate of change in the rejuvenating effects of births and the rate of change in the rejuvenating effects of deaths are the same; consequently, the rates of change in the mean age of the population reach a local maximum.[13]

At *Stage 4*, populations observe rejuvenating effects of deaths between 0.2 and 0.6, and rejuvenating effects of births between 0.6 and 0.4. The combined rejuvenating effects of births and deaths are still less than one, that is, populations still age. Mortality continues to decline, now mostly at middle and old ages, and thus the rise of the age selectivity of deaths decelerate (Figure 9). The crude death rates (d) increase (Figure 10). Consequently, the rejuvenating effects of deaths increase from the rise both of the age selectivity of deaths and of the crude death rates (d). Fertility also continue to decline, but now with more gradual reductions. The rejuvenating effects of births decrease slower than the rejuvenating effects of deaths increase; consequently,

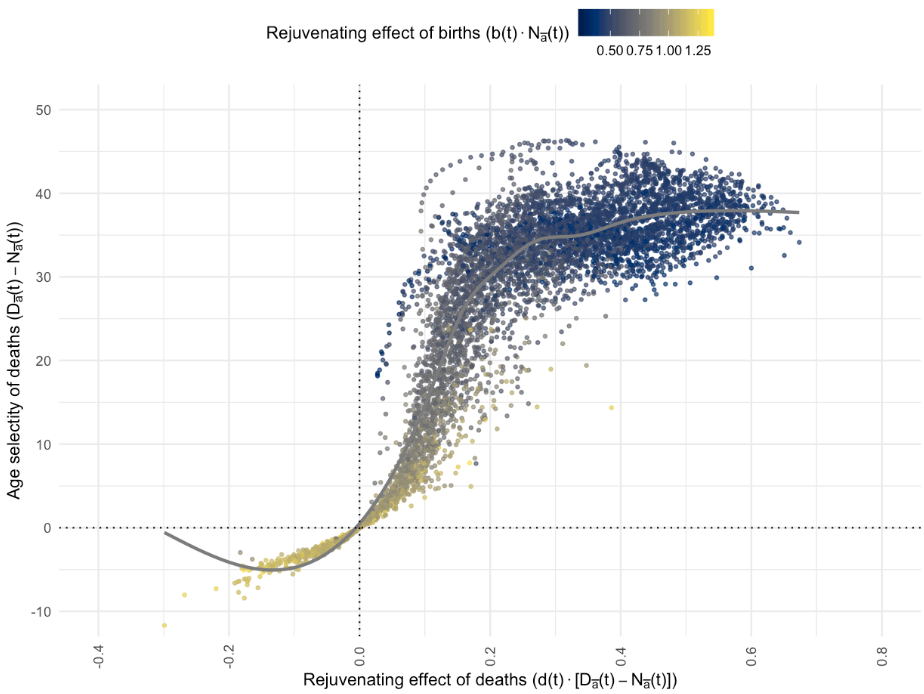
12 $-db(t)/dt \cdot N_{\overline{a}}(t) > dd(t)/dt \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] \iff d^2N_{\overline{a}}(t)/dt^2 > 0.$

13 $-db(t)/dt \cdot N_{\overline{a}}(t) = dd(t)/dt \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] \iff d^2N_{\overline{a}}(t)/dt^2 = 0.$

the rates of change in the mean age of the population decrease.[14] At the end of *Stage 4*, the combined rejuvenating effects of births and deaths cross the mean age stability line again, and the mean ages of the populations ($N_{\bar{a}}$) reach a local maximum. We indicate this moment as *Stage 4A* at Table 4.

Finally, at *Stage 5*, populations observe rejuvenating effects of births around 0.4, and rejuvenating effects of deaths above 0.6. The combined rejuvenating effects of births and deaths are again greater than one, and thus populations rejuvenate anew. Mortality continues to decline, this time mostly at old ages, and thus the age selectivity of deaths stabilize (Figure 9). The crude death rates (d) still increase (Figure 10). Consequently, the rejuvenating effects of deaths increase entirely from the rise of the crude death rates (d). Fertility stabilizes. The rejuvenating effects of births stabilize, and the rejuvenating effects of deaths still increase; consequently, the rates of change in the mean age of the population still decrease.

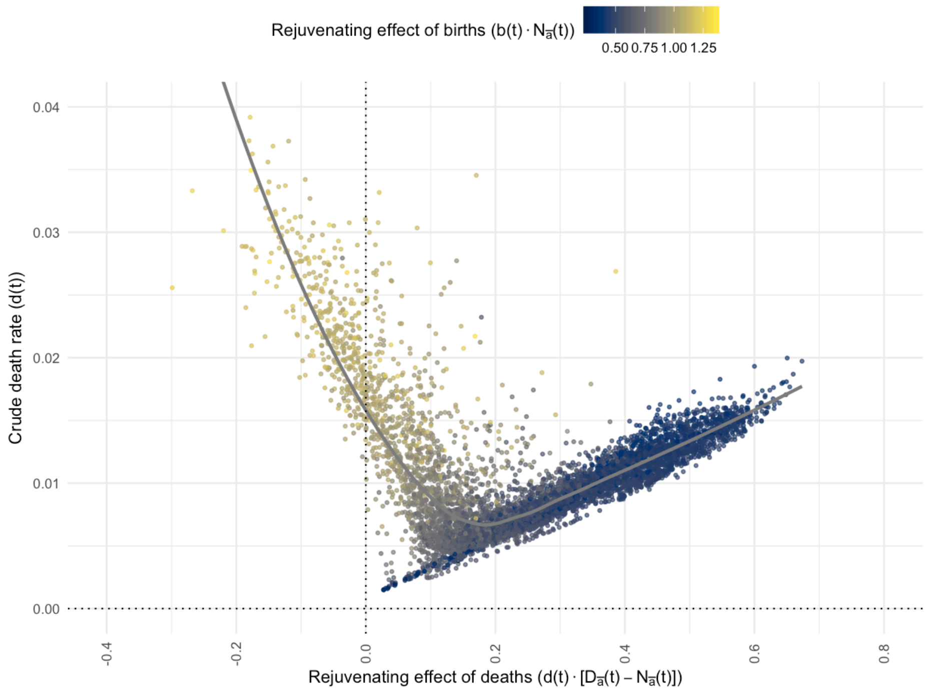
Figure 9 – Rejuvenating effect of deaths ($d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)]$) by age selectivity of deaths ($D_{\bar{a}}(t) - N_{\bar{a}}(t)$)



Source: Author's calculations, based on United Nations (2017c).

[14] $-db(t)/dt \cdot N_{\bar{a}}(t) < dd(t)/dt \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)] \iff d^2N_{\bar{a}}(t)/dt^2 < 0.$

Figure 10 – Rejuvenating effect of deaths ($d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)]$) by crude death rate ($d(t)$)

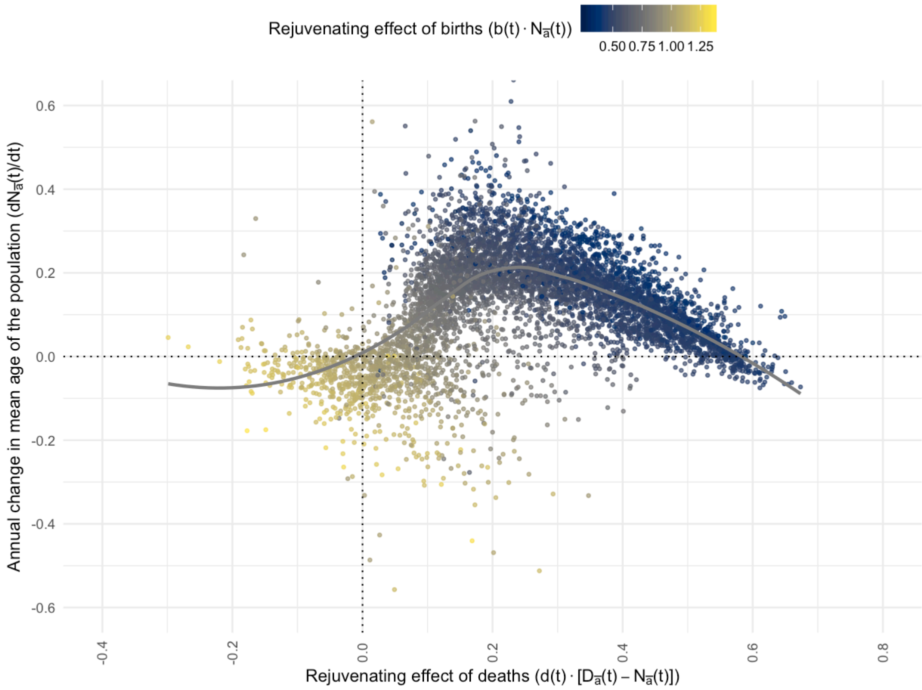


Source: Author's calculations, based on United Nations (2017c).

3.7 CONCLUSION

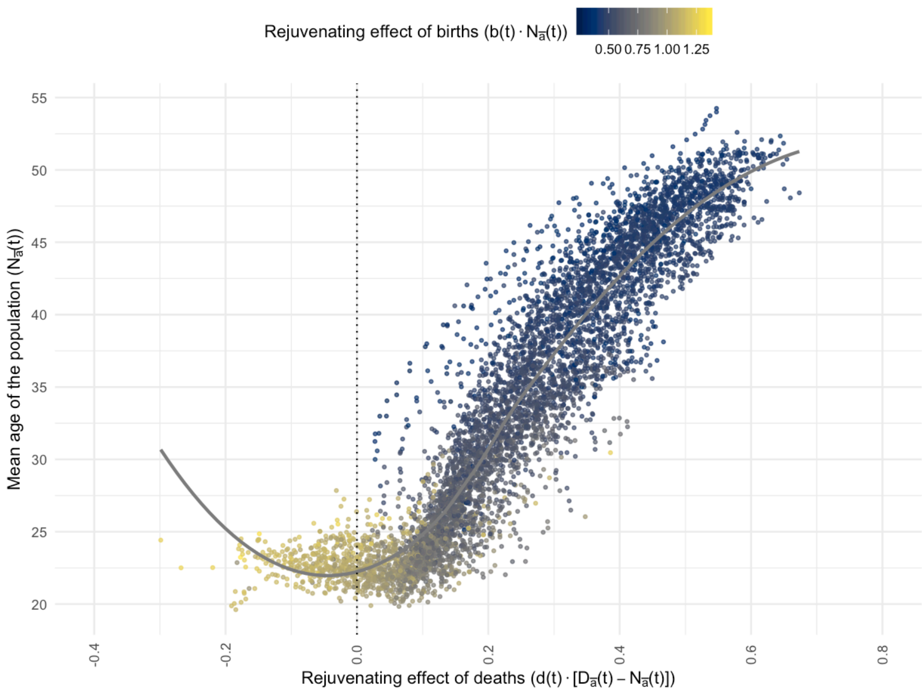
Earlier in this chapter, we acknowledge that, across the globe, the demographic transition has varied with respect to the onset, pace, and scale of mortality and fertility declines, leading to different processes of population aging. Yet now we argue that despite these variations, demographic transitions differ alongside a general concerted pattern between the rejuvenating effect of births and the rejuvenating effect of deaths. We propose a categorization of the stages of the demographic transition based on levels and indicators of the rejuvenating effects of births and deaths (Table 4). Across the globe, population aging varies alongside the same pattern and stages, and we may determine these stages by either only the rejuvenating effect of births, or only the rejuvenating effect of deaths as in Figure 11 and Figure 12.

Figure 11 – Rejuvenating effect of deaths $(d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)])$ by annual rate of change in the mean age of the population $(dN_{\bar{a}}(t)/dt)$



Source: Author's calculations, based on United Nations (2017c).

Figure 12 – Rejuvenating effect of deaths $(d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)])$ by mean age of the population $(N_{\bar{a}}(t))$



Source: Author's calculations, based on United Nations (2017c).

4 Period balances, age rebalances

4.1 INTRODUCTION

POPULATION AGING impacts retirement systems primarily because it changes the relation between beneficiaries and contributors. In the case of retirement systems structured on period financial balances, known as pay-as-you-go (PAYG) systems, sustainability is directly affected by variations in the old age dependency ratio (OADR), that is, the ratio of the population 65 years of age and older (i.e., potential beneficiaries) to the population 20 to 64 years of age (i.e., potential contributors). For example, in 1889, Germany approved the law that implement the world's first national disability and old-age social security system[1], which set the retirement age for old-age pensions at 70 years of age (STOLLEIS, 2013).[2] By that time, in 1890, Germany's[3] proportion of the total population with 65 years of age and older was 5.1%, and its OADR was 8.5% (RAHLF et al., 2015). One hundred and twenty-five years later, in 2015, these indicators had respectively changed to 21.1% and 34.8% (UNITED NATIONS, 2017c). Likewise, when the United States approved the Social Security Act of 1935, its proportion of the total population with 65 years of age and older was 6.1%, and its OADR was 10.7% (U.S. CENSUS BUREAU, 2016).[4] Eighty years later, in 2015, the same indicators were equal to 14.6% and 24.6% (UNITED NATIONS, 2017c). Similarly, the world's OADR has grown from 9.9% in 1950 to 12.8% in 2000, and is expected to be 28.3% in 2050, reaching 41.8% in 2100 (UNITED NATIONS, 2017c).

External demographic or economic factors that impact retirement systems' financial balances and, consequently, their contributions, benefits, or both are denominated "uninsurable risks" (SETTERGREN, 2001, p. 4). Uninsurable risks cannot be avoided by definition, and are pervasive, they exist in every retirement system, private or public, structured on period or cohort financial balances. Nevertheless, retirement systems can and should safeguard against the impact of these risks (SETTERGREN, 2001). Many policies may buffer the burden of population aging in PAYG systems, such as varying contributions, benefits, or both, and changing the normal ages of contribution and retirement.[5] Among the alternative policies, there is an increasing debate among actuaries, demographers, and economists about adjusting the retirement age based on gains in longevity. But since PAYG systems are established primarily on period financial balances, adjusting the retirement age based on gains in longevity, a life cycle characteristic, may not be

[1] Law Concerning Disability and Old-Age Insurance of 22 June 1889. *Gesetz. betr. die Invaliditäts- und Altersversicherung vom 22. Juni 1889* (STOLLEIS, 2013, p. 74).

[2] Only in 1916 the retirement age was lowered to 65 years of age. *Gesetz betr. Renten in der Invalidenversicherung v. 12. Juni 1916* (STOLLEIS, 2013, p. 90).

[3] Deutsche Zollverein.

[4] Excludes Alaska and Hawaii.

[5] Policies that automatically adjust benefits to uninsurable risks are "automatic stabilizers" of the type "automation of the first order" (SCHERMAN, 2011, p.18–22). Scherman (2011) defined two other types of automatic stabilizers: a) notional defined contribution (NDC) designs, specifically, PAYG systems where the contributions of each individual determines one's benefits; and b) automation of the second order, which are based on the PAYG system's financial balance itself.

effective in lessening the impact of population aging if the contribution of mortality to changes in the population age distribution is only moderate. Therefore, PAYG systems should contemplate the role of the rejuvenating effect of deaths in the definition of retirement age policies. Moreover, when old-age mortality declines, adjusting the retirement age based on gains in life expectancy, a mean age at death measure, may be less effective than based on gains in the modal age at death.

We use a *stylized demographic model* to analyze the burden of population aging in PAYG systems in the world from 1950 to 2100. In our stylized demographic model, all population (N) between the age of entry into the labor force (L) and the age of entry into retirement (R) works and contributes to the PAYG system; all population older than the age of entry into retirement (R) is retired and receives benefits from the PAYG system; contributions are equal to contribution rates (con) times wages (w); benefits are equal to benefit rates (ben) times wages (w); age of entry into the labor force (L) and age of entry into retirement (R) are initially fixed; contribution rates (con), benefit rates (ben), and wages (w) are not age-specific; and wages (w) do not vary in response to either the labor market dynamics or productivity changes.

We investigate to what degree the burden of population aging befalls contributors and beneficiaries in different period policy designs of PAYG systems. We benefit from using data that represent all stages of the demographic transition and a variety of demographic trajectories, to elucidate the primacy of changes in period population age structures to the equilibrium of PAYG systems. First, we present the main attributes of PAYG systems in three alternative policy designs: two classic and a third proposed by Musgrave (1981). Second, we review different approaches for measuring the old-age threshold or adjusting the retirement age based on gains in longevity, including a method introduced by Bayo and Faber (1981); and present policies of a selected group of countries that adjust the normal retirement age or retirement pensions based on gains in longevity. Third, we detail our methods and assumptions. Fourth, we estimate the distribution of the burden of population aging between contributors and beneficiaries in different policy designs of PAYG systems. Fifth, we assess the change in the retirement age based on gains in longevity, and how much it alleviates the burden of population aging on contributors and beneficiaries. Sixth, we propose a framework to investigate the effectiveness of policies that adjust the retirement age based on gains in longevity. Last, we propose adjusting the retirement age based on gains in the modal age at death and evaluate its effectiveness.

4.2 PERIOD BALANCES: PAY-AS-YOU-GO SYSTEMS

Pay-as-you-go (PAYG) systems are based on *period financial balances* and have no funding of assets. At every period, benefits are honored from contributions made in the same period, that is, each period pays for itself. The period financial balance ensues that the OADR must equal the ratio of the contribution rate (con) to the benefit rate (ben) (FERNANDES, 1993, p. 18–20, 93–97;

KEYFITZ, 1977, p. 262–265; KEYFITZ; GÓMEZ DE LÉON, 1980). Let a be age; and t, time:

$$\int_L^R N(a,t) \cdot con(t) \cdot w \, da = \int_R^\infty N(a,t) \cdot ben(t) \cdot w \, da \tag{4.1a}$$

$$\frac{con(t)}{ben(t)} = \frac{\displaystyle\int_R^\infty N(a,t)\, da}{\displaystyle\int_L^R N(a,t)\, da} \tag{4.1b}$$

Altogether, PAYG systems are built on *intergenerational solidarity*, for today's contributors honor the benefits of today's retirees, taking for granted that the benefits of tomorrow's retirees will be honored by tomorrow's contributors (FERNANDES, 1993, p. 18–20). In PAYG systems, it is implicitly assumed that *intergenerational transfers* are unlimited and unbreakable (KEYFITZ, 1982, 1985, 1988; KEYFITZ; GÓMEZ DE LÉON, 1980; LAPKOFF, 1991) and that, otherwise, the state will have the power and disposition to impose it (KEYFITZ, 1985, p. 29), that is, the "funding mechanism" is the altruism of future generations (LAPKOFF, 1991, p. 160).[6] Therefore, the policy designs of PAYG systems reflect the nature of their social *intergenerational contracts* upon which rest their credibility, long-term political viability, and uninterrupted acceptance as fair by both contributors and beneficiaries (MUSGRAVE, 1981, p. 96–98). Traditionally, we can structure PAYG systems upon two classic policy designs. In the one, the benefit rate (*ben*) is fixed and at every period total contributions adjust via the contribution rate (*con*) to the total benefits honored by the system. This policy design is known as defined benefit (DB) pay-as-you-go (PAYG) system,

$$con(t) = \frac{\displaystyle\int_R^\infty N(a,t)\, da}{\displaystyle\int_L^R N(a,t)\, da} \cdot ben(t) \tag{4.2}$$

In the other, the contribution rate (*con*) is fixed and at every period total benefits adjust via the benefit rate (*ben*) to the total contributions made to the system. This policy design is known as defined contribution (DC) pay-as-you-go (PAYG) system,

$$ben(t) = \frac{\displaystyle\int_L^R N(a,t)\, da}{\displaystyle\int_R^\infty N(a,t)\, da} \cdot con(t) \tag{4.3}$$

Therefore, different period policy designs imply distinct life cycle perspectives. In DB PAYG systems, the *promise* is if from L to R each individual contributes a proportional share of the total benefits honored by the system, from R until one's death each individual will receive a fixed percentage of the average wage. In DC PAYG systems, the *promise* is if from L to R each individual

6 On the contrary, fully funded retirement systems, also known as actuarial or reserve, are based on *cohort financial balances* and do have funding of assets. For every cohort, benefits are honored from contributions made by the same cohort, that is, each cohort pays for itself. There are no intergenerational contracts or solidarity (BOURGEOIS-PICHAT, 1978; FERNANDES, 1993, p. 69–74, 93–97; KEYFITZ, 1977, p. 47–48, 262–265; KEYFITZ; GÓMEZ DE LÉON, 1980).

contributes a fixed percentage of one's wage, from R until one's death each individual will receive a proportional share of the total contributions made to the system (FERNANDES, 1993, p. 18–20).

Moreover, different policy designs also lead to distinct distributions of the uninsurable risk of population aging. In DB PAYG systems, this risk befalls contributors via rising *con*, and thus per capita benefits ($ben \cdot w$) improve relative to per capita net wages ($[1-con]\cdot w$). In DC PAYG systems, this risk befalls beneficiaries via declining *ben*, and thus per capita benefits deteriorate relative to per capita net wages (MUSGRAVE, 1981, p. 99–104). As a "fair and practicable solution" for the distribution of the risk of population aging, Musgrave (1981, p. 97, 104) proposed a new policy design that holds constant the ratio of per capita benefits to per capita net wages (ϕ) by adjusting both *con* and *ben* at every period. He named it fixed relative position (FRP) pay-as-you-go (PAYG) system,[7]

$$\phi(t) = \frac{ben(t)}{1 - con(t)} \tag{4.4a}$$

$$con(t) = \frac{\phi \cdot \int_R^\infty N(a,t)\,da}{\int_L^R N(a,t)\,da + \phi \cdot \int_R^\infty N(a,t)\,da} \tag{4.4b}$$

$$ben(t) = \frac{\phi \cdot \int_L^R N(a,t)\,da}{\int_L^R N(a,t)\,da + \phi \cdot \int_R^\infty N(a,t)\,da} \tag{4.4c}$$

In FRP PAYG systems, consequently, the life cycle perspective or *promise* is if from L to R each individual contributes a proportional share of the total benefits honored by the system, and if from R until one's death each individual receives a proportional share of the total contributions made to the system, from L until one's death the ratio of per capita benefits to per capita net wages (ϕ) will hold constant. The risk of population aging befalls both contributors via rising *con*, and beneficiaries via declining *ben*, but ϕ neither improves nor deteriorates. Therefore, FRP PAYG systems have more flexibility for *ad hoc* policy changes to *con* or *ben*, observing that ϕ holds constant, favor greater credibility, long-term viability, and acceptance than DB and DC PAYG systems, that is, a stronger foundation for the social intergenerational contract.

4.3 AGE REBALANCES: EQUIVALENT RETIREMENT AGES

Ryder (1975) was the first demographer to propose "a new index of old age" based on changing the concept of age from the number of years elapsed since birth to the numbers of years until

[7] Musgrave (1981, p. 97) identified five PAYG policy designs: a) ad hoc provision, which is a loose agreement where at every period voters decide the level of support; b) fixed replacement rate, which is equivalent to DB; c) fixed contribution rate, which is equivalent to DC; d) fixed replacement rate adjusted, which is similar to DB, but the wage base of beneficiaries is adjusted for productivity and wage increases of contributors; and e) fixed relative position (FRP). In our stylized demographic model, there is no difference between the wage bases of beneficiaries and contributors; consequently, fixed replacement rate adjusted is equivalent to DB.

death. Entry into old-age would be determined not by chronological age, but by the age where life expectancy is equal to "[...] some arbitrary length of time, such as 10 years [...]" (RYDER, 1975, p. 15–17).

Siegel (1980) drew attention to Ryder (1975)'s concept of old-age and its "[...] economic, social, legal, and ethical implications [...]" (SIEGEL, 1980, p. 346), that is, should socioeconomic groups who have higher mortality or morbidity have earlier access to old-age benefits? He observed that demographers generally use chronological classifications to define the limits of old-age, whereas its cultural definitions vary depending upon the longevity of a population (SIEGEL, 1980, p. 345–346). He later applied Ryder (1975)'s concept while reviewing new measurements of aging in a work on the aspects of the older population in the United States (SIEGEL; DAVIDSON, 1984).

The first important addition to Ryder (1975)'s view was the independent work of Bayo and Faber (1981). Their motivation was the public interest, debate, and recommendations of federal government commissions for a gradual increase of the United States Social Security normal retirement age. Gains in life expectancy since 1940 would endorse the proposed gradual increase, however there was no foundation to decide what would be an equitable increase. They recognized that it would be unfair or unreasonable to expect that all extra years of life be spent either in work or in retirement, and proposed a method to measure "equivalent retirement ages" based on declines in mortality "which will be equitable to future retirees relative to past or present retirees" (BAYO; FABER, 1981, p. 1). Bayo and Faber (1981)'s measures of equivalent age are built on three pillars: a) which characteristic or combination of characteristics of a person's life determine equivalence; b) at which point in a person's life we should measure equivalence; and c) which base year we should select as a standard. First, they emphasized that any characteristic should be related to the retirement age only, because if the characteristic were dependent on any other provisions (e.g., contributions or benefits designs) it could neutralize deliberate changes to the social security program. Therefore, they proposed two characteristics to measure equivalence: the "expected number of years spent in retirement" as a limiting case, for it assumes that all gains in life expectancy are spent working; and the "ratio of the expected number of years spent in retirement to the expected number of years spent in the labor force" that equitably distributes the gains in life expectancy between working and retirement years (BAYO; FABER, 1981, p. 3). Second, they advocated measuring equivalence at the age of entry into the labor force (L), for it is an equitable approach as it factors in the experience of people who do not survive to retirement. They also considered measuring equivalence at the age of entry into retirement (R), but only because it is a viable approach. Third, they recommended adopting as base the year when social security benefits were first paid (1940 in their United States context) because it acknowledges "that a specific decision to set the retirement age [...] was made when the program started" (BAYO; FABER, 1981, p. 4). Accordingly, Bayo and Faber (1981) presented four measures of equivalent retirement ages (ERA) that express different perspectives of equity. Their equations are based on the life table functions life expectancy at age a ($\mathring{e}_a$) and number of survivors to age a (l_a), and assume that the age of entry into the labor force is 20 years. Table 5 presents Bayo and

Faber (1981) equations, but with L as the age of entry into the labor force, and R as the age of entry into retirement.

Table 5 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement

Point of measurement	Characteristic of measurement			
	Expected years in retirement		Ratio of expected years in retirement to expected years in work	
Entry into retirement (R)	$\mathring{e}_R$	(4.5)	$\frac{\mathring{e}_R}{R - L}$	(4.6)
Entry into labor force (L)	$\frac{l_R}{l_L} \cdot \mathring{e}_R$	(4.7)	$\frac{l_R / l_L \cdot \mathring{e}_R}{\mathring{e}_L - l_R / l_L \cdot \mathring{e}_R}$	(4.8)

Source: Adapted from table in Bayo and Faber (1981, p. 4).

Changes in mortality after R influence all four measures of equivalent retirement ages (ERA). Changes in mortality only between L and R influence ERA measured at L and by the ratio of expected years in retirement to expected years in work (Equation 4.8), but may or may not influence ERA measured at L and by the expected years in retirement (Equation 4.7). For example, let a change in adult mortality between L and R result in a decline in the life expectancy at age L ($\mathring{e}_L$), but not in a change of the number of survivors to age R (l_R) and, consequently, not in the probability of surviving between L and R (l_R / l_L). Let also old-age mortality after R remain the same and $\mathring{e}_R$ do not change. In this case, the retirement age would remain the same if determined by Equations 4.5, 4.6, and 4.7, even though it would increase if measured by Equation 4.8. Notwithstanding, in contexts of declines both in adult and old-age mortality, Equation 4.7 results in the highest ERA, for it allocates both gains in $\mathring{e}_R$ and in the probability of surviving between L and R (l_R / l_L) to more working years; Equation 4.5 produces the next to highest, because it allocates gains in $\mathring{e}_R$ to more working years, but does not include gains in l_R / l_L; Equation 4.8 yields the next to lowest, for it distributes both gains in $\mathring{e}_R$ and in l_R / l_L between working and retirement years; and Equation 4.6 renders the lowest, because it distributes gains in $\mathring{e}_R$ between working and retirement years, but does not include gains in l_R / l_L. Yet when declines in adult mortality after the base year are minimum, Equations 4.7 and 4.5 yield close and the highest ERA; and equations 4.8 and 4.6 produce adjoining and the lowest ERA.

Kotlikoff (1981) analyzed the economic effects of gains in longevity.[8] His perspective is of gains in longevity that keep "people young for longer periods of time", and not that keep "old people alive for longer periods" (KOTLIKOFF, 1981, p. 98). These "youthful" gains in longevity expand the consumption of commodities and leisure by individuals, which demands extra income, and thus increased work. He developed stylized economic models under two demographic scenarios: increasing the expected years in work, keeping the expected years in retirement constant

[8] Although published in the same year as Bayo and Faber (1981)'s paper, Kotlikoff (1981)'s work was based on a paper originally presented at a workshop held in June 1979 (MCGAUGH; KIESLER, 1981, p. xx).

(Equation 4.5); and keeping the ratio of expected years in retirement to expected years in work constant (Equation 4.8). For Kotlikoff (1981), gains in longevity accompanied by increases of expected years in work are beneficial to social security systems via higher ratios of workers (i.e., contributors) to retirees (i.e., beneficiaries), guaranteed that institutional changes eliminate incentives to early retirement, such as the implicit taxation of the work of elderly or retirees.

Bayo and Faber (1981) believed that the mortality trends for many subgroups of the old-age population in the United States had not been or would not be thereafter considerably different from those of the total old-age population. That is, mortality differentials between subgroups of the old-age population had not and would not change substantially over time. Therefore, they argued, adjustments to the retirement age from trends in mortality of the total population would be equitable for its subgroups (BAYO; FABER, 1981, p. 6). Nevertheless, McMillen (1984) showed that there were significant differences between equivalent retirement ages (ERA) estimated separately for men, women and the total population. She used the same data as Bayo and Faber (1981) and likewise assumed 20 years as the age of entry into the labor force (L), and 65 years as the age of entry into retirement (R). As an illustration, for the base year 1940, ERA measured by Equation 4.5 would be, respectively for men, women and the total population, about 71, 75 and 74 years in 2000, and 73, 78 and 77 years in 2050 (MCMILLEN, 1984, p. 7). ERA measured by Equation 4.8 would be, accordingly, around 70, 73 and 72 years in 2000, and 72, 75 and 74 years in 2050 (MCMILLEN, 1984, p. 10). McMillen (1984) reasoned about the impacts of the selection of the base year on contexts of increasing mortality differentials; specifically, when the base year changes to subsequent years, more of the mortality differential is included in the baseline and, therefore, future differentials in ERA are smaller (MCMILLEN, 1984, p. 8–9). She concluded observing that to comprehend the differences in ERA by sex is relevant not to set distinct retirement ages for men and for women, but to assist retirement age policies.

Castro and Fernandes (1997) estimated Bayo and Faber (1981)'s four measures for Brazil from 1950 to 2050, separately for men and women, and two retirement benefit scenarios: old-age and length of service. They compared the results with the Brazilian Social Security System and with its projected evolution if the then retirement reform proposals were implemented.[9] Castro and Fernandes (1997) assumed the age of entry into the labor force (L) to be 15 years, and the age of entry into retirement (R) to be 65 years for men and 60 years for women in the old-age retirement scenario, and 50 years for men and 45 years for women in the length of service retirement scenario.[10] In the old-age retirement benefit scenario and base year 1950, ERA measured by Equation 4.5 would be, respectively for men and women, 70 and 66 years in 2000 (i.e., increases of 5 and 6 years), and 75 and 72 years in 2050 (i.e., increases of 10 and 12 years). Equivalent retirement ages (ERA) measured by Equation 4.8 would be, correspondingly, 70 and 66 years in 2000 (i.e.,

9 Specifically: a) replace length of service with length of contribution; b) eliminate special length of service requirements for teachers, journalists, and airline crews; c) set the minimum retirement age at 60 years for men and 55 years for women (CASTRO; FERNANDES, 1997, p. 4).

10 Ultimately, length of service retirement after 35 years of service for men and 30 years of service for women, in agreement with the Brazilian social security legislation at that time.

increases of 5 and 6 years), and 74 and 71 years in 2050 (i.e., increases of 9 and 11 years) (CASTRO; FERNANDES, 1997, p. 8–12). In the length of service retirement scenario, none of the ERA measures but one would reach either for men or women the respective base ages of 65 and 60 years of entry into retirement of the old-age retirement scenario; the exception were ERA for women measured by Equation 4.6 for base year 1950 (60 years in 2020) and for base year 1960 (60 years in 2050) (CASTRO; FERNANDES, 1997, p. 12–15).

Lee and Goldstein (2003) analyzed the consequences of gains in life expectancy for the timing of life cycle stages or events. Their benchmark is the "proportional rescaling of the life cycle" in which all life cycle stages or events change in proportion to variations in life expectancy (LEE; GOLDSTEIN, 2003, p. 183). Proportional rescaling has two forms: one is "strong proportionality", which modifies both the average and the distribution of timing of life cycle events or stages; the other is "weak proportionality" where only the mean timing of life cycle events or stages change, while their distribution (i.e., variance) does not (LEE; GOLDSTEIN, 2003, p. 184). Moreover, proportional rescaling can be "flow constrained" where rate or flow variables (e.g., income) are constant and stock variables (e.g., life cycle income) adjust, or "stock constrained" where stock variables (e.g., completed fertility) do not change and flow variables (e.g., fertility rates) adjust (LEE; GOLDSTEIN, 2003, p. 185).[11] Lee and Goldstein (2003, p. 188–190) observed that historical gains in life expectancy have not been distributed equally along the life cycle (see also Horiuchi (1999)), and thus are inconsistent with proportional rescaling. They also emphasized that the correct reference to rescale retirement ages over the life cycle is not the expected years in retirement (Equation 4.5), but the ratio of expected years in retirement to expected years in work (Equation 4.8) (LEE; GOLDSTEIN, 2003, p. 198, p. 204 note 10).

Sanderson and Scherbov (2013) proposed the formal structure for a methodology to measure population aging which translates the values of population characteristics into "alpha-ages". They name this methodology "the characteristic approach". This general and unifying framework was based on their previous studies (LUTZ; SANDERSON; SCHERBOV, 2008; SANDERSON; SCHERBOV, 2005, 2007, 2008, 2010), which extended Ryder (1975) and Lee and Goldstein (2003) independently of Bayo and Faber (1981) and Kotlikoff (1981), and was further explored and developed in Sanderson and Scherbov (2014, 2015, 2017). Alpha-ages with remaining life expectancy as characteristic are named "prospective ages", and measures based on prospective ages are "prospective measures" (e.g., prospective old age dependency ratio (POADR)). Sanderson and Scherbov estimated alpha-ages based on several characteristics, such as remaining life expectancy (Equation 4.5), ratio of expected years in retirement to expected years in work (Equation 4.8), and probability of surviving for the next five years; and measures based on alpha-ages that included median age, OADR, and proportions old.

In Table 6, we summarize most of the authors we referenced previously, their concepts or methodologies and longevity criteria for adjusting the age of entry into retirement (R).

[11] In Lee and Goldstein (2003)'s framework, Bayo and Faber (1981)'s equivalent retirement ages (ERA) are weak proportional rescaling of the age of entry into retirement, which is analog to Figure 125 in section D.1, Appendix D.

Table 6 – Adjusting the age of entry into retirement (R) based on gains in longevity: authors, years, concepts or methodologies, and longevity criteria

Author	Year	Concept / Methodology	Longevity criteria
Ryder (1975)	1975	New index of old-age	Age where life expectancy is equal to some arbitrary length of time
Bayo and Faber (1981)	1981	Equivalent retirement ages	a) expected years in retirement; b) ratio of expected years in retirement to expected years in work. (1)
Kotlikoff (1981)	1981 (2)	Youthful gains in longevity	a) expected years in retirement; b) ratio of expected years in retirement to expected years in work.
Lee and Goldstein (2003)	2003	Proportional rescaling of the life cycle	a) ratio of expected years in retirement to expected years in work.
Sanderson and Scherbov (2005) (3)	2005	Characteristic approach (4)	a) expected years in retirement; b) ratio of expected years in retirement to expected years in work (5).

Source: Author's creation, based on the listed references.
(1): Both measured at either the age of entry into the labor force or the age of entry into retirement.
(2): Based on a paper originally presented at a workshop held in June 1979 (MCGAUGH; KIESLER, 1981, p. xx).
(3): Further explored and developed in Lutz, Sanderson, and Scherbov (2008) and Sanderson and Scherbov (2007, 2008, 2010, 2013, 2014, 2015, 2017).
(4): Named in Sanderson and Scherbov (2013).
(5): Developed in Sanderson and Scherbov (2014).

4.3.1 *Equivalent retirement ages: policies of a selected group of countries*

Some countries have already implemented or plan to implement policies that adjust the normal retirement age or retirement pensions based on gains in life expectancy. A few of these policies may explicitly mention and establish a minimum age for full retirement, whereas others may do so indirectly and let policyholders chose to retire at the same age but with reduced pensions, that is, partial retirement. This flexibility may indicate that in most cases politicians avoid debating age limits of social programs and, therefore, do not expressly emphasize or enact statutory retirement ages (SCHERMAN, 2011).

In Brazil, length of contribution retirement pensions require a minimum of 35 years of contribution for men, and 30 years for women. Policyholders may retire based on the 85/95 formula or on the *social security factor*. The 85/95 formula refers to the sum of years of contribution and age; originally, if it were at least 85 for women and 95 for men, policyholders were entitled for full length of contribution pensions. The 85/95 formula increased to 86/96 for the biennium 2019/2020, and will continue to increase up to 90/100 in year 2027. If the sum of years of contribution and age is less than the 85/95 formula, the social security factor shall be applied to retirement pensions. The social security factor is based on age, length of contribution, a contribution index equal to 0.31, and life expectancy at the age of entry into retirement (BRASIL, 2017, 2019; OECD, 2015, p. 222–224). For example, as of April 2019 the social security factor could be between 0.187 (for 15 years of contribution, age 43 years and $\mathring{e}_{43}$=36.6) and 2.094 (for 55 years of contribution, age 70 years and $\mathring{e}_{70}$=15.2). Social security factors that are at least equal to 1.0 may be obtained by combining, for example, 47 years of contribution and age 57 years (factor=1.01 and $\mathring{e}_{57}$=24.8), 42 years of contribution and age 60 years (factor=1.005 and $\mathring{e}_{60}$=22.4), 35 years of contribution and age 65 years (factor=1.022 and $\mathring{e}_{65}$=18.7), and 28 years of contribution and age 70 years (factor=1.019 and $\mathring{e}_{70}$=15.2) (BRASIL, 2018).

In Finland, since 2010 earnings-related retirement pensions have been adjusting for gains in longevity by the *life expectancy coefficient*, which is calculated for each cohort at age 62, and is determined by increases in life expectancy since 2009 and a yearly discount rate of 2%. By 2060, the life expectancy coefficient is projected to reduce pensions to 79.2% of their pre-reform values. Starting in 2017, the normal retirement age for earnings-related pensions will raise from 63 to 65 years in increments of 3 months every calendar year. After that, it will be adjusted for gains in life expectancy and limited to 2 months per calendar year (OECD, 2015, p. 251–255, 2017, p. 34).

In Italy, since 1995 earnings-related retirement pensions are calculated from notional account balances that are converted into annuities by the *transformation coefficient*. The transformation coefficient is estimated based on the life expectancy at age of entry into retirement, the probability that the individual will leave a widow or widower, and the life expectancy of the widow or widower at the pensioner's death. Starting in 2010, the transformation coefficient has been adjusting for changes in life expectancy every three years. Since 2013, the normal retirement age has been automatically adjusting based on $\mathring{e}_{65}$ every three years until 2019, and every two years afterwards. By 2019, the normal retirement age will be 67 years both for men and women. As of 2014, policyholders could retire earlier from age 62 if the length of contribution was at least 42 years and 6 months for men, and 41 years and 6 months for women. Lengths of contribution requirements also increase based on life expectancy (CHŁOŃ-DOMIŃCZAK; FRANCO; PALMER, 2012; OECD, 2015, p. 290–294).

In the Netherlands, the normal retirement age for the state basic old-age pension has been gradually increasing from age 65 years and 2 months in 2014, to age 66 in 2018, 67 in 2021, 67 years and 3 months in 2022, and after that it will be adjusted for gains in life expectancy (OECD, 2015, p. 310–312, 2017, p. 38).

In Norway, since 2011 income retirement pensions are calculated from "pension entitlements" divided by the *life expectancy divisor*, which is calculated for each cohort at age 61 and based essentially on the remaining life expectancy. Each cohort has different life expectancy divisors from age 62 to age 75. Basic guarantee pensions are adjusted for the life expectancy divisor at age 67 (OECD, 2015, p. 316–319).

In Portugal, the normal retirement age was increased from 65 to 66 years in 2014 and, since 2016, it has been adjusting for gains in longevity, specifically, by the ratio between $\mathring{e}_{65}$ in the first two of the previous three years and $\mathring{e}_{65}$ in the year 2000. The normal retirement age can be reduced by four months for each year of contribution that surpasses 40 years if the individual has reached age 65. Since 2007, earnings-related retirement pensions are the product of reference earnings, an accrual rate, and a *sustainability factor*. The sustainability factor is applied for retirements below the normal retirement age, and is calculated based on gains in $\mathring{e}_{65}$ between the year 2000 and the year before the entry into retirement (OECD, 2015, p. 325–331, 2019, p. 58–65).

In Sweden, there is no formal retirement age. Policyholders may retire, fully or partially, from the age of 61. Pensions are calculated at the time of retirement by dividing each individual's notional account balance by an *annuity divisor*. The annuity divisor is determined by each cohort's $\mathring{e}_{65}$ and by a discount interest rate of 1.6 percent. Consequently, when $\mathring{e}_{65}$ increases, individuals have to retire later than previous cohorts to receive full pensions; otherwise they receive partial pensions (OECD, 2015, p. 352–353; SCHERMAN, 2011; SETTERGREN, 2001, 2003). For example, according to the Annual Report of 2002 of the Swedish Pension System, for the cohort born in 1940, in the year 2005 when it reached age 65, the projected annuity divisor would be 15.7 and $\mathring{e}_{65}$ would be 18 years and 6 months. For the 1965 cohort, in the year 2030 these values would be, respectively, 17.2 and 20 years and 6 months, and its individuals would have to retire 16 months later than the 1940 cohort to have the same proportional pensions. Eventually, for the 1990 cohort, in the year 2055 the projected annuity divisor would be be 18.2 and $\mathring{e}_{65}$ would be 21 years and 11 months, and its individuals would have to retire 26 months later than the 1940 cohort to be entitled for the same proportional pensions (SETTERGREN, 2003, p. 104).

In the Slovak Republic, in 2015 the normal retirement age was 62 with a minimum of 15 years of contribution. Starting in 2017, the normal retirement age would be adjusted for gains in life expectancy. Women with children have reduced normal retirement ages (e.g., in 2014, women with five or more children could retire at 57 years and 6 months), but these retirement ages were increasing and projected to be at least 62 years in 2024 (OECD, 2015, p. 338–341).

In Spain, in 2014 the normal retirement age for full pension was 65 years and two months for those with less than 35 years and 6 months of contribution, and 65 years for those with at least 38 years and 6 months of contribution. Starting in 2019, earnings-related retirement pensions will be adjusted by a *sustainability factor*, which will be determined by the growth in life expectancy of new pensioners. By 2027, the normal retirement age will be 67 years both for men and women (OECD, 2015, p. 348–351).

In the United Kingdom, in 2015 the normal retirement age was 65 years for men and 62 years and 6 months for women, and it was planned to increase to 65 years for women until November 2018. Legislation had been approved to increase the normal retirement age to 66 years by October 2020, and to 67 years between 2026 and 2028. The Government had proposed that later increases in the normal retirement age should be calculated from changes in life expectancy (OECD, 2015, p. 368–371).

In Table 7, we summarize the countries we referenced above, their longevity criteria for adjusting the age of entry into retirement (R) or retirement pensions, and respective policies start years.

Table 7 – Adjusting the age of entry into retirement or retirement pensions based on gains in longevity: countries, longevity criteria, and policy start years

Country	Longevity criteria	Policy start year
Brazil	($) Life expectancy at the age of entry into retirement	1999
Finland	(*R*) Increases in life expectancy ($) Increases in life expectancy at age 62	(*R*) 2026 ($) 2010
Italy	(*R*) Life expectancy at age 65. ($) Life expectancy at the age of entry into retirement; probability that the pensioner will leave a widow or widower; life expectancy of the widow or widower at the pensioner's death.	(*R*) 2013, ($) 1995
The Netherlands	(*R*) Increases in life expectancy	2023
Norway	($) Life expectancy at age 61	2011
Portugal	(*R*, $) Increases in life expectancy at age 65	(*R*) 2016, ($) 2007
Sweden	($) Life expectancy at age 65	1998
Slovak Republic	(*R*) Increases in life expectancy	2017
Spain	($) Increases in life expectancy of new pensioners	2019
United Kingdom	(*R*) Increases in life expectancy	2029

Source: Author's creation, based on Brasil (2017, 2019), Chłoń-Domińczak, Franco, and Palmer (2012), OECD (2015, 2017, 2019), Scherman (2011), and Settergren (2001, 2003).
(*R*): Age of entry into retirement.
($): Retirement pensions.

4.4 METHODS AND ASSUMPTIONS

We use our stylized demographic model to analyze the burden of population aging on defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems in the world from 1950 to 2100. We adopt 1950–1955 as the base period, 20 years as the age of entry into the labor force (L), and 65 years as the age of entry into retirement (R). Benefit rates (ben) in DB PAYG systems are 100%; contribution rates (con) in DC PAYG systems are 15%; and ratios of per capita benefits to per capita net wages (ϕ) in FRP PAYG systems are equal to 100%.

Equivalent retirement ages (ERA) are for the five-year periods of the 2017 UN REVISION, specifically, in the period 1950–1955 all countries R and ERA are 65 years, then from 1955–1960 to 2095–2100 ERA are determined by the respective five-year period life tables. Still, since we would strictly need to estimate ERA for 2100–2105 to calculate OADR from ERA for 2100 and because the 2017 UN REVISION presents no life tables for 2100–2105, we assume that life tables and, consequently, ERA for 2100–2105 are the same as those for 2095–2100. Likewise, we presume that the rejuvenating effects of births, deaths and migration for 2100–2105 are the same as those for 2095–2100.

We reference equivalent retirement ages (ERA) in terms of the number of person-years lived above age a (T_a) and the number of survivors to age a (l_a) as in Table 8. Also, we follow Bayo and Faber (1981), Kotlikoff (1981), Lee and Goldstein (2003), and Sanderson and Scherbov (2014), and use for ERA the ratio of expected years in retirement to expected years in work measured at the age of entry into the labor force (L), specifically, we use Equation 4.12.

Table 8 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement in terms of T_a and l_a

Point of measurement	Characteristic of measurement			
	Expected years in retirement		Ratio of expected years in retirement to expected years in work	
Entry into retirement (R)	$\frac{T_R}{l_R}$	(4.9)	$\frac{T_R}{l_R \cdot (R-L)}$	(4.10)
Entry into labor force (L)	$\frac{T_R}{l_L}$	(4.11)	$\frac{T_R}{T_L - T_R}$	(4.12)

Source: Author's creation, based on table in Bayo and Faber (1981, p. 4).

Last, analog to the "prospective old age dependency ratio (POADR)" term introduced in Sanderson and Scherbov (2007, p. 48), and the terminology "prospective ages" vis-à-vis "prospective measures" in Sanderson and Scherbov (2008) and their later works; we name old age dependency ratios (OADR) calculated from equivalent retirement ages (ERA), equivalent old age dependency ratios (EOADR).

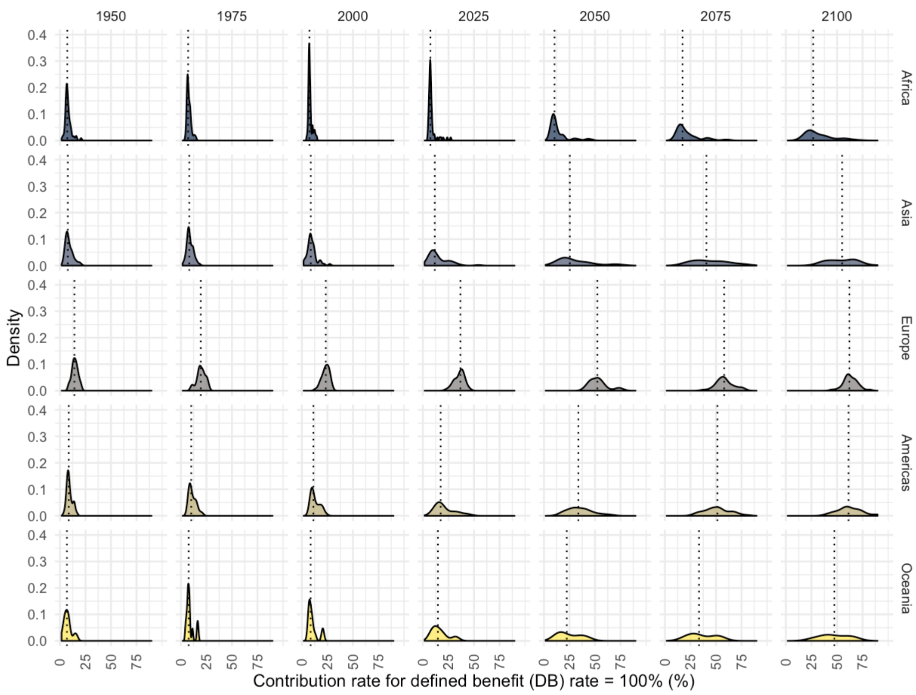
4.5 PAY-AS-YOU-GO SYSTEMS: DISTRIBUTION OF THE BURDEN OF WORLD POPULATION AGING

We estimate the distribution of the burden of population aging between contributors, via contribution rates (*con*), and beneficiaries, via benefit rates (*ben*), of defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems. Figures 13 to 16 plot the densities of contribution rates (*con*) and benefit rates (*ben*) of DB, DC, and FRP PAYG systems for selected years and all regions; and Figures 17 to 20 plot their distribution in 2100 by subregions.

Between 1950 and 2100, the median contribution rate (*con*) of DB PAYG systems increases from 7.2% to 26.7% in Africa, from 7.7% to 55.0% in Asia, from 14.2% to 62.0% in Europe, from 8.7% to 61.2% in the Americas, and from 6.9% to 47.4% in Oceania. In 2100, the median *con* of DB PAYG systems is above 60% in as many as nine of the twenty-two subregions: Southern Europe (68.8%), Eastern Asia (66.5%), Southern Asia (66.4%), Western Europe (64.9%), Caribbean (64.7%), Central America (63.4%), Australia/New Zealand (61.5%), Northern Europe (60.0%) and Eastern Europe (59.6%). In the same period, the median benefit rate (*ben*) of DC PAYG systems decreases from 206.9% to 56.1% in Africa, from 193.4% to 27.3% in Asia, from 105.5% to 24.2% in Europe, from 172.7% to 24.5% in the Americas, and from 217.7% to 31.7% in Oceania. In 2100, the median *ben* of DC PAYG systems is below 25% for the same nine subregions: Southern Europe (21.8%), Eastern Asia (22.6%), Southern Asia (22.6%), Western Europe (23.1%), Caribbean (23.2%), Central America (23.8%), Australia/New Zealand (24.4%), Northern Europe (25.0%) and Eastern Europe (25.2%).

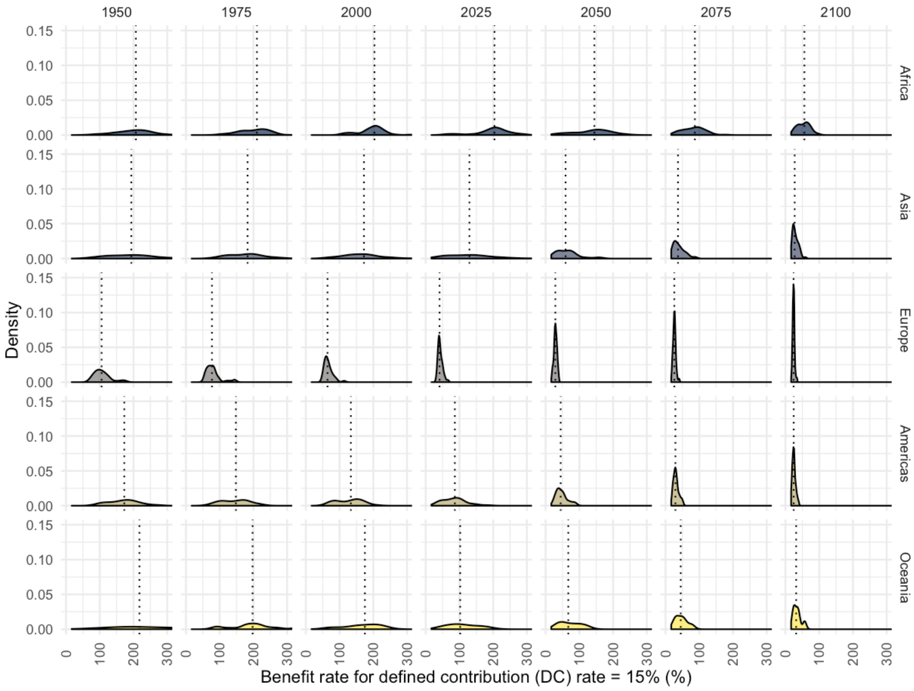
Except for Africa in general, these figures demonstrate the long-term unfeasibility of DB and DC PAYG systems in aging populations. Yet adopting FRP PAYG systems, and thus distributing the risk of population aging between contributors and beneficiaries, may lead to more credible, nevertheless still demanding, scenarios. Between 1950 and 2100, the median contribution rate (*con*) of FRP PAYG systems increases from 6.7% to 21.1% in Africa, from 7.2% to 35.5% in Asia, from 12.4% to 38.3% in Europe, from 8.0% to 38.0% in the Americas, and from 6.4% to 32.1% in Oceania. Still in the same period, the median benefit rate (*ben*) of FRP PAYG systems decreases from 93.2% to 78.9% in Africa, from 92.8% to 64.5% in Asia, from 87.5% to 61.7% in Europe, from 92.0% to 62.0% in the Americas, and from 93.5% to 67.8% in Oceania. In 2100, for the said nine subregions with median *con* of DB PAYG systems above 60% and median *ben* of DC PAYG systems below 25% in FRP PAYG systems, the median *con* is between 40.8% (Southern Europe) and 37.3% (Eastern Europe), and the median *ben* is between 59.2% (Southern Europe) and 62.7% (Eastern Europe).

Figure 13 – Density of the contribution rate (*con*) of DB by selected periods and regions



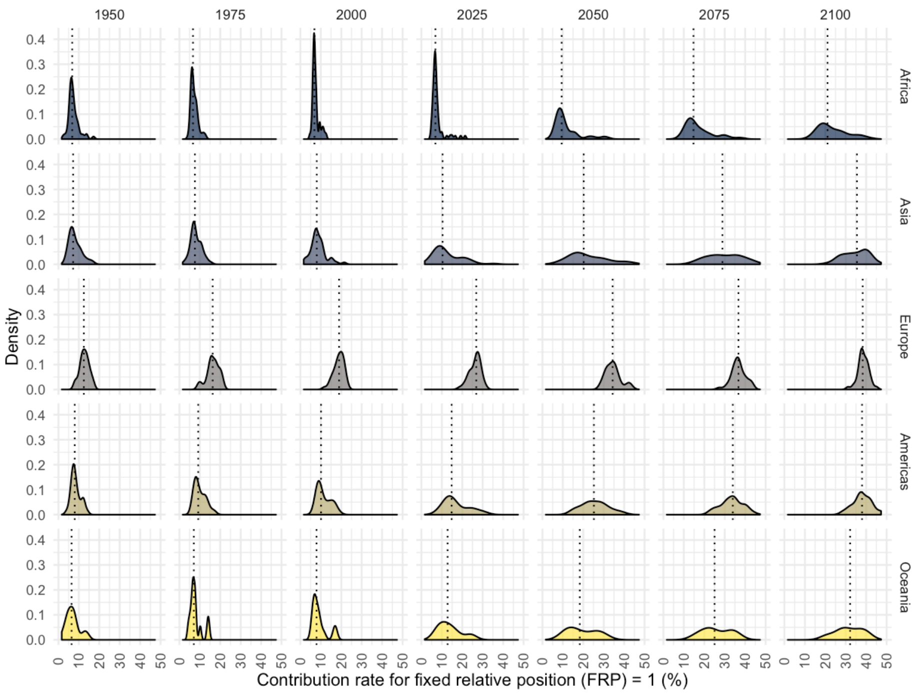
Source: Author's calculations, based on United Nations (2017c).
Notes: Benefit rate (*ben*) = 100%. Vertical dotted line indicates the median of the distribution.

Figure 14 – Density of the benefit rate (*ben*) of DC by selected periods and regions



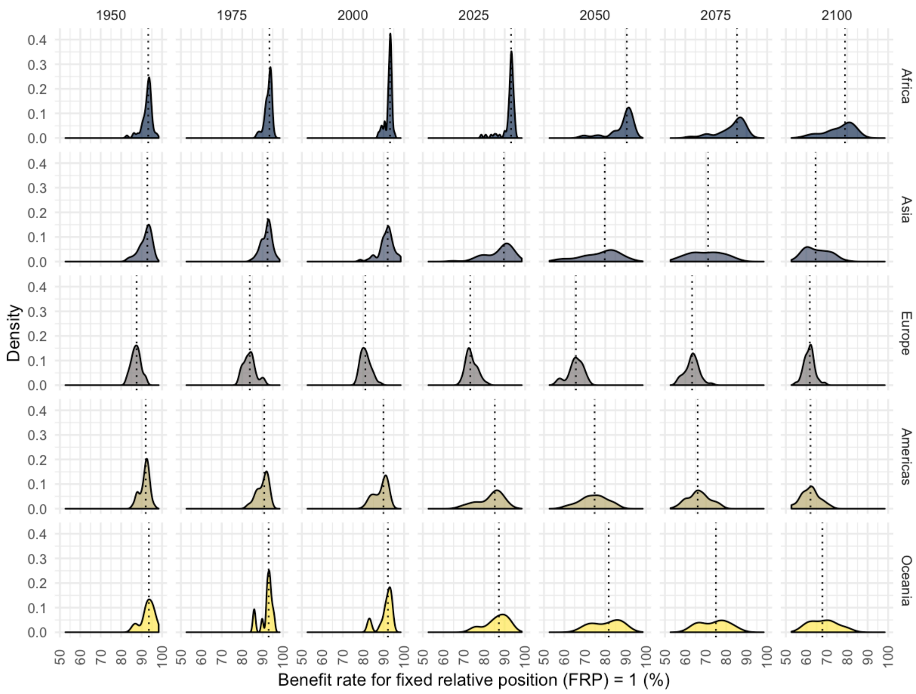
Source: Author's calculations, based on United Nations (2017c).
Notes: Contribution rate (*con*) = 15%. Vertical dotted line indicates the median of the distribution.

Figure 15 – Density of the contribution rate (*con*) of FRP by selected periods and regions



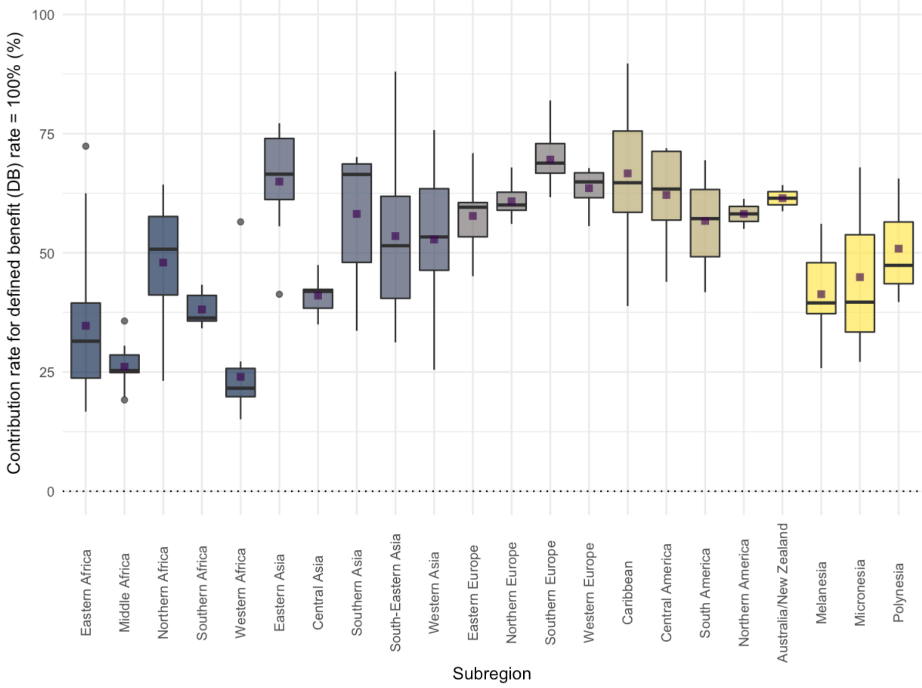
Source: Author's calculations, based on United Nations (2017c).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Vertical dotted line indicates the median of the distribution.

Figure 16 – Density of the benefit rate (*ben*) of FRP by selected periods and regions



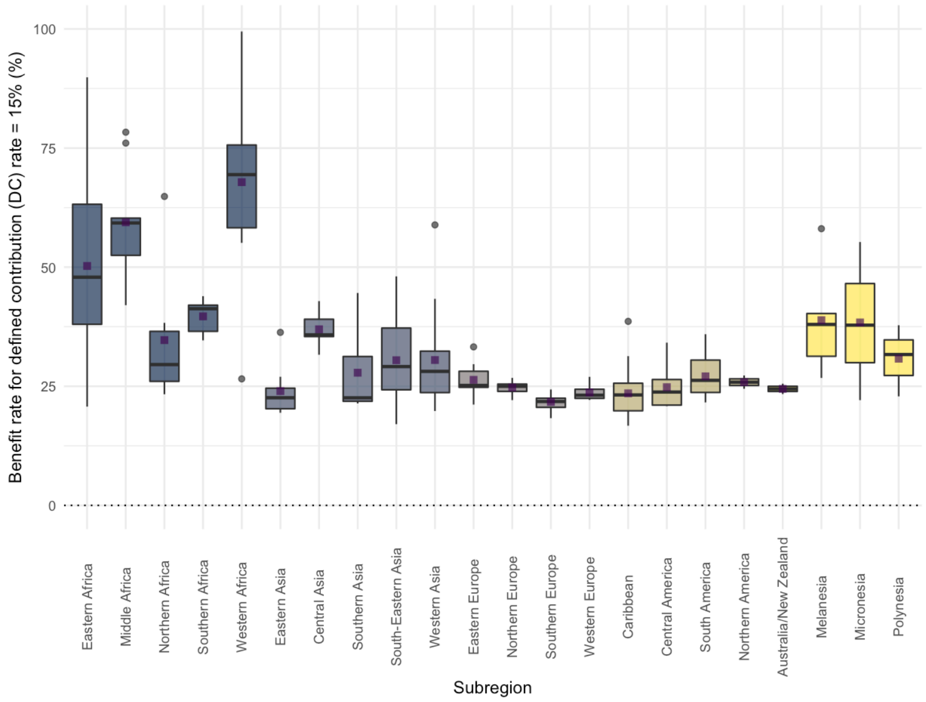
Source: Author's calculations, based on United Nations (2017c).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Vertical dotted line indicates the median of the distribution.

Figure 17 – Contribution rate (*con*) of DB in 2100 by subregions



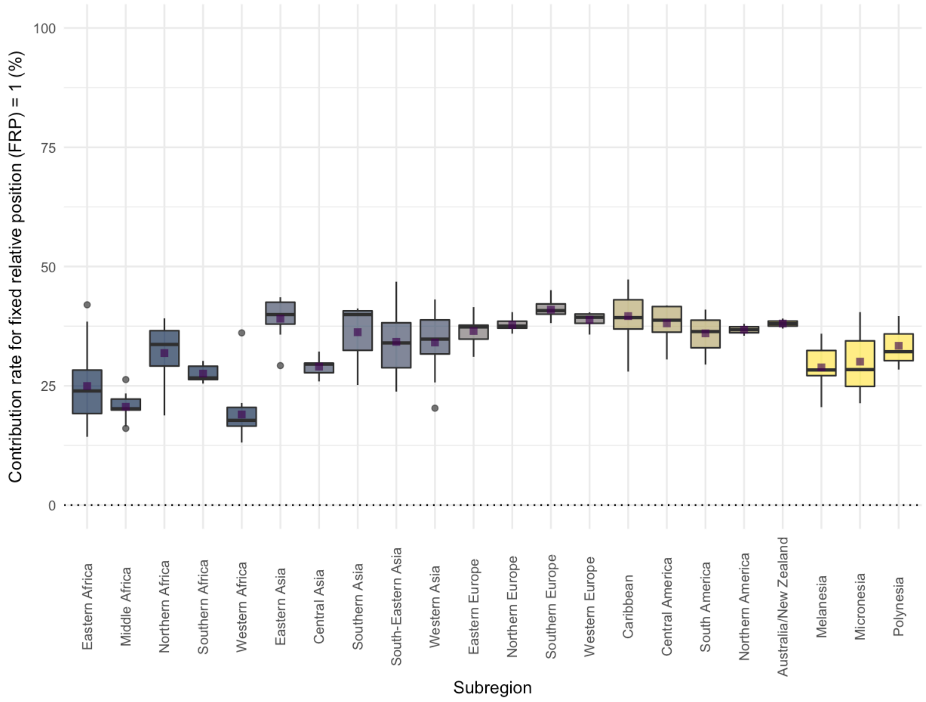
Source: Author's calculations, based on United Nations (2017c).
Notes: Benefit rate (*ben*) = 100%. Square indicates the mean of the distribution.

Figure 18 – Benefit rate (*ben*) of DC in 2100 by subregions



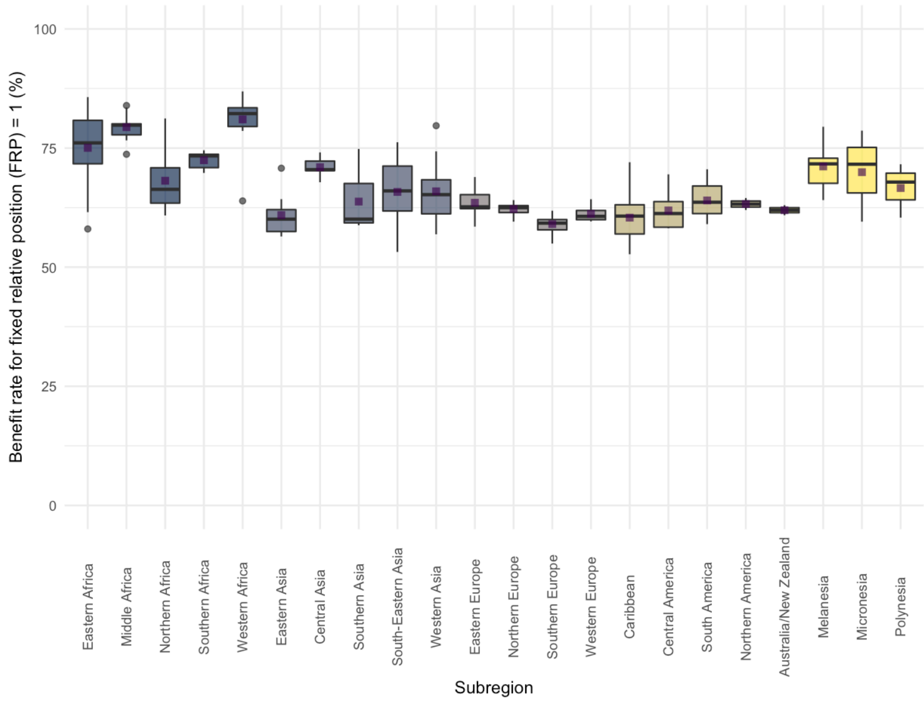
Source: Author's calculations, based on United Nations (2017c).
Notes: Contribution rate (*con*) = 15%. Square indicates the mean of the distribution.

Figure 19 – Contribution rate (*con*) of FRP in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Square indicates the mean of the distribution.

Figure 20 – Benefit rate (*ben*) of FRP in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Square indicates the mean of the distribution.

4.6 EQUIVALENT RETIREMENT AGES, PAY-AS-YOU-GO SYSTEMS AND WORLD POPULATION AGING

We assess the equivalent retirement age (ERA) given by the ratio of expected years in retirement to expected years in work (Equation 4.12), and analyze how much it buffers the burden of population aging on contributors and beneficiaries in PAYG systems. Figure 21 plots the density of equivalent retirement ages (ERA) for selected periods and all regions with a reference vertical line at 65 years; and Figure 22 details its distribution in 2100 by subregions. Figures 23 to 26 present the densities of contribution rates (*con*) and benefit rates (*ben*) of DB, DC and FRP PAYG systems and ERA for selected years and all regions; and Figures 27 to 30 detail their distribution in 2100 by subregions.

As a consequence of increasing ratios of expected years in retirement to expected years in work, equivalent retirement ages (ERA) rise in all regions from 65 years in 1950–1955 to median values around 70 years in 2000–2005, to about 75 years in 2050–2055, and eventually 77.8 years in Africa, 79.3 years in Asia, 78.3 years in Europe, 78.9 years in the Americas, and 79.3 years Oceania. In 2100, the subregions with the highest median ERA are Northern Africa in Africa (81.0 years), Southern Asia and Eastern Asia (82.7 years) in Asia, Southern Europe and Western Europe in Europe (79.4 years), Central America in the Americas (80.8 years), and Polynesia in Oceania (83.9 years).

Much of the debate about adjusting the retirement age based on gains in longevity focus on both levels and changes in the levels of the equivalent retirement age (ERA). But the noteworthy characteristic from our results is both the homogeneity of ERA among subregions that have different rejuvenating effect of deaths, and the heterogeneity of ERA among subregions that have similar rejuvenating effect of deaths. For example, in 2100, homogeneity of ERA and different cumulative rejuvenating effect of deaths (see Figure 4) are observed for Northern America and Australia/New Zealand compared with Easter Africa and Middle Africa, for Eastern Europe and Northern Europe compared to Southern Africa and Western Africa, and in that twelve of the twenty-two subregions have median ERA within the range 77 years to 80 years.[12] Also in 2100, heterogeneity of ERA and similar cumulative rejuvenating effect of deaths are noted within the four Europe subregions, and between Melanesia, Micronesia and Polynesia.

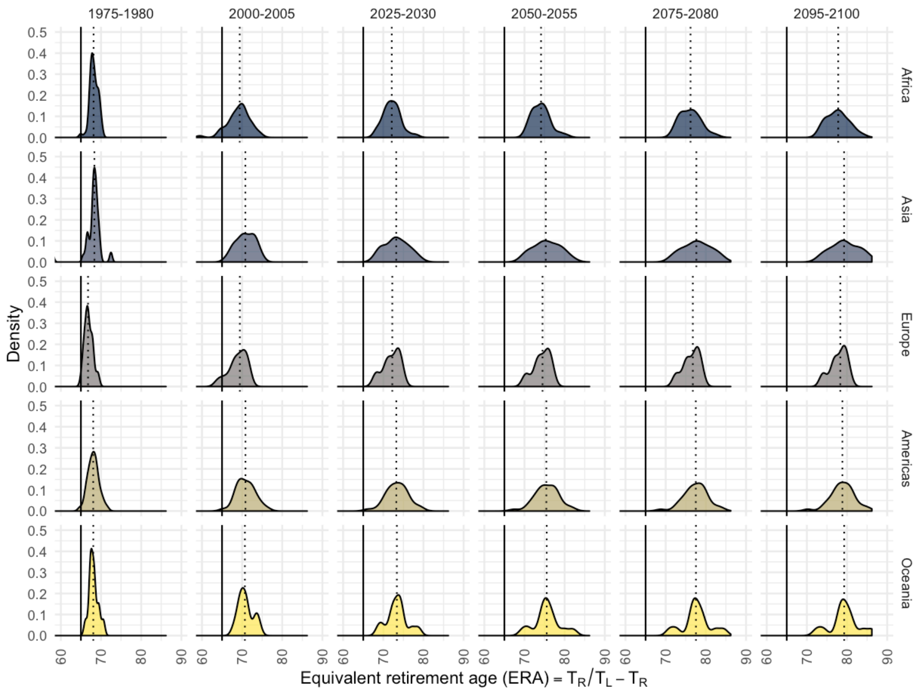
Although equivalent retirement ages (ERA) do buffer the burden of population aging, our results indicate that ERA frequently *over-buffer* this burden when there are mismatches between ERA and the rejuvenating effect of deaths, that is, ERA increase the age of entry into retirement (R) more than necessary, resulting in lower variable contribution rates (*con*) and higher variable benefit rates (*ben*) than in the base year. Actually, between 1950 and 2100, Africa, Asia, the Americas and Oceania observe median variable *con* that first decrease then increase, and median variable *ben* that initially increase then decrease. In defined benefit (DB) PAYG systems, the median *con* in Africa decreases from 7.2% in 1950 to 3.3% in 2025 then increases to 7.8% in 2100, in Asia decreases from 7.8% in 1950 to 4.3% in 2010 then increases to 17.3% in 2100, in the Americas decreases from 8.7% in 1950 to 6.9% in 1990 then increases to 22.1% in 2100, and in Oceania decreases from 6.9% in 1950 to 4.5% in 2010 then increases to 14.4% in 2100. In Europe, the median *con* remains between 13.7% and 17.2% from 1950 to 2020, then increases to 25.7% in 2100. Likewise, in defined contribution (DC) PAYG systems, the median *ben* in Africa increases from 206.9% in 1950 to about 454.0% in 2025 then decreases to 191.9% in 2100, in Asia increases from 193.4% in 1950 to 349.3% in 2010 then decreases to close 86.7% in 2100, in the Americas increases from 172.7% to 219.0% in 1990 then decrease to 68.0% in 2100, and in Oceania increases from 217.7% in 1950 to 332.4% in 2010 then decreases to 104.5% in 2100. In Europe, the median *ben* varies between 109.0% and 87.0% from 1950 to 2020, then decreases to 58.4% in 2100.

The same happens in fixed relative position (FRP) PAYG systems, yet to a lesser degree because FRP PAYG systems distribute the risk of population aging between contributors and beneficiaries, and thus dilute any over-buffering of population aging between *con* and *ben*. In FRP PAYG systems, the median *con* in Africa decreases from 6.6% to in 1950 to 3.2% in 2025 then increases to 7.2% in 2100, in Asia decreases from 7.2% to in 1950 to 4.1% in 2010 then increases to 14.8% in 2100, in the Americas decreases from 8.0% in 1950 to 6.4% in 1990 then increases to 18.1% in 2100, and in

[12] In decreasing order of ERA (median ERA, median cumulative rejuvenating effect of deaths): Western Europe (79.4 years, 58.5 years), Southern Europe (79.4 years, 60.6 years), Melanesia (79.3 years, 29.1 years), South America (79.2 years, 38.8 years), South-Eastern Asia (79.0 years, 36.8 years), Australia/New Zealand (78.9 years, 49.1 years), Western Asia (78.7 years, 27.8 years), the Caribbean (78.2 years, 49.5 years), Eastern Africa (78.1 years, 17.5 years), Northern America (77.9 years, 51.2 years), Middle Africa (77.7 years, 16.8 years), and Northern Europe (77.1 years, 59.5 years).

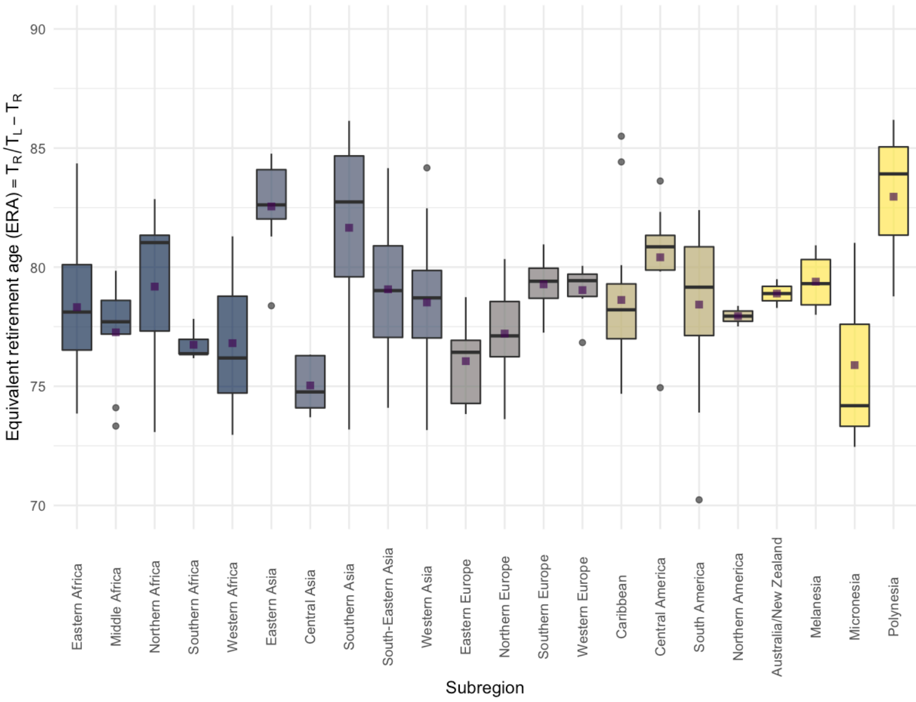
Oceania decreases from 6.4% in 1950 to just below 4.3% in 2010 then increases to 12.6% in 2100. In Europe, the median *con* remains between 12.1% and 14.7% from 1950 to 2020, then increases to 20.4% in 2100. Likewise, still in FRP PAYG systems, the median *ben* in Africa increases from 93.2% in 1950 to 96.8% in 2025 then decreases to 92.8% in 2100, in Asia increases from 92.8% in 1950 to 95.9% in 2010 then decreases to 85.2% in 2100, in the Americas increases from 92% to 93.6% in 1990 then decreases to 81.9% in 2100, and in Oceania increases from 93.6% in 1950 to 95.7% in 2010 then decreases to 87.4% in 2100. In Europe, the median *ben* varies between 87.9% and 85.3% from 1950 to 2020, then decreases to 79.6% in 2100.

Figure 21 – Density of equivalent retirement age (ERA) by selected periods and regions



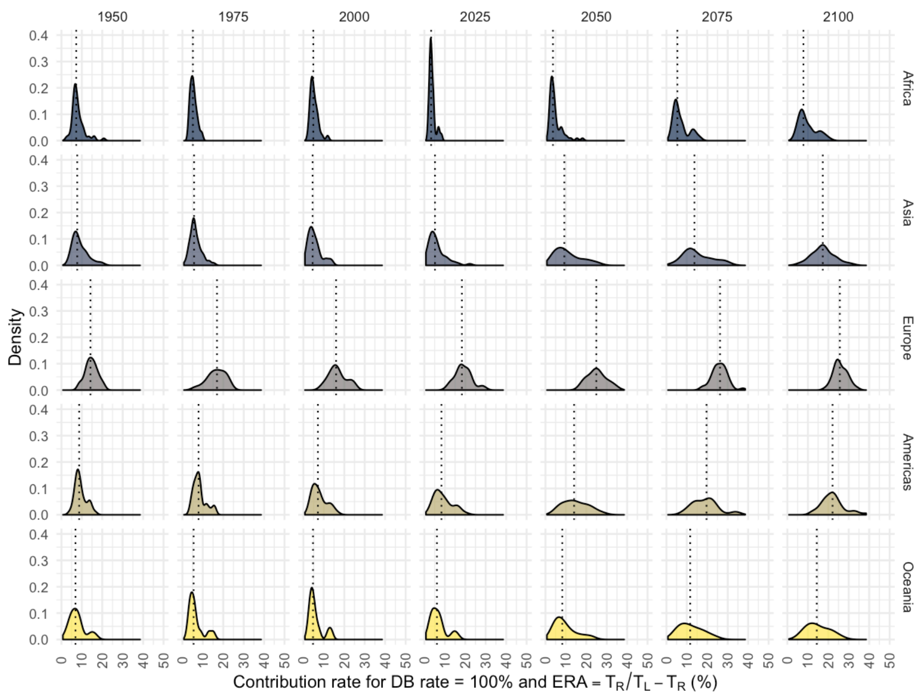
Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

Figure 22 – Equivalent retirement age (ERA) in 2100 by subregions



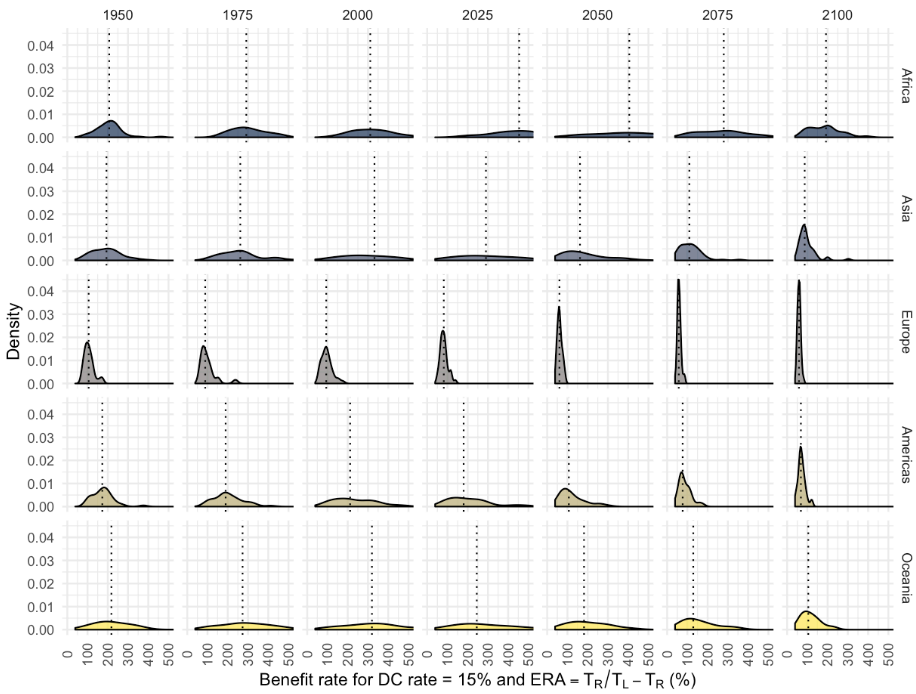
Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

Figure 23 – Density of the contribution rate (*con*) of DB and ERA by selected periods and regions



Source: Author's calculations, based on United Nations (2017c).
Notes: Benefit rate (*ben*) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

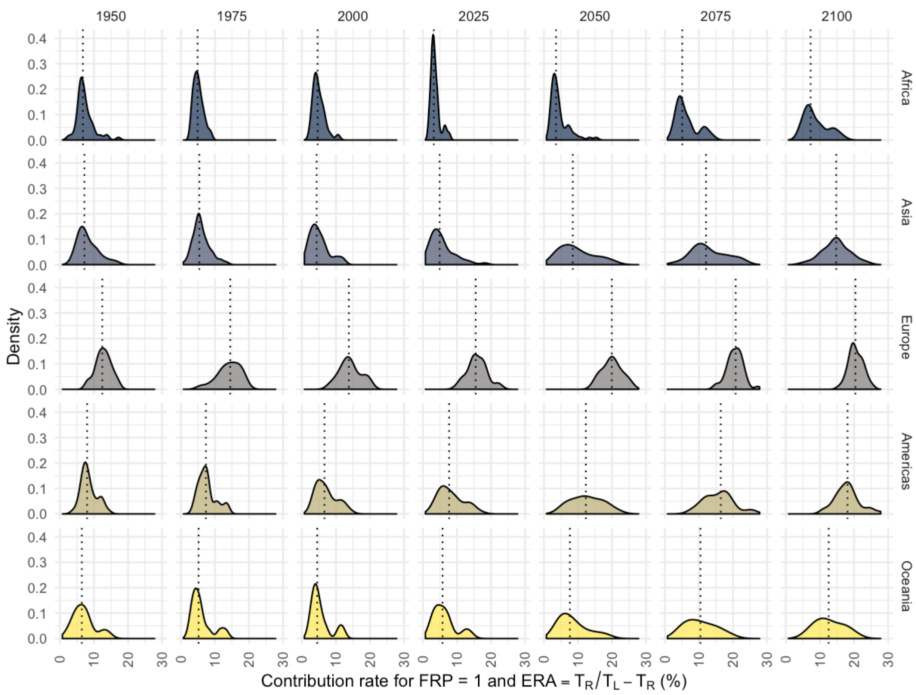
Figure 24 – Density of the benefit rate (*ben*) of DC and ERA by selected periods and regions



Source: Author's calculations, based on United Nations (2017c).

Notes: Contribution rate (*con*) = 15%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

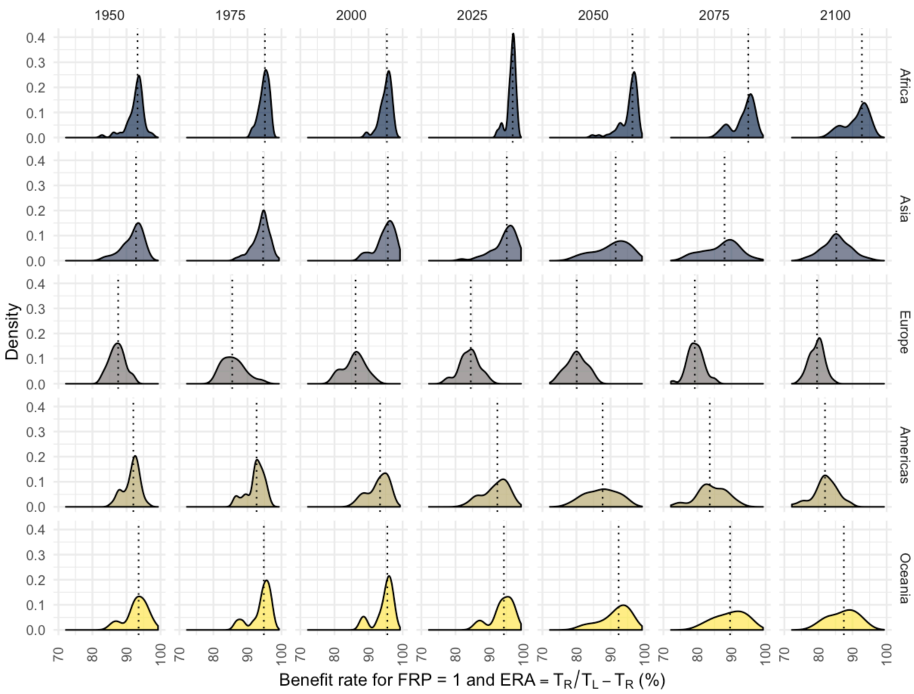
Figure 25 – Density of the contribution rate (*con*) of FRP and ERA by selected periods and regions



Source: Author's calculations, based on United Nations (2017c).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

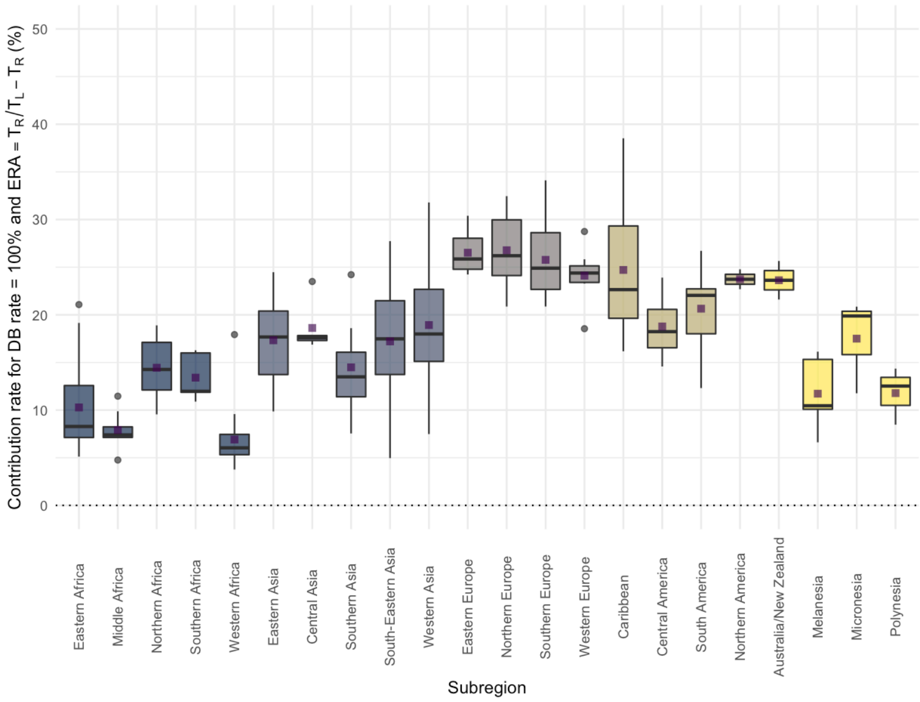
Figure 26 – Density of the benefit rate (*ben*) of FRP and ERA by selected periods and regions



Source: Author's calculations, based on United Nations (2017c).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

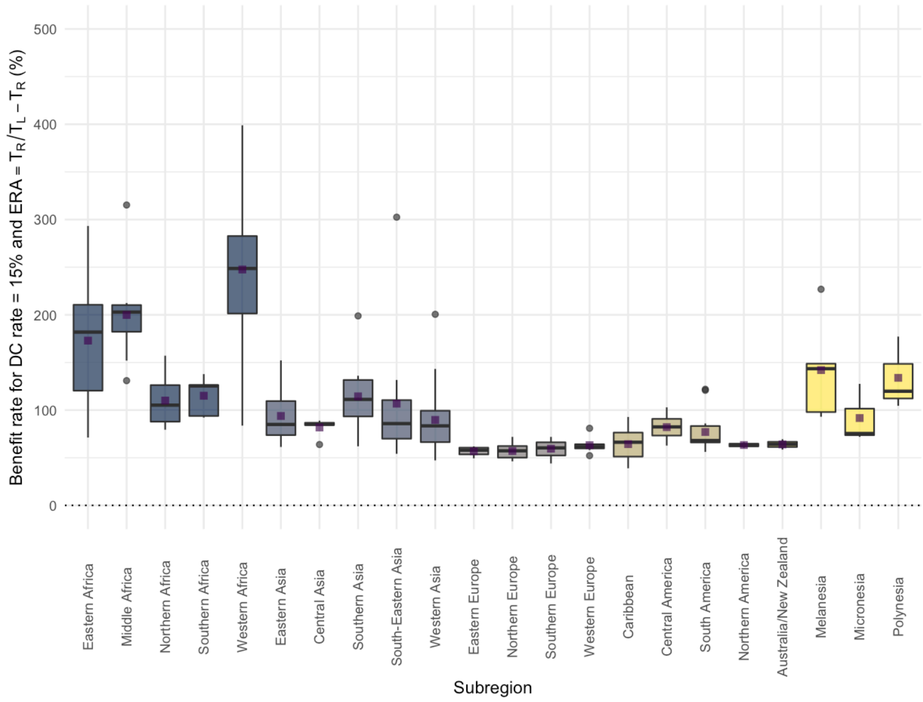
Figure 27 – Contribution rate (*con*) of DB and ERA in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).

Notes: Benefit rate (*ben*) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

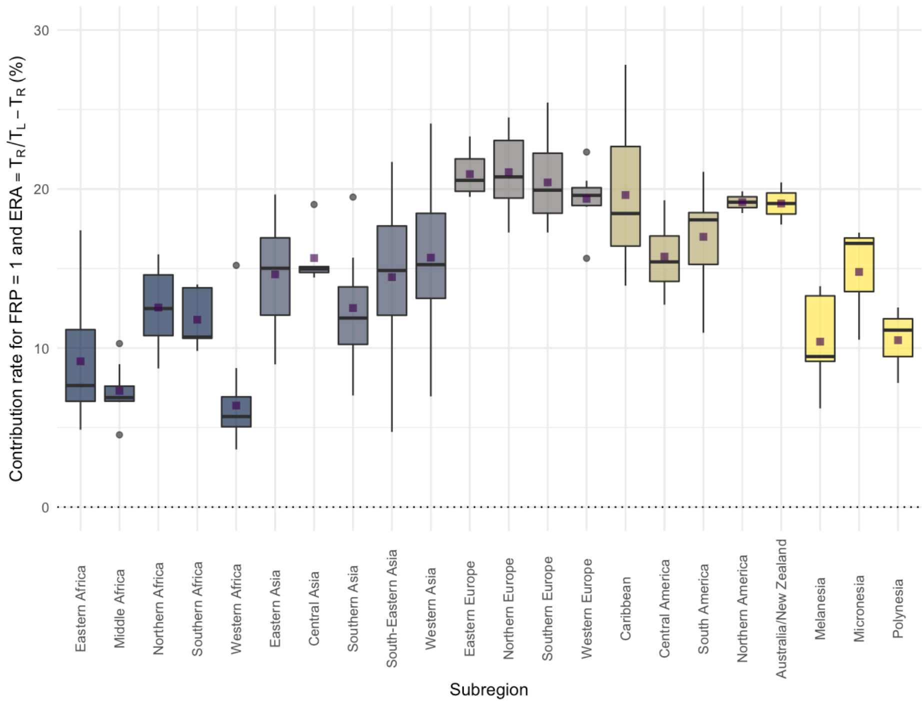
Figure 28 – Benefit rate (*ben*) of DC and ERA in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).

Notes: Contribution rate (*con*) = 15%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

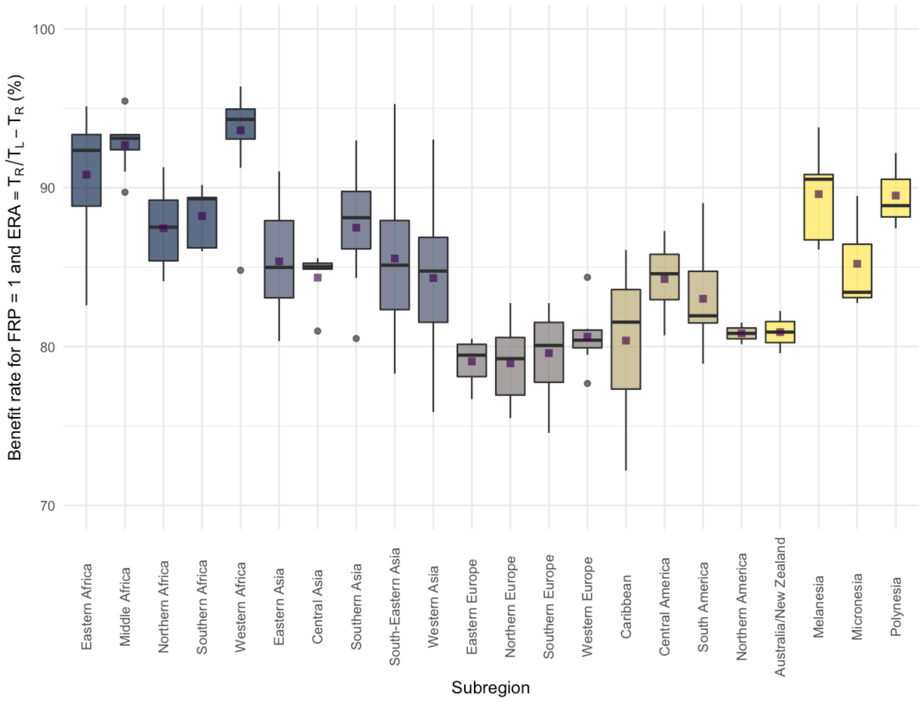
Figure 29 – Contribution rate (*con*) of FRP and ERA in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

Figure 30 – Benefit rate (*ben*) of FRP and ERA in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

4.7 EQUIVALENT RETIREMENT AGES EFFECTIVENESS

Equivalent retirement ages (ERA) ineffectively over-buffer population aging because PAYG systems are established on population characteristics, while ERA are based on life cycle characteristics. Fundamentally, population characteristics change because mortality, fertility or migration change, while life cycle characteristics change because mortality or migration change (PRESTON, 1982; PRESTON; COALE, 1982).[13] Thus, equivalent retirement ages (ERA) may not effectively buffer the impact of population aging if the contribution of mortality to changes in the population age distribution is only moderate.[14] Therefore, PAYG systems should contemplate the role of the rejuvenating effect of deaths and, by extension, the stages of the demographic transition, in the definition of retirement age policies.

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13 Preston (1982) formulated measures for both the prevalence of an attribute G in a population at a moment in time (G_P) and the prevalence of an attribute G over the course of the life cycle (G_L) according to stable populations that are closed to migration. Nevertheless, the formulas of G_P and G_L may be extended to accommodate non-stable populations and migration. Essentially, the effect of migration on a cohort's or a population's size is analogous to the effect of mortality (PRESTON; COALE, 1982).

14 Basically, old age dependency ratios (OADR) are Preston (1982)'s G_P, while equivalent retirement ages (ERA) are Preston (1982)'s G_L. Rigorously, old age dependency ratios (OADR) are the ratio between two G_P's (G_P^{20-64}/G_P^{65+}), and ERA measured by the ratio of expected years in retirement to expected years in work are the ratio between two G_L's (G_L^{20-64}/G_L^{65+}). In the one (G_P^{20-64} or G_L^{20-64}) the characteristic G is to be 20 to 64 years of age, in the other (G_P^{65+} or G_P^{65+}) the characteristic G is to be 65 years of age and older.

We may measure the level of population aging relative to the base year 1950 by calculating the ratio of the old age dependency ratio (OADR) observed at year t ($OADR(t)$) to the OADR observed in 1950 ($OADR(1950)$),

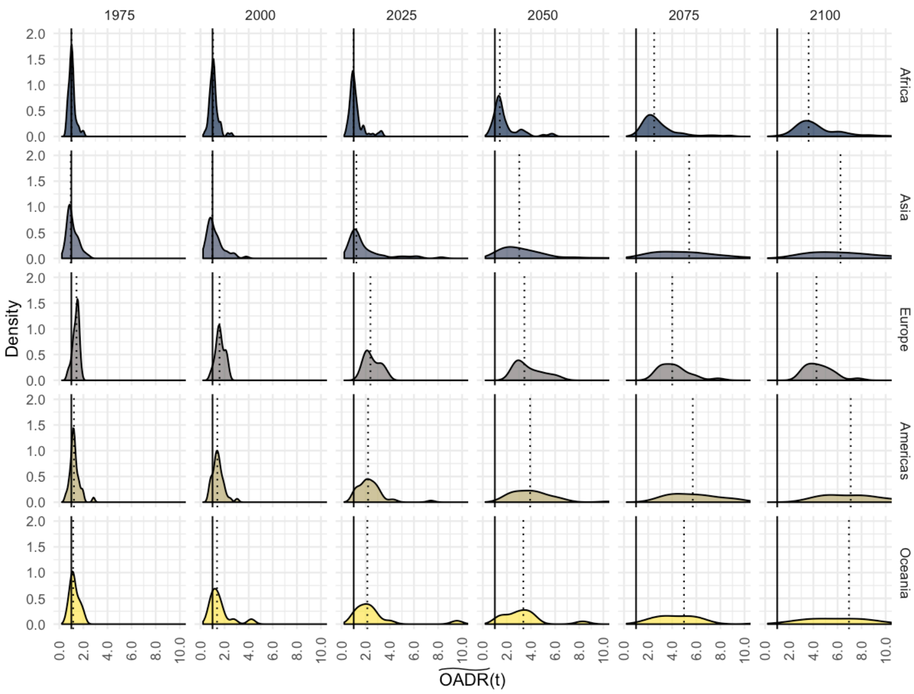
$$\widetilde{OADR}(t) = \frac{OADR(t)}{OADR(1950)} \tag{4.13}$$

Likewise, we may assess the effectiveness of equivalent retirement ages (ERA) relative to the base year 1950 by calculating the ratio of the equivalent old age dependency ratio (EOADR) observed at year t ($EOADR(t)$) to the EOADR observed in 1950 ($EOADR(1950)$),

$$\widetilde{EOADR}(t) = \frac{EOADR(t)}{EOADR(1950)} \tag{4.14}$$

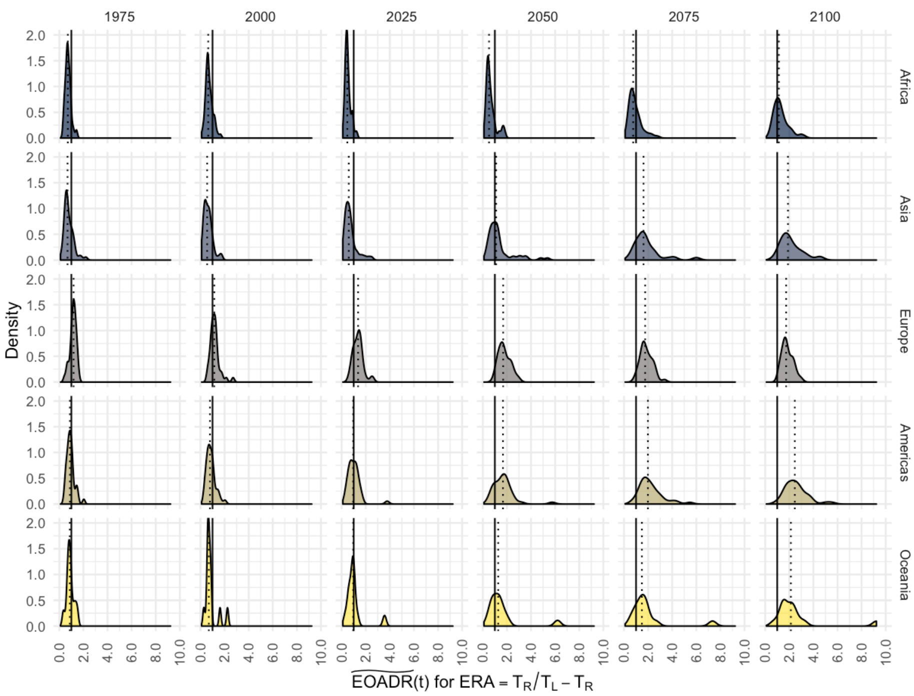
Figures 31 and 32 respectively present the density of $\widetilde{OADR}(t)$ and $\widetilde{EOADR}(t)$ for selected years and all regions with a reference vertical line at 1. Eventually in 2100, the median of $\widetilde{OADR}(t)$ is 3.6 in Africa, 6.2 in Asia, 4.3 in Europe, 7.1 in the Americas, and 7.0 in Oceania. As we could anticipate, $\widetilde{EOADR}(t)$ are quite lower than $\widetilde{OADR}(t)$; in 2100, its median is 1.04 in Africa, 1.9 in Asia, 1.7 in Europe, 2.5 in the Americas, and 2.1 in Oceania.

Figure 31 – Density of $\widetilde{OADR}(t)$ by selected periods and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

Figure 32 – Density of $\widetilde{EOADR}(t)$ by selected periods and regions



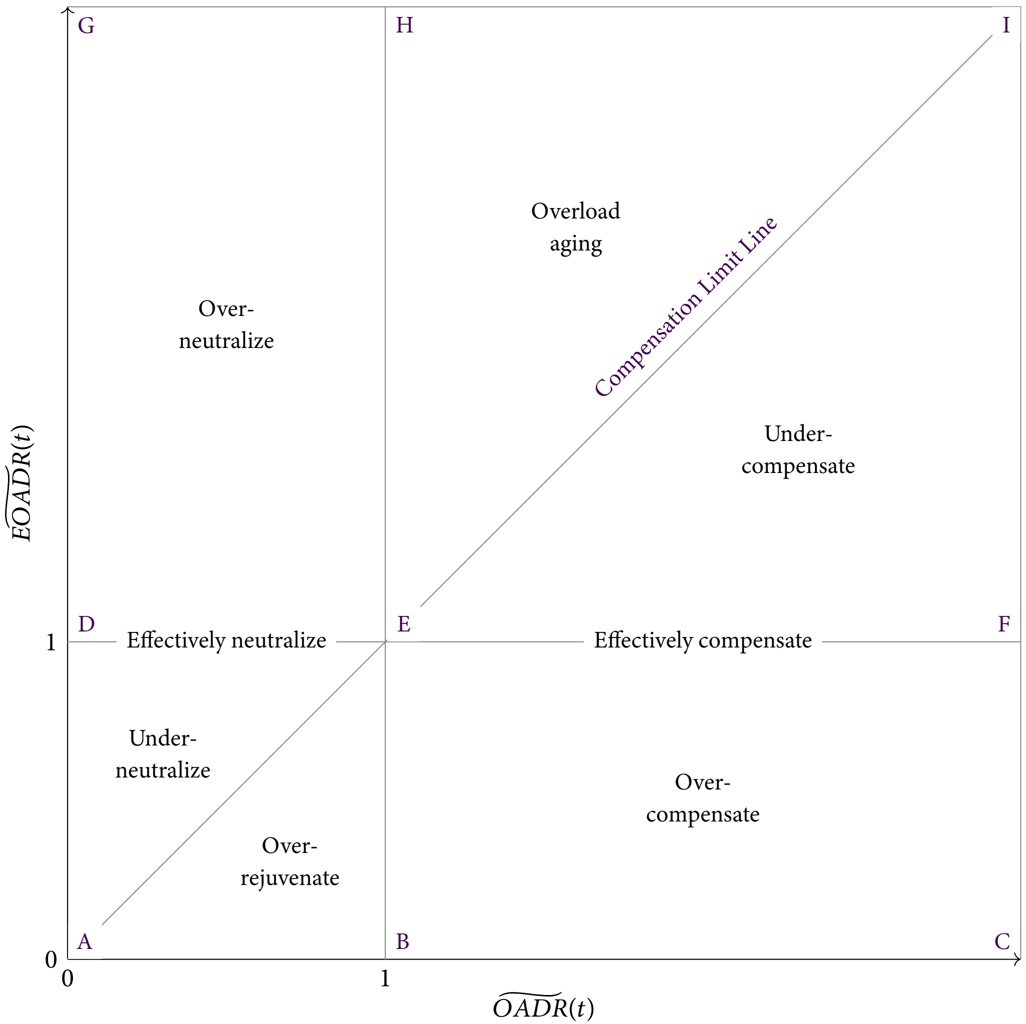
Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

Fundamentally, if $\widetilde{EOADR}(t) = 1$ then equivalent retirement ages (ERA) sustain in year t the same equivalent old age dependency ratios (EOADR) that were observed in the base year 1950, that is, they *effectively* compensate changes in populations age distributions relatively to 1950. But a proper analysis of ERA effectiveness should compare $\widetilde{EOADR}(t)$ with $\widetilde{OADR}(t)$ into *effectiveness categories* as we depict in Figure 33. Populations that are in the line BEH are neither aging nor rejuvenating, those that are above this line are aging, and those that are below this line are rejuvenating, always relatively to ERA's base year. Populations that are in the *compensation limit line*[15] (line AEI) have ERA equal to R, ultimately there is no compensation whatsoever to any changes in populations age distributions relatively to ERA's base year. If populations are aging, those that are above the compensation limit line[16] (triangle EHI) observe ERA which are less than R and thus *overload* the burden of population aging; populations that are below this line[17] observe ERA which are higher than R and thus either *under-compensate*, that is, increase R less than necessary (triangle EFI), or *effectively compensate*, namely, increase R as much as necessary, (line EF), or *over-compensate*, specifically, increase R more than necessary (square BEFC). Similarly, if populations are rejuvenating and R do not need to change, those that are below the compensation limit line (triangle ABE) observe ERA which are higher than R and thus *over-rejuvenate* populations; populations that are above this line observe ERA which are lower than R and thus either *under-neutralize*, that is, decrease R less than necessary to increase EOADR (triangle ADE), or *effectively neutralize*, specifically, decrease R as much as necessary to sustain the same EOADR (line DE), or *over-neutralize*, namely, decrease R more than necessary to increase EOADR (square DGHE).

We plot $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ in Figure 34 and detail it by subregions in Figure 35. Most populations under-compensate, over-compensate, or over-rejuvenate changes in the OADR, with very few discernible populations in the other effectiveness categories. Over-compensate and over-rejuvenate populations are more evident in Africa, Asia, Southern Europe, the Caribbean, South America and Melanesia, and are mostly associated with lower rejuvenating effects of deaths. Populations in Eastern Europe and Northern Europe markedly under-compensate and are linked to higher rejuvenating effects of deaths. Northern America and Australia/New Zealand are closer to effectively compensate.

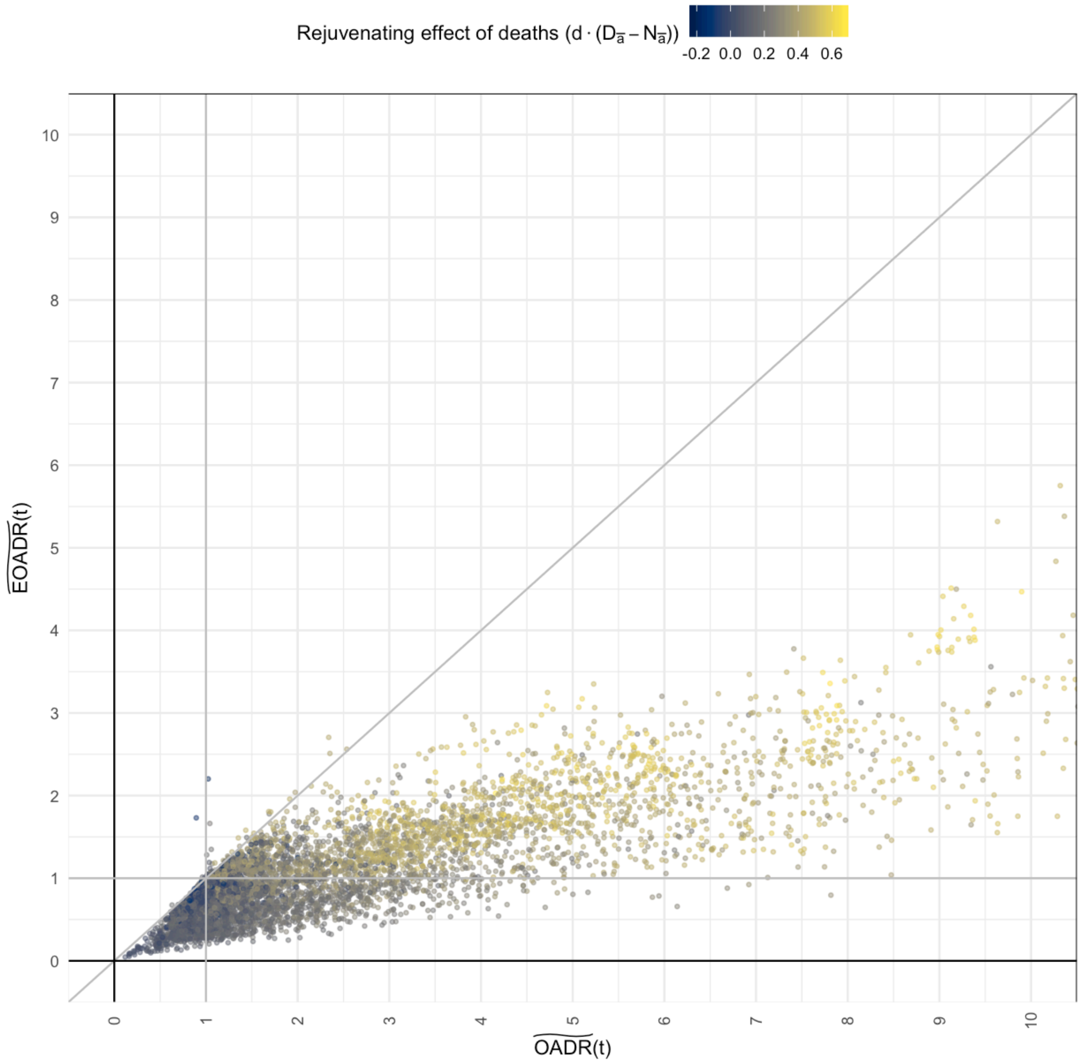
15 $\widetilde{EOADR}(t) = \widetilde{OADR}(t) \implies EOADR(t) = OADR(t)$.
16 $\widetilde{EOADR}(t) > \widetilde{OADR}(t) \implies EOADR(t) > OADR(t)$.
17 $\widetilde{EOADR}(t) < \widetilde{OADR}(t) \implies EOADR(t) < OADR(t)$.

Figure 33 – Equivalent retirement age (ERA) effectiveness categories measured via $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$



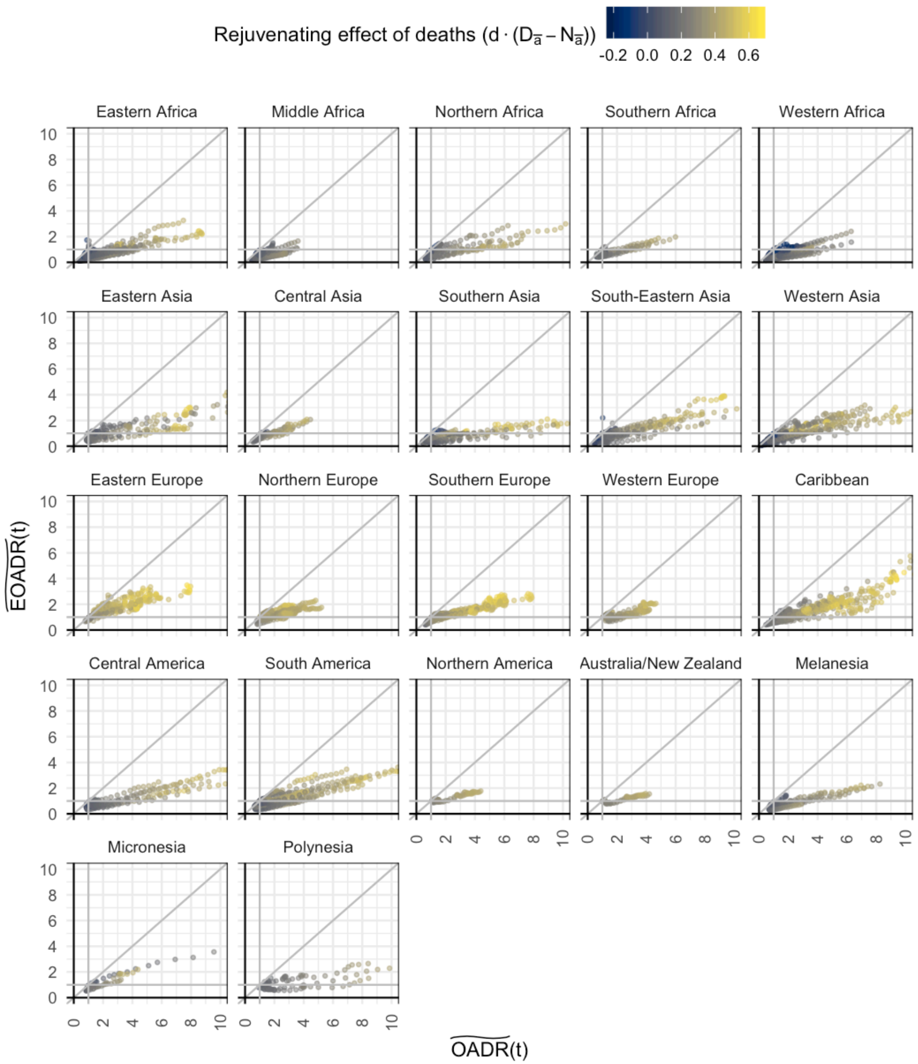
Source: Author's creation.

Figure 34 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 35 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ and subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

We group the number of countries by ERA effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition in Table 9.[18] For a total of 6,030 populations, ERA over-rejuvenate 19% (1,173), over-compensate 30% (1,794), effectively compensate 6% (353), and under-compensate 44% (2,634), with 1% (76) in the remaining categories. For 391 populations that have rejuvenating effects of deaths between -0.2 and 0.0 (i.e., stages 1 to 1A of the demographic transition), ERA over-rejuvenate 57% (223) and over-compensate 28% (109). For 2,251 populations that have rejuvenating effects of deaths between 0.0 and 0.2 (i.e., stages 2 to 3), ERA over-rejuvenate 39% (1,091) and over-compensate 46% (1,151). For 1,925 populations that have rejuvenating effects of deaths between 0.2 and 0.4 (i.e., first half of stage 4), ERA over-compensate 32% (625), effectively compensate, 10% (194), and under-compensate 52% (999). For 1,383 populations that have rejuvenating effects of deaths between 0.4 and 0.6 (i.e., second half of stage 4), ERA under-compensate 96% (1,326). Particularly, for the 353 populations that ERA effectively compensate, 55% (194) are in the first half of stage 4. Thus, if the OADR is aging relatively to ERA's base year, on the one hand, the lower the rejuvenating effect of deaths, the higher the probability that ERA over-compensate; on the other hand, the higher the rejuvenating effect of deaths, the higher the likelihood that ERA effectively compensate or under-compensate.

[18] In Table 9, the stage of the demographic transition is determined exclusively by rejuvenating effect of deaths. Also, to prevent classifying populations with minimum changes both in the OADR and the EOADR relatively to 1950 in any of the effectiveness categories, we classify populations that have $0.95 =< \widehat{OADR}(t) <= 1.05$ and $0.95 =< \widehat{EOADR}(t) <= 1.05$ as neutral-aging. Also, to avert zero counting of populations in the effectiveness categories effectively neutralize and effectively compensate, albeit populations close to be classified as so, we classify populations that observe $\widehat{OADR}(t) > 1.05$ and $0.95 =< \widehat{EOADR}(t) <= 1.05$ as effectively compensate; and populations that observe $\widehat{OADR}(t) < 0.95$ and $0.95 =< \widehat{EOADR}(t) <= 1.05$ as effectively neutralize. We adjust the other ERA effectiveness categories accordingly.

Table 9 – Number of countries by equivalent retirement age (ERA) effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition

Equivalent retirement age (ERA) effectiveness category	Rejuvenating effect of deaths: −0.2, 0.0	0.0	0.0, 0.2	0.2	0.2, 0.4	0.4, 0.6	0.6	> 0.6	Total	Example Countries
	Stage of the demographic transition: 1	1A	2	3	4	4	4A	5		
Neutral-aging	2	0	20	2	17	1	0	0	42	Estonia (1990–1995) New Zealand (1970–1975) Zambia (2000–2005)
Over-neutralize	1	0	0	0	0	0	0	0	1	Rwanda (1990–1995)
Effectively neutralize	0	0	0	0	0	0	0	0	0	⋯
Under-neutralize	0	0	2	1	0	0	0	0	3	South Africa (1980–1985) Guyana (1990–1995) Botswana (2000–2005)
Over-rejuvenate	139	84	786	82	82	0	0	0	1,173	Uruguay (1960–1965) China (1970–1975) Philippines (2010–2015)
Over-compensate	49	60	751	291	625	18	0	0	1,794	Mexico (1995–2000) United Kingdom (2015–2020) India (2030–2035)
Effectively compensate	18	10	74	30	194	27	0	0	353	United States (2020–2025) Chile (2025–2030) France (2030–2035)
Under-compensate	19	8	135	67	999	1,326	62	18	2,634	Switzerland (1985–1990) Italy (2020–2025) Brazil (2035–2040)
Overload aging	1	0	5	5	8	11	0	0	30	Norway (1965–1970) Russian Federation (2000–2005) Zimbabwe (2005–2010)
Total	229	162	1,773	478	1,925	1,383	62	18	6,030	

Source: Author's creation and calculations, based on United Nations (2017c).

Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Neutral-aging = 0.95 =< $\widehat{OADR}(t)$ <= 1.05 and 0.95 =< $\widehat{EOADR}(t)$ <= 1.05; Effectively compensate = $\widehat{OADR}(t)$ > 1.05 and 0.95 =< $\widehat{EOADR}(t)$ <= 1.05; Effectively neutralize = $\widehat{OADR}(t)$ < 0.95 and 0.95 =< $\widehat{EOADR}(t)$ <= 1.05. The other ERA effectiveness categories are adjusted accordingly. Stage of the demographic transition determined exclusively by rejuvenating effect of deaths $(d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)])$.

⋯: Not applicable.

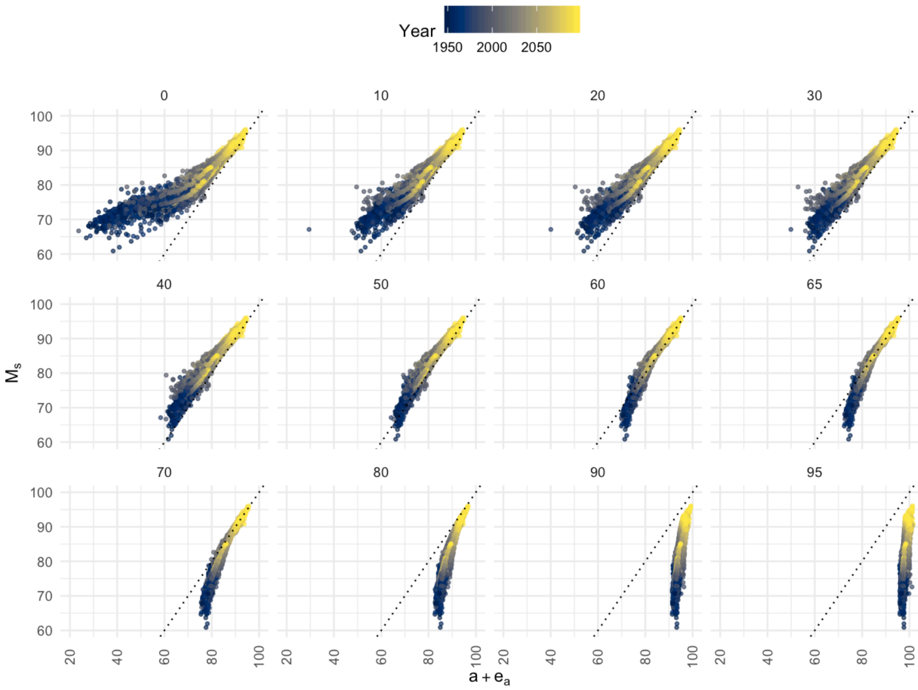
4.8 EQUIVALENT RETIREMENT AGES BASED ON THE MODAL AGE AT DEATH

The equivalent ages presented earlier in this chapter reflect indicators of longevity based on life expectancy at age a ($\mathring{e}_a$), that is, the mean length of life or the mean age at death at age a. But as we note in subsection B.2.2, the modal age at death (M) is a recommended alternative indicator of longevity, because it is only determined by old-age mortality, it is free from bias caused by arbitrary selections of age limits for old-age, and it is also a valuable public health policy guide (e.g., demand for health infrastructures and services are accentuated at ages around M) (HORIUCHI; OUELLETTE, et al., 2013; KANNISTO, 2001; MISSOV et al., 2015).

From birth up to middle-ages, the modal age at death (M) is older than total life expectancy at age a ($a + \mathring{e}_a$) due to the general bimodal and left-skewed age distribution of deaths. At birth, this difference may vary from 30 years in high mortality contexts, to 5 years in low mortality conditions (CANUDAS-ROMO, 2010; HORIUCHI; OUELLETTE, et al., 2013). This variance is the result of the combination of two factors: first, while M is determined by old-age mortality only, $\mathring{e}_a$ is dictated by mortality at all ages equal to and above a; second, the general pattern of mortality decline, with its initial steep decline in infant and child mortality, followed by a reduction in young-age and middle-age adult mortality, ultimately followed by a decline in old-age mortality (HORIUCHI, 1999; WILMOTH, 2000). Accordingly, from birth up to middle-ages, $\mathring{e}_a$ at first rapidly rises at birth and young ages, and next improves at middle-ages, alongside negligible gains in M. Then, as declines in mortality shift to older ages, improvements in $\mathring{e}_a$ decelerates and M increases. Consequently, from birth up to middle-ages, the difference between M and $a+\mathring{e}_a$ first observes a strong decline, and later stabilizes (HORIUCHI; OUELLETTE, et al., 2013). At ages older than middle-ages, however, the patterns and trends between M and $a + \mathring{e}_a$ are quite different. Specifically, when old-age mortality is high, M is nearly equal to $a + \mathring{e}_a$ at 60 years of age, and approximately 5 years younger than $a+\mathring{e}_a$ at 75 years of age. As old-age mortality declines, not only M increases faster than $\mathring{e}_a$, but also the higher the age the slower the increase in $\mathring{e}_a$ (e.g., $\mathring{e}_{75}$ increases slower than $\mathring{e}_{65}$), which ultimately results in M higher than $a + \mathring{e}_a$ (HORIUCHI; OUELLETTE, et al., 2013). Our estimates of the modal age at death from senescent mortality (M_s) (see Appendix B) are consistent to the patterns and trends above, as in Figure 36 that plots total life expectancy at age a ($a + \mathring{e}_a$) by M_s and selected ages.[19]

[19] M_s is practically equal to while somewhat higher than M, assuming that at old ages the proportional level of premature mortality given by the Makeham term (γ) is very low (HORIUCHI; OUELLETTE, et al., 2013, p. 54). See subsection B.2.2 and Figure 113 (Density of Makeham term ratio ($\tilde{\gamma}^{\text{FINAL}}$) for the final mathematical mortality models and selected periods) in Appendix B.

Figure 36 – Total life expectancy at age a ($a + \mathring{e}_a$) by modal age at death from senescent mortality (M_s) and selected ages



Source: Author's calculations, based on United Nations (2017c).

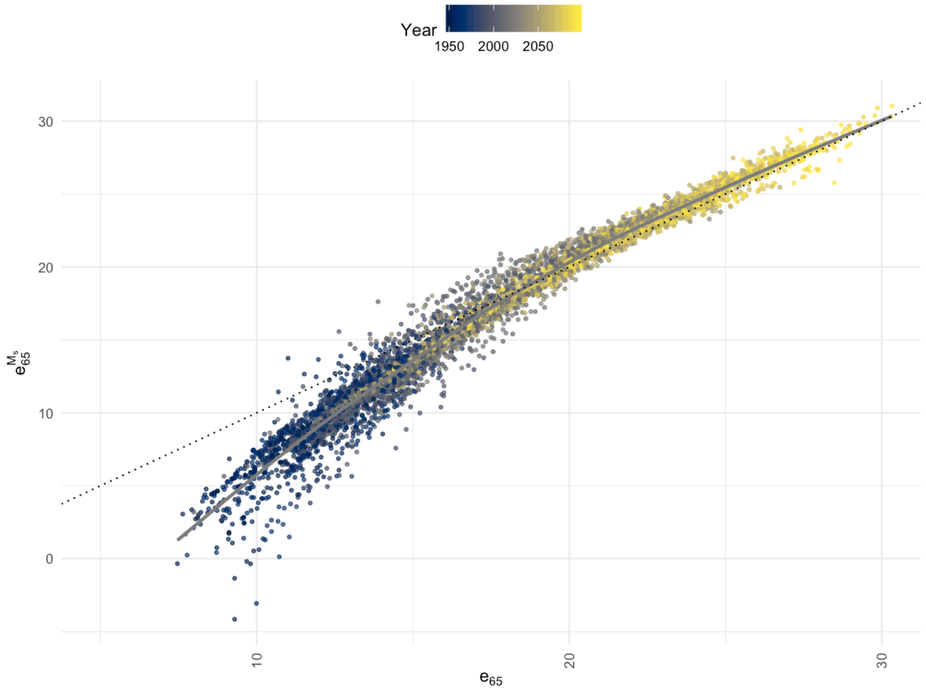
Comparing the total life expectancy at age a ($a + \mathring{e}_a$) with the modal age at death from senescent mortality (M_s) is equivalent to corresponding life expectancy at age a ($\mathring{e}_a$) to $M_s - a$. Consequently, let the difference between M_s and age a be the *modal life expectancy* at age a,

$$\mathring{e}_a^{M_s} = M_s - a \tag{4.15}$$

As an illustration, for age 65, the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) changes from values that are initially 5 to 10 years younger than $\mathring{e}_a$ to values that are around 1 to 2 years older than life expectancy at age a ($\mathring{e}_a$), as we observe in Figures 37 and 38.[20]

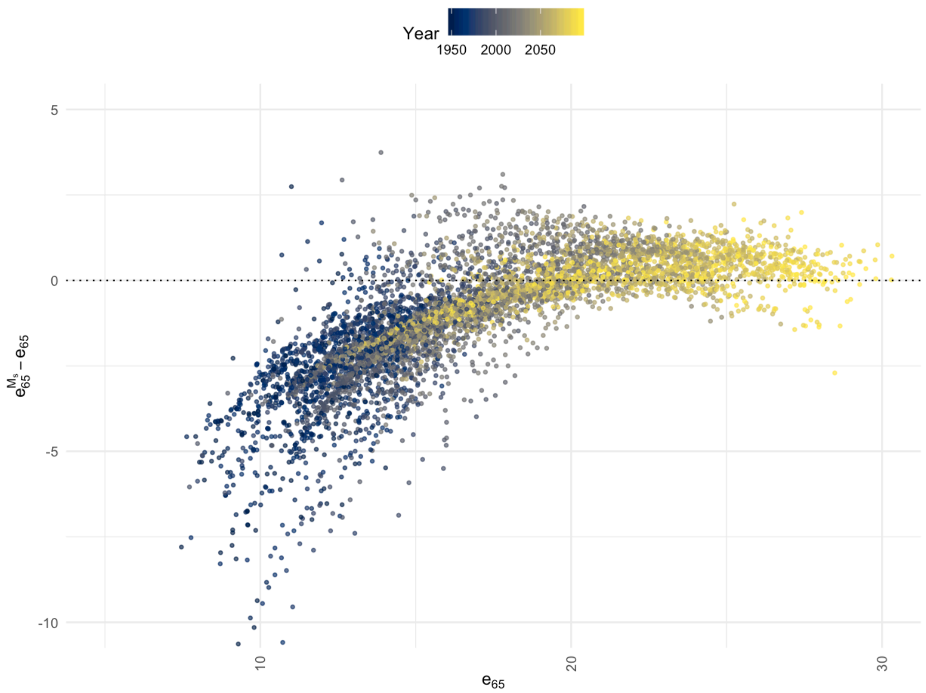
[20] Figure 126 in section D.2, Appendix D, details Figure 38 by subregions.

Figure 37 – Modal life expectancy at age a ($\mathring{e}_a^{M_s}$) by life expectancy at age a ($\mathring{e}_a$) for age 65



Source: Author's calculations, based on United Nations (2017c).

Figure 38 – Life expectancy at age a ($\mathring{e}_a$) by modal life expectancy at age a minus life expectancy at age a ($\mathring{e}_a^{M_s} - \mathring{e}_a$) for age 65



Source: Author's calculations, based on United Nations (2017c).

Thus, when old-age mortality declines, adjusting the age of entry into retirement (R) based on gains in the life expectancy at age a ($\mathring{e}_a$) may be less effective than based on gains in the modal life expectancy at age a ($\mathring{e}_a^{M_s}$), because increases in the age of entry into retirement (R) will be slower than the gains in longevity for *most beneficiaries* of the PAYG system. Accordingly, we propose equivalent retirement age (ERA) measures based on the modal life expectancy at age a ($\mathring{e}_a^{M_s}$). Table 10 presents these measures in terms of the modal number of person-years lived above age a ($T_a^{M_s}$) and the number of survivors to age a (l_a).[21,22]

Table 10 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement based on $\mathring{e}_a^{M_s}$ and in terms of $T_a^{M_s}$ and l_a

Point of measurement	Characteristic of measurement			
	Modal expected years in retirement		Ratio of modal expected years in retirement to modal expected years in work	
Entry into retirement (R)	$\frac{T_R^{M_s}}{l_R}$	(4.16)	$\frac{T_R^{M_s}}{l_R \cdot (R - L)}$	(4.17)
Entry into labor force (L)	$\frac{T_R^{M_s}}{l_L}$	(4.18)	$\frac{T_R^{M_s}}{T_L^{M_s} - T_R^{M_s}}$	(4.19)

Source: Author's creation, based on table in Bayo and Faber (1981, p. 4).

We evaluate the equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s}$ given by the ratio of expected modal years in retirement to expected modal years in work measured at the age of entry into the labor force (L) (Equation 4.19). Figure 39 plots the density of ERA based on $\mathring{e}_a^{M_s}$ for selected periods and all regions with a reference vertical line at 65 years; and Figure 40 details its distribution in 2100 by subregions. Figure 41 presents the density of the difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA based on $\mathring{e}_a$ for selected periods and all regions with a vertical reference line at zero; and Figure 42 plots its distribution in 2100 by subregions.

In 2100, the median of the difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA based on $\mathring{e}_a$ is 1.1 years in Africa, 2.2 years in Asia, 1.3 years in Europe, and 1.6 years in the Americas and Oceania. In 2100, the lowest median of the differences are in Western Africa (0.4 year) and Micronesia (0.9 year), and the highest are in Polynesia (3.9 years), Central Asia (3.0 years), and Eastern Asia (2.8 years). Still in 2100, extreme high differences are observed in Cambodia (5.4 years, 87.1 vs 81.7), China (4.8 years, 89.3 vs 84.5), and Mauritius (4.8, 86.8 vs 82.0). Negative differences are present in eight countries, with only two relevant, Honduras (−1.4 years, 79.6 vs 81.0) and Papua New Guinea (−0.7 year, 77.3 vs 78.0).[23]

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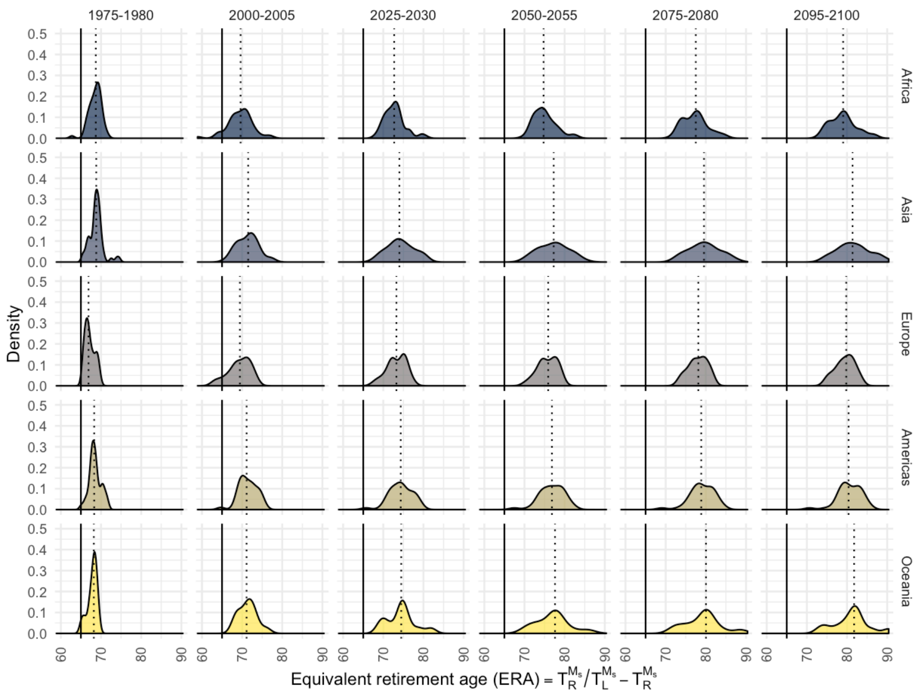
21 $T_a^{M_s} = \mathring{e}_a^{M_s} \cdot l_a$.

22 Table 17 in section D.2, Appendix D, presents the same measures, in terms of the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) and the number of survivors to age a (l_a).

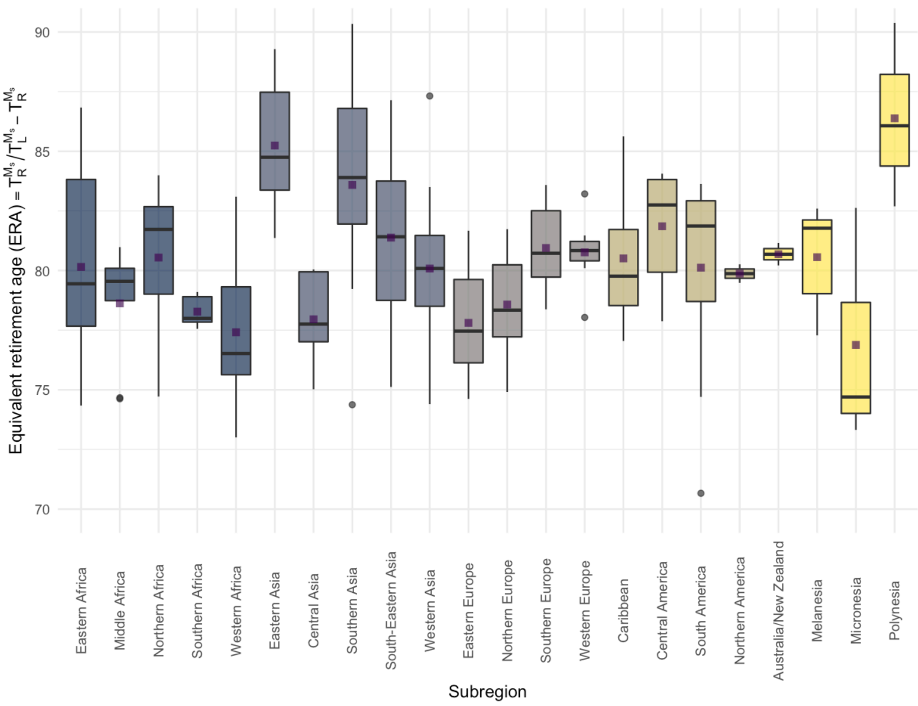
23 Honduras (−1.37 years), Papua New Guinea (−0.71 years), Mali (−0.31 year), Martinique (−0.23 year), Afghanistan (−0.16 year), Gambia (−0.13 year), Montenegro (−0.046 year), and Comoros (−0.033 year).

Figure 39 – Density of ERA based on $\mathring{e}_a^{M_s}$ by selected periods and regions



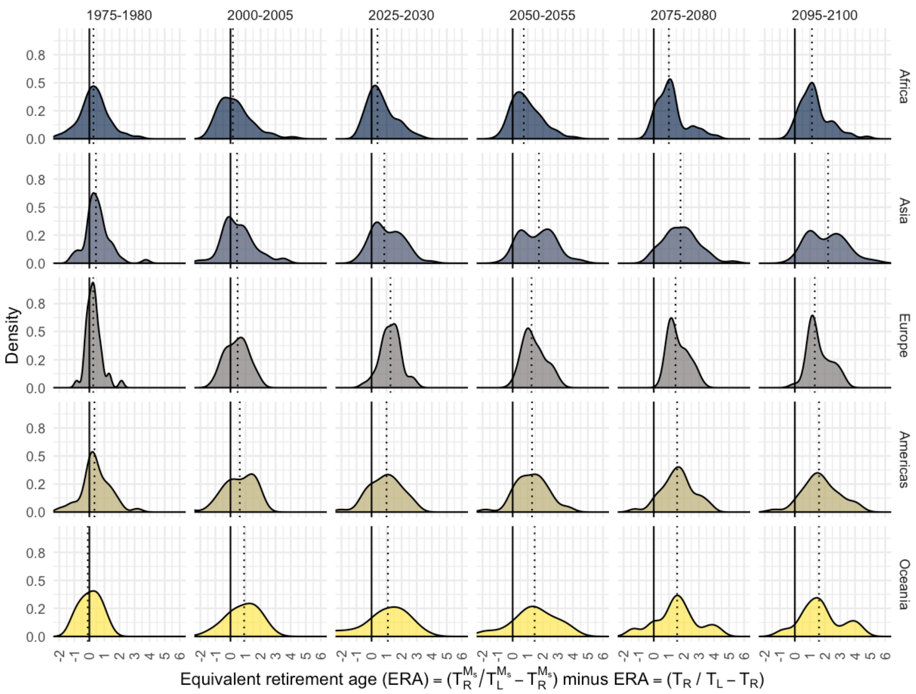
Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Vertical dotted line indicates the median of the distribution.

Figure 40 – Equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s}$ in 2100 by subregions



Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Square indicates the mean of the distribution.

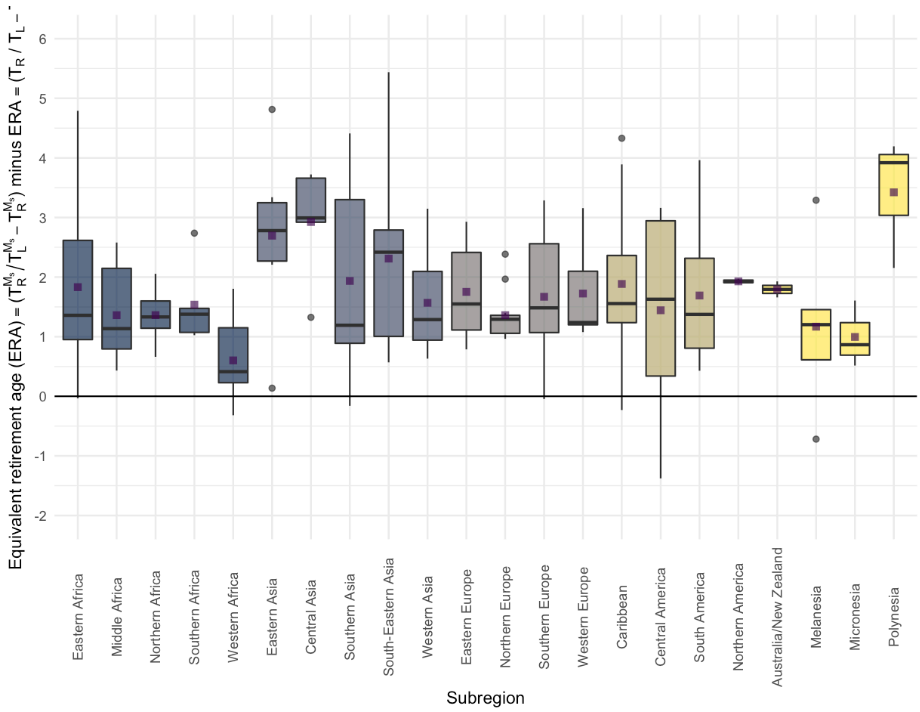
Figure 41 – Density of difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA selected periods and regions



Source: Author's calculations, based on United Nations (2017c).

Notes: Equivalent retirement age (ERA) $= (T_R)/(T_L - T_R)$. Equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s} = (T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Vertical dotted line indicates the median of the distribution.

Figure 42 – Difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA in 2100 by subregions



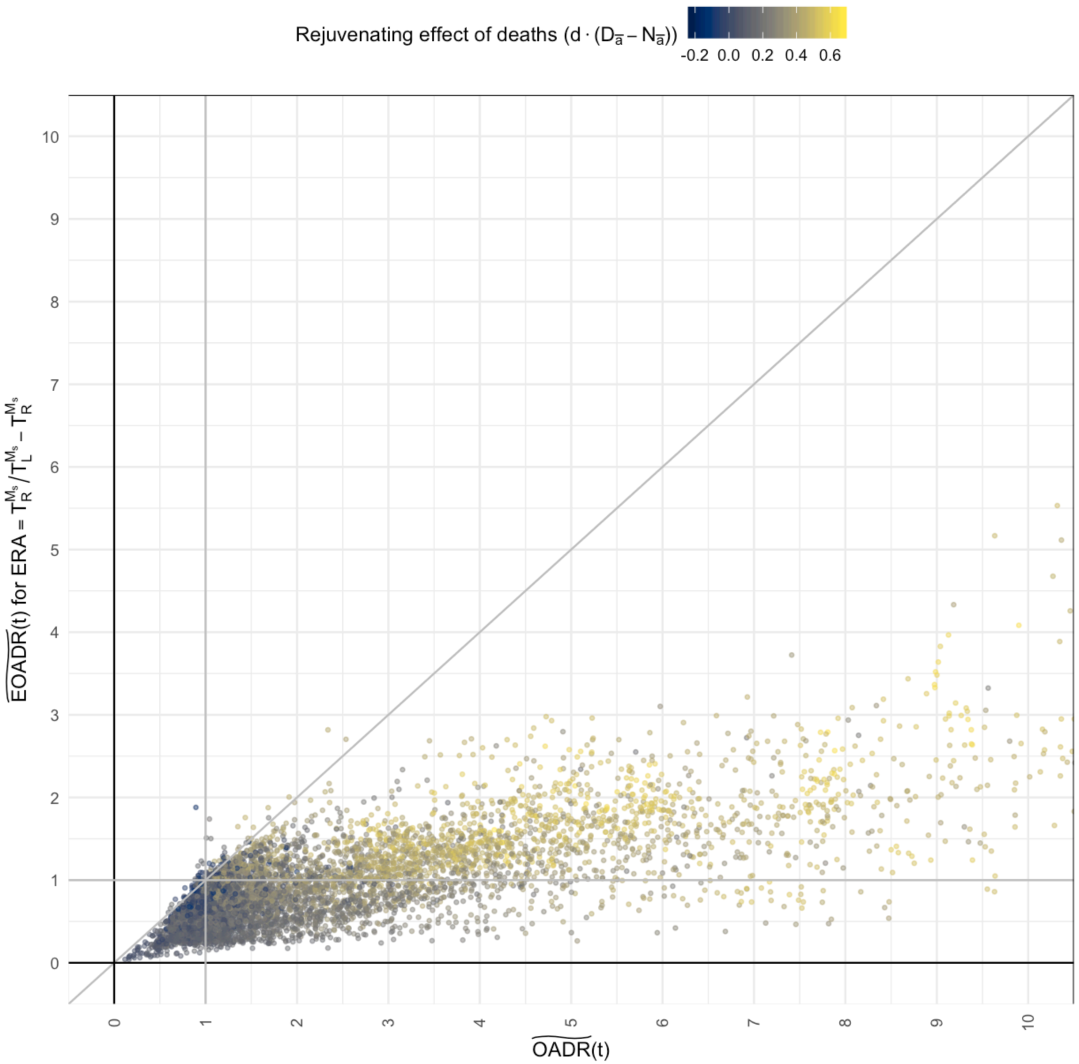
Source: Author's calculations, based on United Nations (2017c).

Notes: Equivalent retirement age (ERA) $= (T_R)/(T_L - T_R)$. Equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s} = (T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Square indicates the mean of the distribution.

We plot $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ in Figure 43.[24] Equivalent retirement ages (ERA) based on $\mathring{e}_a^{M_s}$ are generally higher than ERA based on $\mathring{e}_a$; consequently, $\widetilde{EOADR}(t)$ for ERA based on $\mathring{e}_a^{M_s}$ are mostly lower than $\widetilde{EOADR}(t)$ for ERA based on $\mathring{e}_a$. Therefore, if the OADR is aging relatively to ERA's base year, we may expect that ERA based on $\mathring{e}_a^{M_s}$ increase the number of populations that over-compensate and decrease the number of populations that under-compensate. We group the number of countries by ERA based on $\mathring{e}_a^{M_s}$ effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition in Table 11. For a total of 6,030 populations, ERA based on $\mathring{e}_a^{M_s}$ maintains over-rejuvenate at 19% (1,173 to 1,141), increases over-compensate from 30% to 36% (1,794 to 2,191), holds effectively compensate at 6% (353 to 380), and decreases under-compensate from 44% to 37% (2,634 to 2,205), with from 1% to 2% (76 to 113) in the remaining categories. The most relevant changes happen for populations that have rejuvenating effects of deaths between 0.2 and 0.6 (stage 4 of the demographic transition). For 1,925 populations that have rejuvenating effects of deaths between 0.2 and 0.4 (i.e., first half of stage 4), over-compensate increases from 32% to 45% (625 to 871), effectively compensate virtually remains the same from 10% to 11% (194 to 217), and under-compensate decreases from 52% to 38% (999 to 733). For 1,383 populations that have rejuvenating effects of deaths between 0.4 and 0.6 (i.e., second half of stage 4), under-compensate decreases 96% to 87% (1,326 to 1,209). Therefore, although ERA based on $\mathring{e}_a^{M_s}$ increase the number and level of over-compensate, and decrease the number and level of under-compensate (see Figures 34 and 43), for rejuvenating effects of deaths above 0.4, they improve the likelihood to effectively compensate.

[24] Figure 127 in section D.2, Appendix D, details Figure 43 by subregions.

Figure 43 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ for ERA based on $\mathring{e}_a^{M_s}$



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$.

Table 11 – Number of countries by equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s}$ effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition

Equivalent retirement age (ERA) effectiveness category	Rejuvenating effect of deaths								Total
	−0.2, 0.0	0.0	0.0, 0.2	0.2	0.2, 0.4	0.4, 0.6	0.6	> 0.6	
	Stage of the demographic transition								
	1	1A	2	3	4	4	4A	5	
Neutral-aging	5	1	9	3	12	1	0	0	31
Over-neutralize	3	2	3	0	0	0	0	0	8
Effectively neutralize	4	0	4	1	0	0	0	0	9
Under-neutralize	10	2	6	4	0	0	0	0	22
Over-rejuvenate	122	80	778	78	83	0	0	0	1,141
Over-compensate	52	59	791	320	871	98	0	0	2,191
Effectively compensate	7	10	64	16	217	66	0	0	380
Under-compensate	18	7	107	51	733	1,209	62	18	2,205
Overload aging	8	1	11	5	9	9	0	0	43
Total	229	162	1,773	478	1,925	1,383	62	18	6,030

Source: Author's creation and calculations, based on United Nations (2017c).

Notes: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Neutral-aging = 0.95 =< $\widetilde{OADR}(t)$ <= 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05; Effectively compensate = $\widetilde{OADR}(t)$ > 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05; Effectively neutralize = $\widetilde{OADR}(t)$ < 0.95 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05. The other ERA effectiveness categories are adjusted accordingly. Stage of the demographic transition determined exclusively by rejuvenating effect of deaths ($d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)]$).

4.9 CONCLUSION

Population aging is unavoidable, pervasive, and an uninsurable risk to any pay-as-you-go (PAYG) retirement system because it changes the relation between beneficiaries and contributors. An alternative policy that may buffer the burden of population aging in PAYG systems is to adjust the age of entry into retirement (R) based on gains in life expectancy, also known as equivalent retirement age (ERA). This policy has been implemented by several countries across the globe. Nevertheless, we argue that equivalent retirement ages (ERA) are intrinsically ineffective because PAYG systems are structured on population characteristics, while equivalent retirement ages (ERA) are based on life cycle characteristics. We propose effectiveness categories for equivalent retirement ages (ERA) that are based on the change of both the old age dependency ratio (OADR) and the equivalent old age dependency ratio (EOADR) relatively to ERA's base year. We demonstrate that if the old age dependency ratio (OADR) is aging relatively to the equivalent retirement age (ERA)'s base year, on the one hand, the lower the rejuvenating effect of deaths, the higher the probability that the equivalent retirement age (ERA) increases the age of entry into retirement (R) more than necessary; on the other hand, the higher the rejuvenating effect of deaths, the higher the likelihood that the equivalent retirement age (ERA) increases the age of entry into retirement (R) less than necessary. Also, we argue that when old-age mortality declines, equivalent retirement ages (ERA) based on gains in the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) may be less ineffective than equivalent retirement ages (ERA) based on gains in the life expectancy at age a ($\mathring{e}_a$).

From a policy guidance standpoint for PAYG systems that adopt equivalent retirement ages (ERA), if the old age dependency ratio (OADR) is not aging relatively to the equivalent retirement age (ERA)'s base year, policymakers should not increase the age of entry into retirement (R). If the old age dependency ratio (OADR) is aging and the equivalent old age dependency ratio (EOADR) is not aging both relatively to the equivalent retirement age (ERA)'s base year, policymakers should either increase the age of entry into retirement (R) to less than determined by the equivalent retirement age (ERA), or increase the age of entry into retirement (R) to the age bounded by the equivalent retirement age (ERA) and also decrease the contribution rate (*con*), increase the benefit rate (*ben*), or both. If the old age dependency ratio (OADR) and the equivalent old age dependency ratio (EOADR) are both aging relatively to the equivalent retirement age (ERA)'s base year, policymakers should increase the age of entry into retirement (R) to more than delimited by the equivalent retirement age (ERA), or increase the age of entry into retirement (R) to the age regulated by the equivalent retirement age (ERA) and increase the contribution rate (*con*), decrease the benefit rate (*ben*), or both. Fixed relative position (FRP) PAYG systems have the best policy design to face any of these scenarios, not only because they offer greater flexibility for *ad hoc* policy changes to contribution rates (*con*) or benefit rates (*ben*), but because they dilute any risk of equivalent retirement age (ERA) ineffectiveness between contribution rates (*con*) and benefit rates (*ben*) as well.

5 *Period balances, generational imbalances*

5.1 INTRODUCTION

POPULATION AGING impacts the return of pay-as-you-go (PAYG) systems to birth cohorts, because changes in period relations between beneficiaries and contributors lead to distinct life cycle contribution rates (*con*) and life cycle benefit rates (*ben*) among different birth cohorts. That is, PAYG systems are built on intergenerational transfers that are intrinsically redistributive, and likely favor some birth cohorts to the detriment of others, leading to inequitable returns among generations. In short, PAYG systems treat generations differently, by definition. We acknowledged (see section 4.2) that the long-term political viability of PAYG systems rest on their policy designs, that is, on the nature of their *intergenerational contracts* (MUSGRAVE, 1981, p. 96–98). Yet now we also agree that period financial balances of PAYG systems fundamentally concern short-term fiscal equilibria, whereas their life cycle intergenerational transfers relate to long-term equality among generations, and thus that the long-term stability of PAYG systems hinge on their *intergenerational equality* as well (KEYFITZ, 1985, p. 7).

We use a *stylized demographic model* to investigate the effects of population aging on the returns of PAYG systems to birth cohorts in the world from 1950 to 2100, specifically, from the 1925–1930 birth cohort to the 2000–2005 birth cohort. Identically to chapter 4, in our stylized demographic model, all population (N) between the age of entry into the labor force (L) and the age of entry into retirement (R) works and contributes to the PAYG system; all population older than R is retired and receives benefits from the PAYG system; contributions are equal to contribution rates (*con*) times wages (w); benefits are equal to benefit rates (*ben*) times wages (w); age of entry into the labor force (L) and age of entry into retirement (R) are initially fixed; contribution rates (*con*), benefit rates (*ben*), and wages (w) are not age-specific; and wages (w) do not vary in response to either the labor market dynamics or productivity changes.

We analyze the returns of PAYG systems to birth cohorts under different policy designs and retirement ages of PAYG systems, and how equivalent retirement age (ERA) policies influence them. First, we present a mathematical expression introduced by Samuelson (1958) and expanded by Aaron (1966) to measure the return of PAYG systems to birth cohorts. Second, we detail our methods and assumptions. Third, we estimate the return of PAYG systems to birth cohorts for defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems. Fourth, we evaluate how different fixed retirement ages, and equivalent retirement ages (ERA) influence the returns of defined benefit (DB) PAYG systems and defined contribution (DC) PAYG systems to birth cohorts. Last, we analyze the influence of the demographic determinants of population aging on the returns of PAYG systems to birth cohorts.

5.2 GENERATIONAL IMBALANCES: INTRINSIC RATE OF RETURN

In a pay-as-you-go (PAYG) system, the intrinsic rate of return (IRR) of an individual or birth cohort is the rate of interest that equals the life cycle flow of contributions with the life cycle flow of benefits when both are discounted to any given date.[1] Expanding on Samuelson (1958)'s seminal work, Aaron (1966) proposed the equation for the intrinsic rate of return (IRR) as a measure of the welfare (i.e., return) of PAYG systems to individuals (i.e., birth cohorts). Keyfitz (1985, 1988) and Lapkoff (1983, 1991) extended and further explored the IRR with the analysis of intergenerational inequality in PAYG systems.[2] The intrinsic rate of return (IRR) has advantages over other measures of the return of PAYG systems to birth cohorts:[3] it warrants unbiased comparisons of demographic scenarios, policy designs, and retirement age policies of PAYG systems (KEYFITZ, 1985); it favors measuring PAYG systems against the rates of return of investments, fully funded retirement systems, or other transfer systems (KEYFITZ, 1985, 1988); and it does not demand the choice of a discount rate. Still, the main advantage of the IRR is that it incorporates into a single measure the interactions of periods and birth cohorts in a PAYG system, specifically, the interactions of the relation between beneficiaries and contributors of periods and the population structure of the birth cohort (i.e., the mortality level and structure of the birth cohort) (FERNANDES, 1993, p. 21). Let a be age; t, time; c, cohort; t^c, the year of birth of cohort c; B, births; $p(a)$, the probability of surviving from birth to age a; and ρ, the intrinsic rate of return:

$$\int_L^R N^c(a) \cdot con(t^c + a) \cdot w \cdot e^{-\rho^c \cdot a}\, da = \int_R^\infty N^c(a) \cdot ben(t^c + a) \cdot w \cdot e^{-\rho^c \cdot a}\, da \tag{5.1a}$$

$$N^c(a) = B^c \cdot p^c(a) \tag{5.1b}$$

$$\therefore \int_L^R p^c(a) \cdot con(t^c + a) \cdot w \cdot e^{-\rho^c \cdot a}\, da = \int_R^\infty p^c(a) \cdot ben(t^c + a) \cdot w \cdot e^{-\rho^c \cdot a}\, da \tag{5.1c}$$

In a stable population, the intrinsic rate of return (IRR) equals the population growth rate, which is equivalent to Samuelson (1958)'s *biological interest rate*. Also, in a stable population whose economy (wages) grows exponentially at a constant rate, the IRR equals the population growth rate plus the wages growth rate; consequently, if the sum of the growth rates of population and wages is greater than the market rate of interest, a PAYG system raises everyone's welfare compared with a fully funded retirement system, which is Aaron (1966)'s *social insurance paradox*. Therefore, economic growth and population growth have the same effect in the return of

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1 The chosen date (e.g., birth of the cohort, implementation of the PAYG system) does not influence the intrinsic rate of return (IRR).

2 This rate was named "implicit rate of interest" in Keyfitz (1985), "implicit rate of return" in Keyfitz (1985, 1988) and Lapkoff (1983, 1991), and "implied rate of return" in Lapkoff (1991). We follow the term intrinsic rate of return (IRR) used elsewhere (FERNANDES, 1993).

3 E.g., net present value, benefits to contributions ratio.

PAYG systems to birth cohorts; yet an economy can produce higher growth rates than a population can (KEYFITZ, 1985, p. 6–7).[4]

We argued (see section 4.2) that different policy designs of PAYG systems imply distinct life cycle perspectives. Consequently, different policy designs result in distinct life cycle flow of contributions and life cycle flow of benefits, and thus distinct intrinsic rates of return (IRR) for each birth cohort. Consider Equation 5.1a, in defined benefit (DB) PAYG systems, between L and R, the IRR results from the old age dependency ratio (OADR) of periods (via *con*) and the population structure of the birth cohort (via $N_c(a)$), and after R, the IRR derives only from the population structure of the birth cohort. Conversely, in defined contribution (DC) PAYG systems, between L and R, the IRR results only from the population structure of the birth cohort, and after R, the IRR derives from the old age dependency ratio (OADR) of periods (via *ben*) and the population structure of the birth cohort.[5] Besides, when populations age, birth cohorts observe increasing OADR as they move from working to retirement years; consequently, when populations age, the IRR derives from lower OADR in DB PAYG systems than in DC PAYG systems, and thus the intrinsic rate of return (IRR) is higher in DB PAYG systems than in DC PAYG systems. Moreover, when populations age, more birth cohorts benefit from lower OADR in DB PAYG systems than in DC PAYG systems, and thus, when populations age, the intrinsic rate of return (IRR) decreases more smoothly and have lower intergenerational inequalities in DB PAYG systems than in DC PAYG systems. In sum, when populations age, defined benefit (DB) PAYG systems are preferable to defined contribution (DC) PAYG systems because the former yields higher and less variable intrinsic rates of return (IRR) (FERNANDES, 1993, p. 32–41).

Similarly, in fixed relative position (FRP) PAYG systems, both between L and R, and after R, the intrinsic rate of return (IRR) results from both *con* and *ben* of periods, and the population structure of the birth cohort. Yet in FRP PAYG systems, *con* and *ben* derive not only from the relation between beneficiaries and contributors of periods, but also from the constant ratio of

[4] In a stable population with given population growth rate (r), wages growth rate (ψ), and intrinsic rate of return (ρ), the following equality stands (FERNANDES, 1993, p. 24–25; LAPKOFF, 1983):

$$\frac{\int_L^\infty p(a)\cdot e^{-ra}\,da}{\int_L^R p(a)\cdot e^{-ra}\,da} = \frac{\int_L^\infty p(a)\cdot e^{\psi a}\cdot e^{-\rho a}\,da}{\int_L^R p(a)\cdot e^{\psi a}\cdot e^{-\rho a}\,da} \tag{5.2a}$$

$$\therefore \rho = r + \psi \tag{5.2b}$$

[5] The fixed levels of either *ben* in DB PAYG systems or *con* in DC PAYG systems do not influence the IRR, because if we multiply the fixed *ben* or *con* by any constant k, then the variable *con* or *ben* are also respectively multiplied by k, formally, from Equation 5.1a (FERNANDES, 1993, p. 28):

$$\int_L^R N_c(a)\cdot k\cdot con(t_c+a)\cdot w\cdot e^{-\rho_c a}\,da = \int_R^\infty N_c(a)\cdot k\cdot ben(t_c+a)\cdot w\cdot e^{-\rho_c a}\,da \tag{5.3a}$$

$$\therefore \int_L^R N_c(a)\cdot con(t_c+a)\cdot w\cdot e^{-\rho_c a}\,da = \int_R^\infty N_c(a)\cdot ben(t_c+a)\cdot w\cdot e^{-\rho_c a}\,da \tag{5.3b}$$

per capita benefits to per capita net wages (ϕ). Let us revisit the equations for *con* and *ben* in FRP PAYG systems:

$$con(t) = \frac{\phi \cdot \int_R^\infty N(a,t)\, da}{\int_L^R N(a,t)\, da + \phi \cdot \int_R^\infty N(a,t)\, da} \qquad \text{(4.4b revisited)}$$

$$ben(t) = \frac{\phi \cdot \int_L^R N(a,t)\, da}{\int_L^R N(a,t)\, da + \phi \cdot \int_R^\infty N(a,t)\, da} \qquad \text{(4.4c revisited)}$$

Equations 4.4b and 4.4c give:

$$\phi \to 0 = \begin{cases} con & \to \phi \cdot OADR \\ ben & \to \phi \\ \therefore \rho & \to \text{DB} \end{cases} \qquad (5.4a)$$

$$\phi \to \infty = \begin{cases} con & \to 1 \\ ben & \to 1 \,/\, OADR \\ \therefore \rho & \to \text{DC} \end{cases} \qquad (5.4b)$$

That is, the lower ϕ, the closer FRP PAYG systems are to DB PAYG systems; the higher ϕ, the closer FRP PAYG systems are to DC PAYG systems. Consequently, in FRP PAYG systems, the level and structure of intrinsic rates of return (IRR) are intermediary to those of DB PAYG systems and DC PAYG systems. Therefore, counterintuitive as it may seem, when populations age, increasing the ratio of per capita benefits to per capita net wages (ϕ) may not raise fixed relative position (FRP) PAYG systems' welfare but, on the contrary, may lead to lower and faster decreases of intrinsic rates of return (IRR).

Different retirement ages tend to change the intrinsic rate of return (IRR) distinctively in defined benefit (DB) PAYG systems and in defined contribution (DC) PAYG systems. In DB PAYG systems, a higher age of entry into retirement (R) renders the birth cohort more working years with lower contribution rates (*con*) and less retirement years with the same benefit rate (*ben*). Analogously, in DC PAYG systems, a higher age of entry into retirement (R) ensues the birth cohort more working years with the same contribution rate (*con*) and less retirement years with higher benefit rates (*ben*). But in DB PAYG systems, the birth cohort enjoys the compensations from a higher R (i.e., lower *con*) immediately after the age of entry into the labor force (L); whereas in DC PAYG systems, the birth cohort enjoys the compensations from a higher R (i.e., higher *ben*) only after the age of entry into retirement (R). Therefore, a higher age of entry into retirement (R) tends to increase the intrinsic rate of return (IRR) in defined benefit (DB) PAYG systems because the lower *con* from a higher R reflect upon a larger proportion of the birth cohort's life cycle (i.e., working years) and also because the birth cohort is not dependent on surviving to older ages to enjoy them. Contrarily, a higher age of entry into retirement (R) tends to decrease the intrinsic rate of return (IRR) in defined contribution (DC) PAYG systems because the higher *ben* from a higher R reflect upon a smaller proportion of the birth cohort's life cycle (i.e., retirement years) and also because the birth cohort is dependent on surviving to yet older ages to enjoy them. Further, when populations age, the scenario may be still less promising for defined contribution (DC) PAYG systems because birth cohorts observe reductions in benefit rates (*ben*) (i.e., growth in the OADR), and thus the birth cohort may experience equal or lesser *ben* from a higher age of entry into retirement (R). Ultimately, if the age of entry into retirement (R) is extremely high (i.e., $R \to \infty$), the probability of surviving to R approaches zero (i.e., $p_{a,R} \to 0$), then, in defined benefit (DB) PAYG systems, the life cycle flow of contributions approaches zero, the life cycle flow of benefits also approach zero, and the intrinsic rates of return (IRR) approaches zero; and, in defined contribution (DC) PAYG systems, the life cycle flow of contributions does not change, the life cycle flow of benefits approaches zero, and the intrinsic rates of return (IRR) approach minus infinity (FERNANDES, 1993, p. 32–41).

Boskin and Puffert (1987) analyzed the financial impact for birth cohorts, households and system finances of the retirement segment of the United States Social Security under alternative scenarios for fertility, mortality, wage growth, and financing assumptions. They estimated real intrinsic rates of return (IRR) of retirement for cohorts from before–1912 to 1983–1992. In their base case, IRR were 11.61% for the before–1912 cohort, 5.74% for the 1913–1922 cohort, 3.72% for the 1923–1932 cohort, and between 2.75% and 1.96% for the younger cohorts. Their fertility scenarios would only impact the IRR of future birth cohorts that were not included in their study. Their mortality scenarios other than the base case were for high mortality and low mortality.[6] For before 1923–1932 cohorts, the IRR would decrease around 0.15% in the high mortality scenario, and would increase about 0.05% in the low mortality scenario; for the 1933–1942 cohort the IRR would decrease 0.16% in the high mortality scenario and would increase 0.22% in the low mortality scenario; and for cohorts from 1942–1952, the IRR would decrease around 0.3% in the high mortality scenario, and would increase about 0.4% in the low mortality scenario. Their real wages growth scenarios assumed 1.5% per year in the base case, 2.5% per year in the high wage growth, and 0.0% per year in the low wage growth. For before–1912 and 1913–1922 cohorts, high or low wage growth scenarios would not change the IRR; for the 1923–1932 cohort the IRR would increase 0.16% in the high wage growth scenario and would decrease 0.12% in the low wage growth scenario; and for cohorts from 1933–1942, the IRR would increase around 0.6% in the high wage growth scenario, and would decrease about 0.3% in the low wage growth scenario. Boskin and Puffert (1987) considered six alternative scenarios for the trust fund surplus, and two of these scenarios assumed that taxes and benefits would be balanced starting in 1990: the "pay-as-you-go tax rates" scenario assumed that tax rates would adjust to balance benefit payments (i.e., defined benefit (DB)); and the "pay-as-you-go benefits" scenario assumed that benefit rates

6 Boskin and Puffert (1987) did not detail any implicit indicators for the mortality scenarios (i.e., life expectancy at birth).

would adjust to match tax receipts (i.e., defined contribution (DC)). For cohorts from before 1912 to 1933–1942, the IRR of the "pay-as-you-go benefits" scenario would be on average 0.25% higher than those of the "pay-as-you-go tax rates" scenario, because in DC PAYG systems they would be favored by the larger fixed taxes paid by the baby boom cohort; and for cohorts from 1942–1952, the IRR of the "pay-as-you-go tax rates" scenario would be on average 0.6% higher than those of the "pay-as-you-go benefits" scenario, because DB PAYG systems would postpone the effects of population aging on intrinsic rates of return (IRR).

Fernandes (1993) used a stylized demographic model to estimate intrinsic rates of return (IRR) for Brazil for birth cohorts from 1930–1935 to 2145–2150, separately for men and women, for both defined benefit (DB) and defined contribution (DC) PAYG systems, and retirement ages 60, 65 and 70 years. The estimated intrinsic rates of return (IRR) were higher for older cohorts, higher for women than for men, higher for DB PAYG systems than for DC PAYG systems, higher in DB PAYG systems for upper retirement ages, and lower in DC PAYG systems for upper retirement ages. For example, for retirement age at 60 years and the 1930–1935 cohort, the IRR for men were 3.01% in DB PAYG systems and 2.12% in DC PAYG systems; the IRR for women were 3.76% in DB PAYG systems and 2.84% in DC PAYG systems. For the 2035–2040 cohort and retirement age at 60, 65 and 70 years respectively, the IRR for men were 0.60%, 0.65% and 0.72% in DB PAYG systems and −0.08%, −0.11% and −0.15% in DC PAYG systems; the IRR for women were 0.93%, 1.02% and 1.14% in DB PAYG systems and 0.26%, 0.26% and 0.27% in DC PAYG systems. Fernandes (1993) estimated that defined benefit (DB) PAYG systems compared with defined contribution (DC) PAYG systems would postpone for about 70 years the effect of population aging over intrinsic rates of return (IRR). For instance, for retirement age at 65 years, an intrinsic rate of return (IRR) of 2.0% would be observed for men by the 1990–1995 cohort in DB PAYG systems and by the 1930–1935 cohort in DC PAYG systems; and for women by the 2005–2010 cohort in DB PAYG systems and by the 1950–1955 cohort in DC PAYG systems. Also, for retirement age at 65 years, an intrinsic rate of return (IRR) of 0.5% would be observed for men by the 2045–2050 cohort in DB PAYG systems and by the 1960–1965 cohort in DC PAYG systems; and for women by the 2070–2075 cohort in DB PAYG systems and by the 2000–2005 cohort in DC PAYG systems.

Afonso (2016) investigated the distributional aspects and progressivity of old-age and length of contribution retirement benefits in the Brazilian Social Security System. Among other indicators, he estimated intrinsic rates of return (IRR) for birth cohorts from 1930 to 1960. For example, the estimated mean value of the IRR was 4.34% for the 1930 cohort, 5.53% for the 1945 cohort, and 7.70% for the 1960 cohort, that is, IRR were higher for younger cohorts. Intrinsic rates of return (IRR) were higher for women (6.33%) than for men (4.62%), higher for low than for high education level groups (from 6.54% to 2.01%), and higher for low than for high income groups (from 6.66% to 3.85%). For Afonso (2016), the estimated intrinsic rates of return (IRR) would support additional evidences that would indicate progressivity of the Brazilian Social Security System.

Clingman, Burkhalter, and Chaplain (2019) estimated theoretical intrinsic rates of return (IRR) for birth cohorts of hypothetical workers under the Old-Age, Survivors and Disability Insurance Program (OASDI) of the United States Social Security.[7] They presented hypothetical workers with four levels of pre retirement earnings, and considered three financing scenarios: "present law scheduled", which assume taxes (i.e., contribution rates) and benefits under the current law;[8] "increased payroll tax", which assume that payroll taxes will increase after 2033 to match scheduled benefits (i.e., defined benefit (DB)); and "payable benefits", which assume that benefits will proportionally reduce after 2033 to match scheduled contributions (i.e., defined contribution (DC)). Intrinsic rates of return (IRR) would decrease for virtually all earnings level from the 1920 to the 1949 cohort, and then would increase from the 1955 to the 2004 cohort. For example, for the two-earner couple very-low/very-low earnings from 6.67% to 4.84% and then from 4.86% to 5.15%; and for the two-earner couple high/high earnings from 3.53% to 1.96% and then from 2.03% to 2.40%. The exception would be maximum earnings level workers, for whom the IRR would decrease until the 1964 cohort because of historical increases in the taxable maximum, and then would increase from the 1973 to the 2004 cohort. For instance, for the two-earner couple maximum from 3.27% to 1.04% and then from 1.17% to 1.41%. When compared with the "present law scheduled", the "increased payroll tax" scenario would decrease the IRR starting with the 1973 cohort, specifically by cohort, the IRR would change on average by −0.05% for 1973, −0.25% for 1985, −0.50% for 1997, and −0.70% for 2004. Similarly, the "payable benefits" scenario would also decrease the IRR, but by larger values and starting with the 1943 cohort, for example by cohort, the IRR would change on average by −0.01% for 1943, −0.07% for 1949, −0.16% for 1955, −0.45% for 1964, −0.62% for 1973, −0.77% for 1985, −0.89% for 1997, and −0.95% for 2004. Thus, their results corroborate our argument that, when populations age, intrinsic rates of return (IRR) are higher in DB PAYG systems than in DC PAYG systems, in other words, that defined benefit (DB) PAYG systems postpone the burden of population aging on intrinsic rates of return (IRR).

7 Birth cohorts are 1920, 1930, 1937, 1943, 1949, 1955, 1964, 1973, 1985, 1997, and 2004.

8 Although the "present law scheduled" projected a deficit between scheduled income (i.e., contributions) and scheduled benefits after 2033.

5.3 METHODS AND ASSUMPTIONS

We use our stylized demographic model to investigate the effects of population aging on the returns of defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems to members of different birth cohorts in the world from 1950 to 2100. As in chapter 4, we adopt 1950–1955 as the base period, 20 years as the age of entry into the labor force (L), and 65 years as the age of entry into retirement (R). Likewise, benefit rates (*ben*) in DB PAYG systems are 100%; contribution rates (*con*) in DC PAYG systems are 15%; and ratios of per capita benefits to per capita net wages (ϕ) in FRP PAYG systems are equal to 100%. Correspondingly, variable benefit rates (*ben*) and contribution rates (*con*) are those estimated in chapter 4, and are for the five-year periods of the 2017 UN REVISION, for instance, in the period 1950–1955 *ben* and *con* are those estimated for year 1950.

Equivalent retirement ages (ERA) are also those estimated in chapter 4 from the ratio of expected years in retirement to expected years in work (Equation 4.12), and are for the five-year periods of the 2017 UN REVISION; specifically, in the period 1950–1955 all countries R and ERA are 65 years, then from 1955–1960 to 2095–2100 ERA are determined by the respective five-year period life tables. Similarly to chapter 4, we assume that life tables, ERA, and the rejuvenating effects of births, deaths and migration for 2100–2105 are the same as those for 2095–2100. Likewise, benefit rates (*ben*) and contribution rates (*con*) for 2100–2105 are those estimated for year 2100.

We estimate intrinsic rates of return (IRR) from Equation 5.1a for the five-year birth cohorts that have complete working life cycles (i.e., from 20–24 years to 60–64 years) and retirement life cycles (i.e., 65–69 years to 95+ years) within the 150 years from 1950 to 2100 of the 2017 UN REVISION, that is, sixteen birth cohorts from the 1925–1930 birth cohort, which had 20–24 years in 1950, to the 2000–2005 birth cohort, which will have 95+ years in 2100.[9] We use the 95+ age group of periods as a proxy for the last age group of the five-year birth cohorts.

We also investigate how different fixed retirement ages and equivalent retirement ages (ERA) influence intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems and defined contribution (DC) PAYG systems. Particularly, we verify Fernandes (1993)'s claim that higher ages of entry into retirement (R) tend to increase intrinsic rates of return (IRR) of DB PAYG systems, and to decrease intrinsic rates of return (IRR) of DC PAYG systems.

Last, we are also interested in analyzing the relationship between the intrinsic rate of return (IRR) and the demographic determinants of population aging. We propose to estimate a life cycle demographic determinants of population aging characteristic measure based on both Preston (1982)'s life cycle characteristics and Preston, Himes, and Eggers (1989)'s demographic determinants of population aging. Consider an individual attribute G that is age-specific, and let $g(a)$ be the proportion of persons aged a with attribute G. The prevalence of attribute G over the course

[9] For a total of 3,216 birth cohorts from 16 five-year birth cohorts multiplied by 201 countries and areas.

of the life cycle (G_L) is (PRESTON, 1982):

$$G_L = \frac{\int_0^{\infty} p(a) \cdot g(a)\, da}{\int_0^{\infty} p(a)\, da} \tag{5.5}$$

Now, consider that we may determine the stages of population aging by either only the rejuvenating effect of births or only the rejuvenating effect of deaths (see chapter 3). Therefore, we may estimate a birth cohort's life cycle demographic determinant of population aging characteristic via only period rejuvenating effect of deaths ($d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)]$) (PRESTON; HIMES; EGGERS, 1989). The *life cycle rejuvenating effect of deaths ($G_L^{\mathcal{D}}$)* is:

$$G_L^{\mathcal{D}} = \frac{\int_0^{\infty} N^c(a) \cdot d(t^c + a) \cdot [D_{\overline{a}}(t^c + a) - N_{\overline{a}}(t^c + a)]\, da}{\int_0^{\infty} N^c(a)\, da} \tag{5.6}$$

But since different policy designs of PAYG systems imply distinct life cycle perspectives, the intrinsic rate of return (IRR) relates to period rejuvenating effects of deaths only between L and R via contribution rates (con) in defined benefit (DB) PAYG systems, only after R via benefit rates (ben) in defined contribution (DC) PAYG systems, and both between L and R via contribution rates (con) and after R via benefit rates (ben) in fixed relative position (FRP) PAYG systems. Therefore, we integrate both the numerator and the denominator in Equation 5.6 between R and L in DB PAYG systems, between R and ∞ in DC PAYG systems, and between L and ∞ in FRP PAYG systems, as we detail in Table 12.

Table 12 – Life cycle rejuvenating effect of deaths by pay-as-you-go (PAYG) system policy design

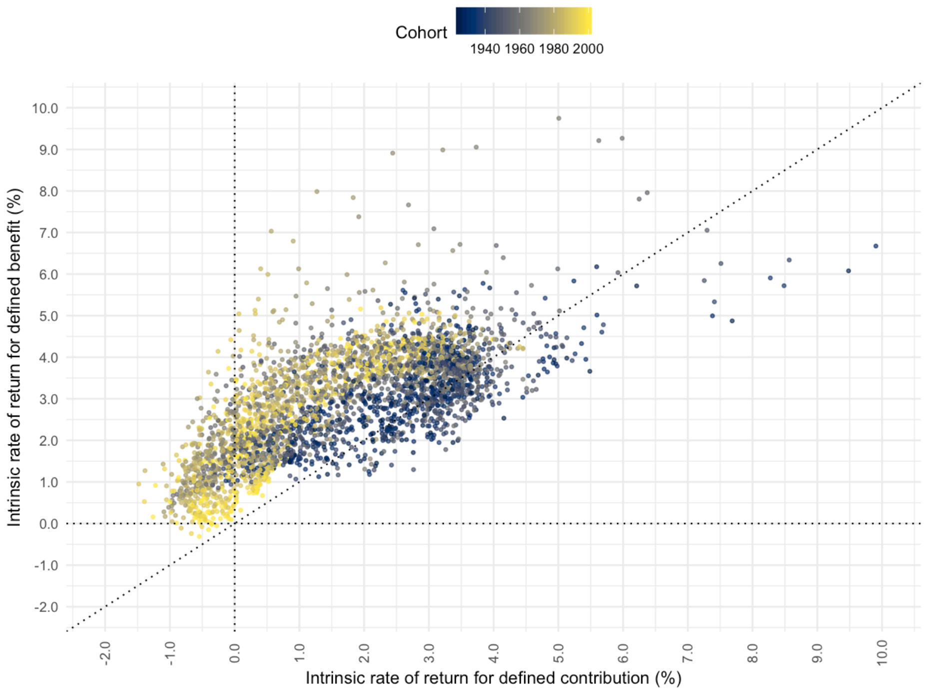
PAYG system		Life cycle rejuvenating effect of deaths	
Defined benefit (DB)	working life cycle rejuvenating effect of deaths	$G_W^{\mathcal{D}} = \frac{\int_L^R N^c(a) \cdot d(t^c + a) \cdot [D_{\overline{a}}(t^c + a) - N_{\overline{a}}(t^c + a)]\, da}{\int_L^R N^c(a)\, da}$	(5.7)
Defined contribution (DC)	retirement life cycle rejuvenating effect of deaths	$G_R^{\mathcal{D}} = \frac{\int_R^{\infty} N^c(a) \cdot d(t^c + a) \cdot [D_{\overline{a}}(t^c + a) - N_{\overline{a}}(t^c + a)]\, da}{\int_R^{\infty} N^c(a)\, da}$	(5.8)
Fixed relative position (FRP)	working and retirement life cycle rejuvenating effect of deaths	$G_{WR}^{\mathcal{D}} = \frac{\int_L^{\infty} N^c(a) \cdot d(t^c + a) \cdot [D_{\overline{a}}(t^c + a) - N_{\overline{a}}(t^c + a)]\, da}{\int_L^{\infty} N^c(a)\, da}$	(5.9)

Source: Author's creation, based on Preston (1982) and Preston, Himes, and Eggers (1989).

5.4 INTRINSIC RATE OF RETURN AND POLICY DESIGNS OF PAY-AS-YOU-SYSTEMS

We estimate the return of PAYG systems to birth cohorts via intrinsic rates of return (IRR) of defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems. Figure 44 presents IRR of DC PAYG systems by IRR of DB PAYG systems. Figure 45 plots IRR of FRP PAYG systems by IRR of DB PAYG systems. Figure 46 shows IRR of FRP PAYG systems by IRR of DC PAYG systems.[10] Intrinsic rates of return (IRR) are mostly higher in DB PAYG systems than in DC PAYG systems. In DB PAYG systems, IRR are as high as 9.74% (Qatar, 1965–1970), as low as −0.31% (Bulgaria, 2000–2005), and only 7 (0.22%) of 3,216 birth cohorts present negative intrinsic rates of return (IRR).[11] In DC PAYG systems, IRR are as high as 9.90% (United Arab Emirates, 1930–1935), as low as −1.48% (Republic of Moldova, 1990–1995), and, by contrast, 499 (15.5%) birth cohorts present negative intrinsic rates of return (IRR). In FRP PAYG systems with ratios of per capita benefits to per capita net wages (ϕ) equal to 100%, intrinsic rates of return (IRR) are close to, while lower than those of DB PAYG systems; whereas distant to, while higher than those of DC PAYG systems.

Figure 44 – Intrinsic rate of return (IRR) of defined contribution (DC) by IRR of defined benefit (DB)

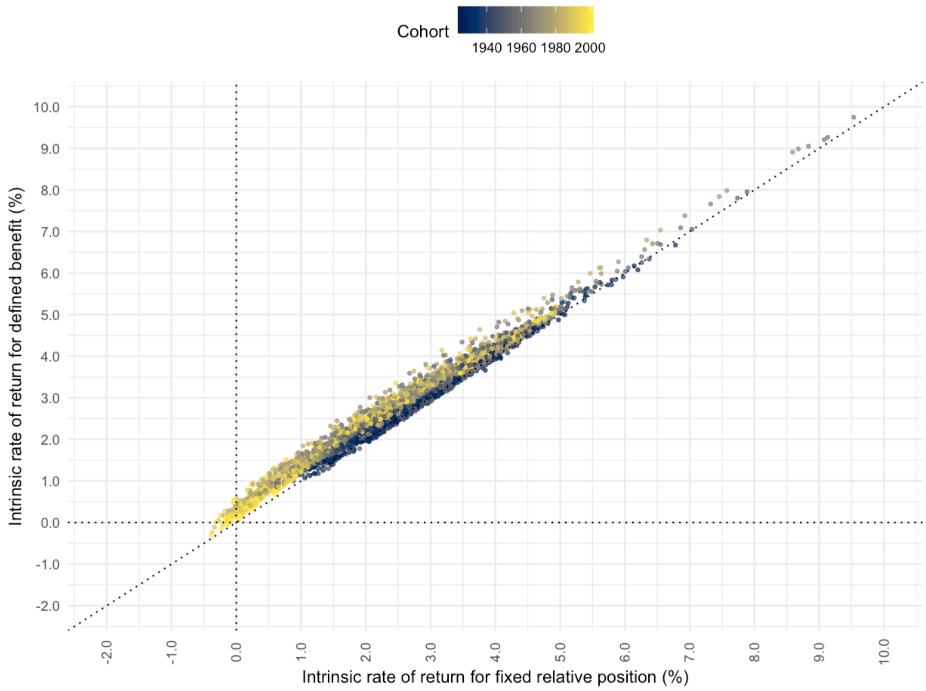


Source: Author's calculations, based on United Nations (2017c).

[10] Figures 128, 129 and 130 in section E.1, Appendix E, detail these results by subregions.

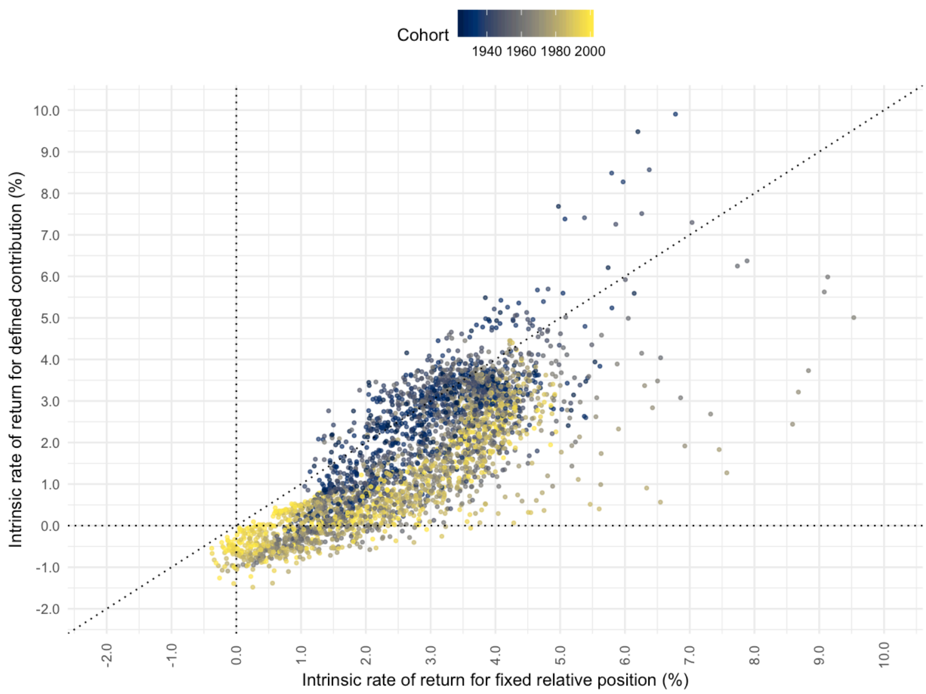
[11] Bulgaria (2000–2005, −0.31%), Bulgaria (1995–2000, −0.24%), Latvia (2000–2005, −0.15%), Bulgaria (1990–1995, −0.11%), Latvia (1995–2000, −0.075%), Japan (2000–2005, −0.056%), and Lithuania (2000–2005, −0.026%).

Figure 45 – Intrinsic rate of return (IRR) of fixed relative position (FRP) by IRR of defined benefit (DB)



Source: Author's calculations, based on United Nations (2017c).

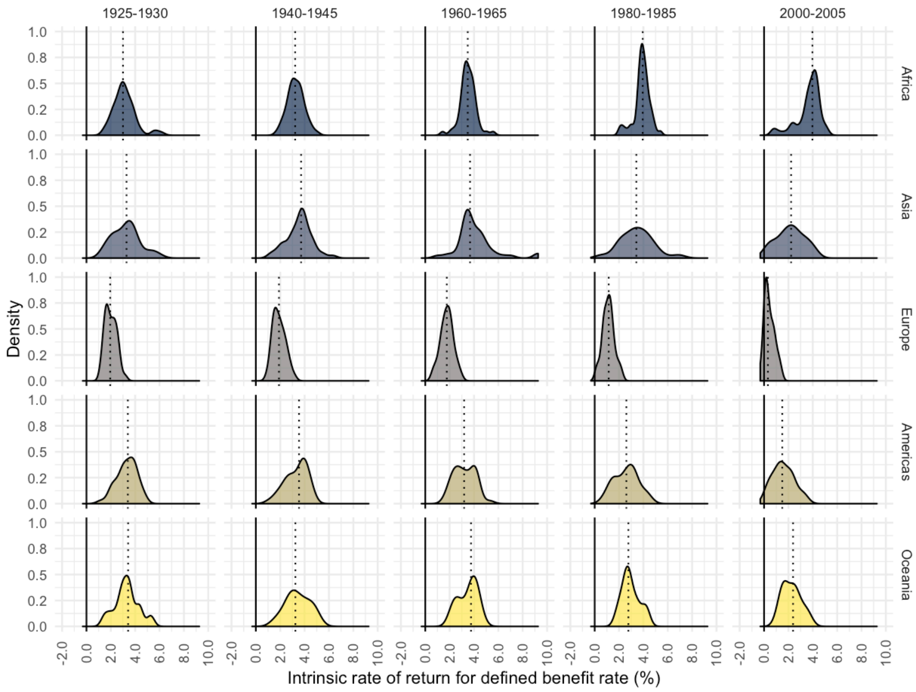
Figure 46 – Intrinsic rate of return (IRR) of fixed relative position (FRP) by IRR of defined contribution (DC)



Source: Author's calculations, based on United Nations (2017c).

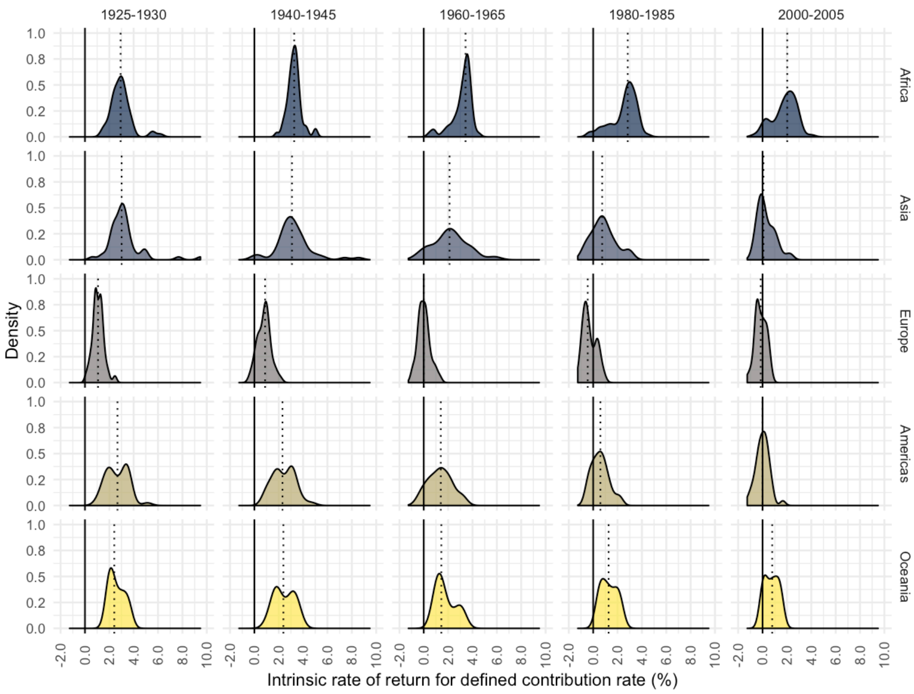
Figures 47 to 49 plot the densities of the intrinsic rates of return (IRR) of DB, DC, and FRP PAYG systems for selected birth cohorts and all regions. From the 1925–1930 to the 2000–2005 cohort, in defined benefit (DB) PAYG systems, the median intrinsic rate of return (IRR) in Africa increases from 3.00% to 4.04% for the 1985–1990 cohort then decreases to 3.96%, in Asia increases from 3.28% to 3.83% for the 1955–1960 cohort then decreases to 2.22%, in Europe decreases from 1.94% to 0.31%, in the Americas increases from 3.40% to 3.54% for the cohort 1940–1945 then decreases to 1.49%, and in Oceania from 3.42% to 3.82% for the 1955–1960 cohort then decreases to 2.38%. In defined contribution (DC) PAYG systems, the median intrinsic rate of return (IRR) in Africa increases from 2.92% to 3.42% for the 1960–1965 cohort then decreases to 2.02%, in Asia remains stable around 3.10% up to the 1945–1950 cohort then decreases to 0.07%, in Europe decreases from 1.07% to −0.45% for the 1980–1985 cohort then increases to −0.15%, in the Americas decreases from 2.67% to −0.00%, and in Oceania increases from 2.41% to 2.67% for the 1930–1935 cohort then decreases to 0.79%. In fixed relative position (FRP) PAYG systems, median intrinsic rates of return (IRR) are similar to those of DB PAYG systems, yet somewhat lower, in Africa increases from 2.98% to 3.89% for the 1990–1995 cohort then decreases to 3.63%, in Asia increases from 3.27% to 3.67% for the 1950–1955 cohort then decreases to 1.82%, in Europe decreases from 1.81% to 0.18%, in the Americas increases from 3.26% to 3.39% for the 1940–1945 cohort then decreases to 1.00%, and in Oceania increases from 3.29% to 3.65% for the 1950–1955 cohort then decreases to 2.08%.

Figure 47 – Density of the intrinsic rate of return (IRR) of defined benefit (DB) by selected cohorts and regions



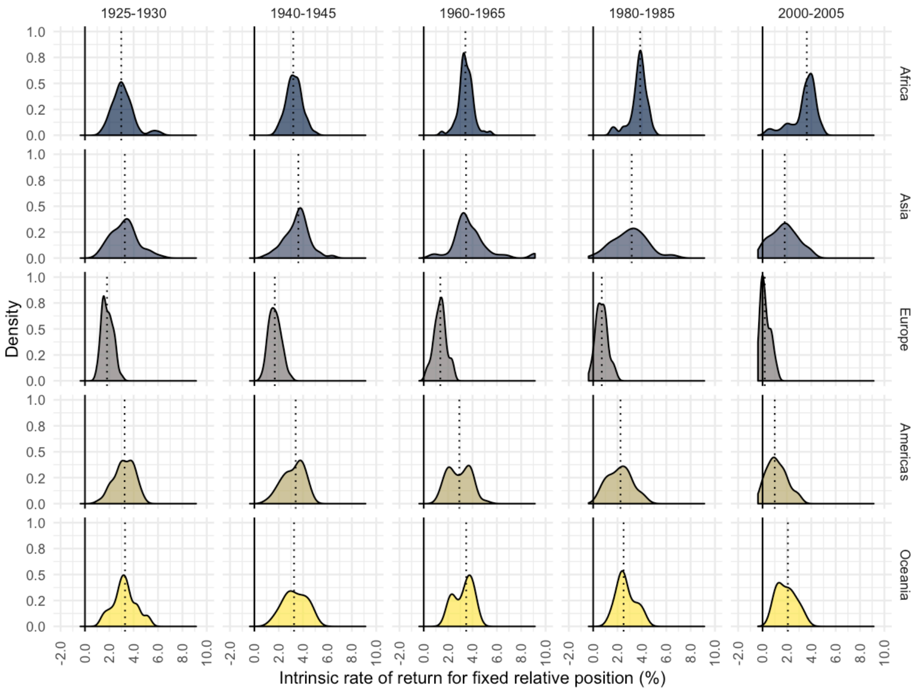
Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

Figure 48 – Density of the intrinsic rate of return (IRR) of defined contribution (DC) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

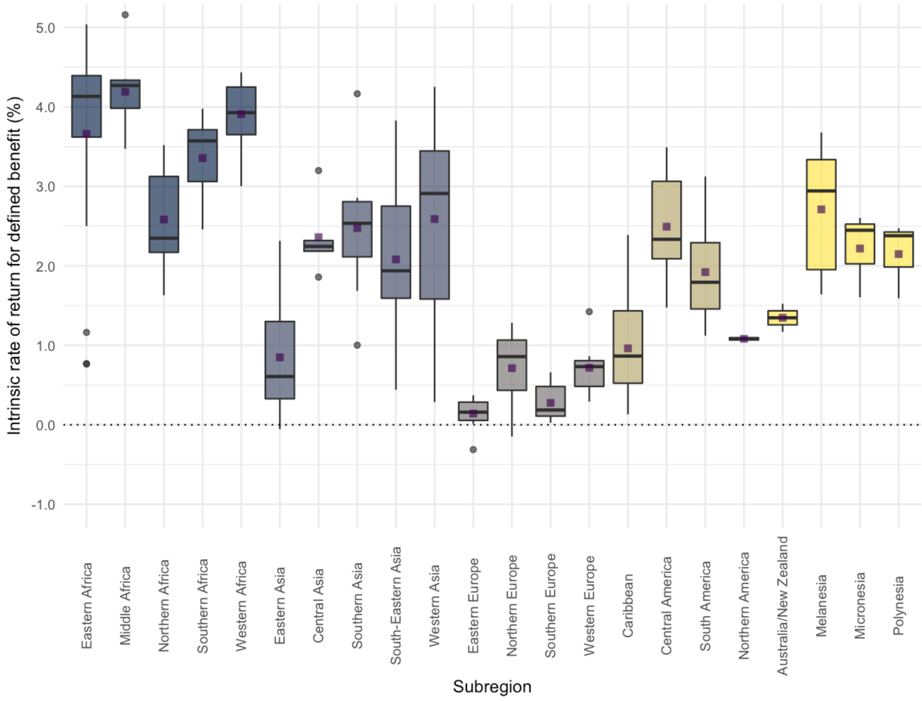
Figure 49 – Density of the intrinsic rate of return (IRR) of fixed relative position (FRP) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

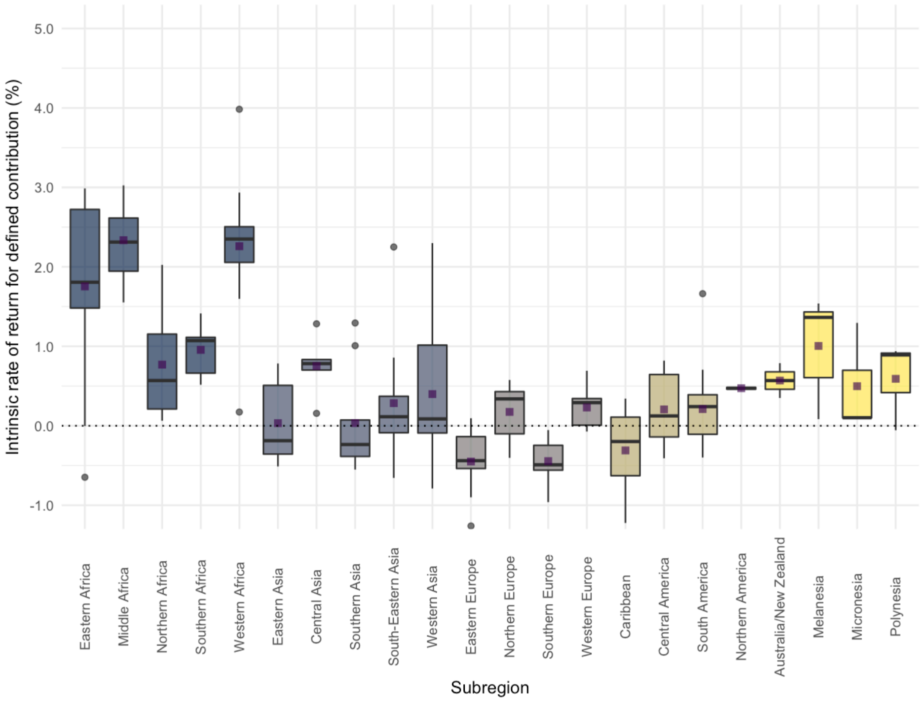
Figures 50 to 52 present the distribution of the intrinsic rates of return (IRR) of DB, DC, and FRP PAYG systems for the 2000–2005 birth cohort by subregions. For the 2000–2005 cohort, the median intrinsic rate of return (IRR) of defined benefit (DB) PAYG systems is close to 4.00% in three subregions: Middle Africa (4.27%), Eastern Africa (4.13%), and Western Africa (3.93%); and below 1.00% in four subregions: Caribbean (0.86%), Northern Europe (0.85%), Western Europe (0.73%), Eastern Asia (0.61%), Southern Europe (0.18%), and Eastern Europe (0.16%). The median intrinsic rate of return (IRR) of defined contribution (DC) PAYG systems is above 2.00% in only two subregions: Western Africa (2.35%), and Middle Africa (2.31%); and negative in five subregions: Eastern Asia (−0.19%), Caribbean (−0.20%), Southern Asia (−0.24%), Eastern Europe (−0.44%), and Southern Europe (−0.49%). Similar to other birth cohorts, for the 2000–2005 cohort, the median intrinsic rate of return (IRR) of fixed relative position (FRP) PAYG systems is a little lower than those of DB PAYG systems, that is, close to 4.00% in two subregions: Middle Africa (4.09%), and Eastern Africa (3.93%); and below 1.00% in five subregions: Northern America (0.89%), Northern Europe (0.69%), Western Europe (0.59%), Caribbean (0.44%), and Eastern Asia (0.41%); and close to zero in two subregions: Southern Europe (0.02%), and Eastern Europe (−0.06%).

Figure 50 – Intrinsic rate of return (IRR) of defined benefit (DB) and 2000–2005 cohort by subregions



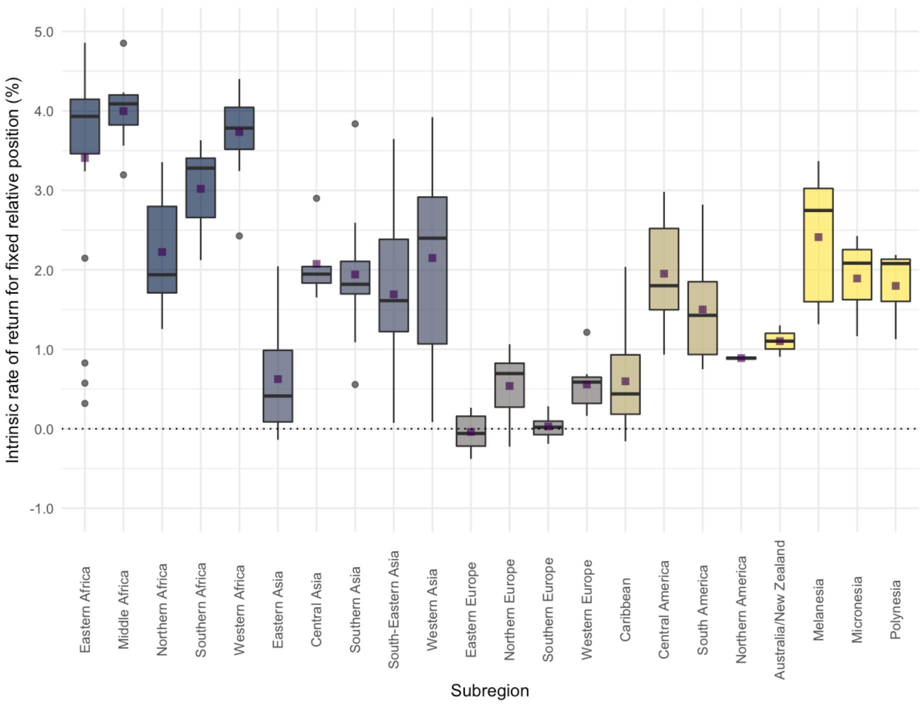
Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

Figure 51 – Intrinsic rate of return (IRR) of defined contribution (DC) and 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

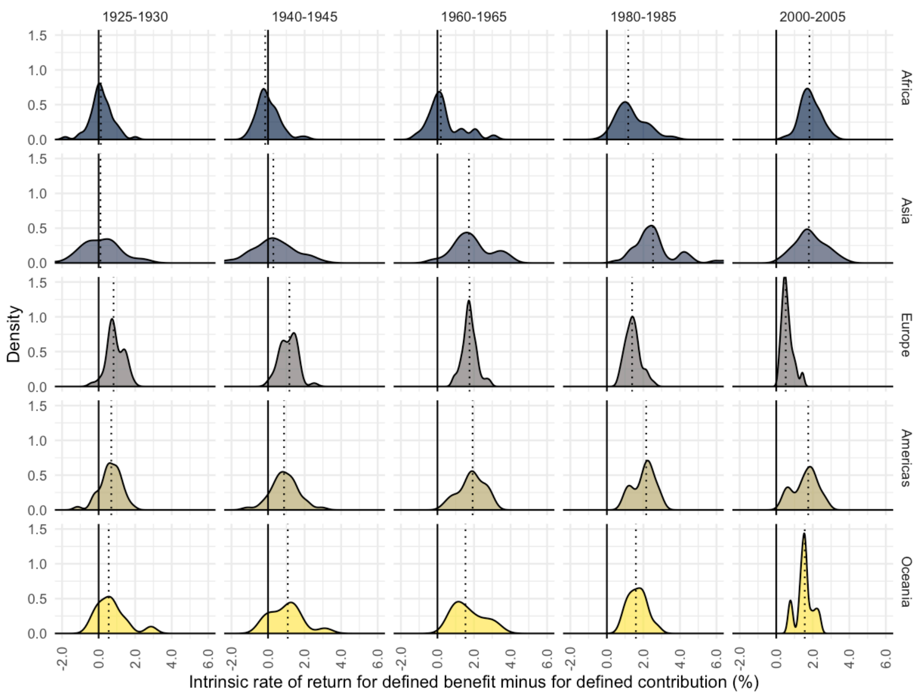
Figure 52 – Intrinsic rate of return (IRR) of fixed relative position (FRP) and 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

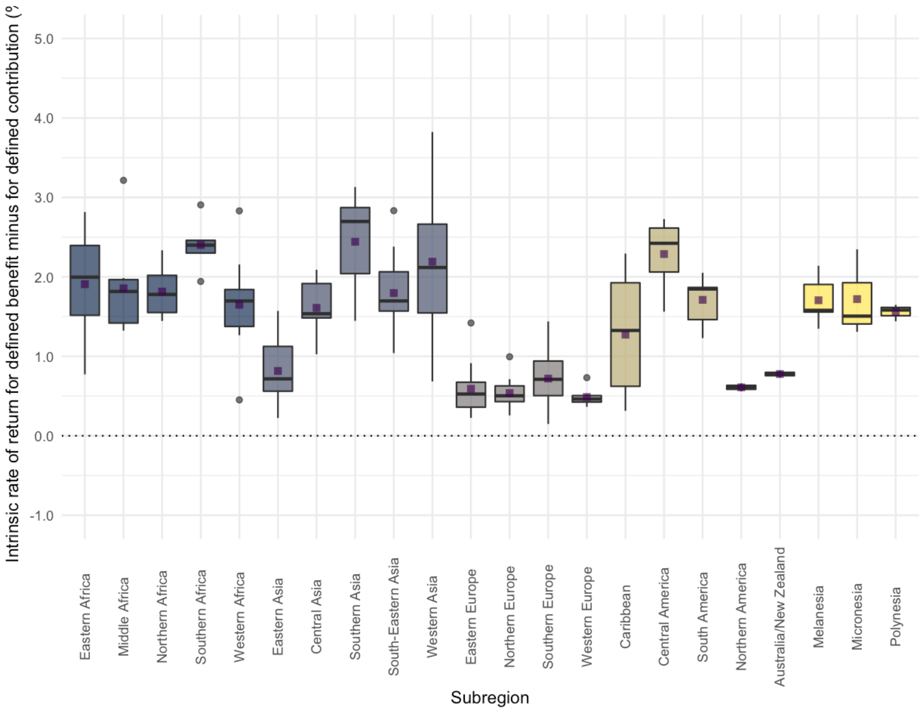
Figure 53 presents the density of the difference between intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems and of defined contribution (DC) PAYG systems for selected birth cohorts and all regions. Figure 54 plots its distribution for the 2000–2005 birth cohort by subregions. From the 1925–1930 to the 2000–2005 cohort, the median of the difference between IRR of DB PAYG systems and of DC PAYG systems in Africa remains around zero and slightly negative (minimum of −0.16%) until the 1955–1960 cohort and then increases to 1.81%, in Asia increases from 0.10% to 2.53% for the 1980–1985 cohort then decreases to 1.78%, in Europe increases from 0.82% to 1.75% for the 1960–1965 cohort then decreases to 0.51%, in the Americas increases from 0.69% to 2.15% for the 1980–1985 cohort then decreases to 1.75%, and in Oceania increases from 0.55% to 1.65% for the 1970–1975 cobort then decreases to 1.56%. For the 2000–2005 cohort, the median of the difference between IRR of DB PAYG systems and of DC PAYG systems is above 2.00% in five subregions: Southern Asia (2.70%), Central America (2.42%), Southern Africa (2.40%), Western Asia (2.12%), and Eastern Africa (2.00%); and close to 0.5% in three subregions: Eastern Europe (0.52%), Northern Europe (0.50]%, and Western Europe (0.46%).

Figure 53 – Density of the difference between intrinsic rate of return (IRR) of defined benefit (DB) and IRR of defined contribution (DC) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

Figure 54 – Difference between intrinsic rate of return (IRR) of defined benefit (DB) and IRR of defined contribution (DC), for 2000–2005 cohort by subregions

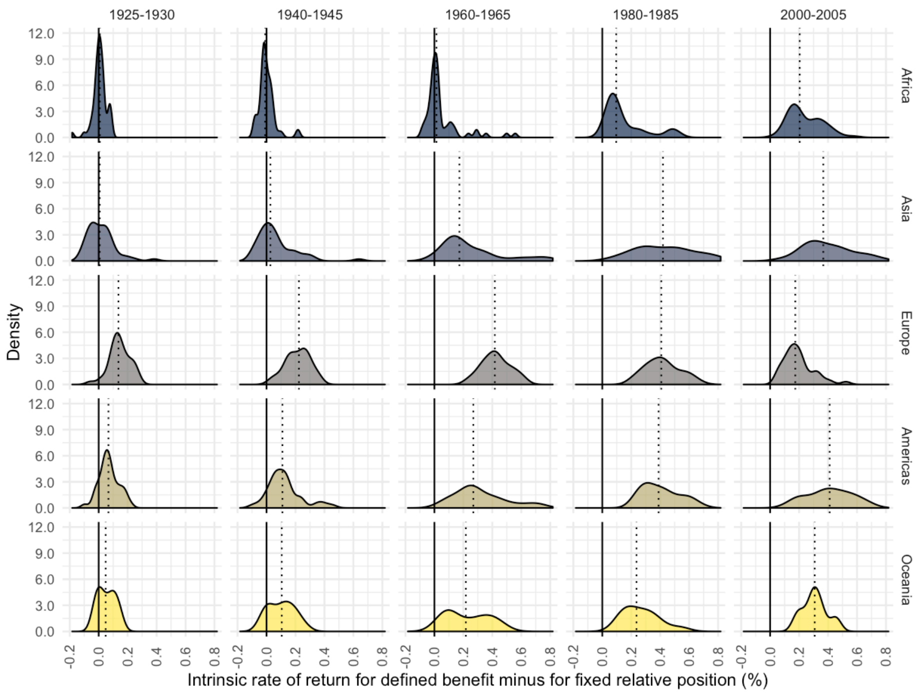


Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

Figure 55 plots the density of the difference between intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems and of fixed relative position (FRP) PAYG systems for selected birth cohorts and all regions. Figure 56 presents its distribution for the 2000–2005 birth cohort by subregions. From the 1925–1930 to the 2000–2005 cohort, the median of the difference between IRR of DB PAYG systems and of FRP PAYG systems in Africa remains around zero until the 1975–1980 cohort then increases to 0.20%, in Asia remains around zero until the 1950–1955 cohort, increases to 0.45% for the 1990–1995 cohort then decreases to 0.37%, in Europe increases from 0.14% to 0.45% for the 1970–1975 cohort then decreases to 0.17%, in the Americas increases to 0.42% for the 1985–1990 cohort then remains around this level, and in Oceania increases from around zero to 0.31%. For the 2000–2005 cohort, the median of the difference between IRR of DB PAYG systems and of FRP PAYG systems is above 0.40% in five subregions: Southern Asia (0.60%), Central America (0.56%), Western Asia (0.46%), South America (0.43%), and South-Eastern Asia (0.39%). Compared to the difference between intrinsic rates of return (IRR) of DB PAYG systems and of DC PAYG systems, the profile by subregions is similar — except for Northern Africa and Eastern Asia, whereas the medians are between three to eleven times lower.[12]

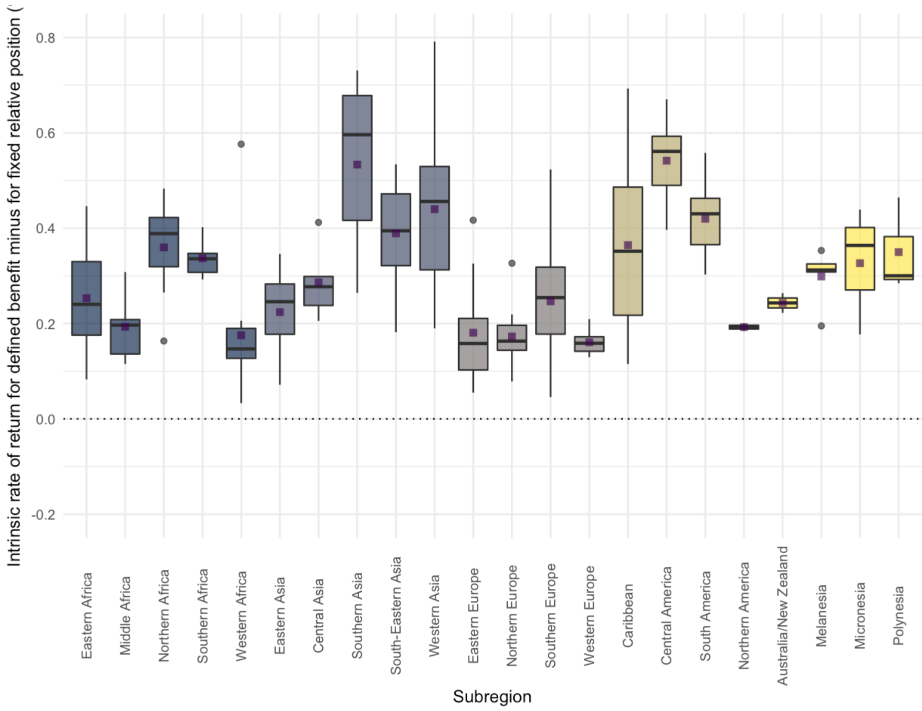
[12] In increasing order, Southern Europe (2.79), Western Europe (2.91), Eastern Asia (2.91), Northern Europe (3.08), Northern America (3.17), Australia/New Zealand (3.20), Eastern Europe (3.32), Caribbean (3.77), Micronesia (4.14), South America (4.29), South-Eastern Asia (4.30), Central America (4.32), Southern Asia (4.52), Northern Africa (4.58), Western Asia (4.64), Melanesia (5.05), Polynesia (5.27), Central Asia (5.54), Southern Africa (7.14), Eastern Africa (8.31), Middle Africa (9.24), and Western Africa (11.57).

Figure 55 – Density of the difference between intrinsic rate of return (IRR) of defined benefit (DB) and IRR of fixed relative position (FRP) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

Figure 56 – Difference between intrinsic rate of return (IRR) of defined benefit (DB) and IRR of fixed relative position (FRP), for 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

Figures 57 to 60 present the intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems and of defined contribution (DC) PAYG systems by birth cohorts for all 201 countries and areas. These figures reinforce our argument that, when population age, intrinsic rates of return (IRR) are higher and decrease more smoothly in DB PAYG systems than in DC PAYG systems, that is, DB PAYG systems postpone the burden of population aging on intrinsic rates of return (IRR).[13] Likewise, when populations rejuvenate, intrinsic rates of return (IRR) are lower and increase more slowly in DB PAYG systems than in DC PAYG systems, that is, DC PAYG systems anticipate the dividend of population rejuvenation on intrinsic rates of return (IRR). Specifically, when populations rejuvenate and then age, IRR of DC PAYG systems are initially higher because older birth cohorts incorporate the dividends of population rejuvenation on their IRR, and then decrease as younger birth cohorts incorporate the effects of population aging on their IRR; while IRR of DB PAYG systems are initially lower because older birth cohorts do not incorporate the dividends of population rejuvenation on their IRR, then increase as younger birth cohorts incorporate these dividends on their IRR, and next decrease as even younger birth cohorts finally incorporate the effects of population aging on their IRR. This generates a *crossover* between the IRR of defined benefit (DB) PAYG systems and the IRR of defined contribution (DC) PAYG systems.[14]

Figures 61 to 64 plot the intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems and of fixed relative position (FRP) PAYG systems by birth cohorts for all 201 countries and areas. When both intrinsic rates of return (IRR) are stable or increase, differences between the IRR of DB PAYG systems and of FRP PAYG systems are mostly inconsequential. Nevertheless, negative differences (i.e., IRR of DB PAYG systems smaller than IRR of FRP PAYG systems) occur when there are *crossovers* between the IRR of DB PAYG systems and the IRR of DC PAYG systems, because the higher IRR of DC PAYG systems for cohorts born before the crossover influence and increase the IRR of FRP PAYG systems.[15] Also, when both intrinsic rates of return (IRR) decrease, relevant positive differences (i.e., IRR of DB PAYG systems higher than IRR of FRP PAYG systems) are noticeable because the IRR of DC PAYG systems influence and accelerate the downturn of the IRR of FRP PAYG systems, which therefore decrease faster than those of defined benefit (DB) PAYG systems.[16]

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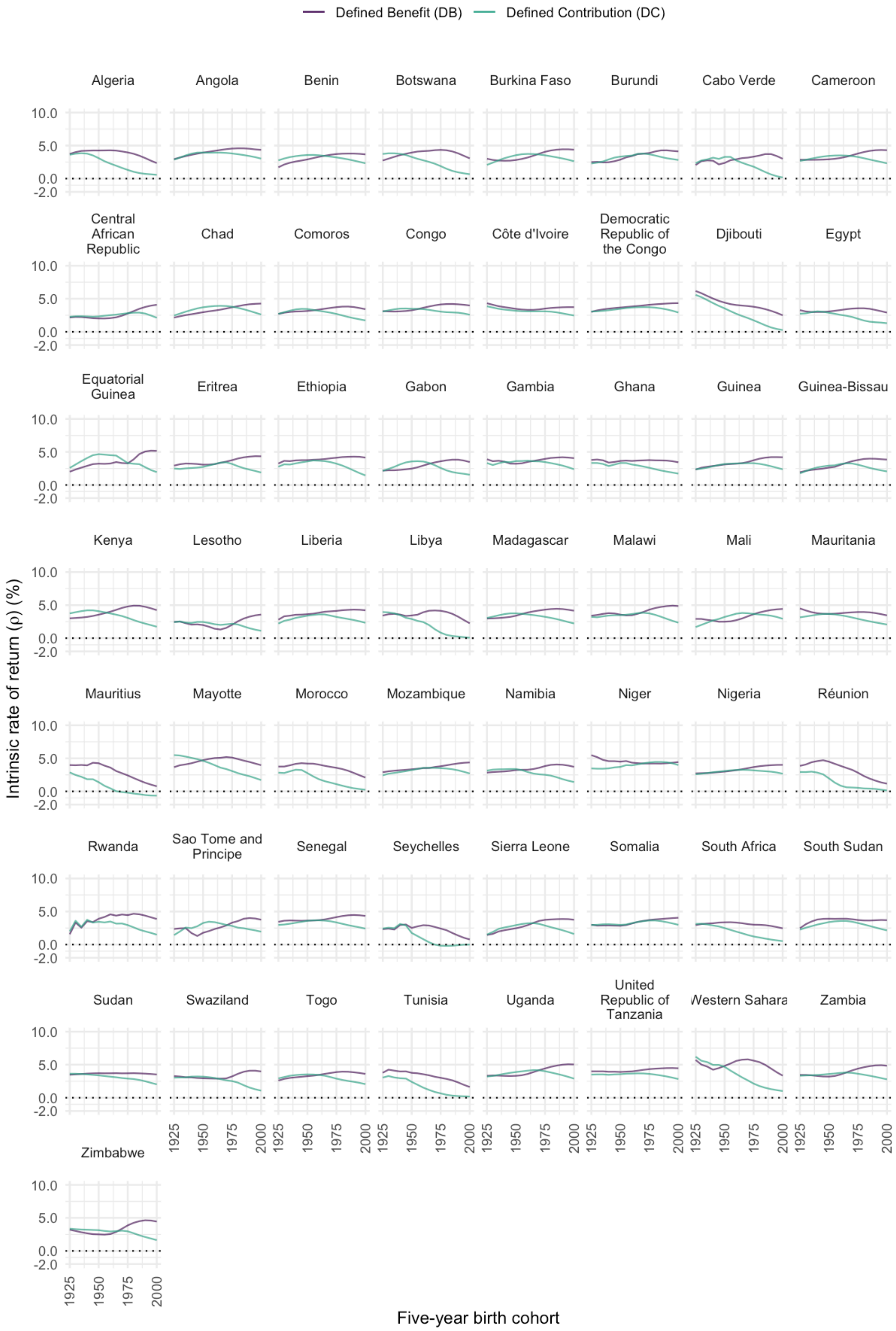
[13] For instance, Algeria, Djibouti, Libya, Mauritius, Morocco, South Africa, Tunisia and Western Sahara in Africa; Afghanistan, Bhutan, China, Indonesia, Japan, Republic of Korea, Singapore, and Thailand in Asia; Albania, Bosnia and Herzegovina, Finland, Italy, Poland, Portugal, Slovenia and Spain in Europe; Brazil, Canada, Chile, Colombia, Honduras, Mexico, Nicaragua and Peru in the Americas; and Fiji, French Polynesia, Papua New Guinea, and Solomon Islands in Oceania.

[14] For example, Benin, Equatorial Guinea, Gabon, Kenya, Mayotte, and São Tomé and Príncipe in Africa; Bahrain, Brunei Darussalam, Kuwait, Mongolia, Oman, Qatar, State of Palestine, United Arab Emirates, and Uzbekistan in Asia; Ireland in Europe; and Belize, and French Guiana in the Americas.

[15] For instance, Benin 1925–1930 (−0.10%, 1.95% vs. 2.05%), Gabon 1950–1955 (−0.11%, 2.47% vs. 2.58%), Mayotte 1925–1930 (−0.18%, 3.66% vs. 3.84%), and São Tomé and Príncipe 1945–1950 (−0.13%, 1.30% vs. 1.42%) in Africa; State of Palestine 1925–1930 (−0.10%, 1.95% vs. 2.05%), United Arab Emirates 1925–1930 (−0.12%, 6.08% vs. 6.20%), and Uzbekistan 1935-1940 (−0.10%, 2.05% vs. 2.15%) in Asia; and French Guiana 1925–1930 (−0.10%, 3.98% vs. 4.08%) in the Americas.

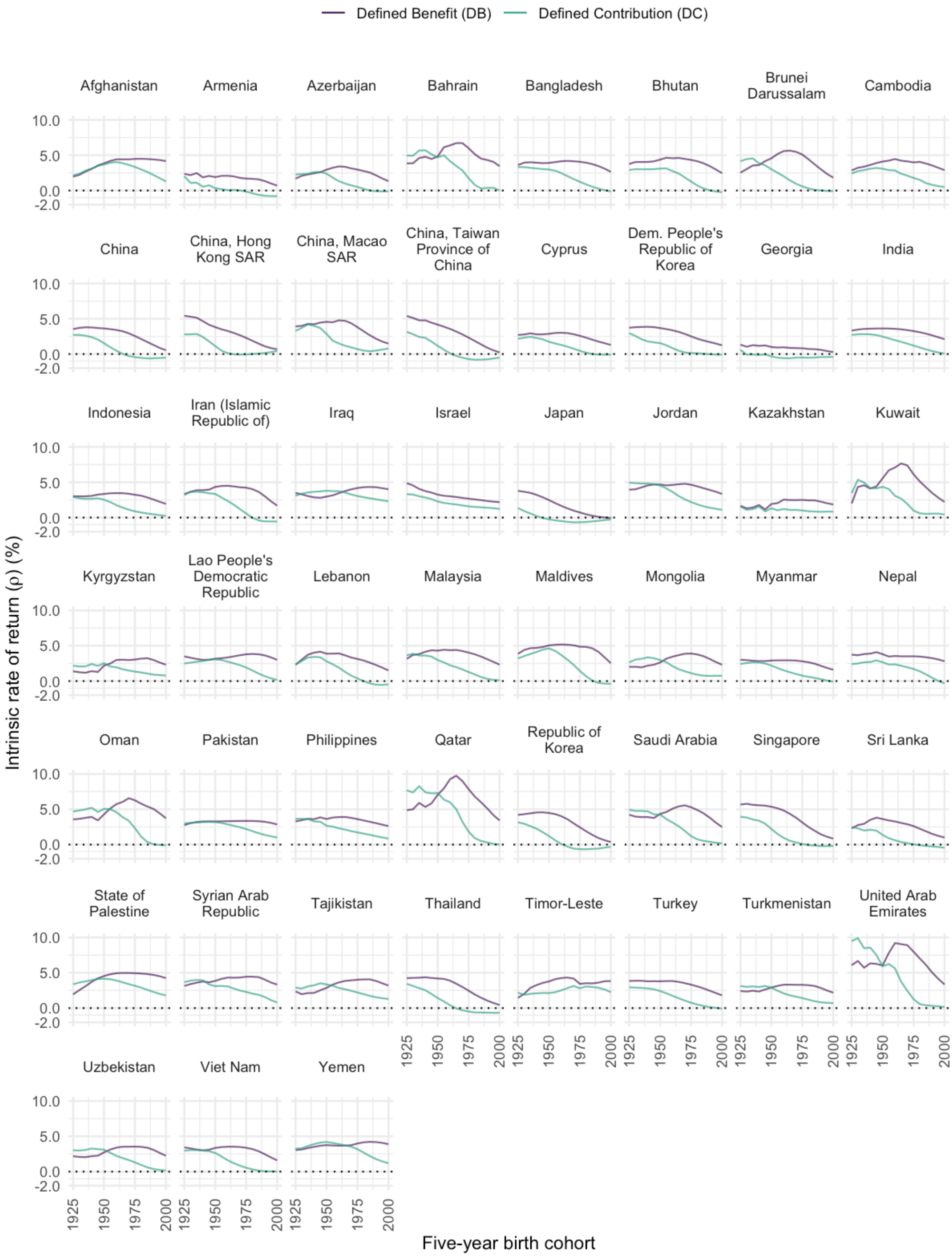
[16] For example, China, Taiwan Province of China 1975–1980 (0,87%, 2.32% vs. 1.45%), Iran (Islamic Republic of) 1985–1990 (0.90%, 3.64% vs. 2.74%), Maldives 1990–1995 (0.97%, 4.14% vs. 3.17%), Oman 1995–2000 (0.80%, 4.42% vs. 3.62%), Republic of Korea 1970–1975 (0.94%, 3.00% vs. 2.06%), Singapore 1970–1975 (0.88%, 3.80% vs. 3.92%), in Asia; and Cuba 1970–1975 (0,76%, 2.29% vs. 1.53%) in the Americas.

Figure 57 – Intrinsic rate of return (IRR) of defined benefit (DB) and of defined contribution (DC) by cohorts and country — Africa



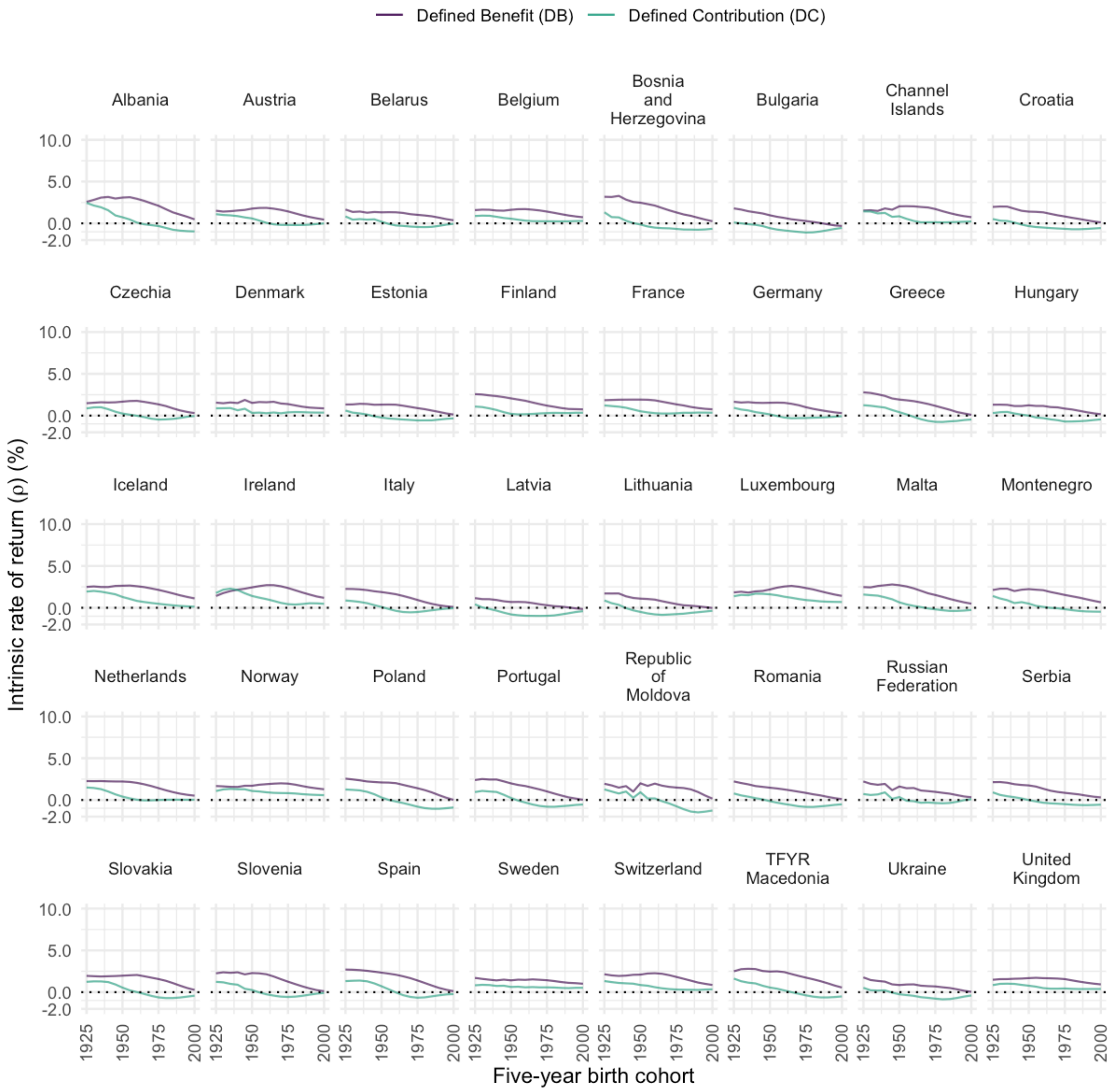
Source: Author's calculations, based on United Nations (2017c).

Figure 58 – Intrinsic rate of return (IRR) of defined benefit (DB) and of defined contribution (DC) by cohorts and country — Asia



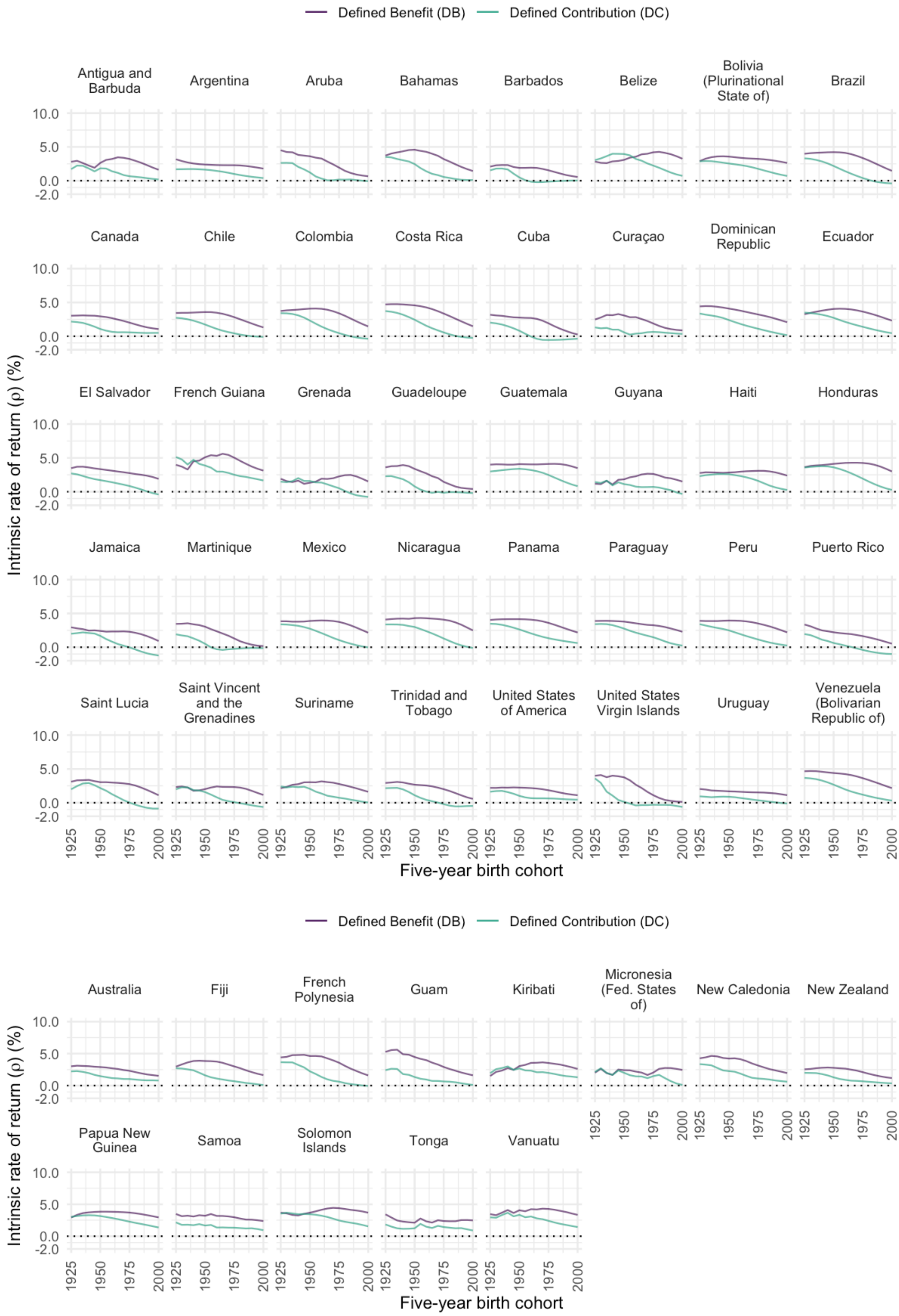
Source: Author's calculations, based on United Nations (2017c).

Figure 59 – Intrinsic rate of return (IRR) of defined benefit (DB) and of defined contribution (DC) by cohorts and country — Europe



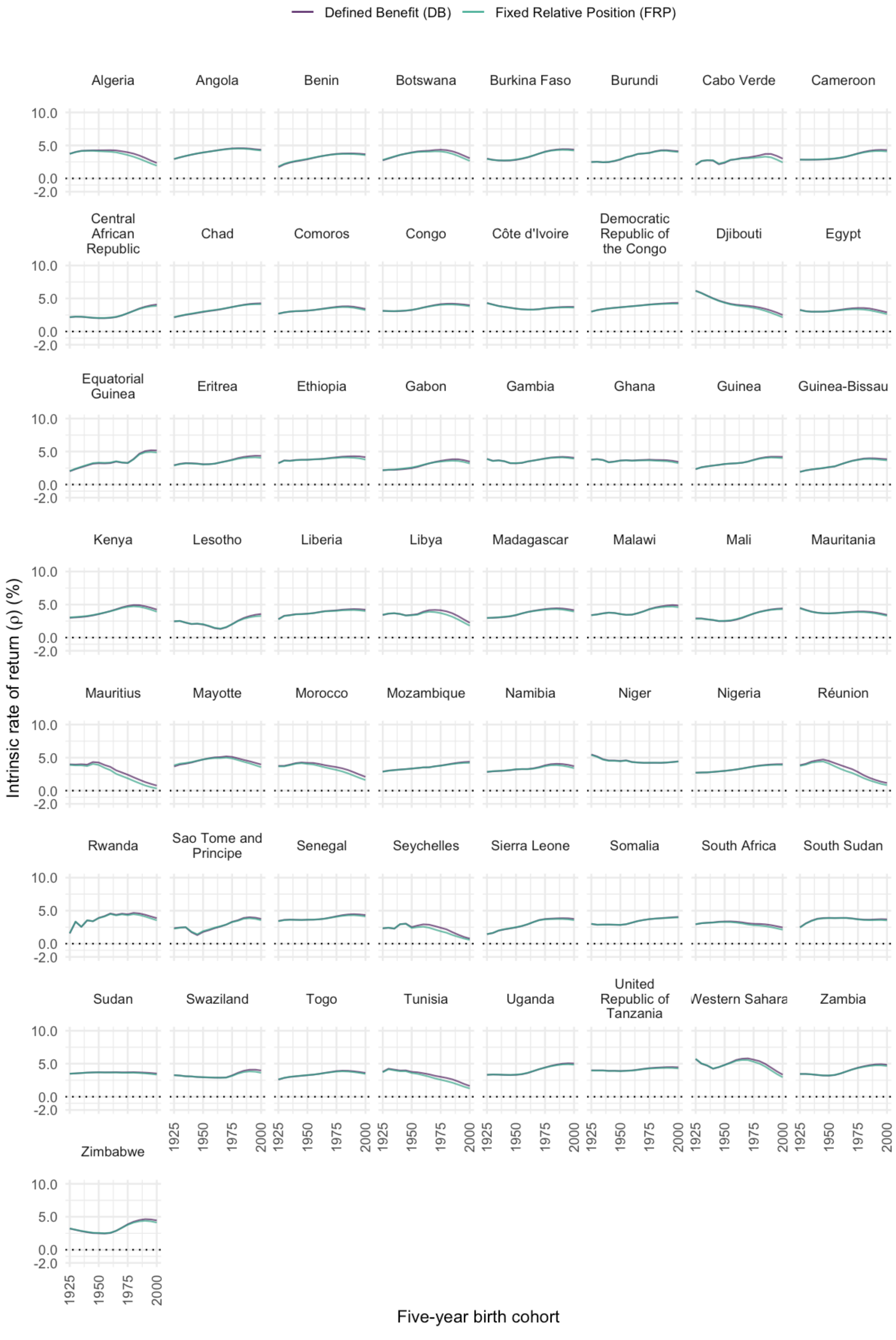
Source: Author's calculations, based on United Nations (2017c).

Figure 60 – Intrinsic rate of return (IRR) of defined benefit (DB) and of defined contribution (DC) by cohorts and country — Americas and Oceania



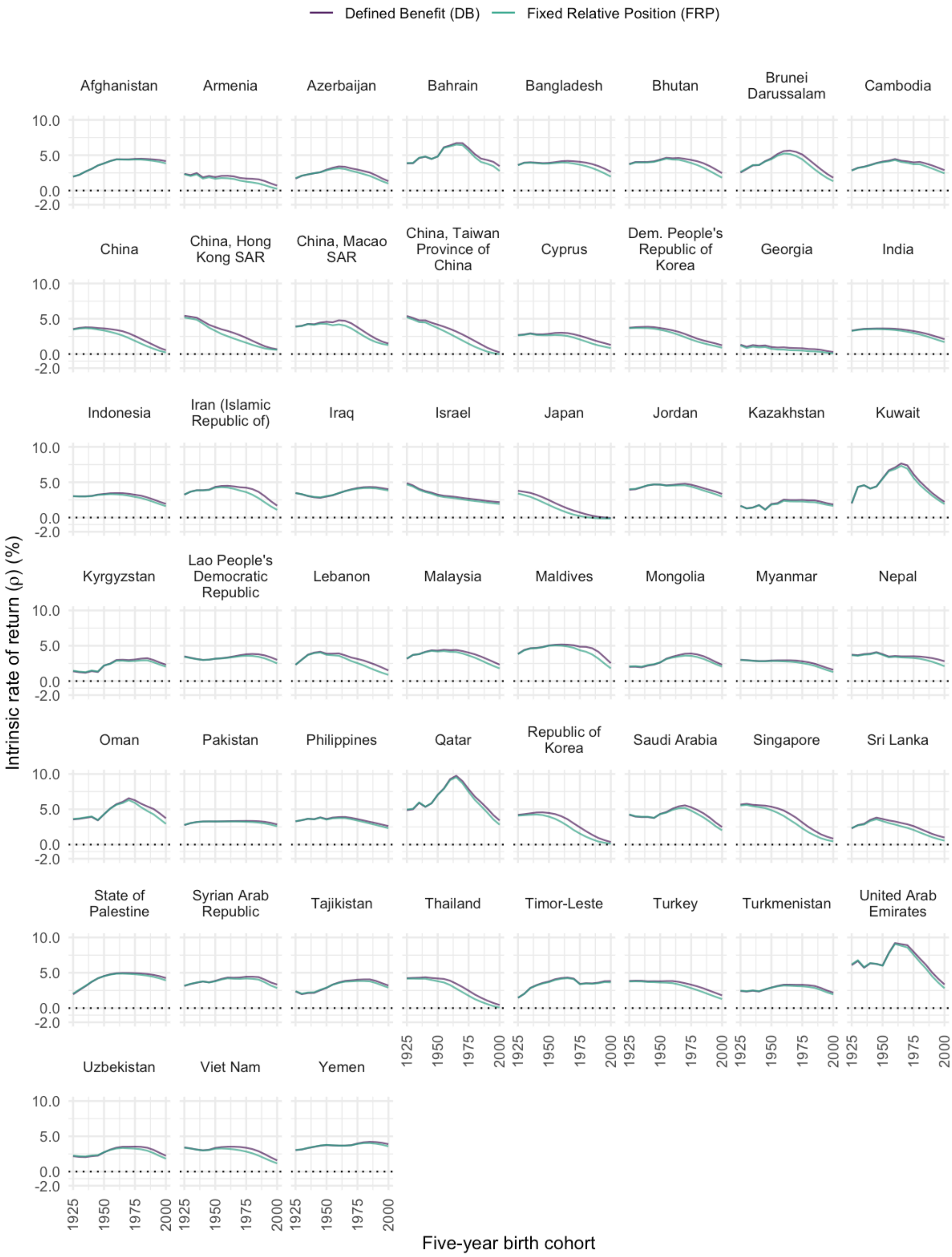
Source: Author's calculations, based on United Nations (2017c).

Figure 61 – Intrinsic rate of return (IRR) of defined benefit (DB) and of fixed relative position (FRP) by cohorts and country — Africa



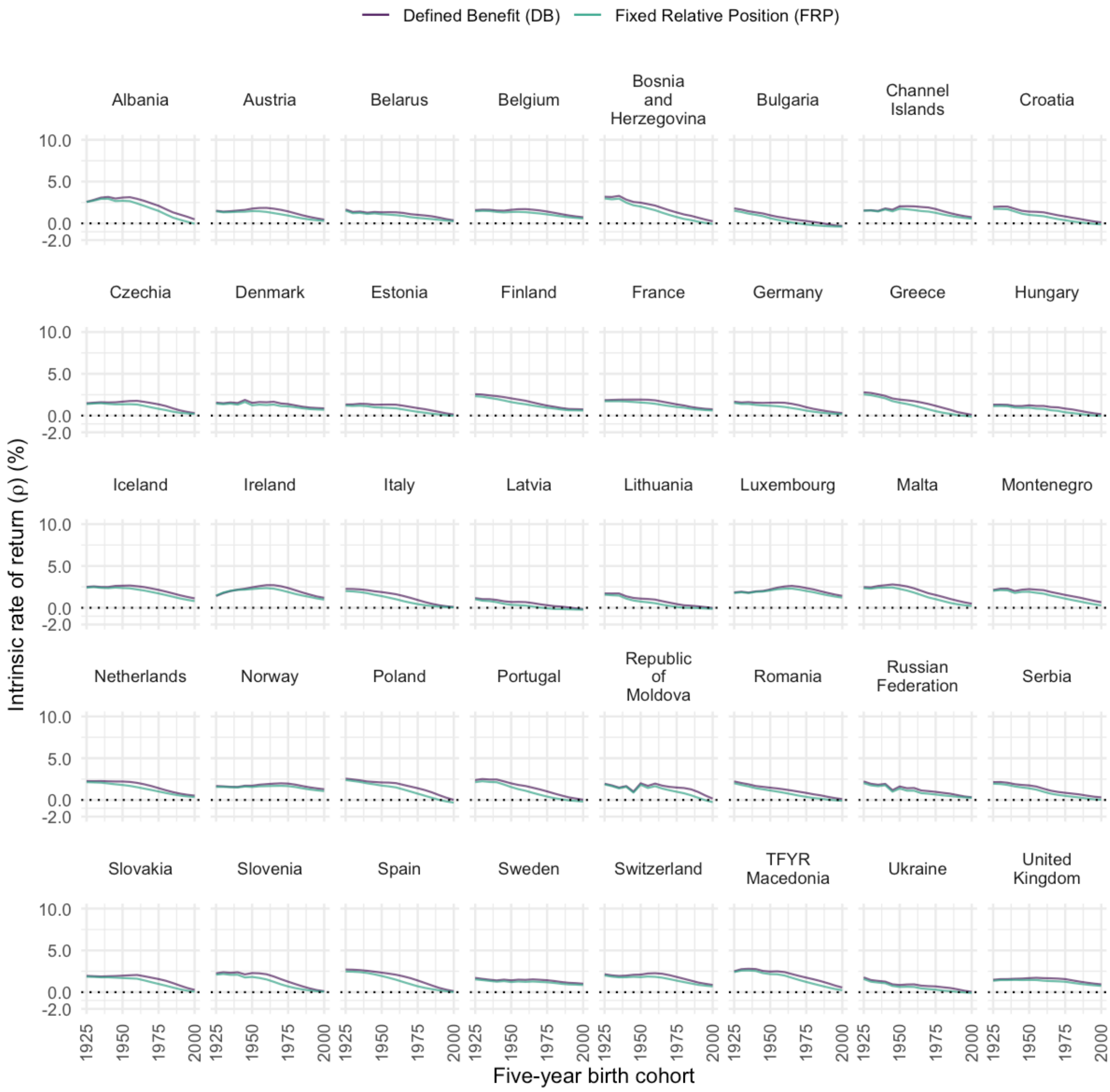
Source: Author's calculations, based on United Nations (2017c).

Figure 62 – Intrinsic rate of return (IRR) of defined benefit (DB) and of fixed relative position (FRP) by cohorts and country — Asia



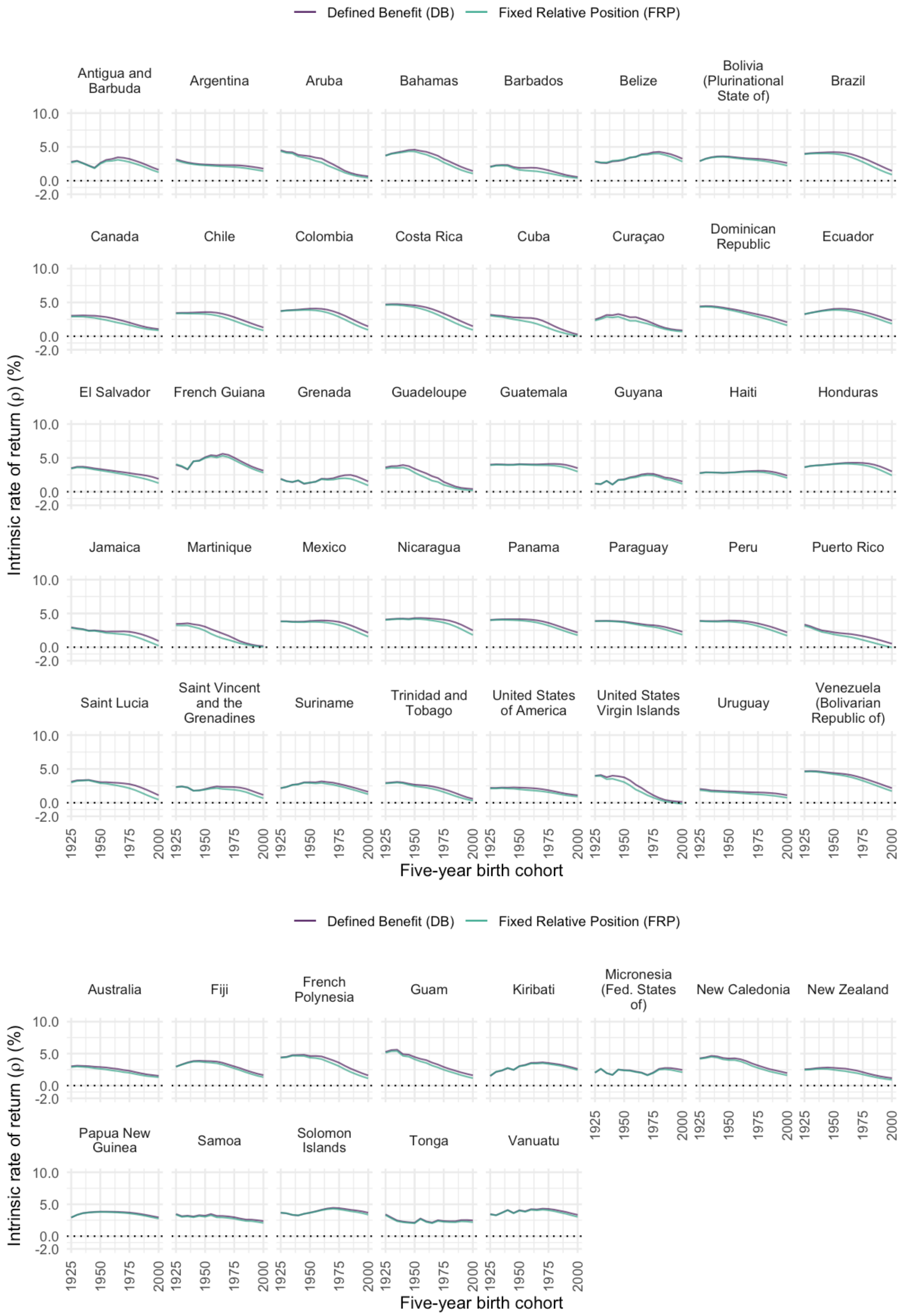
Source: Author's calculations, based on United Nations (2017c).

Figure 63 – Intrinsic rate of return (IRR) of defined benefit (DB) and of fixed relative position (FRP) by cohorts and country — Europe



Source: Author's calculations, based on United Nations (2017c).

Figure 64 – Intrinsic rate of return (IRR) of defined benefit (DB) and of fixed relative position (FRP) by cohorts and country — Americas and Oceania



Source: Author's calculations, based on United Nations (2017c).

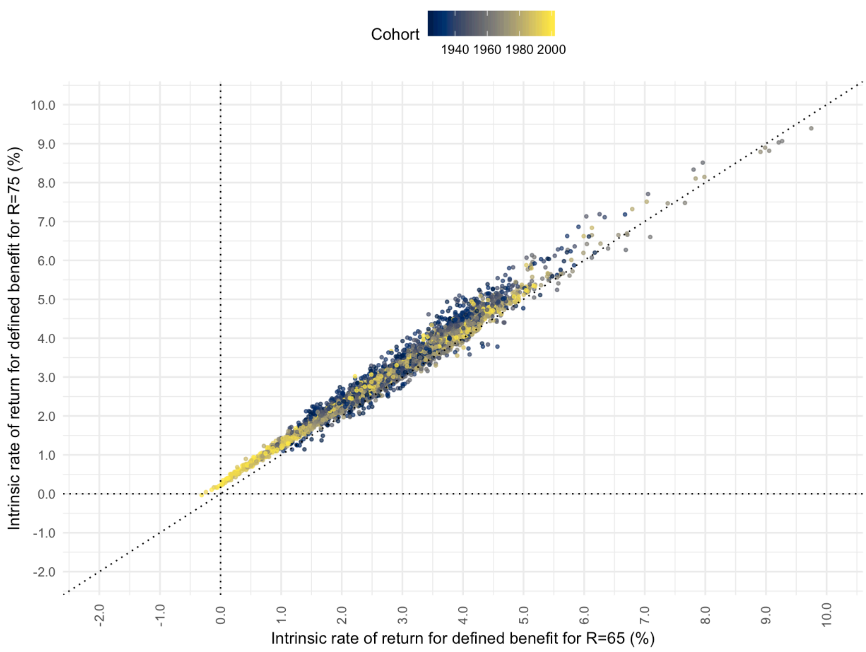
5.5 INTRINSIC RATE OF RETURN AND RETIREMENT AGES

We evaluate how the age of entry into retirement (R) influence intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems and defined contribution (DC) PAYG systems. We compare IRR of R equal to 65 years with IRR of R equal to 75 years; and also IRR of R equal to 65 years with IRR of equivalent retirement ages (ERA). Particularly, we verify Fernandes (1993)'s claim that higher ages of entry into retirement (R) tend to increase IRR of defined benefit (DB) PAYG systems, and to decrease IRR of defined contribution (DC) PAYG systems.

5.5.1 *Intrinsic rate of return and fixed retirement ages*

Figures 65 and 66 plot intrinsic rates of return (IRR) of R equal to 65 years by intrinsic rates of return (IRR) of R equal to 75 years, of defined benefit (DB) PAYG systems and defined contribution (DC) PAYG systems respectively.[17] Initially, our data suggests that Fernandes (1993)'s claim is correct. In defined benefit (DB) PAYG systems, intrinsic rates of return (IRR) are predominantly higher for R equal to 75 years than for R equal to 65 years, for instance, 3,015 (93.75%) of 3,126 birth cohorts present IRR for R equal to 75 years higher than IRR for R equal to 65 years. On the contrary, in defined contribution (DC) PAYG systems, intrinsic rates of return (IRR) are mostly lower for R equal to 75 years than for R equal to 65 years, specifically, 2,912 (90.55%) birth cohorts present IRR for R equal to 75 years lower than IRR for R equal to 65 years.

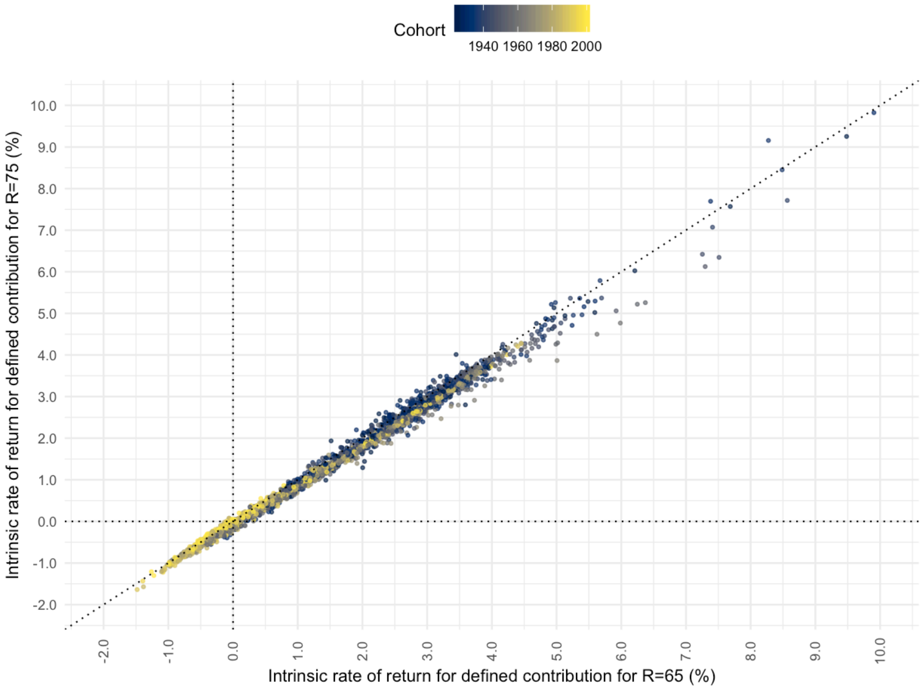
Figure 65 – Intrinsic rate of return (IRR) of $R = 65$ by IRR of $R = 75$ of defined benefit (DB)



Source: Author's calculations, based on United Nations (2017c).

[17] Figures 131 and 132 in section E.2, Appendix E, detail these results by subregions.

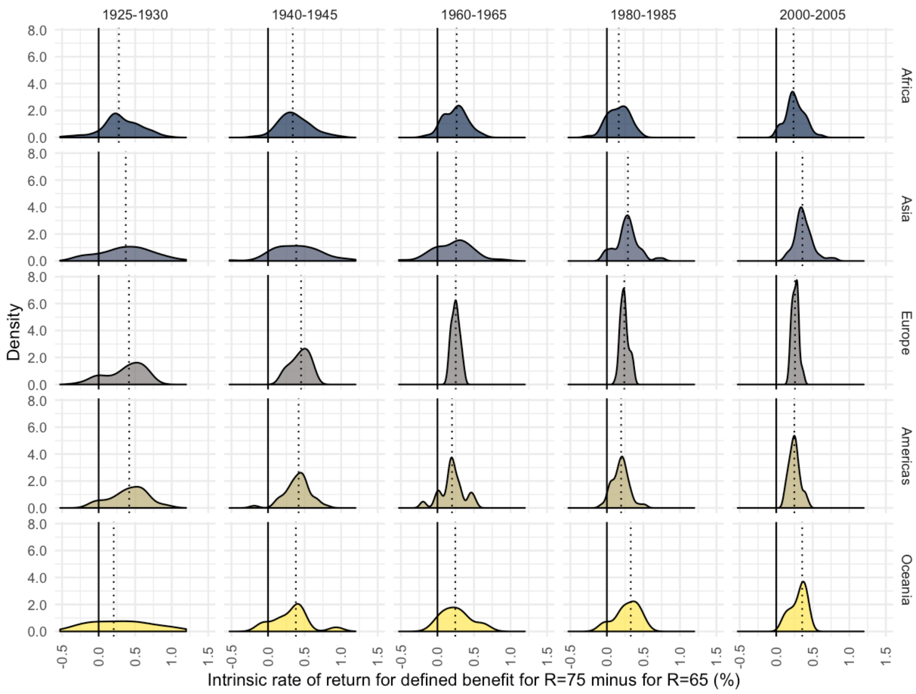
Figure 66 – Intrinsic rate of return (IRR) of R = 65 by IRR of R = 75 of defined contribution (DC)



Source: Author's calculations, based on United Nations (2017c).

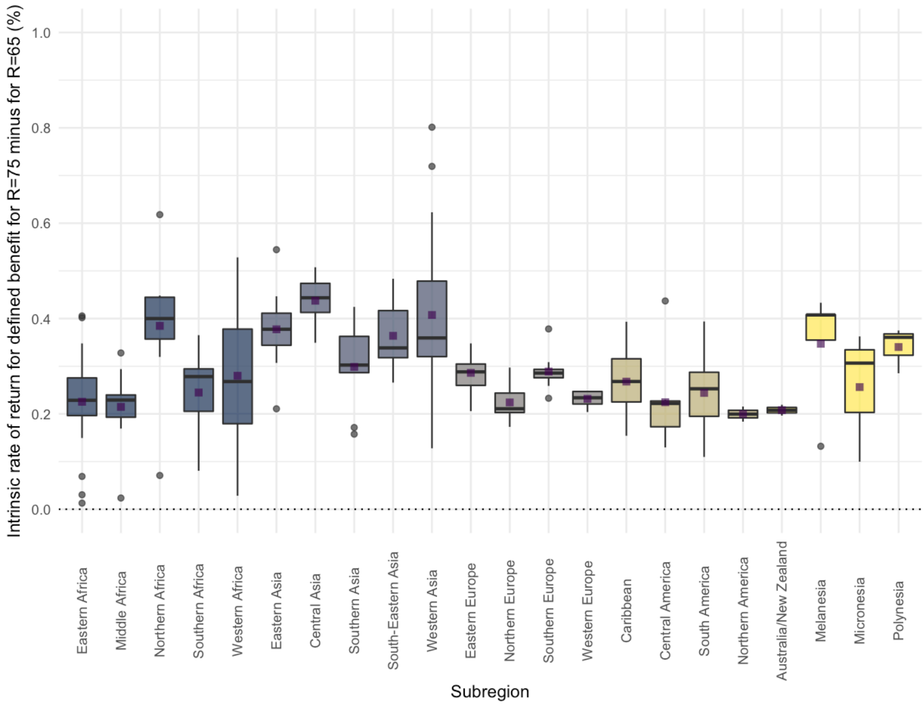
Figure 67 plots the density of the difference between intrinsic rates of return (IRR) of R equal to 75 years and intrinsic rates of return (IRR) of R equal to 65 years of defined benefit (DB) PAYG systems for selected birth cohorts and regions. Figure 68 shows its distribution for the 2000–2005 birth cohort by subregions. In defined benefit (DB) PAYG systems, from the 1925–1930 to the 2000–2005 cohort, the median of the difference between IRR of R equal to 75 years and of R equal to 65 years in Africa increases from 0.28% to 0.34% for the 1935–1940 cohort then decreases to 0.16% for the 1980–1985 cohort then increases to 0.23%, in Asia increases from 0.37% to 0.44% for the 1935–1940 cohort then decreases to 0.25% for the 1965–1970 cohort then increases to 0.36%, in Europe increases from 0.42% to 0.48% for the 1935–1940 cohort then decreases to around 0.25% for the 1960–1965 and younger cohorts, in the Americas increases from 0.41% to 0.45% for the 1935–1940 cohort then decreases to 0.19% for the 1975–1980 cohort then increases to 0.25%, and in Oceania increases from 0.21% to 0.38% for the 1940–1945 cohort then decreases to 0.23% for the 1965–1970 cohort then increases to 0.35%. For the 2000–2005 cohort, the median of the difference between IRR of R equal to 75 years and of R equal to 65 years is above 0.40% in three subregions: Central Asia (0.44%), Melanesia (0.41%), and Northern Africa (0.40%); and close to 0.20% in three subregions: Northern Europe (0.21%), Australia/New Zealand (0.21%), and Northern America (0.20%).

Figure 67 – Density of the difference between intrinsic rate of return (IRR) of $R = 75$ and IRR of $R = 65$ of defined benefit (DB) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

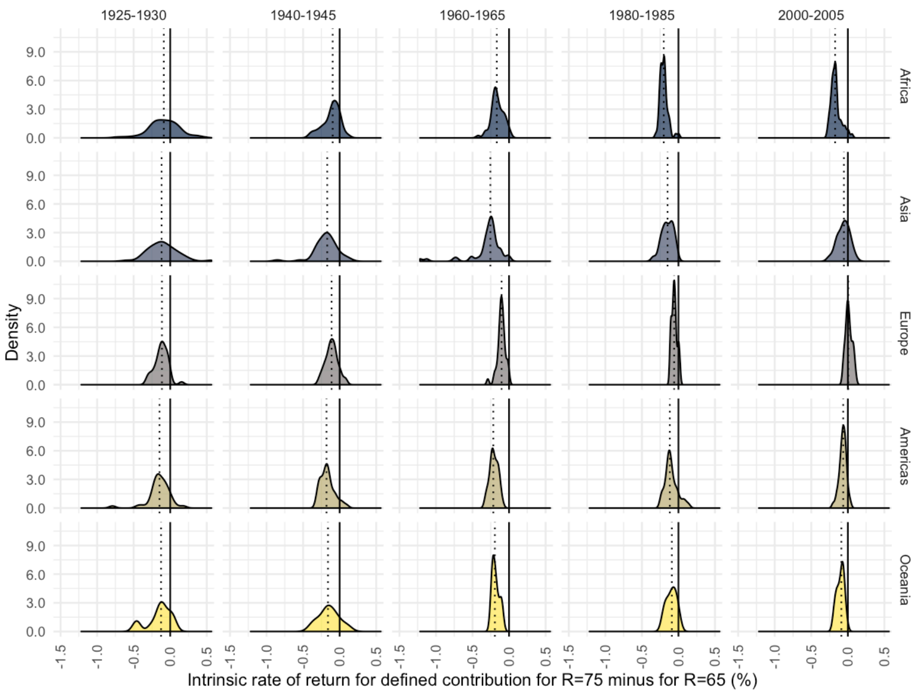
Figure 68 – Difference between intrinsic rate of return (IRR) $R = 75$ and IRR of $R = 65$ of defined benefit (DB) for 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

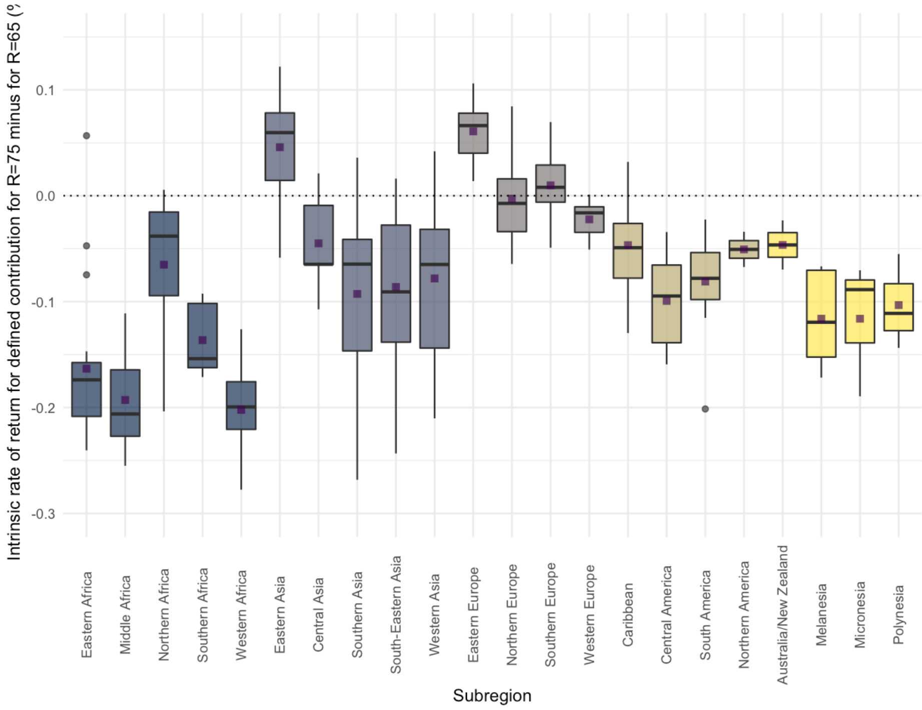
Figure 69 presents the density of the difference between intrinsic rates of return (IRR) of R equal to 75 years and intrinsic rates of return (IRR) of R equal to 65 years of defined contribution (DC) PAYG systems for selected birth cohorts and regions. Figure 70 plots its distribution for the 2000–2005 birth cohort by subregions. In defined contribution (DC) PAYG systems, from the 1925–1930 to the 2000–2005 cohort, the median of the difference between IRR of R equal to 75 years and of R equal to 65 years in Africa decreases from −0.08% to −0.21% for the 1985–1990 cohort then increases to −0.17%, in Asia decreases from −0.12% to −0.26% for the 1955–1960 cohort then increases to −0.05%, in Europe decreases from −0.11% to −0.14% for the 1935–1940 cohort then increases to 0.005%, in the Americas decreases from −0.15% to −0.21% for the 1960–1965 cohort then increases to −0.06%, and in Oceania decreases from −0.13% to −0.19% for the 1960–1965 cohort then increases to −0.08%. For the 2000–2005 cohort, the median of the difference between IRR of R equal to 75 years and of R equal to 65 years is below −0.20% in only two subregions: Middle Africa (−0.21%), and Western Africa (−0.20%); and marginally positive in three subregions: Southern Europe (0.01%), Eastern Asia (0.06%), and Eastern Europe (0.07%).

Figure 69 – Density of the difference between intrinsic rate of return (IRR) of $R = 75$ and IRR of $R = 65$ of defined contribution (DC) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Note: Vertical dotted line indicates the median of the distribution.

Figure 70 – Difference between intrinsic rate of return (IRR) $R = 75$ and IRR of $R = 65$ of defined contribution (DC) for 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Note: Square indicates the mean of the distribution.

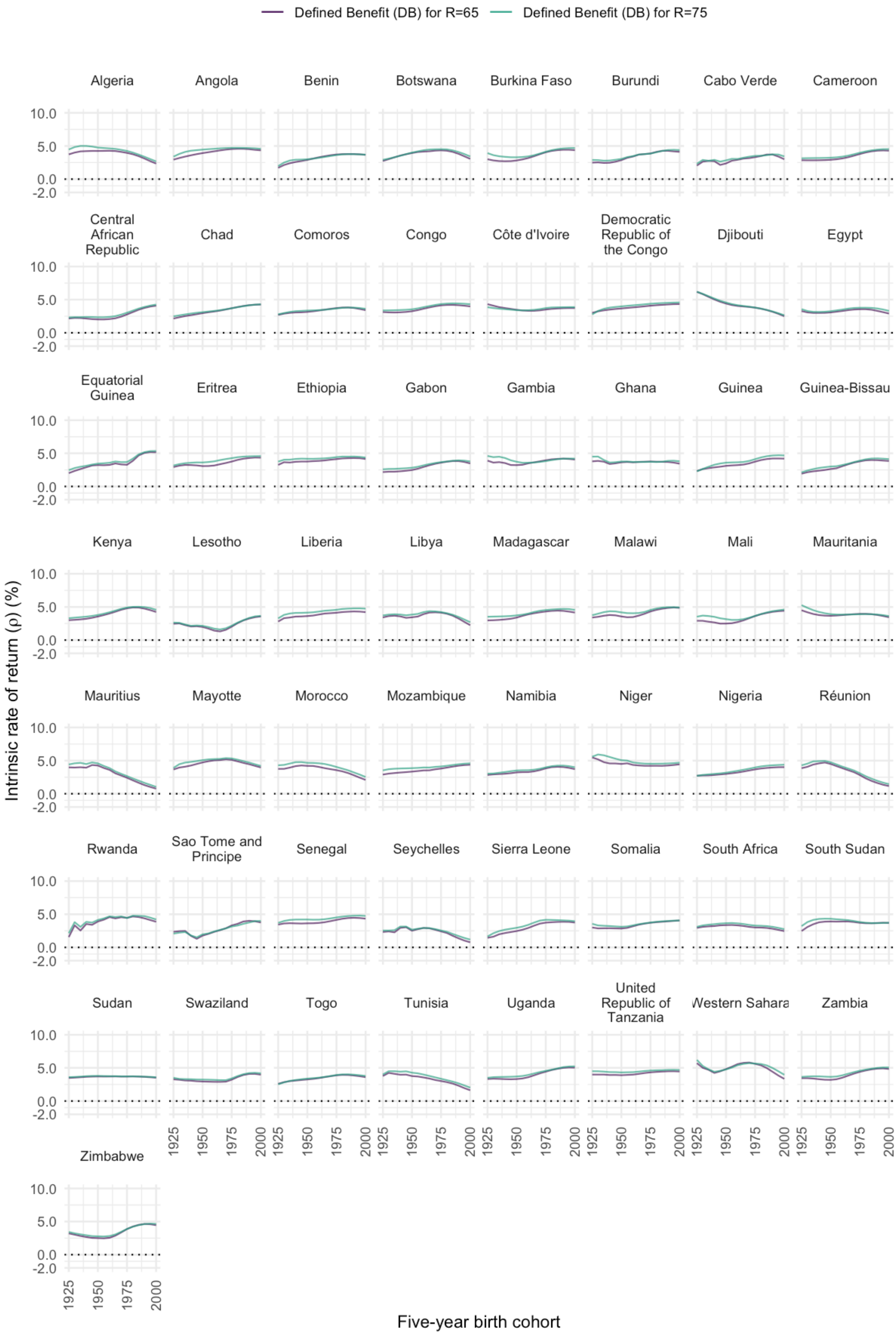
Figures 71 to 74 present the intrinsic rates of return (IRR) of R equal to 65 years and of R equal to 75 years, of defined benefit (DB) PAYG systems by birth cohorts for all 201 countries and areas. These figures strengthen the argument that higher ages of entry into retirement (R) tend to increase intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems; particularly for older cohorts which benefit the most from higher R and corresponding much lower contribution rates (*con*) in young populations.[18]

Figures 75 to 78 plot the intrinsic rates of return (IRR) of R equal to 65 years and of R equal to 75 years, of defined contribution (DC) PAYG systems by birth cohorts for all 201 countries and areas. Likewise, these figures reinforce the claim that higher ages of entry into retirement (R) tend to decrease intrinsic rates of return (IRR) of defined contribution (DC) PAYG systems. Still, absolute differences between intrinsic rates of return (IRR) of R equal to 75 years and of R equal to 65 years are smaller in DC PAYG systems than in DB PAYG systems. Therefore, these differences are much less noticeable in DC PAYG systems, but are nevertheless prominent for a few cohorts that experience equal or lesser benefit rates (*ben*) despite higher R.[19]

[18] For instance, Algeria, Burkina Faso, Eritrea, Gambia, Mali, Morocco, and Niger in Africa; Bangladesh, Bhutan, China, Maldives, Nepal, Oman, Republic of Korea, United Arab Emirates, and Viet Nam in Asia; Austria, Belgium, Luxembourg, Poland, Slovenia, and United Kingdom in Europe; Bolivia (Plurinational State of), Brazil, Nicaragua, and Peru in the Americas; and New Caledonia, and Samoa in Oceania.

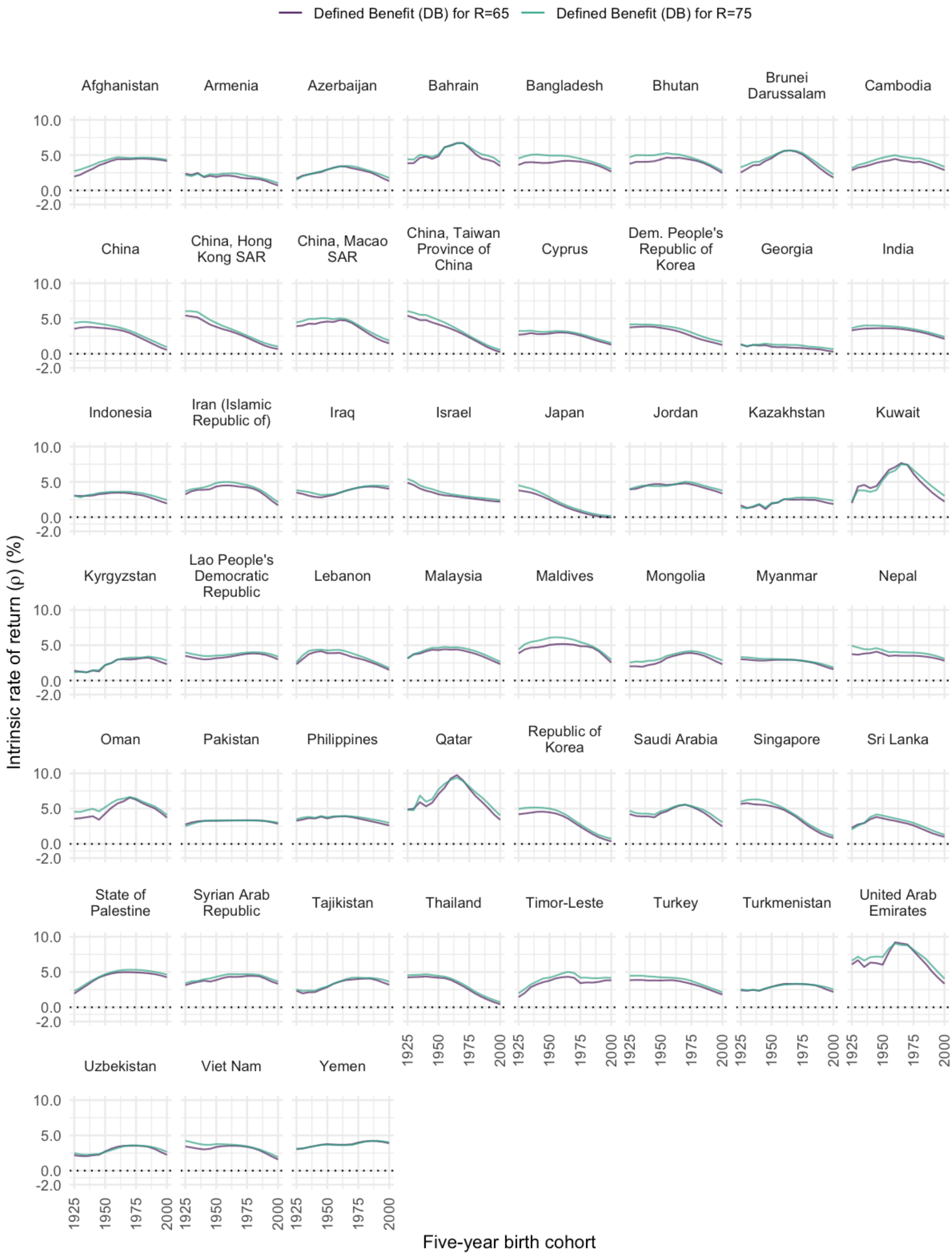
[19] For example, Bahrain, Kuwait, Maldives, Oman, Qatar, and United Arab Emirates in Asia.

Figure 71 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined benefit (DB) by cohorts and country — Africa



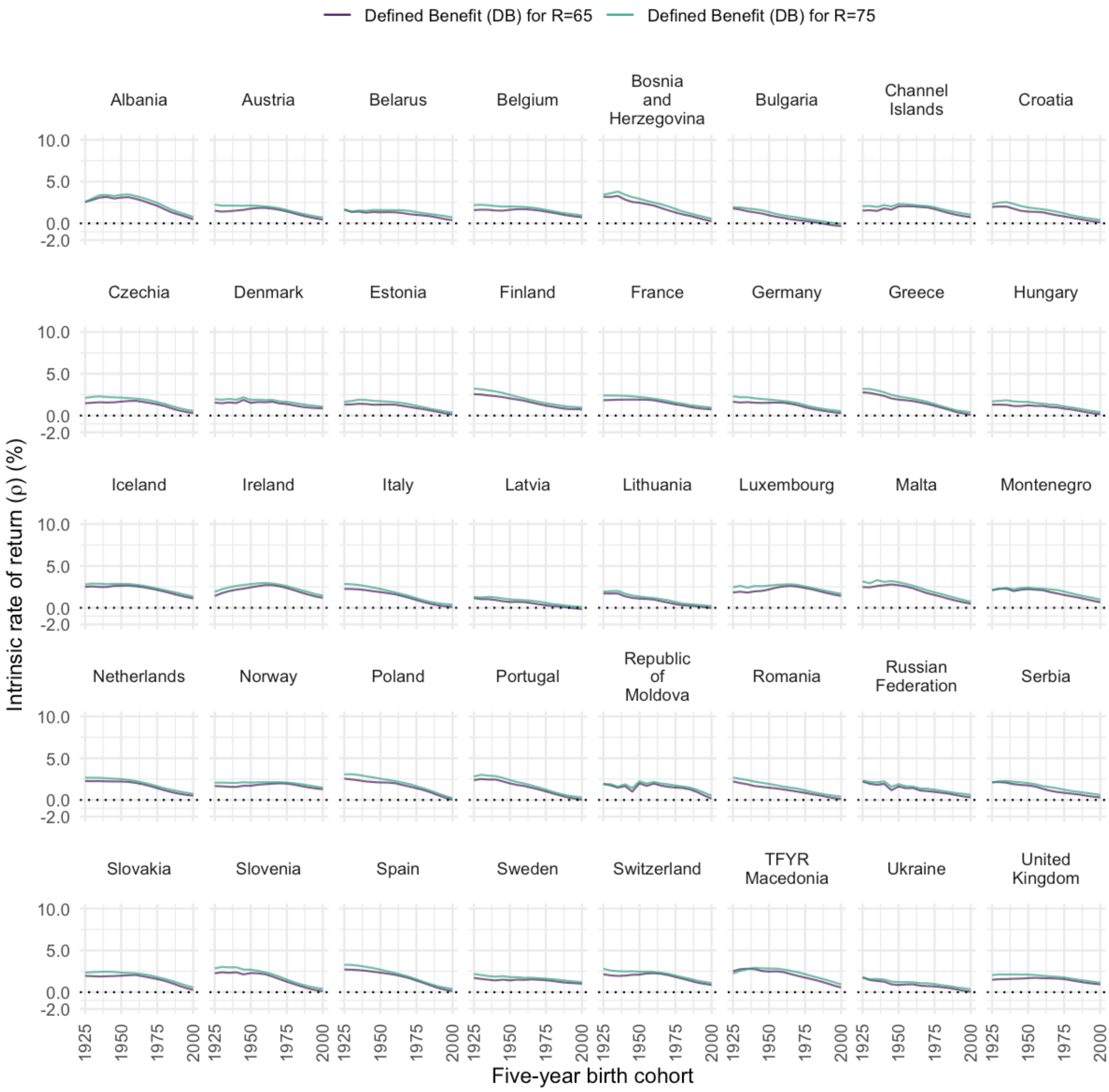
Source: Author's calculations, based on United Nations (2017c).

Figure 72 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined benefit (DB) by cohorts and country — Asia



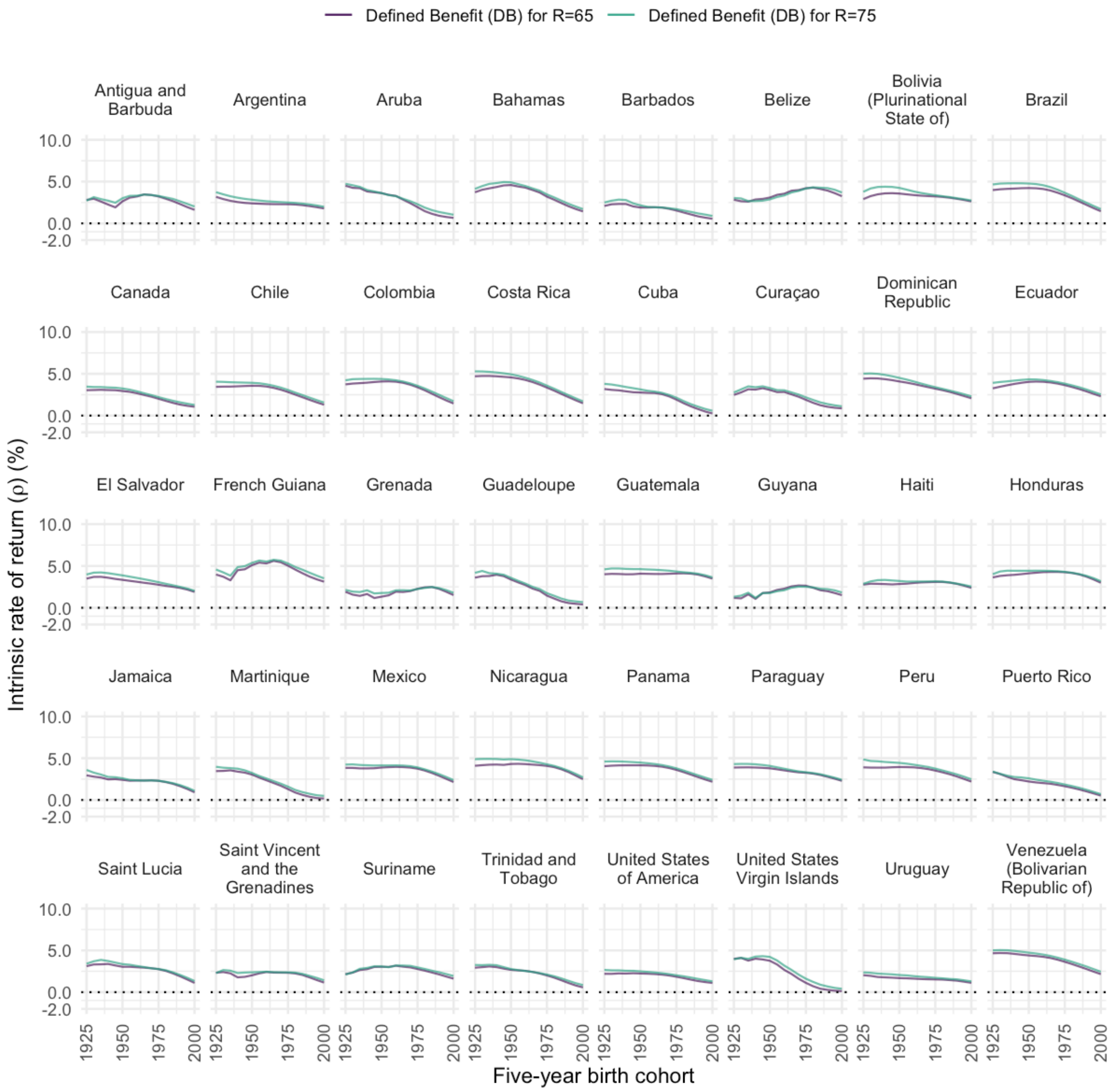
Source: Author's calculations, based on United Nations (2017c).

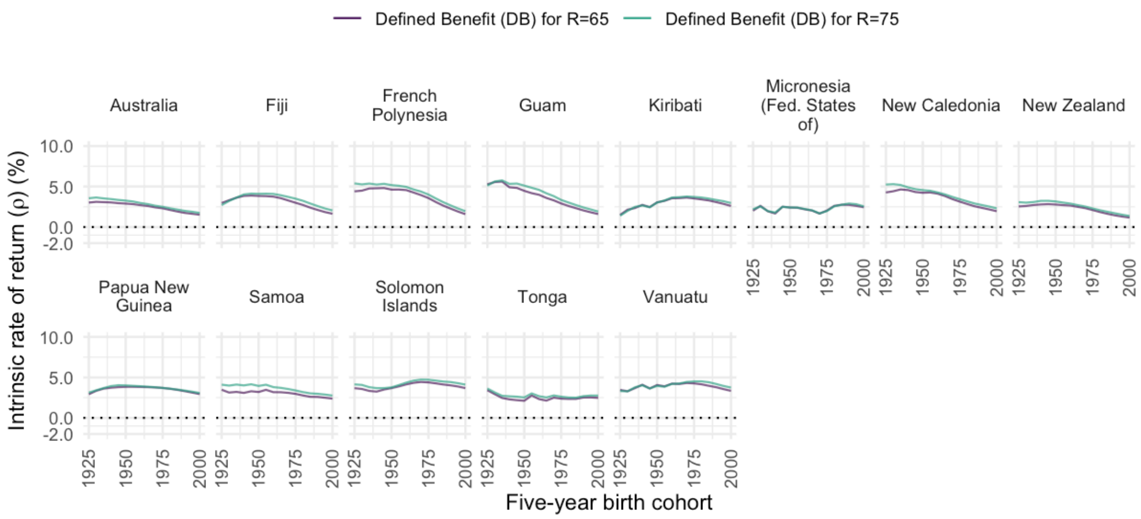
Figure 73 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined benefit (DB) by cohorts and country — Europe



Source: Author's calculations, based on United Nations (2017c).

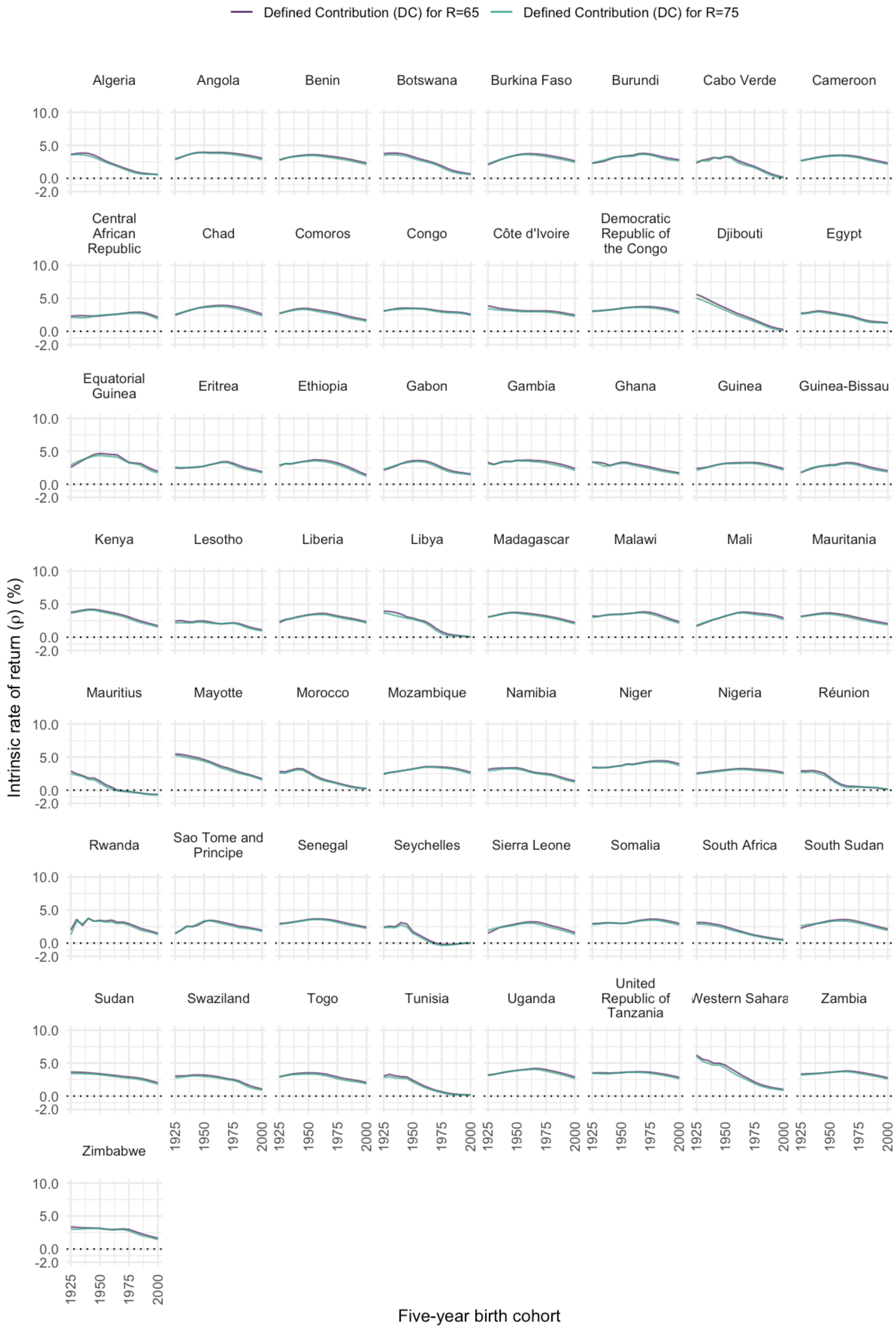
Figure 74 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined benefit (DB) by cohorts and country — Americas and Oceania





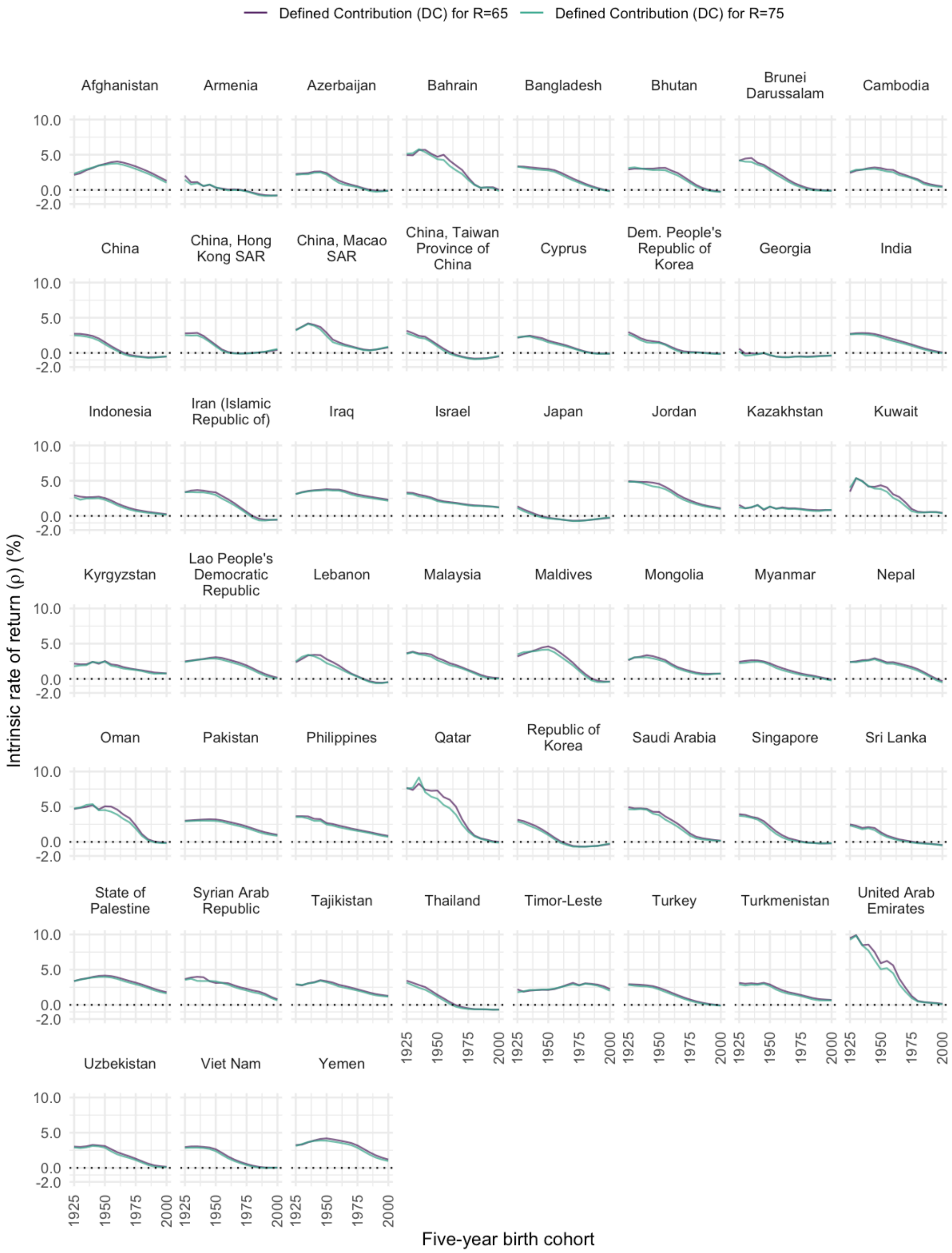
Source: Author's calculations, based on United Nations (2017c).

Figure 75 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined contribution (DC) by cohorts and country — Africa



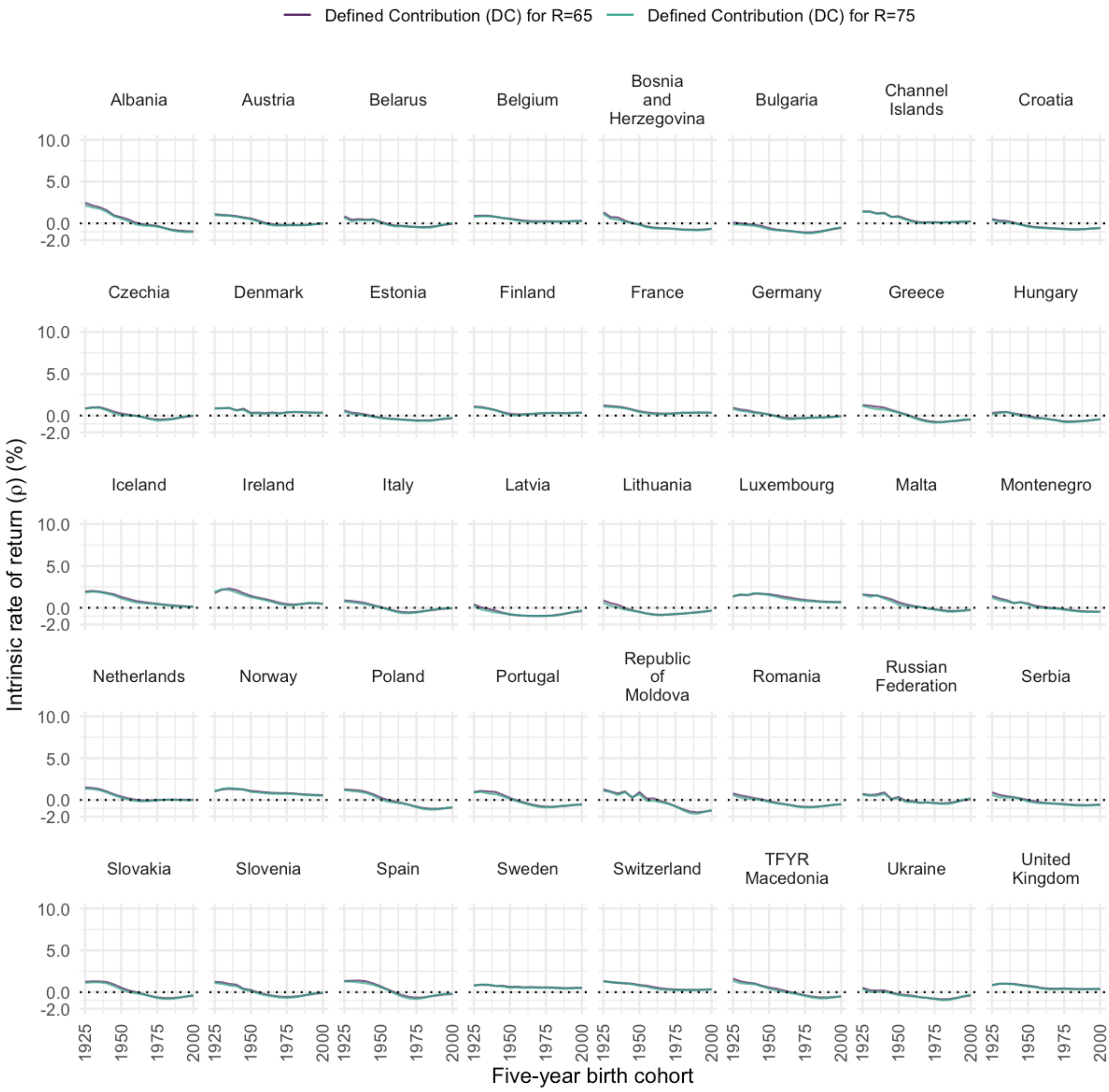
Source: Author's calculations, based on United Nations (2017c).

Figure 76 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined contribution (DC) by cohorts and country — Asia



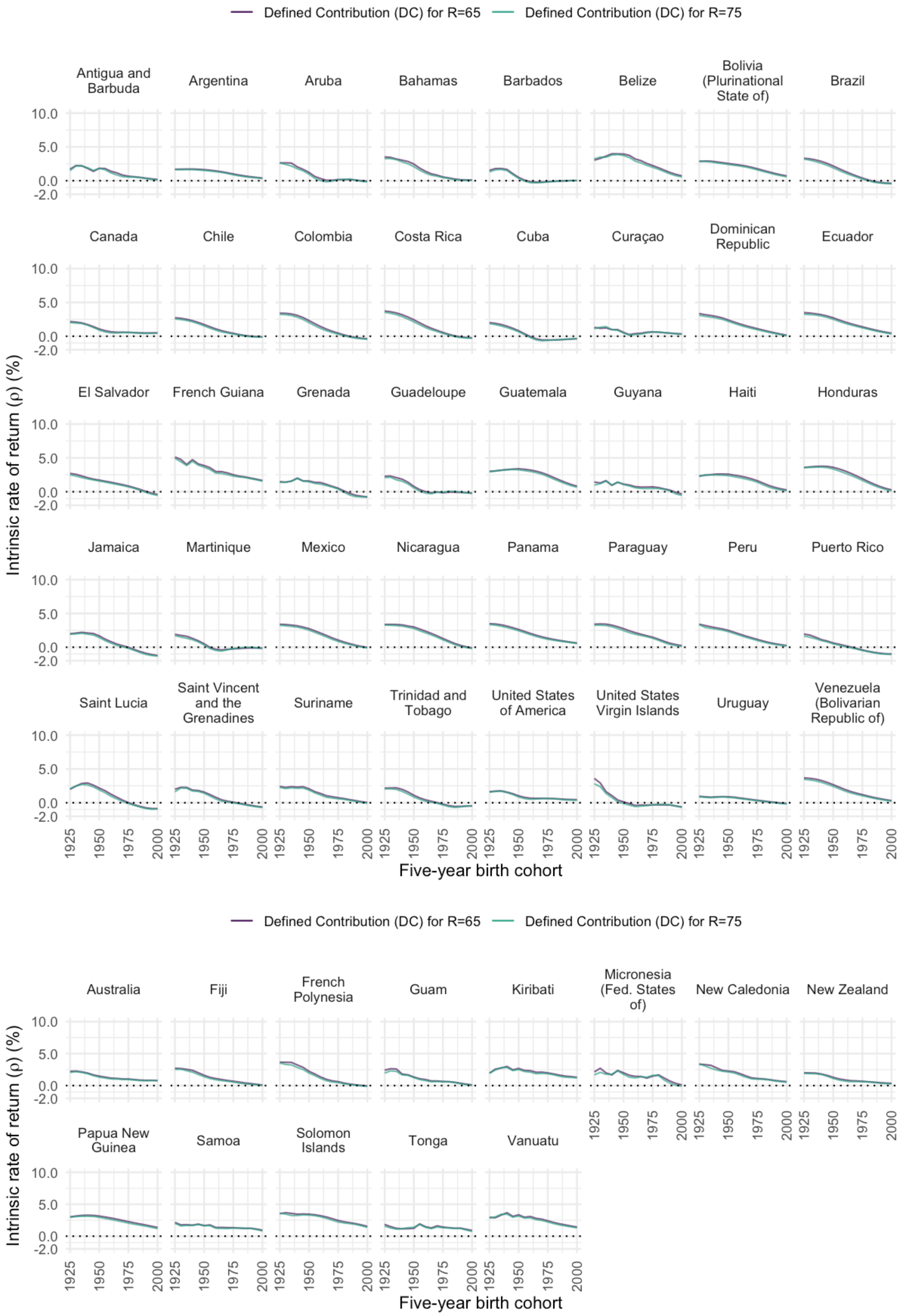
Source: Author's calculations, based on United Nations (2017c).

Figure 77 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined contribution (DC) by cohorts and country — Europe



Source: Author's calculations, based on United Nations (2017c).

Figure 78 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of $R = 75$ of defined contribution (DC) by cohorts and country — Americas and Oceania

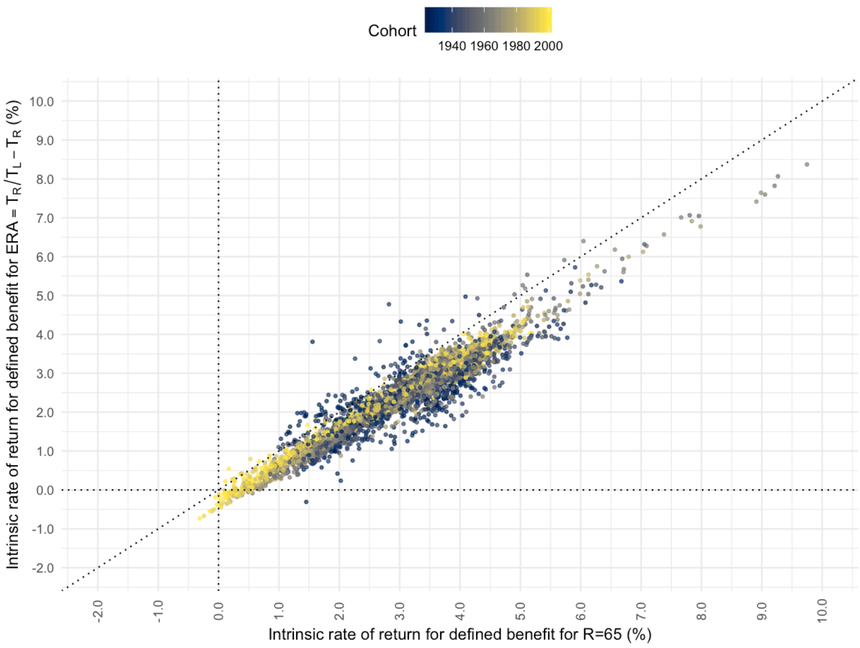


Source: Author's calculations, based on United Nations (2017c).

5.5.2 *Intrinsic rate of return and equivalent retirement ages*

Figures 79 and 80 present intrinsic rates of return (IRR) of R equal to 65 years by intrinsic rates of return (IRR) of equivalent retirement ages (ERA), of defined benefit (DB) PAYG systems and defined contribution (DC) PAYG systems respectively.[20] At first, our results contradict Fernandes (1993)'s claim that higher ages of entry into retirement (R) tend to increase IRR of defined benefit (DB) PAYG systems, and to decrease IRR of defined contribution (DC) PAYG systems. In defined benefit (DB) PAYG systems, IRR are predominantly lower for ERA than for R equal to 65 years. Specifically, only 119 (3.70%) of 3,126 birth cohorts present IRR for ERA higher than IRR for R equal to 65 years. Of these 119 cohorts, the difference between the IRR for ERA and the IRR for R equal to 65 years is greater than 0.70% for only eight cohorts, and has a median equal to 0.22%.[21] In defined contribution (DC) PAYG systems, IRR are mostly higher for ERA than for R equal to 65 years. For instance, only 630 (19.59%) birth cohorts present IRR for ERA lower than IRR for R equal to 65 years. Of these 630 cohorts, the difference between IRR for ERA and for R equal to 65 years is less than −0.70% for only six cohorts, and between zero and −0.05% for 307 cohorts.[22]

Figure 79 – Intrinsic rate of return (IRR) of R = 65 by IRR of ERA of defined benefit (DB)



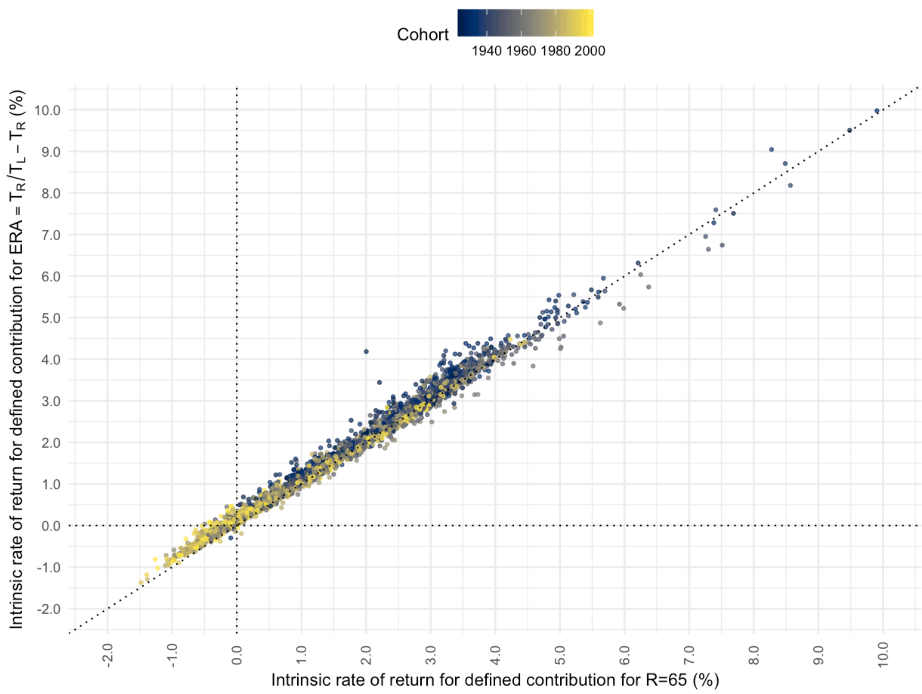
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

[20] Figures 133 and 134 in section E.2, Appendix E, detail these results by subregions.

[21] Rwanda 1925–1930 (2.25%, 3.81% vs. 1.56%), Zimbabwe 1935–1940 (1.95%, 4.77% vs. 2.82%), Zimbabwe 1930–1935 (1.31%, 4.33% vs. 3.02%), Zimbabwe 1940–1945 (1.16%, 3.83% vs. 2.67%), Lesotho 1935–1940 (1.12%, 3.38% vs. 2.26%), Uganda 1925–1930 (0.92% 4.25% 3.33%), Zambia 1930–1935 (0.90% 4.35% 3.45%), Côte d'Ivoire 1930–1935 (0.88%, 4.97% vs. 4.09%), and Oman 1945–1950(0.72%, 4.15% vs. 3.43%).

[22] United Arab Emirates 1945–1950 (−0.77%, 6.74% vs. 7.51%), Qatar 1960–1965 (−0.76%, 5.22% vs. 5.98%), Qatar 1965–1970 (−0.75%, 4.25% vs. 5.00%), United Arab Emirates 1960–1965 (−0.75%, 4.87% vs. 5.62%), Oman 1960–1965 (−0.75%, 3.83% vs. 4.58%), and Oman 1955–1960 (−0.73%, 4.29% vs. 5.02%).

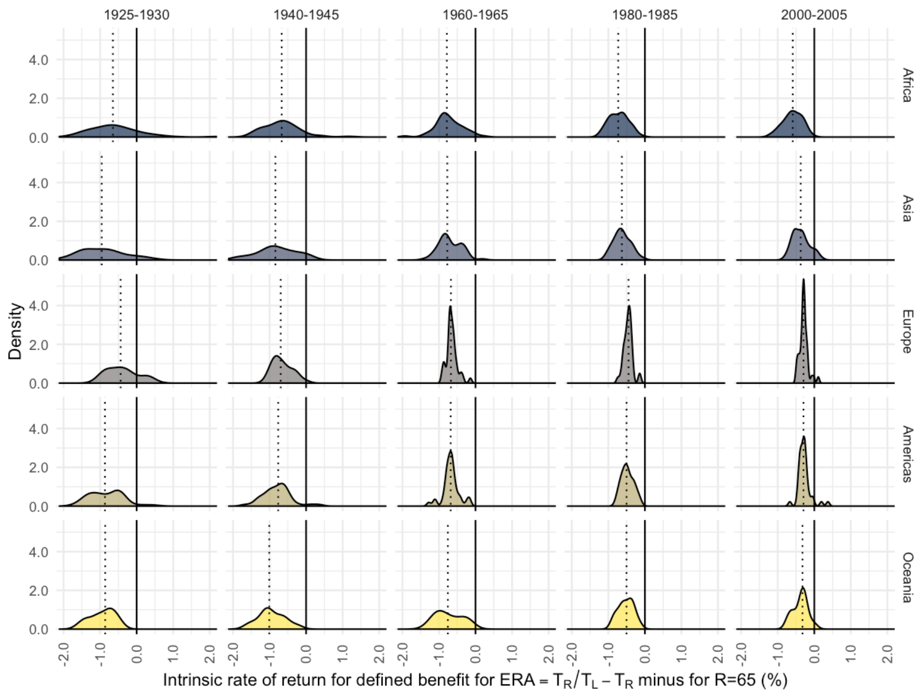
Figure 80 – Intrinsic rate of return (IRR) of R = 65 by IRR of ERA of defined contribution (DC)



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

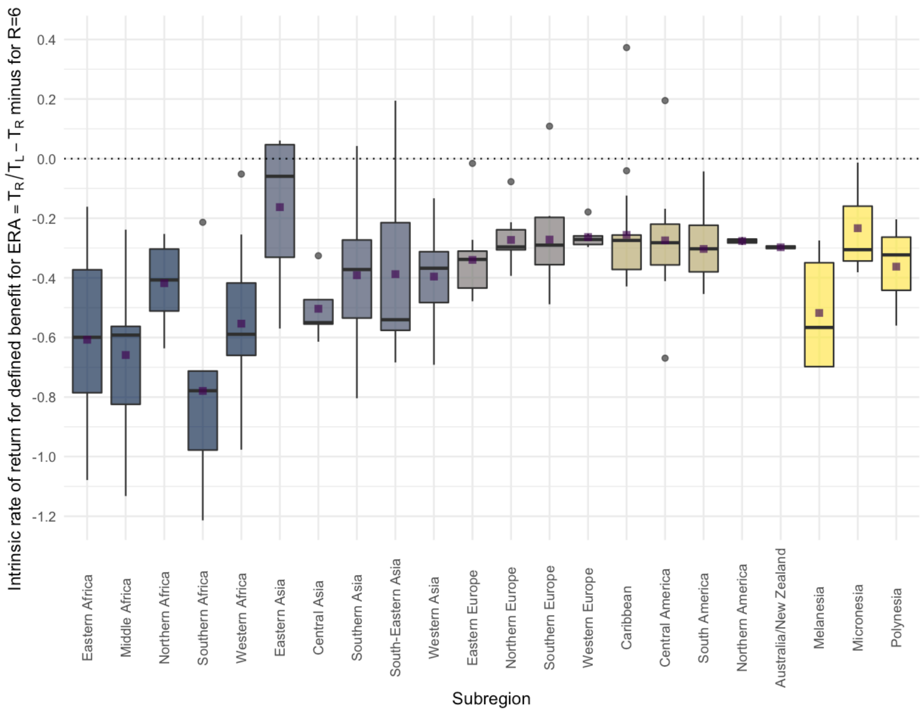
Figure 81 plots the density of the difference between intrinsic rates of return (IRR) of equivalent retirement ages (ERA) and intrinsic rates of return (IRR) of R equal to 65 years of defined benefit (DB) PAYG systems for selected birth cohorts and regions. Figure 82 shows its distribution for the 2000–2005 birth cohort by subregions. In defined benefit (DB) PAYG systems, from the 1925–1930 to the 2000–2005 cohort, the median of the difference between IRR of ERA and of R equal to 65 years is mostly more negative for the older cohorts, and varies between −0.81% (1970–1975) and −0.58% (1930–1935) in Africa, −0.96% (1925–1930) and −0.37% (2000–2005) in Asia, −0.70% (1945–1950) and −0.30% (2000–2005) in Europe, −0.87% (1925–1930) and −0.30% (2000–2005) in the Americas, and −1.00% (1940–1945) and −0.32% (2000–2005) in Oceania. In general, this difference is as low as −2.13% (Bhutan 1930–1935, 1.91% vs. 4.04%) and as high as 2.25% (Rwanda 1925–1930, 3.81% vs. 1.56%). For the 2000–2005 cohort, the median of the difference between IRR of ERA and of R equal to 65 years varies between −0.78% (Southern Africa) and −0.06% (Eastern Asia), and is closer to zero (around −0.30%) in the subregions of Europe, the Americas, and Oceania except Melanesia.

Figure 81 – Density of the difference between intrinsic rate of return (IRR) of ERA and IRR of $R = 65$ of defined benefit (DB) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

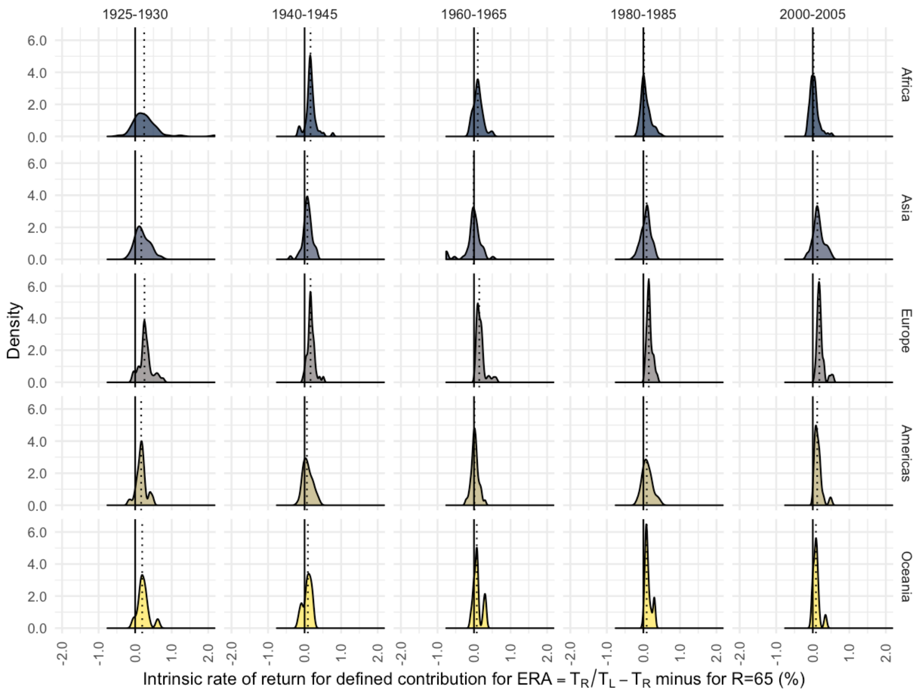
Figure 82 – Difference between intrinsic rate of return (IRR) of ERA and IRR of $R = 65$ of defined benefit (DB) for 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

Figure 83 plots the density of the difference between intrinsic rates of return (IRR) of equivalent retirement ages (ERA) and intrinsic rates of return (IRR) of *R* equal to 65 years of defined contribution (DC) PAYG systems for selected birth cohorts and regions. Figure 84 presents its distribution for the 2000–2005 birth cohort by subregions. In defined contribution (DC) PAYG systems, from the 1925–1930 to the 2000–2005 cohort, the median of the difference between IRR of ERA and of *R* equal to 65 years varies between −0.002% (1990–1995) and 0.26% (1930–1935) in Africa, −0.007% (1960–1965) and 0.17% (1925–1930) in Asia, 0.14% (1955–1960) and 0.26% (1925–1930) in Europe, 0.013% (1960–1965) and 0.16% (1925–1930) in the Americas, and 0.04% (1950–1955) and 0.19% (1925–1930) in Oceania. Overall, this difference remains between −0.77% (United Arab Emirates 1945–1950, 6.74% vs. 7.51%) and 0.97% (Zimbabwe 1935–1940, 6.74% vs. 7.51%).[23] For the 2000–2005 cohort, the median of the difference between IRR of ERA and of *R* equal to 65 years varies between −0.014% (Middle Africa) and 0.33% (Eastern Asia). Except for Eastern Asia (0.33%), Eastern Europe (0.23%), and Southern Europe (0.18%), in absolute values, this median is below 0.15% for all other nineteen subregions, and below 0.10% for ten subregions.

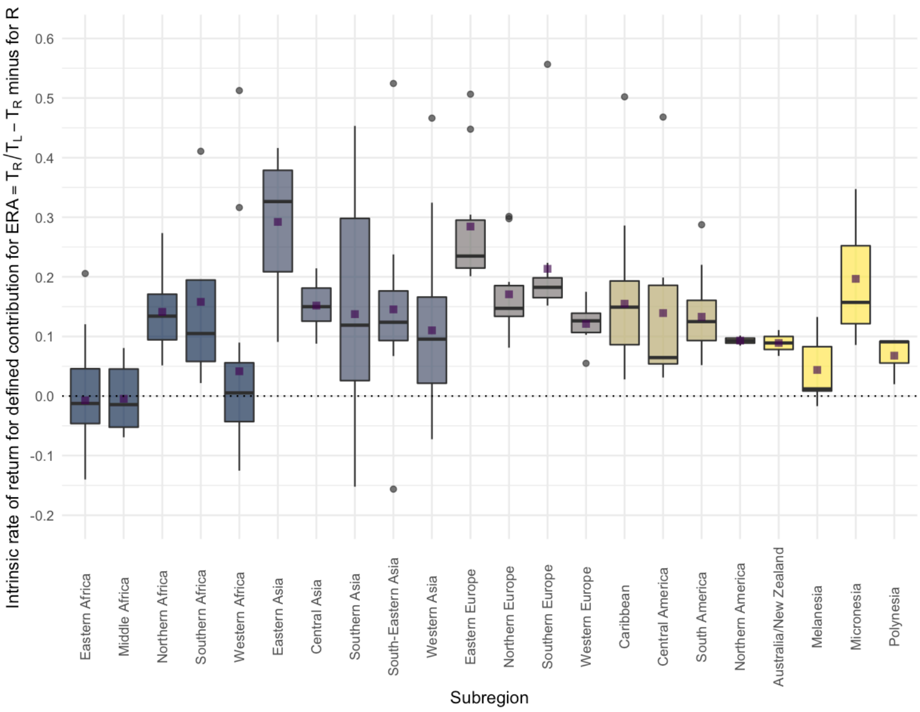
Figure 83 – Density of the difference between intrinsic rate of return (IRR) of ERA and IRR of *R* = 65 of defined contribution (DC) by selected cohorts and regions



Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

23 Except for two outliers: 2.18% (Rwanda 1925–1930, 4.18% vs. 2.00%), and 1.23% (Liberia 1925–1930, 3.44% vs.2.20%).

Figure 84 – Difference between intrinsic rate of return (IRR) of ERA and IRR of R = 65 of defined contribution (DC) for 2000–2005 cohort by subregions



Source: Author's calculations, based on United Nations (2017c).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

Although these results seem to contradict Fernandes (1993)'s claim, the explanations for these apparent paradoxical effects of equivalent retirement ages (ERA) on intrinsic rates of return (IRR) of defined benefit (DB) and defined contribution (DC) PAYG systems are on Fernandes (1993)'s reasoning itself. Specifically, in defined benefit (DB) PAYG systems, a higher age of entry into retirement (R) renders the birth cohort more working years with lower contribution rates (*con*). But in equivalent retirement ages (ERA) DB PAYG systems, the lower contribution rates (*con*) *phase in*, that is, as R *gradually* increase, the contribution rates (*con*) *slowly* decrease over the working life cycle. In ERA DB PAYG systems the age of entry into retirement (R) for the cohort may be the same as those in fixed retirement age DB PAYG systems, but the working life cycle contribution rates (*con*) are higher. Analogously, in defined contribution (DC) PAYG systems, a higher age of entry into retirement (R) ensues the birth cohort less retirement years with higher benefit rates (*ben*). But in equivalent retirement ages (ERA) DC PAYG systems, the higher benefit rates (*ben*) *keep on*, that is, as R *gradually* increase, the benefit rates (*ben*) *steadily* increase over the retirement life cycle. In ERA DC PAYG systems the age of entry into retirement (R) for the cohort may be the same as those in fixed retirement age DC PAYG systems, but the retirement life cycle benefit rates (*ben*) are higher. Therefore, higher ages of entry into retirement (R) in *equivalent retirement age (*ERA*)* PAYG systems tend to *decrease* intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems, and to *increase* intrinsic rates of return (IRR) of defined contribution (DC) PAYG systems.

Figures 85 to 88 present the intrinsic rates of return (IRR) of R equal to 65 years and of equivalent retirement ages (ERA), of defined benefit (DB) PAYG systems by birth cohorts for all 201 countries and areas. We observe four noteworthy patterns of the effect of equivalent retirement ages (ERA) on intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems. First, ERA decrease more the IRR of older cohorts, and less the IRR of younger cohorts, that is, both IRR converge.[24] Second, ERA decrease evenly the IRR of all cohorts, impacting all cohorts equally, that is, both IRR are in parallel.[25] Third, ERA decrease less the IRR of older cohorts, and more the IRR of younger cohorts, that is, both IRR diverge.[26] Fourth, ERA increase the IRR of older cohorts because of contemporary increases in adult mortality and respective decreases in $(T_R)/(T_L - T_R)$ and in ERA.[27] Consider Zimbabwe in Africa, equivalent retirement ages (ERA) increase the IRR of cohorts from 1925–1930 to 1940–1945[28] because ERA are closer to or lower than 65 years from 1990–1995 to 2005–2010, specifically, ERA is equal to 66.23 years in 1990–1995, 62.45 years in 1995–2000, 59.45 years in 2000–2005, and 62.31 years in 2005–2010.

Figures 89 to 92 plot the intrinsic rates of return (IRR) of R equal to 65 years and of equivalent retirement ages (ERA), of defined contribution (DC) PAYG systems by birth cohorts for all 201 countries and areas. The absolute differences between IRR of ERA and of R equal to 65 years are smaller in DC PAYG systems than in DB PAYG systems. Nevertheless, we note two noteworthy patterns of the effect of equivalent retirement ages (ERA) on intrinsic rates of return (IRR) of defined contribution (DC) PAYG systems. First ERA increase the IRR of older cohorts not only because of contemporary increases in adult mortality and respective decreases in ERA, as in DB PAYG systems, but also because of thereafter increases in ERA that eventually provide these cohorts with increasing benefit rates (*ben*).[29] Second, ERA decrease the IRR of intermediate cohorts because the large sizes of these cohorts hinder the steadily increase of *ben* over their retirement life cycle.[30]

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24 For instance, Algeria, Angola, Mali, Morocco, Réunion, South Sudan, and Tunisia in Africa; Afghanistan, Bhutan, China, India, Iran (Islamic Republic of), Japan, Maldives, Nepal, Singapore, Thailand, Timor-Leste, and Turkey in Asia; Austria, Finland, Greece, Ireland, Italy, Portugal, Slovenia, Spain, and Switzerland in Europe; Bolivia (Plurinational State of), Brazil, Chile, Colombia, Costa Rica, Guadeloupe, Martinique, and Nicaragua in the Americas; and Australia, French Polynesia, New Zealand, Papua New Guinea, and Samoa in Oceania.

25 For example, Burkina Faso, Cabo Verde, Democratic Republic of Congo, Equatorial Guinea, Guinea, Madagascar, Mozambique, and Senegal in Africa; Brunei Darussalam, Jordan, Lao People's Democratic Republic, State of Palestine, and Syrian Arab Republic in Asia; Bosnia and Herzegovina in Europe; Argentina, Barbados, Jamaica, Suriname, and Trinidad and Tobago in the Americas; and Solomon Islands, Tonga, and Vanuatu in Oceania.

26 For instance, Cameroon, Central African Republic, Egypt, Nigeria, and Seychelles in Africa; Armenia, Kyrgyzstan, and Uzbekistan in Asia; Bulgaria, Montenegro, and Serbia in Europe; and Belize, and Guyana in the Americas.

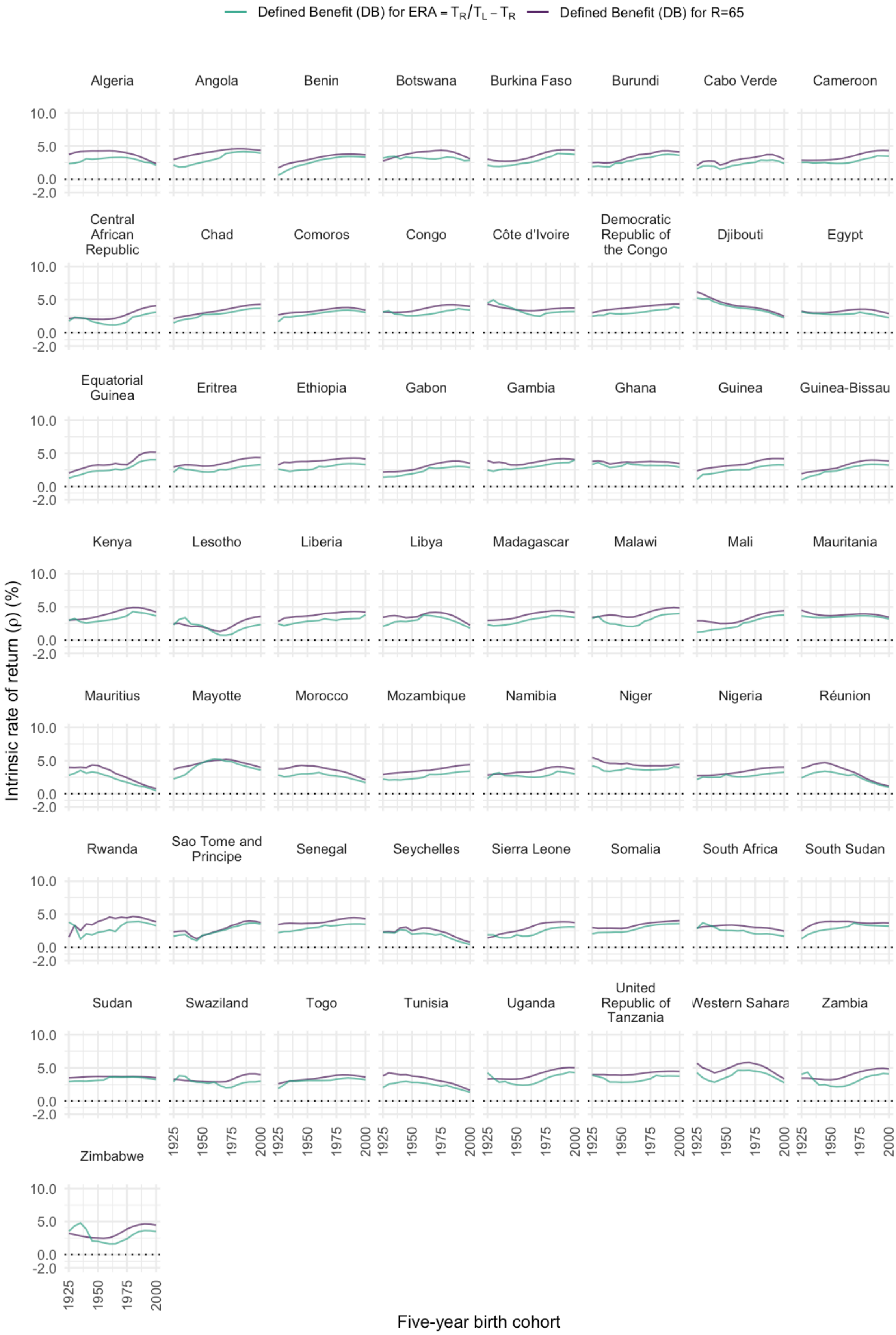
27 For example, Botswana, Rwanda, Sierra Leone, South Africa, Uganda, Zambia, and Zimbabwe in Africa; Kazakhstan in Asia; and Bulgaria, Lithuania, Russian Federation, and Ukraine in Europe.

28 Respectively, cohorts 1925–1930, 3.52% vs. 3.21%; 1930–1935, 4.33% vs. 3.02%; 1935–1940, 4.77% vs. 2.82%; and 1940–1945, 3.82% vs. 2.67%.

29 See footnote 27.

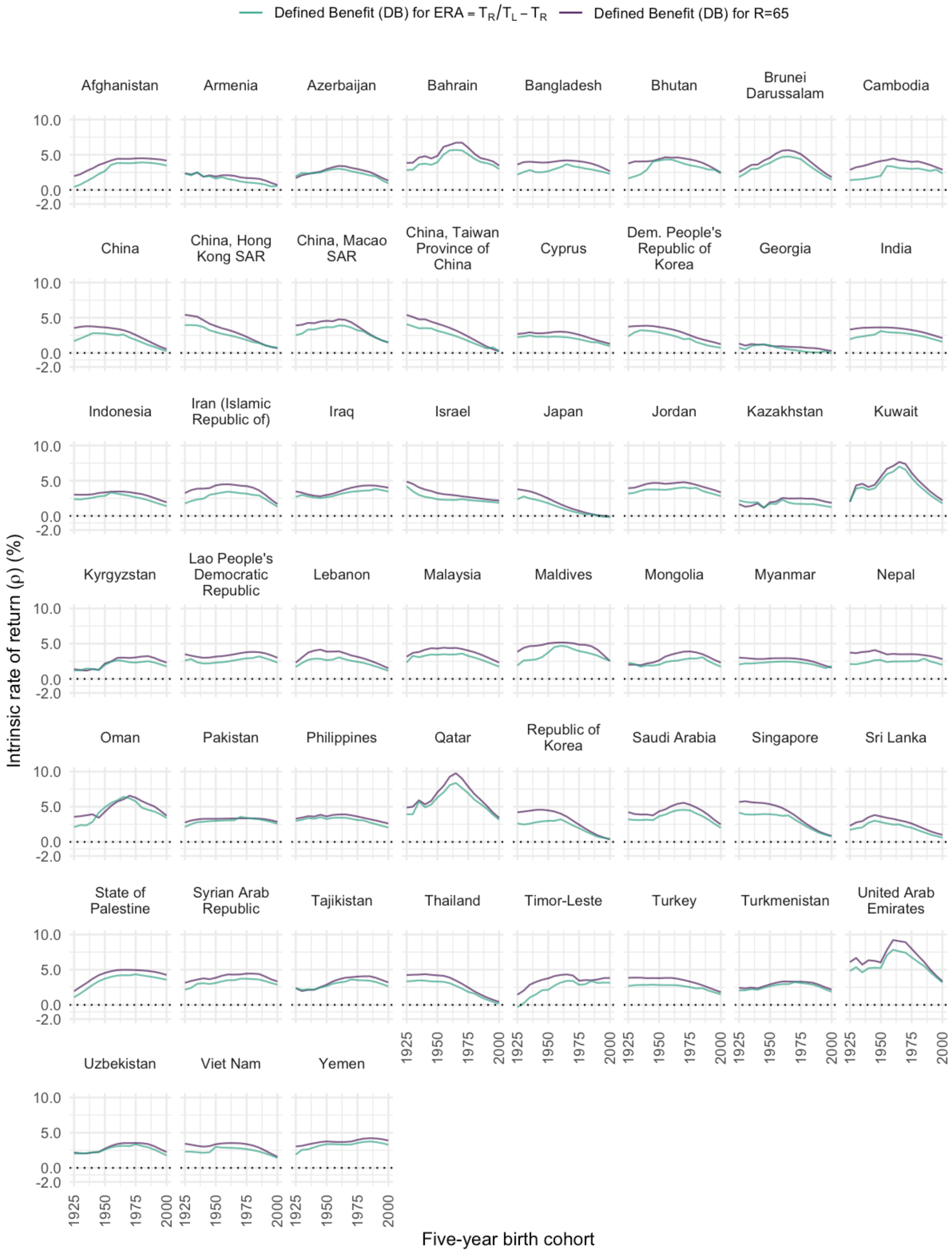
30 For instance, Maldives, Oman, Qatar, and United Arab Emirates in Asia.

Figure 85 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined benefit (DB) by cohorts and country — Africa



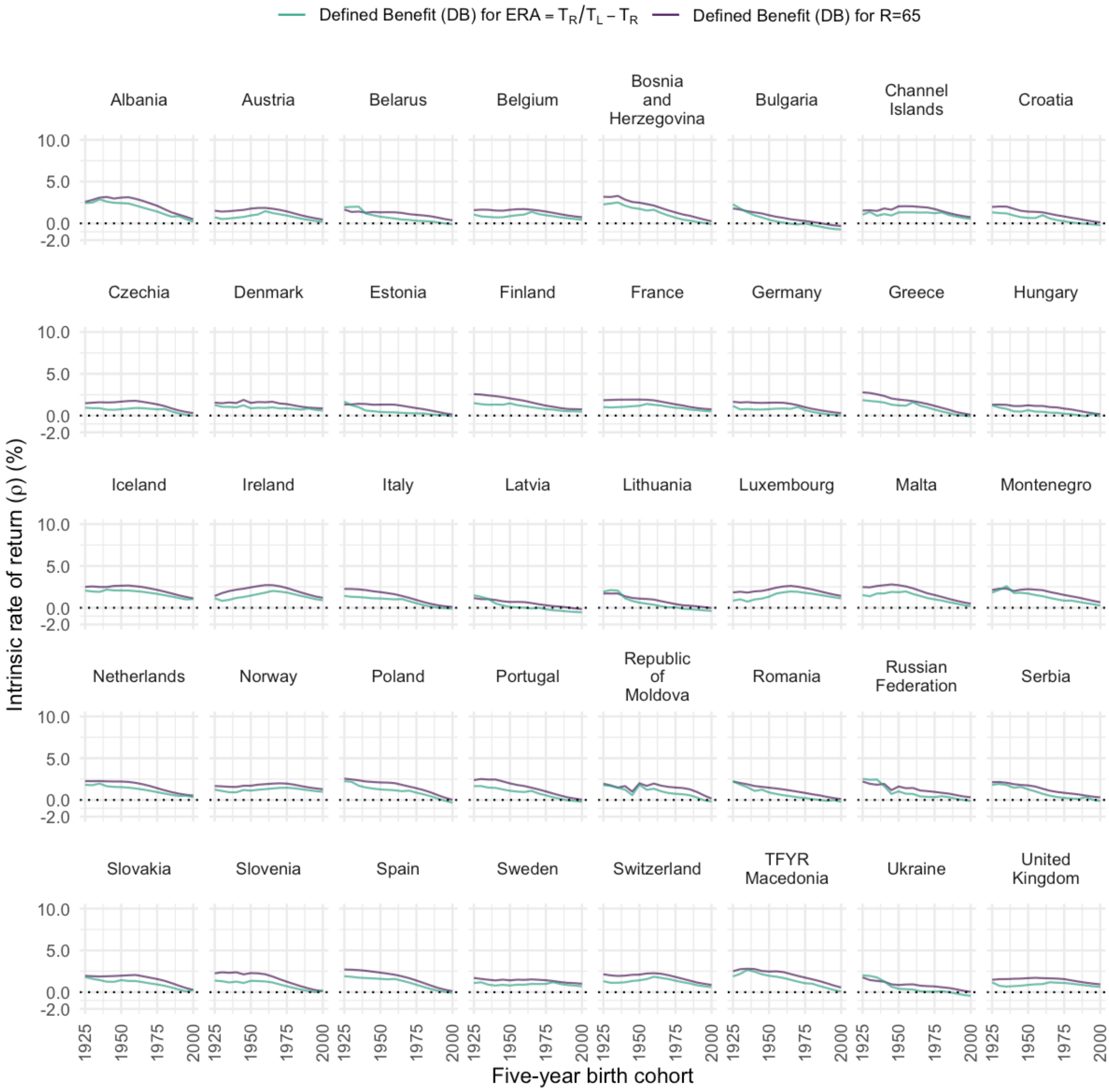
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 86 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined benefit (DB) by cohorts and country — Asia



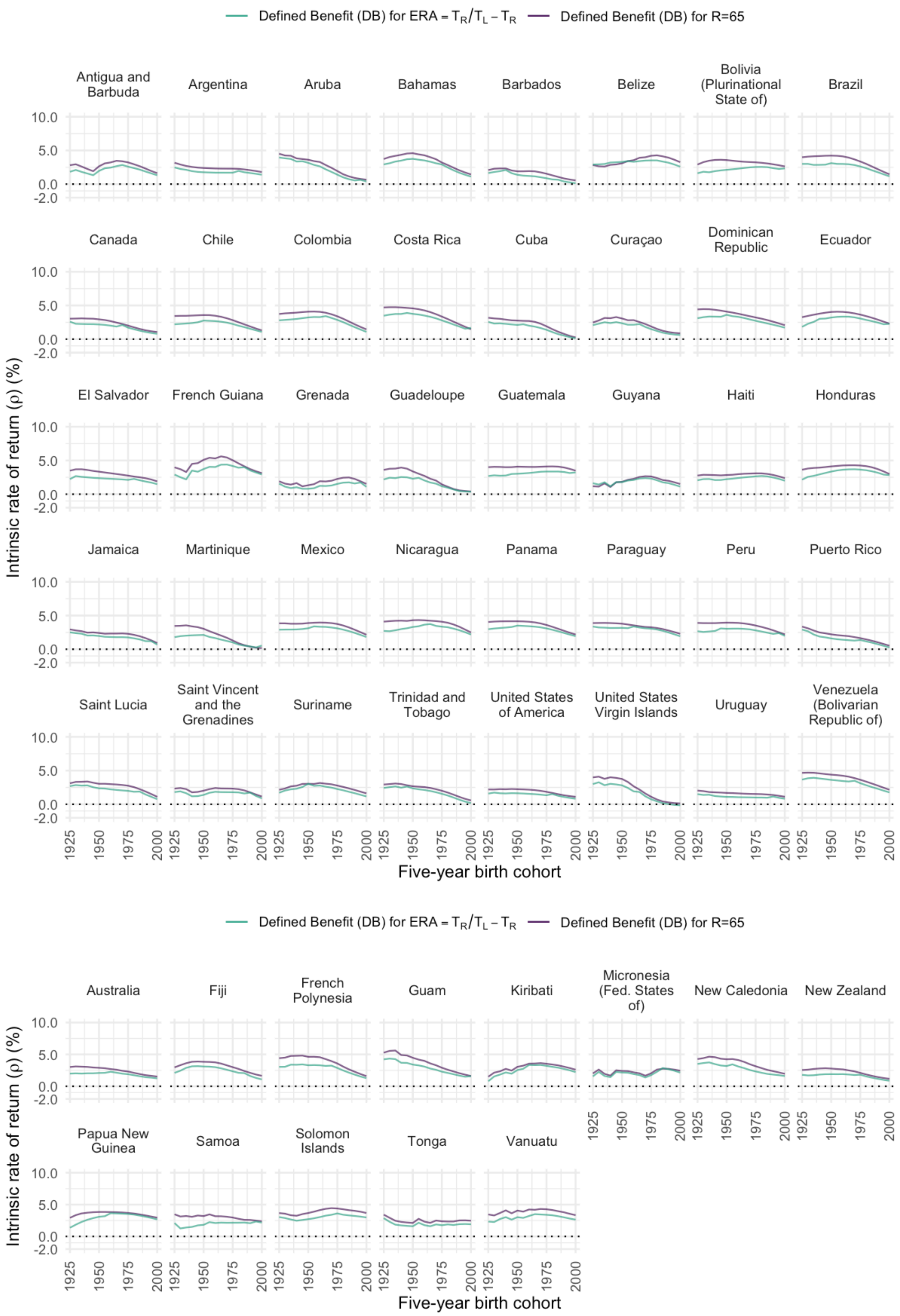
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 87 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined benefit (DB) by cohorts and country — Europe



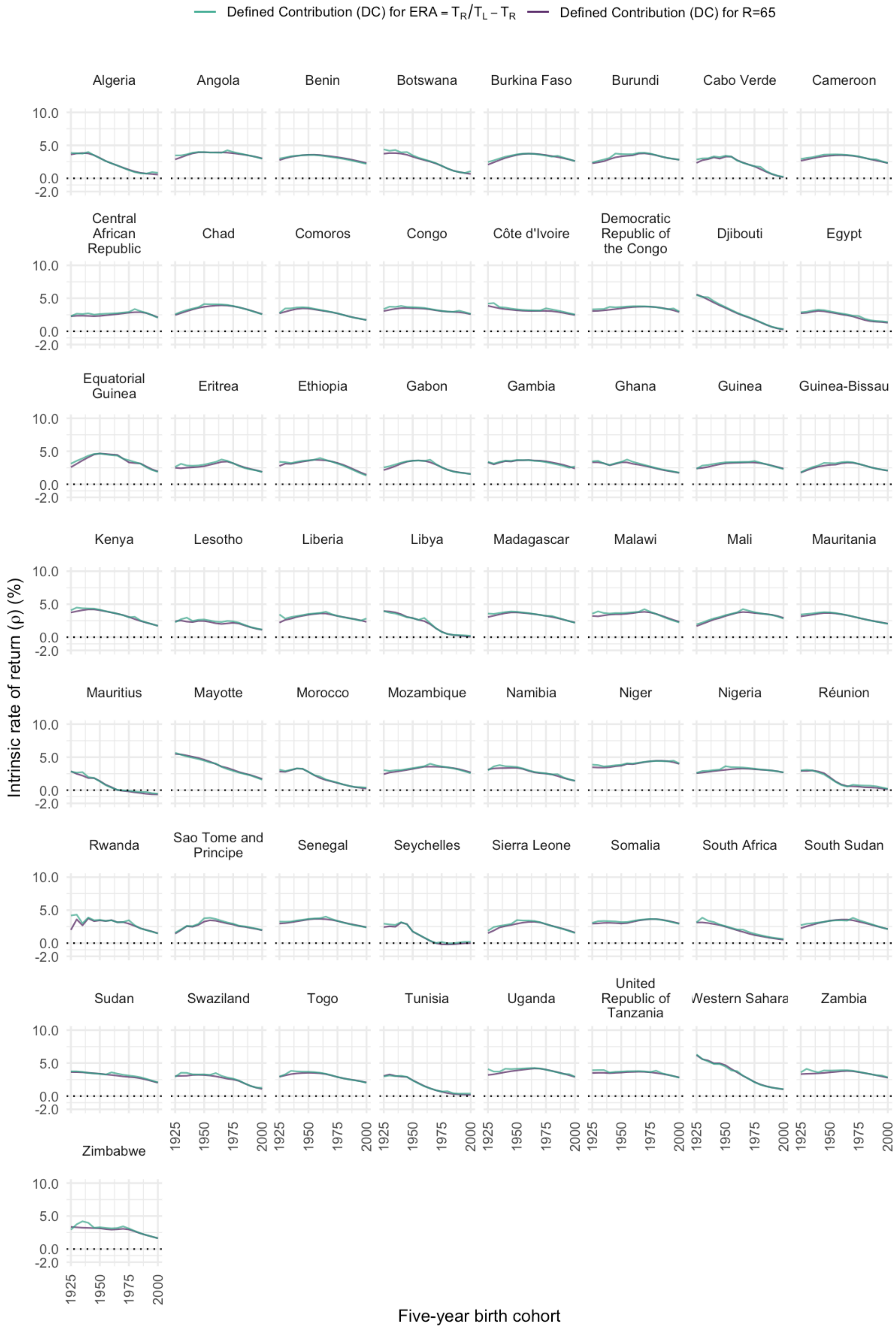
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 88 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined benefit (DB) by cohorts and country — Americas and Oceania



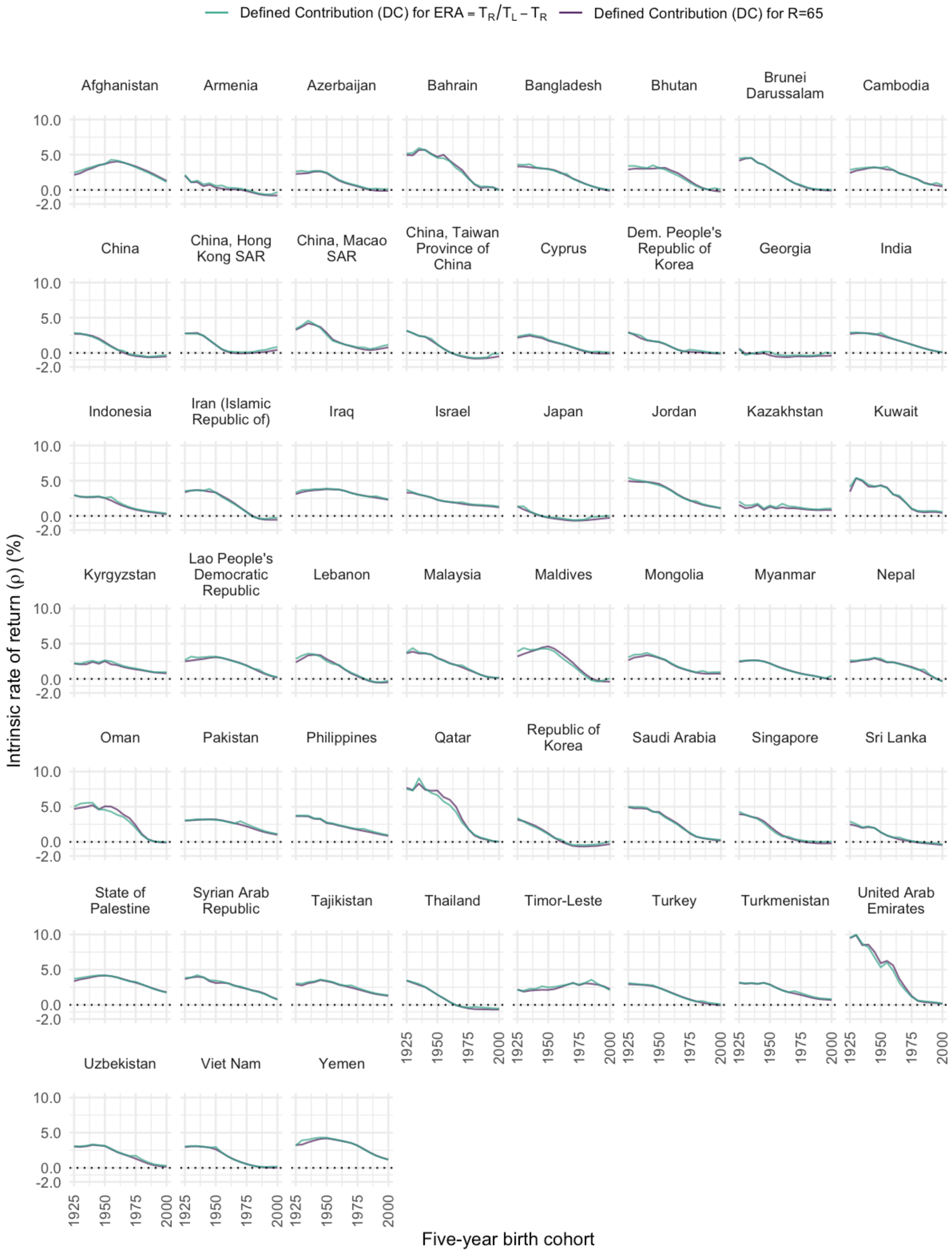
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 89 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined contribution (DC) by cohorts and country — Africa



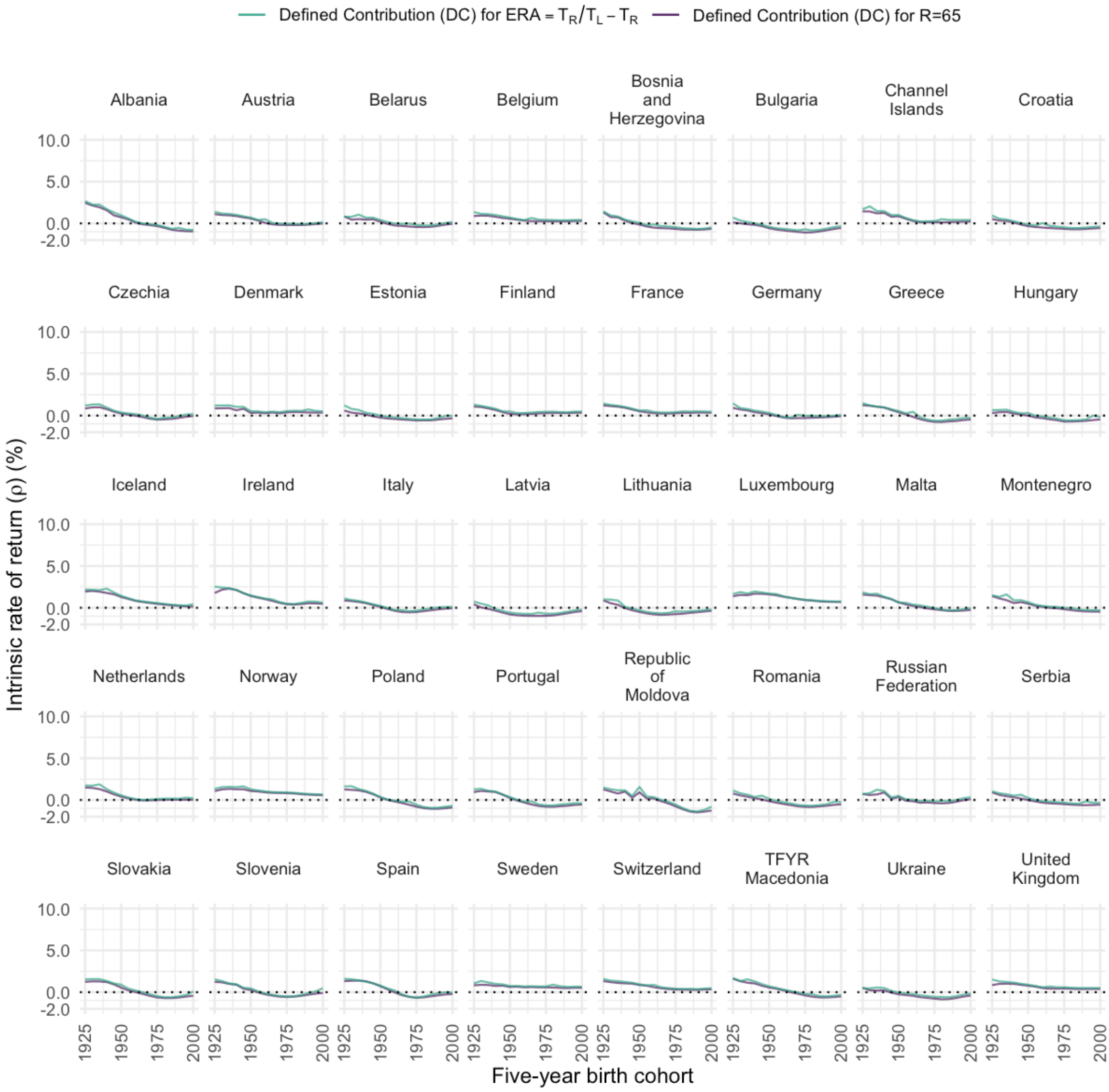
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 90 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined contribution (DC) by cohorts and country — Asia



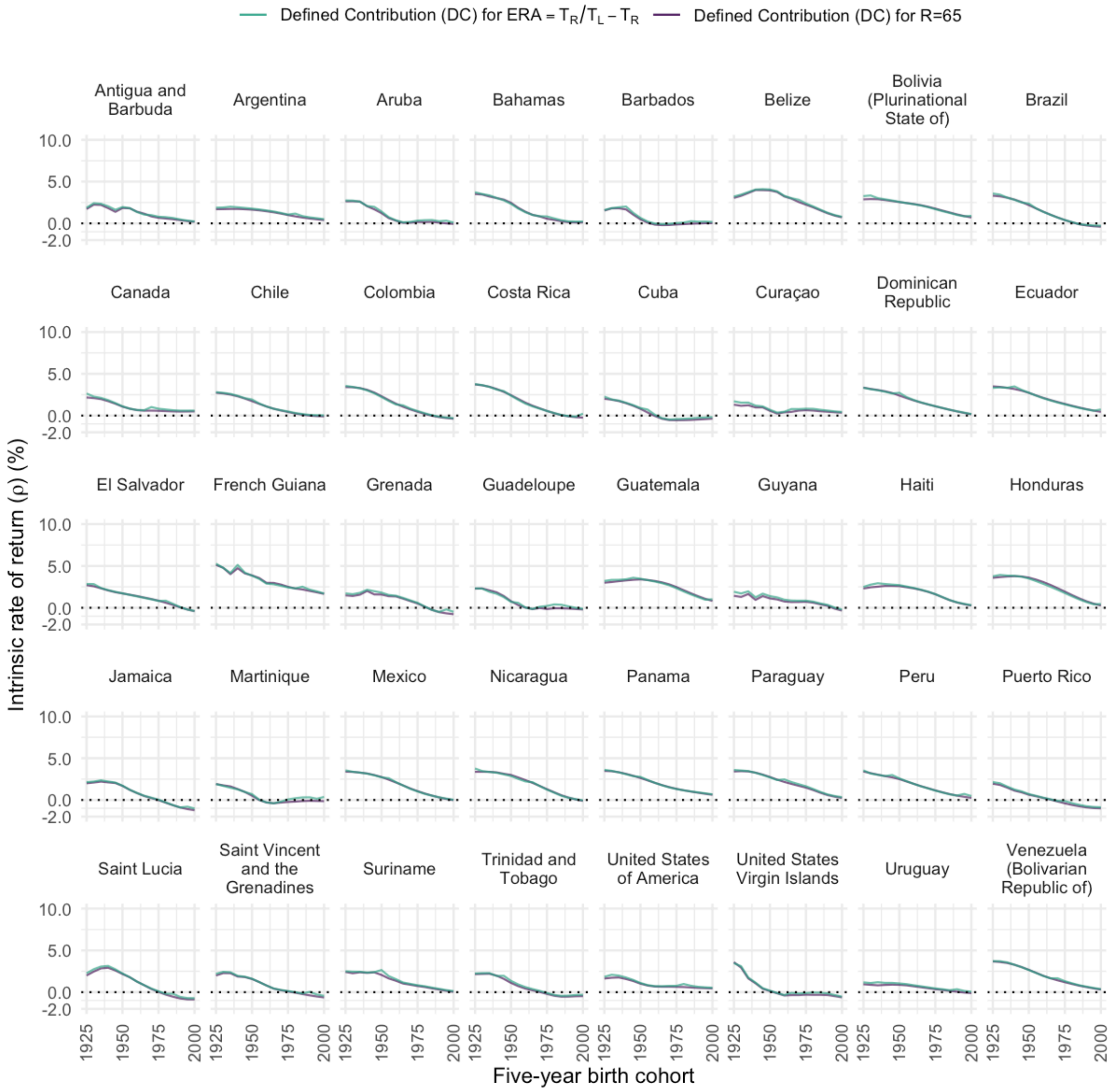
Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

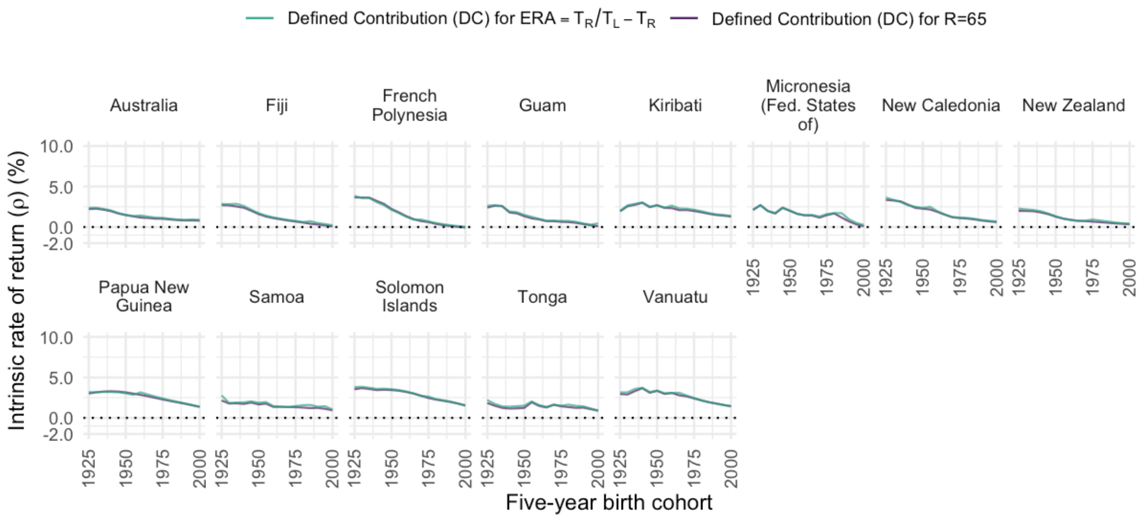
Figure 91 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined contribution (DC) by cohorts and country — Europe



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 92 – Intrinsic rate of return (IRR) of $R = 65$ and IRR of ERA of defined contribution (DC) by cohorts and country — Americas and Oceania





Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

5.6 INTRINSIC RATE OF RETURN AND THE LIFE CYCLE REJUVENATING EFFECT OF DEATHS

Last, we analyze the relationship between life cycle rejuvenating effects of deaths and intrinsic rates of return (IRR). Figure 93 plots the intrinsic rates of return (IRR) of defined benefit (DB) PAYG systems by working life cycle rejuvenating effects of deaths ($G_W^{\mathcal{D}}$). Figure 94 presents the intrinsic rates of return (IRR) of defined contribution (DC) PAYG systems by retirement life cycle rejuvenating effects of deaths ($G_R^{\mathcal{D}}$). Figure 95 plots the intrinsic rates of return (IRR) of fixed relative position (FRP) PAYG systems by working and retirement life cycle rejuvenating effects of deaths ($G_{WR}^{\mathcal{D}}$).[31] For all PAYG systems there is a consistent negative correlation between their respective intrinsic rates of return (IRR) and life cycle rejuvenating effect of deaths.[32] For defined benefit (DB) and fixed relative position (FRP) PAYG systems, this negative correlation is more pronounced when the life cycle rejuvenating effects of deaths are greater than 0.1, that is, from the middle of Stage 2 of the demographic transition when populations start to age (see section 3.6).[33] Ultimately, the higher the life cycle rejuvenating effect of deaths, the higher the period rejuvenating effects of deaths observed by the birth cohort, the higher the old age dependency ratios (OADR) observed by the birth cohort, the lower the intrinsic rate of return (IRR).

Besides, these figures suggest that for the same level of life cycle rejuvenating effects of deaths, IRR of defined benefit (DB) PAYG systems tend to be higher than IRR of defined contribution (DC) PAYG systems.[34] Consider the different spans of working and retirement life cycles, we may expect that working life cycle rejuvenating effects of deaths ($G_W^{\mathcal{D}}$) incorporate period rejuvenating effects of deaths that are distributed over a broader range of values around $G_W^{\mathcal{D}}$ than retirement life cycle rejuvenating effects of deaths ($G_R^{\mathcal{D}}$) which consolidate period rejuvenating effects of deaths that are mostly close to $G_R^{\mathcal{D}}$. Consequently, for the same level of $G_W^{\mathcal{D}}$ and $G_R^{\mathcal{D}}$, IRR of DB PAYG systems incorporate period rejuvenating effects of deaths lower than $G_W^{\mathcal{D}}$ into a large proportion of the birth cohorts' life cycles, whereas IRR of DC PAYG systems incorporate period rejuvenating effects of deaths mostly close to $G_R^{\mathcal{D}}$ into a small proportion of the birth cohorts' life cycles. Therefore, for the same level of $G_W^{\mathcal{D}}$ and $G_R^{\mathcal{D}}$, IRR of DB PAYG systems tend to be higher than IRR of DC PAYG systems. Table 13 presents examples of countries, birth cohorts and intrinsic rates of return (IRR) by PAYG system policy design and life cycle rejuvenating effect of deaths.

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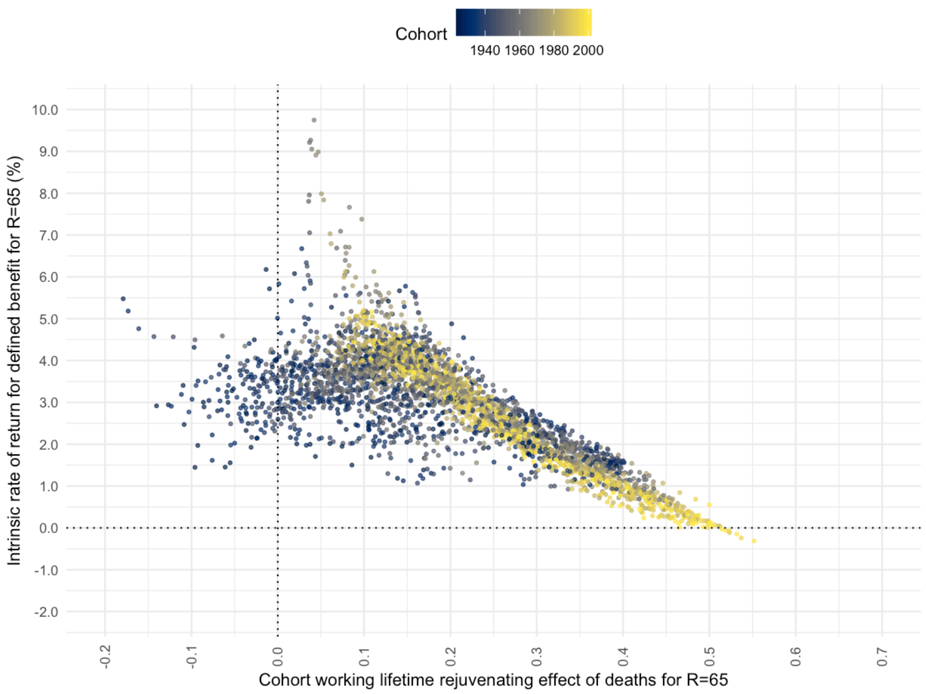
31 Figures 135, 136 and 137 in section E.3, Appendix E, detail these results by subregions.

32 IRR of DB PAYG systems and $G_W^{\mathcal{D}}$ are correlated at −0.758 (Pearson) and −0.765 (control for year and country/area); IRR of DC PAYG systems and $G_R^{\mathcal{D}}$ are correlated at −0.936 (Pearson) and −0.917 (control for year and country/area); and IRR of FRP PAYG systems and $G_{WR}^{\mathcal{D}}$ are correlated at −0.803 (Pearson, and control for year and country/area).

33 For life cycle rejuvenating effects of deaths greater than or equal to 0.1, IRR of DB PAYG systems and $G_W^{\mathcal{D}}$ are correlated at −0.892 (Pearson) and −0.891 (control for year and country/area); IRR of DC PAYG systems and $G_R^{\mathcal{D}}$ are correlated at −0.961 (Pearson) and −0.952 (control for year and country/area); and IRR of FRP PAYG systems and $G_{WR}^{\mathcal{D}}$ are correlated at −0.908 (Pearson, and control for year and country/area).

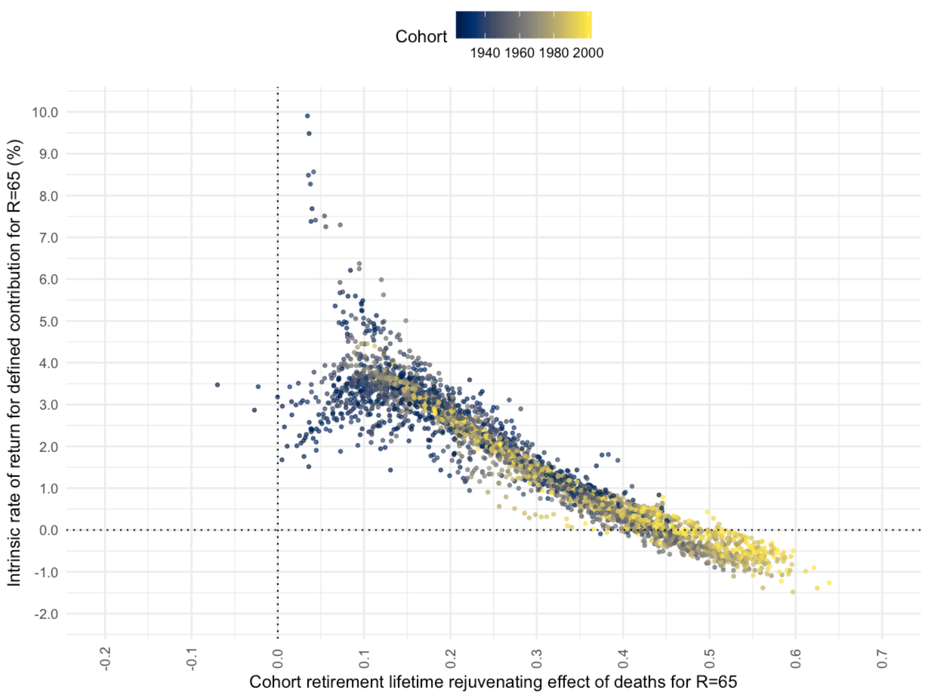
34 As an illustration, for both $G_W^{\mathcal{D}}$ and $G_R^{\mathcal{D}}$ equal to 0.2, IRR of DB PAYG systems are around 3.5%, and IRR of DC PAYG system are close to 3.0%; and for both $G_W^{\mathcal{D}}$ and $G_R^{\mathcal{D}}$ equal to 0.4, IRR of DB PAYG systems are around 1.0%, and IRR of DC PAYG system are close to 0.5%.

Figure 93 – Intrinsic rate of return (IRR) of defined benefit (DB) by cohort working life cycle rejuvenating effect of deaths ($G_W^{\mathcal{D}}$)



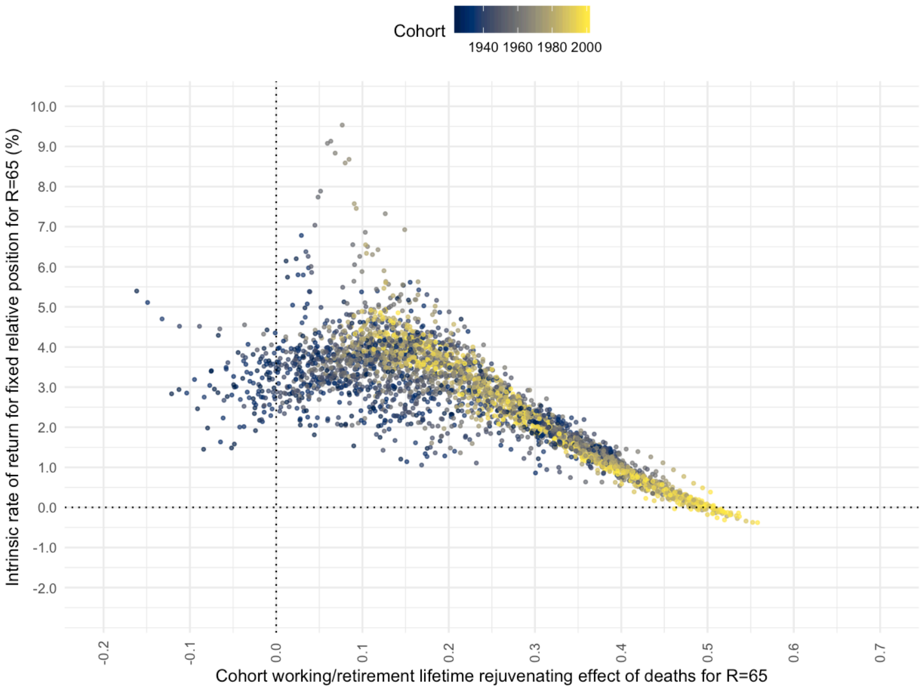
Source: Author's calculations, based on United Nations (2017c).

Figure 94 – Intrinsic rate of return (IRR) of defined contribution (DC) by cohort retirement life cycle rejuvenating effect of deaths ($G_R^{\mathcal{D}}$)



Source: Author's calculations, based on United Nations (2017c).

Figure 95 – Intrinsic rate of return (IRR) of fixed relative position (FRP) by cohort working and retirement life cycle rejuvenating effect of deaths ($G^{\mathcal{D}}_{WR}$)



Source: Author's calculations, based on United Nations (2017c).

Table 13 – Example countries, birth cohorts and intrinsic rates of return (IRR) by pay-as-you-go (PAYG) system policy design and life cycle rejuvenating effect of deaths

Life cycle rejuvenating effect of deaths (1)	Pay-as-you-go (PAYG) system		
	Defined benefit (DB)	Defined contribution (DC)	Fixed relative position (FRP)
−0.1	Côte d'Ivoire, 1925–1930, 4.32% Malawi, 1925–1930, 3.40% Burkina Faso, 1925–1930, 3.00%	···	Niger, 1940–1945, 4.52% Angola, 1930–1935, 3.19% Mali, 1930–1935, 2.85%
0.0	Western Sahara, 1930–1935, 5.02% Nepal, 1930–1935, 3.66% Togo, 1935–1940, 3.00%	Niger, 1935–1940, 3.43% Somalia, 1925–1930, 2.96% Chad, 1925–1930, 2.47%	Maldives, 1930–1935, 4.35% Timor-Leste, 1945–1950, 3.49% Madagascar, 1925–1930, 2.98%
0.1	Brunei Darussalam, 1955–1960, 5.18% Brazil, 1935–1940, 4.13% Philippines, 1930–1935, 3.45%	Jordan, 1935–1940, 4.82% French Guiana, 1935–1940, 4.02% Pakistan, 1935–1940, 3.13%	Mayotte, 1970–1975, 5.00% Guatemala, 1955–1960, 4.00% Eritrea, 1955–1960, 3.08%
0.2	Republic of Korea, 1960–1965, 4.02% Peru, 1975–1980, 3.52% Kyrgyzstan, 1975–1980, 3.05%	Singapore, 1940–1945, 3.38% Viet Nam, 1940–1945, 2.99% Chile, 1930–1935, 2.65%	New Caledonia, 1945–1950, 4.10% China, 1940–1945, 3.61% India, 1970–1975, 3.15%
0.3	Spain, 1935–1940, 2.63% Uruguay, 1925–1930, 2.04% Canada, 1980–1985, 1.76%	Armenia, 1925–1930, 2.03% Israel, 1980–1985, 1.51% Mexico, 1975–1980, 1.06%	New Zealand, 1935–1940, 2.58% Iceland, 1965–1970, 2.06% Argentina, 1985–1990, 1.85%
0.4	Sweden, 1965–1970, 1.50% United States of America, 2000–2005, 1.11% Portugal, 1980–1985, 0.77%	Ireland, 1970–1975, 0.59% Cambodia, 2000–2005, 0.50% France, 1955–1960, 0.38%	United Kingdom, 1930–1935, 1.45% Germany, 1955–1960, 1.12% Brazil, 2000–2005, 0.90%
0.5	Bulgaria, 1975–1980, 0.28% Italy, 2000–2005, 0.09% Japan, 2000–2005, −0.05%	Netherlands, 1980–1985, 0.03% Spain, 1970–1975, −0.57% Armenia, 1990–1995, −0.73%	Czechia, 2000–2005, 0.19% Lithuania, 1985–1990, −0.01% Latvia, 1985–1990, −0.15%
> 0.6	···	Poland, 2000–2005, −0.90% Poland, 1995–2000, −0.99% Republic of Moldova, 1995–2000, −1.39%	···

Source: Author's creation and calculations, based on United Nations (2017c).
(1): ± 0.02. Working life cycle rejuvenating effect of deaths ($G_W^{\mathcal{D}}$) for defined benefit (DB), retirement life cycle rejuvenating effect of deaths ($G_R^{\mathcal{D}}$) for defined contribution (DC), and working and retirement life cycle rejuvenating effect of deaths ($G_{WR}^{\mathcal{D}}$) for fixed relative position (FRP).
···: No observations.

5.7 CONCLUSION

Population aging impacts the return of pay-as-you-go (PAYG) systems to birth cohorts, because changes in period relations between beneficiaries and contributors lead to distinct life cycle contribution rates (*con*) and life cycle benefit rates (*ben*) among different birth cohorts. A measure for the welfare or return of PAYG systems to individuals or birth cohorts is the intrinsic rate of return (IRR). We show that when populations age, defined benefit (DB) PAYG systems are preferable to defined contribution (DC) PAYG systems because the former yields higher and less variable intrinsic rates of return (IRR). In addition, when populations age, increasing the ratio of per capita benefits to per capita net wages (ϕ) of fixed relative position (FRP) PAYG systems may lead to lower and faster decreases of intrinsic rates of return (IRR). We vindicate Fernandes (1993)'s claim that a higher age of entry into retirement (R) tends to increase the intrinsic rate of return (IRR) of defined benefit (DB) PAYG systems, and to decrease the intrinsic rate of return (IRR) of defined contribution (DC) PAYG systems. Yet we argue that in equivalent retirement age (ERA) PAYG systems a higher age of entry into retirement (R) tends to decrease the intrinsic rate of return (IRR) of defined benefit (DB) PAYG systems, and to increase the intrinsic rate of return (IRR) of defined contribution (DC) PAYG systems. Also, we demonstrate that for all policy designs of PAYG systems there is a consistent negative correlation between the life cycle rejuvenating effects of deaths and intrinsic rates of return (IRR).

From an intrinsic rate of return (IRR) policy guidance standpoint for PAYG systems, if the policy design is defined benefit (DB) PAYG system, policymakers should implement a higher age of entry into retirement (R) instead of an equivalent retirement age (ERA) policy. If the policy design is defined contribution (DC) PAYG system, the opposite is recommended, policymakers should implement an equivalent retirement age (ERA) policy instead of a higher age of entry into retirement (R). Last, if the policy design is fixed relative position (FRP) PAYG system, the recommendation is the same as for defined benefit (DB) PAYG systems, combined with judicious increases of the ratio of per capita benefits to per capita net wages (ϕ).

6 Conclusion

Population aging is the primary consequence of the demographic transition, it is unavoidable, pervasive, and impacts any retirement system, private or public, structured on period or cohort financial balances. In the case of pay-as-you-go (PAYG) systems, population aging affects period financial sustainability and intergenerational equality, with important economic, political, and social consequences.

In this dissertation, we first examine the contribution of births, deaths, and migrations to world population aging from 1950 to 2100 (chapter 3, Population aging). We argue that demographic transitions differ alongside a general concerted pattern between the rejuvenating effect of births and the rejuvenating effect of deaths. We propose a categorization of the stages of the demographic transition based on levels and indicators of the rejuvenating effects of births and deaths. We demonstrate that, across the globe, population aging varies alongside the same pattern and stages, and we may determine these stages by either only the rejuvenating effect of births, or only the rejuvenating effect of deaths.

Next, in chapter 4 (Period balances, age rebalances), we use a stylized demographic model to analyze the burden of population aging in PAYG systems in the world from 1950 to 2100. We argue that equivalent retirement ages (ERA) are intrinsically ineffective because PAYG systems are structured on population characteristics, while equivalent retirement ages (ERA) are based on life cycle characteristics. We propose effectiveness categories for equivalent retirement ages (ERA) that are based on the change of both the old age dependency ratio (OADR) and the equivalent old age dependency ratio (EOADR) relatively to the equivalent retirement ages (ERA)' base year. We demonstrate that if the old age dependency ratio (OADR) is aging relatively to the equivalent retirement age (ERA)'s base year, on the one hand, the lower the rejuvenating effect of deaths, the higher the probability that the equivalent retirement age (ERA) increases the age of entry into retirement (R) more than necessary; on the other hand, the higher the rejuvenating effect of deaths, the higher the likelihood that the equivalent retirement age (ERA) increases the age of entry into retirement (R) less than necessary. Also, we argue that when old-age mortality declines, equivalent retirement ages (ERA) based on gains in the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) may be less ineffective than equivalent retirement ages (ERA) based on gains in the life expectancy at age a ($\mathring{e}_a$).

Last, in chapter 5 (Period balances, generational imbalances), we apply the same stylized demographic model to investigate the effects of population aging on the returns of PAYG systems to birth cohorts in the world from 1950 to 2100, specifically, from the 1925–1930 birth cohort to the 2000–2005 birth cohort. We show that when populations age, defined benefit (DB) PAYG systems are preferable to defined contribution (DC) PAYG systems because the former yields higher and less variable intrinsic rates of return (IRR). In addition, when populations age, increasing the ratio of per capita benefits to per capita net wages (ϕ) of fixed relative position (FRP) PAYG systems may lead to lower and faster decreases of intrinsic rates of return (IRR). We vindicate Fernandes (1993)'s claim that a higher age of entry into retirement (R) tends to increase the intrinsic rate of return (IRR) of defined benefit (DB) PAYG systems, and to decrease the intrinsic rate of return (IRR) of defined contribution (DC) PAYG systems. Yet we argue that in equivalent retirement age (ERA) PAYG systems a higher age of entry into retirement (R) tends to decrease the intrinsic rate of return (IRR) of defined benefit (DB) PAYG systems, and to increase the intrinsic rate of return (IRR) of defined contribution (DC) PAYG systems. Also, we demonstrate that for all policy designs of PAYG systems there is a consistent negative correlation between the life cycle rejuvenating effects of deaths ($G_L^{\mathcal{D}}$) and intrinsic rates of return (IRR).

According to our findings, there are two main policy implications of population aging to pay-as-you-go (PAYG) systems. The first one is associated with the effectiveness of equivalent retirement ages (ERA). Policymakers should use an effectiveness framework for equivalent retirement ages (ERA) to decide if the age of entry into retirement (R) should increase, the level of the increase, and if the increase should be combined with adjusting the contribution rate (con), the benefit rate (ben), or both. Fixed relative position (FRP) PAYG systems have the best policy design to face any of these scenarios, not only because they offer greater flexibility for *ad hoc* policy changes to contribution rates (con) or benefit rates (ben), but because they dilute any risk of equivalent retirement ages (ERA) ineffectiveness between contribution rates (con) and benefit rates (ben) as well.

The second policy implication is related to intergenerational inequalities measured by the intrinsic rate of return (IRR). Policymakers should account for the policy design of PAYG systems when deciding between fixed retirement ages or equivalent retirement ages. Specifically: a) in defined benefit (DB) PAYG system, they should implement a higher age of entry into retirement (R) instead of an equivalent retirement age (ERA) policy; b) in defined contribution (DC) PAYG system, the opposite is recommended, policymakers should implement an equivalent retirement age (ERA) policy instead of a higher age of entry into retirement (R); and c) in fixed relative position (FRP) PAYG system, the recommendation is the same as for defined benefit (DB) PAYG systems, combined with judicious increases of the ratio of per capita benefits to per capita net wages (ϕ).

One should be aware of the limitations of this dissertation that may alter the consequences of population aging to PAYG systems. First, since we adopt a global view, we loose in local specificity such as institutional arrangements. Second, because we focus on the total population, we do not investigate intragenerational inequalities as done elsewhere (FERNANDES, 1993). Third, since we use a stylized demographic model, we do not consider age-specific profiles of employment, retirement, wages (w), contribution rates (con), and benefit rates (ben). Fourth, because our stylized demographic model is purely demographic, we do not consider the interactions between demographic and economic variables such as the effects of population aging on human capital, and thus on labor market and productivity. Last, since we focus on retirement systems only, we do not incorporate other types of transfers — public (e.g., education, health care) or private (e.g., familial support, bequests) — that may affect the return of social security to birth cohorts.

Finally, our results and limitations leave us with research directions for additional work. First, on population aging, we hope to extend the investigation of the demographic transition and the general concerted pattern between the rejuvenating effects of births and deaths to both periods prior to 1950 and country subdivisions. Second, on age rebalances, we plan to expand the analysis of the effectiveness of equivalent retirement ages (ERA) based on gains in the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) to other scenarios, such as private and local (e.g., state, municipal) retirement systems. Third, on generational imbalances, we expect to extend the analysis to intragenerational inequalities, at least separately for men and women. Last, future analysis should expand our stylized demographic model to an economic-demographic model with age-specific profiles of employment, retirement, wages (w), contribution rates (con) and benefit rates (ben), and interactions between demographic and economic variables.

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APPENDIXES

Appendix A – Supplement to chapter 2

A.1 SUPPLEMENT TO SECTION 2.1

Table 14 – Period indicators for the medium fertility projection variant of the 2017 UN REVISION, for five-year periods, from 1950–1955 to 2095–2100

Indicator	Unit of measure
Total fertility rate	live births per woman
Net reproduction rate	surviving daughters per woman
Crude birth rate	births per 1,000 population
Number of births, both sexes combined	thousands
Life expectancy at birth for both sexes combined	years
Male life expectancy at birth	years
Female life expectancy at birth	years
Infant mortality rate for both sexes combined ($q_{0,1}$)	infant deaths per 1,000 live births
Under-five mortality for both sexes combined ($q_{0,5}$)	deaths under age five per 1,000 live births
Crude death rate	deaths per 1,000 population
Number of deaths, both sexes combined	thousands
Number of male deaths	thousands
Number of female deaths	thousands
Net migration rate	per 1,000 population
Net number of migrants, both sexes combined	thousands
Average annual rate of population change	percentage
Rate of natural increase	per 1,000 population
Sex ratio at birth	male births per female births
Female mean age of childbearing	years

Source: United Nations (2017c).
Note: `https://esa.un.org/unpd/wpp/DVD/Files/1_Indicators(Standard)/CSV_FILES/WPP2017_period_indicators_medium.csv`.

Appendix B – Model old-age mortality

B.1 INTRODUCTION

We model old-age mortality to obtain life tables with the same open-ended age groups as populations and deaths (see subsection 2.1.5). We also equalize the first age group between life tables (0 and 1–4 years originally) and populations and deaths (0–4 years). First, we model old-age mortality to estimate age-specific death rates for the 85–89 to 105–109 age groups. Second, we use these age-specific death rates to calculate life table functions for the 85–89, 90–94 and 95+ age groups. Last, we calculate life table functions for the 80+, and 0–4 age groups.

B.2 MODEL OLD-AGE MORTALITY

Mathematical mortality models, or *laws of mortality*, are mathematical formulas in which the force of mortality at age a ($\mu(a)$), the continuous form of the age-specific death rate between ages a and $a+n$ ($m_{a,n}$), depends on age. Most mathematical mortality models are either descriptive or explanatory: descriptive models only define the formulas and do not explain why they are so; explanatory models justify the biological mechanism behind the formulas. Altogether, we should choose explanatory models over descriptive ones (THATCHER; KANNISTO; VAUPEL, 1998). We use explanatory mathematical mortality models to estimate $m_{a,n}$ for the 85–89 to 105–109 age groups from 1950–1955 to 2095–2100. First, we discuss explanatory mathematical mortality models and the parameter of the overall level of mortality. Second, we estimate additional life table $m_{a,n}$ for the 85–89 and 90–94 age groups from 1990–1995 to 2095–2100. Last, we apply explanatory mathematical mortality models and estimate life table $m_{a,n}$ for the 85–89 to 105–109 age groups.

B.2.1 Explanatory mathematical models for old-age mortality

We follow Thatcher, Kannisto, and Vaupel (1998) who worked with the following explanatory mathematical mortality models:[1]

Gompertz's law (GOMPERTZ, 1825),

$$\mu(a) = \alpha\, e^{\beta a} \tag{B.1}$$

Makeham's law (MAKEHAM, 1860),

$$\mu(a) = \gamma + \alpha\, e^{\beta a} \tag{B.2}$$

the logistic model, initially proposed by Perks (1932),

$$\mu(a) = \gamma + \frac{\alpha\, e^{\beta a}}{1 + k\,\alpha\, e^{\beta a}} \tag{B.3}$$

[1] Thatcher, Kannisto, and Vaupel (1998) also worked with two descriptive models: Heligman and Pollard model (HELIGMAN; POLLARD, 1980), and the quadratic model (COALE; KISKER, 1990).

the Kannisto model (Kannisto, 1992 as cited in THATCHER; KANNISTO; VAUPEL, 1998, p. 16),

$$\mu(a) = \gamma + \frac{\alpha\, e^{\beta a}}{1 + \alpha\, e^{\beta a}} \tag{B.4}$$

and the Weibull model (WEIBULL, 1951),

$$\mu(a) = \alpha\, a^{\beta}. \tag{B.5}$$

Although these models have distinct mathematical formulations, their parameters are comparable: α indicates the overall level of mortality; β is the tempo of age-related mortality increase;[2] γ is the mortality from causes that do not depend on age, known as the Makeham term; and $1/k$ is the upper limit of the mortality curve in the logistic model. Gompertz's law and the Weibull model assume that there is only an age-dependent component to mortality; Makeham's law incorporates to Gompertz's an age-independent component; the logistic model adds to Makeham's an upper limit to mortality (HORIUCHI; OUELLETTE, et al., 2013; THATCHER; KANNISTO; VAUPEL, 1998; WILMOTH, 1995); and the Kannisto model assumes that this upper limit to mortality is equal to 1 (THATCHER; KANNISTO; VAUPEL, 1998; THATCHER, 1999). That is, Gompertz's law for $\gamma = 0$ and $k = 0$, Makeham's law for $k = 0$, and the Kannisto model for $k = 1$ are special cases of the logistic model.[3] If the mathematical mortality model has a Makeham term, we assume that mortality is the sum of premature mortality, constant over ages and given by γ, and senescent mortality, result of age-related deterioration of physiological functions and represented by the remaining parts of the mathematical formulations (HORIUCHI; WILMOTH, 1998, p. 400; HORIUCHI; OUELLETTE, et al., 2013, p. 52–54). Appropriately, we can have a Makeham variant for the Weibull model (HORIUCHI; OUELLETTE, et al., 2013, p. 53–54):

$$\mu(a) = \gamma + \alpha\, a^{\beta} \tag{B.6}$$

Thatcher, Kannisto, and Vaupel (1998) worked with data from low mortality countries in the period 1960–1990,[4] and argued that in modern data the Makeham term is minimal, and thus at old ages the difference between Gompertz's and Makeham's laws is inconsequential. For example, for Swedish women aged 55–95 in the period 1973–1977, Horiuchi, Ouellette, et al. (2013, p. 54) estimated that only two percent of the force of mortality at the modal age at death was due to premature mortality. Therefore, Thatcher, Kannisto, and Vaupel (1998) selected Gompertz's law

[2] Three different mortality aging patterns: exponential for Gompertz's and Makeham's laws; logistic for the logistic and Kannisto models; and power-function for the Weibull model (HORIUCHI; OUELLETTE, et al., 2013; MISSOV et al., 2015; THATCHER; KANNISTO; VAUPEL, 1998).

[3] We adopt a slightly different parameter for the upper limit of the mortality curve in the logistic model than Thatcher, Kannisto, and Vaupel (1998)'s $\mu(x) = c + \frac{a\, e^{bx}}{1 + \alpha\, e^{bx}}$, and Horiuchi, Ouellette, et al. (2013)'s $\mu(x) = c + \frac{a\, e^{bx}}{1 + (a/g)\, e^{bx}}$, because with k it is more direct to model and compare Gompertz's law, Makeham's law, the Kannisto model, and the logistic model.

[4] Austria, Denmark, England and Wales, Finland, France, West Germany, Iceland, Italy, Japan, the Netherlands, Norway, Sweden and Switzerland.

and, despite abandoning Makeham's law, worked with the logistic and Kannisto models with Makeham terms.

In fact, Beard (1959, 1971), and later and independently Vaupel, Manton, and Stallard (1979), showed that the logistic model may result from Makeham's law acting individually in a heterogeneous population. In such population, each *ith* individual is subject to a force of mortality equal to $\mu_i(a) = \gamma + \alpha_i e^{\beta a}$, and the average value of $\mu_i(a)$ conforms to the logistic model given by Equation B.3. In this model, known as fixed frailty model or Gamma Makeham model, α_i, frailty, is age-independent, varies among individuals following a gamma distribution at birth, and results from the joint effects of genetic, environmental, and lifestyle characteristics; while β, age-related exponential mortality increase, and γ, the Makeham term, are the same for all individuals (HORIUCHI; WILMOTH, 1998; THATCHER; KANNISTO; VAUPEL, 1998; THATCHER, 1999). Therefore, we can justify the exact same population mortality pattern starting from contrasting theoretical interpretations of the mortality process (YASHIN; VAUPEL; IACHINE, 1994). In our case, from different combinations of the population mortality composition and the individual force of mortality. But is it important if there is heterogeneity in mortality? Yes, if the unobserved heterogeneity is significant and alters the result of the research (VAUPEL; YASHIN, 1985). For us, it is not important if the population mortality patterns are different from the individual mortality patterns that are underneath. Specifically, it is not important if the final population mortality pattern is the result of: a) population mortality homogeneity with individual forces of mortality equal to Makeham's law; b) population mortality heterogeneity with individual forces of mortality equal to Makeham's law; or c) population mortality homogeneity with individual forces of mortality equal to the logistic model with a Makeham term (see Table 15). Yet it is important that we have in either circumstance Makeham's law or the logistic model among our models to properly represent the final form of the population force of mortality. We assume that in many contexts of the 2017 revision of the official United Nations population estimates and projections (2017 UN REVISION) premature mortality may be non-negligible and an important component of total old-age mortality (WILMOTH, 1995), and that in the contexts it is negligible it will reflect in a lower proportional value of the model's Makeham term. For these reasons, we choose the following four models: Makeham, and Makeham variants of logistic, Kannisto and Weibull.

Table 15 – Population mortality law from combination of population mortality composition and individual force of mortality

Population mortality composition	Individual force of mortality	Population mortality law
Homogeneous	Makeham	Makeham
Homogeneous	Logistic	Logistic
Heterogeneous	Makeham	Logistic

Source: Author's creation, based on Beard (1959, 1971), Vaupel, Manton, and Stallard (1979), and Yashin, Vaupel, and Iachine (1994).

B.2.2 *Parameters of the mathematical models and the modal age at death*

We follow Horiuchi, Ouellette, et al. (2013) to define the parameters of our Makeham variant mathematical mortality models. They reformulated Gompertz's law, the logistic and Weibull models, and its Makeham variants using the old-age modal age at death (M) as the parameter for the overall level of mortality. The modal age at death (M) (i.e., the location of the old-age death heap) is an indicator of lifespan together with life expectancy (i.e., the mean age at death or the mean length of life), and the median age at death (i.e., the probable length of life reached by half individuals) (Acsádi and Nemeskéri, 1970 as cited in HORIUCHI; OUELLETTE, et al., 2013, p. 40). The modal age at death (M) has been suggested as alternative indicator in the study of longevity (HORIUCHI; OUELLETTE, et al., 2013; KANNISTO, 2001). While life expectancy is highly dependent on young-age mortality, M is only determined by old-age mortality (KANNISTO, 2001), granted that mortality risk follows a bathtub curve (HORIUCHI; OUELLETTE, et al., 2013, Appendix A). The modal age at death (M) is also free from bias caused by arbitrary selections of age limits for old-age (KANNISTO, 2001). Besides, M is useful from a health policy perspective, when we consider that the demand for health infrastructures and services are accentuated at ages around M (MISSOV et al., 2015).

Expressing mathematical mortality models in terms of M instead of the overall level of mortality (α) has advantages: α is a hypothetical value, the force of mortality at a 'reference age' extrapolated from the age pattern of old-age mortality (HORIUCHI; OUELLETTE, et al., 2013), while M is more descriptive, comprehensible and comparable (HORIUCHI; OUELLETTE, et al., 2013; MISSOV et al., 2015); in circumstances where the tempo of age-related mortality increase (β) is far different between two populations, α may be paradoxically lower in the population with higher adult mortality, whereas lower values of M mostly reflect higher adult mortality (HORIUCHI; OUELLETTE, et al., 2013); still, the correlation between parameter estimators for α and β in Gompertz's law is very high, while it is much lower if we reformulate it in terms of M and β (MISSOV et al., 2015, p. 1035–1040).[5] In all our Makeham variants mathematical mortality models, the modal age at death is from senescent mortality (M_s), which is practically equal to while somewhat higher than the modal age at death (M), assuming that at old ages the proportional level of premature mortality given by the Makeham term (γ) is very low. For instance, for Swedish women aged 55–95 in the period 1973–1977, M was 84.3, M_s was 84.6 and premature mortality represented 2% of $\mu(M)$ (HORIUCHI; OUELLETTE, et al., 2013, p. 54). The parameters of our mathematical mortality models are:

[5] For typical human mortality, the correlation can be reduced from absolute values above 0.95, for α and β, to below 0.40, for M and β.

Gompertz-Makeham,

$$\mu(a) = \gamma + \beta\, e^{\beta(a-M_s)} \tag{B.7}$$

Logistic-Makeham,[6]

$$\mu(a) = \gamma + \frac{\beta\, e^{\beta(a-M_s)}}{1 + k\,\beta\, e^{\beta(a-M_s)}} \tag{B.8}$$

Kannisto-Makeham,[7]

$$\mu(a) = \gamma + \frac{\beta\, e^{\beta(a-M_s)}}{1 + \beta\, e^{\beta(a-M_s)}} \tag{B.9}$$

and Weibull-Makeham,

$$\mu(a) = \gamma + \frac{\beta}{M_s}\left(\frac{a}{M_s}\right)^{\beta}. \tag{B.10}$$

B.2.3 Additional age-specific death rates for the 85–89 and 90–94 age groups

We estimate additional life table age-specific death rates between ages a and $a + n$ ($m_{a,n}$) for the 85–89 and 90–94 age groups from 1990–1995 to 2095–2100. Initially, we assume that life table $m_{a,n}$ are equal to observed age-specific death rates between ages a and $a + n$ ($M_{a,n}$) (PRESTON; HEUVELINE; GUILLOT, 2001, p. 42–44, 48, 61–63):

$$m_{a,n}(t, t+h) \simeq M_{a,n}(t, t+h) \tag{B.11}$$

We calculate $M_{a,n}$ from the number of observed deaths between ages a and $a + n$ ($D_{a,n}$) and the period observed number of person-years lived between ages a and $a + n$ ($PY_{a,n}$):

$$M_{a,n}(t, t+h) = \frac{D_{a,n}(t, t+h)}{PY_{a,n}(t, t+h)} \tag{B.12}$$

We estimate period $PY_{a,n}$ for 5-year periods $t, t + h$ from annual populations from age a to age $a + n$ ($N_{a,n}$):

$$PY_{a,n}(t, t+h) = \sum_{x=t}^{t+h-1} \frac{N_{a,n}(x+1) - N_{a,n}(x)}{\ln \dfrac{N_{a,n}(x+1)}{N_{a,n}(x)}} \tag{B.13}$$

In Equation B.13, we implicitly assume that the instantaneous growth rate is constant in each five 1-year periods within our 5-year periods, which imposes compatibility between the crude growth rate and the mean annualized growth rate (PRESTON; HEUVELINE; GUILLOT, 2001, p. 8–17).[8]

6 As in the traditional equation, we adopt a slightly different parameter for the upper limit of the mortality curve in the logistic model than Horiuchi, Ouellette, et al. (2013)'s $\mu(x) = c + \dfrac{b\, e^{b(x-M_s)}}{1 + (b/g)\, e^{b(x-M_s)}}$.

7 Horiuchi, Ouellette, et al. (2013) did not work with or reformulate the Kannisto model in terms of M or M_s. Nevertheless, we derive it as a special case of the logistic model ($k = 1$).

8 When $N_{a,n}(x) = N_{a,n}(x+1)$, we presume that the population growth is linear, and use $\dfrac{N_{a,n}(x+1) + N_{a,n}(x)}{2}$ instead of $\dfrac{N_{a,n}(x+1) - N_{a,n}(x)}{\ln(N_{a,n}(x+1)/N_{a,n}(x))}$ in Equation B.13.

To guarantee consistency between our estimates and the 2017 UN REVISION $m_{85,+}$, we calculate a correction factor between $m_{85,+}$ and $M_{85,+}$ (ζ):

$$\zeta(t, t+h) = \frac{m_{85,+}(t, t+h)}{M_{85,+}(t, t+h)} \tag{B.14}$$

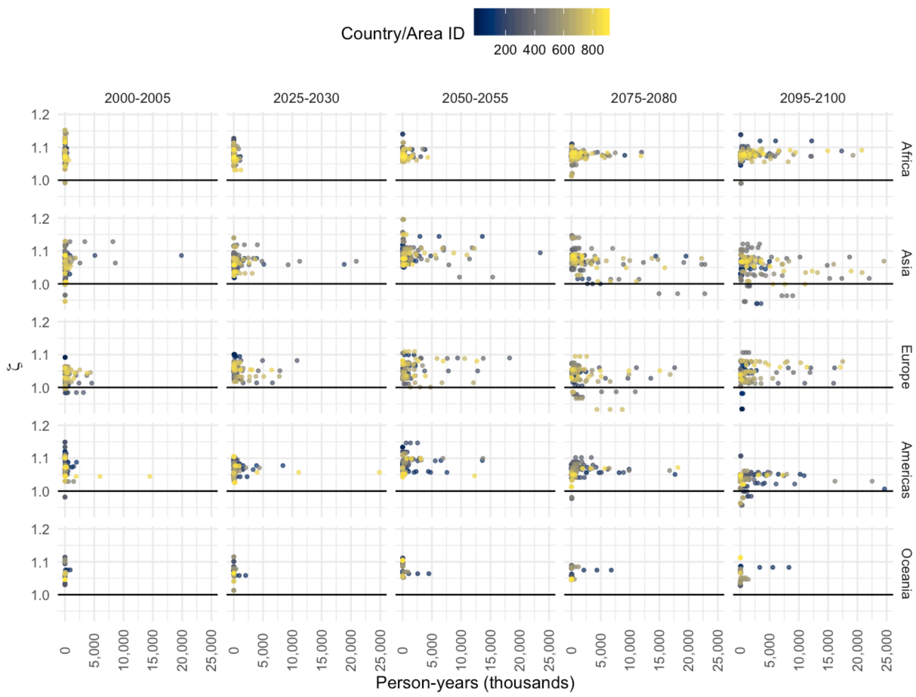
Finally, we apply ζ to $M_{85,5}$ and $M_{90,5}$:

$$m_{85,5}(t, t+h) = M_{85,5}(t, t+h) \cdot \zeta(t, t+h) \tag{B.15}$$

$$m_{90,5}(t, t+h) = M_{90,5}(t, t+h) \cdot \zeta(t, t+h) \tag{B.16}$$

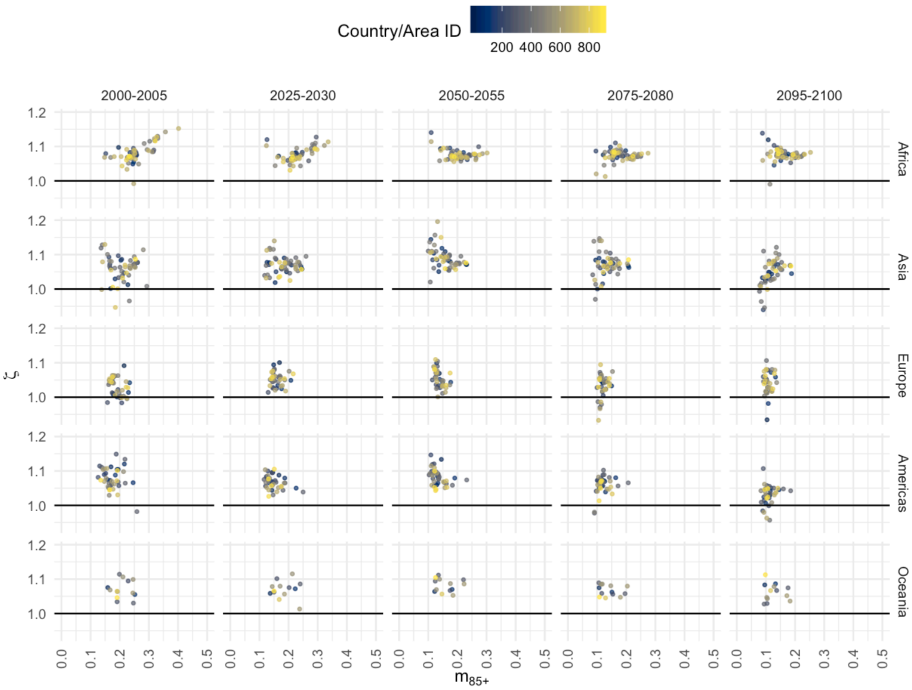
In Figure 96, we plot the period observed number of person-years lived between ages a and $a+n$ ($PY_{a,n}$) by the correction factor between $m_{85,+}$ and $M_{85,+}$ (ζ) for the 85–89, 90–94 and 95+ age groups, and in Figure 97 we present $m_{85,+}$ by ζ, for selected periods and all regions. We observe that ζ presents no bias from the size of the number of person-years lived or the life table 85+ age group mortality levels.

Figure 96 – Observed number of person-years lived between ages a and $a+n$ ($PY_{a,n}$) by correction factor between $m_{85,+}$ and $M_{85,+}$ (ζ), for the 85–89, 90–94 and 95+ age groups



Source: Author's calculations, based on United Nations (2017c).
Note: From a total of 13,266 observations, we exclude 137 that have extreme values of $PY_{a,n}$ over 25 million.

Figure 97 – Age-specific death rates between ages a and $a+n$ ($m_{a,n}$) for age group 85+ ($m_{85,+}$) by correction factor between $m_{85,+}$ and $M_{85,+}$ (ζ) and selected periods



Source: Author's calculations, based on United Nations (2017c).

B.2.4 *Model age-specific death rates for the 85–89 to 105–109 age groups*

We apply the Gompertz-Makeham (Equation B.7), Logistic-Makeham (Equation B.8), Kannisto-Makeham (Equation B.9), and Weibull-Makeham (Equation B.10) mathematical mortality models to estimate life table $m_{a,n}$ for the 85–89 to 105–109 age groups from 1950–1955 to 2095–2100. We fit these models to the six oldest available five-year age groups of $m_{a,n}$: the 55–59 to 80–84 age groups from 1950–1955 to 1985–1990, and the 65–69 to 90–94 age groups from 1990–1995 to 2095–2100. For the 55–59 to 80–84 age groups, we use the 2017 UN REVISION life tables $m_{a,n}$. For the 85–89 and 90–94 age groups, we use the additional estimates of $m_{a,n}$ from subsection B.2.3.

We employ the R language and environment (`Rstats`) (R CORE TEAM, 2018) with the MortalityLaws `R` package (`MortalityLaws`) (PASCARIU; CANUDAS-ROMO, 2017; PASCARIU, 2018), and use the `MortalityLaws` feature that let us define our own parametrized mortality functions. The estimation procedure assumes that deaths at age a ($D(a)$) have a Poisson distribution with parameters $E(a)\mu(a)$, where $E(a)$ is exposure to risk (i.e., person-years lived) (BRILLINGER, 1986; MISSOV et al., 2015). Appropriately, we adopt the loss function method of `MortalityLaws` that minimizes the Poisson log-likelihood function ($\ln L$) (PASCARIU, 2018):

$$\ln L = \sum_{a} -[D(a)\ln \mu(a) - E(a)\mu(a)] \tag{B.17}$$

Now, we turn to analyze the goodness of fit of our models, and then we go through the criteria to choose the final best models for each geographic area and period. Let $m_{a,n}^{\text{LAW}}(t, t+h)$ denote the life table age-specific death rates fitted by any of the models (LAW) for any 5-year period $t, t+h$. We calculate the following differences for each five-year age group and model

$$m_{a,n}^{\Delta\text{LAW}}(t, t+h) = m_{a,n}(t, t+h) - m_{a,n}^{\text{LAW}}(t, t+h), \tag{B.18}$$

and their absolute relative differences:

$$m_{a,n}^{|\Theta\text{LAW}|}(t, t+h) = \left| \frac{m_{a,n}^{\Delta\text{LAW}}(t, t+h)}{m_{a,n}(t, t+h)} \right| \tag{B.19}$$

We calculate the maximum absolute relative differences over age groups:

$$m_{\vee a}^{|\Theta\text{LAW}|}(t, t+h) = \max_{a} m_{a,n}^{|\Theta\text{LAW}|}(t, t+h) \tag{B.20}$$

Let ν be the total number of age groups used to fit $m_{a,n}$ to the mortality models. We calculate the arithmetic average absolute relative differences over age groups:[9]

$$m_{\overline{a}}^{|\Theta\text{LAW}|}(t, t+h) = \frac{m_{\cdot}^{|\Theta\text{LAW}|}(t, t+h)}{\nu(t, t+h)} = \frac{\sum_a m_{a,n}^{|\Theta\text{LAW}|}(t, t+h)}{\nu(t, t+h)} \tag{B.21}$$

Figure 98 shows death rates absolute relative differences ($m_{a,n}^{|\Theta\text{LAW}|}$) by age groups, for selected periods and all regions. Figure 99 and Figure 100 respectively present death rates maximum absolute relative differences ($m_{\vee a}^{|\Theta\text{LAW}|}$) and death rates average absolute relative differences ($m_{\overline{a}}^{|\Theta\text{LAW}|}$) by the minimized Poisson log-likelihood function ($\ln L$). Figure 101 plots death rates maximum absolute relative differences ($m_{\vee a}^{|\Theta\text{LAW}|}$) by death rates average absolute relative differences ($m_{\overline{a}}^{|\Theta\text{LAW}|}$). We observe that the first age group[10] has higher absolute relative differences, and that a low value of the minimized Poisson log-likelihood function ($\ln L$) does not guarantee either low maximum absolute relative differences or low average absolute relative differences. But maximum absolute relative differences and average absolute relative differences are highly correlated, that is, a low maximum absolute relative difference generally leads to a low average absolute relative difference.[11] Therefore, we choose as the final best model for each geographic area and 5-year period $t, t+h$, the one that has the minimum arithmetic average absolute relative differences calculated over the oldest five age groups that were used to fit the models in that period:[12]

$$m_{\overline{a}}^{\wedge|\Theta\text{LAW}|}(t, t+h) = \min_{\text{LAW}} m_{\overline{a}}^{|\Theta\text{LAW}|}(t, t+h) \tag{B.22}$$

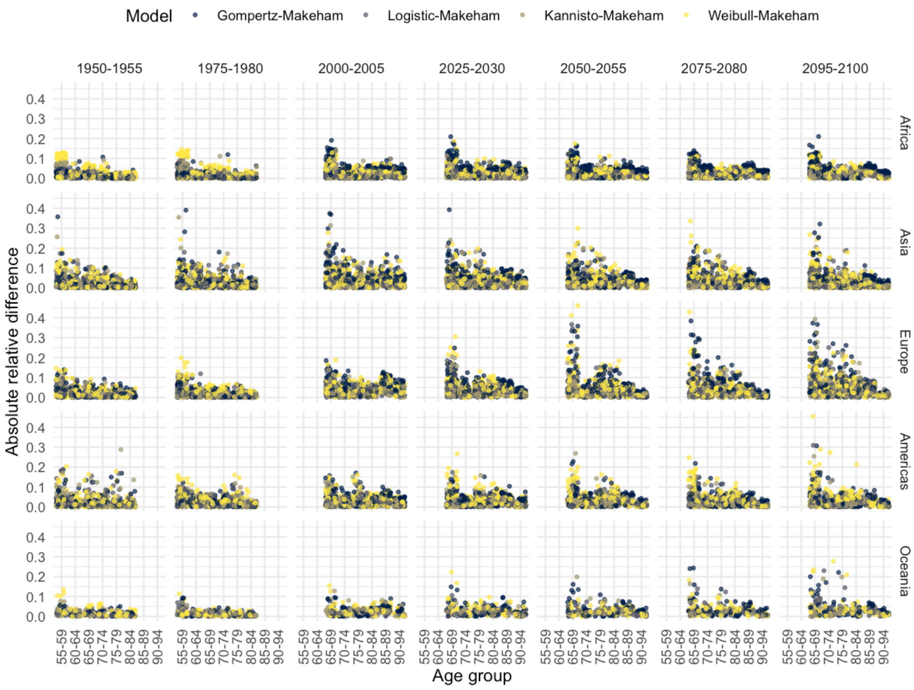
9 $\nu(t, t+h) = 6$, for the six oldest available five-year age groups of $m_{a,n}$.

10 55–59 from 1950–1955 to 1985–1990 and 65–69 from 1990–1995 to 2095–2100.

11 Death rates maximum absolute relative differences and death rates average absolute relative differences are correlated at 0.938 (Pearson), and partially correlated at 0.938 (control for country/area), 0.936 (control for year), and 0.936 (control for year and country/area).

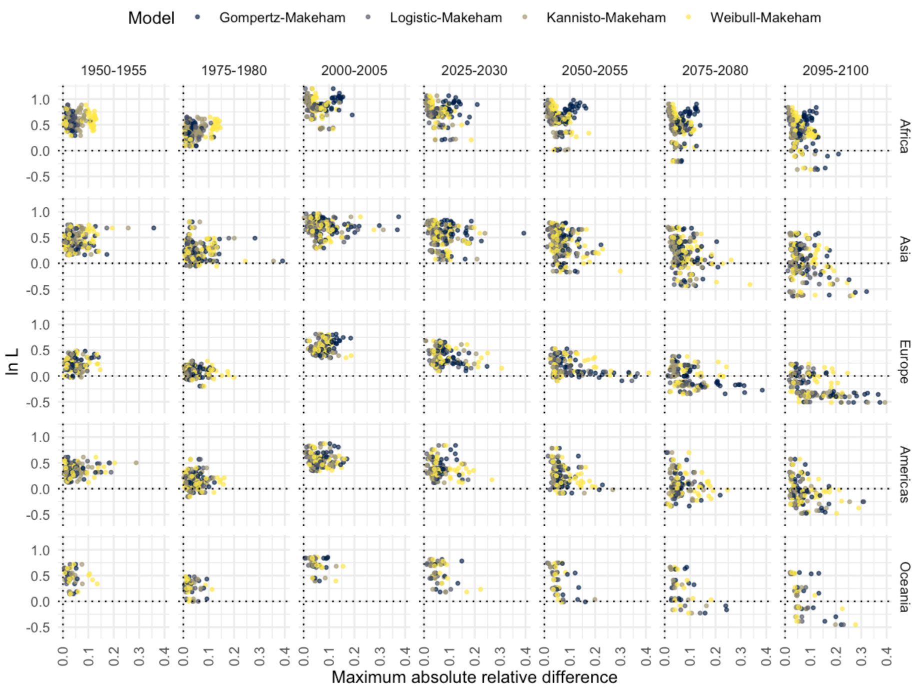
12 60–64 to 80–84 from 1950–1955 to 1985–1990, and 70–74 to 90–94 from 1990–1995 to 2095–2100.

Figure 98 – Death rates absolute relative differences ($m_{a,n}^{|\Theta_{\text{LAW}}|}$) by age groups and selected periods



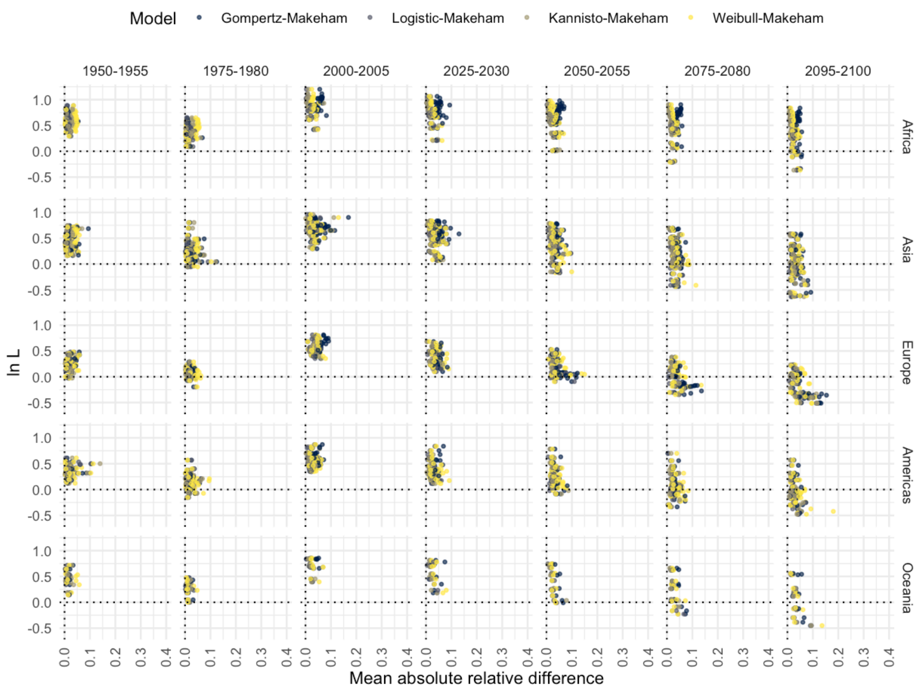
Source: Author's calculations, based on United Nations (2017c).

Figure 99 – Death rates maximum absolute relative differences ($m_{\forall a}^{|\Theta_{\text{LAW}}|}$) by minimized Poisson log-likelihood function ($\ln L$) and selected periods



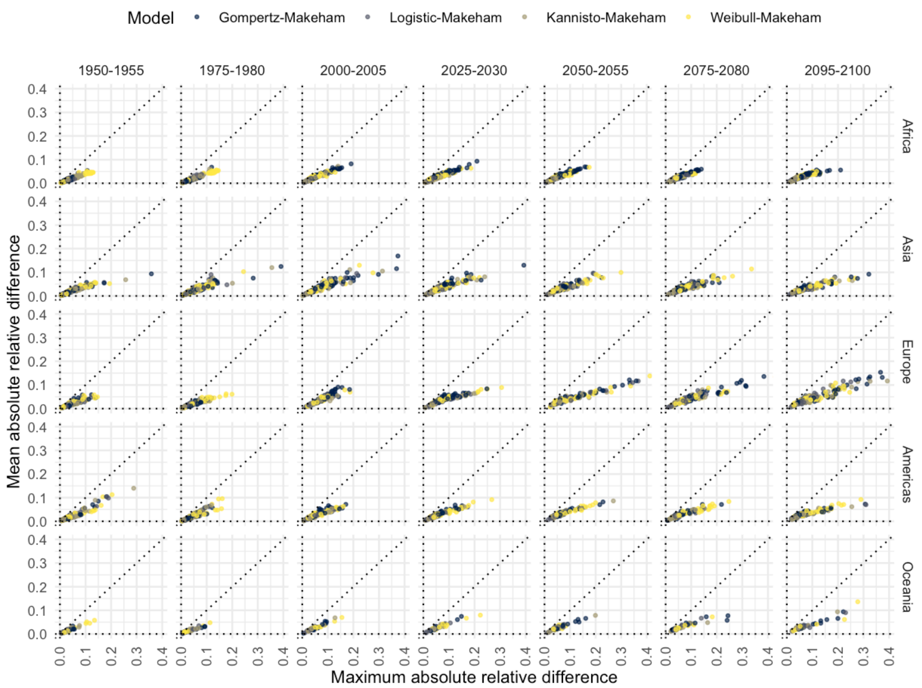
Source: Author's calculations, based on United Nations (2017c).

Figure 100 – Death rates average absolute relative differences ($m_{\bar{a}}^{|\Theta_{LAW}|}$) by minimized Poisson log-likelihood function ($\ln L$) and selected periods



Source: Author's calculations, based on United Nations (2017c).

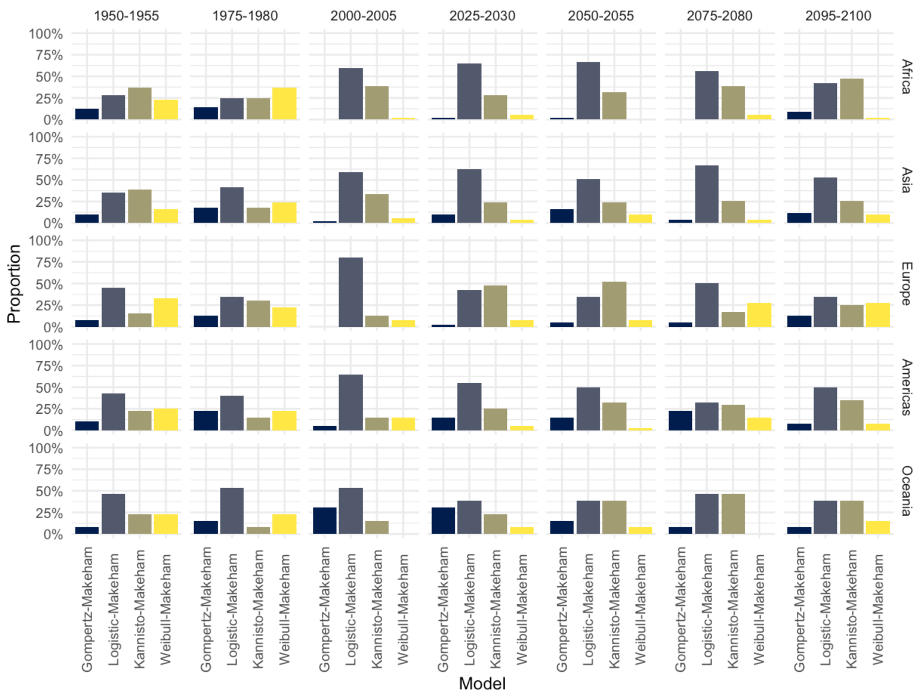
Figure 101 – Death rates maximum absolute relative differences ($m_{\forall a}^{|\Theta_{LAW}|}$) by death rates average absolute relative differences ($m_{\bar{a}}^{|\Theta_{LAW}|}$) and selected periods



Source: Author's calculations, based on United Nations (2017c).

For the final mathematical mortality models (FINAL) and selected periods and all regions, Figure 102 presents the proportions of each model, Figure 103 shows death rates absolute relative differences ($m_{a,n}^{|\Theta_{\text{FINAL}}|}$) by age groups, Figure 104 presents death rates average absolute relative differences ($m_{\overline{a}}^{|\Theta_{\text{FINAL}}|}$) by the minimized $\ln L$, and Figure 105 plots death rates maximum absolute relative differences ($m_{\forall a}^{|\Theta_{\text{FINAL}}|}$) by death rates average absolute relative differences ($m_{\overline{a}}^{|\Theta_{\text{FINAL}}|}$). The majority of our final models are either Logistic-Makeham or Kannisto-Makeham, and the Logistic-Makeham is the final model for almost all regions and periods. Thatcher, Kannisto, and Vaupel (1998) observed that since the Kannisto model is a special case of the logistic model, for the same set of data we may expect the logistic to have a better fit, and the Kannisto to result in better extrapolations to older ages. They concluded that, all things considered, the logistic and the Kannisto are the best-fitting models for ages from 80 to 120.

Figure 102 – Proportions of each final mathematical mortality models for selected periods

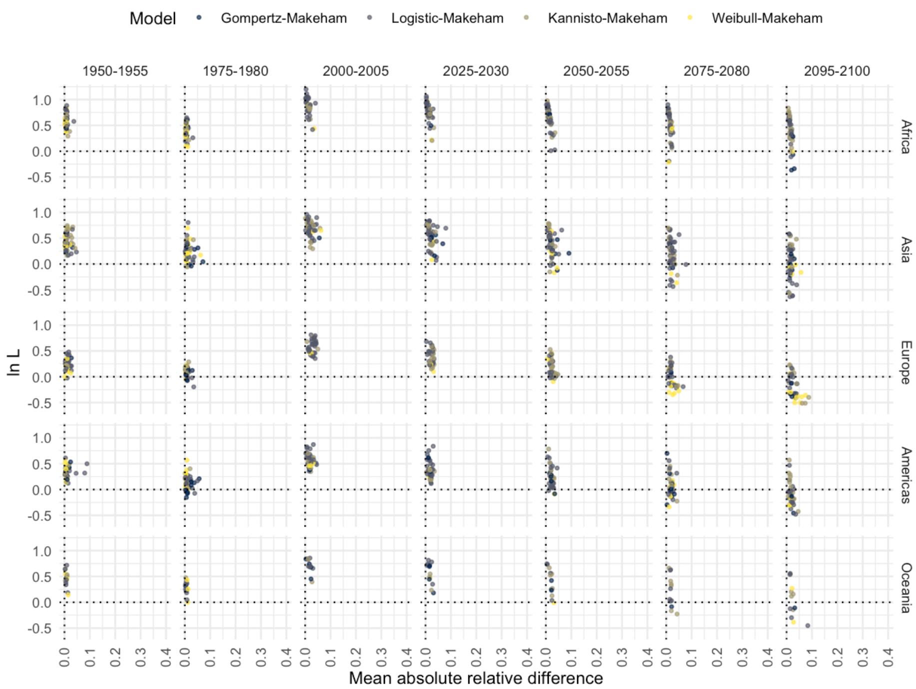


Source: Author's calculations, based on United Nations (2017c).

Figure 103 – Death rates absolute relative differences ($m_{a,n}^{|\Theta \text{FINAL}|}$) by age groups for the final mathematical mortality models

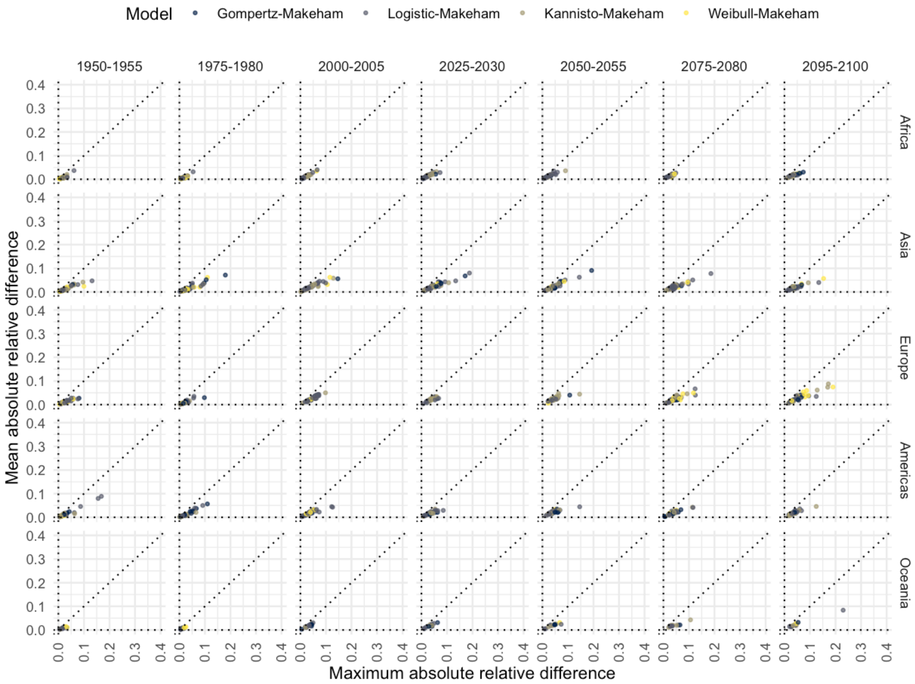
Source: Author's calculations, based on United Nations (2017c).

Figure 104 – Death rates average absolute relative differences for the final mathematical mortality models ($m_{\overline{a}}^{|\Theta \text{FINAL}|}$) by minimized Poisson log-likelihood function (ln L)



Source: Author's calculations, based on United Nations (2017c).

Figure 105 – Death rates maximum absolute relative differences ($m_{\forall a}^{|\Theta\text{FINAL}|}$) by death rates average absolute relative differences ($m_{\bar{a}}^{|\Theta\text{FINAL}|}$) for the final mathematical mortality models

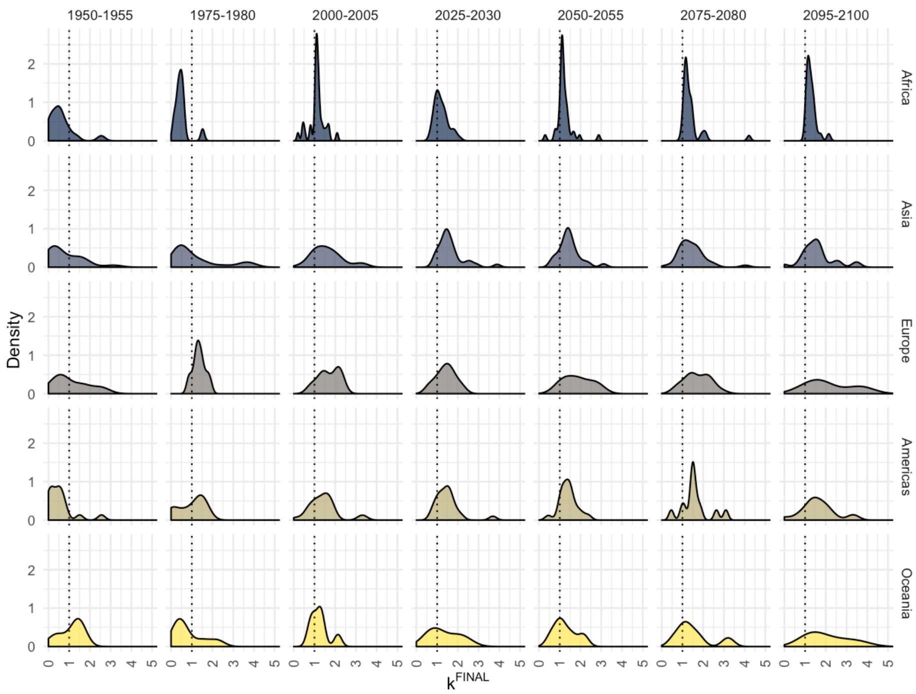


Source: Author's calculations, based on United Nations (2017c).

We initially evaluate if our final Logistic-Makeham models are close to Gompertz-Makeham ($k = 0$) or to Kannisto-Makeham ($k = 1$) in Figure 106 that presents the density of k for selected periods and all regions with a vertical reference line at $k = 1$. Similarly, in Figure 107 we plot the density of modal age at death from senescent mortality (M_s) for selected periods and all regions with a vertical reference line at age 80. The values of M_s by β for the final mathematical mortality models (except Weibull-Makeham models) are in Figure 108, and detailed by subregion in Figure 110. These values are plotted apart for the Weibull-Makeham model in Figure 109 that has a different scale for β. Our results show that generally as M_s increases β also increases, which is along with the results presented by Missov et al. (2015, p. 1042, Figure 4) for all Human Mortality Database (HMD) countries.[13] That is, old-age mortality declines not because senescence (deterioration with age) slows, but because old-age mortality levels are postponed (VAUPEL, 2010). Our results in Figure 111 and Figure 112 further show that β increases over the years and varies across regions. These results would reject the hypothesis of Vaupel (2010, p. 538–539, 541) (also cited in Missov et al. (2015, p. 1040)) that populations would differ in their levels of health (M_s), but not in their rates of increase in old-age mortality (β), which would be constant over time.

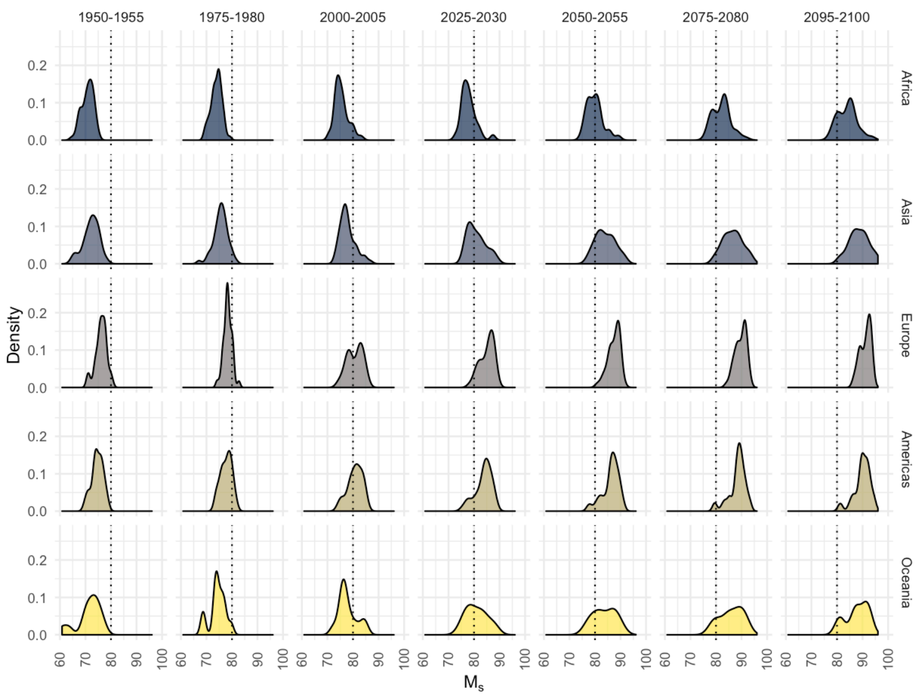
[13] Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at `www.mortality.org` or `www.humanmortality.de`.

Figure 106 – Density of k^{FINAL} for the final Logistic-Makeham models and selected periods



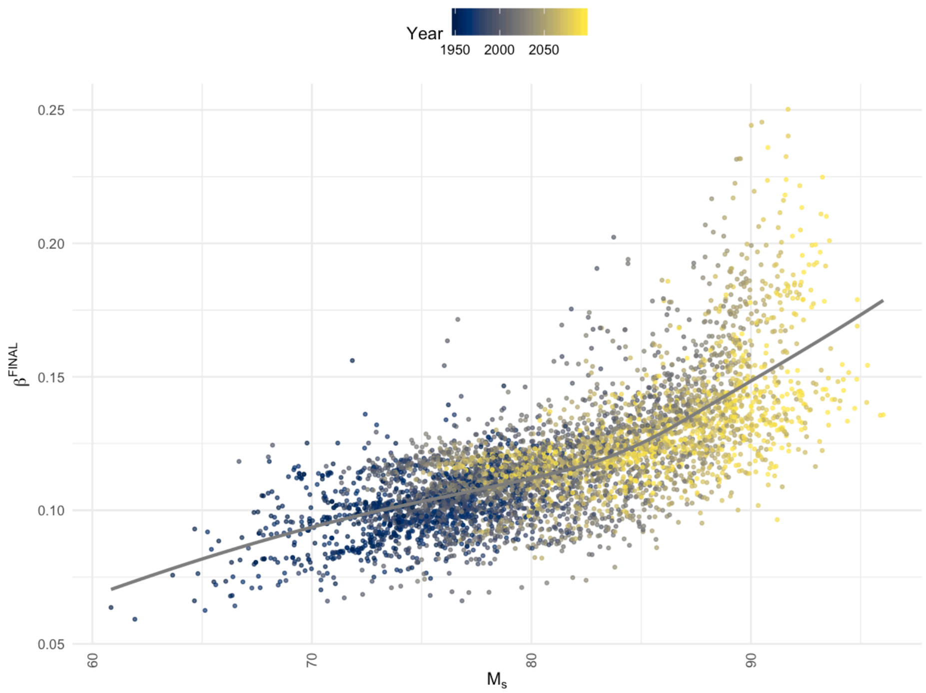
Source: Author's calculations, based on United Nations (2017c).

Figure 107 – Density of the modal age at death from senescent mortality (M_s^{FINAL}) for the final mathematical mortality models and selected periods



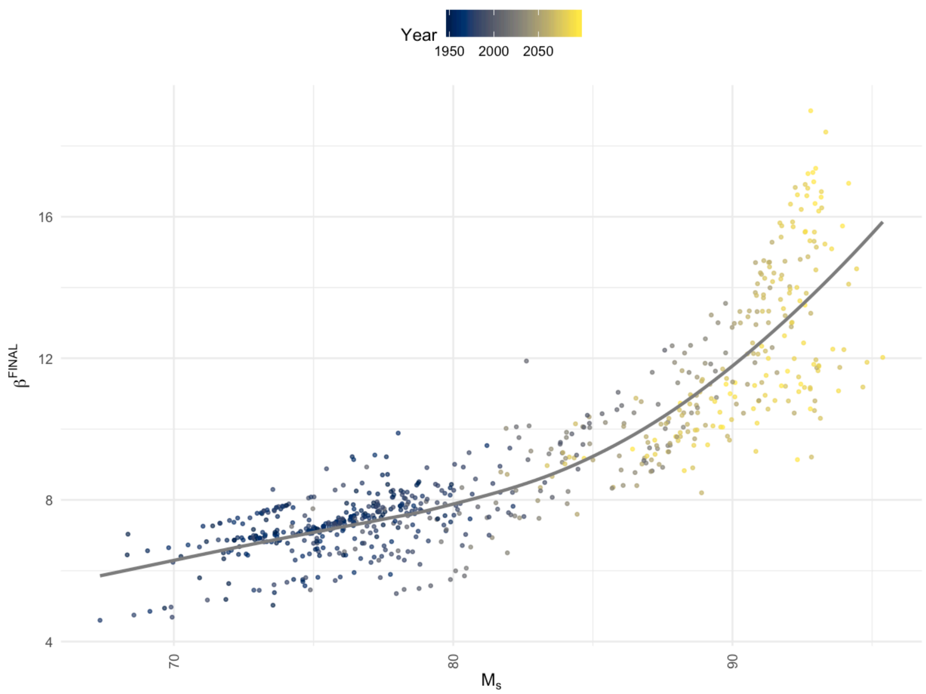
Source: Author's calculations, based on United Nations (2017c).

Figure 108 – Modal age at death from senescent mortality (M_s^{FINAL}) by tempo of age-related mortality increase (β^{FINAL}) for the final mathematical mortality models



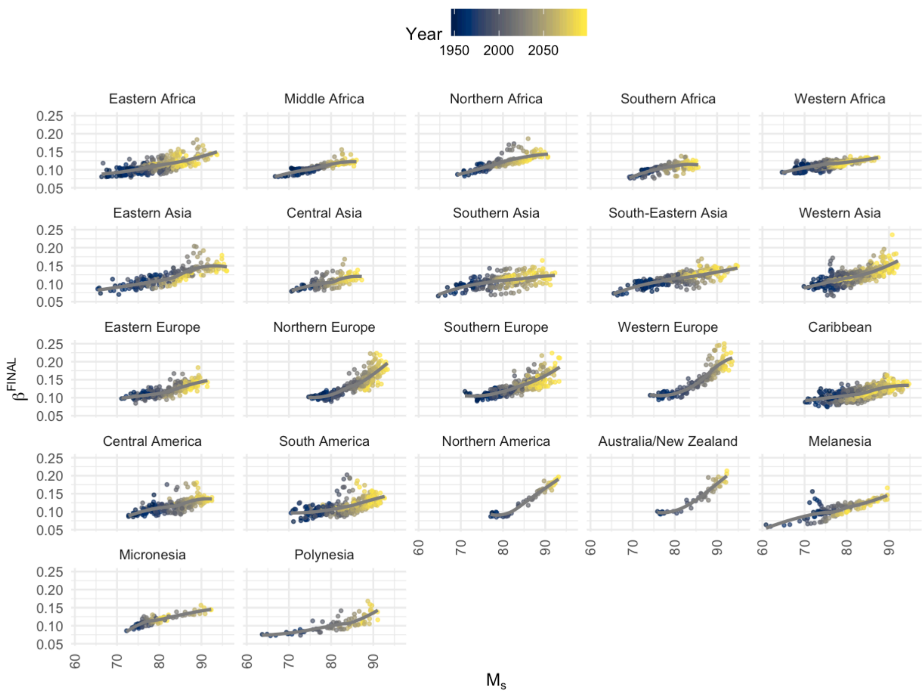
Source: Author's calculations, based on United Nations (2017c).
Note: Does not include the Weibull-Makeham model that has a different scale for β.

Figure 109 – Modal age at death from senescent mortality (M_s^{FINAL}) by tempo of age-related mortality increase (β^{FINAL}) for the final Weibull-Makeham models



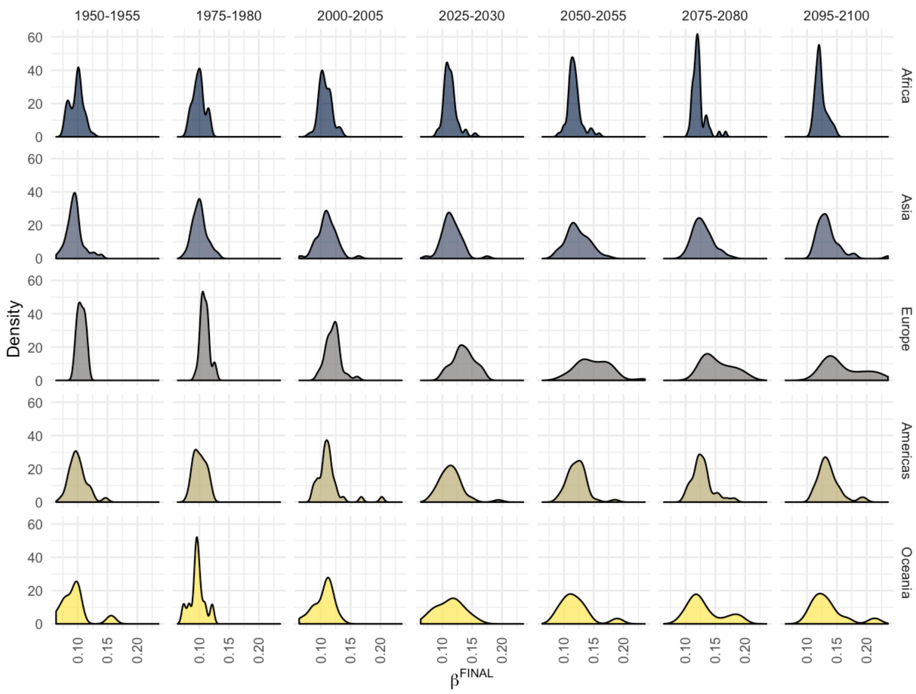
Source: Author's calculations, based on United Nations (2017c).

Figure 110 – Modal age at death from senescent mortality (M_s^{FINAL}) by tempo of age-related mortality increase (β^{FINAL}) and subregions for the final mathematical mortality models



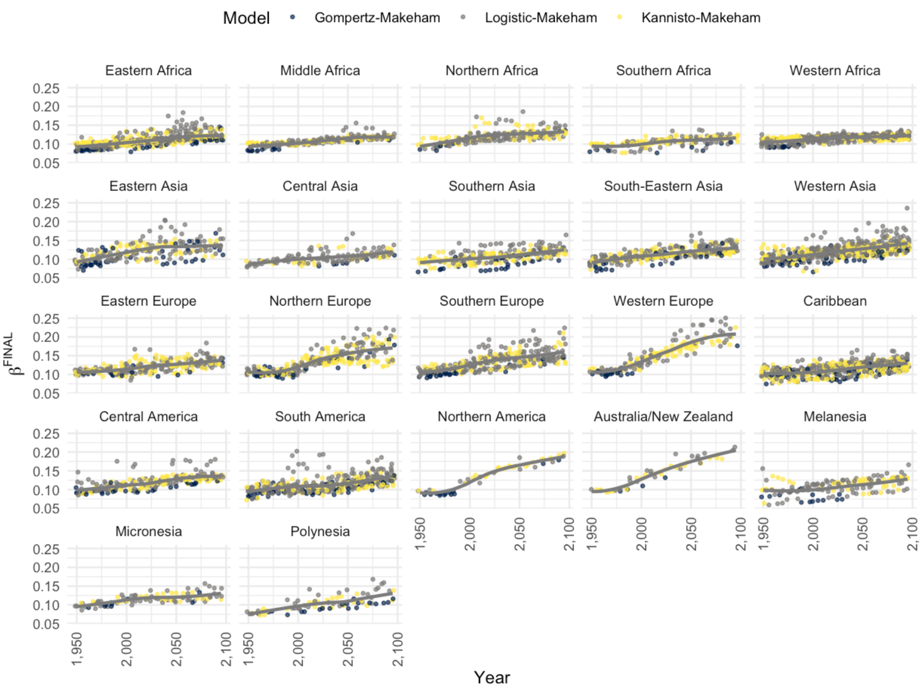
Source: Author's calculations, based on United Nations (2017c).
Note: Does not include the Weibull-Makeham model that has a different scale for β.

Figure 111 – Density of tempo of age-related mortality increase (β^{FINAL}) for the final mathematical mortality models and selected periods



Source: Author's calculations, based on United Nations (2017c).
Note: Does not include the Weibull-Makeham model that has a different scale for β.

Figure 112 – Tempo of age-related mortality increase (β^{FINAL}) by periods and subregions for the final mathematical mortality models



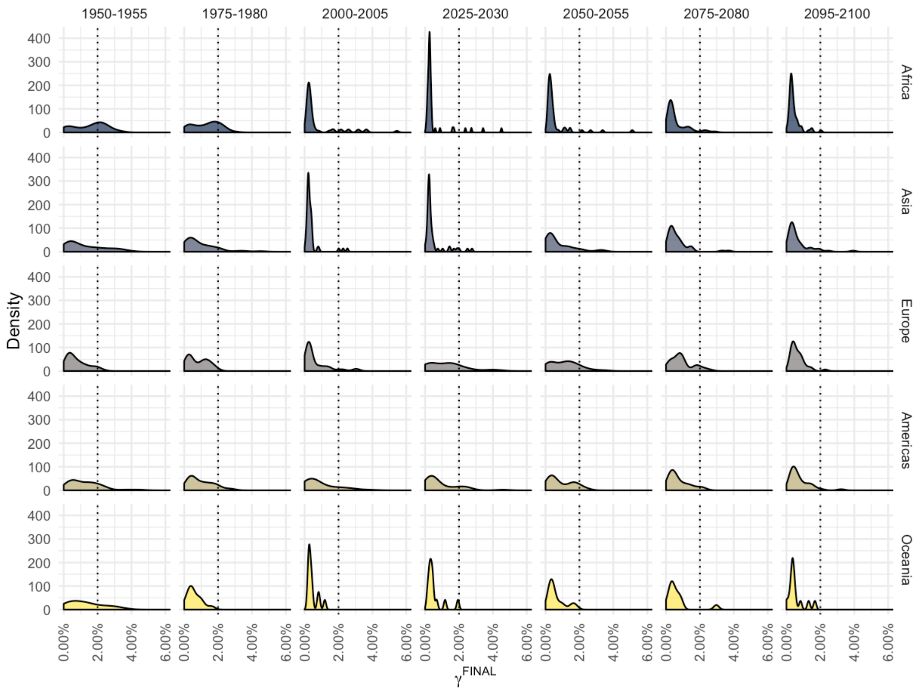
Source: Author's calculations, based on United Nations (2017c).
Note: Does not include the Weibull-Makeham model that has a different scale for β.

Now, we assess Thatcher, Kannisto, and Vaupel (1998)'s and Horiuchi, Ouellette, et al. (2013)'s claim that premature mortality, given by γ, the Makeham term, is low or negligible at old ages. We calculate at each 5-year period $t, t+h$ the ratio between γ and the arithmetic average of the life table $m_{a,n}$ over age groups 75–79 to 105–109 for the final mathematical mortality models:

$$\tilde{\gamma}^{\text{FINAL}}(t,t+h) = \frac{\gamma^{\text{FINAL}}(t,t+h)}{m_{\bar{a}}^{\text{FINAL}}(t,t+h)} = \frac{\gamma^{\text{FINAL}}(t,t+h)}{\frac{\sum_{a=75}^{105} m_{a,n}^{\text{FINAL}}(t,t+h)}{\nu(t,t+h)}} \tag{B.23}$$

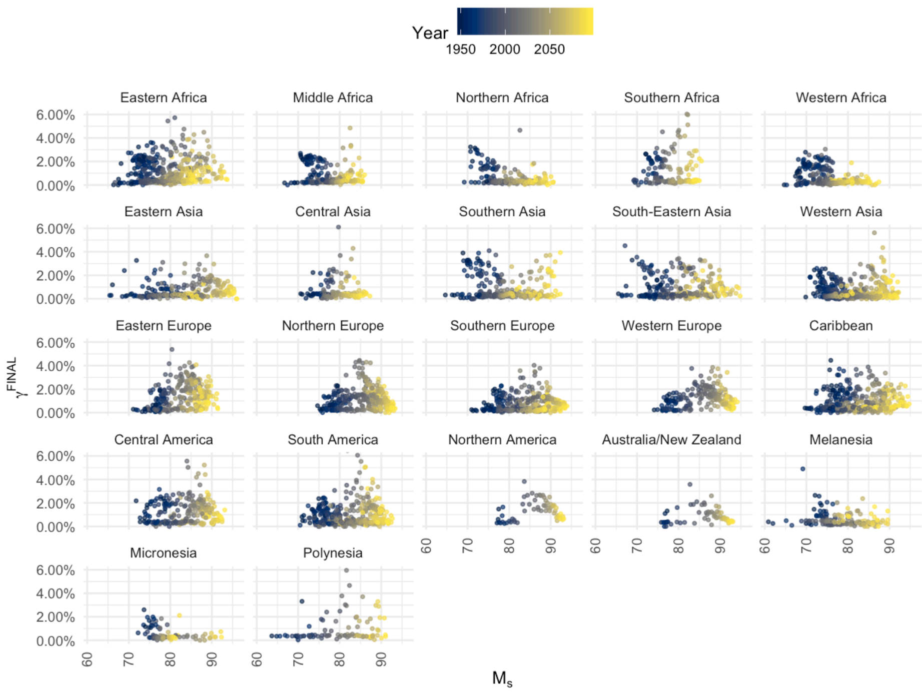
Figure 113 presents the density of $\tilde{\gamma}^{\text{FINAL}}$ for selected periods and all regions with a vertical reference line at 2%. Overall, $\tilde{\gamma}^{\text{FINAL}}$ is below 2% for most periods and regions, with a few exceptions; and around 0.5% for most of the twenty-first century. Yet Figure 114 shows that there is no indication whatsoever that higher M_s lead to lower $\tilde{\gamma}^{\text{FINAL}}$.

Figure 113 – Density of Makeham term ratio ($\tilde{\gamma}^{\text{FINAL}}$) for the final mathematical mortality models and selected periods



Source: Author's calculations, based on United Nations (2017c).

Figure 114 – Modal age at death from senescent mortality (M_s^{FINAL}) by Makeham term ratio ($\tilde{\gamma}^{\text{FINAL}}$) and subregions for the final mathematical mortality models



Source: Author's calculations, based on United Nations (2017c).

B.3 CALCULATE LIFE TABLE FUNCTIONS

We calculate life table functions from 1950–1955 to 2095–2100: first, for the 85–89, 90–94, and 95+ age groups; second, for the 80+ age group; and last, for the 0–4 age group. The formulas we use in the following sections (B.3.1, B.3.2, and B.3.3) are from Keyfitz (1977, chapter 2), Keyfitz and Beekman (1984, chapter II), and Preston, Heuveline, and Guillot (2001, chapter 3).

B.3.1 *Life table functions for the 85–89, 90-94 and 95+ age groups*

The life table functions for the 85–89, 90-94 and 95+ age groups are based on the life table $m_{a,n}$ of the final mathematical mortality models for the 85–89 to 105–109 age groups ($m_{a,n}^{\text{FINAL}}$) that we estimated in subsection B.2.4. Let the life table functions that are originally from the 2017 UN REVISION continue with no extra subscripts or superscripts (e.g., $m_{a,n}$, l_a), and the life table functions based on the $m_{a,n}$ of the final mathematical mortality models have FINAL superscripts (e.g., $m_{a,n}^{\text{FINAL}}$, l_a^{FINAL}). Our calculations take the following order:

a) compute the probability of dying between ages a and $a+n$ ($q_{a,n}$):

$$q_{a,n}^{\text{FINAL}} = \begin{cases} 1 - e^{(-n \cdot m_{a,n}^{\text{FINAL}})} & \text{assume that } \mu(a) \text{ is constant between } a \text{ to } a+n \\ 1 & \text{for the 95+ age group } (q_{95,+}^{\text{FINAL}}) \end{cases} \tag{B.24}$$

b) derive the probability of surviving from age a to $a+n$ ($p_{a,n}$):

$$p_{a,n}^{\text{FINAL}} = 1 - q_{a,n}^{\text{FINAL}} \tag{B.25}$$

c) compute the number of survivors to age a (l_a):

$$l_a^{\text{FINAL}} = \begin{cases} l_{85} & \text{for } a \text{ equal to 85} \\ l_{a-n}^{\text{FINAL}} \cdot p_{a-n,n}^{\text{FINAL}} & \text{for } a \text{ equal to 90, 95, 100, 105} \end{cases} \tag{B.26}$$

d) calculate the number of deaths between ages a and $a+n$ ($d_{a,n}$):

$$d_{a,n}^{\text{FINAL}} = \begin{cases} l_a^{\text{FINAL}} - l_{a+n}^{\text{FINAL}} & \text{for the 85–89 to 100–104 age groups} \\ l_a^{\text{FINAL}} & \text{for the 95+ and 105–109 age groups} \end{cases} \tag{B.27}$$

e) compute the average number of person-years lived between ages a and $a+n$ by those dying in the age interval ($a_{a,n}$):

$$a_{a,n}^{\text{FINAL}} = \left\{ n + \frac{1}{m_{a,n}^{\text{FINAL}}} - \frac{n}{q_{a,n}^{\text{FINAL}}} \quad \text{for the 85–89 to 100–104 age groups} \right. \tag{B.28}$$

See Equation B.34 for $a_{a,n}^{\text{FINAL}}$ for the 95+ and 105–109 age groups.

f) derive the number of person-years lived between ages a and $a+n$ ($L_{a,n}$):

$$L_{a,n}^{\text{FINAL}} = \begin{cases} (n \cdot l_{a+n}^{\text{FINAL}}) + (a_{a,n}^{\text{FINAL}} \cdot d_{a,n}^{\text{FINAL}}) & \text{For the 85–89 to 100–104 age groups} \\ \dfrac{l_{105}^{\text{FINAL}}}{m_{105,5}^{\text{FINAL}}} & \text{For the 105–109 age group } (L_{105,5}^{\text{FINAL}}) \\ \sum_{a=95}^{105} L_{a,n}^{\text{FINAL}} & \text{For the 95+ age group } (L_{95+}^{\text{FINAL}}) \end{cases} \tag{B.29}$$

g) calculate the number of person-years lived above age a (T_a):

$$T_a^{\text{FINAL}} = \sum_{y=a}^{\infty} L_{y,n}^{\text{FINAL}} \tag{B.30}$$

h) derive $m_{95,+}$, the $m_{a,n}$ for the open-ended age group 95+:

$$m_{95,+}^{\text{FINAL}} = \frac{l_{95}^{\text{FINAL}}}{T_{95}^{\text{FINAL}}} \tag{B.31}$$

i) calculate the probability of surviving from age group a, n to age group $a+n, n$ ($S_{a,n}$):

$$S_{a,n}^{\text{FINAL}} = \begin{cases} \dfrac{L_{a+n,n}^{\text{FINAL}}}{L_{a,n}^{\text{FINAL}}} & \text{For the 85–89 to 100–104 age groups} \\ 0 & \text{For the 105–109 age group} \\ \dfrac{T_{100}^{\text{FINAL}}}{T_{95}^{\text{FINAL}}} & \text{For the 95+ age group} \end{cases} \tag{B.32}$$

j) derive life expectancy at age a ($\mathring{e}_a$):

$$\mathring{e}_a^{\text{FINAL}} = \frac{T_a^{\text{FINAL}}}{l_a^{\text{FINAL}}} \tag{B.33}$$

k) compute $a_{a,n}$ for the age groups 95+ and 105–109:

$$a_{a,n}^{\text{FINAL}} = \mathring{e}_a^{\text{FINAL}} \tag{B.34}$$

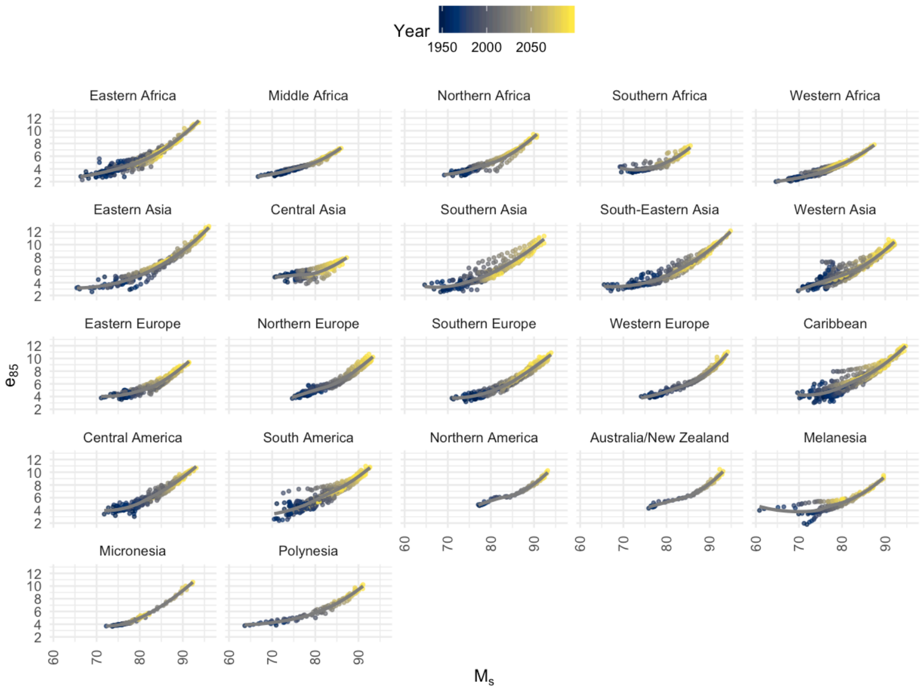
l) assure consistency between the values of life table functions based on $m_{a,n}^{\text{FINAL}}$ and those from the 2017 UN REVISION. We compute a correction factor between $m_{85,+}$ and $m_{85,+}^{\text{FINAL}}$ (λ). Since our calculated l_{85}^{FINAL} are equal to the 2017 UN REVISION l_{85} (Equation B.26), then:

$$\lambda = \frac{m_{85,+}}{m_{85,+}^{\text{FINAL}}} = \frac{\dfrac{l_{85}}{T_{85}}}{\dfrac{l_{85}^{\text{FINAL}}}{T_{85}^{\text{FINAL}}}} = \frac{T_{85}^{\text{FINAL}}}{T_{85}} \tag{B.35}$$

We apply λ to our estimates of $m_{a,n}^{\text{FINAL}}$ for the 85–89 to 105–109 age groups ($m_{a,n}^{\text{FINAL}} \cdot \lambda$) and redo the calculations from Equation B.24 to Equation B.35. We repeat the whole process until we have λ equal to 1.0 with six decimal places of arithmetic precision.

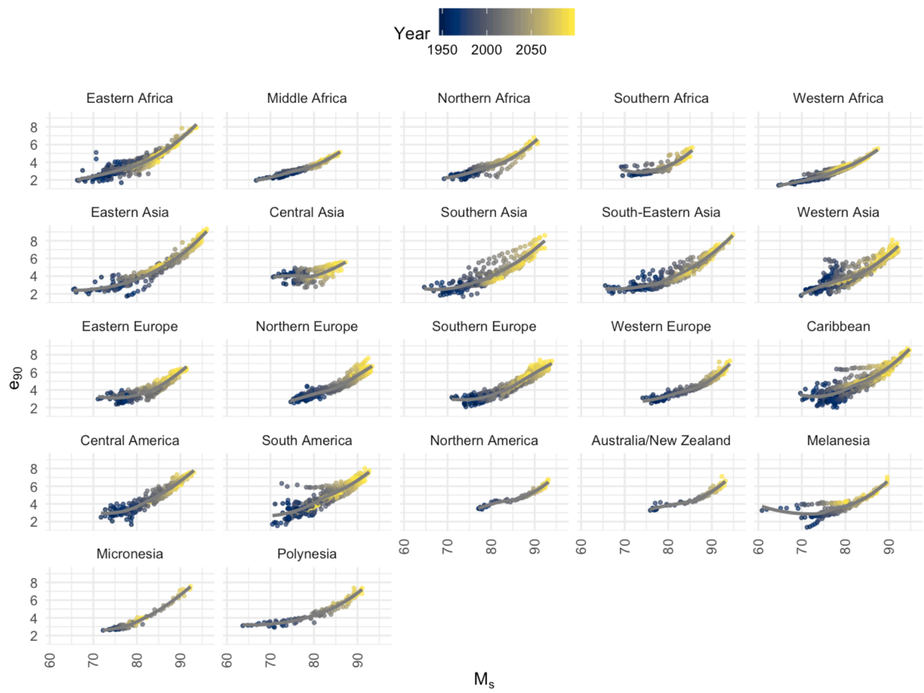
Figures 115, 116, and 117 show M_s for the final mathematical models by, respectively, $\mathring{e}_{85}$, $\mathring{e}_{90}$, and $\mathring{e}_{95}$ for the final life tables, detailed by subregions.

Figure 115 – Modal age at death from senescent mortality for the final mathematical mortality models (M_s^{FINAL}) by life expectancy at age 85 ($\mathring{e}_{85}$) for the final life tables



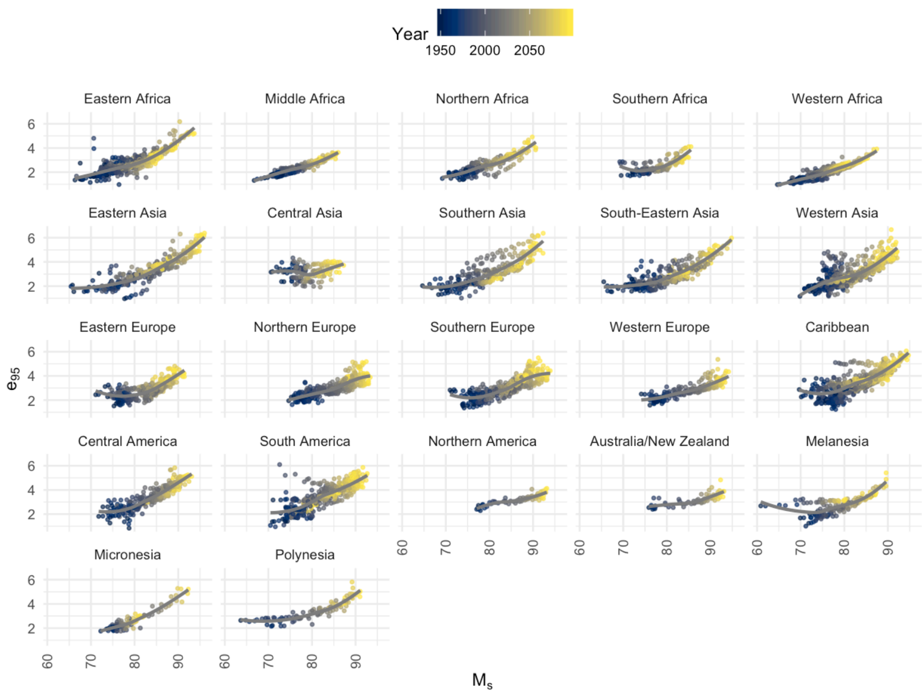
Source: Author's calculations, based on United Nations (2017c).

Figure 116 – Modal age at death from senescent mortality for the final mathematical mortality models (M_s^{FINAL}) by life expectancy at age 90 ($\mathring{e}_{90}$) for the final life tables



Source: Author's calculations, based on United Nations (2017c).

Figure 117 – Modal age at death from senescent mortality for the final mathematical mortality models (M_s^{FINAL}) by life expectancy at age 95 ($\mathring{e}_{95}$) for the final life tables



Source: Author's calculations, based on United Nations (2017c).

B.3.2 *Life table functions for the 80+ age group*

We use the following formulas to calculate the life table functions for the 80+ age group from 1950–1955 to 2095–2100:

$$l_{80}^{\text{FINAL}} = l_{80} \tag{B.36}$$

$$p_{80,+}^{\text{FINAL}} = 0 \tag{B.37}$$

$$q_{80,+}^{\text{FINAL}} = 1 \tag{B.38}$$

$$d_{80,+}^{\text{FINAL}} = d_{80,5} + d_{85,+} \tag{B.39}$$

$$L_{80,+}^{\text{FINAL}} = L_{80,5} + L_{85,+} \tag{B.40}$$

$$T_{80}^{\text{FINAL}} = T_{80} \tag{B.41}$$

$$m_{80,+}^{\text{FINAL}} = d_{80,+}^{\text{FINAL}} \,/\, L_{80,+}^{\text{FINAL}} \tag{B.42}$$

$$S_{80+,5}^{\text{FINAL}} = T_{85} \,/\, T_{80}^{\text{FINAL}} \tag{B.43}$$

$$\mathring{e}_{80}^{\text{FINAL}} = \mathring{e}_{80} \tag{B.44}$$

$$a_{80,+}^{\text{FINAL}} = \mathring{e}_{80}^{\text{FINAL}} \tag{B.45}$$

B.3.3 Life table functions for the 0–4 age group

Finally, we use the subsequent formulas to compute the life table functions for the 0–4 age group from 1950–1955 to 2095–2100:

$$l_0^{\text{FINAL}} = l_0 \tag{B.46}$$

$$p_{0,5}^{\text{FINAL}} = \frac{l_5}{l_0} \tag{B.47}$$

$$q_{0,5}^{\text{FINAL}} = 1 - p_{0,5}^{\text{FINAL}} \tag{B.48}$$

$$d_{0,5}^{\text{FINAL}} = d_{0,1} + d_{1,4} \tag{B.49}$$

$$L_{0,5}^{\text{FINAL}} = L_{0,1} + L_{1,4} \tag{B.50}$$

$$T_0^{\text{FINAL}} = T_0 \tag{B.51}$$

$$m_{0,5}^{\text{FINAL}} = d_{0,5}^{\text{FINAL}} \,/\, L_{0,5}^{\text{FINAL}} \tag{B.52}$$

$$S_{0,5}^{\text{FINAL}} = L_{0,5}^{\text{FINAL}} \,/\, (5 \cdot l_0) \tag{B.53}$$

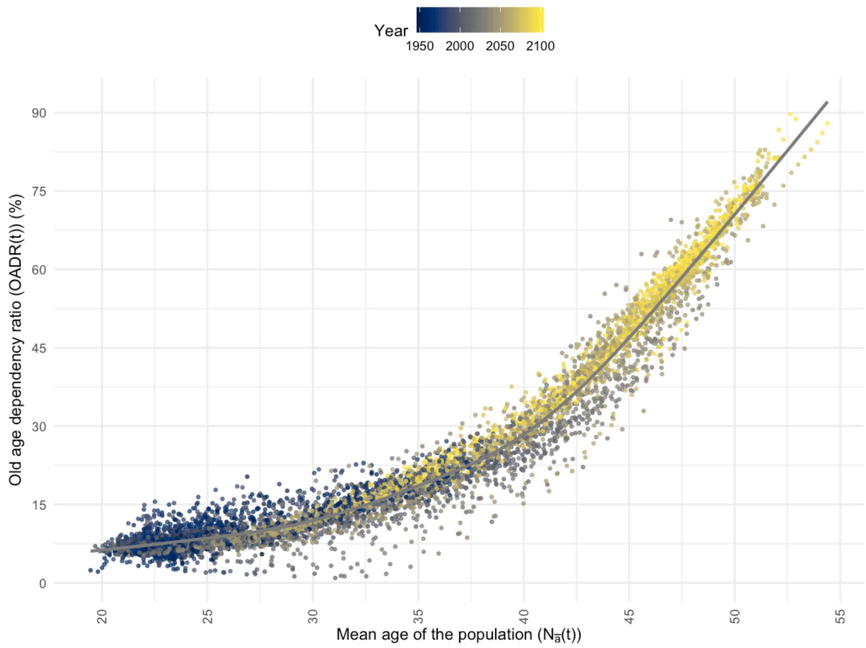
$$\mathring{e}_0^{\text{FINAL}} = \mathring{e}_0 \tag{B.54}$$

$$a_{0,5}^{\text{FINAL}} = \frac{L_{0,5}^{\text{FINAL}} - (5 \cdot l_5)}{l_0 - l_5} \tag{B.55}$$

Appendix C – Supplement to chapter 3

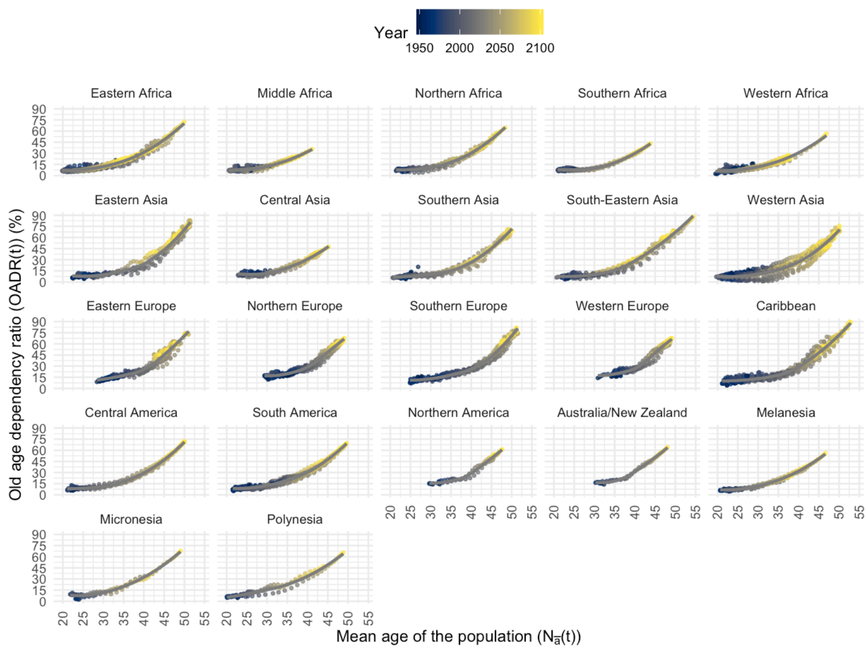
C.1 SUPPLEMENT TO SECTION 3.2

Figure 118 – Mean age of the population ($N_{\bar{a}}$) by old age dependency ratio (OADR)



Source: Author's calculations, based on United Nations (2017c).

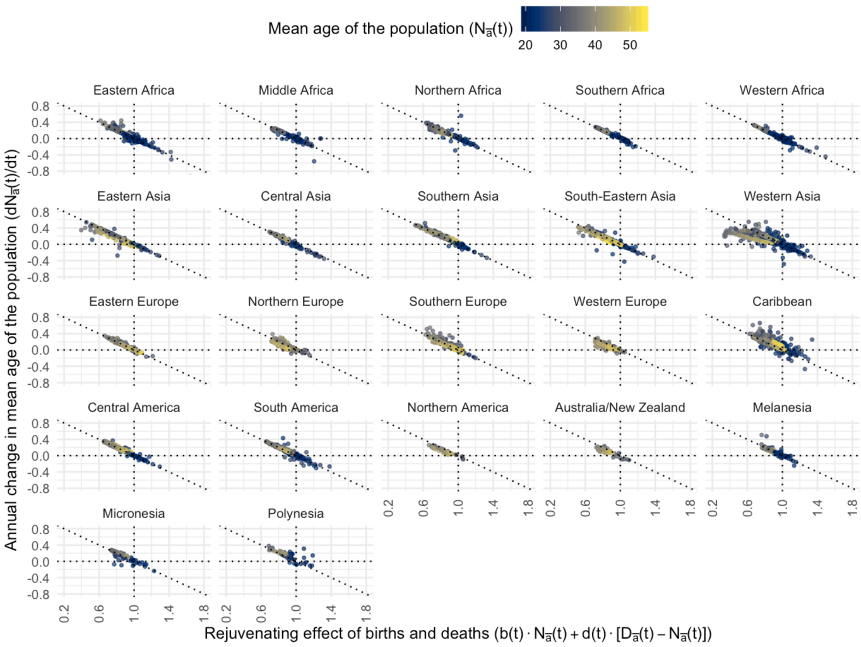
Figure 119 – Mean age of the population ($N_{\bar{a}}$) by OADR and subregions



Source: Author's calculations, based on United Nations (2017c).

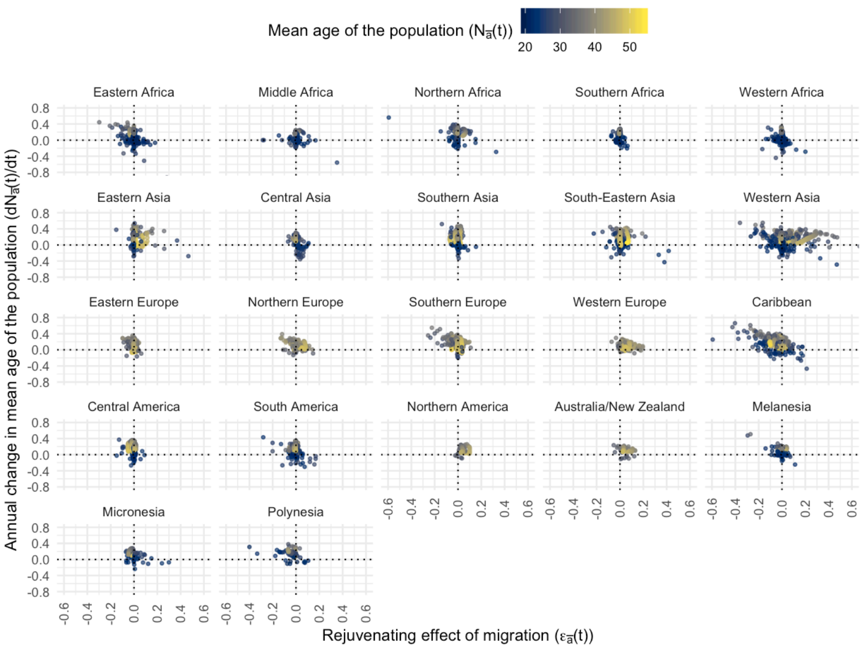
C.2 SUPPLEMENT TO SECTION 3.5

Figure 120 – Combined rejuvenating effect of births and deaths ($b \cdot N_{\bar{a}} + d \cdot [D_{\bar{a}} - N_{\bar{a}}]$) by annual rate of change in the mean age of the population ($dN_{\bar{a}}(t)/dt$) and subregions



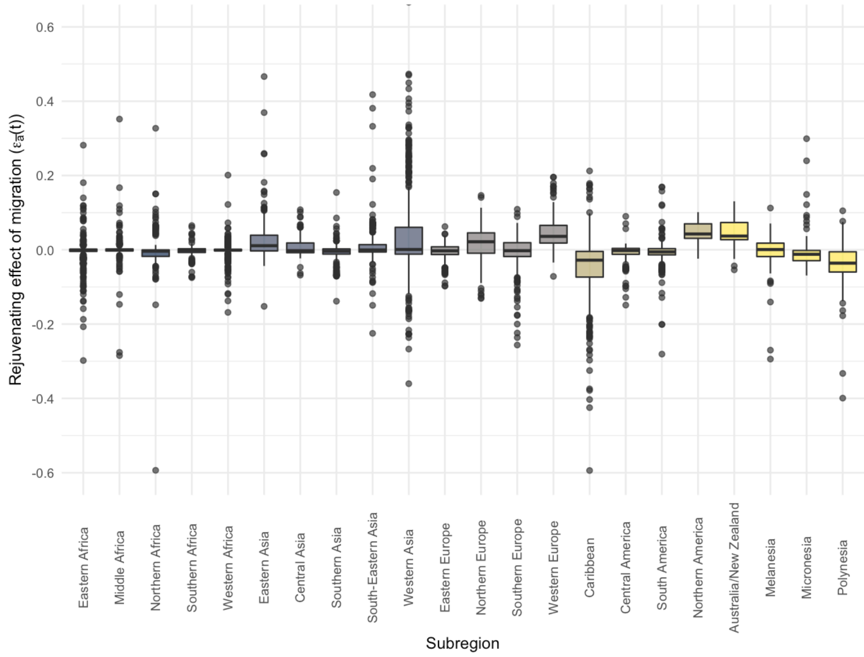
Source: Author's calculations, based on United Nations (2017c).

Figure 121 – Rejuvenating effect of net migration ($\epsilon_{\bar{a}}$) by annual rate of change in the mean age of the population ($dN_{\bar{a}}(t)/dt$) and subregions



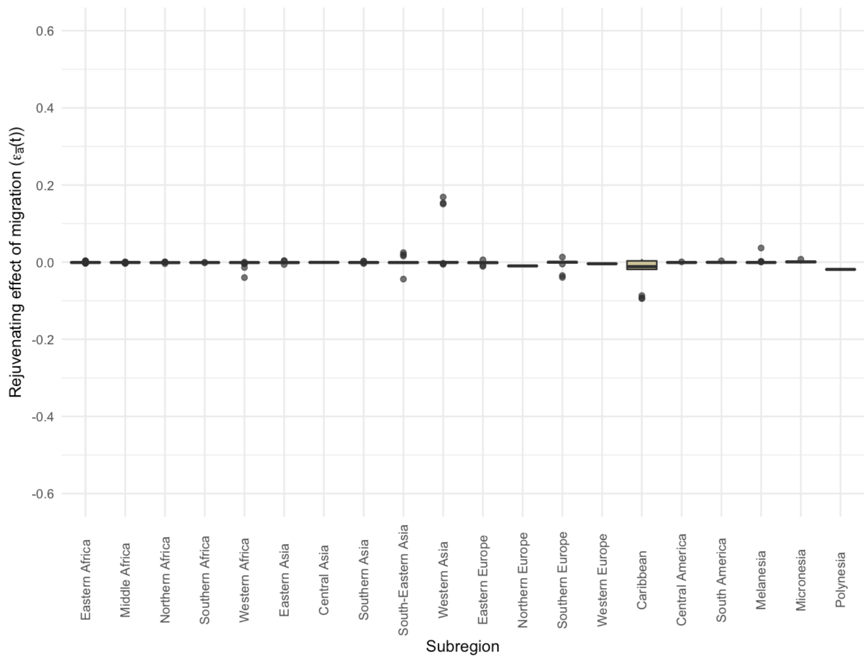
Source: Author's calculations, based on United Nations (2017c).

Figure 122 – Rejuvenating effect of net migration ($\epsilon_{\overline{a}}$) by subregions, for observations with an absolute net migration rate more than or equal to 0.0001



Source: Author's calculations, based on United Nations (2017c).

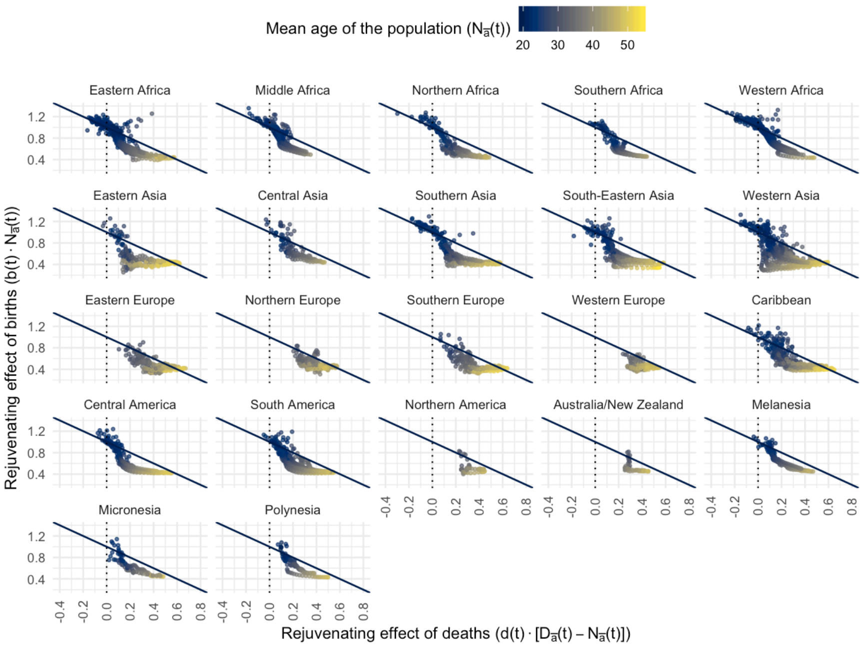
Figure 123 – Rejuvenating effect of net migration ($\epsilon_{\overline{a}}$) by subregions, for observations with an absolute net migration rate less than 0.0001



Source: Author's calculations, based on United Nations (2017c).

C.3 SUPPLEMENT TO SECTION 3.6

Figure 124 – Rejuvenating effect of deaths ($d \cdot [D_{\bar{a}} - N_{\bar{a}}]$) by rejuvenating effect of births ($b \cdot N_{\bar{a}}$) and subregions



Source: Author's calculations, based on United Nations (2017c).

Table 16 – Indicators of the mean age of the population, rejuvenating effect of births, and rejuvenating effect of deaths by stage of the demographic transition

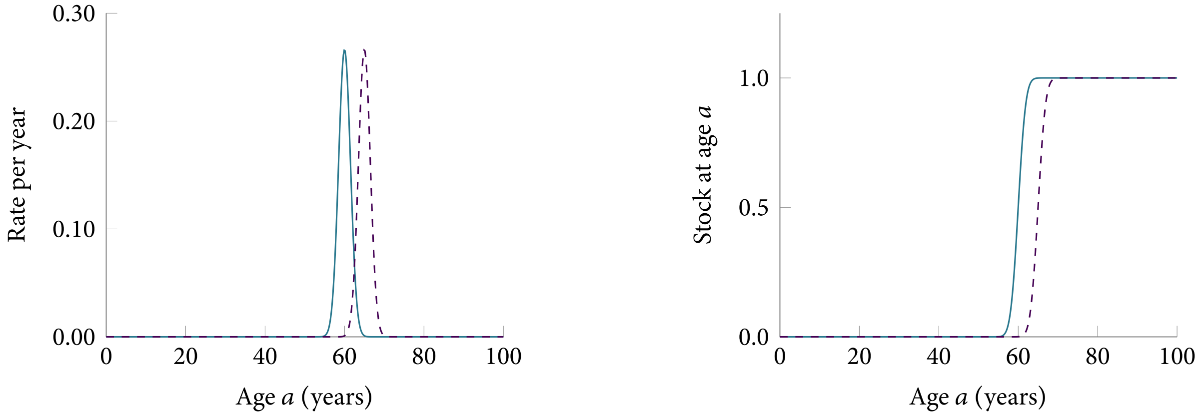
Stage	Mean age of the population			Rejuvenating effect of births $(b(t) \cdot N_{\bar{a}}(t))$	Rejuvenating effect of deaths $(d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)])$	Rejuvenating effect of births and rejuvenating effect of deaths		Example Countries
	$N_{\bar{a}}(t)$	$\frac{d}{dt}N_{\bar{a}}(t)$	$\frac{d^2}{dt^2}N_{\bar{a}}(t)$			Combined	$-\frac{d}{dt}b(t) \cdot N_{\bar{a}}(t)$ versus $\frac{d}{dt}d(t) \cdot [D_{\bar{a}}(t) - N_{\bar{a}}(t)]$	
1	decrease	negative	positive	1.0, 1.2	-0.2, 0.0	> 1	>	Turkey (1950–1955) Afghanistan (1980–1985) Somalia (1990–1995)
1A	minimum	zero	positive	1.0	0.0	= 1	>	Peru (1965–1970) Pakistan (1970–1975) Chad (2005–2010)
2	increase	positive	positive	1.0, 0.6	0.0, 0.2	< 1	>	Bulgaria (1950–1955) China (1955–1960) Angola (2010–2015)
3	increase	maximum	zero	0.6	0.2	< 1	=	Japan (1970–1975) Philippines (2025–2030) Niger (2095–2100)
4	increase	positive	negative	0.6, 0.4	0.2, 0.6	< 1	<	Austria (1950–1955) United States (1965–1970) Brazil (2025–2030)
4A	maximum	zero	negative	0.4	0.6	= 1	<	Portugal (2060–2065) Spain (2060–2065) Jamaica (2080–2085)
5	decrease	negative	negative	0.4	> 0.6	> 1	<	Poland (2070–2075) Albania (2085–2090) Puerto Rico (2090–2095)

Source: Author's creation and calculations, based on Preston, Himes, and Eggers (1989) and United Nations (2017c).

Appendix D – Supplement to chapter 4

D.1 SUPPLEMENT TO SECTION 4.3

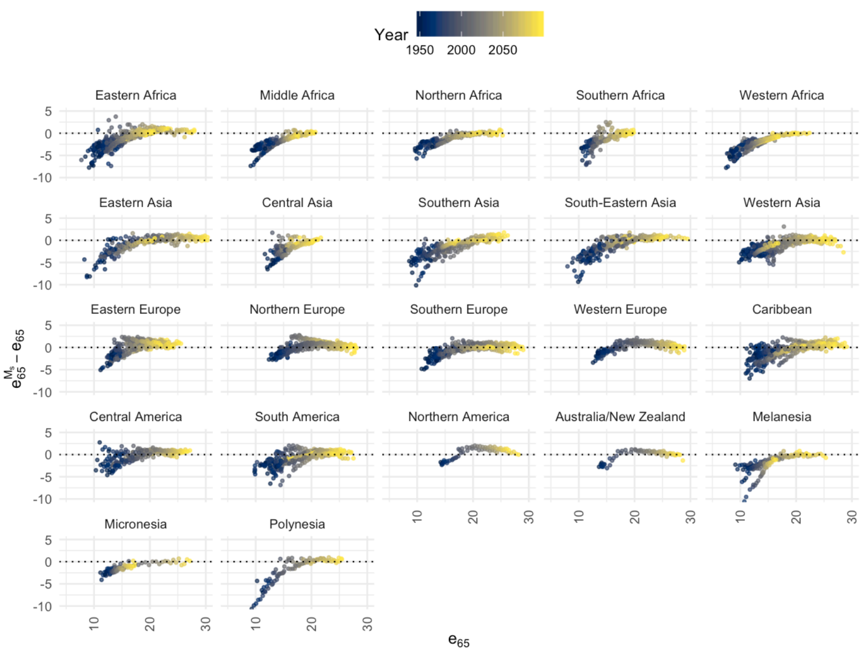
Figure 125 – Proportional rescaling of the age of entry into retirement from 60 years to 65 years under weak proportionality



Source: Author's creation, based on Figure 1 in Lee and Goldstein (2003, p. 186).
Note: Curves are for variance (σ^2) equal to 1.5 years.

D.2 SUPPLEMENT TO SECTION 4.8

Figure 126 – Life expectancy at age a ($\mathring{e}_a$) by modal life expectancy at age a minus life expectancy at age a ($\mathring{e}_a^{M_s} - \mathring{e}_a$) and subregions for age 65



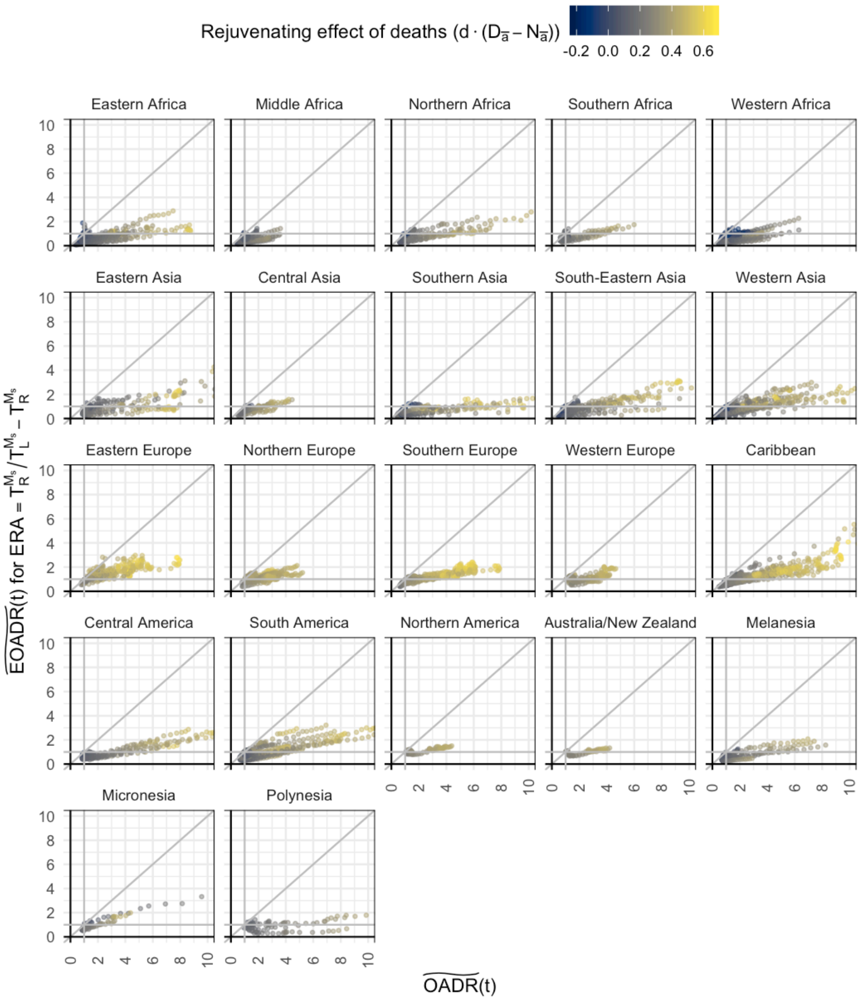
Source: Author's calculations, based on United Nations (2017c).

Table 17 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement based on $\mathring{e}_a^{M_s}$

Point of measurement	Characteristic of measurement			
	Modal expected years in retirement		Ratio of modal expected years in retirement to modal expected years in work	
Entry into retirement (R)	$\mathring{e}_R^{M_s}$	(D.1)	$\frac{\mathring{e}_R^{M_s}}{R-L}$	(D.2)
Entry into labor force (L)	$\frac{l_R}{l_L} \cdot \mathring{e}_R^{M_s}$	(D.3)	$\frac{l_R / l_L \cdot \mathring{e}_R^{M_s}}{\mathring{e}_L^{M_s} - l_R / l_L \cdot \mathring{e}_R^{M_s}}$	(D.4)

Source: Author's creation, based on table in Bayo and Faber (1981, p. 4).

Figure 127 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ for $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$ and subregions

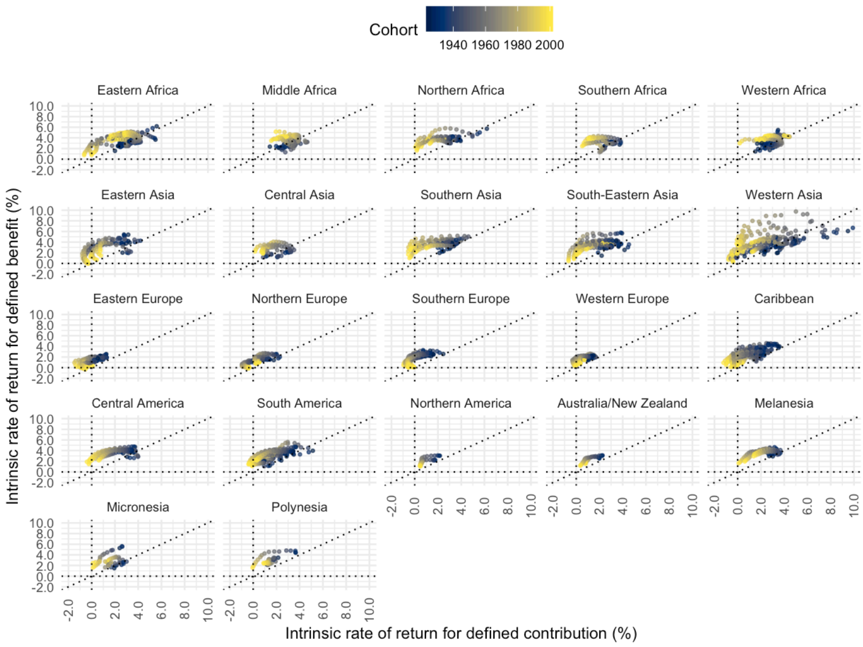


Source: Author's calculations, based on United Nations (2017c).

Appendix E – Supplement to chapter 5

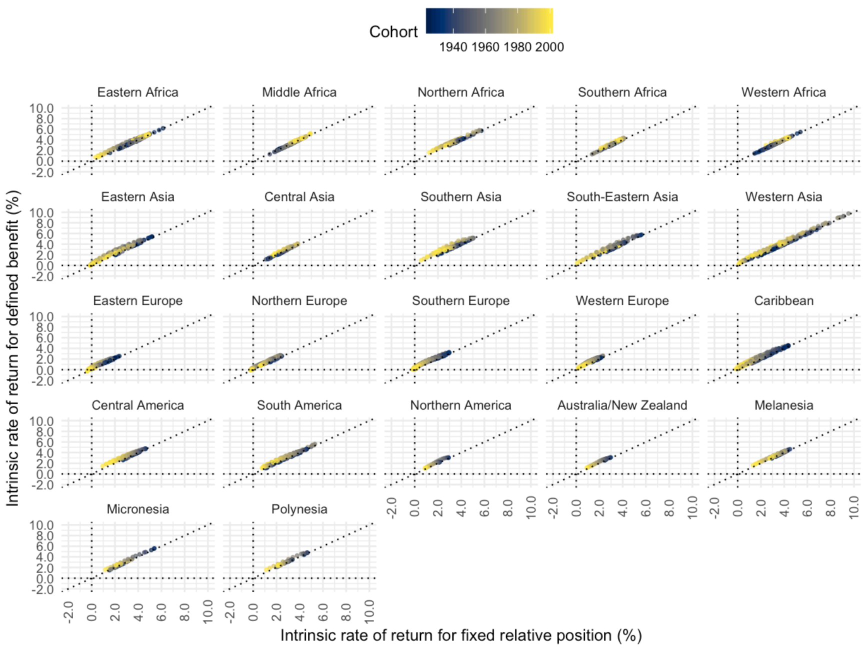
E.1 SUPPLEMENT TO SECTION 5.4

Figure 128 – Intrinsic rate of return (IRR) of DC by IRR of DB and subregions



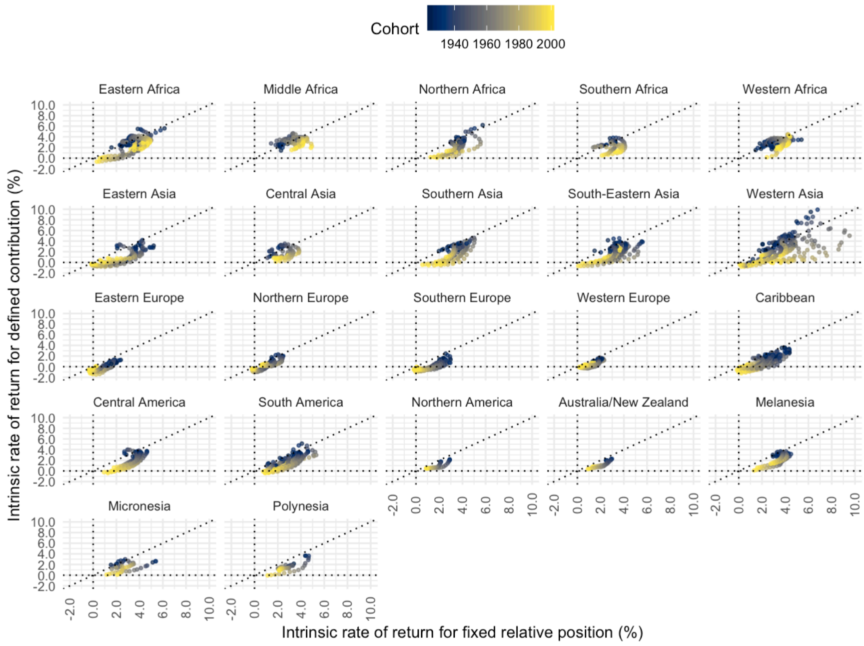
Source: Author's calculations, based on United Nations (2017c).

Figure 129 – Intrinsic rate of return (IRR) of FRP by IRR of DB and subregions



Source: Author's calculations, based on United Nations (2017c).

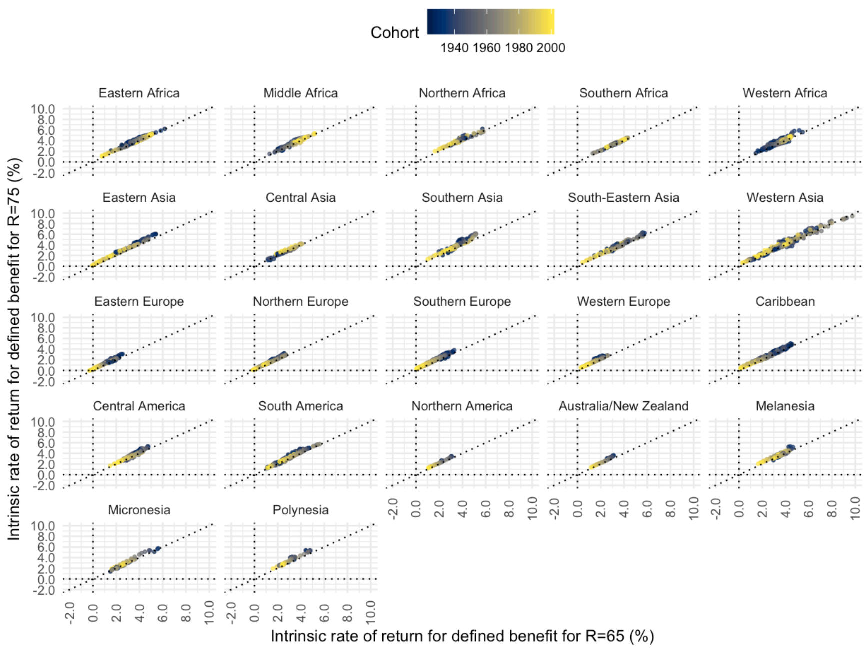
Figure 130 – Intrinsic rate of return (IRR) of FRP by IRR of DC and subregions



Source: Author's calculations, based on United Nations (2017c).

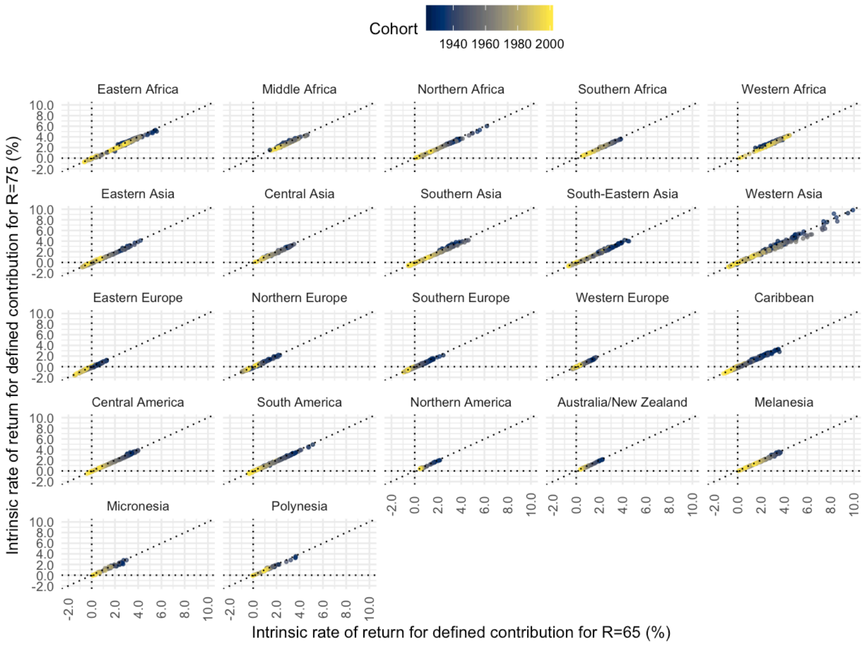
E.2 SUPPLEMENT TO SECTION 5.5

Figure 131 – Intrinsic rate of return (IRR) of $R = 65$ by IRR of $R = 75$ and subregions of defined benefit (DB)



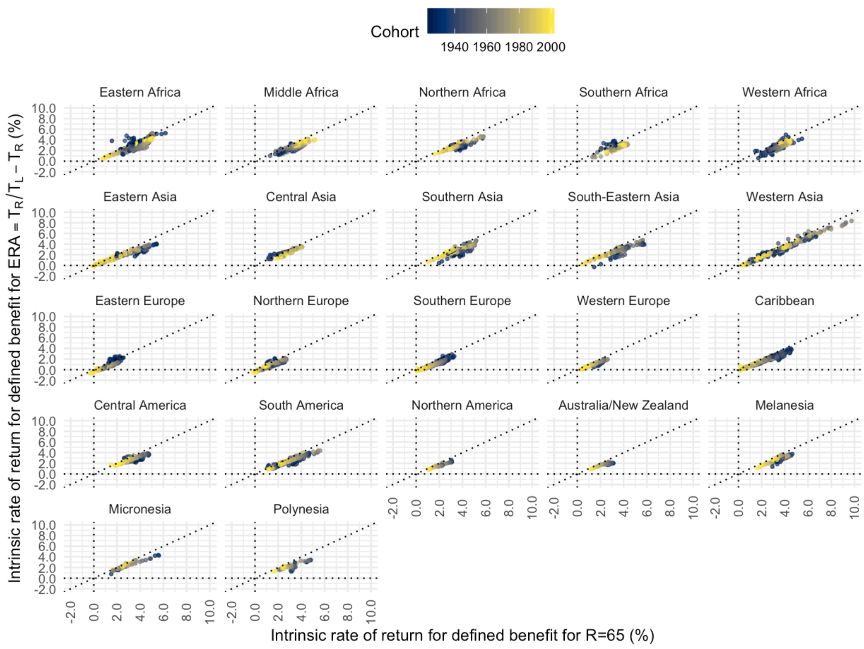
Source: Author's calculations, based on United Nations (2017c).

Figure 132 – Intrinsic rate of return (IRR) of $R = 65$ by IRR of $R = 75$ and subregions of defined contribution (DC)



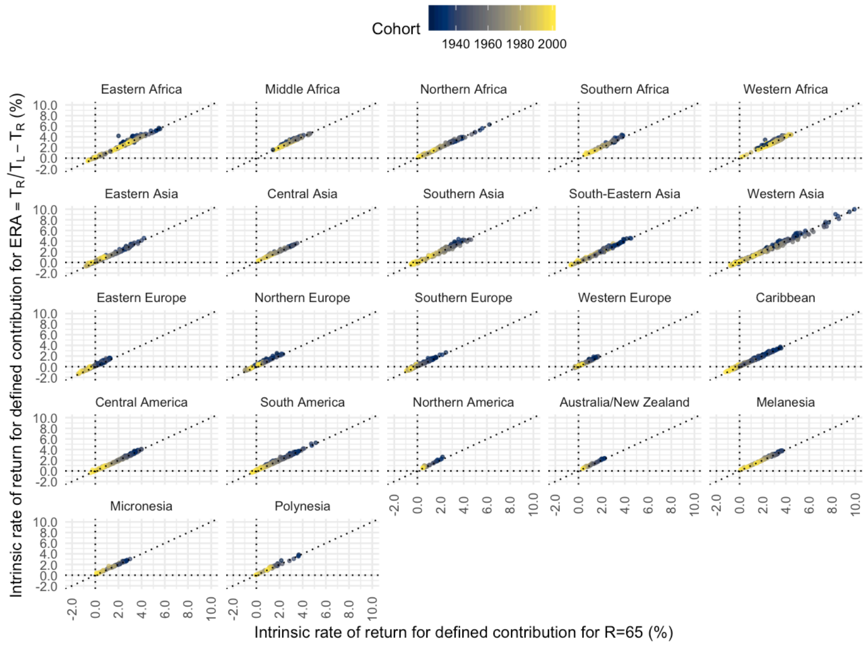
Source: Author's calculations, based on United Nations (2017c).

Figure 133 – Intrinsic rate of return (IRR) of $R = 65$ by IRR of ERA and subregions of defined benefit (DB)



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

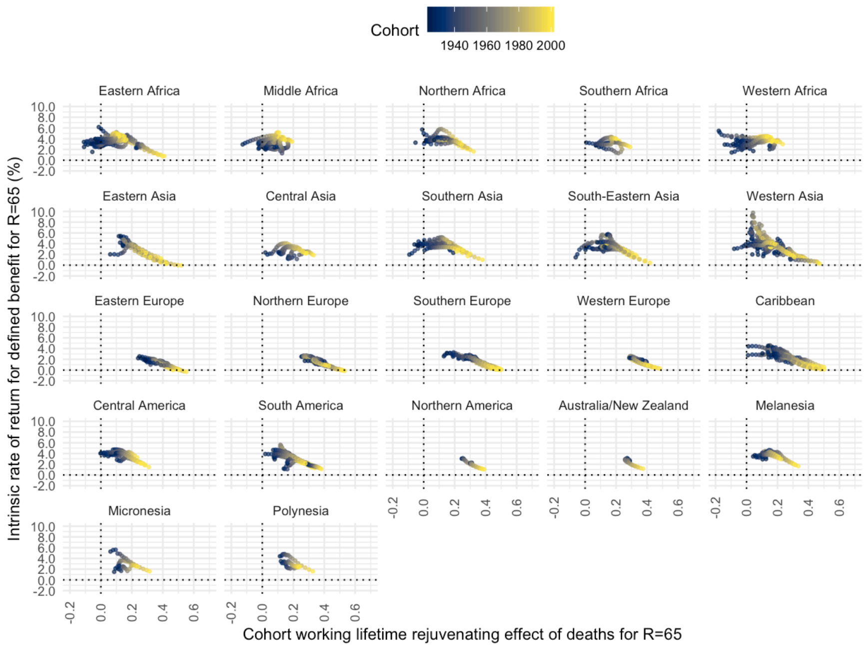
Figure 134 – Intrinsic rate of return (IRR) of $R = 65$ by IRR of ERA and subregions of defined contribution (DC)



Source: Author's calculations, based on United Nations (2017c).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

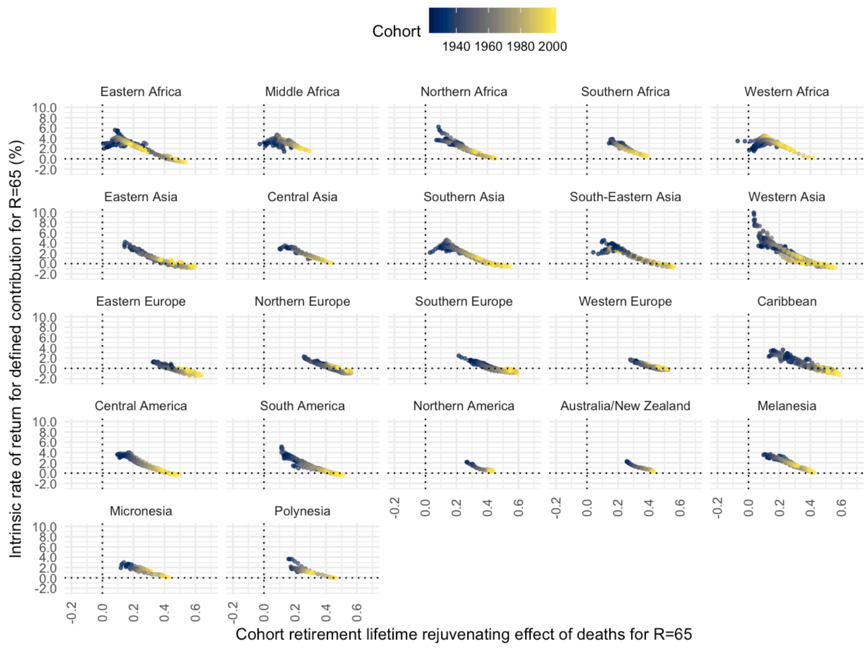
E.3 SUPPLEMENT TO SECTION 5.6

Figure 135 – Intrinsic rate of return (IRR) of defined benefit (DB) by cohort working life cycle rejuvenating effect of deaths ($G_W^{\mathcal{D}}$) and subregions



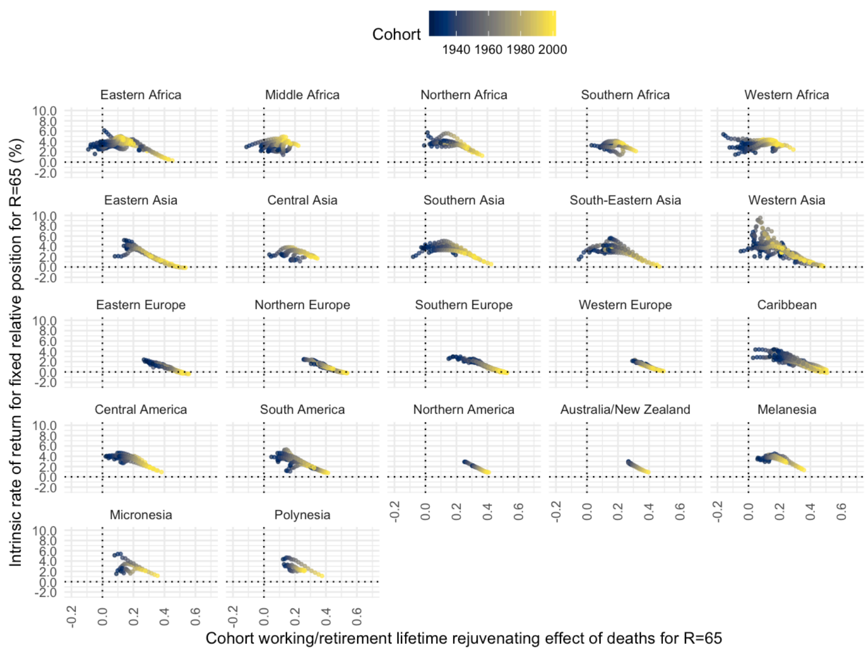
Source: Author's calculations, based on United Nations (2017c).

Figure 136 – Intrinsic rate of return (IRR) of defined contribution (DC) by cohort retirement life cycle rejuvenating effect of deaths ($G_R^{\mathcal{D}}$) and subregions



Source: Author's calculations, based on United Nations (2017c).

Figure 137 – Intrinsic rate of return (IRR) of fixed relative position (FRP) by cohort working and retirement life cycle rejuvenating effect of deaths ($G_{WR}^{\mathcal{D}}$) and subregions



Source: Author's calculations, based on United Nations (2017c).

ANNEXES

Annex A – 2017 UN REVISION, *Classification of countries*

Source: UNITED NATIONS. **World Population Prospects: The 2017 Revision, Classification of Countries by Region, Income-Group and Subregion of the World.** New York: United Nations, Department of Economic and Social Affairs, Population Division, 2017b. Available from: https://esa.un.org/unpd/wpp/General/Files/Definition_of_Regions.pdf

UNITED NATIONS NATIONS UNIES

POPULATION DIVISION
Department of Economic and Social Affairs

World Population Prospects: The 2017 Revision

CLASSIFICATION OF COUNTRIES BY REGION, INCOME GROUP AND SUBREGION OF THE WORLD

Africa

Eastern Africa

Burundi
Comoros
Djibouti
Eritrea
Ethiopia
Kenya
Madagascar
Malawi
Mauritius [1]
Mayotte
Mozambique
Réunion
Rwanda
Seychelles
Somalia
South Sudan
Uganda
United Republic of Tanzania[3]
Zambia
Zimbabwe

Middle Africa

Angola
Cameroon
Central African Republic
Chad
Congo
Democratic Republic of the Congo
Equatorial Guinea
Gabon
São Tomé and Príncipe

Northern Africa

Algeria
Egypt
Libya
Morocco
Sudan
Tunisia
Western Sahara

Southern Africa

Botswana
Lesotho
Namibia
South Africa
Swaziland

Western Africa

Benin
Burkina Faso
Cabo Verde
Côte d'Ivoire
Gambia
Ghana
Guinea
Guinea-Bissau
Liberia
Mali
Mauritania
Niger
Nigeria
Saint Helena[2] *
Senegal
Sierra Leone
Togo

[1] Including Agalega, Rodrigues, and Saint Brandon.
[2] Including Ascension, and Tristan da Cunha.
[3] Including Zanzibar.

CLASSIFICATION OF COUNTRIES (*continued*)

Asia

Eastern Asia	*South-Central Asia*[4]	*South-Eastern Asia*	*Western Asia*
	Central Asia		
China[5]	Kazakhstan	Brunei Darussalam	Armenia
China, Hong Kong SAR[6]	Kyrgyzstan	Cambodia	Azerbaijan[7]
China, Macao SAR [8]	Tajikistan	Indonesia	Bahrain
China, Taiwan Province of China	Turkmenistan	Lao People's Democratic Republic	Cyprus[9]
Democratic People's Republic of Korea	Uzbekistan	Malaysia[10]	Georgia[11]
		Myanmar	Iraq
Japan	*Southern Asia*	Philippines	Israel
Mongolia		Singapore	Jordan
Republic of Korea	Afghanistan	Thailand	Kuwait
	Bangladesh	Timor-Leste	Lebanon
	Bhutan	Viet Nam	Oman
	India		Qatar
	Iran (Islamic Republic of)		Saudi Arabia
	Maldives		State of Palestine[12]
	Nepal		Syrian Arab Republic
	Pakistan		Turkey
	Sri Lanka		United Arab Emirates
			Yemen

[4] The regions Southern Asia and Central Asia are combined into South-Central Asia.
[5] For statistical purposes, the data for China do not include Hong Kong and Macao, Special Administrative Regions (SAR) of China, and Taiwan Province of China.
[6] As of 1 July 1997, Hong Kong became a Special Administrative Region (SAR) of China.
[7] Including Nagorno-Karabakh.
[8] As of 20 December 1999, Macao became a Special Administrative Region (SAR) of China.
[9] Refers to the whole country.
[10] Including Sabah and Sarawak.
[11] Including Abkhazia and South Ossetia.
[12] Including East Jerusalem.

CLASSIFICATION OF COUNTRIES (*continued*)

Europe

Eastern Europe	*Northern Europe*	*Southern Europe*	*Western Europe*
Belarus	Channel Islands[13]	Albania	Austria
Bulgaria	Denmark	Andorra*	Belgium
Czechia	Estonia	Bosnia and Herzegovina	France
Hungary	Faeroe Islands*	Croatia	Germany
Poland	Finland[14]	Gibraltar*	Liechtenstein*
Republic of Moldova[15]	Iceland	Greece	Luxembourg
Romania	Ireland	Holy See[16] *	Monaco*
Russian Federation	Isle of Man*	Italy	Netherlands
Slovakia	Latvia	Malta	Switzerland
Ukraine[17]	Lithuania	Montenegro	
	Norway[18]	Portugal	
	Sweden	San Marino*	
	United Kingdom of Great Britain and Northern Ireland[20]	Serbia[19]	
		Slovenia	
		Spain[21]	
		The former Yugoslav Republic of Macedonia[22]	

[13] Refers to Guernsey, and Jersey.
[14] Including Åland Islands.
[15] Including Transnistria.
[16] Refers to the Vatican City State.
[17] Including Crimea.
[18] Including Svalbard and Jan Mayen Islands.
[19] Including Kosovo.
[20] Also referred to as United Kingdom.
[21] Including Canary Islands, Ceuta and Melilla.
[22] Also referred to as TFYR Macedonia.

CLASSIFICATION OF COUNTRIES (*continued*)

Latin America and the Caribbean

Caribbean

Anguilla*
Antigua and Barbuda
Aruba
Bahamas
Barbados
British Virgin Islands*
Caribbean Netherlands*[23]
Cayman Islands*
Cuba
Curaçao
Dominica*
Dominican Republic
Grenada
Guadeloupe[25]
Haiti
Jamaica
Martinique
Montserrat*
Puerto Rico
Saint Kitts and Nevis*
Saint Lucia
Saint Vincent and the Grenadines
Sint Maarten (Dutch part)*
Trinidad and Tobago
Turks and Caicos Islands*
United States Virgin Islands

Central America

Belize
Costa Rica
El Salvador
Guatemala
Honduras
Mexico
Nicaragua
Panama

South America

Argentina
Bolivia (Plurinational State of)
Brazil
Chile
Colombia
Ecuador
Falkland Islands (Malvinas)*[24]
French Guiana
Guyana
Paraguay
Peru
Suriname
Uruguay
Venezuela (Bolivarian Rep. of)

[23] Refers to Bonaire, Saba and Sint Eustatius.
[24] A dispute exists between the Governments of Argentina and the United Kingdom of Great Britain and Northern Ireland concerning sovereignty over the Falkland Islands (Malvinas).
[25] Including Saint-Barthélemy and Saint-Martin (French part).

CLASSIFICATION OF COUNTRIES (*continued*)

Northern America

Bermuda*
Canada
Greenland*
Saint Pierre and Miquelon*
United States of America

Oceania

Australia/New Zealand	*Melanesia*	*Micronesia*	*Polynesia*[26]
Australia[27]	Fiji	Guam	American Samoa*
New Zealand	New Caledonia	Kiribati	Cook Islands*
	Papua New Guinea	Marshall Islands*	French Polynesia
	Solomon Islands	Micronesia (Federated States of)	Niue*
	Vanuatu	Nauru*	Samoa
		Northern Mariana Islands*	Tokelau*
		Palau*	Tonga
			Tuvalu*
			Wallis and Futuna Islands*

Sub-Saharan Africa

Angola	Côte d'Ivoire	Guinea-Bissau	Namibia	South Africa
Benin	Democratic Republic of the Congo	Kenya	Niger	South Sudan
Botswana	Djibouti	Lesotho	Nigeria	Swaziland
Burkina Faso	Equatorial Guinea	Liberia	Réunion	Togo
Burundi	Eritrea	Madagascar	Rwanda	Uganda
Cameroon	Ethiopia	Malawi	Saint Helena	United Republic of Tanzania
Cabo Verde	Gabon	Mali	São Tomé and Príncipe	Zambia
Central African Republic	Gambia	Mauritania	Senegal	Zimbabwe
Chad	Ghana	Mauritius	Seychelles	
Comoros	Guinea	Mayotte	Sierra Leone	
Congo		Mozambique	Somalia	

[26] Including Pitcairn.
[27] Including Christmas Island, Cocos (Keeling) Islands, and Norfolk Island.

CLASSIFICATION OF COUNTRIES (*continued*)

Least developed countries

Afghanistan
Angola
Bangladesh
Benin
Bhutan
Burkina Faso
Burundi
Cambodia
Central African Republic
Chad
Comoros
Democratic Republic of the Congo
Djibouti
Eritrea
Ethiopia
Gambia
Guinea
Guinea-Bissau
Haiti
Kiribati
Lao People's Democratic Republic
Lesotho
Liberia
Madagascar
Malawi
Mali
Mauritania
Mozambique
Myanmar
Nepal
Niger
Rwanda
São Tomé and Príncipe
Senegal
Sierra Leone
Solomon Islands
Somalia
South Sudan
Sudan
Timor-Leste
Togo
Tuvalu
Uganda
United Republic of Tanzania
Vanuatu
Yemen
Zambia

NOTE: Countries with a population of less than 90,000 in 2017 are indicated by an asterisk (*).

CLASSIFICATION OF COUNTRIES (*continued*)

High-income countries

Andorra
Antigua and Barbuda
Aruba
Australia
Austria
Bahamas
Bahrain
Barbados
Belgium
Bermuda
British Virgin Islands
Brunei Darussalam
Canada
Cayman Islands
Channel Islands
Chile
China, Hong Kong SAR
China, Macao SAR
China, Taiwan Province of China
Curaçao
Cyprus
Czechia
Denmark
Estonia
Faroe Islands
Finland
France
French Polynesia
Germany
Gibraltar
Greece
Greenland
Guam
Hungary
Iceland
Ireland
Isle of Man
Israel
Italy
Japan
Kuwait
Latvia
Liechtenstein
Lithuania
Luxembourg
Malta
Monaco
Netherlands
New Caledonia
New Zealand
Northern Mariana Islands
Norway
Oman
Palau
Poland
Portugal
Puerto Rico
Qatar
Republic of Korea
Saint Kitts and Nevis
San Marino
Saudi Arabia
Seychelles
Singapore
Sint Maarten (Dutch part)
Slovakia
Slovenia
Spain
Sweden
Switzerland
Trinidad and Tobago
Turks and Caicos Islands
United Arab Emirates
United Kingdom of Great Britain and Northern Island
United States of America
United States Virgin Islands
Uruguay

CLASSIFICATION OF COUNTRIES (*continued*)

Upper-middle-income countries

Albania
Algeria
American Samoa
Argentina
Azerbaijan
Belarus
Belize
Bosnia and Herzegovina
Botswana
Brazil
Bulgaria
China
Colombia
Costa Rica
Croatia
Cuba
Dominica
Dominican Republic
Ecuador
Equatorial Guinea
Fiji
Gabon
Grenada
Guyana
Iran (Islamic Republic of)
Iraq
Jamaica
Kazakhstan
Lebanon
Libya
Malaysia
Maldives
Marshall Islands
Mauritius
Mexico
Montenegro
Namibia
Nauru
Panama
Paraguay
Peru
Romania
Russian Federation
Saint Lucia
Saint Vincent and the Grenadines
Samoa
Serbia
South Africa
Suriname
The former Yugoslav Republic of Macedonia
Thailand
Tonga
Turkey
Turkmenistan
Tuvalu
Venezuela (Bolivarian Rep. of)

Lower-middle-income countries

Angola
Armenia
Bangladesh
Bhutan
Bolivia (Plurinational State of)
Cabo Verde
Cambodia
Cameroon
Congo
Côte d'Ivoire
Djibouti
Egypt
El Salvador
Georgia
Ghana
Guatemala
Honduras
India
Indonesia
Jordan
Kenya
Kiribati
Kyrgyzstan
Lao People's Democratic Republic
Lesotho
Mauritania
Micronesia (Fed. States of)
Mongolia
Morocco
Myanmar
Nicaragua
Nigeria
Pakistan
Papua New Guinea
Philippines
Republic of Moldova
Sao Tome and Principe
Solomon Islands
Sri Lanka
State of Palestine
Sudan
Swaziland
Syrian Arab Republic
Tajikistan
Timor-Leste
Tunisia
Ukraine
Uzbekistan
Vanuatu
Viet Nam
Yemen
Zambia

CLASSIFICATION OF COUNTRIES (*continued*)

Low-income countries

Afghanistan
Benin
Burkina Faso
Burundi
Central African Republic
Chad
Comoros
Democratic People's Republic of Korea
Democratic Republic of the Congo
Eritrea
Ethiopia
Gambia
Guinea
Guinea-Bissau
Haiti
Liberia
Madagascar
Malawi
Mali
Mozambique
Nepal
Niger
Rwanda
Senegal
Sierra Leone
Somalia
South Sudan
Togo
Uganda
United Republic of Tanzania
Zimbabwe