PRICING THE HIGHER ORDER CO-MOMENTS IN THE BRAZILIAN STOCK MARKET

Carolina Magda da Silva Roma

Email: carolina.magda.adm@gmail.com

Aluna do Curso de Doutorado em Finanças/Administração, CEPEAD/UFMG

Ivika Jäger

Email: ivika.jager@phdstudent.hhs.se

Aluna do Curso de Doutorado em Finanças na Stockholm School of Economics, SEE

Robert Aldo Iquiapaza

Email: riquiapaza@gmail.com

Professor e Pesquisador do CEPEAD/UFMG

Aureliano Angel Bressan

Email: bressan@face.ufmg.br

Professor e Pesquisador do CEPEAD/UFMG

ABSTRACT

The aim of this article is to investigate whether assets' co-skewness and co-kurtosis with the market are priced on the Brazilian stock market. The Fama-MacBeth (1973) regression method is used to empirically test the pricing of the higher order co-moments on a cross-section of portfolio returns. The analysis further expands the model by including the size and value factors proposed by Fama and French (1993) and the momentum factor introduced by Carhart (1997). The time series results taking into account the the higher co-moments along with the four-factor variables point out that co-skewness and co-kurtosis capture some variance in the asset returns beyond the size, value and momentum factors. Moreover, the cross-sectional estimation results give partial support for co-skewness being priced in the Brazilian stock market, but only in case the model controls for the size, value and momentum factors. Moreover, the cross-sectional estimation results give partial support for co-skewness being priced in the Brazilian stock market. Controlling for up and down markets turns out to be important and results in strong support for beta pricing while also providing partial evidence of existing premia for co-skewness and co-kurtosis.

Keywords: Higher co-moments; Carhart model; Fama-MacBeth regression; Factors; CAPM; 4-moment CAPM

1 INTRODUCTION

In the 1950s, Markowitz (1952) formulated the underpinnings of modern portfolio theory which established the mean and variance of portfolio returns as the two key parameters in investment decisions. The mean-variance criteria have consequently been adopted as a basis for models and theories in a wide range of studies. Inspired by the mean-variance idea, the Capital Asset Pricing Model (CAPM) was independently developed by Sharpe (1964), Lintner (1965) and Mossin (1966) roughly a decade later.

The idea behind the CAPM is simple – an asset's expected return is a linear function of the market risk premium and its size depends solely on the systematic covariance risk (beta) of the asset. The model, hence, assumes that the investors have a quadratic utility function and the only things that matter for investment decisions are the expected mean and variance of asset returns. Consequently, the only source of risk in the model which is priced is the stock return's co-movement with the market return that cannot be diversified away. Largely due to its simple and straightforward interpretation, the CAPM has become one of the most adopted approaches for estimating the cost of capital for a company or an investment project. It also remains an industry standard despite the empirical evidence showing that the model fails to capture the stylized facts in asset returns.

Meanwhile, regardless of the practicioners' reluctance to adopt more complex equations, academia has been eagerly developing extensions of the CAPM to better fit the empirical data. Using the US stocks, Fama and French (1993) constructed two factors, based on market capitalization and book-to-market ratio, and demonstrated that their three-factor model is superior to the CAPM. Neither of these factors have theoretical foundations, but the authors claim that the size and value premiums are related to a firm's probability of distress. Based on Jegadeesh and Titman (1993), Carhart (1997) added another factor to the Fama-French model that reflects the market momentum effects. The Fama-French (1993) three-factor and Carhart (1997) four-factor models have been shown to capture a large portion of the variation of stock returns in the US as well as a wide range of developed and emerging markets around the world.

In addition to including more empirically motivated factors, a separate strand of literature relaxes the linearity assumption of the model by incorporating higher co-moment terms, such as systematic skewness and systematic kurtosis – also called co-skewness and co-kurtosis according to Christie-David and Chaudry (2001). These terms are included based on the existence of non-normality in the asset return distributions. For example, Kraus and Litzenberger (1976), Fang and Lai (1997), Dittmar (2002) have studied higher order co-moment effects in the context of asset pricing. The authors, in general, find that the third and fourth systematic moments are priced. Furthermore, the importance of the higher moments and whether investors price them has been investigated in different capital markets around the globe: Hung, Shackleton and Xu (2004) in the United Kingdom, Javid (2009) in Pakistan, Hasan et al. (2013) in Bangladesh, Messis, Iatridis and Blanas (2007) in Greece, Doan, Lin and Zurbruegg (2010) in Australia and United States, etc.

Moreover, Chung, Johnson and Schill (2006) look at whether the the Fama and French (1993) factors are the true risk factors or just proxies to the higher order co-moments and find that the Fama and French (1993) factors lose explanatory power (become insignificant or less significant) when three to ten systematic co-moments are included to the model. Hung, Azad and Fang (2014) investigate the size, value, momentum and liquidity factors as well as higher co-moments separately in periods around and between the financial crises and point to some evidence regarding co-skewness being priced during the market crashes while other variables lose significance.

This article contributes to the literature analyzing the role of systematic covariance, systematic skewness and systematic kurtosis in the Brazilian stock market. We use the Fama and MacBeth's (1973) two-stage regression method to analyze the higher order co-moments from two perspectives. First, we employ a non-linear return-generating process that consistently proxies for the higher order co-moments (beta, co-skewness and co-kurtosis) with a quadratic market model (including the market premium and the squared market premium) and also a cubic market model (including the market premium and its squared and cubed alterations). Then, we estimate the cross-sectional regressions to examine whether the higher order co-moments are priced in the Brazilian stock market. We conduct the analysis on portfolios sorted by size and the book-to-market ratio as well as double sorted portfolios based on both size and the book-to-market ratio. For robustness, we use both value and equal weighing of stocks when computing portfolio returns. Second, we verify how the inclusion of the empirical factors proposed by Fama and French (1993) and Carhart (1997) affect the price of the risk factors when all of them are analyzed together. Third, we follow Pettengill, Sundaram and Mathur (1995) and examine the different models separately in Bull and Bear markets.

Our findings indicate that the Fama-French-Carhart factors should not be considered as proxies for the higher order co-moments since the latter capture some variance in the asset returns beyond the size, value and momentum factors. However, we find that in general the co-skewness and co-kurtosis with the market is not priced in the Brazilian stock market. The co-skewness has a significantly negative risk premium only when using value-weighted portfolios that are formed based on market capitalization ranking of stocks. Controlling for up and down markets turns out to be important and results in strong support for beta pricing while also providing partial evidence of existing premia for co-skewness and co-kurtosis.

The article is organized as follows: Section 2 discusses previous theoretical and empirical literature on the CAPM, models with higher moments and models containing the empirical factors. Section 3 presents the data and the methodological setup of the study. Section 4 reports the empirical results together with a discussion of the findings. Finally, the concluding remarks highlight the key findings and future research possibilities in this area.

2 THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 The CAPM

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) remains to this day a benchmark model in asset pricing (SHIH et al.; 2014). Largely due to its simple and straightforward interpretation, the CAPM has also become one of the most utilized theoretical equations in practice. It is still widely used by market professionals and academia (Levy, 2012) for estimating the cost of capital for a company or an investment project, performance assessment for individual assets as well as investment funds, portfolio diversification, and investment evaluation, among other purposes (GALAGEDERA, 2007).

The idea behind the CAMP is simple – an asset's expected return is a linear function of the market risk premium. The scaling factor (β), which determines the size of the return, is the systematic risk of the asset, i.e. the covariance between the asset's return and the market return normalized by the market return's variance. Consequentely, the only source of risk in the model is the portion of return variance that cannot be diversified away by the investor. The CAPM equation is as follows:

$$\mathrm{E}(r_{it}) - r_{ft} = \beta_{im}[\mathrm{E}(r_{mt}) - r_{ft}] \quad \text{, where } \beta_{im} = \frac{\mathrm{E}[(r_{it} - \mathrm{E}(r_{it}))(r_{mt} - \mathrm{E}(r_{mt}))]}{\mathrm{E}[(r_{mt} - \mathrm{E}(r_{mt}))^2]}$$

Equation 1

Where r_{it} represents the return on asset *i* in month *t*, r_{ft} is the risk free return in month *t* and r_{mt} is the return on the market portfolio in month *t*.

Sharpe (1964) argues that before CAPM there existed no theory that tied the price of an asset to the investors' preferences and asset attributes, among other factors, that determine the asset's price. The lack of such a theory made it difficult to establish the relationship between the price and risk of an asset. So he proposed an equilibrium pricing theory under certain risk conditions based on two underlying assumptions: a common interest rate at which all investors can lend and borrow without restrictions, and that the expectations of all investors are homogeneous. The first assumption is equivalent to assuming a frictionless market and the second implies that the investors construct exactly the same efficient frontier since there is complete agreement with regards to the distribution of asset returns. Thus, the optimal portfolio is the same for all investors.

As highlighted by Fama and French (1992, p. 427), the central point of the model lies in the fact that the market portfolio is efficient in terms of the mean-variance criteria, according to the concept proposed by Markowitz (1959). Chung, Johnson and Schill (2004, p. 924) affirm that: "if the CAPM holds, only the second-order systematic comoment (beta) should be priced. [...] Without normality, the CAPM is unlikely to hold". Mandlerbrot (1963) has provided evidence of non-normality in asset returns, so it may be that the underlying assumptions required by the CAPM are too restrictive and do not conform to reality. Due to the empirical failure of the CAPM, alternative models incorporating higher order co-moments and additional factors have emerged.

2.2 Higher moments

The third and fourth moments of a probability distribution are skewness and kurtosis, respectively. Skewness measures asymmetry, i.e. the extent to which the distribution is tilted towards the positive or negative end of the distribution. Meaning that, given a positively skewed return distribution, there is a greater probability that a positive demeaned return of a specific magnitude is realized compared to a negative demeaned return of the same magnitude. The opposite holds for a negatively skewed return distribution and skewness of zero corresponds to the normal (symmetric) distribution (HARVEY, SIDDIQUE, 1999). Kurtosis represents the extent to which a distribution is concentrated in the center or the tails, i.e. whether extreme events are more or less likely compared to the normal distribution (FANG; LAI, 1997) – a value greater than three indicates fatter tails than the normal distribution (leptokurtosis).

It is empirically verified that the asset returns, in general, do not conform to the normal distribution (SU; LEE; CHIU, 2014; FEUNOU; JAHAN-PARVAR; TÉDONGAP, 2014; JONDEAU; ROCKINGER, 2006; VERHOEVEN; MCALEER, 2004) which is the theoretical underpinning of Markowitz' (1952) portfolio theory (JONDEAU; ROCKINGER, 2006) as well as the underlying assumption for the CAPM (VERHOEVEN; MCALEER, 2004; PEIRÓ, 1999). Furthermore, Brooks et al. (2005) point out that the variance is a measure of risk only under the nornality assumption, but if the return distributions have heavy tails then the true risk of the portfolio is underestimated.

Damodaran (1985) attributes the asymmetry in the asset return distributions to the dissemination of good and bad news about the companies, while Chen, Hong and Stein (2001) claim it is due to investor heterogeneity and Bae, Lim and Wei (2006) claim that the cause is the difference in the quality of corporate governance across companies. The presence of leptokurtosis in return series was first observed by Mandelbrot (1963) and Fama (1965). This has subsequently become a well-known characteristic displayed by the financial series (CONT, 2001) and Bai, Russell and Tiao (2003) argue that leptokurtosis can be induced by volatility clustering.

Some studies point out that the financial returns exhibit negative skewness (HARVEY; SIDDIQUE, 1999, 2000; KIM; WHITE, 2004; ÑIGUEZ; PEROTE; RUBIA, 2009) and the unconditional distribution of asset returns is leptokurtic (CONT, 2001), the latter has become a stylized fact in finance. Hence, positive skewness implies lower risk which translates into lower returns, while greater kurtosis means more risk and hence higher returns. In this sense, Scott and Horvath (1980) discuss the investors' preferences for the higher moments. Under general assumptions, the investors tend to like odd moments (mean and skewness, for example) which reduce risk and dislike even moments (like variance and kurtosis) which increase risk. Denoting an investor's utility with U and wealth with W, the following hold for the first four derivatives of the utility function with respect to wealth:

$$U'(W) > 0 \qquad U''(W) < 0 \qquad U'''(W) > 0 \qquad U''''(W) < 0$$

There is a number of recent studies applying the higher (co)moments to portfolio construction or optimization problems (BOUDT; LU; PEETERS, 2015, JONDEAU; ROCKINGER, 2012; HARVEY, et al., 2010), pricing of shares and options (DOAN; LIN; CHNG, 2014; HUNG; AZAD; FANG, 2014; HASAN et al., 2013; CHANG, CHRISTOFFERSEN; JACOBS, 2013; CHIAO; HUNG; SRIVASTAVA, 2003) and expanding risk measures (GALAGEDERA, BROOKS, 2007; ELING; SCHUHMACHER, 2007). The portfolio optimization problems seek to select the assets while taking into account their higher moments, the asset pricing literature aims to quantify the price of covariance, co-skewness and co-kurtosis and includes skewness and kurtosis measures to price derivatives, and finally, the risk measures literature includes the higher moments to compute the traditional performance indicators, such as the Sharpe ratio, Value-at-Risk (VaR) and downside risk.

Whether and how many higher moments to add to a model has been a topic for debate among researchers. On one hand, Arditti (1967), studying the total skewness, does support the idea of including only up to the third moment of the distribution as risk measures, and justifies his choice based on Kaplansky (1945) who argues that the unconditional fourth moment does not generate any relevant information to investors. Levy (1969), on the other hand, claims that the only two cases which allow disregarding higher moments beyond the third are i) if all moments tend to zero, and ii) if the utility function is cubic (the derivatives do not appear for higher than the third moments). In the same line, other researchers started to discuss and include higher moments than the second as Jean (1971) follows the previously mentioned authors and expands the theoretical portfolio analysis to include the third moment and Rubinstein (1973) constructs a theoretical equilibrium model with and without the homogeneous subjective probabilities and including up to the skewness.

Kraus and Litzenberger (1976) derived and tested the extended CAPM model that includes moments up to systematic co-skewness, γ_{im}, which is known as the 3-Moment CAPM. The authors mention that the results of previous researchers, like Blume and Friend (1973), Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), which bring out some flaws in the CAPM theory are due to the specification problems, i.e. not considering the third moment. The co-skewness represents the marginal contribution of an asset to the portfolio's overall skweness and risk-averse investors have less preference for assets that contribute to a decrease in the portfolio' skewness requiring higher expected return to hold these assets (BALI; DEMIRTAS; LEVY, 2009).

Fiend and Westerfield (1980) test the Kraus and Litzenberger (1976) model by considering different specifications of the market portfolio, e.g. adding bond returns to the sample of stock returns. They find that whether co-skewness has significant power for explaining the asset returns is dependent on the test construction and the used market index measure. Further, Lim (1989) tests the 3-moment CAPM using the generalized method of moments (GMM) methodology and finds support to the idea that systemic co-skewness is priced by investors.

Fang and Lai (1997) provide evidence supporting the importance of systematic skewness as well as systematic kurtosis in explaining the asset returns. However, their 4-moment CAPM model assumes perfect capital markets. Christie-David and Chauldry (2001) study the 4-moment CAPM on the futures market and find that both co-skewness and co-kurtosis have significant explanatory power for the futures returns and the result is not dependent on the chosen proxy for the market portfolio. Dittman (2002) confirms the significance of skewness and kurtosis in explaining the asset returns when the return on aggregate wealth is used as a risk factor. The co-kurtosis represents the marginal contribution of an asset to the portfolio's overall kurtosis and risk-averse investors have less preference for assets that contribute to an increase in the portfolio's kurtosis requiring higher expected returns to hold these assets (BALI; DEMIRTAS; LEVY, 2009).

The 4-Moment CAPM in equation form is as follows:

$$\mathrm{E}(r_i) - r_f = b_\beta \beta_{im} + b_\gamma \gamma_{im} + b_\delta \delta_{im}$$ Equation 2

, with:

$$\beta_{im} = \frac{\mathrm{E}[(r_{it} - \mathrm{E}(r_{it}))(r_{mt} - \mathrm{E}(r_{mt}))]}{\mathrm{E}[(r_{mt} - \mathrm{E}(r_{mt}))^2]}$$ Equation 3

$$\gamma_{im} = \frac{\mathrm{E}[(r_{it} - \mathrm{E}(r_{it}))(r_{mt} - \mathrm{E}(r_{mt}))^2]}{\mathrm{E}[(r_{mt} - \mathrm{E}(r_{mt}))^3]}$$ Equation 4

$$\delta_{im} = \frac{\mathrm{E}[(r_{it} - \mathrm{E}(r_{it}))(r_{mt} - \mathrm{E}(r_{mt}))^3]}{\mathrm{E}[(r_{mt} - \mathrm{E}(r_{mt}))^4]}$$ Equation 5

Where β_{im} stands for the systematic risk (variance) known from the CAPM, γ_{im} is the systematic skewness and δ_{im} the systematic kurtosis. The coefficients b_β, b_γ and b_δ are known as the risk premia or market prices related to each of the higher co-moments. The market prices of the higher co-moments are defined as follows:

$$b_\beta = \frac{d\mathrm{E}(r_{it})}{d\sigma^2(r_{it})}\sigma^2(r_{mt}), \quad b_\gamma = \frac{d\mathrm{E}(r_{it})}{d\gamma^3(r_{it})}\gamma^3(r_{mt}) \quad \text{and} \quad b_\delta = \frac{d\mathrm{E}(r_{it})}{d\theta^4(r_{it})}\delta^4(r_{mt})$$

Equation 6

If $\alpha_3 = 0$, the model is reduced to the Kraus and Litzenberger (1976) model. If also $\alpha_2 = 0$, then it refers to the standard two-moment CAPM. In line with Scott and Horvath (1980), according to Fang and Lai (1997), Hwang and Satchell (1999), Ranaldo and Favre (2005) and Liow and Chan (1995), theoretically, the following is expected for the premia:

- positive signs for the coefficients associated with systematic variance and systematic kurtosis because these variables imply higher risk and, hence, the investor demands a higher return for holding an asset with higher covariance or co-kurtosis;
- negative sign for the coefficient associated with systematic skewness as positive skewness implies lower risk and, hence, the investor demands a lower return for holding an asset with higher co-skewness.

Additionally, Chung, Johnson and Schill (2006) look at whether the the Fama and French (1993) factors are the true risk factors or just proxies to the higher order co-moments and find that the Fama and French (1993) factors lose explanatory power (become insignificant or less significant) when three to ten systematic co-moments are included to the model. Harvey and Siddique (2000) report that the size and value factors capture to some extent similar information as skewness, however adding skewness to the model captures something beyond the two empirical factors. Hung, Azad and Fang (2014) investigate the performance of the empirical factors and co-moments during the major crisis periods in history – the stock market crashes of 1929 and 1987 and the dot com bubble followed by the credit crunch. The authors show that the size, value, momentum and liquidity factors lose explanatory power during the crisis periods while there is evidence of co-skewness pricing around the big stock market crashes.

The studies looking at the Brazilian capital markets include Silva (2005) and Castro Junior (2008). Silva (2005) tests the models including up to the tenth co-moment and the size, value and momentum factors on the BM&F Bovespa data. His findings, in line with Chung, Johnson and Schill (2006), indicate that when including the empirical factors as explanatory variables, the explanatory power of the higher co-moments remains modest; in fact, only co-kurtosis maintains significance across the different models. Castro Junior (2008), on the other hand, includes co-skewness and co-kurtosis together with a set of corporate control variables, such as net revenue, B/M ratio, leverage and time-sector fixed effects to test the pricing of higher moments on all stocks listed on the Brazilian stock exchange over the period 2003 to 2007. He concludes that both co-skewness and co-kurtosis are relevant risk factors for asset pricing.

2.3 The empirical factors: size, value and momentum

Given the rather restrictive and unrealistic assumptions, the CAPM has been subject to extensive empirical testing, for example Black, Jensen and Scholes (1972), Blume and Friend (1973) and Fama and MacBeth (1973). If the CAMP holds, then the Jensen's alpha, defined as $\alpha = \left(E[r_i] - r_f\right) - \left(E[r_m] - r_f\right) * \beta$, should not be statistically significantly different from zero. Black, Jensen and Scholes (1972), however, find that high betas were associated with negative alphas and low betas were related to positive alphas. Roll's (1977) critique relating to the CAPM is that it is impossible to construct the market portfolio which is truly diversified since it should include every single asset on the market; that is, everything that has a marketable value, including commodities, collectibles etc. Meanwhile, the model continues to be applied using the stock market indices as a proxy.

Further empirical anomalies not explained by the CAPM are presented by Banz (1981) and Bhandari (1988). The prior argues that the CAPM is a misspecified model and observes a size premium: firms with smaller market capitalization (MCap) tend to have higher returns than firms with larger MCap. The latter shows that that the debt-to-equity (leverage) ratio captures the return variation (positive relationship) when controlling for the beta and size. Using data from the US capital markets, Rosenberg, Reid and Lanstein (1985) are among the first to identify that stocks with high book-to-market equity (B/M) ratios, the so called value stocks, have higher returns than stocks with low B/M ratios, the growth stocks. The return differential between value and growth stocks gives the High Minus Low (HML) factor. Chan, Hamao and Lakonishok (1991) test four fundamental variables (earnings yield, size, book-to-market ratio and cash flow yield) on the Japanese stock market and also find that the B/M ratio has significant explanatory power for the asset returns.

In line with the arguments raised by Stattman (1980), Basu (1983) and Bhandari (1988), which identify other significant factors explaining the returns, Fama and French (1993) construct two new risk factors in addition to the beta, one based on size in terms of market capitalization (MCap) and the other based on value as measured by the book-to-market (B/M) ratio. They demonstrate that their three-factor model performs better than the CAPM on the US data. Fama and French (2004) suggest that the poor performance of the CAPM could be related to the simplifying assumptions or difficulties to empirically test them.

As a response to the evidence against the relevance of the beta, Pettengill, Sundaram and Mathur (1995) claim that the relationship between the beta and asset returns is not independent across the varying market risk premium. The positive relation predicted by the CAPM uses ex-ante expectations while it is estimated using ex-post observations. After controlling for the sign of the realized market excess return (up or down market), the authors find cross-sectional support for beta pricing, hence, showing that a significant positive risk-return relationship can be found conditional on the market trends.

The size and value factors do not have theoretical foundations, but Fama and French (1993) claim that their premiums are related to firms' probability of distress. The size factor measures the expected additional risk premium for holding small stocks as compared to large stocks and the value factor measures the expected additional risk premium for investing in firms with high B/M ratios as compared to firms with low B/M ratios. The three-factor model in equation form is:

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p (r_{m,t} - r_{f,t}) + s_p SMB_t + h_p HML_t + \varepsilon_t \qquad \text{Equation 7}$$

where:

$r_{i,t} - r_{f,t}$ = Excess return on portfolio *p* over the return on the risk-free asset in month *t*;

$r_{m,t} - r_{f,t}$ = Excess return on the market portfolio *m* over the return on the risk-free asset in month *t*;

SMB_t = Small Minus Big – size factor in month *t*;

HML_t = High Minus Low – value factor in month *t*;

The coefficients β_p, s_p and h_p are known as the risk premia or market prices related to each of the risk factors.

Another anomaly in the asset return data is the momentum effect investigated by Jegadeesh and Titman (1993). The authors demonstrate that stocks with good performance in the last months tend to sustain high returns in the short term and stocks with poor performance tend to have low returns in the near future. They investigate strategies of buying stocks from the first group, the winners, and sell stocks from the second group, the losers, and verify that this strategy generates significant positive returns. The return differential between the winners portfolio and the losers portfolio comprises the momentum factor.

Based on Jegadeesh and Titman (1993), Carhart (1997) adds the momentum factor to the Fama and French (1993) model, thus developing the four factor model and shows that it captures part of the variation in stock returns that is not explained by the size and value factors. The Fama-French (1993) three-factor and Carhart (1997) four-factor models have been shown to capture a large portion of the variation in stock returns in the US as well as a wide range of developed and emerging markets around the world. The four-factor model, also known as the Carhart (1997) model, is represented by the following equation:

$$r_{i,t} - r_{f,t} = \alpha_p + \beta_p (r_{m,t} - r_{f,t}) + s_p SMB_t + h_p HML_t + m_p Mom_t + \varepsilon_t$$

Equation 8

with

Mom_t = momentum factor in month t

m_p = risk premium or market price related to the momentum factor

The evidence from the Brazilian capital markets with regards to the momentum effects are contradictory. Kimura (2003) shows that when adjusting for systematic risk, the momentum strategies do not lead to statistically significant gains. Mussa et al. (2007) investigate the 16 momentum strategies with different formation and holding periods applied by Jegadeesh and Titman (1993) and conclude that only three of these strategies yield positive and significant returns. Flister, Bressan and Amaral (2010) set out to replicate the results using a 3-month formation period and 6-month holding period and fail to find a significant difference between the winners and losers portfolios. The authors associate the contradiction in the findings to the exclusion of the financial sector companies in their sample. Improta (2012) tries out 1296 different strategies (all combinations of 1 to 36 months formation and holding periods) and finds only one (2 months formation + 2 months holding) with significant gains. In general, Improta (2012) verifies that when controlling for the risk exposure using the Fama and French (1993) factors, the return on the momentum strategy is not statistically different from zero. More recently, Picolli et al. (2015) show that the momentum strategies become profitable after controlling for the crisis periods, even when taking into account the risk exposure using the CAPM and the three-factor model.

3 METHODOLOGY

3.1 Data

The sample used in this study covers the period from December 1995 to February 2016 and includes all shares listed on the São Paulo Stock Exchange (BM&FBOVESPA) available in the Economática® software.The December 1995 data is needed to create the HML risk factor and the period was chosen to correspond to a period in which the Brazilian economy has arguably also been more stable, i.e. after the implementation of the Plano Real ("Real Plan") in 1994. The continuous stock return is defined as the difference between the natural logarithms of the adjusted closing prices at the end of each month. The risk-free asset is proxied by the Interbank Deposit Certificate (Certificado de Depósito Interbancário – CDI) rate and the market portfolio return by the return of Índice Bovespa (Ibovespa), the market index of the stock exchange containing roughly 60-65 most traded stocks.

3.2 Sample selection

The portfolios for the analysis as well as the portfolios which are then used to compute the size, value and momentum factors are constructed on June 30th every year. The initial sample covers 913 stocks from different sectors. When selecting the sample of stocks to be included in the final sample in a given year, we require the following:

i. positive stock market capitalization on the preceding June 30th for computing the size factor
ii. positive stock market capitalization and book value of equity on the preceding December 31st for computing the value factor
iii. non-missing returns on the preceding April 30th and May 31st for constructing the momentum factor

Furthermore, following Fama and French (1993) and Carhart (1997), we exclude financial institutions from the analysis. It is important to note that this paper differs from from Mussa, Rogers and Securato (2009) and Mussa, Fama and Santos (2012) in the way the local empirical factors are formed. The latter papers use the market capitalization (MCap) and book-to-market (B/M) ratio on the company level to construct the factors, meaning that the common and preferred stocks are aggregated for each company. However, due to the small number of shares listed on the Brazilian stock exchange, we use the information on the share level, meaning that the common and preferred stocks are treated as two different assets implying that, for instance, two stocks of the same company may be in different percentiles and then be used to compose different risk factors.

3.3 Portfolio construction procedures

To create the portfolios which are used for the analysis in order to check whether investors price the higher order co-moments, we sort the stocks based on the size and value measures. Evey year, the stocks are split into 15 groups given their market capitalization (as of the end of June) and book-to-market ratio (as of the end of December). The portfolio returns are computed on a value-weighted basis. The weights are defined at the end of June every year and kept constant throughout the holding period (one year). In case a stock in a portfolio has a missing monthly return during the holding period, the weights are proportionally redistributed among the remaining stocks such that the weights sum up to one. To further investigate the problem, the same analysis is conducted using equal weighting of stocks in the portfolios.

3.4 Construction of the Risk Factors

The first factor, the market risk premium, $r_m - r_{ft}$, is computed as the difference between the monthly return on the market portfolio and the monthly return on the risk-free asset.

The construction of the SMB, HML and MOM factors is similar to that suggested by Fama and French (1993) and Carhart (1997). The value-weighted portfolios are formed at the end of June each year and rebalanced annually, i.e. every 12 months. If it happens that a stock has a missing observation during a holding period then the weights are redistributed within the portfolio for the respective month when computing the monthly portfolio return. In that way, the missing observations are not treated as observations of zero return, but are simply excluded.

The steps to estimate the factors are:

i. at the end of June each year, the sample of eligible stocks is divided into two groups based on the median market capitalization;

ii. within each of the two *size* groups, the stocks are divided into three groups according to the book-to-market ratio (as of the preceding December), the groups are split based on the 30^{th} and 70^{th} percentiles;

iii. within each of the six *size-B/M* groups, the stocks are ordered based on the past 11-month accumulated returns (-12 months until -1 month) and divided into two groups (winners and losers) based on the median of the accumulated return.

This method gives us 12 value-weighted portfolios for computing the empirical factors: 6 portfolios of small stocks and 6 portfolios of large stocks; 4 portfolios of high B/M ratio stocks, 4 portfolios of medium B/M ratio stocks and 4 portfolios of low B/M ratio stocks; 6 portfolios of winners and 6 portfolios of losers.

The size factor, SMB, is defined as the difference between the average monthly return on the six portfolios of small stocks and the average monthly return on the six portfolios of large stocks. The value factor, HML, is computed as the difference between the average monthly return on the four portfolios of high B/M ratio stocks and the average monthly return on the four portfolios with low B/M ratio stocks. The momentum factor, MOM, is calculated as the difference between the average monthly return on the six portfolios of winners and the average monthly return on the six portfolios of losers.

3.5 Higher Co-moment Models

This section presents the econometric model that we use to estimate the assets' covariance, co-skewness and co-kurtosis with the market excess return so that the higher order co-moments can be added to the CAPM as risk factors. The model is in line with the one adopted in earlier studies, such as Ranaldo and Favre (2005), Liow and Chan (2005) and Hwang and Satchell (1999). The latter show that the higher moment CAPM has issues with multicollinearity and present the option to use the quadratic and cubic market models to overcome the issue. The authors argue that this method is consistent with the 3-moment and 4-moment CAPM models.

The cubic market model which allows investigating investors' preferences towards skewness and kurtosis is as follows:

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p(r_{m,t} - r_f) + \gamma_p[r_{m,t} - \mathrm{E}(r_m)]^2 + \delta_p[r_{m,t} - \mathrm{E}(r_m)]^3 + \varepsilon_t$$

Equation 9

Where β_p, γ_p and δ_p are the asset's estimated sensitivities with respect to the covariance and higher co-moments. From Equation 9, we can see that the traditional CAPM is a particular case where $\gamma_p = \delta_p = 0$ (LIOW; CHAN, 2005). Barone-Adesi (1985), Barone-Adesi, Gagliardini and Urga (2004) and Hung (2008) show that the quadratic model is consistently equal to the CAPM model which includes co-skewness. Similarly, the cubic model is consistent with the CAPM model that includes co-kurtosis, as can be found in Hwang and Satchell (1999).

3.6 Fama-MacBeth regression

Equation 10 presents the full empirical model including the higher order co-moments and the Fama-French-Carhart (FFC) factors. We test different specifications of this model to investigate how the estimates change when one or the other factor is included or excluded, e.g. the three market models, the Fama and French (1993) and Carhart (1997) models and the full model.

To conduct our analysis, we use the two-stage regressions first introduced by Fama and MacBeth (1973), henceforth FM, to test the pricing of the higher order co-moments.

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p(r_{m,t} - r_f) + \gamma_p[r_{m,t} - \mathrm{E}(r_m)]^2 + \delta_p[r_{m,t} - \mathrm{E}(r_m)]^3 +$$

$$s_p SMB_t + h_p HML_t + m_p Mom_t + \varepsilon_t$$

Equation 10

The first step in the process involves estimating the time series of the coefficients β_p, γ_p, θ_p, s_p, h_p and m_p through an extended model which includes the FFC factors. This is done using a 5-year rolling window for the monthly returns. The second step includes running cross-sectional regressions for each month where the previously estimated coefficients are used as explanatory variables for the asset returns as shown in Equation 11.

Second stage cross-sectional regressions for each month *t*:

$$r_{p,t} - r_{f,t} = b_0 + b_\beta \beta_p + b_\gamma \gamma_p + b_\delta \delta_p + b_s s_p + b_h h_p + b_m m_p + \varepsilon_p$$

Equation 11

Where $r_{p,t}$ is the monthly asset return and $\hat{b}_{0t}$ and $\hat{b}_{\beta t}, \hat{b}_{\gamma t}, \hat{b}_{\delta t}, \hat{b}_{st}, \hat{b}_{ht}, \hat{b}_{mt}$ are the estimated risk premia for each risk factor for each month *t*; the risk premia are then averaged over time and the t-statistic is computed to evaluate their significance. The formulas used for this purpose are presented below:

$$b_j = \frac{1}{n}\sum_{t=x}^{T} \hat{b}_{jt} \quad \text{for } j = 0, \beta, \dots. m$$

Equation 12

$$t_{jt} = \frac{b_j}{std_{bj}/\sqrt{T}}$$

Equation 13

Where std_{bj} is the standard deviation of b_j, and T is the number of cross-sectional regression carried out in the second stage.

To identify any potential issues with our estimates, we test for heteroskedasticity in the asset return series using the Breusch-Pagan (1979) test and for autocorrelation using the Durbin-Watson (1951) and Breusch (1978) - Godfrey (1978) tests. In case the series exhibits heteroskedasticity and/or autocorrelation, the Newey-West (1987) standard errors are used for computing the t-statistics of the estimates instead of the default robust standard errors.

3.7 Bull versus Bear markets

The cross-sectional model following Pettengill, Sundaram and Mathur (1995) to investigate the behavior of co-skewness and co-kurtosis separately in up and down markets is shown in Equation 14.

$$r_{p,t} - r_{f,t} = b_0 + b_\beta^\pm D^\pm \beta_p + b_\gamma^\pm D^\pm \gamma_p + b_\delta^\pm D^\pm \delta_p + b_s^\pm D^\pm s_p + b_h^\pm D^\pm h_p + b_m^\pm D^\pm m_p + \varepsilon_t$$

Equation 14

The dummy variable D takes the value 1 if the market premium is positive and 0 otherwise, and the six coefficients capturing the risk premia related to covariance, co-skewness, co-kurtosis, size, value and momentum, $b_\beta^\pm$, $b_\gamma^\pm$, $b_\delta^\pm$, $b_s^\pm$, $b_h^\pm$ and $b_m^\pm$, are estimated both in the Bull market (+) and the Bear market (-).

4 RESULTS

4.1 Descriptive Statistics

The distributional properties of the explanatory variables and stock portfolios used in the analysis are presented in Table 1. Panel B reports the characteristics of the portfolios formed based on market capitalization – from 1 (big) to 15 (small). Panel C shows the same statistics for the portfolios formed based on the book-to-market ratio – from 1 (high B/M ratio) to 15 (low B/M ratio). Panel D presents the characteristics of the 25 double sorted portfolios.

Table 1 – Descriptive statistics

Panel A: Explanatory variables

Risk Factor	Mean	Std. deviation	Min	Max	Skewness	Kurtosis	Jarque - Bera stat
$(r_m - r_f)$	-0,0004	0,0839	-0,4102	0,2247	-0,6622	5,3073	71,892***
$(r_m - E(r_m))^2$	0,0071	0,0146	0,0000	0,1661	6,4808	62,8190	37 493,042***
$(r_m - E(r_m))^3$	-0,0003	0,0050	-0,0677	0,0119	-10,5163	139,6399	191 213,038***
SMB	-0,0010	0,0513	-0,1573	0,3042	0,8737	7,9770	280,684***
HML	0,0055	0,0453	-0,1066	0,1782	0,2700	3,8623	10,784***
MOM	0,0076	0,0353	-0,0997	0,1129	0,1059	3,0725	0,54

Panel B: Returns on 15 size sorted portfolios

Size portfolio	Value-weighted Mean	Value-weighted Std. deviation	Value-weighted Skewness	Value-weighted Kurtosis	Equal-weighted Mean	Equal-weighted Std. deviation	Equal-weighted Skewness	Equal-weighted Kurtosis
Big	0,008	0,088	-1,758	13,445	0,008	0,082	-1,514	10,840
2	0,003	0,077	-0,699	6,413	0,003	0,077	-0,680	5,884
3	0,006	0,072	-0,570	5,692	0,007	0,071	-0,530	5,951
4	0,007	0,074	-0,481	5,082	0,008	0,073	-0,453	4,837
5	0,010	0,073	-0,728	6,282	0,010	0,073	-0,671	6,234
6	0,004	0,077	-0,542	4,914	0,004	0,077	-0,556	4,997
7	0,007	0,072	-0,545	5,553	0,007	0,072	-0,520	5,465
8	0,009	0,076	-0,038	6,529	0,009	0,075	-0,041	6,568
9	0,008	0,077	-0,532	4,864	0,008	0,077	-0,518	4,746
10	0,006	0,069	-0,010	3,401	0,006	0,069	-0,015	3,391
11	0,009	0,071	-0,436	4,980	0,009	0,071	-0,438	4,909
12	0,010	0,067	0,301	3,485	0,009	0,067	0,311	3,399
13	0,005	0,080	0,148	3,626	0,005	0,082	0,064	3,974
14	0,006	0,095	0,250	4,451	0,006	0,096	0,359	4,726
Small	0,003	0,114	0,457	4,322	0,007	0,108	0,511	4,160

Panel C: Returns on 15 value sorted portfolios

Value portfolio	Value-weighted Mean	Value-weighted Std. deviation	Value-weighted Skewness	Value-weighted Kurtosis	Equal-weighted Mean	Equal-weighted Std. deviation	Equal-weighted Skewness	Equal-weighted Kurtosis
High	0,009	0,102	0,355	4,685	0,017	0,091	0,776	5,062
2	0,009	0,107	-0,037	4,615	0,013	0,078	0,027	3,912
3	0,006	0,098	-0,484	4,009	0,011	0,071	-0,161	4,276
4	0,012	0,092	-0,613	5,450	0,011	0,069	-0,312	3,766
5	0,004	0,102	-1,567	13,080	0,010	0,073	-0,381	4,630
6	0,012	0,087	-0,554	7,141	0,013	0,070	-0,005	4,270
7	0,012	0,096	0,001	5,662	0,008	0,065	-0,488	5,248
8	0,010	0,089	-1,091	7,416	0,010	0,071	-0,702	5,899
9	0,010	0,082	-0,060	4,734	0,011	0,068	-0,464	6,202
10	0,009	0,086	-0,877	6,573	0,009	0,073	-0,183	5,593
11	0,006	0,103	-0,145	24,294	0,003	0,067	-0,863	6,344
12	0,010	0,101	-1,350	13,317	0,006	0,079	-0,364	4,746
13	0,005	0,079	-0,534	4,007	-0,001	0,073	-0,532	4,928
14	-0,001	0,086	-0,619	8,054	-0,002	0,075	-0,677	8,214
Low	0,004	0,081	-1,017	5,717	-0,002	0,087	-0,593	5,475

Panel D: Returns on 25 double sorted portfolios

Size	Book-to-Market	Value-weighted Mean	Value-weighted Std. deviation	Value-weighted Skewness	Value-weighted Kurtosis	Equal-weighted Mean	Equal-weighted Std. deviation	Equal-weighted Skewness	Equal-weighted Kurtosis
Big	High	0,005	0,096	-0,454	5,317	0,007	0,088	-0,616	7,936
	2	0,009	0,092	-1,635	12,028	0,009	0,082	-0,727	6,571
	3	0,008	0,096	-0,869	9,266	0,007	0,084	-0,645	5,634
	4	0,006	0,093	-1,528	13,411	0,004	0,078	-0,369	5,218
	Low	0,006	0,080	-0,989	6,946	0,002	0,079	-0,930	7,357
2	High	0,011	0,078	-0,055	3,930	0,011	0,077	-0,156	3,833
	2	0,016	0,083	-0,047	4,319	0,016	0,082	-0,065	4,277
	3	0,004	0,076	-0,375	4,715	0,004	0,079	-0,319	4,996
	4	0,006	0,084	-0,722	5,380	0,006	0,082	-0,591	5,265
	Low	0,002	0,072	-0,674	5,543	0,002	0,074	-0,707	5,702
3	High	0,011	0,091	-0,366	4,374	0,012	0,091	-0,429	4,441
	2	0,011	0,080	-0,385	7,194	0,012	0,077	-0,344	7,109
	3	0,012	0,080	-1,065	8,612	0,012	0,079	-0,871	7,691
	4	0,005	0,069	-0,244	4,516	0,006	0,071	-0,162	5,255
	Low	-0,007	0,093	-0,686	5,454	-0,007	0,094	-0,603	5,438
4	High	0,011	0,092	-0,059	3,969	0,013	0,086	0,045	3,921
	2	0,007	0,078	-0,092	4,312	0,007	0,077	0,014	4,092
	3	0,013	0,083	0,167	4,741	0,012	0,079	-0,270	5,093
	4	0,004	0,089	0,049	5,357	0,003	0,086	-0,037	4,924
	Low	0,001	0,085	-0,176	4,798	0,001	0,085	-0,178	4,568
Small	High	0,017	0,108	0,283	4,178	0,012	0,103	0,836	4,989
	2	0,023	0,086	0,593	4,734	0,019	0,091	0,444	3,892
	3	0,001	0,085	-0,507	6,479	0,001	0,079	-0,227	6,133
	4	0,010	0,083	0,343	4,642	0,004	0,087	0,191	4,168
	Low	0,001	0,091	0,264	4,730	-0,001	0,096	0,762	7,854

Panel A shows that based on the Jarque-Bera test, all of the risk factors except MOM are non-normally distributed. The market premium and cubed excess market return have a small negative mean return, negative skewness and excess kurtosis (above 3) while the squared excess market return is on average slightly positive, is skewed to the right and also exhibits excess kurtosis. The SMB and HML factors are slightly positively skewed and exhibit some excess kurtosis while SMB has a small negative average return and HML has a positive mean return.

11 out of the 15 size sorted portfolios presented in Panel B of Table 1 are skewed towards the negative end of the return distribution, implying that negative returns are more likely compared to positive returns of the same magnitude. All the portfolios exhibit excess kurtosis, meaning that the return distributions have fatter tails compared to the normal distribution. The characteristics do not vary notably between value-weighted and equal-weighted portfolios. The key takeaway from this table is that the portfolios with the highest risk (standard deviation and hence variance) do not have the highest average returns.

Similar characteristics hold for value sorted as well as double sorted portfolios as seen in Panel C and Panel D of Table 1. 13 out of the 15 value sorted portfolios have small negative skeweness and all 15 display excess kurtosis, 19 out of the 25 double sorted portfolios aer skewed to the left and all 25 exhibit excess kurtosis. Unreported Jarque-Bera tests reject the null hypothesis of normality for all but 4 out of the 85 portfolios presented in Panel B, Panel C and Panel D of Table 1.

4.2 Time series regressions

Table 2 reports the results of the time series regressions of the monthly portfolio returns on the explanatory variables. We analyze four different models – the standard CAPM (Equation 1), the 4-moment CAPM (Equation 2), the Carhart model (Equation 8) and the 4-moment CAPM together with the Fama-French-Carhart empirical risk factors (Equation 10). The table indicates the number of portfolios for which the respective coefficient is significantly different from zero, the plus and minus next to the coefficients indicate the sign of the significant coefficients; for example, the number 15 for β+ in the first line of Panel A indicates that the beta coefficient is positive and significant for all 15 value-weighted, size-sorted portfolios.

Table 2 – Number of significant coefficients in the time series regressions

Panel A: Value-weighted portfolios

15 size sorted portfolios

Model	α+	α-	β+	β-	γ+	γ-	δ+	δ-	*s+*	*s-*	*h+*	*h-*	*m+*	*m-*	*Adj. R-square*
CAPM	0	2	15	0											0,469
4-mom CAPM	0	1	15	0	0	0	1	1							0,473
Carhart	0	3	15	0					13	1	0	3	0	2	0,608
4-mom + Carhart	0	1	15	0	0	1	2	0	13	1	0	3	0	1	0,621

15 value sorted portfolios

Model	α+	α-	β+	β-	γ+	γ-	δ+	δ-	*s+*	*s-*	*h+*	*h-*	*m+*	*m-*	*Adj. R-square*
CAPM	0	3	15	0											0,5193
4-mom CAPM	0	1	15	0	0	2	2	2							0,5365
Carhart	0	3	15	0					8	0	5	4	2	1	0,5586
4-mom + Carhart	0	1	15	0	0	2	1	1	7	0	4	4	2	1	0,5775

25 double sorted portfolios

Model	α+	α-	β+	β-	γ+	γ-	δ+	δ-	*s+*	*s-*	*h+*	*h-*	*m+*	*m-*	*Adj. R-square*
CAPM	1	6	25	0											0,3985
4-mom CAPM	1	1	25	0	0	0	1	1							0,4038
Carhart	1	6	25	0					20	0	4	10	1	2	0,5287
4-mom + Carhart	1	0	25	0	0	2	1	0	20	0	4	9	1	1	0,5403

Panel B: Equal-weighted portfolios

15 size sorted portfolios

Model	α+	α-	β+	β-	γ+	γ-	δ+	δ-	*s+*	*s-*	*h+*	*h-*	*m+*	*m-*	*Adj. R-square*
CAPM	0	2	15	0											0,4655
4-mom CAPM	0	0	15	0	0	0	0	1							0,4680
Carhart	0	3	15	0					13	1	0	3	1	2	0,6090
4-mom + Carhart	0	0	15	0	0	1	2	0	13	0	0	3	1	2	0,6210

15 value sorted portfolios

Model	α+	α-	β+	β-	γ+	γ-	δ+	δ-	*s+*	*s-*	*h+*	*h-*	*m+*	*m-*	*Adj. R-square*
CAPM	0	4	15	0											0,4577
4-mom CAPM	0	2	15	0	0	1	0	0							0,4590
Carhart	0	4	15	0					15	0	3	8	1	1	0,6026
4-mom + Carhart	0	1	15	0	0	4	3	0	15	0	3	8	1	1	0,6170

25 double sorted portfolios

Model	α+	α-	β+	β-	γ+	γ-	δ+	δ-	*s+*	*s-*	*h+*	*h-*	*m+*	*m-*	*Adj. R-square*
CAPM	0	7	25	0											0,3992
4-mom CAPM	0	2	25	0	0	0	0	1							0,4006
Carhart	0	5	25	0					20	0	4	10	0	2	0,5354
4-mom + Carhart	0	1	25	0	0	3	1	0	20	0	4	9	0	1	0,5454

The beta coefficient remains positive and significant throughout all model specifications and portfolio sorting and weighting methods. While the intercept (alpha) is significantly negative for a few portfolios for the standard CAPM specification, the number of significant intercepts decreases when the higher order co-moments are added to the model, indicating a better fit of the latter model to the data. This does not happen notably when only the empirical factors are included as in the Carhart model.

A key result in Table 2 is that the co-skewness (gamma) and co-kurtosis (delta) measures remain mostly insignificant, indicating that the squared and cubed excess market return lack strong explanatory power for the asset returns. Consistently with previous findings of Chung, Johnson and Schill (2006) in the UK market and Silva (2005) in the Brazilian market, the size factor (*s*) is more important in explaining the variation in asset returns than the value factor (*h*). Additionally, we find that the momentum factor (*m*) adds little in terms of explanatory power as it remains mostly insignificant.

Comparing Panel A (value-weighted portfolios) and Panel B (equal-weighted portfolios) shows notable differences only when looking at value sorted portfolios. In that case, the size and the value factors have better explanatory power when equal weighting is used. There is no major difference between the panels regarding the significance of co-skewness and co-kurtosis, though the number of portfolios with a significant co-skewness coefficient is slightly higher in Panel B while the number of portfolios with a significant co-kurtosis coefficient is higher in Panel A.

It is important to note that the co-skewness and co-kurtosis still remain somewhat significant when the empirical risk factors are added to the 4-moment CAPM. This gives indication that the size, value and momentum factors are not proxies for the higher order co-moments and the latter capture variation beyond the empirical risk factors.

4.3 Fama-MacBeth regressions

The Fama-MacBeth (1973) cross-sectional regression results are presented in Table 3 for value-weighted portfolios and Table 4 for equal-weighted portfolios. As before, we analyze four different models and look at portfolios with different weighting and sorting schemes to investigate whether the portfolios' covariance, co-skewness and co-kurtosis with the market and the three empirical risk factors are priced in the Brazilian stock market.

Table 3 – Fama-MacBeth regressions – value-weighted portfolios

Panel A: Size sorted portfolios

Model	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	*Adj. R-square*
CAPM	-0,0104	0,0069						0,0600
	(-1,6602)*	(0,7909)						
4-mom CAPM	-0,0085	0,0030	-0,0028	-0,0002				0,1509
	(-1,3261)	(0,3107)	(-1,7346)*	(-0,4978)				
Carhart	-0,0089	0,0039			0,0017	-0,0065	-0,0004	0,2140
	(-0,9651)	(0,3104)			(0,4211)	(-0,7640)	(-0,0658)	
4-mom+Carhart	-0,0164	0,0095	-0,0056	0,0007	0,0035	0,0017	-0,0052	0,3103
	(-1,5837)	(0,7018)	(-2,6327)***	(1,2211)	(0,846)	(0,1783)	(-0,6625)	

Panel B: Value sorted portfolios

Model	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	*Adj. R-square*
CAPM	-0,0004	-0,0050						0,0425
	(-0,0553)	(-0,6041)						
4-mom CAPM	-0,0037	-0,0005	-0,0024	0,0002				0,0853
	(-0,5494)	(-0,0597)	(-1,5735)	(0,7244)				
Carhart	-0,0081	0,0053			-0,0128	0,0076	0,0119	0,1552
	(-1,1546)	(0,6562)			(-2,2264)**	(1,6886)*	(3,105)***	
4-mom+Carhart	-0,0078	0,0033	-0,0024	0,0000	-0,0070	0,0020	0,0053	0,2026
	(-1,0577)	(0,3628)	(-1,5085)	(0,0815)	(-1,2245)	(0,4539)	(1,0598)	

Panel C: Double sorted portfolios

Model	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	*Adj. R-square*
CAPM	0,0022	-0,0114						0,0559
	(0,461)	(-1,6053)						
4-mom CAPM	0,0035	-0,0140	-0,0014	-0,0002				0,0918
	(0,6927)	(-1,8521)*	(-1,1343)	(-0,6237)				
Carhart	0,0130	-0,0221			-0,0013	0,0048	0,0078	0,1715
	(2,5184)**	(-2,9314)***			(-0,3874)	(1,1751)	(2,1086)**	
4-mom+Carhart	0,0122	-0,0232	-0,0011	-0,0001	-0,0014	0,0065	0,0090	0,1925
	(2,2866)**	(-3,0055)***	(-0,8685)	(-0,403)	(-0,4155)	(1,5965)	(2,3867)**	

Table 4 – Fama-MacBeth regressions – equal-weighted portfolios

Panel A: Size sorted portfolios

Model	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	*Adj. R-square*
CAPM	-0,0079 (-1,148)	0,0026 (0,2697)						0,0717
4-mom CAPM	-0,0071 (-1,107)	0,0033 (0,3392)	0,0007 (0,4112)	-0,0004 (-0,8750)				0,1548
Carhart	-0,0066 (-0,6813)	-0,0004 (-0,0262)			0,0037 (0,9415)	-0,0040 (-0,4575)	0,0011 (0,1731)	0,2260
4-mom+Carhart	-0,0131 (-1,0818)	0,0064 (0,3901)	-0,0030 (-1,4297)	0,0002 (0,2404)	0,0026 (0,638)	-0,0006 (-0,0616)	-0,0033 (-0,4093)	0,3212

Panel B: Value sorted portfolios

Model	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	*Adj. R-square*
CAPM	0,0136 (2,1098)**	-0,0292 (-2,583)***						0,0520
4-mom CAPM	0,0132 (2,0418)**	-0,0302 (-2,6608)***	0,0014 (0,8537)	-0,0009 (-1,6108)				0,0599
Carhart	0,0128 (1,7503)*	-0,0188 (-1,7882)*			0,0033 (0,4374)	0,0082 (1,8039)*	-0,0061 (-1,1116)	0,1301
4-mom+Carhart	0,0084 (1,1448)	-0,0139 (-1,2229)	0,0007 (0,477)	-0,0008 (-1,6791)*	0,0052 (0,566)	0,0098 (1,9817)**	-0,0066 (-1,079)	0,1465

Panel C: Double sorted portfolios

Model	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	*Adj. R-square*
CAPM	-0,0004 (-0,0807)	-0,0080 (-1,0520)						0,0465
4-mom CAPM	-0,0007 (-0,1424)	-0,0101 (-1,3246)	-0,0012 (-0,9581)	-0,0002 (-0,5351)				0,0800
Carhart	0,0096 (1,8653)*	-0,0176 (-2,2232)**			-0,0008 (-0,2264)	0,0081 (1,9814)**	0,0083 (2,3217)**	0,1596
4-mom+Carhart	0,0117 (2,0887)**	-0,0223 (-2,7233)***	-0,0015 (-1,1293)	-0,0001 (-0,2902)	-0,0012 (-0,3341)	0,0089 (2,2344)**	0,0097 (2,4598)**	0,1809

The results presented in Table 3 and Table 4 indicate that, overall, neither the covariance, co-skewness and co-kurtosis with the market nor the size, value and momentum factors are priced in the Brazilian stock market. The price of beta (covariance with the market) is significant and negative for some model specifications only when looking at the double sorted portfolios (Panel C in both tables) or equal-weighted value sorted portfolios (Table 4, Panel B).

The results provide partial support for the co-skewness being priced, but only when we look at Panel A of Table 3, i.e. using value-weighted portfolios formed based on size sorting. The coefficient for co-skewness is negative, as expected. Weak support is also found for the co-kurtosis being priced, but only for equal-weighted value sorted portfolios in Panel A of Table 4. The sign of the co-kurtosis premium is negative which is contrary to what theory predicts.

Regarding the empirical risk factors, we find evidence supporting the significance of the positive momentum premium in double sorted portfolios (Panel C in both tables) and the positive value premium in equal-weighted value or double sorted portfolios (Table 4, Panel B and C). All three factors are priced for the Carhart model when using value-weighted value sorted portfolios, however, the coefficients become insignificant when the higher order moments are added to the Carhart model, even though model fit (adjusted R-square) increases.

A possible explanation for these results may be the choice of the time period, and more specifically, the averaging of estimates from the time series regressions. It may be that such averaging of the estimates removes some time-dependent variation which is why the risk factors turn out not to be priced. Time variation may be especially important in a rapidly developing environment such as Brazil. That is why we follow Pettengill, Sundaram and Mathur (1995) and separate the estimates for up and down (Bull and Bear) markets.

4.4 Bull versus Bear markets

Tables 5 and 6 present the results of the two-step Fama and MacBeth (1973) regressions for the size, value and double sorted portfolios following the methodology of Pettengill, Sundaram and Mahur (1995) that separates the up and down markets according to the realized market excess return.

Table 5 – Fama-MacBeth regressions – bull vs. bear – value-weighted portfolios

Panel A: Size sorted portfolios

Model	Status	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	Adjusted R
CAPM	Up	0,0026	0,0463						0,0766
		(0,2841)	(3,6909)***						
	Down	-0,0226	-0,0303						0,0442
		(-2,6820)***	(-2,8294)***						
4-mom CAPM	Up	-0,0036	0,0517	-0,0033	0,0010				0,1731
		(-0,3569)	(3,8183)***	(-1,4394)	(1,8039)*				
	Down	-0,0131	-0,0431	-0,0023	-0,0013				0,1300
		(-1,6455)*	(-3,7223)***	(-1,0156)	(-2,3783)**				
Carhart	Up	0,0088	0,0370			-0,0071	-0,0011	-0,0003	0,2401
		(0,6694)	(2,0545)**			(-1,2726)	(-0,0857)	(-0,0262)	
	Down	-0,0257	-0,0274			0,0100	-0,0116	-0,0006	0,1894
		(-2,0024)**	(-1,6407)			(1,7652)*	(-1,0244)	(-0,0695)	
4-mom+Carhart	Up	-0,0046	0,0519	-0,0032	0,0013	-0,0050	0,0088	-0,0046	0,3337
		(-0,3187)	(2,7606)***	(-1,1023)	(1,626)	(-0,8639)	(0,6287)	(-0,3997)	
	Down	-0,0275	-0,0306	-0,0078	0,0001	0,0114	-0,0051	-0,0057	0,2882
		(-1,8632)*	(-1,6452)*	(-2,5617)***	(0,1537)	(1,9824)**	(-0,4190)	(-0,5359)	

Panel B: Value sorted portfolios

Model	Status	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	Adjusted R
CAPM	Up	0,0258	0,0171						0,0459
		(2,8051)***	(1,5019)						
	Down	-0,0251	-0,0259						0,0392
		(-2,7669)***	(-2,2013)**						
4-mom CAPM	Up	0,0129	0,0340	-0,0020	-0,0003				0,0980
		(1,3526)	(2,8376)***	(-0,9737)	(-0,8539)				
	Down	-0,0193	-0,0331	-0,0027	0,0006				0,0733
		(-2,1170)**	(-3,0619)***	(-1,2325)	(1,5422)				
Carhart	Up	0,0136	0,0316			-0,0130	0,0153	0,0134	0,1711
		(1,3351)	(2,7524)***			(-1,6607)*	(2,4924)**	(2,3259)**	
	Down	-0,0285	-0,0195			-0,0125	0,0003	0,0105	0,1402
		(-3,1023)***	(-1,7995)*			(-1,495)	(0,0397)	(2,0482)**	
4-mom+Carhart	Up	0,0076	0,0406	-0,0013	-0,0008	-0,0132	0,0101	0,0013	0,2327
		(0,6888)	(3,1282)***	(-0,6117)	(-1,6362)	(-1,5193)	(1,6114)	(0,1793)	
	Down	-0,0223	-0,0320	-0,0035	0,0008	-0,0011	-0,0057	0,0091	0,1742
		(-2,3002)**	(-2,8054)***	(-1,4532)	(1,6058)	(-0,1527)	(-0,9434)	(1,2931)	

Panel C: Double sorted portfolios

Model	Status	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	Adjusted R
CAPM	Up	0,0106	0,0347						0,0504
		(1,6105)	(4,0379)***						
	Down	-0,0058	-0,0549						0,0611
		(-0,8812)	(-6,0697)***						
4-mom CAPM	Up	0,0084	0,0364	-0,0037	0,0002				0,0847
		(1,2261)	(4,1501)***	(-2,3249)**	(0,5356)				
	Down	-0,0011	-0,0617	0,0008	-0,0006				0,0985
		(-0,145)	(-6,2168)***	(0,4546)	(-1,63)				
Carhart	Up	0,0150	0,0296			-0,0059	0,0123	0,0019	0,1625
		(2,3049)**	(3,6365)***			(-1,2228)	(2,0113)**	(0,3464)	
	Down	0,0110	-0,0709			0,0031	-0,0023	0,0133	0,1801
		(1,3888)	(-7,0000)***			(0,7152)	(-0,433)	(2,7037)***	
4-mom+Carhart	Up	0,0130	0,0297	-0,0037	0,0005	-0,0068	0,0123	0,0007	0,1911
		(1,9052)*	(3,7511)***	(-2,1312)**	(1,0983)	(-1,41)	(2,0258)**	(0,1187)	
	Down	0,0113	-0,0731	0,0013	-0,0007	0,0038	0,0011	0,0169	0,1937
		(1,3968)	(-6,8823)***	(0,7364)	(-1,8563)*	(0,8519)	(0,2041)	(3,3366)***	

Table 6 – Fama-MacBeth regressions – bull vs. bear - equal-weighted portfolios

Panel A: Size sorted portfolios

Model	Status	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	Adjusted R
CAPM	Up	0,0096	0,0345						0,0747
		(0,9532)	(2,3672)**						
	Down	-0,0245	-0,0274						0,0689
		(-2,6843)***	(-2,1948)**						
4-mom CAPM	Up	0,0031	0,0452	-0,0009	0,0006				0,1636
		(0,3232)	(3,3728)***	(-0,3702)	(1,1916)				
	Down	-0,0167	-0,0363	0,0022	-0,0013				0,1464
		(-2,0084)**	(-2,8190)***	(0,9481)	(-2,2604)**				
Carhart	Up	0,0040	0,0404			-0,0025	0,0061	0,0030	0,2476
		(0,2995)	(2,1503)**			(-0,4536)	(0,4465)	(0,3256)	
	Down	-0,0167	-0,0389			0,0096	-0,0135	-0,0007	0,2055
		(-1,1985)	(-2,0124)**			(1,7577)*	(-1,2225)	(-0,0848)	
4-mom+Carhart	Up	-0,0043	0,0525	-0,0017	0,0010	-0,0056	0,0138	-0,0014	0,3383
		(-0,2538)	(2,3503)**	(-0,5443)	(1,1060)	(-0,9482)	(0,8521)	(-0,1285)	
	Down	-0,0215	-0,0372	-0,0043	-0,0006	0,0105	-0,0142	-0,0050	0,3050
		(-1,2381)	(-1,607)	(-1,4702)	(-0,6702)	(1,8317)*	(-1,0990)	(-0,4344)	

Panel B: Value sorted portfolios

Model	Status	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	Adjusted R
CAPM	Up	0,0245	0,0122						0,0421
		(2,6807)***	(0,8203)						
	Down	0,0034	-0,0683						0,0613
		(0,3726)	(-4,2882)***						
4-mom CAPM	Up	0,0151	0,0201	-0,0024	-0,0001				0,2318
		(1,6365)	(1,3755)	(-1,1867)	(-0,1210)				
	Down	0,0114	-0,0777	0,0049	-0,0016				0,0945
		(1,2514)	(-4,9505)***	(2,0905)**	(-1,9633)**				
Carhart	Up	0,0192	0,0156			0,0090	0,0055	-0,0087	0,1024
		(1,8072)*	(1,063)			(0,8787)	(0,9051)	(-1,1026)	
	Down	0,0067	-0,0514			-0,0022	0,0108	-0,0036	0,1562
		(0,6675)	(-3,5798)***			(-0,2053)	(1,5931)	(-0,4697)	
4-mom+Carhart	Up	0,0122	0,0231	-0,0025	-0,0001	0,0063	0,0058	-0,0137	0,0981
		(1,1526)	(1,4633)	(-1,3224)	(-0,1493)	(0,4877)	(0,8621)	(-1,5383)	
	Down	0,0048	-0,0489	0,0038	-0,0014	0,0042	0,0136	0,0002	0,1923
		(0,4712)	(-3,1635)***	(1,6725)*	(-2,1144)**	(0,3184)	(1,8738)*	(0,0221)	

Panel C: Double sorted portfolios

Model	Status	b_0	b_β	b_γ	b_δ	b_s	b_h	b_m	Adjusted R
CAPM	Up	0,0101	0,0353						0,0452
		(1,4252)	(3,5421)***						
	Down	-0,0104	-0,0490						0,0478
		(-1,4593)	(-5,0230)***						
4-mom CAPM	Up	0,0077	0,0350	-0,0036	0,0006				0,0678
		(1,0605)	(3,5989)***	(-2,3674)**	(1,2394)				
	Down	-0,0088	-0,0526	0,0010	-0,0009				0,0916
		(-1,172)	(-5,4280)***	(0,4725)	(-1,7114)*				
Carhart	Up	0,0119	0,0353			-0,0055	0,0155	0,0031	0,1505
		(1,6821)*	(3,8058)***			(-1,0658)	(2,5513)**	(0,6225)	
	Down	0,0075	-0,0675			0,0037	0,0012	0,0132	0,1682
		(0,9976)	(-6,6330)***			(0,8055)	(0,2163)	(2,6046)***	
4-mom+Carhart	Up	0,0131	0,0304	-0,0038	0,0007	-0,0069	0,0142	0,0019	0,1711
		(1,709)*	(3,1680)***	(-2,4156)**	(1,3941)	(-1,3432)	(2,5578)**	(0,3425)	
	Down	0,0104	-0,0721	0,0008	-0,0009	0,0043	0,0038	0,0171	0,1902
		(1,2713)	(-6,7039)***	(0,3871)	(-1,7331)*	(0,9289)	(0,6792)	(3,0958)***	

Just as Hung, Shackleton and Xu (2004) find strong support for beta pricing in the UK when up and down markets are analyzed separately, we find consistently strong support for beta pricing in the Brazilian market as seen in Table 5 and Table 6. The premium for covariance with the market is positive when the market is up and negative when the market is down. This means that the usefulness of beta as a measure of risk that reflects a positive tradeoff between the risk and the return can be assessed through this conditional model. The result holds throughout the different model specifications and sorting and weighting methods, though the evidence is the weakest when looking at equal-weighted value sorted portfolios (Table 6, Panel B) and strongest for value-weighed double sorted portfolios (Table 5, Panel C).

Panel A in Table 5 as well as Table 6 (size sorted portfolios) provides some support for the co-skewness being priced in the down market (negative risk premium), but only when controlling for the size, value and momentum factors in the model (full specification). There is also some indication for co-kurtosis being priced (positive premium in the up market, negative in the down market) but the effect is weaker and disappears when controlling for the empirical risk factors. The empirical factors are not priced for the size sorted portfolios.

When considering value sorted portfolios (Panel B in both tables), the higher co-moments are not priced for value-weighted portfolios. But for equal-weighted portfolios, this is the only sorting scheme that leads to co-skewness and co-kurtosis being priced simultaneously in the down market. Furthermore, the results provide partial support for the pricing of the momentum factor and rather weak evidence for the size and value factors being priced only for value-weighted portfolios. It shows that when the market return has performance inferior to the return on the risk-free asset, the higher co-moments may matter. In general, the results do not support that co-skewness and co-kurtosis premia capture the same cross-sectional variation as the size, value and momentum factors as the higher co-moments are more present in the value and double sorted portfolios than in size sorted portfolios.

Using double sorted portfolios provides further evidence for the pricing of higher order co-moments (Panel C in both tables). We see that co-skewness has a significant negative risk premium in the up market and co-kurtosis has a significant negative premium in the down market. In general, it seems that the risk premia for co-skewness and co-kurtosis do not lose significance in the presence of the size, value and momentum factors. This indicates that in good times when the market is going up and the market return exhibits positive skewness, then the investors require a lower return from an asset that has positive co-skewness with the market. Similarly, when the markets are going down and the market return has excess kurtosis, then the investors require a lower return from an asset that has positive co-kurtosis with the market.

5 CONCLUSION

In this paper, we revisit one of the major empirical questions in finance – whether higher order co-moments with the market are priced in the equity markets. The analysis includes testing the 4-moment CAPM, developed by and Fang and Lai (1997), to find empirical evidence for the pricing of covariance, co-skewness and co-kurtosis in the Brazilian stock market.

The evidence from the time series regressions shows that the beta (coefficient for the market premium factor) remains a highly significant explanatory variable for asset returns even in the presence of higher co-moments and the empirical size, value and momentum factors. The results further indicate that the quadratic and cubed excess market return are not very relevant explanatory variables for the asset returns but provide support to the notion that the empirical factors do not proxy the higher order co-moments, indicating that co-skewness and co-kurtosis capture a part of the variance in asset returns that the empirical factors fail to incorporate.

The unconditional cross-sectional analysis provides weak support for the pricing of co-skewness and momentum on the Brazilian stock market. The co-skewness pricing is evident only for value-weighted size sorted portfolios and co-kurtosis is marginally priced in the equal-weighted value sorted portfolios. Conditional models controlling for up and down markets reveal strong support for beta pricing while also providing partial evidence of existing premia for co-skewness and co-kurtosis.

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