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Goodwin cycles and the BoPC growth paradigm: A macrodynamic model of growth and fluctuations☆

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Abstract

The paper builds a non-linear macrodynamic model to study the relation between the functional distribution of income, technological progress and economic growth. In the short-term, the interaction between the productivity regime, the demand regime and the distributive conflict generates cyclical paths *a la* Goodwin. In the long-term, output growth rate is constrained by the balance of payments *a la* Thirlwall, in which the elasticities of foreign trade are modeled as a function of the complex relation between the wage-share and the innovation capabilities of theeconomy.

JEL classification: E12; E32; O11

Keywords: Cyclical growth; Goodwin cycles; Distributive cycles; Thirlwall's law; Balance of payments constraint

Resumo

O artigo constrói um modelo dinâmico não-linear para estudar a relação entre a distribuição funcional da renda, o progresso técnico e o crescimento econômico. No curto prazo, a interação entre o regime de produtividade, o regime de demanda e o conflito distributivo gera trajetórias cíclicas *a la* Goodwin. No longo prazo, o crescimento do produto é restrito pelo balanço de pagamentos *a la* Thirlwall, em que as elasticidades de comércio exterior são modeladas pela complexa relação entre o *wage-share* e a capacidade inovativa da economia.

Palavras chave: Crescimento cíclico; Ciclos de Goodwin; Ciclos distributivos; Lei de Thirlwall; Restrição externa

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1. Introduction

A fundamental characteristic of the capitalist economy is its cyclical and irregular growth behavior. The heterodox tradition in macroeconomics has a variety of models which seek to capture this cycle-tendency relation, emphasizing different aspects of the real world. An important exercise has to be done in order to solve the fundamental differences between then, delimiting a basic structure for the dynamics of accumulation and distribution.

The relation between effective demand and income distribution is a central aspect in the heterodox theories of distributive conflict (Barbosa and Taylor, 2006). The effective demand influences the functional distribution of income through fluctuations in nominal wages and labor productivity. Income distribution in turn influences consumption and investment through cyclical changes in the level of capacity utilization and the wage-share. Distributive conflict models explain inflation, distribution and growth juxtaposing wage demands and price behavior where each part seeks to protect its share on income (Rezai, 2012).

In the other hand, Thirlwall's law – one of the most successful empirical regularities in non-conventional growth theory – proposes that in the long run growth is Balance-of-Payments Constrained (BoPC) (Alonso and Garcimartín, 1999; Thirlwall, 2011). Since countries cannot finance BoP imbalances permanently, there is an adjustment in aggregate demand that constrain its expansion and consequently output's growth (Setterfield, 2011a,b; McCombie, 2011).

Taking as inspiration Goodwin's synthesis,[1] this study offers a modeling structure that adds up to other efforts by integrating key elements of heterodox tradition. The model is compatible with concepts like distributive conflict, autonomous investment function, cumulative causation, balance-of-payments-constraint growth, and the difference between science and technology.

Integrating different economic approaches is always challenging, not just because these contributions are disperse spatially and temporally, but also because they have unique richness and complexity. Efforts in this direction inevitable lead to losses of information as a collateral effect. However, we consider that the exercise is of great benefit since it allows, beyond its mathematical beauty and elegance, a broader view of our study object. It also has pedagogic purposes showing how different concepts can dialogue one with the other.

The paper also explores the interaction between the functional distribution of income, technological progress and economic growth. We built a *KG* (Kaldor-Goodwin) model of endogenous growth that generates cyclical trajectories *a la* Goodwin and a balance-of-payments-constrained growth *a la* Thirlwall. The income elasticities of foreign trade are modeled as a function of the complex relation between the *wage-share* and the innovation capacity of the economy.

Formally, Goodwin distributive cycle dynamics have been used as a basic framework to study different dimensions of capitalism structural instability. Classical contributions include Desai (1973) focusing on inflation, Van der Ploeg (1983) in its relation with neoclassical growth, and Shah and Desai (1981) on induced innovation. In order to study its relation with the Minskyan Financial Instability Hypothesis, Keen (1995), Keen (2013), Sordi and Vercelli (2006, 2012, 2014) among others have built a series of growth-cycle models exploring the non-linear interactions between financial and distributive variables.

Other important contributions include Sasaki (2013) combining Goodwin, Kaleckian and Marxian features, Schoder (2014) on the Harrodian instability and the Keynesian principle of effective demand, and Flaschel (2015) on the recent formalization of the Marx-Keynes-Schumpeter model. However, to the best of our knowledge, all this contributions concern closed economies. Our exercise is a first attempt to show that a marriage between Goodwin and Thirlwall may be possible in an open economy framework. Moreover this marriage is also justified on the grounds of the empirical support to both Goodwin (for example Harvie, 2000; Mohun and Veneziani, 2008; Tarassow, 2010; Zipperer and Skott, 2011; Kiefer and Rada, 2015) and Thirlwall's law (for example Bagnai, 2010; Gouvea and Lima, 2010, 2013; Cimoli et al., 2010; Romero and McCombie, 2016).

The paper's next section is dedicated to present a dynamic non-linear *KG* model with the properties discussed earlier. The last section brings our conclusions and some considerations about future research.

[1] According to Punzo (2006) we can credit to Richard M. Goodwin the great and visionary synthesis in which income distribution, as seen in Marxist analysis, interacts with innovation, as seen by Schumpeter, and the Keynesian effective demand principle, generating typically dynamics of a capitalist economy. The economy is modeled in a way that national production follows the aggregate demand restriction, but the engine of the trajectory is the accumulation made possible by innovation (Di Matteo and Sordi, 2015).

2. A macrodynamic model of growth and fluctuations

There are two ways to formalize growth and fluctuations. The first one consists in build two different theories in which short and long run are independent. The other takes cycle and tendency as indissolubly fused being generated from a unique dynamic system. Our approach is in the middle of these alternatives. In one hand we present separate theories to explain cycle and long-run growth. However, even though not fully integrated, short and long run interact through an adjustment mechanism capable to also generate permanent fluctuations.

The exercise developed in this section explicitly incorporates the principle of effective demand and the existence of distributive conflict in a non-linear macrodynamic model. Since we are working with a "real" economy in the sense that there is no money, the existence of fundamental uncertainty is treated implicitly in the determination of social conventions that sustain the current institutional framework.

Our model uses one of the conceptual elements of the classical macrodynamics, namely the adoption of a formalized structure formed by a collection of functional relations with given parameters.[2] However, inspired by Harrod and Goodwin, we reject the idea of an inherent stable economy in which the dynamic is purely generated by an exogenous impulse that activates a propagation mechanism – the structure of the system.

The model is structurally unstable in two dimensions.[3] First, because of its non-linearity, it generates the possibility of bifurcations. Second, because it is subject to continuous disturbances that come from the interaction between the short and long run dynamics.

Following Setterfield and Cornwall (2002) the model is based in three pillars: (i) Productivity Regime; (ii) Demand Regime and (iii) Distributive conflict. We will proceed by presenting each pillar, and then we will present the set of equations that form the dynamic system.

2.1. The productivity regime

Let us consider an economy with the follow aggregate production function[4]:

$$X_t = F(K_t; L_t) = \min\left\{\frac{K_t}{a_t}u_t; \frac{L_t}{b_t}\right\} \tag{1}$$

where X_t corresponds to total output and results from the combination of capital, K_t, and labor, L_t, weighted by their technical coefficients, a_t and b_t. Variable u_t stands for the level of capacity utilization and it is equal to the ratio of current Y, and potential output, Y^*. If inputs are efficiently used the economy operates with a level of output that satisfies the following condition:

$$X_t = \frac{K_t}{a_t}u_t = \frac{L_t}{b_t}$$

In this exercise the production function has two main purposes. First of all it determines the balance condition between capital accumulation and the labor productivity growth. Second, the productivity regime based on the Kaldor-Verdoorn (KV) law depends on it. Taking the technical coefficient of capital as constant, in terms of rates we have:

$$\frac{\dot{X}}{X_t} = \frac{\dot{K}}{K_t} + \frac{\dot{u}}{u_t} = \frac{\dot{L}}{L_t} + \frac{\dot{q}}{q_t} = y \tag{2}$$

Where q_t corresponds to labor productivity and is given by the inverse of labor technical coefficient ($q_t = 1/b_t$). The supply side efficient condition establishes that: $\frac{\dot{K}}{K_t} + \frac{\dot{u}}{u_t} = \frac{\dot{L}}{L_t} + \frac{\dot{q}}{q_t}$.

[2] For a review about the classical research program in macrodynamics see Punzo (2009).

[3] For a discussion about the structural instability in macrodynamic models see Vercelli (1985), Vercelli (2000) and Sordi and Vercelli (2006).

[4] Even though heterodox authors usually reject the neoclassical production function and even avoid the utilization of the production function concept itself, we can argue that implicitly is adopted a Leontief type. This proposition comes from the assumption that output's growth rate equals capital's accumulation growth rate or the sum between labor productivity and population growth rates.

In order to capture the relation between economic growth and increasing returns to scale we state a linear formulation of the Kaldor-Verdoorn's law:

$$\frac{\dot{q}}{q_t} = \Omega_0(T_t) + \Omega_1 \frac{\dot{K}}{K_t} \tag{3}$$

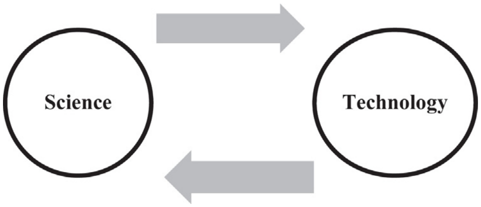
where Ω_0 represents *disembodied* productivity gains, Ω_1 correspond to Verdoorn's coefficient and T_t is a variable that captures the technological conditions of the economy.[5]

Substituting (3) in (2) we obtain the employment growth rate as a difference between the accumulation and labor productivity growth rates:

$$\frac{\dot{L}}{L_t} = (1 - \Omega_1)\frac{\dot{K}}{K_t} - \Omega_0(T_t) + \frac{\dot{u}}{u_t} \tag{4}$$

Employment adjusts to the difference between the output's growth rate and the productivity growth rate. The last one depends on the accumulation itself through increasing returns to scale, and on *disembodied* technological change.

Before continue we have to make some considerations about how technology works in this economy. According to Bernardes and Albuquerque (2003) the National Innovation Systems literature emphasizes the existence of an institutional division of labor between science and technology.[6] In general lines, while universities and research institutes produce science, firms produce technology.[7] Both groups interact and influence each other:



Even though we recognize that the relation between science and technology is not linear, we propose a simple system in order to represent the complex interaction the Schumpeterian literature suggests exists between those variables. So, be:

$$TECH = \tau_0 + \tau_1 SCIE \tag{5}$$

$$SCIE = \tau_0 + \tau_1 TECH \tag{6}$$

where *TECH* represents the technological production and *SCIE* the scientific infrastructure. The parameters τ_1 and π_1 capture the sensibility of *TECH* to changes in *SCIE* and the sensibility of *SCIE* to variations in *TECH*, respectively. Finally τ_0 and π_0 are exogenous effects.

Solving the system formed by Eqs. (5) and (6) we have that:

$$TECH^* = \frac{\tau_0 + \tau_1 \tau_0}{1 - \tau_1 \pi_1} \tag{7}$$

$$SCIE^* = \frac{\pi_0 + \pi_1 \tau_0}{1 - \tau_1 \pi_1} \tag{8}$$

[5] Recently McCombie and Spreafico (2015) have argued that the intercept cannot and should not be interpreted as an exogenous technical change contribution to growth, and Verdoorn's coefficient does not represent increasing returns *per se*. However, we will follow the traditional interpretation given to both components.

[6] The complex network of interactions and cooperation between agents that contribute to innovation – researchers, engineers, suppliers, producers, users and institutions – while the technological system evolves in a National State has been conceptualize as National System of Innovation (NSI) (Lundvall, 1992; Perez, 2010). Following Metcalfe (1995), the NSI corresponds to the conjunction of institutions that contribute to the development and diffusion of new technologies and operates as a referential that government uses in order to formulate innovation policies.

[7] Technology influences science through several channels that include but are not limited to the formation of a research agenda, as empirical knowledge repository, and source of equipment and research instruments (Rosemberg, 1982). On the other hand, science influences technology as a source of technological opportunities and through labor market (Pavitt, 1991; Klevorick et al., 1995). Ribeiro et al. (2010) and Castellacci and Natera (2013) suggest that the channels connecting the scientific infrastructure and the technological production change in coevolution along the growth path.

From the combination of $TECH^*$ and $SCIE^*$ we obtain T_t that represents the technology conditions (or capabilities) of the economy, so $T_t = T(TECH^*;SCIE^*)$. The degree of technological development gathers the vector of capabilities of an economy and determines the trajectories that firms can chose in the migration process to more complex productive structures (Hidalgo et al., 2007; Hidalgo and Hausmann, 2011).

2.2. The aggregate demand curve

The accounting identity of aggregate demand for an open economy without government is given by:

$$Y_t \equiv C_t + I_t + XL_t$$

where Y_t corresponds to total output, C_t is consumption, I_t is investment, and XL_t corresponds to net exports. Dividing this expression by the capital stock in t we have:

$$\frac{Y_t}{K_t} = \frac{C_t}{K_t} + \frac{I_t}{K_t} + \frac{XL_t}{K_t} \tag{9}$$

Our economy has two social classes, namely, workers and entrepreneurs (or capitalists). Workers consume all their income and the entrepreneurs save part of their income. Total consumption is given by:

$$C_t = v_t L_t + c_K r_t K_t \tag{10}$$

where v_t corresponds to real wages, c_K is the propensity to consume of entrepreneurs, and r_t corresponds to the profit rate. Total wages are given by $v_t L_t$ while total profits are given by $r_t K_t$. Defining the wage-share as $\varpi_t = v_t L_t / Y_t$ we can rewrite Eq. (10) as:

$$C_t = \varpi_t Y_t + c_K (1 - \varpi_t) Y_t \tag{11}$$

Dividing by the capital stock we have:

$$\frac{C_t}{K_t} = [c_K + (1 - c_K)\varpi_t] u_t \tag{12}$$

where $u_t = Y_t/K_t$ and the capital technical coefficient was normalized to 1.

Investment in the Kaleckian tradition is represented as a linear function of the profit-share and the level of capacity utilization (Bhaduri and Marglin, 1990). But since the profit-share is the complementary of the wage-share we write:

$$\frac{I_t}{K_t} = \gamma_t - \gamma_1 \varpi_{t-1} + \gamma_2 u_{t-1} \tag{13}$$

where γ_t represents the "autonomous" investment component,[8] γ_1 captures the investment sensibility to wage-share variations, and γ_2 captures the sensibility of investment to variations in the level of capacity utilization. An increase in the wage-share reduces investment, while an increase in the level of capacity utilization always increases it.

Eq. (13) has two fundamental differences in relation to the usually employed in the Kaleckian growth literature. Investment is not a function of the profit-share but instead responds to the wage-share. This allows us to standardize the model as usually done by the literature of cycles that follows Goodwin (e.g. Goodwin, 1967; Keen, 1995; Barbosa and Taylor, 2006; Rezai, 2012; Sordi and Vercelli, 2014).

A second difference concerns the position of the variables on time. While consumption, for example, depends on the current functional distribution of income and the current level of capacity utilization, we considerer that investment depends on ϖ and u in $t-1$. The economic intuition for that is in the nature of investment. Since I is a crucial variable that links aggregate demand/supply and short/long run we consider that entrepreneurs planned the investment with a lag of one period. Therefore, investment in t results of a decision taken in $t-1$.

[8] Some times in the neo-kaleckian literature this component is associated with the Keynesian animal spirits. This is not the case here. It basically aggregates all components that determine investment and are not included in the function. One could think for example of public investment or the influence of variables as the exchange rate.

Net exports are modeled following Oreiro and Araújo (2013) and the properties described by Bhaduri and Marglin (1990) and Porcile and Lima (2013), so that:

$$\frac{XL_t}{K_t} = \xi_0 + \xi_1 \varepsilon_t - \xi_2 u_t + \xi_3 u_{t-1}^f \tag{14}$$

where ξ_0 is a constant, ε_t corresponds to real exchange rate and is exogenous, and ξ_1, ξ_2 and ξ_3 are sensibility parameters. Taking the Marshall-Lerner condition as granted, currency devaluation allows an increase in exports and a reduction of imports, increasing net exports. An increase in the domestic capacity utilization level increases imports reducing net exports. Finally, an increase in foreign capacity utilization increases net exports.

Eq. (14) presents an important difference in relation to the formulation usually employed in the literature. Net exports in t depends on the foreign capacity utilization level in $t-1$. The intuition for this formulation is that while the decision to import is immediate, the decision to export demands planning. The entrepreneurs look to the foreign level of capacity utilization in one period to decide if exports the next one.

Substituting Eqs. (12), (13) and (14) in (9) we obtain the aggregate demand as a proportion of the capital stock:

$$u_t = [c_K + (1 - c_K)\varpi_t]u_t + \gamma_t - \gamma_1 \varpi_{t-1} + \gamma_2 u_{t-1} + \xi_0 + \xi_1 \varepsilon_t - \xi_2 u_t + \xi_3 u_{t-1}^f \tag{15}$$

Rearranging the expression and isolating u_t we find the level of capacity utilization as a function of the wage-share and the capacity utilization of the last period:

$$u_t = \frac{\gamma_t + \xi_0 + \xi_1 \varepsilon_t - \gamma_1 \varpi_{t-1} + \gamma_2 u_{t-1} + \xi_3 u_{t-1}^f}{1 - c_K - (1 - c_K)\varpi_t + \xi_2} \tag{16}$$

Advancing Eq. (16) in one period and subtracting u_t from both sides:

$$u_{t+1} - u_t = \frac{\gamma_t + \xi_0 + \xi_1 \varepsilon_t - \gamma_1 \varpi_t + \gamma_2 u_t + \xi_3 u_t^f}{1 - c_K - (1 - \varpi_t) + \xi_2} - u_t \tag{17}$$

For mathematical convenience, the difference equation above can be approximated by a differential equation so we have $\dot{u} = u_{t+1} - u_t$. Calling $\Lambda_u = 1 - c_K - (1 - \varpi_t) + \xi_2$, as the inverse of the Keynesian multiplier, then:

$$\dot{u} = \alpha_0 + \alpha_1 u_t + \alpha_2 \varpi_t + \alpha_3 u_t^f \tag{18}$$

where $\alpha_0 = (\gamma_t + \xi_0 + \xi_1 \varepsilon_t)/\Lambda_u > 0$, $\alpha_1 = (\gamma_2 - \Lambda_u)/\Lambda_u < 0$, $\alpha_2 = -\gamma_1/\Lambda_u < 0$, and $\alpha_3 = \xi_3/\Lambda_u > 0$. Since the Keynesian multiplier is necessarily positive, $\Lambda_u > 0$. The Keynesian stability condition demands that $\alpha_1 < 0$, so $\gamma_2 - \Lambda_u < 0$. This means that the sensibility of investment to an increase of the level of capacity utilization has to be lower than the multiplier. It is important to notice that the Keynesian multiplier also depends on the functional distribution of income, $\partial \Lambda_u / \partial \varpi_t > 0$. However, for the sake of simplicity, we will take it as constant.

Eq. (18) represents our *aggregate demand curve*. Variations in the level of capacity utilization are a function of the income distribution and the domestic and foreign level of capacity utilization. Traditionally, aggregate demand curve is obtained through the difference between the desired and guaranteed growth rates (e.g. Bhaduri, 2008; Sasaki, 2013; Schoder, 2014). Our exercise proposes an alternative way to derive the problem through the aggregate demand fundamental identity.

2.3. The distributive curve

Models with distributive conflict try to explain inflation and distributive aspects juxtaposing the demands for increases in nominal wages and the price behavior in a way that workers and capitalists try to protect their income share (Rezai, 2012). In our model capitalists are responsible for fixing prices, while workers are responsible for changes in nominal wages. So that:

$$\frac{\dot{w}}{w_t} = \lambda_0 + \lambda_1 u_{t-1} + \lambda_2 \frac{\dot{q}}{q_t} + \lambda_3 \frac{\dot{p}}{p_t} \tag{19}$$

$$\frac{\dot{p}}{p_t} = \zeta_0 + \zeta_1 u_{t-1} - \zeta_2 \frac{\dot{q}}{q_t} + \zeta_3 \frac{\dot{w}}{w_t} \tag{20}$$

where $\dot{w}/w_t$ is the rate of change of nominal wages and $\dot{p}/p_t$ represents inflation. The parameters λ_1 and ζ_1 capture the sensibility of wages and inflation to changes in capacity utilization, respectively. Parameters λ_2 and ζ_2 represent the sensibility of wages and inflation to changes in labor productivity. An increase in productivity increases wages and reduces prices. Distributive conflict is represented by the capacity of workers to replenish inflation, weighted by coefficient λ_3, and the capacity of capitalists to replenish wage increases, weighted by coefficient ζ_3. Finally, λ_0 and ζ_0 are exogenous parameters that capture the other components of distributive conflict.

Traditionally both $\dot{w}/w_t$ and $\dot{p}/p_t$ are modeled as functions of the difference between the current functional distribution of income and the distribution desired by each social class. We do not agree with it. In fact, unions and capitalists are not conscious, explicitly or implicitly, of the level of income distribution. However, they do look directly to the level of capacity utilization, to the adjustment in prices and wages, and to productivity gains.

The relation between entrepreneurs and workers in the distributive conflict depends on the capacity of appropriation of the fruits of technical progress, i.e. the increases in labor productivity, for each group. Through the Kaldor-Verdoorn mechanism $\dot{q}/q_t = \Omega_0 + \Omega_1(\dot{K}/K)$. But since $\dot{K}/K = I/K$ and employing Eq. (13) we have that:

$$\frac{\dot{q}}{q_t} = \Omega_0 + \Omega_1(\gamma_t - \gamma_1\varpi_{t-1} + \gamma_2 u_{t-1}) \tag{21}$$

Substituting Eqs. (20) and (21) in (19)

$$\frac{\dot{w}}{w} = \frac{\Phi_1}{\Lambda_w} + \frac{\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\Omega_1\gamma_2}{\Lambda_w}u_{t-1} + \frac{(\lambda_3\zeta_2 - \lambda_2)\Omega_1\gamma_1}{\Lambda_w}\varpi_{t-1} \tag{22}$$

where $\Lambda_w = 1 - \lambda_3\zeta_3 > 0$ corresponds to the inverse of the wages multiplier and $\Phi_1 = \lambda_0 + (\lambda_2 - \lambda_3\zeta_2)(\Omega_1\gamma_t + \Omega_0) + \lambda_3\zeta_0$ is a constant term. Eq. (22) gives the nominal wages growth rate as a function of the level of capacity utilization and the wage-share.

Substituting Eq. (21) and (22) in (20) we obtain the inflation rate:

$$\frac{\dot{p}}{p} = \Phi_2 + \left\{\zeta_1 - \zeta_2\Omega_1\gamma_2 + \frac{\zeta_3\left[\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\,\Omega_1\gamma_2\right]}{\Lambda_w}\right\}u_{t-1} + \left[\frac{\zeta_3\Omega_1\gamma_1\,(\lambda_3\zeta_2 - \lambda_2)}{\Lambda_w} + \zeta_2\Omega_1\gamma_1\right]\varpi_{t-1} \tag{23}$$

where $\Phi_2 = \zeta_0 - \zeta_2(\Omega_1\gamma_t + \Omega_0) + \zeta_3(\Phi_1/\Lambda_w)$ is a constant term. Eq. (23) gives the inflation rate as a function of the level of capacity utilization and the wage-share.

We have defined the wage-share as $\varpi_t = v_t L_t/Y_t$. However, real wages are defined by the ratio between nominal wages and the price level. On the other hand, $L_t/Y_t = 1/q_t$. Therefore, in rate of changes:

$$\frac{\dot{\varpi}}{\varpi_t} = \frac{\dot{w}}{w_t} - \frac{\dot{p}}{p_t} - \frac{\dot{q}}{q_t} \tag{24}$$

Substituting Eqs. (21), (22) and (23) in (24) we have that:

$$\frac{\dot{\varpi}}{\varpi_t} = \frac{\Phi_1}{\Lambda_w} - \Phi_2 - (\Omega_1\gamma_t + \Omega_0) + \left\{\frac{(1-\zeta_3)\left[\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\,\Omega_1\gamma_2\right]}{\Lambda_w} - (1-\zeta_2)\Omega_1\gamma_2 - \zeta_1\right\}u_{t-1} + \frac{\Omega_1\gamma_1}{\Lambda_w}[(1-\zeta_3)(\lambda_3\zeta_2 - \lambda_2) + (1-\zeta_2)\Lambda_w]\varpi_{t-1} \tag{25}$$

Considering small intervals, Eq. (25) can be rewrite for mathematical convenience in continuous form as:

$$\dot{\varpi} = \varpi(\beta_0 + \beta_1 u_t + \beta_2\varpi_t) \tag{26}$$

where $\beta_0 = \Phi_1/\Lambda_w - \Phi_2 - (\Omega_1\gamma_t + \Omega_0) \lesseqgtr 0$, $\beta_1 = ((1-\zeta_3)[\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\Omega_1\gamma_2])/\Lambda_w - (1-\zeta_2)\Omega_1\gamma_2 - \zeta_1 \lesseqgtr 0$ corresponds to the sensibility of $\dot{\varpi}$ to changes in the level of capacity utilization, and $\beta_2 = \Omega_1\gamma_1/\Lambda_w[(1-\zeta_3)(\lambda_3\zeta_2 - \lambda_2) + (1-\zeta_2)\Lambda_w] \lesseqgtr 0$ is given by the sensibility of $\dot{\varpi}$ to changes in the distribution of income.

Eq. (26) represents our *distributive curve*. Variations in the wage-share are modeled as a function of the functional distribution of income and the level of capacity utilization. It is the result of the distributive conflict between capitalists and workers intermediate by the productivity regime of the economy.

2.4. The distributive system

Suppose the existence of two regions or countries.[9] The global distributive system is formed by the distributive and demand curves of each region:

$$\dot{u} = \alpha_0 + \alpha_1 u_t + \alpha_2 \varpi_t + \alpha_3 u_t^f$$

$$\dot{u}^f = \alpha_0^f + \alpha_1^f u_t^f + \alpha_2^f \varpi_t^f + \alpha_3^f u_t$$

$$\frac{\dot{\varpi}}{\varpi} = \beta_0 + \beta_1 u_t + \beta_2 \varpi_t$$

$$\frac{\dot{\varpi}^f}{\varpi^f} = \beta_0^f + \beta_1^f u_t^f + \beta_2^f \varpi_t^f$$

where the superscript *f* corresponds to the region consider "foreign". The level of capacity utilization and the wage-share of each region are determined jointly. We assumed that the variables that correspond to the foreign economy are exogenous and included then in the constant term. So, the distributive system can be represented by:

$$\begin{pmatrix} \dot{u} \\ \dot{\varpi}/\varpi \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix} \begin{pmatrix} u_t \\ \varpi_t \end{pmatrix} + \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix} \tag{27}$$

It is a system of differential equations 2×2 with a linear and a non-linear equation. In *steady-state* $\dot{\varpi} = \dot{u} = 0$. The solution with economic meaning is defined and given by:

$$u^* = \frac{\beta_2}{\beta_1}\left(\frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\alpha_1\beta_2 - \alpha_2\beta_1}\right) - \frac{\beta_0}{\beta_1} \tag{28}$$

$$\varpi^* = \frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\alpha_1\beta_2 - \alpha_2\beta_1} \tag{29}$$

To investigate the stability of the system, we linearized it around the fixed point and named it "implicit equilibrium":

$$\begin{pmatrix} \dot{u} \\ \dot{\varpi} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} u - u^* \\ \varpi - \varpi^* \end{pmatrix} \tag{30}$$

$$J_{11} = \alpha_1 < 0 \tag{31}$$

$$J_{12} = \alpha_2 < 0 \tag{32}$$

$$J_{21} = \beta_1 \varpi^* \lesseqgtr 0 \tag{33}$$

$$J_{22} = 2\beta_2\varpi^* + \beta_1 u^* + \beta_0 \lesseqgtr 0 \tag{34}$$

Assumption 1. The condition to topological equivalence between the systems is satisfied, that is, $\alpha_0\beta_1 \neq \alpha_1\beta_0$.

[9] The growth path of an economy describes the process of income creation inserted in a specific historic and institutional context. That means that the economies are structurally distinct between then. There are two ways to represent their differences. The first one rests on the assumption that is possible to model both economies using a unique model with distinct parameters. The second considers that we need a specific model for each economy. In this study we adopted the first strategy.

A system of non-linear differential equations, N, can me mapped by a linear equivalent, L, so that the qualitative properties of L in the neighbor of the critical point are similar of N in the same point. In this case we say that both systems are topological equivalent. It is required that the Jacobian matrix must be invertible (Shone, 2002). A sufficient condition for that is $\det(J) \neq 0$. After some algebraic manipulations we can show that it holds if $\alpha_0\beta_1 \neq \alpha_1\beta_0$.

Proposition 1. The economy is always *profit-led* in its cyclical dynamics.

Looking to the demand curve, we say that an economy is *wage-led* if in equilibrium an increase in the wage-share increases the level of capacity utilization. This will happen when $\alpha_2 > 0$. On the other hand, the economy will be *profit-led* if an increase in the wage-share reduces the level of capacity utilization, so that, $\alpha_2 < 0$.

However, since $\alpha_2 = -\gamma_1/\Lambda_u < 0$, the economy is always *profit-led*. The result is driven by two forces. First, because variations in capacity utilization, $\dot{u}$, depend fundamentally on investment behavior. According to our investment function, an increase in the wage-share always has a negative impact on investment. Second, because the Keynesian stability condition imposes that $\alpha_1 < 0$. Since both have a negative impact on $\dot{u}$, in equilibrium they go in opposite directions.

This result has important implications in terms of economic policy that we do not discuss here. The distinction between *profit-led* and *wage-led* growth is a major feature of Post-Keynesian economics and it has triggered an extensive econometric literature.[10] From a theoretical perspective, our results are in line with the original Goodwin literature and, as presented in the next section, dialogue with a particular interpretation given by Blecker (2015) to the Kaleckian dilemma. According to Blecker, a revision of the empirical studies in the *profit-led vs wage-led* controversies suggests that cyclically the economy is *profit-led* while its tendency is *wage-led*.

In the original growth-cycle model wages are the "predator" and employment the "prey". In our exercise, on the other hand, wages are still the "predator", but the "prey" is the level of capacity utilization. The intuition is that during the cycle a reduction in the wage-share allows an increase in investment which in turn implies in an increase in the employment and the capacity utilization of the economy. The increase in the level of capacity utilization brings an increase in wages and of the wage-share. Up to a certain point the increase in the wage-share reduces investment and consequently reduces employment and the level of capacity utilization. Wages then decrease and the cycle is repeated.

Proposition 2. If the distributive stability condition holds, that is, $\beta_2 \leq 0$, the system is stable as long as $\alpha_0\beta_1 > \alpha_1\beta_0$.

The stability condition based on Olech's Theorem imposes that $tr(J) < 0$ and $\det(J) > 0$. So we need to have:

$$tr(J) \equiv J_{11} + J_{22} = \alpha_1 + \beta_2\varpi^* < 0$$

$$\det(J) \equiv J_{11}J_{22} - J_{12}J_{21} = \alpha_0\beta_1 - \alpha_1\beta_0 > 0.$$

As a result of the Keynesian and the Distributive stability conditions α_1 and β_2 are negative, so $tr(J) < 0$. In addition, if $\alpha_0\beta_1 > \alpha_1\beta_0$ the second condition will always be satisfied.

Proposition 3. If the distributive stability condition holds whenever $(\alpha_1 + \beta_2\varpi^*)^2 < 4(\alpha_0\beta_1 - \alpha_1\beta_0)$ the critical point will be a node spiral asymptotically stable that in a sense approximates a Goodwin cycle.

In order to analyze this proposition we need to evaluate the nature of the eigenvalues. It depends on the relation between $tr(J)^2$ and $4\det(J)$. We will have a cyclical spiral if $tr(J)^2 < 4\det(J)$ since the eigenvalues will be imaginary. That means that we will have a spiral node as long as $(\alpha_1 + \beta_2\varpi^*)^2 < 4(\alpha_0\beta_1 - \alpha_1\beta_0)$.

Proposition 4. If the distributive stability condition holds and $\beta_0 < 0$, then a distributive adjustment *labor-market-led* is always stable and the *goods-market-led* adjustment is stable as long as $|\alpha_0\beta_1| < |\alpha_1\beta_0|$.

Following Rezai (2012) nomenclature, if an increase in the level of capacity utilization increases $\dot{\varpi}$ we say the economy is *labor-market-led*, so $\beta_1 > 0$. On the other hand the economy will be *goods-market-led* if an increase in the wage-share reduces $\dot{u}$, so that, $\beta_1 < 0$.

As result of the Keynesian stability condition $\alpha_1 > 0$. We also know that $\alpha_0 = (\gamma_t + \xi_0 + \xi_1\varepsilon_t)/\Lambda_u > 0$ and we are assuming $\beta_0 < 0$. So if $\beta_1 > 0$ then $\alpha_0\beta_1 > \alpha_1\beta_0$ is always true and the system is stable. On the other hand, if $\beta_1 > 0$ then $\alpha_0\beta_1 > \alpha_1\beta_0$ is true as long as $|\alpha_0\beta_1| < |\alpha_1\beta_0|$.

[10] For a review of the empirical and theoretical literature in the field see Palley (2014) and Blecker (2015).

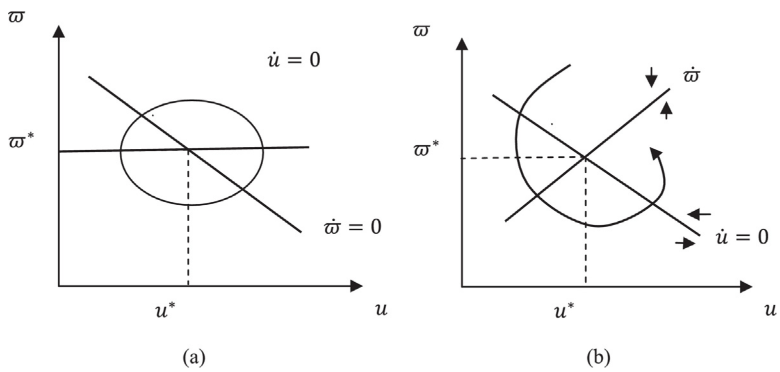


Fig. 1. The distributive system.

Proposition 5. If the distributive stability holds and $\beta_0 > 0$, then a distributive adjustment *labor-market-led* will be stable as long as $\alpha_0\beta_1 > \alpha_1\beta_0$ and the *goods-market-led* adjustment is always unstable.

As result of the Keynesian stability condition $\alpha_1 > 0$. We also know that $\alpha_0 = (\gamma_t + \xi_0 + \xi_1\varepsilon_t)/\Lambda_u > 0$ and we are assuming $\beta_0 > 0$. So if $\beta_1 > 0$, as long as $\alpha_0\beta_1 > \alpha_1\beta_0$ the system will be stable. On the other hand, if $\beta_1 < 0$ then $\alpha_0\beta_1 > \alpha_1\beta_0$ is never true and the system will be unstable.

Proposition 6. If the distributive stability condition does not hold and $\alpha_0\beta_1 > \alpha_1\beta_0$ we will have a periodic orbit *a la* Goodwin as long as $\alpha_1 = -\beta_2\varpi^*$.

The condition for the appearance of a periodic orbit (or a center) is that $tr(J) = 0$ and $\det(J) > 0$. As long as $\alpha_1 = \beta_2\varpi^*$ and $\alpha_0\beta_1 > \alpha_1\beta_0$ both conditions are satisfied.

Proposition 7. If the distributive stability condition does not hold and $\alpha_1 + \beta_2\varpi^* > 0$ the system will always be unstable.

In this case we have $tr(J) > 0$ and Olech's Theorem for stability is violated. An increase in the wage-share implies in an increase in $\dot{\varpi}$ which increase the wage-share again. The process is explosive.

Fig. 1 shows the phase portrait of the two cases that generate cyclical motions *a la Goodwin*. Diagram 1a represents the "Periodic Orbit" case while Diagram 1b represents the "Spiral node" case in a *labor-market-led* economy.

Given the values of u^* and ϖ^* determined by Eqs. (28) and (29) we can find the labor productivity, wages, prices, and employment growth rates. For that we substitute (28) and (29) in (21), (22), (3) and (4) respectively. That give us $\dot{q}^*/q(u^*;\varpi^*)$, $\dot{w}^*/w(u^*;\varpi^*)$, $\dot{p}^*/p(u^*;\varpi^*)$ and $\dot{L}^*/L(u^*;\varpi^*)$.

2.5. The balance of payments constraint

According to Thirlwall's law the BoPC growth rate is given by:

$$y^{BP} = \frac{\varphi}{\rho} y_f \tag{35}$$

where y^{BP} is the balance-of-payments-constraint growth rate, that is, the growth rate allowed by the aggregate demand constraint.[11] Finally y_f is the growth rate of the foreign region.

Looking to advance in the study of the determinants of the foreign trade elasticities ratio we must address the impact of technological capabilities and income distribution on it. The hypothesis that there is a positive relation between technological capabilities and non-price competitiveness (φ/ρ) is strongly supported theoretically and empirically.[12]

[11] Long run growth is directly proportional to the product between the foreign income growth and the ratio between the income elasticities of exports and imports. Growth is balance-of-payments-constrained in the sense that there is a limit of supply currency that the economy can count to satisfy its needs to import. The higher the ratio between the foreign trade elasticities the lower would be the BP constraint.

[12] For a review about the recent literature in this matter see Ribeiro et al. (2016).

On the other hand, the relation between inequality and the foreign trade elasticities is not evident. Structuralist authors like Furtado (1968) and Tavares and Serra (1976) argue that high levels of income inequality in Latin America led to significant differences in consumption patterns between the lower and upper classes. Upper classes demand superfluous and highly technological products that, as result of its small scale, were incapable to induce domestic production. In this sense Bohman and Nilsson (2007) and Dalgin et al. (2008) conclude that, given non-homothetic preferences, more unequal countries tend to export relatively more necessity goods and import more luxury goods.[13] Therefore we should expect that a better income distribution improves non-price competitiveness.

However, one may argue that since workers have a higher propensity to consume, a higher wage-share would lead to greater spending on imports leading to a lower ratio of trade elasticities. Moreover, an increase in the wage share implies in an increase of labor unit costs, making a large number of goods be no profitable produced in the country and therefore reducing non-price competitiveness (Oreiro, 2016).

Controversy also comes from the indirect influence that income distribution may have on the foreign trade elasticities through technology. Acemoglu et al. (2012), for instance, focusing on industrialized economies claim that income inequality is required in order to stimulate innovation. Innovation itself can temporarily raise inequality if innovators dispose of quasi rents (Cozzens, 2008).

Still, Weinhold and Nair-Reichert (2009) analyzed a longer sample of 53 developed and developing countries between 1994 and 2000. They conclude that a more equitable income distribution seems to be positive correlated to innovation via its positive effects on the functioning of domestic institutions. Similar results are provided by Hopkin et al. (2014) that consider that more equitable systems like the Scandinavian economies perform better that U.S. in terms of innovation.[14]

We will follow the assertive that "*statistical evidence generally supports the view that inequality impedes growth [. . .]*" (Ostry et al., 2014). This is not the same as to assume a positive relation between φ/ρ and ϖ (even though the relation exists, as we will show in a non-linear fashion). We suggest that non-price competitiveness change while technological conditions evolve given a wage-share.

Thirlwall (1997) and Setterfield (1997) argued that the elasticity of exports growths as the country moves from the production of primary products to manufactures and decreases when the economy get lock in antiquate industrial structures. As a result we should observe an inverted U relationship. On the other hand, McCombie and Roberts (2002) consider that is the ratio between the foreign trade elasticities that present the inverted U relation. While low growth rates generate pressures to an increase in the elasticities ratio, high growth rates would encourage the lock-in of the productive structure.

In order to capture these insights and specifically the inverted U relation, our approach focuses on the trajectory of the elasticities as the domestic technological conditions evolves due to a logistic function. Some important properties arise from the interaction between T, ϖ and (φ/ρ):

$$\left(\frac{\varphi}{\rho}\right)_{T+1} = G\left[\left(\frac{\varphi}{\rho}\right)_T;\varpi\right] = \varpi\left(\frac{\varphi}{\rho}\right)_T\left[z-\left(\frac{\varphi}{\rho}\right)_T\right] \tag{36}$$

where z corresponds to a technological variable that captures knowledge globally available. The difference equation above takes the ratio between the foreign trade elasticities in $T+1$ as a function of the wage-share and the elasticities ratio itself in T. Notice that here T represent the technological conditions of the economy. The productive structure, represented by the foreign trade elasticities ratio, follows an inverted U associated with the distributive and technological conditions.

In the long run, $(\varphi/\rho)_{T+1} = (\varphi/\rho)_T = (\varphi/\rho)^*$. We have two possible solutions, namely, (i) $(\varphi/\rho)^* = 0$ e (ii) $(\varphi/\rho)^* = (\varpi z - 1)/\varpi$. Applying Taylor's polynomial to Eq. (36) we have:

$$\left(\frac{\varphi}{\rho}\right)_{T+1} = G\left[\left(\frac{\varphi}{\rho}\right)^*;\varpi\right] + \frac{\partial G}{\partial\left(\frac{\varphi}{\rho}\right)^*}\left[\left(\frac{\varphi}{\rho}\right)_T - \left(\frac{\varphi}{\rho}\right)^*\right] \tag{37}$$

[13] Engel's law states that, as income grows, consumers tend to substitute necessity goods by luxury goods, where the latter have income elasticity of demand greater than unity and the first have income elasticity of demand less than unity. Here, non-homothetic preferences basically mean that the proportion of income that consumers spend on luxury and necessity goods varies as income increases (Ribeiro et al., 2016).

[14] For a review about the recent literature in this matter see Weinhold and Nair-Reichert (2009) and Botta (2015).

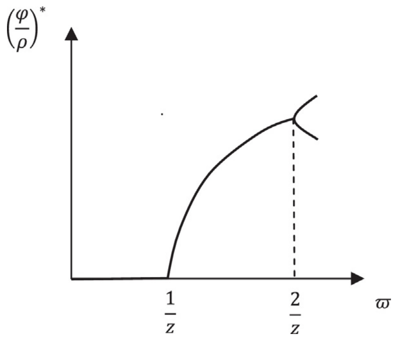


Fig. 2. Foreign trade elasticities and the functional income distribution.

But we know that $G[(\varphi/\rho)^*;\varpi] = (\varphi/\rho)^*$ and we can show that $\partial G/\partial(\varphi/\rho)^* = (2 - \varpi z)$. Substituting both values in (45) and rearranging the terms:

$$\left[\left(\frac{\varphi}{\rho}\right)_{T+1} - \left(\frac{\varphi}{\rho}\right)^*\right] = (2 - \varpi z)\left[\left(\frac{\varphi}{\rho}\right)_T - \left(\frac{\varphi}{\rho}\right)^*\right] \tag{38}$$

Eq. (38) can be rewrite as a simple difference equation in the format $l_{T+1} = (2 - \varpi z)l_T$. The stability condition demands that $0 < 2 - \varpi z < 1$. That implies $1/z < \varpi < 2/z$. Fig. 2 represents $(\varphi/\rho)^*$ as a function of ϖ:

Proposition 8. The points $\varpi = 1/z$ and $\varpi = 2/z$ are bifurcation points where we observe qualitative changes in the behavior of the objective function.

For $\varpi < 1/z$, that is, with a sufficient low wage-share, the productive structure will be in *lock-in*. Technology in this case is not capable to increase the foreign trade elasticities ratio and consequently cannot relieve the external constraint. For $\varpi < 1/z$, a better income distribution favoring wages allows a relief of the external constraint while T evolves. However, for $\varpi = 2/z$ the function presents chaotic behavior.

There is a security band for ϖ linked to the global technological conditions. It is quite reasonable to assume that a wage-share too high reduces the investment capacity of entrepreneurs compromising long run growth. At the same time a wage-share too low can conduct the economy to a "demand trap".

Proposition 9. The economy is always *wage-led* in its BoPC dynamics.

Outside the *lock in* case we have that $(\varphi/\rho)^* = (\varpi z - 1)/\varpi$ and $\partial(\varphi/\rho)^*/\partial\varpi > 0$. An increase in the *wage-share* allows an increase in the BoPC growth rate. Therefore the economy is in a sense *wage-led* in its BoPC dynamics.

We shall notice that while cyclically the accumulation regime is always *profit-led*, in the long run the economy is in a sense always *wage-led*, as the ratio between the foreign trade elasticities is a positive function of the *wage-share*. This is in line with Blecker (2015) that shows that empirical evidence focus in the cycle dynamics usually finds *profit-led* results, while works that focus in aggregate demand find *wage-led* results.

2.6. Growth and fluctuations: when the short-term meets the long run

The model presented gives us two growth rates: (i) the capital accumulation growth rate and (ii) the balance-of-payments-constraint growth rate. Whereas both of them are given quite independently, they can be equal just for coincidence.[15] So we have:

$$y = \gamma_t + \gamma_1\varpi^* + \gamma_2 u^* \tag{39}$$

$$y^{BP} = \frac{\varphi}{\rho} y_f \tag{40}$$

[15] The foreign trade elasticities are a function of the wage-share, so it is not strictly correct to say that both growth rates are independently. However, we consider here that the elasticities change slowly and depend more on the evolution of technology given a wage-share that of the wage-share itself.

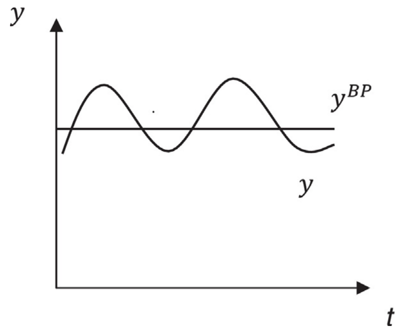


Fig. 3. Adjust mechanism between y_{BP} and y.

We need them to explicitly specify an adjustment mechanism between y and y^{BP} in order to avoid the over determination problem.

Considering that there is plentiful empirical evidence supporting that in the long run growth is balance of payments constrained, the follow dynamic is proposed. If $y > y^{BP}$, capital accumulation exceeds the BoPC and capitalists are forced to reduce y in order to guarantee the BoP equilibrium. On the other hand, if $y < y^{BP}$ there is space to expand accumulation in order to approximate y to y^{BP}. Since economic agents are immersed in an environment of fundamental uncertainty in both cases their calculations are subjective.

While the first derivative of y in t corresponds to variations in the rhythm of capital accumulation in time, the second derivative corresponds to the intensity of those variations. Put another way, $\ddot{y}$ captures the intensity of the adjustment of y on time. We propose that the intensity of the adjustment is a linear function of the difference between y and y^{BP}, that is:

$$\ddot{y} = j_t(y^{BP} - y) \tag{41}$$

Where $j_t > 0$ is an exogenous variable that captures the subjective perception of the necessity of adjustment. Eq. (41) shows that the higher the difference between y^{BP} and y the higher the intensity of the adjustment will be.

One particular solution of the differential equation above is[16]:

$$y = y^{BP} + sen(t\sqrt{j}) \tag{42}$$

Fig. 3 represents the mechanism so far described.

When $y > y^{BP}$, the capital accumulation growth rate exceeds the external constraint, there has to be an adjustment in investment. In our model it happens through a reduction of γ_t. This could be due to a current account crisis, which forces the government and capitalists to cut investments. Since is a decentralized decision and subject to fundamental uncertainty, y will be reduced to the point that $y < y^{BP}$. In some moment and depending on the value of j_t, capital accumulation starts raising through an increase in γ_t. It is important to notice that changes in γ_t imply in a constant movement of the "implicitly solution". This story corresponds basically to a current account crisis mechanism.

Proposition 10. The capitalist system is inherently unstable as result of the interaction between the short-term and long-term dynamics–reflected in y and y^{BP} – that generate continuous fluctuations through "autonomous" investment, γ_t.

The appropriate way to treat the problem would be include $\gamma_t = (\varphi/\rho)y_f + sen(t\sqrt{j_t}) + \gamma_1\varpi_t - \gamma_2 u_t$ in the distributive system and proceed to the dynamic analysis. Doing that we will fully integrated cycle and tendency and there will be no more distinction between short/long run. However, this leads us to a non-autonomous non-linear dynamic system. We avoid this road even though recognize that further research has to be done in order to enrich and complement the exercise.

[16] The complete solution is given by $y = y^{BP} + c_1 sen(t\sqrt{j}) + c_2 \cos(t\sqrt{j})$. For simplification we consider the case when $c_1 = 1$ and $c_2 = 0$.

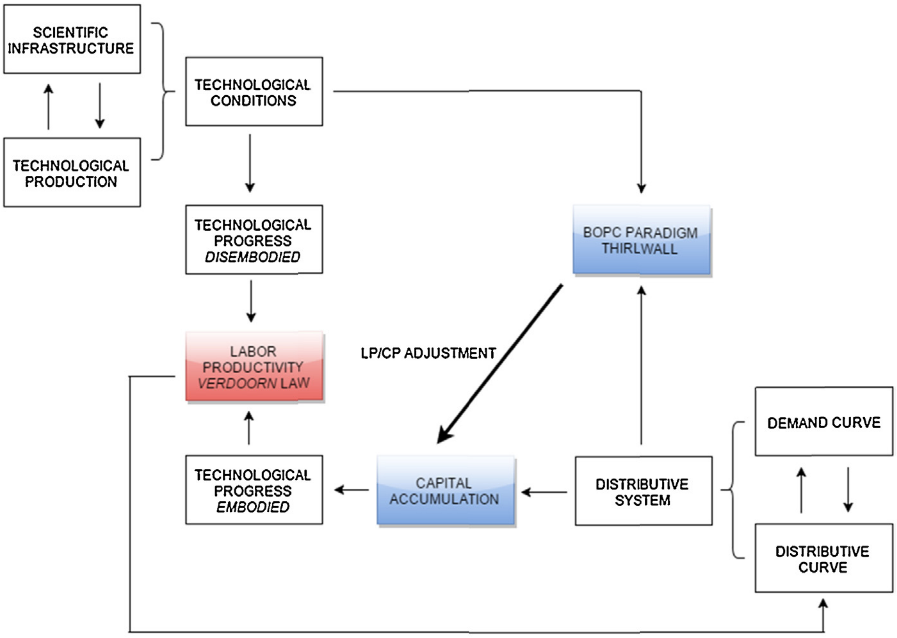


Fig. 4. A synthetic diagram of the model.

Proposition 11. If the conditions establish by Proposition 6 are fulfilled, we will have two endogenous sources of instability. First, the possibility of a periodic orbit between ϖ^* and u^* that generates a cyclical accumulation path. Second, the interaction between y and y^{BP} that also will generate continuous and permanent fluctuations.

Even though is not possible to eliminate the cyclical fluctuations, the model suggests that a development strategy depends on technological and distributive variables. On the one hand, the strengthening of the scientific and technological capabilities relieves the external constraint on growth and allows a higher rate of capital accumulation. In parallel we recommend a better distribution between capital and labor in order to obtain smooth distributive conflict and avoid distributive growth traps.

There are four key variables that, immersed in a determined institutional context, form the system: (i) Scientific Infrastructure, *SCIEN*; (ii) Technological Production, *TECH*; (iii) Wage-share, ϖ and (iv) Level of capacity utilization, u.

Technological progress expresses itself through increases in labor productivity. However, since part of it cannot be dissociate from the capital accumulation process, we have that capital accumulation itself influences positively labor productivity. Departing from our investment function it depends directly on ϖ and u. Moreover, since growth is balanced-of-payments-constrained it depends indirectly on the *SCIEN* and *TECH*.

From a macroeconomic perspective we suggest that growth and labor productivity are consolidated from two large blocks that interact with each other. The first one appears from the direct link between science and technology (that is, its technological capabilities) that influences the external constraint. The second appears from the indirect link between the functional distribution of income, the level of capacity utilization and the external constraint. The ultimate expression of growth and technological progress is the increase in labor productivity. Fig. 4 presents a synthetic diagram of the model.

3. Conclusion

This paper builds a dynamic *KG* model in order to study the relation between functional distribution of income, technological progress and economic growth. In the short run, the distributive conflict between capital and labor interacts with the productivity and demand regimes generating cyclical paths *a la* Goodwin. In the long run, the elasticities of foreign trade were modeled as a function of the technological conditions and the wage-share, so that economic growth is balance-of-payments-constrained.

The model proposes a way in which *lock in* traps could appear associated with high levels of inequality. It allows us to better understand the relation between distributive, technological variables and growth, combining elements of Marxist, Kaleckian, Goodwin and Kaldorian traditions. The main element that unifies this "*strange conjunction of stars"* is the conception of the capitalist economy as a structurally unstable system.

The distributive cycle dynamics has been used as departure point to study different aspects of the capitalist economies. However, to the best of our knowledge, all those exercises concern closed economies. Our model is a first attempt to show that a marriage between Goodwin and Thirlwall may be possible in an open economy framework.

We consider that the main limitation of our effort is its failure to fully integrate growth and cycle from a unique dynamic system. A complete integration would allow long and short-run to interact permanently, with the external constraint and the distributive cycles being determined together. This development involves some mathematical complications that were avoided but at the same time would enrich the exercise. The adjustment mechanism proposed leads us to a non-autonomous system of differential equations that we do not solve here.

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