UNIVERSIDADE FEDERAL DE MINAS GERAIS
FACULDADE DE MEDICINA
PROGRAMA DE PÓS-GRADUAÇÃO EM SAÚDE DA CRIANÇA E DO ADOLESCENTE

LARISSA DE SOUZA SALVADOR

Steps towards modeling the multilevel heterogeneity of math learning difficulties

BELO HORIZONTE – MINAS GERAIS
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Co-Orientador: Prof. Dr. Ricardo Moura

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STEPS TOWARDS MODELING THE MULTILEVEL HETEROGENEITY OF MATH LEARNING DIFFICULTIES

LARISSA DE SOUZA SALVADOR

Tese submetida à Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação em Ciências da Saúde, Saúde da Criança e do Adolescente, como requisito para obtenção do grau de Doutor em Ciências da Saúde, Saúde da Criança e do Adolescente, área de concentração em Ciências da Saúde.

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Prof. Vitor Geraldi Haase - Orientador
UFMG

Prof.ª Débora Marques de Miranda
UFMG

Prof.ª Beatriz Vargas Dorneles - Videoconferência
UFGRS

Prof.ª Julia Bahnmüller - Videoconferência
Leibniz-Institut für Wissensmedien

Prof.ª Hedwig Gasteiger - Videoconferência
Universität Osnabrück

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RESUMO

O desempenho na matemática é um assunto muito complexo que envolve vários fatores individuais e ambientais. Variáveis individuais como processamento de magnitude, habilidades verbais e habilidades visuoespaciais podem interagir com outros fatores, como sexo, status socioeconômico e ambiente escolar, impactando de maneira complexa a performance matemática. Essa interação pode resultar em vários subtipos de dificuldades de aprendizagem de matemática (DAM). Para investigar se os subtipos de DAM podem ser encontrados no nível individual e a interação entre os níveis individual e ambiental, realizamos três estudos. Primeiro, foi realizada uma revisão para investigar quais variáveis estão mais fortemente associadas ao desempenho em matemática e para construir um modelo teórico com o intuito de orientar a condução dos estudos experimentais. No segundo estudo, uma análise de cluster foi realizada para investigar a heterogeneidade cognitiva do desempenho da matemática. Neste estudo, foram encontrados dois grupos principais de DAM associados a déficits no processamento de magnitude e habilidades visuoespaciais, respectivamente. No terceiro estudo, foi realizada uma comparação transcultural entre o Brasil e a Itália para investigar as interações entre variáveis individuais e ambientais no desempenho da matemática. Investigamos as habilidades verbais e visuoespaciais e o desempenho em matemática em crianças do ensino fundamental de escolas particulares do Brasil e de escolas públicas do Brasil e da Itália. Diferenças nas habilidades verbais, habilidades visuoespaciais e desempenho matemático foram encontradas entre os diferentes tipos de escola. Foram encontradas interações entre sexo e desempenho matemático nas escolas públicas no Brasil e na Itália, mas não nas escolas particulares no Brasil. Vários fatores cognitivos estão envolvidos no desempenho da matemática, contribuindo para a heterogeneidade da DAM. Vários desses fatores cognitivos que contribuem para o desempenho em matemática podem interagir com fatores ambientais e o sexo, aumentando a complexidade das habilidades matemáticas. Todos esses fatores devem ser considerados em estudos futuros.

Palavras chave: Processamento de magnitude, habilidades visuoespaciais, habilidades verbais, diferenças de gênero, nível socioeconômico, tipos de escola, performance matemática

ABSTRACT

Math performance is a very complex subject that involves several individual and environmental factors. Individual variables such as magnitude processing, verbal skills, and visuospatial skills can interact with other factors such as sex, socioeconomic status and school settings impacting in complex ways on math performance. This interaction can result in several subtypes of math learning difficulties (MLD). To investigate whether subtypes of MLD can be found at the individual level and the interaction between individual and environmental levels we conducted three studies. First, a review was conducted to investigate which variables are most strongly associated with math performance and to build a theoretical model to guide the experimental studies. In the second study, a cluster analysis was performed to investigate the cognitive heterogeneity of math performance. In this study, we founded two main groups of MLD associated with deficits on magnitude processing and visuospatial skills respectively. In the third study, cross-cultural comparison between Brazil and Italy was conducted to investigate the interactions between individual and environmental variables on math performance. We investigated the verbal and visuospatial skills and math performance in elementary school children from private schools in Brazil and State-run schools in Brazil and Italy. Differences in verbal skills, Visuospatial skills, and math performance were found among school settings. Interaction between sex and math performance on the State-run schools in Brazil and Italy, but not on private schools in Brazil were found. Several cognitive factors are involved in math performance contributing to the heterogeneity of MLD. Several of these cognitive factors that contribute to math performance can further interact with environmental factors and sex, increasing the complexity of math skills. All these factors should be considered in future studies.

Keywords: Magnitude processing, visuospatial skills, verbal skills, sex differences, socioeconomic status, school settings, math performance

FIGURE LIST

Study 1: The different levels of interaction between the environment and individual variables in math performance: a review

Study 2: Cognitive Heterogeneity of Math Difficulties: A Bottom-up Classification Approach

Study 3: Educational settings, gender differences, and cognitive processes in math performance: A cross-cultural study with Brazilian and Italian children

TABLE LIST

Study 2: Cognitive Heterogeneity of Math Difficulties: A Bottom-up Classification Approach

Study 3: Educational settings, gender differences, and cognitive processes in math performance: A cross-cultural study with Brazilian and Italian children

LIST OF ABBREVIATIONS AND ACRONYMS

DD	Developmental Dyscalculia
MLD	Math learning difficulty
DAM	Dificuldade de aprendizagem na matemática
IQ	Intelligence Coefficient
TDE	School Performance Test
w	Weber Fraction
SES	Socioeconomic Status
PMA	Primary Mental ability
PMA VM	Primary Mental Ability Verbal Meaning
PMA SR	Primary Mental Ability Spatial Relation

SUMMARY

1. INTRODUCTION

Math performance has aroused great interest and has been the subject of research for decades (Siegler & Braithwaite, 2017). Knowing which individual or environmental factors are most strongly associated with math performance could help us better predict outcomes related to math learning and its specific difficulties, plan more specific interventions, and contribute more effectively to the organization of the school curriculum. Math performance is also a gateway to employment in more profitable science, technology, engineering, and mathematics (STEM) professions (Ceci, Williams, & Barnett, 2009; Halpern et al., 2007), which increases research interest in the topic.

Several individual and environmental factors, organized in complex ways, are involved in math performance (Rubinsten & Henik, 2009). These factors at the individual level can be divided into domain-specific, such as quantity representation, and domain-general, such as verbal skills, visuospatial skills, and working memory/executive functions. Emotion and motivation also play an important role in math learning. Both specific and general domains can interact with sex and environmental factors such as culture, socioeconomic status (SES) and school type. Impairments in each of these domains can result in different subtypes of dyscalculia.

An important domain-specific factor is related to number magnitude processing. The representation of magnitudes depends on two main systems, which involve the non-symbolic representation of quantities and the symbolic representations of quantity (Dehaene, 1992). The non-symbolic representation of the quantity system is called the Approximate Number System (ANS). ANS is innate and is often understood as a starting point for the development of the symbolic comprehension of quantity. The accuracy of ANS is associated with the development of math skills (Halberda, Mazzocco & Feigenson, 2008; Mazzocco, Feigenson & Halberda, 2011). A core deficit hypothesis says that deficits in the ANS would be responsible for the developmental dyscalculia. This hypothesis assumes that the ANS is the basis for the development of comprehension of exact quantity, first when children began to use number words to represent quantities and later to comprehend the concept of the Arabic numbers (Wilson & Dehaene, 2007). An access deficit, or difficulty in establishing links or accessing magnitude representations in the ANS from the corresponding symbolic representations has also been discussed as a central deficit in developmental dyscalculia. In this case, impairments would be found in the symbolic processing of quantities and not in the ANS (Lyons, Price, Vaessen, Blomert & Ansari, 2014; Chen & Li, 2014). However, it is unclear which impairment in symbolic representation or impairment in the link between the nonsymbolic and the symbolic representations is stronger associated with developmental dyscalculia.

At the individual level, domain-general abilities such as verbal and visuospatial skills are also important to understand the development of math skills. Verbal skills are important in several processes related to math skills such as counting, transcoding of numbers in different symbolic representations, acquisition, and retrieval of arithmetic facts, word problem solving, etc (Lopes-Silva et al., 2014, 2016; De Smedt & Boets, 2010; Swanson & Sachse-Lee, 2001). On the other hand, visuospatial abilities have been associated with representation and manipulation of numerical quantities on a number line, place-value understanding, multi-digit calculation, and geometry (Raghubar, Barnes & Hecht, 2010; Bachot, Gevers, Fias & Roeyers, 2005; Li & Geary, 2013). The relationship between specific domain and general domain contributes to a heterogeneous picture of math learning disabilities (MLD).

Importantly, verbal and visuospatial skills are also related to sex variability, which could influence several math skills. Different studies have shown an advantage for girls in tasks involving verbal skills while boys perform better on tasks involving visuospatial skills (Van Tetering, Van der Donk, De Groot & Jolles, 2019). Such findings on sex differences in verbal and visuospatial skills could be involved in the sex differences found by some studies regarding math performance (Kucian, Loenneker, Dietrich, Martin, Von Aster, 2005) and math anxiety (Van Mier., Schleepen, & Van den Berg, 2018).

Sex differences in math performance are also influenced by environmental variables such as cultural and socioeconomic differences. An important hypothesis about sex differences suggests that in more modern societies that present greater gender equality these patterns of sex differences that derive from a socially constructed belief that women are better at verbal skills and worse at math performance could disappear (Newson & Richerson, 2009). On the other hand, a hypothesis about biological differences suggests that these sex differences are controlled by evolutionary adaptiveness (Stoet & Geary, 2018). Some evidence indicates that, in countries with higher gender equality and lower gender competitiveness, biological sex differences are more clearly expressed (Stoet & Geary, 2018). In the present study, we tested these hypotheses comparing countries with different levels of socioeconomic development and gender equality on verbal and visuospatial skills and math performance.

Socioeconomic status also influences math performance. Demir, Prado & Booth (2015) showed that children from different socioeconomic levels tend to activate different brain regions, involved in verbal or visuospatial skills, to solve the same arithmetic problems. This finding suggests that SES can play an important role in individual variables that are associated with math performance. The complex interactions of individual and environmental factors are difficult to model.

Several studies have investigated the cognitive and individual variables involved in math performance, however, most of them do not investigate the interaction between the several levels and variables that play an important role in math performance. To investigate whether subtypes of cognitive deficits can be found at the individual level and the interaction between individual and environmental levels we conducted three studies. First, a review was conducted to investigate which variables are most strongly associated with math performance and to build a theoretical model to guide the testing of hypotheses in the next studies. In the second study, a multivariable classification of the cognitive variables underlying math performance was conducted to investigate its heterogeneity and subtyping. Two main groups composed of children with deficits in visuospatial skills and another group composed of children with deficits in number magnitude processing were found. In the third study, a cross-cultural study was conducted to investigate individual and environmental variables that, according to the theoretical model, could impact on math performance. All studies are described in the following sections.

References

Bachot, J., Gevers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychology science*, *47*(1), 172.

Ceci, S. J., Williams, W. M., & Barnett, S. M. (2009). Women's underrepresentation in science: sociocultural and biological considerations. *Psychological bulletin*, *135*(2), 218.

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta psychologica*, *148*, 163-172.

De Smedt, B., & Boets, B. (2010). Phonological processing and arithmetic fact retrieval: evidence from developmental dyslexia. *Neuropsychologia*, *48*(14), 3973-3981.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1-2), 1-42.

Demir, Ö. E., Prado, J., & Booth, J. R. (2015). Parental socioeconomic status and the neural basis of arithmetic: differential relations to verbal and visuo-spatial representations. *Developmental science*, *18*(5), 799-814.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665.

Halpern, D. F., Benbow, C. P., Geary, D. C., Gur, R. C., Hyde, J. S., & Gernsbacher, M. A. (2007). The science of sex differences in science and mathematics. *Psychological science in the public interest*, *8*(1), 1-51.

Kucian, K., Loenneker, T., Dietrich, T., Martin, E., & Von Aster, M. (2005). Gender differences in brain activation patterns during mental rotation and number related cognitive tasks. *Psychology Science*, *47*(1), 112.

Li, Y., & Geary, D. C. (2013). Developmental gains in visuospatial memory predict gains in mathematics achievement. *PloS one*, *8*(7), e70160.

Lopes-Silva, J. B., Moura, R., Júlio-Costa, A., Geraldi Haase, V., & Wood, G. (2014). Phonemic awareness as a pathway to number transcoding. *Frontiers in psychology*, *5*, 13.

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental science*, *17*(5), 714-726.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child development*, *82*(4), 1224-1237.

Newson, L., & Richerson, P. J. (2009). Why do people become modern? A Darwinian explanation. *Population and Development Review*, *35*(1), 117-158.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and individual differences*, *20*(2), 110-122.

Rubinsten, O., & Henik, A. (2009). Developmental dyscalculia: Heterogeneity might not mean different mechanisms. *Trends in cognitive sciences*, *13*(2), 92-99.

Siegler, R. S., & Braithwaite, D. W. (2017). Numerical development. *Annual Review of Psychology*, *68*, 187-213.

Stoet, G., & Geary, D. C. (2018). The gender-equality paradox in science, technology, engineering, and mathematics education. *Psychological science*, *29*(4), 581-593.

Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of experimental child psychology*, *79*(3), 294-321.

Van Mier, H. I., Schleepen, T. M., & Van den Berg, F. C. (2018). Gender differences regarding the impact of math anxiety on arithmetic performance in second and fourth graders. *Frontiers in psychology*, *9*.

van Tetering, M., van der Donk, M., De Groot, R. H. M., & Jolles, J. (2019). Sex Differences in the Performance of 7–12 Year Olds on a Mental Rotation Task and the Relation With Arithmetic Performance. *Frontiers in psychology*, *10*.

Wilson, A. J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. *Human behavior, learning, and the developing brain: Atypical development*, *2*, 212-237.

2. AIMS

Goal

Given the multiplicity of the process involved in math performance, our goal is to investigate how they interact, and they contribute to the development of calculation fluency in different levels.

Objectives:

a. To investigate the factors associated with the acquisition of math skills in elementary school children;

b. To investigate the individual cognitive heterogeneity associated with math achievement and the presence of relatively specific impairments in visuospatial/visuoconstructional, phonological and magnitude processing (non-symbolic and symbolic) in children with math difficulties.

c. To investigate how the interaction between socio-environmental factors such as different cultures, types of school and socioeconomic level and individual factors such as verbal, visuospatial skills and gender interact with each other and influence the acquisition of math skills in elementary school children.

3. METHODS

To investigate the main goals, we conducted three studies. To investigate the factors associated with the acquisition of math skills in elementary school children, we first performed an integrative review and formulated a theoretical model divided into two different levels of interaction with math performance, the environmental level, and the individual level. From this model, it was possible to make better predictions about the outcomes involving math performance.

To investigate the first objective, we conducted a study using a bottom-up strategy to find groups of children with impairments in cognitive abilities previously raised through the theoretical model that could relate to math learning difficulties.

To investigate the second objective, a study of cross-cultural comparison was conducted between Brazil and Italy. In this study, the sex and environmental level factors such as culture and type of school that could influence math performance were investigated.

4. RESULTS

Results will be presented in three sessions with three different studies, one review and two experimental studies. The first experimental study has already been published and is in the format required by the journal.

4.1. The different levels of interaction between environment and individual variables in math performance: a review

The different levels of interaction between environment and individual variables in math performance: a review

Larissa Salvador[1], Vitor Geraldi Haase[2]

1 Department of Psychology, Graduate Program in Children's and Adolescents Health, Federal University of Minas Gerais (UFMG), Belo Horizonte, Brazil

2 Department of Psychology, Graduate Program in Children's and Adolescents Health, Graduate Program in Psychology: Cognition and Behavior, Graduate Program in Neuroscience, Federal University of Minas Gerais (UFMG); National Institute of Science and Technology on Cognition, Behavior, and Teaching (INCT-ECCE), Belo Horizonte, Brazil

Abstract

Many variables involved in the math calculations performance has been a topic of considerable research and debate. We review the literature, with a focus on the individual and environmental variables related to math calculations performance, and consider the following key questions: (1) What are the main variables related to math performance? (2) Math performance can be influenced by individual/ cognitive and environmental levels? (3) What relationships between predictor variables and math performance are still unclear? We build a two-level theoretical model, considering individual and environmental factors, that could directly or indirectly influence math performance. The theoretical model presents the ways in which individual variables such as verbal skills and visuospatial skills interact with sex and socioeconomic level. However, the magnitude processing, considered a specific domain of individual level does not seem to interact with SES and gender. Throughout the review, we consider several points for future research.

Keywords: Magnitude processing, visuospatial skills, verbal skills, sex differences, socioeconomic status, math fluency calculations, math performance

4.1.1 Introduction

Simple and complex arithmetic calculations are important components underlying several math skills. Poor calculation fluency has been studied as one of the most remarkable symptoms in cases of math learning disabilities (Jordan, Hanich & Kaplan, 2003; Geary, 2004**)**. Calculation fluency is considered the ability to solve arithmetic calculations as quickly and accurately as possible. Results of the most elementary, single-digit, commutative operations are stored as arithmetic facts. Children usually acquire good math fluency on simple calculations in the early years of elementary school. When this does not occur, several math skills can later be impaired and difficult to acquire, as in complex calculations that do not directly involve the automatization of these math arithmetic facts.

Several cognitive factors such as numerical magnitude processing, verbal and visuospatial skills seem to be involved in math calculation. Individual variables such as verbal skills play an important role in retrieving facts in long-term memory, contributing to greater proficiency in math. In the complex math fluency calculations, involving more than one digit, visuospatial skills can have an important role. Moreover, verbal and visuospatial skills can be greatly influenced by sex, and socioeconomic status (SES) resulting in an interaction between all these factors.

The interaction between individual factors and environmental factors such as culture, SES and school type can result in a complex pattern of individual and environmental influences on math performance, however, it is not very clear how these several variables that predict math fluency calculations interact with each other.

In the present study, we review the role of the main cognitive variables with sex and socioeconomic level and, tentatively, propose a theoretical model that could explain how these relationships occur. In the next sections, we review the following points: 1- Individual level: 1.1 Number magnitude processing and math calculation, 1.2 Verbal skills and math calculation, 1.3 Visuospatial skills and math calculation, 2: Environmental level: 2.1 Socioeconomic status and educational settings. 3.Interactions among verbal and visuospatial skills and sex on math calculation.

4.1.2 Individual level

Several studies have suggested that math performance can be predicted by a range of individual/cognitive variables that involving the activity of three different circuits on the parietal lobe (Dehaene, Molko, Cohen, & Wilson, 2004). These circuits are related respectively with three different pathways: number magnitude processing, verbal/linguistic skills and visuospatial skills (LeFevre et. al, 2010). The abilities involved in the activation of the parietal lobe traditionally are divided into a specific domain, such as number magnitude processing (Piazza et al, 2010; Shinnider et al., 2017), or general domain, such as verbal skills, visuospatial skills and executive functions/working memory (Fuchs et al., 2010; Simmons, Singleton & Horne, 2008). In the next sessions, we will discuss the main contributions of these variables to the math calculation.

4.1.2.1 Number magnitude processing and math calculation

Numerical cognition has been the target of the growing interest of psychology and cognitive neuroscience researchers in recent decades. This growing interest has brought significant progress on the comprehension of the development of numerical skills and its interface with education. It has great relevance for educational application to know how numerical cognition develops and which skills are most strongly associated with math performance.

There is some important hypothesis about how we develop the concept of number and how it impacts on the development of math skills (Wilson & Dehaene, 2008). Numerical magnitude processing plays an important role in understanding this hypothesis. Numerical magnitude processing comprehends the non-symbolic representation of numerical quantities and the symbolic representations of quantity (Dehaene, 1992). The Approximate Number System (ANS) represents numerical quantities in a non-symbolic and approximate way, is active since infancy, and is shared with other animal species.

Representations in the ANS allow for approximate estimation and number comparison and, probably, correspond to the semantic quantitative content of the numerical representation. An important hypothesis suggests that the ANS could be a *start-up tool* for the development of the symbolic systems (Feigenson, Libertus & Halberda, 2013). *The start-up tools are* domain-specific systems responsible for central knowledge that allow the acquisition of different kinds of knowledge (Piazza, 2010).

Dehaene, Izard, Spelke, and Pica (2008) conducted a study with an Amazonian indigenous population (Munduruku) that does not have an exact symbolic representation for quantities. They showed that although the Mundurukus do not have symbolic numerals that allow accurate representation of numerical quantities, the characteristics of the non-symbolic Munduruku number representations are very similar to those of Europeans controls. The difference is that the non-symbolic representations of the Mundurukus are less accurate than those of the control group. This suggests that ANS acuity can be improved with learning symbols to represent accurately quantities. However, the ANS improvement with formal schooling can be limited and small (Castronovo & Göbel, 2012) and not show an interaction with sex or other environmental factors such as SES.

The acuity of ANS is subject to high inter-individual variation and, according to some evidence, is an important correlated of math performance on tasks such as cardinality, counting and arithmetic calculations (Halberda, Mazzocco & Feigenson, 2008; Mazzocco, Feigenson & Halberda, 2011, Pinheiro-Chagas et. al, 2014). Piazza et al. (2010) compared the ANS-related performance in children with developmental dyscalculia (DD) compared to a control group. The study showed that children with DD at 10 years old have a performance on a non-symbolic task comparison equivalent to that expected for control children at 5 years old, suggesting an important role of ANS in developing math skills. The core deficit hypothesis suggests that deficits in the ANS would impact on the development of the symbolic systems, important to the comprehension of exact quantity, both initially, when children began to use number words to represent quantities, and also later, to comprehend the concept of the Arabic numbers (Wilson & Dehaene, 2007). The core deficit hypothesis predicts that deficits in the ANS could be one underlying cognitive impairment in DD.

On the other hand, some studies did not find a clear-cut association between non-symbolic comparison tasks, assessing the ANS, and math skills (see review in De Smedt, Noel, Gilmore, Ansari, 2013). Moreover, other studies have found an association of the accuracy of symbolic comparison tasks with math performance (Lyons, Price, Vaessen, Blomert & Ansari, 2014; Chen & Li, 2014), suggesting that the access of non-symbolic representations from symbolic representations should be the core deficit in children with DD. The access hypothesis predicts difficulties in tasks involving the connexions between magnitudes and symbols such as counting, fact automatization of simple addition and multiplication, etc. (Linsen, Verschaffel, Reynvoet & De Smedt, 2014). Math tasks that require an association between numerical magnitude comprehension and symbolic processing such as simple calculation share activity in the angular gyrus (van der Ven, Takashima, Segers, Fernández, & Verhoeven, 2016), an important area associated with symbolic representations of quantities (Bloechle et al., 2016; van der Ven et al., 2016).

Currently, it is unclear which kind of impairment, in non-symbolic representation or in the link between the nonsymbolic and the symbolic representations, is strongest associated with developmental dyscalculia. The importance of the ANS or symbolic representation of magnitudes is still much debated (Lourenco & Bonny, 2017; Matejko & Ansari, 2016; Price, Wilkey & Yeo, 2017). In a meta-analysis, Schneider and coworkers (2017) showed that the predictive power of non-symbolic processing in math performance is still unclear. The study evaluated the effect size of the association between symbolic and non-symbolic magnitude comparison tasks and math performance. They showed that the findings are more consistent regarding the symbolic magnitude processing. Moreover, findings regarding non-symbolic magnitude are partly dependent on the type of measurement used to access the number sense acuity, indicating that more empirical studies need to be conducted for a better understanding of the topic.

4.1.2.2 Verbal skills and math calculation

Verbal skills can be divided into a series of interacting subcomponents, such as phonological processing, verbal working memory, vocabulary, among others. Wilson and Dehaene (2007) point to verbal skills related to verbal number representation as one of the three important pathways for developing math skills. They pointed out that verbal skills are mainly related to the symbolic magnitude representation system. The activity of the left angular gyrus in the parietal lobe is important for symbolic verbal representation of numbers (Dehaene, Molko, Cohen, & Wilson, 2004; Piazza, Pinel, Bihan & Dehaene, 2007). These regions are also related to linguistic processing such as reading and phonological processing (Shaywitz et al., 2004).

Libertus, Odic, Feigenson, and Halberda (2016) observed that symbolic verbal representation of numbers is important to the ability to estimate quantities in an approximative way. The results also showed that the variability in the ANS-number word mapping is associated with formal math abilities such as math calculations. This contributes to understanding the link between the variability in the ANS-number word mapping and formal math skills.

Several formal math skills are strongly dependent on verbal skills. Le-Fevre and colleagues presented a pathway model of the main predictors of math performance (LeFevre, et al 2010; LeFevre, Skwarchuk, Kamawar, Bisanz & Smith-Chant, 2015). They observed that verbal skills are related to both initial numerical verbal representation skills involving the verbal number representation and several tasks related to formal math abilities as calculations. According to this model, verbal skills are important for a series of more complex math tasks. The relationship between math performance and complex tasks could be mediated by the system of symbolic representation of quantities. This may explain why several studies have found a contribution of verbal skills to the acquisition of math skills in several age groups throughout development.

However, Dowker and Nuerk (2016) suggested that several types of verbal skills divided into conceptual properties of language, syntactic, semantic, lexical, visuospatial-orthographic such as writing/reading direction of a language, and phonological processing play a specific role in several types of math abilities such as counting, number transcoding and retrieving answers to arithmetic calculations. Moreover, vocabulary acquisition can also play an important role in understanding number- words. Sullivan and Barner (2011) reviewed a series of studies and found that children likely acquire a vocabulary of number words just as they acquire a vocabulary of object-related words. Both are learned by the association between word and object presented or word and the number of objects presented. In this way, they first learn to relate the number one with a single object and more than one with any given amount. Gradually, children acquire more specific vocabularies for a larger number of objects and reach the counting principles until they get to the principle of cardinality when they consequently relate the last counted number to the number of observed objects.

Importantly, verbal skills are also influenced by cultural and socioeconomic differences, involving factors such as the presence of a second language, and language regularity. In addition, sex differences can also influence verbal skills, suggesting an advantage for girls in a range of reading and writing tasks. All these interactions of environmental variables, sex, and verbal skills will be further discussed below. Importantly, empirical studies should consider intra-nation and inter-nation variations for a better understanding of the interplay of environmental factors and verbal skills and their impact on math performance.

4.1.2.3 Visuospatial skills and math calculation

Visuospatial skill is a multidimensional construct involving many kinds of abilities (Mix & Cheng, 2012). These abilities are acquired throughout development and are used to perform tasks of daily life such as predicting distances, tracing routes, identifying forms, doing visual searches, forming mental images, among others (Postma & van der Ham, 2016). Newcombe (2018) proposed the existence of three kinds of spatial cognition, which divided into two dimensions and working memory. The first dimension is related to the intrinsic or extrinsic spatial representation of an object. The second dimension is related to the static or dynamic representations of space. These two dimensions interact with each other and are responsible for several spatial representations. Working memory plays a key role in the ability to manipulate and integrate information between these two dimensions (Newcombe, 2018).

One of the most important functions associated with visuospatial skills is related to math performance. It is important to understand the classification of spatial representations to understand how these dimensions interact with the number comprehension. The integration between number and space has played an important role in the study of numerical cognition. There is strong evidence that numbers are spatially represented on a mental number line, which in Western cultures starts from left to right (Dehaene, Bossini & Giraux, 1993). An evidence for the mental number line is the Spatial-Numerical Association of Response Codes (SNARC) effect (Fischer, 2003). The SNARC effect consists of an association of faster responses with the left-hand for smaller numbers and faster responses with right-hand for larger numbers in tasks of magnitudes comparisons. The SNARC effect suggests a spatial representation of numbers-oriented from smaller numbers on the left to larger numbers on the right.

The SNARC effect has been associated with the ability of number mapping through mental space. Bachot, Gevers, Fias & Roeyers (2005) carried out a study with children, from 7 to 12 years old, to investigate alterations in number representation in individuals with deficits in visuospatial working memory and math difficulties. The researchers evaluated the SNARC effect throughout a digit comparison task and showed that children with deficits in visuospatial working memory presented no SNARC effect. The study concluded that this might be an indication that the math difficulty in children with visuospatial deficits, can be mediated by difficulties in the representation of numerical quantities.

The space-number representations are traditionally assessed by the number line task. In this type of tasks, children are required to locate a number on a continuous line beginning at 0 and ending at 10, 100, or 1000 according to each study. Schneider and colleagues (2018) showed in a meta-analysis that the ability of number space mapping can be associated with the math performance and strongly associated with complex math calculations and fractions. The correlation of math competencies with the number line task was 0.443. This association between math performance and the number line task is higher when compared with other significant predictors of math competence, such as numerical magnitude comparison tasks (Schneider et al., 2017). These findings demonstrate that the association between number and space is a robust tool for diagnosing and predicting broader math performance.

According to Newcombe's model, the spatial representation of numbers in a mental line would be related to the intrinsic and dynamic spatial representations. This type of spatial representation can be accessed by mental rotation tasks. Gunderson and colleagues (2012) examined the relationship between the mental number line, math performance, and mental rotation. They showed that the accuracy of the number line task at age 6 mediated the relation between mental rotation at age 5 and math performance. This suggests that mental rotation assesses a similar pool of abilities required by the representation and manipulation of numerical magnitude in a mental number line.

Numerical magnitude processing on the brain has been associated with the activity of parietal areas which are also involved in intrinsic spatial manipulation. Sokolowski, Ononye, and Ansari (2019) compared differences in the brain activations during tasks of magnitude processing, mental rotation, and arithmetic. They observed that all three cognitive processes shared activation of regions around the intraparietal sulcus (IPS). However, the specific patterns of activation were observed for numerical and arithmetic processing and for arithmetic and mental rotation. Results suggest that different brain regions are specialized in symbolic number representation and mental rotation, both sharing important activation areas with arithmetic performance. This indicates an important role of mental manipulation in math performance.

Several studies have investigated the role of mental rotation on math performance. In a cross-sectional study, Mix and colleagues (2016) observed that mental rotation was the best predictor of math performance in Kindergarten, and visuospatial working memory was associated with the math performance in later grades. Both visuospatial abilities were important to understanding place value, word problems, calculation, fraction concepts and algebra (Mix et. al, 2016). Of note, visuospatial working memory can be associated with a kind of math task that involves some manipulation of spatial information. Caviola, Mammarella, Lucangeli, and Cornoldi (2014) observed that visuospatial working memory was important for children's performance in subtraction tasks involving carrying procedures. However, visuospatial working memory was not important for performance in addition tasks.

Interestingly, several spatial representations are not associated exclusively with the number magnitude processing. In an important way, the intrinsic and dynamic spatial dimensions of visuospatial skills have an important role in the ability to number-mapping in a mental number line. In another way, visuospatial working memory, play also an important role in more complex tasks involving procedures, such as transcoding tasks, which require an understanding of rules about place value, calculating tasks that involve carrying procedures and not just the manipulation of quantities. Visuospatial abilities are also important in tasks involving geometric shapes and their spatial arrangements.

It is undeniable that visuospatial skills play an important correlate on math performance and that these skills are also important in numerical magnitude processing. However, it is still important to understand how visuospatial skills and magnitude processing interact in children with math learning disabilities and how these abilities are affected by sex and environmental variables such as SES.

4.1.3 Environmental Level

Several environmental variables such as poverty, school settings, and family involvement can affect a child's school performance. These environmental variables that affect math performance are the majority associated with socioeconomic status (SES). In the next session, we will discuss the role of SES in child development and its consequent impact on math skills.

4.1.3.1 Interaction between math performance and SES

Children exposed to low socioeconomic status often have worse school performance compared to their peers. This may happen as SES seems to directly influence several cognitive skills such as language and visuospatial skills. In a longitudinal cohort study Hair, Hanson, Wolfe and Pollak (2015) accessed the magnetic resonance Imaging of children from different SES. They demonstrate that environmental factors such as environmental stimulation, parental nurturance, and early life stress affect negatively the school achievement and also the regional gray matter volumes in frontal and parietal lobes and hippocampus. They suggested that poor family resources can result in different brain volume and, consequently in poor children's educational outcomes.

Differences found at the brain level and in the acquisition of academic skills in several SES can be explained in different ways. Data from the Longitudinal Birth Cohort Study observed that children from lower SES had younger mothers, less frequent parent reading, less home computer use, and fewer books at home (Larson, Russ, Nelson, Olson & Halfon, 2015). They also analyzed the variance explained for each factor in reading and math performance. The study found that factors such as parental style/beliefs and home learning environment were the best variables to explain the gap between low SES group and high SES group on reading and math achievement. However other variables such as family background and early education were also important to explain the gap between the two groups of SES. This type of results highlights an important role of parental simulations on school achievement.

Parental SES can also influence cognitive abilities directly related to math achievement such as verbal and visuospatial skills. Demir-Lira, Prado and Booth (2015) examined the link between parental SES and the neural bases of subtraction in school-age children. They observed a different pattern of brain activations during a subtraction task. Children from lower SES levels presented higher recruitment of the right parietal cortex which was also activated during a control visuospatial task. Children from higher SES levels presented a stronger activity of the left temporal cortex on the same subtraction task however they activated regions also during a control verbal task. The study concluded that differences in parental SES can be responsible for children to engage different neural systems based on verbal and visuospatial skills to solve subtraction problems.

Moreover, other factors than parental SES are also involved in environmental effects on school achievement. Another important point is related to school settings. The school environment involves multiple factors such as availability of resources, teacher profile, quality of education that can interact with each other and influence the children's academic acquisition. McCormick, O'Connor & Horn (2017) demonstrated the important role of teacher closeness with children on math performance. The study demonstrated that a close relationship with teachers can be a protective factor for children of low parental SES regarding math performance. In contrast, children who have some kind of conflict with teachers may be more exposed to the negative effects of a low SES.

The quality of education also plays an important role in math achievement. In most countries, schools are divided between State-runand private schools. The quality of education involved in each of them will vary considerably according to the country. For example, in countries such as Brazil and Spain, the best quality of education and higher SES is related to private schools (Zinovyeva, Felgueroso, and Vázquez 2008; Sampaio and Guimarães, 2009). In many countries that presented a high percentage of children in private schools is documented that they present a relatively advantaged position compared to those enrolled in State-run schools (Cebolla-Boado and Medina 2011).

In contrast, countries such as Italy, the Czech Republic, and Norway the public schools represent a high level of quality and theses countries also presented low rates of children in private schools (Bodovski, Byun, Chykina & Chung, 2017). Bodovski and colleagues (2017) analyzed the main factors that could be associated with a higher quality of education. They found that in these countries with the highest scores on standardized math tests, the high quality of education was associated with a higher level of standardization of educational system, greater expenditure on education and the country's income inequality. The findings reinforce the idea that the school setting is associated with several factors involved with the SES as family context, teacher-student relationship and the standardization methods used in teaching.

SES has a complex influence on math performance. Such a statement must be considered in empirical studies. Importantly, the literature points to both important intra-cultural and cross-cultural differences, these aspects should be considered in future studies.

4.1.4 Interactions among verbal and visuospatial skills and sex on math calculation

Sex differences have been a broad field of research when it comes to verbal and visuospatial skills and math abilities (Andreano and Cahill, 2009 for a review; Reilly, 2012). Some studies point to an advantage for girls in tasks involving verbal skills while boys perform better on tasks involving visuospatial skills and mathematics (Van Tetering, Van der Donk, De Groot & Jolles, 2019; Stoet and Geary, 2013; Hoffman, Gneezy & List, 2011). This suggests that these sex differences in visuospatial skills could be the precursors of sex differences in math performance.

Biological hypotheses about sex differences suggest that visuospatial skills develop differently for boys and girls based on a relationship between sex hormone levels and spatial representation (Kelly, Ostrowski, & Wilson 1999; Kimura, 2002). However, more recent theories consider sex differences in visuospatial skills from a biopsychosocial perspective, in which both biological (sex) and psychosocial variables (gender stereotype) modulate spatial abilities (Hausmann, Schoofs, Rosenthal & Jordan, 2009; Levine, 2016). Pletzer & Harris (2019) assessed 41 men and 41 women to investigate the interaction between sex-related hormones and gender stereotype in visuospatial skills. Sex differences were found, with men outperforming women. Several interactive effects between gender stereotype and sex hormones (progesterone, estradiol and testosterone levels) were identified, suggesting that both biological and social influences play important role in visuospatial performance.

Interestingly, sex differences in visuospatial skills are not found for all types of tasks (Newcombe, 2017). The most consistent findings are for mental rotation tasks; differences can also be found in tasks such as navigation, discrimination of line orientation and Piaget’s water level task (Levine et al., 2016). Mental rotation is important as a mediator between other variables and math performance. Maloney, Waechter, Risko, and Fugelsang (2012) showed that mental rotation can mediate the relationship between math anxiety and performance in arithmetic tasks, which could contribute to the sex differences often encountered in math achievement. Mental rotation seems to play an important role as a mediator between sex differences and math performance. Several studies have called attention to sex differences when it involves the local and global perception of a stimulus, however, in visuoconstructional tasks, these differences are not very consistent (Kramer, Ellenberg Leonard & Share, 1996).

Pruden, Levine & Huttenlocher (2011) observed that sex differences can already be found in very young children. Moreover, these differences in both mathematic and visuospatial skills are weaker in early school life and intensify over the years, suggesting a strong environmental influence in maintaining and widening these sex differences (Neuburger, Jansen, Heil & Quaiser-Pohl, 2011). An important environmental factor is the stereotype threat. Girls' math performance is susceptible to the belief that boys are better at tasks involving visuospatial and arithmetic skills (Spencer, Steele & Quinn, 1999). Stereotype threat in both visuospatial and math skills contributes to performance differences between sex (Pennington, Heim, Levy & Larkin, 2016).

It is noteworthy, that several studies did not find these sex differences in math performance and propose a similarity hypothesis. For example, Guiso, Monte, Sapienza, and Zingales (2008) analyzed cross-national data on math and reading scores, classifying countries according to the level of gender equality. It was found that there is considerable variability among countries, but on average, girls perform lower than boys in math and better than them in reading. When gender equality was considered, these differences disappeared suggesting that girls have an advantage in reading scores and perform as well as boys in mathematics (Guiso et. al, 2008). Stoet and Geary (2013) justify these types of finding by showing that gender differences will be most evident at the extremes of the performance distribution, but not on average performance. However, when they analyzed the same data in countries with higher gender equality, they showed that this pattern of differences in extremes are inconclusive, which highlights these sex differences can be influenced strongly from the culture and socioeconomic development status (Quest, Hyde & Linn, 2010)

With respect to verbal skills, sex differences are consistent for skills such as reading and writing showing an advantage for girls. PISA data have shown that these differences tend to be even greater in more developed countries, with higher gender equality. The same is not true for mathematics. Stoet and Geary (2018) evaluated PISA data for reading/writing and math across countries and showed that in countries with higher gender equality the sex differences in verbal and math skills are greater than in countries with lower gender equality. They also performed an intra-individual comparison in these countries with higher gender equality and showed that around 50% of girls have greater potential in mathematics than in verbal skills while for boys this figure rises to 80%. The study concludes that these sex differences are associated with the biological predispositions factors. However, it is noteworthy that even in countries with around 50% of girls with higher potential in mathematics than in verbal skills, the number of women in the STEM disciplines is yet very low.

Despite the still inconclusive studies around the hypothesis about sex differences, it is clear that this is a variable that interacts with the several predictor variables of math performance. It is important that future studies always consider sex as an important covariate when it comes to math skills.

Summary

The present study reviewed the impact of different variables at the individual level, such as numerical magnitude processing, verbal skills and visuospatial skills, and environmental levels, such as SES, school setting and cultural factors that can interact with sex and impact on math performance.

According to Wilson and Dehaene (2007), the development of math skills is related to the three major pathways involving numerical magnitude processing, verbal skills, and visuospatial skills. However, these pathways interact with each other and with environmental variables such as socioeconomic status, school setting, and cultural differences. Moreover, these variables may be further influenced by sex differences.

Figure 1 shows a model proposed by the present study to represent the interactions between cognitive skills and environmental factors and sex. The model shows how these interactions at several levels can influence math performance.

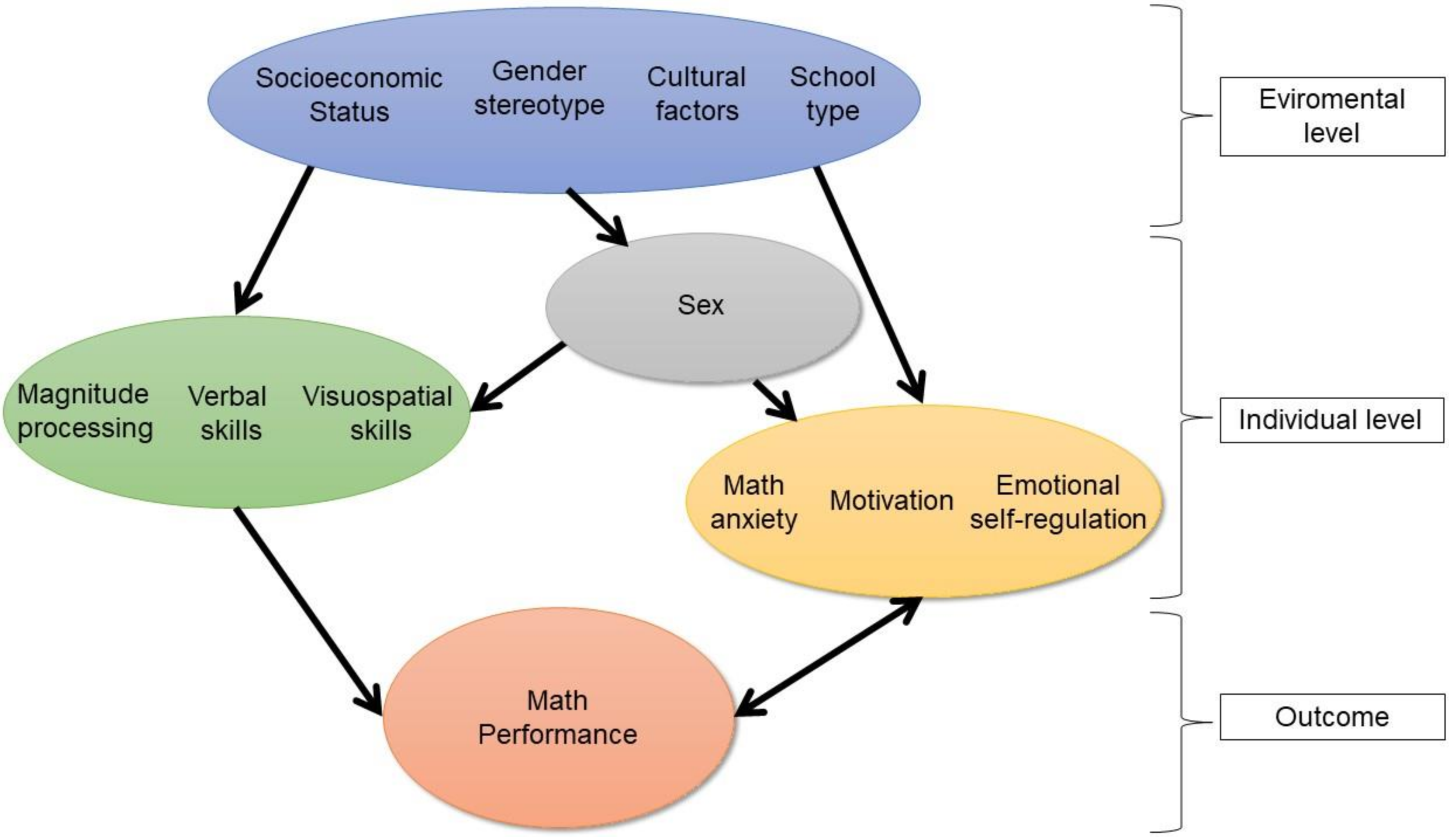

Figure 1: Theoretical model of interaction between variables which contribute to math performance at different levels

The theoretical model, based on the literature review, points to two main important discussions: 1-The need for further studies using multivariate techniques for classifying subtypes of math learning disabilities and, the need to simultaneously consider the interaction of cognitive differences by sex and socio-cultural variables. Further empirical studies are needed to investigate the interaction between these levels.

4.1.6 References

Andreano, J. M., & Cahill, L. (2009). Sex influences on the neurobiology of learning and memory. *Learning & memory*, *16*(4), 248-266.

Bachot, J., Gevers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychology science*, *47*(1), 172.

Bloechle, J., Huber, S., Bahnmueller, J., Rennig, J., Willmes, K., Cavdaroglu, S., ... & Klein, E. (2016). Fact learning in complex arithmetic—The role of the angular gyrus revisited. *Human Brain Mapping*, *37*(9), 3061-3079.

Bodovski, K., Byun, S. Y., Chykina, V., & Chung, H. J. (2017). Searching for the golden model of education: cross-national analysis of math achievement. *Compare: A Journal of Comparative and International Education*, *47*(5), 722-741.

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS one*, *7*(4), e33832.

Caviola, S., Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2014). Working memory and domain-specific precursors predicting success in learning written subtraction problems. *Learning and Individual Differences*, *36*, 92-100.

Cebolla-Boado, H., & Garrido Medina, L. (2010). The impact of immigrant concentration in Spanish schools: school, class, and composition effects. *European Sociological Review*, *27*(5), 606-623.

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta psychologica*, *148*, 163-172.

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*(2), 48-55.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1-2), 1-42.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, *122*(3), 371.

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *science*, *320*(5880), 1217-1220.

Dehaene, S., Molko, N., Cohen, L., & Wilson, A. J. (2004). Arithmetic and the brain. *Current opinion in neurobiology*, *14*(2), 218-224.

Demir-Lira, Ö. E., Prado, J., & Booth, J. R. (2016). Neural correlates of math gains vary depending on parental socioeconomic status (SES). *Frontiers in psychology*, *7*, 892.

Dowker, A., & Nuerk, H. C. (2016). Linguistic influences on mathematics. *Frontiers in psychology*, *7*, 1035.

Else-Quest, N. M., Hyde, J. S., & Linn, M. C. (2010). Cross-national patterns of gender differences in mathematics: a meta-analysis. *Psychological bulletin*, *136*(1), 103.

Fischer, M. (2003). Spatial representations in number processing--evidence from a pointing task. *Visual cognition*, *10*(4), 493-508.

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M., ... & Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities?. *Developmental psychology*, *46*(6), 1731.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of learning disabilities*, *37*(1), 4-15.

Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science*, *320*(5880), 1164-1165.

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: the role of the linear number line. *Developmental psychology*, *48*(5), 1229.

Hair, N. L., Hanson, J. L., Wolfe, B. L., & Pollak, S. D. (2015). Association of child poverty, brain development, and academic achievement. *JAMA pediatrics*, *169*(9), 822-829.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665.

Hausmann, M., Schoofs, D., Rosenthal, H. E., & Jordan, K. (2009). Interactive effects of sex hormones and gender stereotypes on cognitive sex differences—A psychobiosocial approach. *Psychoneuroendocrinology*, *34*(3), 389-401.

Hawes, Z., Sokolowski, H. M., Ononye, C. B., & Ansari, D. (2019). Neural underpinnings of numerical and spatial cognition: An fMRI meta-analysis of brain regions associated with symbolic number, arithmetic, and mental rotation. *Neuroscience & Biobehavioral Reviews*.

Hoffman, M., Gneezy, U., & List, J. A. (2011). Nurture affects gender differences in spatial abilities. *Proceedings of the National Academy of Sciences*, *108*(36), 14786-14788.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child development*, *74*(3), 834-850.

Kelly, S. J., Ostrowski, N. L., & Wilson, M. A. (1999). Gender differences in brain and behavior: hormonal and neural bases. *Pharmacology Biochemistry and Behavior*, *64*(4), 655-664.

Kimura, D. (2002). Sex hormones influence human cognitive pattern. *Neuroendocrinology Letters*, *23*(4), 67-77.

Kramer, J. H., Ellenberg, L., Leonard, J., & Share, L. J. (1996). Developmental sex differences in global-local perceptual bias. *Neuropsychology*, *10*(3), 402.

Larson, K., Russ, S. A., Nelson, B. B., Olson, L. M., & Halfon, N. (2015). Cognitive ability at kindergarten entry and socioeconomic status. *Pediatrics*, *135*(2), e440-e448.

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child development*, *81*(6), 1753-1767.

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child*

development, *81*(6), 1753-1767.

Levine, S. C., Foley, A., Lourenco, S., Ehrlich, S., & Ratliff, K. (2016). Sex differences in spatial cognition: Advancing the conversation. *Wiley Interdisciplinary Reviews: Cognitive Science*, *7*(2), 127-155.

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability?. *Learning and individual differences*, *25*, 126-133.

Libertus, M. E., Odic, D., Feigenson, L., & Halberda, J. (2016). The precision of mapping between number words and the approximate number system predicts children's formal math abilities. *Journal of experimental child psychology*, *150*, 207-226.

Linsen, S., Verschaffel, L., Reynvoet, B., & De Smedt, B. (2014). The association between children's numerical magnitude processing and mental multi-digit subtraction. *Acta Psychologica*, *145*, 75-83.

Lourenco, S. F., & Bonny, J. W. (2017). Representations of numerical and non-numerical magnitude both contribute to mathematical competence in children. *Developmental Science*, *20*(4), e12418.

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental science*, *17*(5), 714-726.

Maloney, E. A., Waechter, S., Risko, E. F., & Fugelsang, J. A. (2012). Reducing the sex difference in math anxiety: The role of spatial processing ability. *Learning and Individual Differences*, *22*(3), 380-384.

Matejko, A. A., & Ansari, D. (2016). Trajectories of symbolic and nonsymbolic magnitude processing in the first year of formal schooling. *PloS one*, *11*(3), e0149863.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS one*, *6*(9), e23749.

McCormick, M. P., O'Connor, E. E., & Horn, E. P. (2017). Can teacher-child relationships alter the effects of early socioeconomic status on achievement in middle childhood?. *Journal of school psychology*, *64*, 76-92.

Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. In *Advances in child development and behavior* (Vol. 42, pp. 197-243). JAI.

Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, *145*(9), 1206.

Neuburger, S., Jansen, P., Heil, M., & Quaiser-Pohl, C. (2011). Gender differences in pre-adolescents' mental-rotation performance: Do they depend on grade and stimulus type?. *Personality and Individual Differences*, *50*(8), 1238-1242.

Newcombe, N. (2017). Harnessing spatial thinking to support stem learning. OECD

Pennington, C. R., Heim, D., Levy, A. R., & Larkin, D. T. (2016). Twenty years of stereotype threat research: A review of psychological mediators. *PloS one*, *11*(1), e0146487.

Piazza, M. (2011). Neurocognitive start-up tools for symbolic number representations. In *Space, Time and Number in the Brain* (pp. 267-285). Academic Press.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., ... & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*(1), 33-41.

Piazza, M., Pinel, P., Le Bihan, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron*, *53*(2), 293-305.

Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., ... & Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation?. *PloS one*, *9*(11), e111155.

Pletzer, B., & Harris, T. (2019). Beyond biological sex: Interactive effects of gender role and sex hormones on spatial abilities. *Frontiers in neuroscience*, *13*, 675.

Postma, A., & van der Ham, I. J. (2016). *Neuropsychology of space: spatial functions of the human brain*. Academic Press.

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter?. *Developmental science*, *14*(6), 1417-1430.

Reilly, D. (2012). Gender, culture, and sex-typed cognitive abilities. *PloS one*, *7*(7), e39904.

Sampaio, B., & Guimarães, J. (2009). Diferenças de eficiência entre ensino público e privado no Brasil. *Economia Aplicada*, *13*(1), 45-68.

Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental science*, *20*(3), e12372.

Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child development*, *89*(5), 1467-1484.

Shaywitz, B. A., Shaywitz, S. E., Blachman, B. A., Pugh, K. R., Fulbright, R. K., Skudlarski, P., ... & Fletcher, J. M. (2004). Development of left occipitotemporal systems for skilled reading in children after a phonologically-based intervention. *Biological psychiatry*, *55*(9), 926-933.

Simmons, F., Singleton, C., & Horne, J. (2008). Brief report—Phonological awareness and visual-spatial sketchpad functioning predict early arithmetic attainment: Evidence from a longitudinal study. *European Journal of Cognitive Psychology*, *20*(4), 711-722.

Sowinski, C., LeFevre, J. A., Skwarchuk, S. L., Kamawar, D., Bisanz, J., & Smith-Chant, B. (2015). Refining the quantitative pathway of the Pathways to Mathematics model. *Journal of Experimental Child Psychology*, *131*, 73-93.

Spencer, S. J., Steele, C. M., & Quinn, D. M. (1999). Stereotype threat and women's math performance. *Journal of experimental social psychology*, *35*(1), 4-28.

Stoet, G., & Geary, D. C. (2013). Sex differences in mathematics and reading achievement are inversely related: Within-and across-nation assessment of 10 years of PISA data. *PloS one*, *8*(3), e57988.

Stoet, G., & Geary, D. C. (2018). The gender-equality paradox in science, technology, engineering, and mathematics education. *Psychological science*, *29*(4), 581-593.

Sullivan, J., & Barner, D. (2011). Number words, quantifiers, and principles of word learning. *Wiley Interdisciplinary Reviews: Cognitive Science*, *2*(6), 639-645.

van der Ven, F., Takashima, A., Segers, E., Fernández, G., & Verhoeven, L. (2016). Non-symbolic and symbolic notations in simple arithmetic differentially involve intraparietal sulcus and angular gyrus activity. *Brain research*, *1643*, 91-102.

van Tetering, M., van der Donk, M., De Groot, R. H. M., & Jolles, J. (2019). Sex Differences in the Performance of 7–12 Year Olds on a Mental Rotation Task and the Relation With Arithmetic Performance. *Frontiers in psychology*, *10*.

Wilson, A. J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. *Human behavior, learning, and the developing brain: Atypical development*, *2*, 212-237.

Yeo, D. J., Wilkey, E. D., & Price, G. R. (2017). The search for the number form area: A functional neuroimaging meta-analysis. *Neuroscience & Biobehavioral Reviews*, *78*, 145-160.

Zinovyeva, N., Felgueroso, F., & Vega, P. V. (2008). *Immigration and students' achievement in Spain* (No. 2008-37).

4.2. Cognitive heterogeneity of math difficulties: a bottom-up classification approach

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Empirical Research

Cognitive Heterogeneity of Math Difficulties: A Bottom-up Classification Approach

Larissa de Souza Salvador* [a], Ricardo Moura[b], Guilherme Wood[c], Vitor Geraldi Haase[de]

[a] Department of Psychology, Graduate Program in Children's and Adolescents Health, Federal University of Minas Gerais (UFMG), Belo Horizonte, Brazil. **[b]** Department of Basic Psychological Processes, Institute of Psychology, University of Brasília, (UnB), Brasília, Brazil. **[c]** Department of Neuropsychology, Institute of Psychology, Karl-Franzens University of Graz, Graz, Austria. **[d]** Department of Psychology, Graduate Program in Children's and Adolescents Health, Graduate Program in Psychology: Cognition and Behavior, Graduate Program in Neuroscience, Federal University of Minas Gerais (UFMG), Belo Horizonte, Brazil. **[e]** National Institute of Science and Technology on Cognition, Behavior and Teaching (INCT-ECCE), Belo Horizonte, Brazil.

Abstract

Math learning difficulties (MD) correspond to math achievement below the 25th percentile and are cognitively heterogeneous. It is not known precisely how cognitive mechanisms underlie distinct subtypes of MD. A bottom-up, cluster-analytic strategy, based on visuoconstructional, visuospatial and phonological working memory, and non-symbolic and symbolic magnitude processing accuracy, was used to form subgroups of children from 3rd to 5th grades according to their math achievement. All children had nonverbal intelligence above the 20th percentile and presented a broad spectrum of variation in math ability. External validity of subgroups was examined considering intelligence and math achievement. Groups did not differ in age. Two groups with a high incidence of MD were associated, respectively, with low visuospatial/visuoconstructional and low magnitude processing accuracy. One group with average cognitive performance also presented above average intelligence and a small incidence of MD. A fourth group with high cognitive performance presented high math performance and high intelligence. Phonological working memory was associated with high but not with low math achievement. MD may be related to complex patterns of associations and dissociations between intelligence and specific cognitive abilities in distinct subgroups. Consistency and stability of these subgroups must be further characterized. However, a bottom-up classification strategy contributes to reducing the cognitive complexity of MD.

Keywords: math learning difficulties, heterogeneity, intelligence, top-down strategy, bottom-up strategy, cluster analysis

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*Corresponding author at: Developmental Neuropsychology Laboratory, Department of Psychology, Federal University of Minas Gerais, Av. Antônio Carlos, 6627, 31270-901 Belo Horizonte, Minas Gerais, Brazil. Tel/Fax: +55-31-3409-6295. E-mail: larissassalvador@gmail.com

Individuals with math learning difficulties (MD) can have problems in a wide range of numerical skills, such as estimating and comparing magnitudes, reading and writing Arabic numbers, mastering the four basic operations, retrieving the math tables, among others (Butterworth, Varma, & Laurillard, 2011; Wilson & Dehaene, 2007). Besides personal, familial, pedagogical and social factors, individual differences among typical and atypical math achievers are also influenced by neurocognitive factors. These factors include basic number, phonological, visuospatial/visuoconstructional, working memory/executive functions processing (Wilson & Dehaene, 2007), and motivational/emotional self-regulation (Dowker, Sarkar, & Looi, 2016). Understanding the cognitive mechanisms underlying MD is an essential goal of the numerical cognitive research, as distinct interventions may be required according to the cognitive profile (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014).

One of the main issues about the neuropsychological research on mathematics is the high variability of cognitive factors associated with math achievement, what is reflected in a high heterogeneity of deficits among children with MD (Rubinsten & Henik, 2009). However, more studies are needed to characterize the cognitive mechanisms underlying MD and their possible connections with subtypes of cognitive impairment.

Different criteria to identify subtypes of MD have been supported by the literature, but this becomes more complex when discussing the difficulty in using an appropriate criterion for the identification of the MD group (see Kaufmann et al., 2013). The most traditional criteria consider the discrepancy between IQ and math abilities, but the use of a cut-off based only on a standardized mathematics task is also a common criterion to find chil- dren with MD. A cut-off based on performance lower than 5th or 10th percentile in a standardized math task would increase the probability of finding children with more severe and clearly defined cognitive deficits. On the other hand, more liberal criteria, such as 25th percentile, could be associated with a difficult to characterize pat- terns of cognitive impairment among these children (Mazzocco, 2007).

Classification systems based only on standardized math tests could present some problems since they do not consider, for example, the importance of the stability and persistence of cognitive profiles associated with MD (Mazzocco & Myers, 2003). Wong, Ho, and Tang (2014) in a longitudinal study using a latent class growth analysis approach were able to find a range of children who had a lower performance in mathematics over three years and a small acquisition of math abilities when compared with their peers. This group was characterised by a cognitive profile compatible with the deficits usually found in children with MD. Another group of children also presented low acquisition of mathematical abilities over years, but this group was associated with a low SES profile and average cognitive skills. A difficult to establish a consistent criterion and methods for finding the MD group could be associated with the high heterogeneity in MD cognitive profile.

This Introduction is divided into four sections. First, we discuss the cognitive heterogeneity underlying math difficulties. Second, we discuss research approaches to subtyping MD, analytically reducing this complexity. Third, we review the literature that used the bottom-up approach of complexity reduction and classification in which subgroups of MD individuals emerged through cluster analysis. Finally, we discuss the approach used in the current study.

Cognitive Heterogeneity of Mathematical Difficulties

Cognitive factors underlying MD have been discussed in the numerical cognitive research and several subtypes of MD have been found (Rubinsten & Henik, 2009). Currently, the MD subtypes have been related to general mechanisms such as working memory/executive functions (Bull & Lee, 2014; Raghubar, Barnes, & Hecht, 2010; Swanson & Jerman, 2006), phonological processing (Lopes-Silva, Moura, Júlio-Costa, Haase, & Wood, 2014; Simmons & Singleton, 2008) and visuospatial processing (Barnes & Raghubar, 2014; Mammarella, Lucangeli, & Cornoldi, 2010; Venneri, Cornoldi, & Garuti, 2003), or math-specific mechanisms related to non- symbolic (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Landerl, Bevan, & Butterworth, 2004; Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014; Schneider et al., 2016) and symbolic number processing (Chen & Li, 2014; De Smedt & Gilmore, 2011; Fazio et al., 2014; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007; Schneider et al., 2016). This variability of cognitive factors results in different MD profiles (Geary, 1993; Karagiannakis et al., 2014; Kosc, 1974; Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015; Wilson & Dehaene, 2007). Nevertheless, the criteria for defining the MD profiles, based on cognitive deficits, have varied according to each study.

Cognitive Factors Associated With Math Achievement

The search for relevant dimensions to identify subtypes of MD can be based on the analysis of patterns of association and dissociation between the relevant cognitive variables. The most investigated cognitive dimen- sions in the math domain are discussed below.

Working memory/executive functions — Math is considered to be the most difficult subject in school (Mazzocco, Hanich, & Noeder, 2012) and every new acquisition in arithmetic places heavy demands on working memory/executive functions (WM/EF, Bull & Lee, 2014; McLean & Rusconi, 2014; Raghubar et al., 2010). This is the case with counting (Camos, Barrouillet, & Fayol, 2001), learning single-digit operations and facts (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), transcoding between numerical notations (Camos, 2008; Moura et al., 2013) and word problem solving (Andersson, 2007; Costa et al., 2011).

Some authors have proposed that EF plays a role also in tasks taping on non-symbolic magnitude representations (Gilmore et al., 2013; Hohol, Cipora, Willmes, & Nuerk, 2017; Merkley, Thompson, & Scerif, 2016). Gilmore et al. (2013), for example, showed that, in trials where the number of dots is negatively correlated to other visual parameters, such as occupied area and luminance, participants use inhibitory control abilities to avoid answering to visual parameters not related to numerosity. Moreover, in this study the authors also indica- ted that inhibitory control, but not the precision of underlying numerical representations, is related to achieve- ment in mathematics. Merkley, Thompson, and Scerif (2016) showed that inhibitory control in preschoolers, measured through a Stroop paradigm task, is highly correlated to the accuracy in a non-symbolic magnitude comparison task.

Impairments in WM/EF could explain the comorbidity between MD and attention-deficit/hyperactivity disorder (ADHD). However, impairments in WM/EF are nonspecific, being present in virtually all neurodevelopmental disorders (Johnson, 2012). To the best of our knowledge, there is no report of cases of MD with impairment restricted to WM/EF.

Phonological processing — The term phonological processing is commonly used to refer to a set of abilities frequently impaired in developmental dyslexia such as rapid automatized naming, phonological working memory and phonemic awareness (Wagner & Torgesen, 1987). Some math-related abilities are also critically dependent on phonological processing. For example, an association with phonological processing has been found for number reading and writing (Lopes-Silva et al., 2014), transcoding (Lopes-Silva et al., 2016), arithmetic fact retrieval (De Smedt & Boets, 2010) and word problem solving (Swanson & Sachse-Lee, 2001). These are all domains of numerical cognition frequently impaired in developmental dyslexia (Simmons & Singleton, 2008). Impairments in phonological processing could explain the comorbidity between MD and dyslexia.

Visuospatial/visuoconstructional processing — Mental spatial representation, manipulation and visuospatially guided control of action are involved in several math-related abilities such as number representation in the mental number line (Dehaene, Bossini, & Giraux, 1993; Wood & Fischer, 2008), place-value understanding (Dietrich, Huber, Dackermann, Moeller, & Fischer, 2016), numerical transcoding (Camos, 2008) and multidigit calculation (Raghubar et al., 2009). Associations have repeatedly been found between math learning difficulties and visuospatial impairments in individuals labeled as having nonverbal learning disability (Bachot, Gevers, Fias, & Roeyers, 2005; Johnson & Myklebust, 1967; Mammarella et al., 2010; Rourke, 1989, 1995; Venneri et al., 2003). However, a specific subtype of MD, related to impaired visuospatial/visuoconstructional processing, has been difficult to characterize (Geary, 1993; Wilson & Dehaene, 2007).

Magnitude processing — The expression numerical processing is applied to identify the ability to quantify numerosities, such as estimating set sizes, comparing sets and counting; also, to identify the ability to use numerical notations. Non-symbolically represented sets of up to 4 elements are quantified rapidly and accurately through subitizing, which seems to depend on visual attentional, rather than on quantitative, processes (Hyde & Spelke, 2011). Larger numbers, above the upper limit of the subitizing range, are represented analogically and approximately. Representational imprecision increases with the number magnitude, describing a logarithmically compressed distribution oriented from left to right (mental number line). It has been postulated that there an approximate number system (ANS) underlying this mental number line (Dehaene, 1992; Dehaene & Cohen, 1995; Hyde & Spelke, 2011). Numbers are also represented symbolically through verbal and visual notation systems. Symbolic notation systems allow the accurate representation of numerosities larger than 4; and the accuracy is largely dependent on counting-based procedures.

Workings of the ANS are characterized by the two basic psychophysical laws of Weber and Fechner, which explain several behavioral effects observed in numerical processing (Dehaene, Dupoux, & Mehler, 1990). The Weber law is operationalized by the distance and ratio effects. As the numerical difference between two numbers decreases, their discriminability also decreases. The corresponding distribution is scalar or ratio. The Weber fraction is the minimal numerical magnitude difference that can be discriminated (just-noticeable differ- ence). The size effect is interpreted as an instance of the Fechner law, which states that discrimination between larger numerical magnitudes is more difficult than between smaller ones. The size effect can be explained as- suming a logarithmically compressed function describing the relationship between the stimuli numerosities and their mental representations. The distance, ratio and size effects agree with the hypothesis of an analogic representation of numbers on a spatially oriented mental number line. Moreover, the fact that it is easier to react to small digits with the left hand and to larger digits with the right hand suggests that the association between magnitude and position on the number line is spatially oriented (SNARC effect, or spatial-numerical association of response codes, Dehaene et al., 1993).

Accuracy of the ANS has been proposed as a predictor of complex arithmetic abilities, and that ANS represents a core system from which human mathematical thinking emerges. ANS accuracy, assessed using the Weber fraction, has been associated with both typical (Halberda & Feigenson, 2008) and atypical math achievement (Mazzocco et al., 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014). Several single-case studies are compatible with the hypothesis of ANS impairment being one marker of math learning difficulties (Davidse, de Jong, Shaul, & Bus, 2014; Haase et al., 2014; Júlio-Costa, Starling-Alves, Lopes-Silva, Wood, & Haase, 2015; Ta'ir, Brezner, & Ariel, 1997).

In some studies, the Weber fraction is strongly associated with a specific measure of basic numerical processing (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016; Anobile, Cicchini, & Burr, 2016). However, the predictive power of ANS accuracy for math achievement has not been always replicated (De Smedt & Gilmore, 2011; Rousselle & Noël, 2007). Other evidence indicates that math ability is critically associated with symbolic rather non-symbolic number processing (Geary et al., 2009; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Vanbinst, Ceulemans, Peters, Ghesquière, & De Smedt, 2018). In any case, meta-analytic results indicate that both non-symbolic and symbolic number processing accuracy are only weakly correlated with math achievement. Correlations are slightly stronger for symbolic processing (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2016).

A performance dissociation was observed by Rousselle and Noël (2007). Children with dyscalculia exhibited impairments in symbolic, but not in non-symbolic, number processing (see also De Smedt & Gilmore, 2011). The access deficit hypothesis of dyscalculia was then proposed (De Smedt & Gilmore, 2011; Noël & Rousselle, 2011; Rousselle & Noël, 2007). Accordingly, deficits observed in children with dyscalculia may be attributed ei- ther to representational inaccuracy in the ANS (representational hypothesis) or to difficulty in automatically ac- cessing non-symbolic quantitative representations from symbolic numerals (access hypothesis).

Subtyping Math Difficulties

Early strategies to identify subtypes of MD were top-down or theoretically oriented (Kosc, 1974; Geary, 1993; Wilson & Dehaene, 2007), inspired by cognitive models of number processing and calculation (Dehaene, 1992; Dehaene & Cohen, 1995; McCloskey, Caramazza, & Basili, 1985). Geary (1993) proposed three types of MD. The first was related to difficulties in retrieving arithmetic facts from semantic memory. The second was related to difficulties in the execution of arithmetic procedures, due to low working memory capacity. The third was associated with difficulties in visuospatial representations, resulting in less accurate strategies for problem-solv- ing. Wilson and Dehaene (2007) further suggest the existence of a core numerical deficit associated with ANS related to non-symbolic magnitude processing. They also proposed two other subtypes of MD, one associated with verbal memory, resulting in difficulties in retrieving arithmetic facts, and the other associated with visuospa- tial attention.

One strategy used to identify subtypes of MD involves cognitive-neuropsychological single-case studies. These studies have helped to identify distinct patterns of performance dissociations, or specific domains of impair- ment, in MD. Specific patterns of impairment were observed in Arabic number reading (Temple, 1989), Arabic number writing (Sullivan, 1996), arithmetic procedures (Temple, 1991), arithmetic facts (De Visscher & Noël, 2013; Kaufmann, 2002; Kaufmann, Lochy, Drexler, & Semenza, 2004; Temple, 1991), phonological processing (Haase et al., 2014; Júlio-Costa et al., 2015) and representational accuracy of ANS (Davidse et al., 2014; Haase et al., 2014; Júlio-Costa et al., 2015; Ta'ir et al., 1997). Results from quasi-experimental, cognitive-neu- ropsychological studies are theoretically meaningful, but their relevance to the garden variety of math difficulties is not known.

Analysis of performance dissociations based on group and case studies may be considered a top-down approach to subtyping, as the relevant cognitive dimensions are previously identified and (ideally) pure cases have then sought that fit into the relevant dimensions. The top-down strategy is characterized by several limita- tions. First and foremost, the subtypes are defined a priori, and this may prevent recognition of patterns when these are not predicted by existing theories. Second, patterns of impairments observed in individual cases are not necessarily consistent with the ideal deficits described in theoretical models. For example, a deficit in ANS would be expected to impair subtraction operations, but this is not always the case (Haase et al., 2014). Third, there is not enough information available regarding the inter-test (DeWind & Brannon, 2016; Dietrich et al.,

2016) and test-retest (DeWind & Brannon, 2016; Inglis & Gilmore, 2014; Júlio-Costa et al., 2015) reliability of ANS-related measures. Judging from the dyslexia literature, the possibility of changing diagnoses and patterns of cognitive impairment is a serious concern (Jordan, Wylie, & Mulhern, 2010; Tannock, 2013).

The Bottom-up Approach

The limitations of the top-down approach have led some researchers to pursue the data-driven or bottom-up approach, which consists of letting the groups emerge from multivariate techniques of classification, such as cluster analysis (Archibald, Cardy, Joanisse, & Ansari, 2013; Bartelet, Ansari, Vaessen, & Blomert, 2014; Gray & Reeve, 2016; Osmon, Smerz, Braun, & Plambeck, 2006; Peake, Jiménez, & Rodríguez, 2017; Pieters, Roeyers, Rosseel, Van Waelvelde, & Desoete, 2015; Reeve, Reynolds, Humberstone, & Butterworth, 2012; Vanbinst et al., 2015; von Aster, 2000). Ideally, subgroups of individuals with intragroup similarities and be- tween-group differences on some criterion variables should emerge. Next, the groups must be validated ac- cording to some external criteria, looking for differential patterns of dissociations and associations within rele- vant cognitive dimensions.

von Aster (2000) was the first to show that the top-down approach could be used to identify subtypes of MD. This study assessed 93 school children who performed poorly in arithmetic skills. Measures of the basic num- ber processing and calculation skills were used in cluster analysis, and a three-cluster solution was identified. The subtypes of MD were labeled “Verbal subtype” with difficulties in counting, “Arabic subtype” with difficulties in reading and writing numbers, and “Pervasive subtype” with difficulties in almost all tasks. However, no gener- al cognitive measures were used in this study, so one cannot specify the effect of, for example, general intelligence.

Results from Reeve et al. (2012), Vanbinst et al. (2015), and Wong, Ho, and Tang (2014) suggest that profiles of performance in very basic number processing tasks, such as dot enumeration, numerical comparison and arithmetic may be consistently identified. The identified profiles were longitudinally stable, independent of general intelligence, and predictive of standardized math achievement performance.

Moreover, a few studies have employed a cross-sectional, bottom-up approach focusing on individuals with low math achievement (Bartelet et al., 2014; Gray & Reeve, 2016; Newton & Penner-Wilger, 2015; Osmon et al., 2006; Peake, Jiménez, & Rodríguez, 2017; Pieters et al., 2015; von Aster, 2000). In a study conducted by Peake et al. (2017), subtypes of MD were investigated and interpreted considering the Triple Code Model (Dehaene & Cohen, 1995). The study accessed a sample of 63 MD elementary school children from two age cohorts (3rd to 4th grades, and 5th to 6th grades). They found different groups for each cohort, 4 groups in the first and 3 groups in the second. Two of these groups were shared between the two cohorts: one with quantity and spatial representational deficits; and the other with verbal deficits in number-fact retrieval. In this study, the clusters were formed from a very small sample, which prevented between-group comparisons. Additionally, the non-symbolic representation of magnitudes was not explicitly evaluated.

Bartelet et al. (2014) used different sampling strategies to investigate subgroup formation in the performance of children with math learning difficulties. Some children were demographically identified by low standardized math achievement, and others were clinically referred because of persistent math learning difficulties. Six sub- groups were identified. One group was characterized by impairments in the number line task and relatively mi- nor impairments in math achievement. Two subgroups presented deficits in ANS, assessed as dot comparison.

In one of these, non-symbolic number processing impairment was associated with visuospatial working memory difficulties. A fourth subgroup presented difficulties in symbolic numerical processing, assessed by counting and transcoding abilities. In the fifth subgroup, no cognitive difficulties were identified. Finally, in the sixth subgroup, math difficulties were associated with lower normal intelligence. However, in this study, the authors chose not to use the internal Weber fraction as a measure of ANS.

Generally, the bottom-up strategy can be an alternative for resolving problems associated with the top-down approach, without using arbitrary cutoff scores to find subtypes of MD. This type of strategy allows examining which cognitive mechanisms are associated with individual differences in math learning. In addition, it offers the possibility of investigating how these mechanisms are associated with working memory, phonological processing, visuospatial/visuoconstructional processing and magnitude processing.

The Current Study

In the current study, a bottom-up, data-driven analytical approach was used to identify profiles of cognitive impairments underlying math difficulties; and, a top-down approach was used for a theoretically-driven analysis of subgroup validity. We wanted to examine the hypothesis that relatively specific impairments in visuospatial/ visuoconstructional, phonological and magnitude processing (non-symbolic and symbolic) are associated with standardized math achievement. We also wanted to compare the relative associations of specific cognitive and/or general intelligence factors with math performance within the subgroups. This strategy has not been fre- quently used in research in numerical cognition. The present study is the first to use a bottom-up approach, to explore different MD subtypes, which also includes the internal Weber fraction and a measure of symbolic mag- nitude processing efficiency as criterion variables.

Methods

Sample

The initial sample comprised 290 children, ages from 8 to 11 years, in the 3rd to 5th grades. Participants were assessed in two distinct phases. First, a screening assessment was performed using the Arithmetic subtest from the Brazilian School Achievement Test (TDE; Oliveira-Ferreira et al., 2012; Stein, 1994) and the Raven's Coloured Progressive Matrices (CPM; Angelini, Alves, Custódio Duarte, & Duarte, 1999). Ninety-eight children were excluded from the sample for the following reasons: 28 children scored below the 20th percentile on the Raven's CPM; 55 children did not complete the entire neuropsychological assessment; 15 children either had a poor adjustment on the fitting procedure to calculate their internal Weber fraction (*w*) in the non-symbolic com- parison task ($R^2 < 0.2$) or they showed an internal Weber fraction that exceeded the limit of discriminability of the non-symbolic magnitude comparison task ($w > 0.6$). The final sample comprised 192 children with a mean age of 9.38 (*SD* = 0.84) years. Children scoring above the 25th percentile on the TDE Arithmetic subtest were classified as Controls (*n* = 150) and those scoring below the 25th percentile as having math difficulties (MD, *n* = 42). Afterwards, all children underwent an individual neuropsychological assessment, described below.

The study was approved by the local research ethics committee (COEP–UFMG) in compliance with the Helsinki principles. Informed consent was obtained in written form from parents and orally from children.

Characterization of Math Difficulties

Different research criteria have been used to identify individuals with MD. According to Mazzocco (2007), individuals with developmental dyscalculia or math learning disability (MLD) are characterized by normal intelligence and math achievement below the 5th percentile. This cutoff score identifies a population with a higher probability of presenting severe, persistent and inherent difficulties, probably of genetic origin. The group of individuals with normal intelligence and math achievement below the 25th percentile is labeled math difficulties (MD). In this group, difficulties may be less severe and more variable, with a higher probability of secondary, psychosocial sources of influence. In the present study, we used the MD criterion of the selection below the 25th percentile. The justification is the need for a large enough sample to conduct multivariate analyses. This cutoff is also justified on the grounds that there is considerable genetic continuity between typical and atypical math performance (Kovas, Haworth, Petrill, & Plomin, 2007).

Instruments

The tasks were selected considering the cognitive factors frequently associated with mathematical perform- ance: visuospatial and phonological working memory (Raghubar et al., 2010), visuospatial and visuoconstruc- tional skills (Barnes & Raghubar, 2014), and non-symbolic and symbolic number representation accuracy (Schneider et al., 2016). Intelligence and performance on the single digit operation tasks of addition, subtraction and multiplication (Costa et al., 2011) were used as external criteria to examine the validity of the emerging clusters.

Raven's Coloured Progressive Matrices (CPM)

Fluid intelligence was assessed using the age-appropriate Brazilian validated version of Raven's Coloured Progressive Matrices (Angelini et al., 1999; Carpenter, Just, & Shell, 1990). Analyses were based on z-scores according to the test manual.

Brazilian School Achievement Test (TDE)

The Brazilian School Achievement Test (Stein, 1994; Oliveira-Ferreira et al., 2012) is a standardized test to assess school achievement in Brazil. Norms include children from the 1st to 6th grades. It comprises three subtests: single word reading, spelling and arithmetic. In this study, we used the Arithmetic subtest. The Arithmetic subtest comprises three simple verbally presented word problems (i.e., which is the largest, 28 or 42?) and 45 written arithmetic calculations of increasing complexity (i.e., very easy: $4 - 1$; easy: $1230 + 150 + 1620$; intermediate: 823×96; hard: $3/4 + 2/8$). This subtest has been used in several numerical cognitive studies in Brazil, presenting both reliability and validity in identifying children with basic arithmetic impairments (Costa et al., 2011; Lopes-Silva et al., 2016; Moura et al., 2013; Pinheiro-Chagas et al., 2014). Analyses were based on z- scores calculated using the parameters provided by Oliveira-Ferreira and colleagues (2012).

Digit Span

Phonological working memory was evaluated using the backward digit span of the Brazilian WISC-III Digits subtest (Figueiredo & Nascimento, 2007). Individual z-scores were calculated using parameters from the present sample.

Corsi Blocks

This test is a measure of the visuospatial component of working memory. We used the backward order to assess it, according to the procedure by Kessels, van Zandvoort, Postma, Kappelle, and de Haan (2000; see also Santos, Mello, Bueno, & Dellatolas, 2005). Individual z-scores were calculated using parameters from the present sample.

Rey Complex Figure

The copy of the Rey figure assesses visuospatial and visuoconstructional abilities. It is based on a complex black and white line drawing that the child must copy as accurately as possible. The accuracy score is based on the presence, distortion or malpositioning of each of the 18 elements of the figure. This task assesses visuo- spatial-representational, executive functions and visuoconstructional abilities (Strauss, Sherman, & Spreen, 2006). Individual z-scores were calculated using parameters from the present sample.

Non-Symbolic Number Magnitude Comparison

In the computerized non-symbolic magnitude comparison task, participants were instructed individually to compare two simultaneously presented sets of dots, indicating which was more numerous. Black dots were presented in a white circle over a black background. In each trial, one of the two white circles contained 32 dots (reference numerosity), and the other contained 20, 23, 26, 29, 35, 38, 41 or 44 dots. Each magnitude of dot sets was presented eight times. The task comprised of 8 learning trials and 64 experimental trials. The maximum stimulus presentation time was 4,000 ms, and the intertrial interval was 700 ms. Between trials, a fixation point appeared on the screen for 500 ms; the fixation point was a cross printed in white with height and width of 3 cm. As a measure of ANS accuracy, the Weber fraction (*w*) was calculated for each child based on the Log- Gaussian model of numerical representation described by Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004) and Dehaene (2007). A higher value indicates worse performance. Previous evidence regarding the validity of this task was obtained by Júlio-Costa et al. (2013), Lopes-Silva et al. (2014), Oliveira et al. (2014) and Pinheiro-Chagas et al. (2014). Individual z-scores were calculated using parameters from the present sample.

Symbolic Number Magnitude Comparison

In the computerized symbolic magnitude comparison task, participants were instructed individually to judge if an Arabic digit presented on the computer screen was larger or smaller than 5. The digits presented on the screen were 1, 2, 3, 4, 6, 7, 8 or 9 (with numerical distances from the reference varying from 1 to 4), printed in white over a black background. If the presented digit was smaller than 5, children should press a predefined key on the left side of the keyboard. Otherwise, if the presented digit was greater than 5, children should press a key on the right side of the keyboard. The task comprised a total of 80 trials, 10 trials for each numerosity. The presented number was shown on the screen for 4000 ms, and the time interval between trials was 700 ms. Before each test trial, there was a fixation trial (a cross) with duration of 500ms. As a measure of symbolic mag- nitude processing efficiency, we used an RT index penalized for inaccuracy: $P = \text{RT}(1 + 2\text{ER})$ according to Lyons, Price, Vaessen, Blomert, and Ansari (2014). In the formula, RT means reaction time and ER stands for error rates, considering reaction time (RT) and errors rates (ER) as measures of performance for each child. ERs were multiplied by 2 because the task was a binary forced choice (ER = 0.5 indicates chance level). High- er scores indicate worse performance. If the performances were perfectly accurate, P would correspond to the child’s average RT ($P = \text{RT}$).

Single Digit Operations

This task comprised addition (27 items), subtraction (27 items), and multiplication (28 items) operations for an individual application, which were printed on separate sheets of paper. Children were instructed to answer as quickly and as accurately as possible, with the time limit per block being 1 min. Arithmetic operations were organized into two levels of complexity and were presented to children in separate blocks: one consisted of simple arithmetic table facts and the other of more complex ones. Simple additions were defined as those operations having results below 10 (i.e., 3 + 5), while complex additions were those having results between 11 and 17 (i.e., 9 + 5). The problems (i.e., 4 + 4) were not used for addition. Simple subtractions comprised problems in which the operands were below 10 (i.e., 9 − 6), while in complex subtractions the first operand ranged from 11 to 17 (i.e., 16 − 9). No negative results were included in the subtraction problems. Simple multiplications consisted of operations with results below 25 or belonging to the 5-table (i.e., 2 × 7, 6 x 5), while in complex multiplication, the results ranged from 24 to 72 (6 × 8). Previous evidence regarding the validity of this task was obtained by Costa et al. (2011), Haase et al. (2014) and Wood et al. (2012). Individual z-scores were calculated using parameters from the present sample.

Procedures

Children were assessed in their schools, in two sessions of approximately 30 minutes each, by specially trained undergraduate psychology students. Intelligence and school achievement assessments were applied to groups of approximately 6 children during the first session, and the other tasks were individually assessed in the second session. The order of the neuropsychological tests was pseudo-randomized in two different sequences.

Statistical Analyses

We performed hierarchical cluster analysis (Ward method with squared Euclidean distance) using measures of phonological and visuospatial working memory, visuospatial and visuoconstructional processing, and symbolic and nonsymbolic magnitude accuracy as the criterion variables for cluster formation. The Ward method considers all possible combinations of clusters and combines clusters which minimize the increase in the error sum of squares in each iteration (Ward, 1963). All raw scores of the criterion variables were transformed into z-scores based on the current sample distribution for each grade separately, to correct for the positive correlations observed among the tasks, age and schooling level.

To characterize the neuropsychological profile of the clusters, we performed a series of variance analysis (ANOVA) with each of the neuropsychological measures as the dependent variables. We also compared the distribution of age and intelligence among clusters. To examine cluster validity, we investigated the frequency of MD (based on TDE scores) in each cluster, and we performed a series of ANCOVAs, with intelligence as a covariate, using the z-scores of the TDE Arithmetic subtest, as well as the single digit operations as dependent variables. We reported appropriate effect sizes indexes (Cohen’s *d* or partial Eta squared).

Results

To identify possible subgroups of cognitive performance that could be eventually associated with math achievement, we used a bottom-up strategy. This type of strategy consists of letting candidate subgroups emerge through cluster analysis and afterwards interpreting and examining their validity.

Emerging Groups

As criteria for cluster membership, we used the performance in the backward forms of digit span and Corsi blocks, Rey figure copy, internal Weber fraction (*w*) in the non-symbolic number comparison task and a meas- ure of symbolic number magnitude comparison efficiency (*P*). Analyses yielded an optimal solution with four clusters of individuals. The dissimilarity coefficients obtained in each stage of the agglomeration processes were used as objective criteria to select the final solution for the analysis. These coefficients reflect the internal heterogeneity of each cluster throughout the process. At first, this agglomeration index is zero, since all cases are still isolated. At the end of this analysis, this index reaches its maximal value, since all the cases are gath- ered into the same group. The cut-off for deciding on the final solution is a drastic increase of these indexes across a few iterations of the optimization function. This function indicates the configuration of cases leading to the maximal heterogeneity between groups. Figure 1 showed the dendrogram obtained during the assignment of individuals to clusters and the distribution of MD and controls in each cluster. The frequency of MD and con- trol cases across clusters will be described later, together with math achievement.

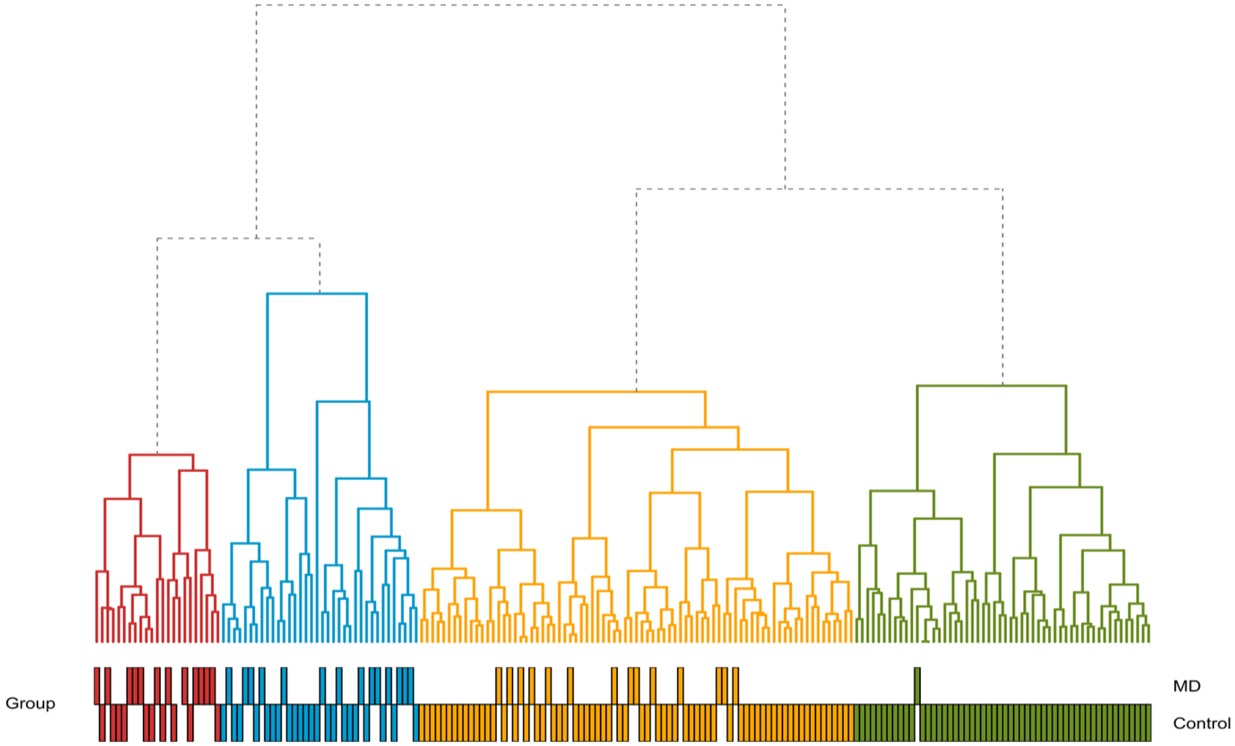

Figure 1. Dendrogram showing the formation of clusters.

Note. Top: Emerging clusters and their interpretation; Cluster 1: Low visuospatial abilities; Cluster 2: Low magnitude processing accuracy; Cluster 3: Average performance; Cluster 4: High performance; Bottom: Classification according to math achievement; Upward bins: MD children; Downward bins: Control group.

Cognitive Characterization of Subgroups

Four clusters were identified. Each cluster was interpreted and characterized according to performance on the respective criterion variable. Figure 2 illustrates the performance of the clusters on each criterion variable.

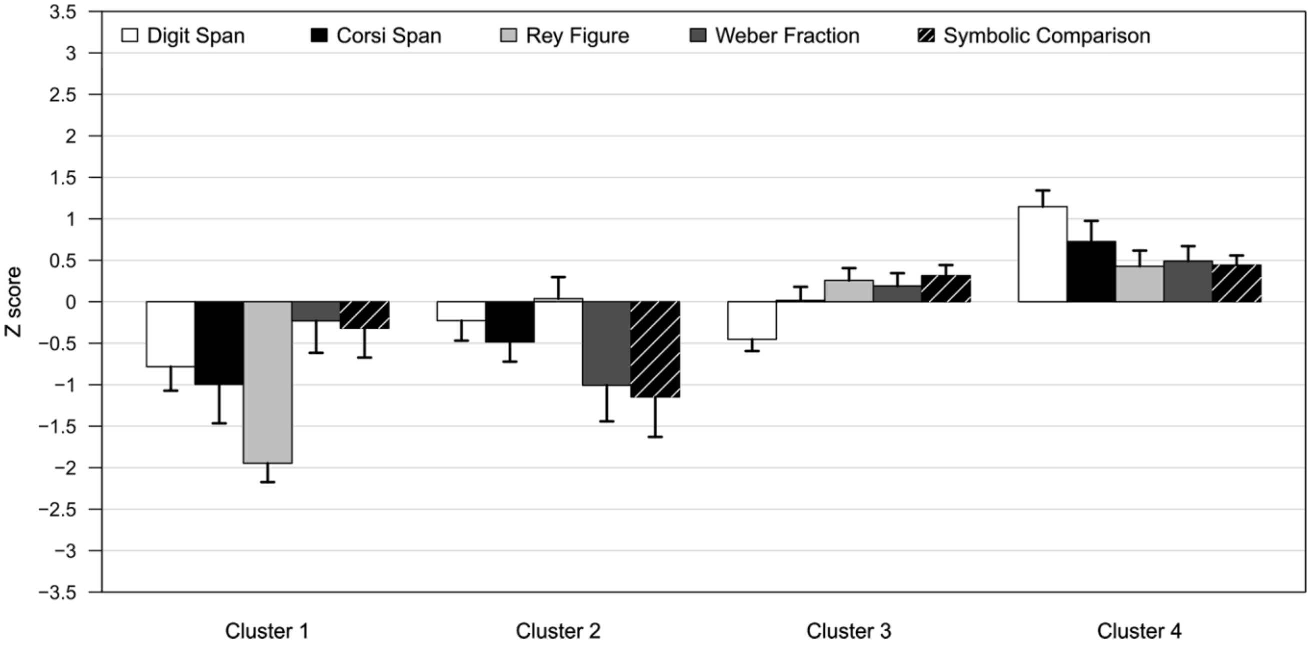

Figure 2. Performance on criterion variables across clusters.

Note. Cluster 1 was characterized by low performance in the Rey figure copy and Corsi blocks (Low visuospatial). Cluster 2 exhibited a lower performance on the non-symbolic and symbolic comparison tasks (Low magnitude processing accuracy). Performance in Cluster 3 was average for all tasks (Average performance). Individuals in Cluster 4 showed high performance across cognitive tasks (High performance).

The descriptive analysis for each cluster with means and standard deviations of each criterion variable is shown in Table 1. To characterize the clusters, we calculated ANOVAs comparing the performance of all clusters in the neuropsychological variables. We conducted post-hoc analyses using the Bonferroni test. Next, we describe the neuropsychological performance in each cluster.

Table 1

Descriptive Data and Analysis of Variance Between Clusters

	Cluster 1 (*n* = 23)		**Cluster 2 (*n* = 36)**		**Cluster 3 (*n* = 79)**		**Cluster 4 (*n* = 54)**		**ANOVA**		**Post-hoc (Bonferroni test)**	n^2_p
Measure	***M***	***SD***	***M***	***SD***	***M***	***SD***	***M***	***SD***	***F*(3, 188)**	***p***		
Criterion variables of cluster formation												
Digit span (backwards)	-0.78	0.66	-0.22	0.71	-0.45	0.63	1.14	0.71	75.24	< .001	4 > 2 > 3 = 1 & 1 = 2	0.546
Corsi Blocks (backwards)	-1.00	1.08	-0.48	0.70	0.01	0.73	0.72	0.91	28.95	< .001	4 > 3 > 2 = 1	0.316
Rey Figure (copy)	-1.94	0.52	0.03	0.76	0.25	0.76	0.42	0.69	73.85	< .001	2 = 3 = 4 > 1	0.541
Weber fraction[a]	0.22	0.89	1.01	1.28	-0.18	0.69	-0.49	0.65	24.76	< .001	2 > 1 > 4 = 3 & 1 = 3	0.283
Symbolic magnitude efficiency[a]	0.32	0.82	1.15	1.42	-0.31	0.57	-0.44	0.42	35.25	< .001	2 > 1 > 3 = 4	0.360
Age and intelligence												
Age (month)	115.91	10.56	116.78	10.03	119.11	10.37	117.70	8.73	0.86	0.450	--	0.014
Raven's CPM (*z* score)	-0.12	0.45	0.50	0.57	0.59	0.68	1.11	0.52	24.20	< .001	4 > 2 & 3 > 1	0.278

[a] Weber fraction and Symbolic magnitude efficiency have an inverse interpretation, the higher the values, the worse the accuracy.

Cluster 1: Low Visuospatial Abilities

Cluster 1 was characterized by low visuospatial performance, both in the Rey figure and Corsi blocks, com- pared to other clusters. Compared to Cluster 2, this Cluster showed lower performance on the digit span ($p < .01$; $d = 0.81$) and the Rey Figure ($p < .001$, $d = 2.91$), but not on the Corsi blocks ($p < 0.24$; $d = 0.57$); and, better performance considering *w* ($p < 0.005$; $d = 0.69$) and *P* ($p < .001$; $d = 0.90$). Compared to Cluster 3, Cluster 1 showed lower performance on the Corsi blocks ($p < .001$; $d = 1.23$), the Rey Figure ($p < .001$, $d = 3.07$), *P* ($p > .006$; $d = 0.79$); and, a non-significant difference regarding digit span ($d = 0.52$) and *w* (d = 0.54). Finally, compared to Cluster 4, Cluster 1 performed poorly in all variables: digit span ($p < .001$; $d = 2.77$), Corsi blocks ($p < .001$; $d = 1.79$), Rey figure ($p < .001$; $d = 3.66$), *w* ($p < .005$; $d = 0.97$) and *P* ($p < .001$; $d = 1.31$).

Cluster 2: Low Magnitude Processing Accuracy

Cluster 2 showed the lowest performance on the tasks which assessed non-symbolic (*w*) accuracy as well as symbolic (*P*) magnitude efficiency. Performance on all other cognitive measures was average. Compared to Cluster 3, Cluster 2 showed worse performance on the Corsi blocks ($p < .018$, $d = 0.68$), *w* ($p < .001$, $d = 1.30$) and *P* ($p < .001$, $d = 1.58$); but, a non-significant difference regarding digit span ($d = 0.35$) and Rey Figure ($d = 0.29$). Compared to Cluster 4, this cluster showed worse performance on the digit span ($p < .001$, $d = 1.92$), the Corsi blocks ($p < .001$, $d = 1.44$), *w* ($p < .001$, $d = 1.58$) and *P* ($p < .001$, $d = 1.67$); but, a non-significant difference on the Rey figure ($d = 0.54$).

Cluster 3: Average Performance

Cluster 3 presented average or near average performance for all variables and was labeled "Average performance". Compared to Cluster 1, Cluster 3 did not differ significantly on the digit span and *w,* but was significantly better on the Corsi blocks, the Rey figure and *P*. Compared to Cluster 2, Cluster 3 showed no significant difference on the digit span and the Rey Figure, but better performance on the Corsi blocks, *w* and *P*. Working memory performance in Cluster 3 was worse than in Cluster 4 regarding the digit span ($p < .001$, $d = 2.40$) and the Corsi blocks ($p < .001$, $d = 0.88$), but still in the average range. There was no significant difference compared to Cluster 4 in the Rey figure ($d = 0.23$), *w* ($d = 0.46$) and *P* ($d = 0.25$).

Cluster 4: High Performance

Cluster 4 showed higher performance than all other clusters on the digit span, the Corsi blocks and *w* ($p < .005$; Table 1). Moreover, Cluster 4 showed better performance than Clusters 1 and 2 on *P* and was labeled "High performance". Compared to Cluster 1, Cluster 4 showed better performance on all measures. Compared to Cluster 2, only the Rey Figure showed no significant difference. Compared to Cluster 3, Cluster 4 performance was similar on the Rey Figure, *w* and *P*.

Cluster Validity

To examine the validity of the emerging clusters, we analyzed age and performance differences in measures of intelligence, standardized math achievement and single digit operations in each cluster.

Age

We calculated ANOVAs to analyze age and intelligence differences among the clusters. Table 1 shows descriptive analyses with means and standard deviations for each cluster and the ANOVA results. There were no significant differences among the clusters regarding age.

Intelligence

All participants had general intelligence scores above the 20th percentile. The Raven's CPM was correlated statistically with the digit span ($r = .41$; $p < .01$), the Corsi blocks ($r = .39$; $p < .01$) and the Rey figure ($r = .45$; $p < .01$). The Raven's CPM showed a weak but significant negative correlation with the Weber fraction ($r = -.15$; $p < .05$) and the Symbolic magnitude efficiency ($r = -.17$; $p < .05$).

We calculated ANOVAs to investigate differences in intelligence among the clusters. As shown in Figure 3 and Table 1, Cluster 1 presented lower intelligence ($p < .001$) than Cluster 2 ($d = 1.18$), 3 ($d = 1.12$) and 4 ($d = 2.46$). Clusters 2 and 3 showed comparable intelligence ($d = 0.14$), and Cluster 4 presented higher intelligence ($p < .001$) than Clusters 1 ($d = 2.46$), 2 ($d = 1.13$), and 3 ($d = 0.84$).

Standardized Math Achievement

Clusters 1 and 2 were characterized by a higher frequency of children with math learning difficulties, defined as performance below the 25th percentile in the Arithmetic subtest of the TDE. Only one child with math difficulties was observed in Cluster 4. The frequency of MD children was 56.5% in Cluster 1, 38.9% in Cluster 2 and 17.7% in Cluster 3. Cluster 4 comprises Control children (see Figure 1). Chi-square analyses reveal that the distribution of MD among Clusters 1, 2 and 3 differs ($X^2 = 14.80$, $df = 2$; $p < .001$).

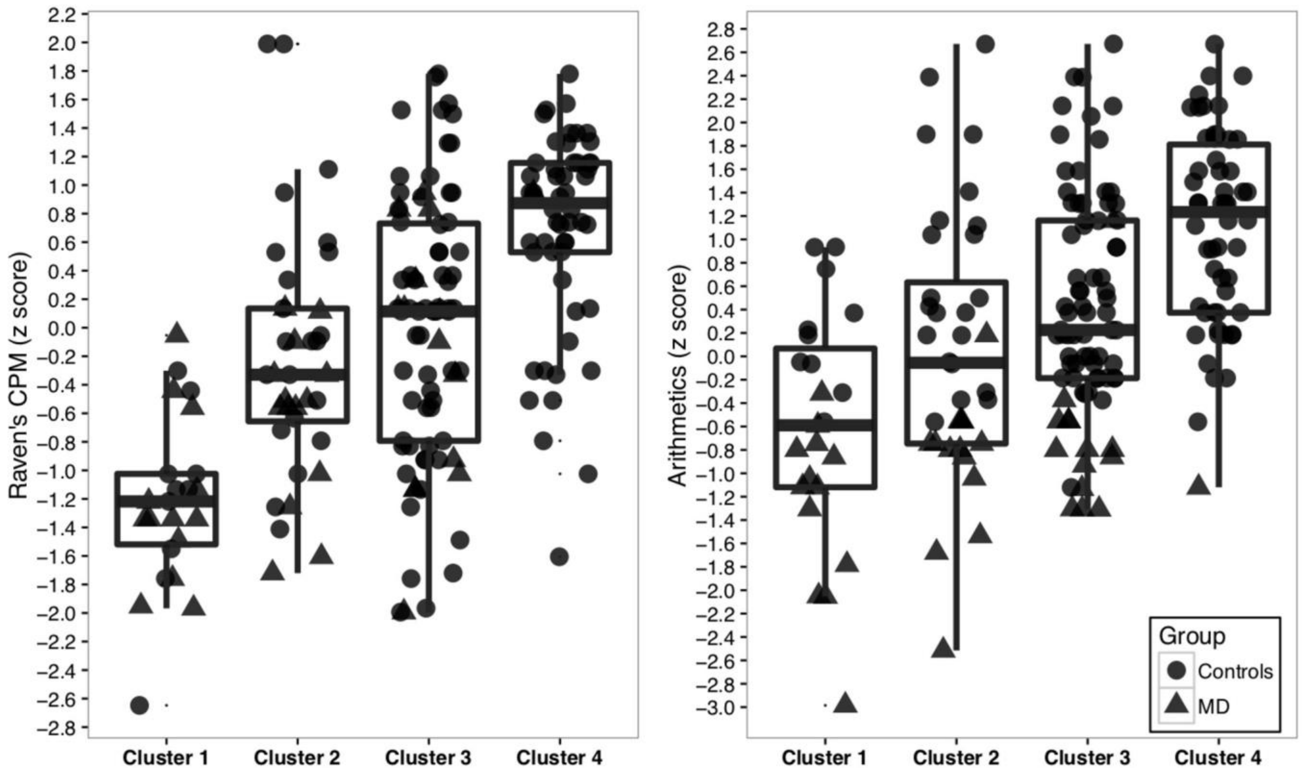

Figure 3. Distribution of intelligence and TDE-Arithmetic subtest performance among the Clusters.

Note. Dispersion of Control and MD cases are depicted by different shapes (Circle for Controls and Triangle for MD).

Figure 3 shows the cluster specific performance on the TDE Arithmetic subtest. We calculated ANCOVAs to compare the performance of all clusters, with intelligence as a covariate. The clusters differed regarding the

TDE Arithmetic subtest ($F = 6.81$, $df = 3; 187$, $p < .001$): a significantly higher performance was obtained in Cluster 4 than in Cluster 1 ($p < .001$, $d = 1.92$) and Cluster 2 ($p < .004$, $d = 1.06$), but Cluster 4 did not significantly different from Cluster 3 ($p > .09$, $d = 0.75$). Cluster 3 presented significantly higher performance than Cluster 1 ($p < .02$, $d = 1.07$), but it was not significantly different from Cluster 2 ($p > 0.65$, $d = 0.35$). Clusters 1 and 2 did not differ in the TDE Arithmetic subtest ($p > .80$, $d = 0.62$).

Single-Digit Operations

As the TDE Arithmetic subtest was used to categorize individuals according to typical or atypical achievement, performance on a different set of single-digit operation tasks was used as an external criterion. ANCOVAs comparing the clusters in single-digit operations were calculated using intelligence as a covariate. Clusters 1 and 2 presented lower performance than Cluster 4 in all operations. Cluster 4 exhibited scores above the mean in all single-digit operations. Even though Clusters 1 and 2 had different cognitive impairments, they presented a similar profile in single-digit operations. Cluster 3 performed slightly below the mean and significantly below Cluster 4 in the more complex single-digit subtraction and multiplication tasks (see Table 2).

Table 2

Comparison of Cluster Performances on Single-Digit Operations

	Cluster 1 (n = 23)		Cluster 2 (n = 36)		Cluster 3 (n = 79)		Cluster 4 (n = 54)		ANCOVA			
Measure	***M***	***SD***	***M***	***SD***	***M***	***SD***	***M***	***SD***	***F* (3, 187)**	***p***	**Post-hoc (Bonferroni test)**	η^2_p
Simple addition	-0.98	1.57	-0.33	1.33	0.20	0.57	0.33	0.45	10.03	< .001	3 = 4 > 1 = 2	0.139
Complex Addition	-0.83	1.02	-0.35	1.01	0.03	0.84	0.54	0.84	8.59	< .001	4 > 1 = 2; 4 = 3; 2 = 3; 3 >1	0.121
Simple subtraction	-0.53	0.99	-0.33	0.99	-0.02	0.97	0.47	0.82	4.46	< .005	4 > 2 = 1; 4 = 3; 1 = 2 = 3	0.067
Complex subtraction	-0.77	0.49	-0.41	0.83	-0.10	0.81	0.75	1.02	15.07	< .001	4 > 1 = 2; 4 > 3; 3 > 1; 3 = 2	0.195
Simple multiplication	-0.84	0.97	-0.28	1.04	-0.02	0.85	0.57	0.81	7.51	< .001	4 > 1 = 2; 4 > 3; 3 > 1; 3 = 2	0.108
Complex multiplication	-0.47	0.75	-0.19	1.09	-0.19	0.83	0.62	0.97	6.96	< .001	4 > 1 = 2 = 3	0.101

In Figure 4, the single-digit operations (addition, subtraction and multiplication) are compared across groups. In all single-digit operations, Cluster 1 had scores below the mean. It was significantly worse than Cluster 4 (all *p's* < .01) in all operations. A similar pattern was observed in Cluster 2: children presented scores below the mean and performance was also significantly worse than Cluster 4 for all operations (all *p's* < .01). Cluster 2 had performance comparable to Cluster 3 in all single-digit operations except simple addition. Cluster 3 performed approximately 0.1 standard deviations below the mean for arithmetic operations. Nevertheless, this cluster performed worse than Cluster 4 in complex subtraction ($p < .001$), and simple ($p < .026$) and complex multiplica- tion ($p < .001$). Cluster 4 showed means around 0.6 standard deviations above the overall mean for all single- digit operations and better performance than the other clusters.

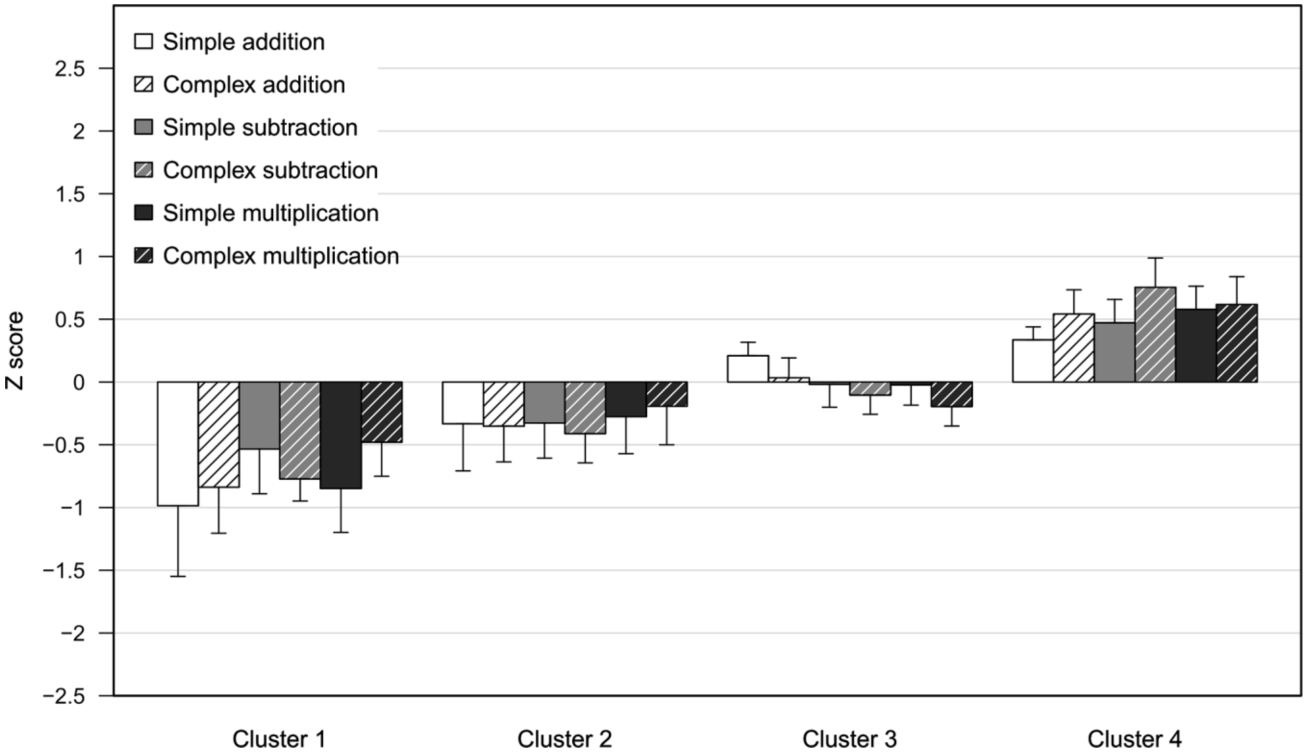

Figure 4. Single-digit operations across clusters. Bars indicate the standard errors of the means.

Discussion

The present study investigated the heterogeneity of mathematics difficulties and its relation to domain-specific and domain-general cognitive skills using a bottom-up or data-driven approach. Using cluster analysis, and based on neuropsychological performance, four groups with specific cognitive profiles were formed. Next, we investigated how these groups performed on math school achievement and single-digit operation tasks, and the effect of general intelligence on performance.

To our knowledge, this is the first study to use a bottom-up approach to explore different math difficulties (MD) subtypes that also includes measures of both non-symbolic and symbolic comparison accuracy as criterion variables. Our final solution was composed of four clusters: Cluster 1 presented low visuospatial performance; Cluster 2, low magnitude processing accuracy, and Clusters 3 and 4 had, respectively, average and high performance in all tasks. Age was comparable across groups and all of the children had normal intelligence. Nevertheless, the clusters presented differences in math performance.

Children with MD were concentrated in Cluster 1 (56.5%) and Cluster 2 (38.9%). Interestingly, Cluster 1 (visuospatial deficits) and Cluster 2 (magnitude processing deficit), performed similarly in the arithmetic school achievement test. The other two clusters were predominantly composed of Control children. In Cluster 3, only 17.7% of children were classified as MD, and in Cluster 4 this percentage decreased to 1.8% (just one child). Arithmetic performance of Clusters 3 and 4 was comparable. It should also be noted that Cluster 4 was composed mostly of children with higher intelligence, which might be associated with the generally higher performance presented in all of the cognitive and mathematical tasks.

A similar scenario of cluster differences was observed for the single-digit operation tasks. Clusters 1 and 2 presented difficulties in the single-digit tasks, while Clusters 3 and 4 presented, respectively, average and high performance. None of the clusters presented selective deficits in any specific kind of single-digit operation. It is important to mention that between-cluster differences regarding standardized math and single-digit operations remained significant after statistically controlling for the effects of intelligence.

These results raise several points of discussion. In the following, we are going to examine the cognitive specificity of clusters, as well as the role of visuospatial/visuoconstructional abilities, magnitude processing and intelligence on math performance.

Cognitive Specificity of Clusters

Cluster 1 presented the lowest performance in the standardized math achievement test. Intelligence was also lower in this group, as compared to the other clusters, but still in the normal range. This group was interpreted as having a significant deficit in visuospatial and visuoconstructional abilities, with the lowest mean scores in the Rey figure copy and backward Corsi blocks. It is possible that this visuospatial impairment is attributable to deficits in the executive components of these tasks, as performance on the backward digit span was also below the sample mean. One interesting feature of Cluster 1 is that the Weber fraction and symbolic magnitude efficiency were average. Also, noteworthy, Cluster 1 presented the highest frequency of MD individuals. Intelligence in Cluster 1 was normal and evenly distributed around the mean. This suggests that intelligence and school achievement may dissociate in a group of individuals with low visuospatial/visuoconstructional performance.

Cluster 2 showed the most specific pattern of cognitive deficits, with low performance only in magnitude processing accuracy (ANS and symbolic magnitude efficiency measures). Performance on the other three cognitive markers was average, and mean intelligence was above the population mean.

Cluster 3, the largest one, was composed of individuals with average to high average performance in most of the cognitive tasks, including intelligence. Interestingly, this cluster presented lower phonological working memory, but still in the normal range. This could be associated with the small MD group allocated in this cluster. Cluster 3 presented MD children in a smaller proportion than Clusters 1 and 2.

Cluster 4 was composed mostly of individuals with high performance in neuropsychological and math tests. The highest cognitive performance in this cluster was related to phonological working memory and intelligence. This cognitive profile is associated with the high math performance in this cluster.

We believe that the clusters that emerged represent relatively specific, consistent and theoretically interpretable patterns of cognitive performance and associations with intelligence and math achievement. As cluster analysis is an exploratory technique, it is not possible to attest to the replicability of these patterns in other samples, their temporal stability or their epidemiological relevance. Cluster analysis is interpreted in this context as a device to identify patterns of association and dissociation between psychological processes, in the same vein as the role played by quasi-experimental studies in cognitive neuropsychology (Temple, 1997). This method is useful to identify hypotheses that deserve further scrutiny.

Important results have emerged in the analysis so far and will be discussed in further detail. Some specific patterns of cognitive performance related to visuospatial and magnitude processing accuracy are related to math performance. At the same time that math achievement is related to intelligence (Clusters 3 and 4) it seems also to dissociate from general cognitive ability in some cases (Clusters 1 and 2).

Association Between Visuospatial Processing and Math Performance

In our study, Cluster 1 with low performance in the Rey figure copy and backward Corsi blocks was the one with the lowest achievement in math and the one that aggregated the largest portion of kids with MD. As intelligence in this group was in the normal range and also partially dissociated from math achievement, one may suppose that specific visuospatial and visuoconstructional deficits are detrimental to math achievement.

As both Rey copy and backward Corsi tasks impose important demands in terms of executive functioning, it is not possible to distinguish between visuospatial representational and access deficits. Low performance could thus be related to task requirements on executive functions. It is important, however, to underline the visuospatial nature of the most severe difficulties, as this group performed below but not far from average on digit span tasks, which also tap into executive functions (Figure 2).

A cluster of low visuospatial working memory performance associated with math difficulties was also observed by Bartelet et al. (2014). Interestingly, in this study, visuospatial working memory deficits occurred together with impairments in the ANS. Results of Bartelet et al. (2014) are easily interpreted in terms of the triple code model (Dehaene, 1992; Dehaene & Cohen, 1995), which assumes that approximate representations of numerical magnitude are represented as a spatially oriented mental number line. It was further proposed that visuospatial attentional mechanisms implemented by the posterior superior parietal cortex are important to process magnitudes in the spatially oriented mental number line (Dehaene, Piazza, Pinel, & Cohen, 2003).

Supporting evidence for an association between visuospatial abilities and ANS-related performance was also obtained by Bachot et al. (2005). They examined the performance of children with both visuospatial and math difficulties on an Arabic digit magnitude comparison task. Results showed that, in comparison to a control group, children with concurrent visuospatial and math deficits exhibited slower reaction times and an absence of spatial orientation of the number line.

In our results, no such association between visuospatial and the non-symbolic numerical processing signature was observed in a second cluster. The discrepancy between the results found by Bartelet et al. (2014), Bachot et al. (2005) and our results can be ascribed to the visuospatial nature of the ANS task used in the previous studies. In the present study, we used a psychophysical index, the Weber fraction, calculated from the performance in a nonsymbolic comparison task. Both Bartelet et al. (2014) and Bachot et al. (2005) measured ANS by means of a mental number line task, in which participants positioned a number on a numerical line.

The influence of visuospatial abilities on math achievement and the existence of subgroups of children with MD who are visuospatially impaired has been postulated since the inception of neuropsychological interest in the area (Geary, 1993; Kosc, 1974; Wilson & Dehaene, 2007). A connection between visuospatial processing deficits and poor math achievement was proposed to underlie the nonverbal learning disability syndrome (Rourke, 1989, 1995). Visuospatial abilities, mainly related to working memory, are relevant to several domains of math achievement. An association has been shown in visuospatial working memory and verbal (Costa et al., 2011) and written problem solving, as well as in verbal to Arabic transcoding abilities (Camos, 2008; Moura et al., 2013; Pixner et al., 2011).

A role for visuospatial processing in single-digit operations has been found in preschool children (LeFevre et al., 2010; McKenzie, Bull, & Gray, 2003). The hypothesis has been advanced that, visuospatial working memo- ry processing is especially relevant for early acquisition of basic arithmetic operations. As mastery grows and problem-results associations are stored as arithmetic facts, phonological working memory becomes more important (McKenzie et al., 2003). Our results suggest that, in a group of kids struggling to learn math, visuospatial processing is still relevant to single-digit operations well beyond the preschool years.

Our results suggest that relatively specific impairments in visuospatial and visuoconstructional processing, partially dissociated from general intelligence and working memory performance, could be associated with difficulties in learning arithmetic. This hypothesis could be corroborated by single-case observations in which visuospatial and visuoconstructional processes would be the only forms of impairment.

Associations Between Magnitude Processing and Math Performance

The most specific and consistent group emerging in our study was Cluster 2, which was characterized by normal intelligence in the face of a large percentage of children presenting MD. The cognitive deficits presented in Cluster 2 were a higher Weber fraction and a higher *P* (the measure of symbolic magnitude comparison efficiency), suggesting the lower accuracy of non-symbolic and symbolic numerical representations. Previous studies have already shown that MD children aged from 8 to 11 years present difficulties in both symbolic and non-symbolic comparison tasks (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin, Mejias, & Noël, 2010; Pinheiro-Chagas et al., 2014).

It is remarkable that intelligence was only weakly correlated with the Weber fraction (r = -.15) and symbolic magnitude efficiency (r = -.17). Almost 70% of individuals in Cluster 2 presented above average intelligence. Considering that in the same Cluster 2, almost 39% of the individuals presented MD, this suggests that intelligence and numerical magnitude processing may represent independent albeit possibly interacting, influences on math achievement.

A role for possibly innate, non-symbolic and approximate numerical representations in the acquisition of arithmetic skills, has been proposed in the triple code model (Dehaene, 1992; Dehaene & Cohen, 1995). This hypothesis has been a source of considerable controversy. Theoretically, the mechanisms by which number sense could influence arithmetic performance are still not clear. One possibility is that non-verbal and even symbolic simple operations require magnitude representations to be executed. A role for magnitude estimation has been proposed in verbal operations, such as the discovery of the “counting on from larger” strategy in addi- tion (Ashcraft, 1992), and in systematically storing of multiplication facts from the larger operand (Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Robinson, Menchetti, & Torgesen, 2002).

Empirically, the role of the ANS in math learning has been also contentious. Some researchers have obtained data suggesting a moderate effect of ANS-related performance on math achievement both in typical (Halberda & Feigenson, 2008) as well as in atypical populations (Landerl et al., 2004; Mazzocco et al., 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014; see meta-analyses in Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2016). Other researchers, however, failed to implicate the ANS in math learning, obtaining instead results that favor a role for symbolic numerical processing (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Rousselle &

Noël, 2007; Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015). The bulk of evidence seems to favor a major role for symbolic numerical processing (De Smedt, Noël, Gilmore, & Ansari, 2013). It is, however, noteworthy that most studies failing to find an effect of the ANS on math performance used simple RT measures on the dot comparison task. Nevertheless, most research showing such an effect mainly employed measures of ANS accuracy such as the internal Weber fraction (Halberda & Feigenson, 2008; Mazzocco et al., 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014).

Recent studies have also proposed that ANS is important for mathematical achievement even when considering other basic symbolic numerical processes, such as counting and cardinality comprehension (Chu, vanMarle, & Geary, 2015; Hirsch, Lambert, Coppens, & Moeller, 2018). Chu, vanMarle, and Geary (2015) showed that in a group of preschoolers children, mathematical skills were best predicted by ANS when it was mediated by cardinality comprehension, thus suggesting that the effect of ANS on math achievement is indirect. Additionally, Hirsch, Lambert, Coppens, and Moeller (2018) did a longitudinal study evaluating a sample of 1700 preschoolers, and demonstrated that a 4 or 5 factor model, composed by basic numerical competences of patterning, seriation, non-symbolic comparison, counting and symbolic number knowledge, are able to predict a large part of the variance in mathematics performance in Grade 6, even controlling for intelligence. Neverthe- less, in these studies, just the accuracy of errors was used how a measure of non-symbolic and symbolic com- petences. These studies have been demonstrating the importance of non-symbolic and symbolic numerical skills to the math performance when they are taken into account together.

As Cluster 2 was the only one exhibiting impairments in both the ANS accuracy and symbolic magnitude efficiency, together with a large proportion of individuals with MD, our results support the hypothesis that low resolution of non-symbolic and symbolic magnitude processing should be considered an important risk factor for MD.

Association Between Intelligence and Math Performance

Intelligence and other highly complex cognitive abilities, such as working memory, are clearly implicated in math learning at every age and ability level (Primi, Ferrão, & Almeida, 2010). Use of IQ as a covariate in neuropsychological studies of children with poor school achievement has been criticized on the grounds that the two measures highly correlate (Dennis et al., 2009). We feel this criticism applies to omnibus measures of intelligence, such as IQ, that include subtests heavily dependent on school experience, like the WAIS-Vocabulary. In this sense, IQ can be interpreted as an outcome of schooling. However, it is also true that some tests of intelligence, such as the Raven's CPM, measure aspects of nonverbal intelligence that are highly inheritable and, to a large extent, independent of school experience (Raven, 2000). The Raven's CPM is considered to be one of the best measures of nonverbal fluid intelligence (Carpenter et al., 1990), and there is considerable quantitative genetic evidence indicating it is a reliable predictor of future school achievement (Hart, Petrill, Thompson, & Plomin, 2009).

Our results clearly point to the importance of nonverbal fluid intelligence, at least as measured by the Raven's CPM, as an important correlate of math achievement, both in typical and atypical individuals. The interactions between intelligence, specific cognitive deficits, and math achievement could be quite complex. Results sug- gest both associations and dissociations. The most salient association was found in the high performing Cluster 4. Only one individual in Cluster 4 exhibited MD and the performance in all math, intelligence and cognitive tasks was well above average. This supports a positive, mutually reinforcing loop between general and specific cognitive abilities and math achievement. This positive association contrasts with the negative association between intelligence and math achievement in one of the clusters described by Bartelet et al. (2014). This difference may be ascribed to the fact that children in the present study presented average to above average intelligence.

Dissociations between intelligence and specific cognitive abilities were observed in Clusters 1 and 2. As a group, both Clusters 1 and 2 were characterized by low average intelligence and low math achievement. How- ever, this association does not hold for all individuals. A significant proportion of individuals in both groups pre- sented MD, although their intelligence was well above average. This suggests math difficulties in highly intelli- gent individuals in Clusters 1 and 2 could be explained, respectively, by specific deficits in executive visuospa- tial abilities and poor resolution of numerical magnitude processing.

A role for intelligence in math difficulties was emphasized by the results of the children with specific cognitive deficits in Clusters 1 and 2. Figure 3 shows a remarkable similarity between intelligence and standardized math profiles across clusters. In Figure 4, it is possible to see that Clusters 1 and 2, with specific cognitive deficits, also presented difficulties in the single-digit operations. Statistical differences remained significant after control- ling for intelligence. Results suggest highly complex relationships between general and specific cognitive abili- ties and math performance.

Both patterns of associations and dissociations were observed, suggesting the existence of specific mechanisms and complex interactions among them. Johnson (2012) suggested that a developmental or specific learning disorder may be characterized when an individual presents a specific deficit associated with insuffi- cient general cognitive resources to compensate for that deficit. If general cognitive resources such as execu- tive functions or intelligence are available, the specific deficit may be compensated, and the difficulties do not go beyond a diagnostic threshold (see also Haase et al., 2014).

In conclusion, this study supports the hypothesis that MDs are a cognitive heterogeneous phenomenon. At the same time, our data suggest that single mechanisms may play specific roles. Other authors, using different clustering criteria, were able to identify a host of distinct subgroups, varying from one study to another (Bartelet et al., 2014; Reeve et al., 2012; Vanbinst et al., 2015; von Aster, 2000). This could underlie the inadequacy of cognitive models based on a general-experimental approach to describe the range of variability expressed in math learning difficulties. This problem could be thornier for math difficulties described by a liberal achievement criterion than in clinically selected cases. Even in clinically referred cases with severe difficulties, the profiles of numerical cognitive impairments do not always conform to the theoretical predictions. For example, Haase et al. (2014) (see also Júlio-Costa et al., 2015) described the case of H.V., a highly intelligent 9-year-old girl with severe and persistent math difficulties associated to stable number sense impairments. According to the triple code model, it would be expected that H.V. would be more impaired in the single-digit subtraction than in the addition and multiplication operations, which are, supposedly, based on verbal representations. Contrary to this expectation, H.V. was more severely and persistently impaired in the multiplication facts. This is noteworthy because multiplication facts are believed to be phonologically represented and H.V.'s phonological processing skills were above average.

Another salient feature in this study was the role of intelligence. Data suggested both associations and dissociations. Higher intelligence was associated with typical cognitive profiles and higher math achievement. Higher math performing groups presented both higher intelligence and lower incidence of cognitive deficits compared to other groups. A cognitive explanation of math difficulties thus requires the concomitant consideration of both general and specific factors, paying attention to complex interactions underlying interindividual variability.

Arithmetic is a very complex subject from the cognitive point of view. Single-case studies in adults and children show that, to a certain degree, arithmetic is composed of relatively segregated modules (Temple, 1997). At the same time, arithmetic is strongly hierarchically organized. General and specific cognitive resources are required at each level of development and degree of complexity. Evidence indicates that controlled processing is important for learning to count (Hecht, 2002), to perform the single-digit operations (LeFevre et al., 2010; McKenzie et al., 2003), to memorize the arithmetic facts (Hecht, Torgesen, Wagner, & Rashotte 2001), to learn to trans- code from one notation to the other (Lopes-Silva et al., 2016), to learn the multidigit algorithms (Venneri, Cornoldi, & Garuti, 2003), word problems and so on.

Specific cognitive requirements, such as number sense, visuospatial or phonological processing, may change according to the level of development or difficulty. Moreover, as the child masters performance on a certain level, based on certain specific abilities, new requirements are imposed in order to reach a higher level. This would result in very complex and recurrent interactions between general and specific cognitive mechanisms throughout development.

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Competing Interests

The authors have declared that no competing interests exist.

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Data Availability

For this study a dataset is freely available (see the Supplementary Materials section).

Supplementary Materials

The dataset and codebook for this study.

Index of Supplementary Materials

Salvador, L. S., Moura, R., Wood, G., & Haase, V. G. (2019). *Supplementary materials to "Cognitive heterogeneity of math difficulties: A bottom-up classification approach".* PsychOpen. https://doi.org/10.23668/psycharchives.2382

References

Andersson, U. (2007). The contribution of working memory to children's mathematical word problem solving. *Applied Cognitive Psychology, 21*, 1201-1216. https://doi.org/10.1002/acp.1317

Angelini, A. L., Alves, I. C. B., Custódio, E. M., Duarte, W. F., & Duarte, J. L. M. (1999). *Padronização brasileira das matrizes progressivas coloridas de Raven. JC Raven. Manual Matrizes Progressivas Coloridas de Raven: Escala especial.* São Paulo, Brasil, SP: Centro Editor de Testes e Pesquisas em Psicologia.

Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental Psychology, 52*(8), 1206-1216. https://doi.org/10.1037/dev0000155

Anobile, G., Cicchini, G. M., & Burr, D. C. (2016). Number as a primary perceptual attribute: A review. *Perception, 45*(1-2), 5-31. https://doi.org/10.1177/0301006615602599

Archibald, L. M., Cardy, J. O., Joanisse, M. F., & Ansari, D. (2013). Language, reading, and math learning profiles in an epidemiological sample of school age children. *PLoS One, 8*(10), Article e77463. https://doi.org/10.1371/journal.pone.0077463

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44*(1-2), 75-106. https://doi.org/10.1016/0010-0277(92)90051-I

Bachot, J., Gevers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychological Science, 47*(1), 172-183.

Barnes, M. A., & Raghubar, K. P. (2014). Mathematics development and difficulties: The role of visual–spatial perception and other cognitive skills. *Pediatric Blood & Cancer, 61*(10), 1729-1733. https://doi.org/10.1002/pbc.24909

Bartelet, D., Ansari, D., Vaessen, A., & Blomert, L. (2014). Cognitive subtypes of mathematics learning difficulties in primary education. *Research in Developmental Disabilities, 35*(3), 657-670. https://doi.org/10.1016/j.ridd.2013.12.010

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36-41. https://doi.org/10.1111/cdep.12059

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science, 332*(6033), 1049-1053. https://doi.org/10.1126/science.1201536

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. R. (2001). Storage and retrieval of addition facts: The role of number comparison. *The Quarterly Journal of Experimental Psychology: Section A, 54*(4), 1005-1029. https://doi.org/10.1080/713756007

Camos, V. (2008). Low working memory capacity impedes both efficiency and learning of number transcoding in children. *Journal of Experimental Child Psychology, 99*(1), 37-57. https://doi.org/10.1016/j.jecp.2007.06.006

Camos, V., Barrouillet, P., & Fayol, M. (2001). Does the coordination of verbal and motor information explain the development of counting in children? *Journal of Experimental Child Psychology, 78*(3), 240-262. https://doi.org/10.1006/jecp.2000.2570

Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review, 97*(3), 404-431. https://doi.org/10.1037/0033-295X.97.3.404

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica, 148*, 163-172. https://doi.org/10.1016/j.actpsy.2014.01.016

Chu, F. W., vanMarle, K., & Geary, D. C. (2015). Early numerical foundations of young children's mathematical development. *Journal of Experimental Child Psychology, 132*, 205-212. https://doi.org/10.1016/j.jecp.2015.01.006

Costa, A. J., Silva, J. B. L., Pinheiro-Chagas, P., Krinzinger, H., Lonneman, J., Willmes, K., . . . Haase, V. G. (2011). A hand full of numbers: A role for offloading in arithmetics learning? *Frontiers in Psychology, 2*, Article 368.

Davidse, N. J., de Jong, M. T., Shaul, S., & Bus, A. G. (2014). A twin-case study of developmental number sense impairment. *Cognitive Neuropsychology, 31*(3), 221-236. https://doi.org/10.1080/02643294.2013.876980

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*(1-2), 1-42. https://doi.org/10.1016/0010-0277(92)90049-N

Dehaene, S. (2007). Symbols and quantities in parietal cortex: Elements of a mathematical theory of number representation and manipulation. In P. Haggard, Y. Rossetti, & M. Kawato (Eds.), *Attention and Performance XXII: Sensori-motor foundations of higher cognition* (pp. 527-574). Oxford, United Kingdom: Oxford University Press.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371-396. https://doi.org/10.1037/0096-3445.122.3.371

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1*(1), 83-120.

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16*(3), 626-641. https://doi.org/10.1037/0096-1523.16.3.626

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*(3-6), 487-506. https://doi.org/10.1080/02643290244000239

Dennis, M., Francis, D. J., Cirino, P. T., Schachar, R., Barnes, M. A., & Fletcher, J. M. (2009). Why IQ is not a covariate in cognitive studies of neurodevelopmental disorders. *Journal of the International Neuropsychological Society, 15*(3), 331-343. https://doi.org/10.1017/S1355617709090481

De Smedt, B., & Boets, B. (2010). Phonological processing and arithmetic fact retrieval: Evidence from developmental dyslexia. *Neuropsychologia, 48*(14), 3973-3981. https://doi.org/10.1016/j.neuropsychologia.2010.10.018

De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of Experimental Child Psychology, 108*(2), 278-292. https://doi.org/10.1016/j.jecp.2010.09.003

De Smedt, B., Noël, M.-P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education, 2*, 48-55. https://doi.org/10.1016/j.tine.2013.06.001

De Visscher, A., & Noël, M. P. (2013). A case study of arithmetic facts dyscalculia caused by a hypersensitivity-to- interference in memory. *Cortex, 49*(1), 50-70. https://doi.org/10.1016/j.cortex.2012.01.003

DeWind, N. K., & Brannon, E. M. (2016). Significant inter-test reliability across approximate number system assessments. *Frontiers in Psychology, 7*, Article 310. https://doi.org/10.3389/fpsyg.2016.00310

Dietrich, J. F., Huber, S., Dackermann, T., Moeller, K., & Fischer, U. (2016). Place-value understanding in number line estimation predicts future arithmetic performance. *British Journal of Developmental Psychology, 34*(4), 502-517. https://doi.org/10.1111/bjdp.12146

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology, 7*, Article 508. https://doi.org/10.3389/fpsyg.2016.00508

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123*, 53-72. https://doi.org/10.1016/j.jecp.2014.01.013

Figueiredo, V. L., & Nascimento, E. (2007). Desempenhos nas duas tarefas do subteste dígitos do WISC-III e do WAIS-III. *Psicologia: Teoria e Pesquisa, 23*, 313-318. https://doi.org/10.1590/S0102-37722007000300010

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*(2), 345-362. https://doi.org/10.1037/0033-2909.114.2.345

Geary, D. C., Bailey, D. H., Littlefield, A., Wood, P., Hoard, M. K., & Nugent, L. (2009). First-grade predictors of mathematical learning disability: A latent class trajectory analysis. *Cognitive Development, 24*(4), 411-429. https://doi.org/10.1016/j.cogdev.2009.10.001

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*(4), 1343-1359. https://doi.org/10.1111/j.1467-8624.2007.01069.x

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., . . . Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS One, 8*(6), Article e67374. https://doi.org/10.1371/journal.pone.0067374

Gray, S. A., & Reeve, R. A. (2016). Number-specific and general cognitive markers of preschoolers' math ability profiles. *Journal of Experimental Child Psychology, 147*, 1-21. https://doi.org/10.1016/j.jecp.2016.02.004

Haase, V. G., Júlio-Costa, A., Lopes-Silva, J. B., Starling-Alves, I., Antunes, A. M., Pinheiro-Chagas, P., & Wood, G. (2014). Contributions from specific and general factors to unique deficits: two cases of mathematics learning difficulties. *Frontiers in Psychology, 5*, Article 102. https://doi.org/10.3389/fpsyg.2014.00102

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the “Number Sense”: The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*(5), 1457-1465. https://doi.org/10.1037/a0012682

Hart, S. A., Petrill, S. A., Thompson, L. A., & Plomin, R. (2009). The ABCs of math: A genetic analysis of mathematics and its links with reading ability and general cognitive ability. *Journal of Educational Psychology, 101*(2), 388-402. https://doi.org/10.1037/a0015115

Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition, 30*(3), 447-455. https://doi.org/10.3758/BF03194945

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology, 79*(2), 192-227. https://doi.org/10.1006/jecp.2000.2586

Hirsch, S., Lambert, K., Coppens, K., & Moeller, K. (2018). Basic numerical competences in large-scale assessment data: Structure and long-term relevance. *Journal of Experimental Child Psychology, 167*, 32-48. https://doi.org/10.1016/j.jecp.2017.09.015

Hohol, M., Cipora, K., Willmes, K., & Nuerk, H. C. (2017). Bringing back the balance: Domain-general processes are also important in numerical cognition. *Frontiers in Psychology, 8*, Article 499. https://doi.org/10.3389/fpsyg.2017.00499

Hyde, D. C., & Spelke, E. S. (2011). Neural signatures of number processing in human infants: Evidence for two core systems underlying numerical cognition. *Developmental Science, 14*(2), 360-371. https://doi.org/10.1111/j.1467-7687.2010.00987.x

Inglis, M., & Gilmore, C. (2014). Indexing the approximate number system. *Acta Psychologica, 145*, 147-155. https://doi.org/10.1016/j.actpsy.2013.11.009

Iuculano, T., Tang, J., Hall, C. W., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science, 11*(5), 669-680. https://doi.org/10.1111/j.1467-7687.2008.00716.x

Johnson, D. J., & Myklebust, H. R. (1967). *Learning disabilities: Educational principles and practices.* New York, NY, USA: Grune & Stratton.

Johnson, M. H. (2012). Executive function and developmental disorders: The flip side of the coin. *Trends in Cognitive Sciences, 16*(9), 454-457. https://doi.org/10.1016/j.tics.2012.07.001

Jordan, J.-A., Wylie, J., & Mulhern, G. (2010). Phonological awareness and mathematical difficulty: A longitudinal perspective. *British Journal of Developmental Psychology, 28*(1), 89-107. https://doi.org/10.1348/026151010X485197

Júlio-Costa, A., Antunes, A. M., Lopes-Silva, J. B., Moreira, B. C., Vianna, G. S., Wood, G., . . . Haase, V. G. (2013). Count on dopamine: influences of COMT polymorphisms on numerical cognition. *Frontiers in Psychology, 4*, Article 531. https://doi.org/10.3389/fpsyg.2013.00531

Júlio-Costa, A., Starling-Alves, I., Lopes-Silva, J. B., Wood, G., & Haase, V. G. (2015). Stable measures of number sense accuracy in math learning disability: Is it time to proceed from basic science to clinical application? *PsyCh Journal, 4*(4), 218-225. https://doi.org/10.1002/pchj.114

Karagiannakis, G., Baccaglini-Frank, A., & Papadatos, Y. (2014). Mathematical learning difficulties subtypes classification. *Frontiers in Human Neuroscience, 8*, Article 57. https://doi.org/10.3389/fnhum.2014.00057

Kaufmann, L. (2002). More evidence for the role of the central executive in retrieving arithmetic facts – A case study of severe developmental dyscalculia. *Journal of Clinical and Experimental Neuropsychology, 24*(3), 302-310. https://doi.org/10.1076/jcen.24.3.302.976

Kaufmann, L., Lochy, A., Drexler, A., & Semenza, C. (2004). Deficient arithmetic fact retrieval—Storage or access problem? A case study. *Neuropsychologia, 42*(4), 482-496. https://doi.org/10.1016/j.neuropsychologia.2003.09.004

Kaufmann, L., Mazzocco, M. M., Dowker, A., von Aster, M., Goebel, S., Grabner, R., . . . Nuerk, H. C. (2013). Dyscalculia from a developmental and differential perspective. *Frontiers in Psychology, 4*, Article 516. https://doi.org/10.3389/fpsyg.2013.00516

Kessels, R. P., van Zandvoort, M. J., Postma, A., Kappelle, L. J., & de Haan, E. H. (2000). The Corsi block-tapping task: Standardization and normative data. *Applied Neuropsychology, 7*(4), 252-258. https://doi.org/10.1207/S15324826AN0704_8

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7*(3), 164-177.

Kovas, Y., Haworth, C. M., Petrill, S. A., & Plomin, R. (2007). Mathematical ability of 10-year-old boys and girls: Genetic and environmental etiology of typical and low performance. *Journal of Learning Disabilities, 40*(6), 554-567. https://doi.org/10.1177/00222194070400060601

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition, 93*(2), 99-125. https://doi.org/10.1016/j.cognition.2003.11.004

Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: Two learning disorders with different cognitive profiles. *Journal of Experimental Child Psychology, 103*(3), 309-324.

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*(6), 1753-1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

Lopes-Silva, J. B., Moura, R., Júlio-Costa, A., Haase, V. G., & Wood, G. (2014). Phonemic awareness as a pathway to number transcoding. *Frontiers in Psychology, 5*, Article 13. https://doi.org/10.3389/fpsyg.2014.00013

Lopes-Silva, J. B., Moura, R., Júlio-Costa, A., Wood, G., Salles, J. F., & Haase, V. G. (2016). What is specific and what is shared between numbers and words? *Frontiers in Psychology, 7*, Article 22. https://doi.org/10.3389/fpsyg.2016.00022

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science, 17*(5), 714-726.

Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2010). Spatial working memory and arithmetic deficits in children with nonverbal learning difficulties. *Journal of Learning Disabilities, 43*(5), 455-468.

Mazzocco, M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 29-47). Baltimore, MD, USA: Paul H Brookes Publishing.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development, 82*(4), 1224-1237. https://doi.org/10.1111/j.1467-8624.2011.01608.x

Mazzocco, M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research, 2012*, Article 170310. https://doi.org/10.1155/2012/170310

Mazzocco, M. M., & Myers, G. F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of Dyslexia, 53*(1), 218-253.

McCloskey, M., Caramazza, A., & Basili, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition, 4*(2), 171-196.

McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual-spatial interference on children's arithmetical performance. *Educational and Child Psychology, 20*(3), 93-108.

McLean, J. F., & Rusconi, E. (2014). Mathematical difficulties as decoupling of expectation and developmental trajectories. *Frontiers in Human Neuroscience, 8*, Article 44. https://doi.org/10.3389/fnhum.2014.00044

Merkley, R., Thompson, J., & Scerif, G. (2016). Of huge mice and tiny elephants: Exploring the relationship between inhibitory processes and preschool math skills. *Frontiers in Psychology, 6*, Article 1903. https://doi.org/10.3389/fpsyg.2015.01903

Moura, R., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: The role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology, 116*(3), 707-727. https://doi.org/10.1016/j.jecp.2013.07.008

Mussolin, C., Mejias, S., & Noël, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition, 115*(1), 10-25.

Newton, A. T., & Penner-Wilger, M. (2015, December). The cognitive and mathematical profiles of children in early elementary school [Abstract]. *Canadian Journal of Experimental Psychology / Revue Canadienne de Psychologie Expérimentale, 69*(4), 348.

Noël, M. P., & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: An explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience, 5*, Article 165.

Nosworthy, N., Bugden, S., Archibald, L., Evans, B., & Ansari, D. (2013). A two-minute paper-and-pencil test of symbolic and nonsymbolic numerical magnitude processing explains variability in primary school children's arithmetic competence. *PLoS One, 8*(7), Article e67918. https://doi.org/10.1371/journal.pone.0067918

Oliveira, L. F., Santos, A. O., Vianna, G. S., Di Ninno, C. Q., Giacheti, C. M., Carvalho, M., . . . Haase, V. G. (2014). Impaired acuity of the approximate number system in 22q11.2 microdeletion syndrome. *Psychology & Neuroscience, 7*(2), 151-158. https://doi.org/10.3922/j.psns.2014.02.04

Oliveira-Ferreira, F., Costa, D. S., Micheli, L. R., Sílvia Oliveira, L. D. F., Pinheiro-Chagas, P., & Haase, V. G. (2012). School Achievement Test: Normative data for a representative sample of elementary school children. *Psychology & Neuroscience, 5*(2), 157-164. https://doi.org/10.3922/j.psns.2012.2.05

Osmon, D. C., Smerz, J. M., Braun, M. M., & Plambeck, E. (2006). Processing abilities associated with math skills in adult learning disability. *Journal of Clinical and Experimental Neuropsychology, 28*(1), 84-95. https://doi.org/10.1080/13803390490918129

Peake, C., Jiménez, J. E., & Rodríguez, C. (2017). Data-driven heterogeneity in mathematical learning disabilities based on the triple code model. *Research in Developmental Disabilities, 71*, 130-142.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., . . . Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition, 116*(1), 33-41. https://doi.org/10.1016/j.cognition.2010.03.012

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron, 44*(3), 547-555.

Pieters, S., Roeyers, H., Rosseel, Y., Van Waelvelde, H., & Desoete, A. (2015). Identifying subtypes among children with developmental coordination disorder and mathematical learning disabilities, using model-based clustering. *Journal of Learning Disabilities, 48*(1), 83-95.

Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., . . . Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation? *PLoS One, 9*(11), Article e111155. https://doi.org/10.1371/journal.pone.0111155

Pixner, S., Zuber, J., Heřmanová, V., Kaufmann, L., Nuerk, H. C., & Moeller, K. (2011). One language, two number-word systems and many problems: Numerical cognition in the Czech language. *Research in Developmental Disabilities, 32*(6), 2683-2689. https://doi.org/10.1016/j.ridd.2011.06.004

Primi, R., Ferrão, M. E., & Almeida, L. S. (2010). Fluid intelligence as a predictor of learning: A longitudinal multilevel approach applied to math. *Learning and Individual Differences, 20*(5), 446-451.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences, 20*(2), 110-122.

Raghubar, K., Cirino, P., Barnes, M., Ewing-Cobbs, L., Fletcher, J., & Fuchs, L. (2009). Errors in multi-digit arithmetic and behavioral inattention in children with math difficulties. *Journal of Learning Disabilities, 42*(4), 356-371.

Raven, J. (2000). The Raven’s progressive matrices: Change and stability over culture and time. *Cognitive Psychology, 41*(1), 1-48. https://doi.org/10.1006/cogp.1999.0735

Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and change in markers of core numerical competencies. *Journal of Experimental Psychology: General, 141*(4), 649-666. https://doi.org/10.1037/a0027520

Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research & Practice, 17*(2), 81-89.

Rourke, B. P. (1989). *Nonverbal learning disabilities: The syndrome and the model*. New York, NY, USA: Guilford Press.

Rourke, B. P. (1995). *Syndrome of nonverbal learning disabilities: Neurodevelopmental manifestations.* New York, NY, USA: Guilford Press.

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*(3), 361-395. https://doi.org/10.1016/j.cognition.2006.01.005

Rubinsten, O., & Henik, A. (2009). Developmental dyscalculia: Heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences, 13*(2), 92-99.

Santos, F. H., Mello, C. B., Bueno, O. F. A., & Dellatolas, G. (2005). Cross-cultural differences for three visual memory tasks in Brazilian children. *Perceptual and Motor Skills, 101*, 421-433. https://doi.org/10.2466/pms.101.2.421-433

Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2016). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20*(3), Article e12372.

Simmons, F. R., & Singleton, C. (2008). Do weak phonological representations impact on arithmetic development? A review of research into arithmetic and dyslexia. *Dyslexia, 14*(2), 77-94. https://doi.org/10.1002/dys.341

Stein, L. M. (1994). *TDE: teste de desempenho escolar: manual para aplicação e interpretação.* São Paulo, Brasil: Casa do Psicólogo.

Strauss, E., Sherman, E. M., & Spreen, O. (2006). *A compendium of neuropsychological tests: Administration, norms, and commentary* (3rd ed.). New York, NY, USA: Oxford University Press.

Sullivan, K. S. (1996). Remediation of Arabic numeral processing in a case of developmental dyscalculia. *Neuropsychological Rehabilitation, 6*(1), 27-54. https://doi.org/10.1080/713755495

Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology, 79*, 294-321.

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research, 76*(2), 249-274. https://doi.org/10.3102/00346543076002249

Ta'ir, J., Brezner, A., & Ariel, R. (1997). Profound developmental dyscalculia: Evidence for a cardinal/ordinal skills acquisition device. *Brain and Cognition, 35*(2), 184-206. https://doi.org/10.1006/brcg.1997.0937

Tannock, R. (2013). Rethinking ADHD and LD in DSM-5 proposed changes in diagnostic criteria. *Journal of Learning Disabilities, 46*(1), 5-25. https://doi.org/10.1177/0022219412464341

Temple, C. M. (1989). Digit dyslexia: A category-specific disorder in development dyscalculia. *Cognitive Neuropsychology, 6*(1), 93-116. https://doi.org/10.1080/02643298908253287

Temple, C. M. (1991). Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology, 8*(2), 155-176. https://doi.org/10.1080/02643299108253370

Temple, C. M. (1997). *Brain damage, behaviour and cognition: Developments in clinical neuropsychology: Developmental cognitive neuropsychology.* Hove, United Kingdom: Psychology Press/Erlbaum (UK) Taylor & Francis.

Vanbinst, K., Ceulemans, E., Ghesquière, P., & De Smedt, B. (2015). Profiles of children's arithmetic fact development: A model-based clustering approach. *Journal of Experimental Child Psychology, 133*, 29-46. https://doi.org/10.1016/j.jecp.2015.01.003

Vanbinst, K., Ceulemans, E., Peters, L., Ghesquière, P., & De Smedt, B. (2018). Developmental trajectories of children's symbolic numerical magnitude processing skills and associated cognitive competencies. *Journal of Experimental Child Psychology, 166*, 232-250.

Venneri, A., Cornoldi, C., & Garuti, M. (2003). Arithmetic difficulties in children with visuospatial learning disability (VLD). *Child Neuropsychology, 9*(3), 175-183.

von Aster, M. (2000). Developmental cognitive neuropsychology of number processing and calculation: Varieties of developmental dyscalculia. *European Child & Adolescent Psychiatry, 9*(Suppl. 2), S41-S57.

Wagner, R. K., & Torgesen, J. K. (1987). The nature of phonological processing and its causal role in the acquisition of reading skills. *Psychological Bulletin, 101*(2), 192-212. https://doi.org/10.1037/0033-2909.101.2.192

Ward, J. H., Jr. (1963). Hierarchical grouping to optimize an objective function. *Journal of the American Statistical Association, 58*(301), 236-244.

Wilson, A. J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. In D. Coch, G. Dawson, & K. W. Fischer (Eds.), *Human behavior, learning, and the developing brain: Atypical development* (pp. 212-238). New York, NY, USA: Guilford Press.

Wong, T. T. Y., Ho, C. S. H., & Tang, J. (2014). Identification of children with mathematics learning disabilities (MLDs) using latent class growth analysis. *Research in Developmental Disabilities, 35*(11), 2906-2920. https://doi.org/10.1016/j.ridd.2014.07.015

Wood, G., & Fischer, M. H. (2008). Numbers, space, and action–from finger counting to the mental number line and beyond. *Cortex, 44*(4), 353-358.

Wood, G., Pinheiro-Chagas, P., Júlio-Costa, A., Micheli, L. R., Krinzinger, H., Kaufmann, L., . . . Haase, V. G. (2012). Math Anxiety Questionnaire: Similar latent structure in Brazilian and German school children. *Child Development Research, 2012*, Article 610192. https://doi.org/10.1155/2012/610192

4.3. Educational settings, gender differences and cognitive processes in math performance: A cross-cultural study with Brazilian and Italian children

Educational settings, gender differences and cognitive processes in math performance: A cross-cultural study with Brazilian and Italian children

Larissa Salvador[1], Sara Caviola[3], Ricardo Basso[4], Ricardo Moura[2], Vitor Geraldi Haase[5], Irene C. Mammarella[6]

1 Department of Psychology, Graduate Program in Children's and Adolescents Health, Federal University of Minas Gerais (UFMG), Belo Horizonte, Brazil

2 Department of Basic Psychological Processes, Institute of Psychology, University of Brasília, (UnB), Brasília, Brazil.

3 School of Psychology, University of Leeds, UK.

4 University of São Paulo (USP). Faculty of Philosophy, Sciences, and Letters of Ribeirão Preto (FFCLRP), Brazil

5 Department of Psychology, Graduate Program in Children's and Adolescents Health, Graduate Program in Psychology: Cognition and Behavior, Graduate Program in Neuroscience, Federal University of Minas Gerais (UFMG); National Institute of Science and Technology on Cognition, Behavior, and Teaching (INCT-ECCE), Belo Horizonte, Brazil

6 Department of Development and Social Psychology, University of Padova, Italy

Abstract

Math performance has been consistently associated with several cognitive variables such as verbal and visuospatial skills. However, little is known about how these variables interact with others such as gender, SES and school settings and how it impacts on math performance. In the present study, we investigated the verbal and visuospatial skills and math performance in elementary school children in two different countries and school settings. 458 children from private schools in Brazil and State-run schoolsin Brazil and Italy were evaluated. Results showed that Italian children had better cognitive and arithmetic performance than Brazilian children. In Brazil, children from private schools performed better than children from State-run schools. Both verbal and visuospatial skills were associated with math fluency performance. There were no gender differences in cognitive measures (verbal and visuospatial skills); however, gender differences were found in math fluency. In particular, Brazilian girls from State-run schoolspresented worse performance than girls from Brazilian private and Italian schools, with the last groups showing no differences between them.

Keywords: Visuospatial skills, Mental rotation, mathematics, math fluency, intelligence, cross-cultural study.

4.3.1 Introduction

Mathematical performance is a complex phenomenon, influenced by multiple variables that interact in complex ways (Bodovski, Byun, Chykina & Chung, 2017). Simply put, it can be said that mathematical performance is influenced by individual and social, cultural and economic differences (Salvador et al., 2019; Hart, Ganley & Purpura, 2016; Else-Quest, Hyde & Linn, 2010; Nosek et al., 2009). Results from international academic performance evaluation programs such as PISA show an important correlation between math performance and socio-economic indicators such as per capita Gross Domestic Product (GDP), social equality, gender equality, among others (Stoet & Geary, 2013; Guiso, Monte, Sapienza & Zingales, 2008). As can be seen in Table 1, developing countries such as Brazil, Uruguay, and Chile present a similar performance on PISA, which is lower than that observed developed countries such as Italy, Portugal, Spain and Norway (PISA, 2015).

Table 1: Socioeconomic and mathematical performance data across countries

Country	GDP per capita	Gini Index	Gender Inequality Index	Women Political empowerment	PISA Math (2015)	Math gender gap (Boys - Girls)
Brazil	11.02	53.3	0.407	0.06	377	15*
Uruguay	14.61	39.5	0.270	0.04	418	12*
Chile	15.13	46.6	0.319	-	423	18*
Italy	35.39	35.4	0.087	0.09	490	20*
Portugal	23.73	35.5	0.088	0.14	492	10*
Spain	33.14	36.2	0.080	0.49	486	16*
Norway	92.12	27.5	0.048	0.49	502	- 2

* p<.05. The difference between male and female scores is significantly different at the .05 level of statistical significance. References: United Nations Development Program- UNDP, 2018; PISA 2015.

Cross-country differences in math achievement can also be analyzed in terms of the gender gap in PISA and gender equality across countries. As suggested by the data presented in Table 1, an important individual trait that can be related to math performance is gender. Some studies point to an advantage for girls in tasks involving verbal skills while boys perform better on tasks involving visuospatial skills and mathematics (Van Tetering, Van der Donk, De Groot & Jolles, 2019). However, a more complex picture is observed when the interactions between socio-environmental variables and gender are considered. Guiso, Monte, Sapienza, and Zingales (2008) analyzed cross-national data on math and reading scores classifying countries according to the level of gender equality. They found that there is considerable variability among countries but, on average, girls perform lower than boys in math and better than boys in reading. When gender equality was considered, these differences disappeared suggesting that girls have an advantage in reading scores and perform as well as boys in mathematics (Guiso et. al, 2008).

Newson and Richerson (2009) proposed the hypothesis that the smaller gender gap in mathematics in more developed countries could reflect the effects of modernization. They compared 174 nations and observed an association between changes on cultural norms and social and economic indicators such as GDP per capita, perception of corruption, gender equity, years spent in education and IQ, that were associated with the decrease in fertility. This is understood in terms of parental investment theory. In adverse ecological conditions, reproductive fitness is associated with a more quantitative strategy of having more children and investing less in each one (Belsky, 2007, Belsky et al., 1991). Otherwise, in more favorable ecological conditions, reproductive fitness may be advanced by a lower progeny and more intense efforts dedicated to each child.

According to Newson and Richerson (2009), the phenomenon of modernization is associated with important changes in social norms caused by changes in kinship influence that could be indicated by the year in which fertility rates began to decline in a country. This decline of fertility then generates a greater availability of women for literacy, emancipation, greater division of labor between the sexes and gender equality. This hypothesis predicts that gender differences in academic achievement should narrow with modernization.

The hypothesis of stereotype threat and modernization of society are concordant (Spencer, Steele & Quinn, 1999). Stereotype threat predicts that stigmatized group members may underperform on tests of ability through concerns about negative societal beliefs as in mathematics, for example, in which boys are believed to perform better than girls (Pennington, Heim, Levy & Larkin, 2016). This hypothesis predicts that girls are more susceptible to the effects of beliefs about gender differences in tasks involving math skills than boys, contributing to the performance differences between gender (Muzzatti & Agnoli, 2007; Galdi, Cadinu & Tomasetto, 2014). Moreover, stereotype threat may be influenced by sociocultural factors such as gender equality across countries (Flore & Wicherts, 2015)

On the other hand, differences in math performance between sexes may be mediated by biological influences such as better visuospatial ability associated with higher testosterone levels in males (Halpern, 2000). Stoet and Geary (2013) suggest that gender differences may persist at the highest performance level of the distribution. They showed that there are no differences in average academic performance between sexes across countries, but differences are observed at the extremes of the distribution, with better female performance in reading/writing tasks and better male performance in math tasks. In a more recent study, Stoet and Geary (2018) suggested that this gender gap in math performance is even greater in more developed countries, which present higher gender equity. They suggest that in these countries, where women do not have to worry so much about wages and social disparities between genders, the biological differences would be more explicit. In developing countries, however, the social factor would be most important for women engaging in tasks involving mathematical performance. Such findings could reflect the influence of biological variables on math performance.

Math performance is also influenced by individual differences in cognitive abilities such as spatial and verbal intelligence. Intelligence is a major predictor of academic achievement and has been strongly related to social outcomes including income, educational attainment, health, general socioeconomic status, fertility and crime rates across several countries (Lynn, Fuerst & Kirkegaard, 2018). In the present study, we investigate spatial and verbal intelligence differences between Brazil and Italy and intra-nationally in Brazil.

The multiple associations identified in math performance suggest that there may be complex interactions between individual and socio-environmental sources of influence on math achievement. In the present article, we investigate the influence of individual and socio-economic-cultural differences on the arithmetic achievement of school children. In order to do so we performed: a) a comparison between two countries, Brazil and Italy, which are located at opposite ends in terms of their socio-economic-educational indicators; b) an intra-national comparison in Brazil between two educational settings with different quality of education and socioeconomic status (SES), the best quality and SES observed in private schools and the poorer quality and SES in State-run schools; c) a comparison between genders. We were interested in finding out how these inter, and intra-national and gender differences are associated with math fluency, visuospatial intelligence, and verbal intelligence.

According to the hypotheses of stereotype threat and society modernization, we can predict: a) better math performance in Italian children and Brazilian private school children than Brazilian State-runschool children; b) better math performance in males than in females in Brazil State-run schoolsthan those in Brazilian private schools and in Italian schools; c) worse performance in arithmetic for Brazilian female children when compared to male ones.

4.3.2 Methods

4.3.2.1 Participants

The initial sample comprised 458 children, from 3rd to 5th grades. Fifty-three children were excluded from the sample because they did not complete the entire protocol. The final sample comprised 410 participants: 206 Italian children, attending State-run schools, with a mean age of 116.41 (sd = 9.53) months. In Brazil, 201 children (mean age = 117.80 [sd = 12.66] months) were recruited from State-run (n = 95, mean age = 115.72 [sd = 12.61] months) and private (n = 106, mean age = 119.66 [sd = 12.47] months) schools. Subdivision of the Brazilian sample into two groups is based on the fact that private schools offer a better quality of teaching than State-run schools, and this reflects in the student's achievement scores.

4.3.2.2 Materials

The instruments were selected considering the need for testing two samples from different countries regarding verbal and visuospatial skills and math performance. The same protocol and procedures were used to assess children in both countries. Assessment was performed using the Verbal Meaning (VM) and Spatial Relations (SR) Primary Mental Ability (PMA) subtests and two arithmetic fluency tasks, a simple task composed of 1-digit operations (Wechsler, 2009) and a complex task composed of 2-digits operations (Caviola, Gerotto, Lucangeli & Mammarella, 2016; Caviola, Gerotto, & Mammarella, 2016).

Primary Mental Abilities (PMA)

For our purposes, two subtests of the Primary Mental Abilities (Thurstone & Thurstone, 1963) were selected: Verbal Meaning (VM) and Spatial Relations (SR). In Brazil, a version based on the PMA was constructed and edited by the group CEPA (Centro Editor de Psicologia Aplicada LTDA, 1993). The CEPA Specific-Factorial Battery is composed of a general intelligence test (g-factor) and seven tests of specific factors. There are no adequate norms for the age group evaluated in Brazil, so it was decided to use z scores based on the whole sample stratified for grades for both countries.

4.3.2.3 Primary mental abilities - Verbal Meaning (VM)

The subtest that evaluates verbal meaning is composed of 60 items in the Brazilian version (Rainho & Ribeiro, 1993), and of 30 items in the original version used to evaluate the Italian children (Thurstone & Thurstone, 1963). The items gradually increase in the word's difficulty and complexity levels. In this subtest, the participant must choose from six options, which is the synonym of the highlighted word. As no current normative standards were available for the Brazilian population, an item-analysis was conducted in the Brazilian sample. Sixteen items were excluded because they presented very high error rates and, consequently, very low correlations with the final result (r around 0.1). After the exclusion of 16 items, the internal consistency of the task changed from $\alpha = 0.78$ for $\alpha = 0.83$. The final Brazilian task was composed of 44 items. The accuracy of the test in the Brazilian version and in the Italian version was based on the percentage of correct answers for each sample. Individual z-scores were calculated using parameters from the whole sample for each grade separately.

4.3.2.4 Primary mental abilities- Spatial Relations (SR).

The PMA SR subtest evaluates visuospatial reasoning skills through the ability to mentally rotate images. Mental rotation involves the ability to rotate the mental images around some axis in two or three-dimensional space. This ability has been associated with numerical processing (Dehaene, Piazza, Pinel & Cohen, 2003; Thompson, Nuerk, Korbinian Moeller & Kadosh, 2013). The PMA SR is composed of 25 items in which an incomplete part of a geometric form is presented, and the participant is requested to choose among four options, which best fits the initial figure, constructing a complete shape of a square. To perform the task, the participant must imagine the complete figure and rotate the parts of the presented figures to find the best one. Raw scores were based on the percentage of correct answers. Individual z-scores were calculated using parameters from the whole sample for each grade.

4.3.2.5 Simple math fluency (WIAT–III)

This task consists of simple calculation problems of addition, subtraction, and multiplication, printed on separate sheets of paper. Addition and subtraction blocks comprise 48 items each one. Multiplication is composed of 40 items. Simple math fluency comprises problems in which the operands were below 10. Children were instructed to answer as quickly and as accurately as possible, with the time limit of 1 min per block. WIAT–III is designed to be used with people from 4 to 19 years old (Wechsler, 2009). The number of correctly answered problems of total operations was considered as the dependent variable.

4.3.2.6 Complex math fluency

Addition, subtraction, and multiplication problems with 24 items each were divided into three blocks (Caviola et al., 2016). The problems consisted of 2-digit operations presented horizontally on a square sheet. The children were instructed to answer as quickly and as accurately as possible, with the time limit of 2 min per block. The number of correctly answered problems for all three operations taken together was considered as the dependent variable.

4.3.2.7 Procedure

Children were assessed in their own schools, in sessions of approximately 40 minutes, by specially trained undergraduate psychology students both in Italy and in Brazil. The assessment was applied to groups of approximately 8 children.

4.3.2.8 Statistical Analyses

First, we conducted descriptive statistical analyses to characterize the participants from Brazil and Italy. Then, we compared the distribution of age and gender among samples. To investigate the association between verbal and visuospatial skills, sex and educational setting (Italy vs. Brazil private vs. Brazil State-run schools) we conducted a series of ANOVAs interaction analysis. Math fluency performance, sex differences, and quality of education were investigated using interaction effect analyzes. Afterward, a multivariate regression analysis was performed using PMA VM and PMA SR as independent variables, controlling for age and gender. This aimed to examine the cognitive structure underlying math fluency performance considering different types of educational settings. The total scores from each math fluency task were used as dependent variables in this model.

All analyses were performed in software R. For investigating significant differences among interaction effects, the “phia” package was used. The multivariate regression analysis was conducted using the “lavaan” package.

4.3.3 Results

Results are presented in four main sections: 1) demographic characteristics of participants; 2) cognitive performance according to gender and educational setting; 3) math performance according to gender and educational setting; 4) cognitive correlates of math achievement according to the educational setting.

4.3.3.1 Demographic characteristics of participants

Demographic data stratified by gender, grade, and nationality are presented in Table 2. No significant age differences were found between Brazilian and Italian children (t = 1.25, df = 408, p = 0.21). However, age differences were significant among the three groups (F= 3.93, df = 2, p = 0.02). Brazilian private school children had a significantly higher mean age than children from Brazilian State-runschool (p = 0.038, d = 0.314) and Italian State-runschool (p = 0.044, d = 0.306). No significant differences regarding the sex distribution were found between the two nationalities ($X^2 = 1.86$, $df = 1$; $p < 0.17$) and among the three educational settings ($X^2 = 3.89$, $df = 2$; $p < 0.14$).

Table 2: Demographic characteristics of participants

	Brazil State-runschool				Brazil Private School				Italy State-runschool			
Grade	n (%)	Male (%)	Female (%)	Age in months Mean (sd)	n (%)	Male (%)	Female (%)	Age in months Mean (sd)	n (%)	Male (%)	Female (%)	Age in months Mean (sd)
3	33 (34.7)	16 (48.5)	17 (51.5)	103.9 (5.31)	38 (37.6)	18 (47.4)	20 (52.6)	106.5 (4.85)	35 (16.7)	18 (51.4)	17 (48.6)	103.4 (4.90)
4	35 (36.8)	20 (57.1)	15 (42.9)	113.5 (4.62)	23 (17.8)	10 (43.5)	13 (56.5)	119.17 (3.48)	113 (54.1)	67 (59.3)	46 (40.7)	114.5 (5.71)
5	27 (28.4)	15 (55.6)	12 (44.4)	132.8 (5.01)	45 (44.6)	19 (42.2)	26 (57.8)	131.0 (7.99)	61 (29.2)	32 (52.5)	29 (47.5)	127.3 (3.55)
Total	95 (100)	44 (53.7)	51 (46.3)		106 (100)	47 (44.3)	59 (55.7)		209 (100)	117 (56.6)	92 (44.0)	

4.3.3.2 Cognitive performance according to gender and educational setting

The performance in both PMA tasks was investigated considering z-scores of a proportion of correctness for each country and educational setting. Significant differences were found among the three groups for PMA VM, in the following order from the best to the worst performance: Italian State-run schools> Brazilian private schools > Brazilian State-run schools. No differences were observed between Italian State-runschool children and Brazilian private school children in the PMA SR, with both performing better than Brazilian State-runschool children. No sex by educational setting interactions was observed either in the PMA VM or PMA SR (Table 3).

Table 3: Performance in PMA tasks for each educational setting

	Brazil State-runschool Mean (sd)	Brazil Private school Mean (sd)	Italy State-runschool Mean (sd)	F (2; 402)	*p*	Post-hoc (Bonferroni test)	η²
PMA VM	-0.68 (0.66)	-0.11 (0.65)	0.37 (1.08)	45.65	< 0.001	2 > 1 > 0	0.183
PMA SR	-0.18 (0.89)	0.18 (0.94)	0.01 (1.05)	3.43	0.033	2 = 1 > 0	0.017

Note: 0 = Brazil State-run schools1= Brazil private schools; 2 = Italy State-run schools; PMA VM: Primary Mental Ability Verbal Meaning; PMA SR: Primary Mental Ability Spatial Relations

4.3.3.3 Math fluency according to gender, country, and educational setting

A series of variance analysis (ANOVAs) was conducted to investigate differences among children from the two Brazilian school types and Italian children (Table 3). In the simple math fluency task, children from Italian State-run schoolsperformed significantly better than Brazilian children from private and State-run schools ($p < 0.001$). In the complex math fluency task, children from Italy and private schools in Brazil performed better than children from Brazilian State-run schools ($p < 0.001$). No differences were found between children from Italian State-runand Brazilian private schools.

Table 4: Math fluency performance according to country and educational setting

	Brazil State-runschool Mean (sd)	Brazil Private school Mean (sd)	Italy State-runschool Mean (sd)	F (2; 407)	*p*	Post-hoc (Bonferroni test)	η^2
Simple math fluency (1 digit)	-0.68 (0.79)	0.01 (0.96)	0.29 (0.95)	37.15	< 0.001	2 > 1 > 0	0.154
Complex math fluency (2 digits)	-0.64 (0.68)	0.10 (0.92)	0.23 (1.02)	29.91	< 0.001	2 = 1 > 0	0.128

	Brazil State-runschool		Brazil Private School		Italy State-runschool	
	Male Mean (sd)	Female Mean (sd)	Male Mean (sd)	Female Mean (sd)	Male Mean (sd)	Female Mean (sd)
Simple math fluency (1 digit)	- 0.44 (0.77)	- 0.95 (0.73)	0.10 (1.05)	- 0.04 (0.87)	0.44 (1.00)	0.11 (0.85)
Complex math fluency (2 digits)	- 0.51 (0.71)	-0.78 (0.64)	- 0.03 (0.89)	0.21 (0.94)	0.26 (1.00)	0.20 (0.92)

Note: 0 = Brazil State-runschool; 1= Brazil Private School; 2 = Italy State-runschool

To investigate the effect of educational setting on math performance we conducted a series of interaction effects analysis considering gender and school type. Figure 1 shows the results of interaction analysis in the simple math fluency task. Figure 1a shows significant between gender differences for the whole sample ($F = 11.86$, $p < 0.001$, $\eta^2 = 0.028$). Figure 1b shows the between gender differences among educational setting groups. The performance of male participants differed significantly across the three educational setting groups ($F = 15.32$, $p < 0.001$, $\eta^2 = 0.126$) in the following order from the best to the worst performance: Italian State-run schools> Brazilian private schools > Brazilian State-run schools. Female participants from Italian and Brazilian private schools did not differ in simple math fluency. Performance in these two groups of female participants differed from that of Brazilian State-runfemales ($F = 25.18$, $p < 0.001$, $\eta^2 = 0.208$)

Figure 1c shows significant gender differences in Brazilian State-run schools ($F = 10.39$, $p < 0.002$, $\eta^2 = 0.101$) and Italian State-run schools ($F = 6.58$, $p < 0.01$, $\eta^2 = 0.031$), with males performing better than females. No significant between gender differences was found in Brazilian private schools.

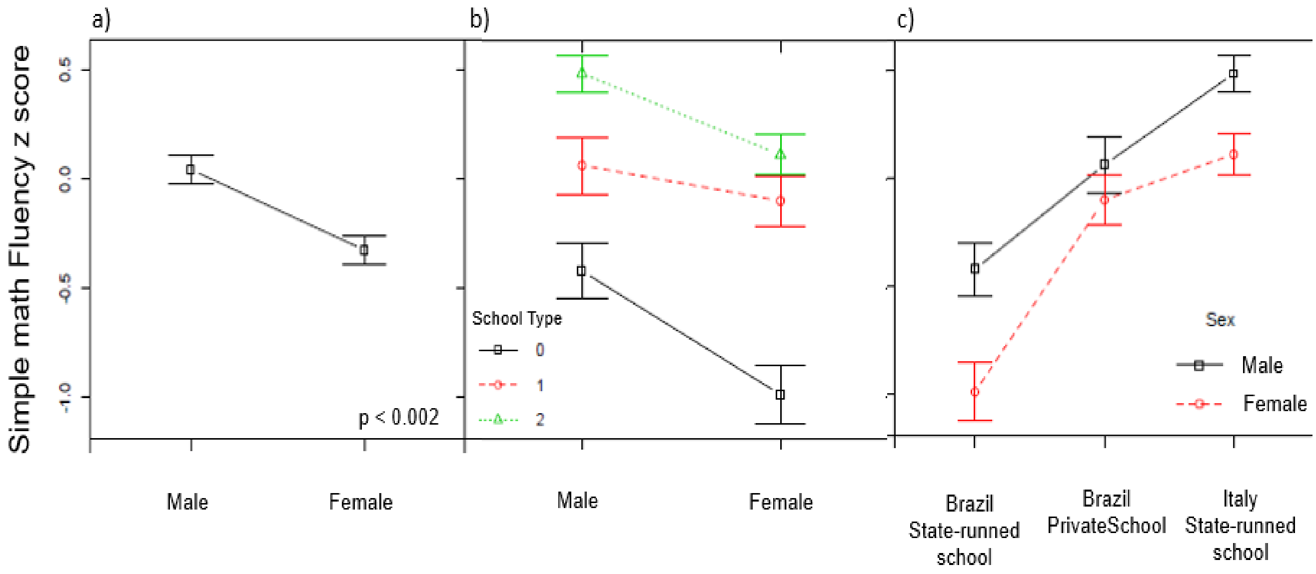

Figure 1: Gender by educational setting interaction effects in the simple math fluency task

Note: 0 = Brazil State-runschool; 1= Brazil Private School; 2 = Italy State-runschool

In complex math fluency, no significant differences were found between gender for the whole sample (Figure 2a). Significant differences in complex math fluency were found among males from the three educational setting groups ($F = 11.40$, $p < 0.001$, $\eta^2 = 0.097$) in the following order from the best to the worst performance: Italian State-run schools> Brazilian private schools > Brazilian State-run schools (Figure 2b). For females, no significant differences were found between Italian and Brazilian Private School children; these two groups outperformed females from Brazilian State-run schools ($F = 21.96$, $p < 0.001$, $\eta^2 = 0.186$). No intra-group gender differences in complex math fluency were found (Figure 2c).

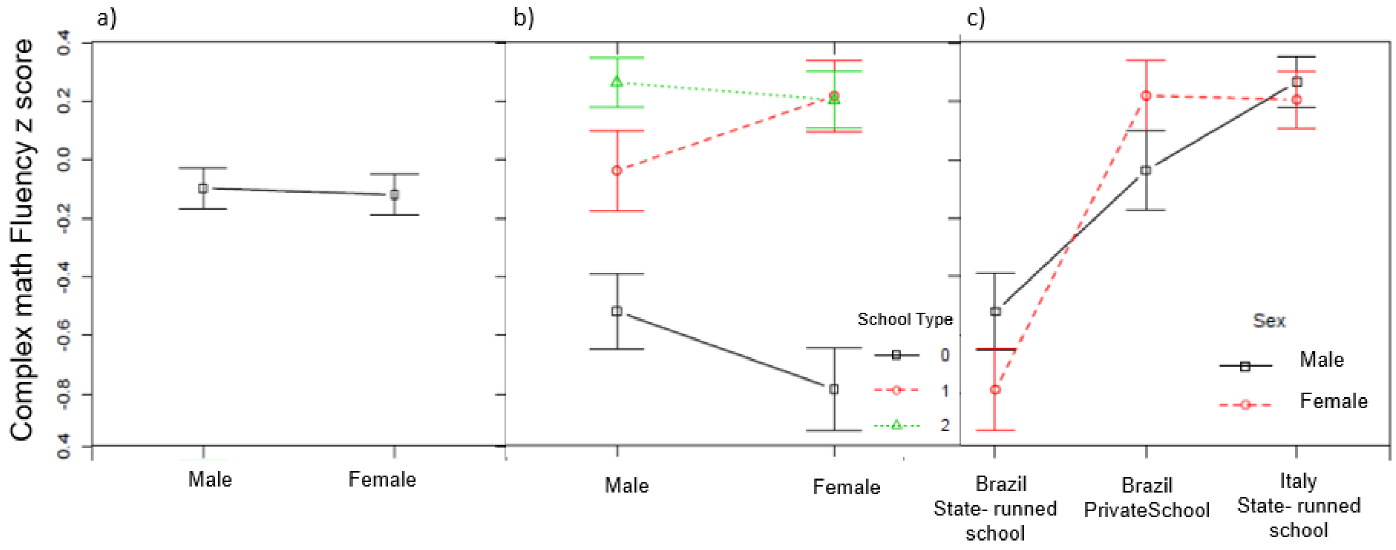

Figure 2: Gender by educational setting interaction effects in the complex math fluency task

Note: 0 = Brazilian State-run schools; 1= Brazilian private schools; 2 = Italian State-runschool

4.3.3.4 Cognitive correlates of math fluency across countries and educational settings

We investigated the association of both types of math fluency according to country and educational setting. First, we conducted a series of multivariate regression analysis using the whole sample. The best-fitting model was chosen based on the goodness-of-fit indexes considered in the literature (Cheung and Rensvold, 2002). These indexes are the comparative fit index (CFI), Tucker-Lewis index (TLI), the standardized root means square residual (SRMR), Akaike information criterion (AIC) and the Bayesian Information Criterion (BIC). For the CFI and TLI values that are >.95 and >.97 have been associated with acceptable values and indicate good fit, respectively. Values of SRMR ≤ 0.08 suggest a good model fit to the data. AIC and BIC are comparative indexes based upon the model chi-square and are interpreted such that the model with the lower value exhibits a better fit of the data.

Figure 3 shows the path analysis of the best multiple regression model considering the country and educational setting. The paths show the relative weight for each variable. All correlations were significant among variables. The model indexes are in accordance with acceptable values for a good fit model. The estimated R^2 for each dependent variable was also evaluated. For the simple math fluency task, the estimate R^2 was 0.452 and, for the complex math fluency task, the estimate R^2 was 0.403. Results indicate that PMA VM, PMA SR, and age contribute to both simple and complex math fluency performance. Otherwise, gender contributes only to simple math fluency performance. Educational setting contributed to both simple and complex math fluency.

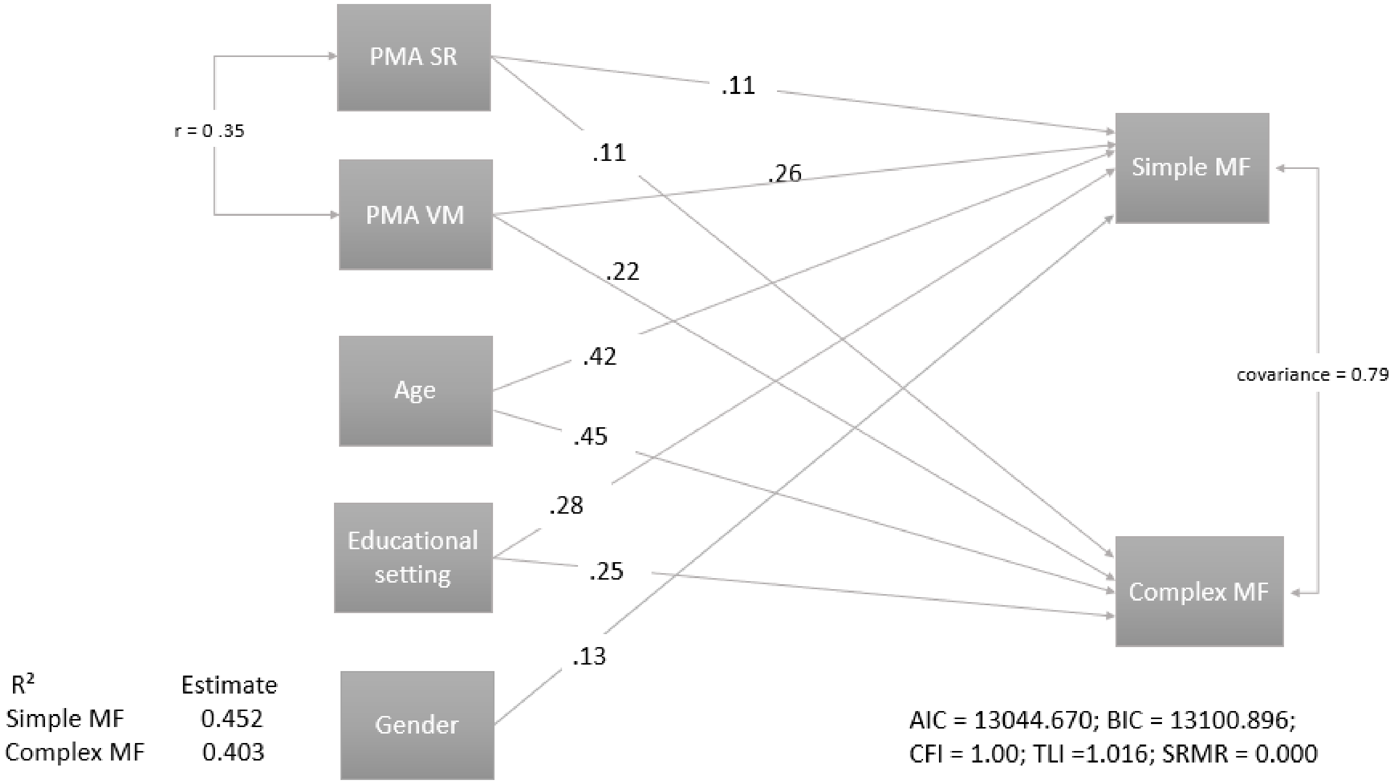

Figure 3: Path Analysis of the best fit model for simple and complex math fluency in the whole sample

4.3.4 Discussion

In this study, we were interested in investigating the influence of socio-economic-cultural factors, gender and cognitive abilities on the math performance of Brazilian and Italian primary school children. In order to investigate this, we compared math performance and cognitive abilities between Brazil and Italy and also between private and state-run Brazilian schools, which represent two educational settings of differing quality and SES.

The results are summarized as follows. In general, Italian children had better cognitive and arithmetic performance than Brazilian children. In Brazil, children from private schools performed better than children from state-run schools. Both PMA-VM and PMA-SR were associated with math fluency performance. There were no gender differences in cognitive measures (PMA VM and PMA SR); however, gender differences were found in math fluency. There was an interaction between gender and school settings regarding math fluency: Brazilian girls from state-run schools presented worse performance than girls from Brazilian private and Italian schools, with the last groups showing no differences between them.

In the next sections, we will discuss a) math performance across countries and educational settings, b) associations between cognitive abilities, math performance, and gender and, c) cognitive correlates of math fluency.

4.3.4.1 Math performance across countries and educational settings

Our most notable findings regarding countries differences are that children from state-run schools in Brazil performed worse than the other two educational setting groups. Moreover, children from Brazilian private schools performed similarly to Italian children in math fluency. Further, important differences were found between children from state-run and private schools in Brazil. These findings can be interpreted based on the hypothesis of society modernization (Newson and Richerson, 2009), which predicts that better countries social and economic indexes are associated with better academic performance.

The role of socioeconomic factors in math performance has been widely described in the literature (Larson, Nelson, Olson, & Halfon, 2015; Bodovski et. al, 2017; Demir, Prado & Booth, 2015). The impact of SES can be seen in the early years, even before formal schooling begins, resulting from environmental factors such as parents' early engagement in math-related activities (Hart, Ganley & Purpura, 2016). Berthelot, Ross, and Tremblay (2001) used an interesting multilevel statistical approach to evaluate factors that influence academic performance and found that SES and its correlates such as the number of books at home, parental involvement with schools, and literacy acquisition in a second language could explain the variance observed. They also reported that 33% of the explained variance was attributed to classrooms and schools. This study has shown that the impact of SES on math performance may contribute to increasing the cultural differences traditionally described in the literature. One strength of our study is that we compared both cross-national linguistic/cultural differences and intra-national SES differences (LeFevre, Cankay, Xu & Lira).

In countries with greater social inequalities, such as Brazil, school performance discrepancies among children from different SES are considerable (Nunes, Schliemann, & Carraher, 1993). Differences between state-run schools and private schools in Brazil largely reflect the country's social inequality (Nunes, Schliemann, & Carraher, 1993). While in Brazil the GINI index is 0.53, in Italy it is 0.35. The private school system serves around 26% of the students mostly from medium to high SES (IBGE, 2017). Although caring for the largest number of students, children from Brazilian state-run schools show a lower math performance than those

educated in private schools (Soares & Candian, 2007; Sampaio and Guimaraes 2009). Results of the Basic Education Development Index in Brazil (IDEB, 2018) show important differences between private and state-run schools and similarities with PISA results.

In Italy, the educational system also differs in terms of state-run schools and private schools; however, the state-run schools are associated with a greater quality of education and most Italian private schools are frequented by children in need of special education. In the present study, only state-run schools in Italy were evaluated. Importantly, even in this context, children may vary in SES. A limitation of the study is that this was not investigated in our sample, which may confuse the interpretation of some results, such as gender differences that will be discussed in the next section.

Despite this limitation, our results suggest that differences in SES and quality of education, represented here by school settings, are strongly associated with the outcomes of math performance and contribute to cultural differences. These findings are in agreement with the literature and the hypothesis of the modernization.

4.3.4.2 Associations between cognitive abilities, math performance and gender

Differences between girls and boys in verbal and visuospatial skills and math performance are one of the most discussed findings when it comes to gender differences (Levine et. al 2016; Hyde, 2016; Halpern et al., 2007). However, these differences remain controversial and the causes of the gender disparities are unclear. In our results, we did not find any gender differences regarding visuospatial and verbal skills; however, discrepancies in math performance were evident.

According to the hypothesis of modernization, we expected to find gender differences in verbal and visuospatial skills between children from different educational settings, with the size of the gap being inversely correlated with modernization. The literature often reports an advantage for girls in verbal skills and for boys in visuospatial skills (Halpern, 2000). However, in the verbal and visuospatial intelligence tasks used here, it was not always possible to find significant differences between genders considering the average performance (Levine et al. 2016). Levine and coworkers reviewed some aspects of gender differences in visuospatial skills and showed that in mental rotation tasks such as PMA SR, gender differences are weaker and not always found

(Cohens'd = 0.26). When it comes to more complex mental rotation tasks as in 3d, the effect of gender differences is very strong (Cohens'd = 0.94). The study, as well as other literature data, argues that girls do not have difficulties in all visuospatial tasks. These difficulties are stronger in visuospatial perception tasks, navigation tasks, and 3d mental rotation tasks. In spatial visualization, visuoconstructional, and 2d mental rotation tasks, girls are able to perform as well as boys (Levine et al,2016; Halpern 2000; Newcombe, 2017).

Another important point regarding gender differences in mental rotation is age. A meta-analysis showed that the effects of differences between boys and girls tend to be higher with increasing age (Voyer, Voyer & Bryden, 1995). This may be happening because the complexity of the tasks increases with age or because of the stereotype beliefs, which also increases with age. Our sample consists of children between 8 and 10 years old, which may justify the absence of gender differences between them.

On the other hand, gender differences were found more consistently in math performance. Interestingly, when the interaction between gender differences and educational settings was analyzed, significant gender differences in simple math fluency were detected only in Italy (Cohen's d = 0.67) and state-run schools in Brazil (Cohen's d = 0.35). In complex math fluency, significant gender differences were found according to educational settings. However, a tendency was observed for girls from Brazilian private schools to present better performance than boys (Cohen's d = 0.26; Figure 2). These results are partially in accordance with the hypothesis of modernization. However, according to the modernization hypothesis, it is not clear why boys outperform girls in math in Italy in this study.

Considering the modernization hypothesis, we predicted that gender differences would be found in Brazil, but not in Italy due to the differences in the development of the two nations. However, gender differences for simple math fluency were detected in children from Italian and Brazilian state-run schools, but not in the case of children from Brazilian private schools. When we compare Italy with other south European countries such as Spain and Portugal, it is possible to observe that although Italy has similar gender equality as these two countries, female representation in political issues is low (Table 1). This discrepancy becomes even greater when we compare Italy with countries such as Norway, which has a high level of gender equality, female political representation, and a non-significant gap between boys and girls in math performance on PISA.

A possible explanation for boys outperforming girls in Italy could be related to gender stereotype threat. According to the modernization hypothesis, gender stereotyping decreases with human development. However, there may be cross-national variability even in developed countries.

This may suggest that, in some countries such as Italy, despite greater economic development compared to Brazil, stereotype beliefs regarding mathematical performance are still present in society. Muzzatti and Agnoli (2007) conducted two experiments to investigate the development of attitudes toward mathematics and stereotype threat susceptibility in Italian kids. They showed that the stereotypes can produce worse effects on math performance in girls but not in boys. Some studies show that susceptibility to stereotype effect may be strongly associated with environmental factors such as SES, parents' and teachers' beliefs (Gunderson, Ramirez, Levine & Beilock, 2012).

Interestingly, our results for both math fluency tasks reveal that girls from Brazilian private schools perform similarly to Italian girls, and far better than Brazilian girls from state-run schools. However, the effect is not the same for boys, as they could be less susceptible to negative gender stereotypes. Brian and colleagues (2009) investigated children's performance in science and mathematics and their relationship with self-reported stereotypes in 34 different countries. As expected, they found that implicit stereotypes associated with these subjects are predictive of the nations’ gender differences at the 8th-grade.

Finally, it is important to emphasize that all gender differences found occurred only for simple math fluency and not for complex tasks. Usually, more complex mathematical tasks are more strongly associated with visuospatial than with verbal skills (Mammarella, Caviola, Giofrè & Szűcs, 2018; Newcombe, Levine, & Mix, 2015; Mix et al., 2016). In contrast, in the current study, both simple and complex math fluency tasks were associated with visuospatial and verbal abilities. However, associations were stronger for verbal abilities, suggesting reliance on verbal retrieval strategies. In line with this hypothesis, previous evidence indicates that girls generally spend more time retrieving arithmetic facts than boys (Royer, Tronsky, Chan, Jackson & Marchant, 1999). Differences in retrieval time could reflect a more cautious behavior from girls in comparison to boys.

Math fluency is a complex skill that depends on the interaction among several factors including cognition, gender, and quality of education. Our results showed that verbal skills are more strongly related to performance in both simple and complex calculations when compared with visuospatial skills. This could be explained by the verbal nature of arithmetic retrieval tasks. In our model, age and school setting variables also entered as explanatory variables for both types of fluency. Gender entered on the model only as an explanatory variable for simple fluency, as previously discussed.

In conclusion, to the best of our knowledge, this is the first study indicating that not only cross-national but also intra-national variability interacts in complex ways with gender in influencing math performance. In general, it is important to highlight that, according to the modernization hypothesis, the education of girls in underprivileged settings should receive special attention. Improvement in numeracy is an important prerequisite of female empowerment.

4.3.5 References

Asante K. O. (2010). Sex Differences in Mathematics Performance among Senior High Students in Ghana. *Gender & Behaviour*; 8(2).

Belsky, J. (2007). Experience in Childhood and the development of reproductive strategies. Acta Psychologica Sinica, 2007, 39, 454-468

Belsky, J., Steinberg, L., & Draper, P. (1991). Childhood experience, interpersonal development, and reproductive strategy: an evolutionary theory of socialization. Child Development, 62, 647-670.

Berthelot, J. M., Ross, N., & Tremblay, S. (2001). Factors affecting Grade 3 student performance in Ontario: A multilevel analysis. *Education Quarterly Review*, *7*(4), 25.

Bodovski, K., Byun, S. Y., Chykina, V., & Chung, H. J. (2017). Searching for the golden model of education: cross-national analysis of math achievement. *Compare: A Journal of Comparative and International Education*, *47*(5), 722-741.

Carnoy, M., Khavenson, T., Fonseca, I., Costa, L., & Marotta, L. (2015). A educação brasileira está melhorando? Evidências do Pisa e do Saeb. *Cadernos de pesquisa*, *45*(157), 450-485.

Casey B. M., Pezaris E., Fineman B., Pollock A., Demers L., Dearing E. (2015). A longitudinal analysis of early spatial skills compared to arithmetic and verbal skills as predictors of fifth-grade girls’ math reasoning. *Learn. Individ. Diff.* 40 90–100. 10.1016/j.lindif.2015.03.028

Caviola, S., Gerotto, G., Lucangeli, D., & Mammarella, I. C. (2016). Nuove prove di fluenza per la matematica [Math fluency tasks]. Trento, Italy: Erickson.

Demir, Ö. E., Prado, J., & Booth, J. R. (2015). Parental socioeconomic status and the neural basis of arithmetic: differential relations to verbal and visuo-spatial representations. *Developmental science*, *18*(5), 799-814.

Else-Quest, N. M., Hyde, J. S., & Linn, M. C. (2010). Cross-national patterns of gender differences in mathematics: a meta-analysis. *Psychological bulletin*, *136*(1), 103.

Flore, P. C., & Wicherts, J. M. (2015). Does stereotype threat influence performance of girls in stereotyped domains? A meta-analysis. *Journal of school psychology*, *53*(1), 25-44.

Galdi, S., Cadinu, M., & Tomasetto, C. (2014). The roots of stereotype threat: When automatic associations disrupt girls' math performance. *Child development*, *85*(1), 250-263.

Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science*, *320*(5880), 1164-1165.

Gunderson, E. A., Ramirez, G., Levine, S. C., & Beilock, S. L. (2012). The role of parents and teachers in the development of gender-related math attitudes. *Sex roles*, *66*(3-4), 153-166.

Halpern, D. F. (2000). *Sex differences in cognitive abilities*. Psychology press.

Halpern, D. F., Benbow, C. P., Geary, D. C., Gur, R. C., Hyde, J. S., & Gernsbacher, M. A. (2007). The science of sex differences in science and mathematics. *Psychological science in the public interest*, *8*(1), 1-51.

Hart, S. A., Ganley, C. M., & Purpura, D. J. (2016). Understanding the home math environment and its role in predicting parent report of children’s math skills. *PloS one*, *11*(12), e0168227.

Hyde, J. S. (2016). Sex and cognition: gender and cognitive functions. *Current opinion in neurobiology*, *38*, 53-56.

Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, M. B., Williams, C. C. (2008). Gender Similarities Characterize Math Performance. *Science*. 321

Kucian, K., Loenneker, T., Dietrich, T., Martin, E., & Von Aster, M. (2005). Gender differences in brain activation patterns during mental rotation and number related cognitive tasks. *Psychology Science*, *47*(1), 112.

Larson, K., Russ, S. A., Nelson, B. B., Olson, L. M., & Halfon, N. (2015). Cognitive ability at kindergarten entry and socioeconomic status. *Pediatrics*, *135*(2), e440-e448.

LeFevre, J. A., Cankaya, O., Xu, C., & Lira, C. J. (2018). Linguistic and Experiential Factors as Predictors of Young Children's Early Numeracy Skills. In *Language and Culture in Mathematical Cognition* (pp. 49-72). Academic Press.

Levine, S. C., Foley, A., Lourenco, S., Ehrlich, S., & Ratliff, K. (2016). Sex differences in spatial cognition: Advancing the conversation. *Wiley Interdisciplinary Reviews: Cognitive Science*, *7*(2), 127-155.

Lynn, R., Fuerst, J., & Kirkegaard, E. O. (2018). Regional differences in intelligence in 22 countries and their economic, social and demographic correlates: A review. *Intelligence*, *69*, 24-36.

Mammarella, I. C., Caviola, S., Giofrè, D., & Szűcs, D. (2018). The underlying structure of visuospatial working memory in children with mathematical learning disability. *British Journal of Developmental Psychology*, *36*(2), 220-235.

McCormick, M. P., O'Connor, E. E., & Horn, E. P. (2017). Can teacher-child relationships alter the effects of early socioeconomic status on achievement in middle childhood?. *Journal of school psychology*, *64*, 76-92.

Moé, A. (2018). Mental rotation and mathematics: Gender-stereotyped beliefs and relationships in primary school children. Learning and Individual Differences 61, 172–180.

Muzzatti, B., & Agnoli, F. (2007). Gender and mathematics: Attitudes and stereotype threat susceptibility in Italian children. *Developmental psychology*, *43*(3), 747.

Newcombe, N. (2017). Harnessing spatial thinking to support stem learning. OECD

Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: The intertwined development of spatial and numerical cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, *6*(6), 491-505.

Newson, L., & Richerson, P. J. (2009). Why do people become modern? A Darwinian explanation. *Population and Development Review*, *35*(1), 117-158.

Nosek, B. A., Smyth, F. L., Sriram, N., Lindner, N. M., Devos, T., Ayala, A., ... & Kesebir, S. (2009). National differences in gender–science stereotypes predict national sex differences in science and math achievement. Proceedings of the National Academy of Sciences, 106(26), 10593-10597.

Nunes, T., Carraher, T. N., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics*. Cambridge University Press.

Pennington, C. R., Heim, D., Levy, A. R., & Larkin, D. T. (2016). Twenty years of stereotype threat research: A review of psychological mediators. *PloS one*, *11*(1), e0146487.

Rainho, O., & Ribeiro, C. C. (1993). Bateria Fatorial de Aptidões Específicas- CEPA. *Rio de Janeiro: CEPA Centro Editor de Psicologia Aplicada LTDA*

Rosselli, M., Ardila, A., Matute, E., Inozemtseva, O. (2009). Gender Differences and Cognitive Correlates of Mathematical Skills in School-Aged Children. *Child Neuropsychology*. 15(3), 216-231.

Royer, J. M., Tronsky, L. N., Chan, Y., Jackson, S. J., & Marchant III, H. (1999). Math-fact retrieval as the cognitive mechanism underlying gender differences in math test performance. *Contemporary educational psychology*, *24*(3), 181-266.

Salvador, L., S, Moura, R., Wood, G., & Haase, V. G. (2019). Cognitive Heterogeneity of Math Difficulties: A Bottom-up Classification Approach. *Journal of Numerical Cognition*, *5*(1), 55-85.

Sampaio, B., & Guimarães, J. (2009). Diferenças de eficiência entre ensino público e privado no Brasil. *Economia Aplicada*, *13*(1), 45-68.

Soares, J. F., & Candian, J. F. (2007). O efeito da escola básica brasileira: as evidências do PISA e do SAEB. *Revista Contemporânea de Educação*, *2*(4), 163-181.

Spencer, S. J., Steele, C. M., & Quinn, D. M. (1999). Stereotype threat and women's math performance. *Journal of experimental social psychology*, *35*(1), 4-28.

Stewart, C., Root, M. M., Koriakin, T., Choi, D., Luria, S. R., Bray, M. A., ... & Courville, T. (2017). Biological gender differences in students' errors on mathematics achievement tests. *Journal of Psychoeducational Assessment*, *35*(1-2), 47-56.

Stoet, G., & Geary, D. C. (2013). Sex differences in mathematics and reading achievement are inversely related: Within-and across-nation assessment of 10 years of PISA data. *PloS one*, *8*(3), e57988.

Stoet, G., & Geary, D. C. (2018). The gender-equality paradox in science, technology, engineering, and mathematics education. *Psychological science*, *29*(4), 581-593.

Thompson, J. M., Nuerk, H. C., Moeller, K., & Kadosh, R. C. (2013). The link between mental rotation ability and basic numerical representations. *Acta psychologica*, *144*(2), 324-331.

Thurstone, L. L., & Thurstone, T. G. (1963). Primary mental abilities. Chicago: Science Research.

van Tetering, M., van der Donk, M., de Groot, R., & Jolles, J. (2019). Sex Differences in the Performance of 7-12 Year Olds on a Mental Rotation Task and the Relation With Arithmetic Performance. *Frontiers in psychology*, *10*, 107. doi:10.3389/fpsyg.2019.00107

Voyer, D., Voyer, S., & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: a meta-analysis and consideration of critical variables. *Psychological bulletin*, *117*(2), 250.

Wechsler D. (2009). Wechsler Individual Achievement Test (3rd ed.). San Antonio, TX: Psychological Corporation.

Zacks, J. M. (2008). Neuroimaging studies of mental rotation: a meta-analysis and review. *Journal of cognitive neuroscience*, *20*(1), 1-19.

5. Conclusions and final considerations

In the present study, we were interested in investigating two main points about math performance: a) The heterogeneity of math learning difficulties and; b) Cross-cultural and intra-cultural factors and their interaction with sex on math skills. The main hypothesis was that these levels of interaction, individual and environmental influence in different ways the math performance.

A theoretical model was built to investigate the main variables that interact with math performance in both individual and environmental levels. Number magnitude processing, verbal and visuospatial skills were investigated at the individual level. The interaction of these variables with sex and SES were also investigated. Interestingly, the review indicated that verbal and visuospatial skills are more influenced by sex and SES than number magnitude processing. This indicates that the deficits in number magnitude processing can be more specific to identifying children with MLD than verbal and visuospatial skills. However, it is important to emphasize that MLD presents a high heterogeneity and can be divided into subtypes according to cognitive deficits.

In the second study, was used a multivariate clusters analyses to investigate the subtypes of MLD. This type of bottom-up strategy is very useful because it allows us to investigate the subgroups of children with math difficulties in a different way than traditionally used in the literature. In this study was possible to identify two profiles of children with MLD. One group of children with number magnitudes processing impairments and another group of children with visuospatial impairments. In this study verbal skills were also investigated. The digit span task was used to evaluate verbal working memory. However, no specific MLD group was associated with deficits in verbal working memory. This may have been due to the low variability presented by the task and not because there is no verbal MLD subtype.

A third study was conducted to investigate the interaction of verbal and visuospatial skills with sex and environmental variables. A comparison at intra-nation and cross-nation level was conducted to test the ways in which these variables relate to each other and with math performance. A complex pattern of interaction between sex, environment and mathematical performance has been identified.

The initial hypothesis that math performance is a complex phenomenon and may be associated with several individuals and environmental variables that interact with sex was corroborated by the present results. Additionally, from these findings, studies of intervention and training skills involved in math performance should consider interaction with all these variables. As discussed throughout the studies, protective factors against the damage of exposure to low SES should be considered in future studies and interventions, such close relationship with teachers, the type of education to which the child is exposed and the family environment.

The conclusion is that several cognitive factors are involved in math performance contributing to the heterogeneity of MLD. Several of these cognitive factors that contribute to math performance can further interact with sex and environmental factors, increasing the complexity of math skills. All these factors should be considered in future studies.