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Márcia Luciana da Costa Peixoto

Static Output-Feedback Control Design for Nonlinear Systems - Polytopic Based Approaches

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A thesis presented to the Graduate Program in Electrical Engineering (PPGEE) of the Federal University of Minas Gerais (UFMG) in partial fulfillment of the requirements to obtain the degree of Doctor in Electrical Engineering.

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"STATIC OUTPUT-FEEDBACK CONTROL DESIGN FOR NONLINEAR SYSTEMS - POLYTOPIC BASED APPROACHES"

MÁRCIA LUCIANA DA COSTA PEIXOTO

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Prof. Dr. Fabrício Gonzalez Nogueira - UFC

Profa. Dra. Grace Silva Deaecto - UNICAMP

Prof. Dr. João Yoshiyuki Ishihara - UNB

Profa. Dra. Vilma Alves de Oliveira - USP/SC

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Márcia Luciana da Costa Peixoto

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RESUMO

Nas últimas décadas, sistemas politópicos como os sistemas lineares com parâmetros variantes no tempo (LPV, do inglês *Linear Parameter-Varying*) e os modelos *fuzzy* Takagi-Sugeno (TS) têm sido amplamente utilizados para representar uma grande classe de modelos não lineares. Além disso, a teoria de Lyapunov vem sendo utilizada com sucesso para desenvolver condições eficientes para análise de estabilidade e projetar controladores para estes sistemas politópicos, geralmente expressas por meio de desigualdades de matriciais lineares (LMIs, do inglês *Linear Matrix Inequalities*). Entre as estratégias de estabilização para sistemas não lineares, o problema de projeto de controle por realimentação estática de saída é conhecido por ser de difícil resolução e tem recebido grande atenção nos últimos anos. No entanto, o controle por realimentação estática de saída permanece sendo um dos tópicos mais desafiadores na teoria de controle. Com base nisso, esta tese aborda dois tópicos principais. i) Novas condições de síntese para o projeto de controladores via realimentação estática de saída para sistemas não lineares discretos no tempo representados por sistemas politópicos. ii) Novas condições dependentes do retardo no tempo para o projeto de controladores por meio de realimentação estática de saída para sistemas não lineares com retardo variante no tempo também representados por sistemas politópicos. Uma característica dos métodos propostos, ao contrário da maioria das técnicas presentes na literatura, é que nenhuma restrição estrutural na matriz de saída é imposta, ou seja, as abordagens propostas podem lidar com variações nas matrizes da dinâmica, da entrada e da saída sem recorrer a transformações de similaridade ou procedimentos iterativos. Ao contrário de outros trabalhos da literatura, outra característica distintiva das abordagens propostas é a estabilidade assintótica local da origem do sistema em malha fechada, que se faz necessária devido à validade do modelo politópico obtido. Isso garante o correto funcionamento do sistema em malha fechada, pois suas trajetórias permanecem dentro da estimativa da região de atração obtida dentro do domínio de validade dos sistemas politópicos. Exemplos numéricos ilustram o potencial e a eficácia das condições propostas. Adicionalmente, outros trabalhos que vêm sendo desenvolvidos durante o doutorado são brevemente apresentados, sendo estes: i) novas condições de estabilidade e estabilização para sistemas LPV utilizando diferentes tipos de funções candidatas de Lyapunov, ii) uma nova abordagem de controle em rede com acionamento por eventos para sistemas sujeitos a ataques cibernéticos estocásticos assim como retardos no tempo induzidos pela rede de comunicação; iii) um novo método para a estimativa de falhas para sistemas não lineares sujeitos a retardos variantes no tempo e a não linearidades não mensuradas.

Palavras-chave: Realimentação estática de saída. Sistemas não lineares. Retardos variantes no tempo. Sistemas lineares com parâmetros variantes no tempo. Desigualdades matriciais lineares.

ABSTRACT

Over the past few decades, polytopic systems such as linear parameter-varying (LPV) and Takagi-Sugeno (TS) fuzzy models have been widely employed to represent a large class of nonlinear systems. The Lyapunov theory has been successfully used to develop efficient conditions for stability analysis and support the design of stabilization controls for polytopic systems, usually expressed through Linear Matrix Inequalities (LMIs). Among stabilization strategies for LPV and TS fuzzy systems, the static output-feedback (SOF) control design problem is known to be harder to solve and has received a lot of attention in the past years. However, SOF control remains one of the most challenging topics in control theory. Based on that, this Thesis addresses two main topics. i) New synthesis conditions for gain-scheduling static output-feedback control of discrete-time nonlinear systems represented by polytopic systems. ii) A novel delay-dependent condition for static output-feedback control of nonlinear systems represented by polytopic systems with time-varying delay. One feature of the proposed methods, unlike most approaches in the literature, is that no structural constraints on the output matrix are imposed, that is, the proposed approaches can handle variation in the dynamics, input, and output matrices without resorting to similarity transformations or iterative procedures. Unlike other works in the related literature, another distinctive feature of the proposed approaches is to ensure the local asymptotic stability of the origin of the closed-loop system, which is necessary due to the validity of the polytopic model obtained. This guarantees the correct operation of the closed-loop system since its trajectories remain inside the guaranteed region of attraction estimation obtained within the validity region of the polytopic systems. Numerical examples illustrate the potential and effectiveness of the proposed conditions. Additionally, further works that have been developed along the Ph.D. are briefly presented: i) novel stability and stabilization conditions for discrete-time LPV systems employing different kinds of Lyapunov functions, ii) results for the problem of periodic event-triggered control co-design for polytopic systems subject to stochastic deception attacks iii) a fault estimation method for a class of nonlinear parameter-varying systems subject to time-varying delay and unmeasured nonlinearities.

Keywords: Static output-feedback control. Nonlinear systems. Time-varying delays. Linear parameter varying systems. Linear matrix inequalities.

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LIST OF ABBREVIATIONS

DoA	Domain of attraction
ETC	Event-triggered control
ETM	Event-triggering mechanism
FTC	Fault-tolerant control
LKF	Lyapunov-Krasovskii functional
LMI	Linear Matrix Inequality
LPV	Linear Parameter-Varying
LTI	Linear Time-Invariant
NCS	Networked control systems
SOF	Static output-feedback
TS	Takagi-Sugeno

LIST OF SYMBOLS

$\mathbb{R}$	The set of real numbers
$\mathbb{R}_{\geq 0\ (>0)}$	The set of non-negative (positive) real numbers
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{m \times n}$	The set of real matrices of order m by n
$\mathbb{N}$	The set of natural numbers, $\mathbb{N} = \{1, 2, \ldots\}$
$\mathbb{N}_{\leq m}$	The set $\{1, 2, \ldots, k\}$ for a given $m \in \mathbb{N}$
$\mathbb{N}_0$	The set $\{0\} \cup \mathbb{N}$
$\mathbb{Z}^+$	The set of positive integers
$\mathcal{C}^n$	Space of the functions mapping the set $\{-\overline{h}, -\overline{h}+1, \ldots, 0\}$ into $\mathbb{R}^n$
$\mathbb{S}^n_+$	The set of $n \times n$ symmetric positive definite matrices
$\lambda_{\max}(X)$	The maximum eigenvalue of a symmetric matrix X
$x^\top$ or $X^\top$	Transpose of a vector x or a matrix X
$\|x\|$	Euclidean norm of a vector x
$\vert x \vert$	Absolute value of a real number x
$\star$	Stands for symmetric blocks in matrices
$\mathrm{He}\{X\}$	Shorthand notation for $X + X^\top$, where X is any matrix
$X > 0\ (\geq 0)$	X is a symmetric positive (semi-)definite matrix
$\mathrm{diag}(X_1, \ldots, X_n)$	The diagonal matrix whose entries are $X_1, \ldots, X_n$, $n \in \mathbb{N}$
I_n	Identity matrix of order n
$0_{n,m},\ 0_n$	Null matrix of order n by m, $0_{n \times n}$

CONTENTS

1 INTRODUCTION

Nonlinear systems play a fundamental role in control systems from an engineering point of view. This is because, in practice, most plants are nonlinear in nature. However, there is no general control methodology for nonlinear systems but only control techniques aimed at specific classes of systems [1]. Nevertheless, it is possible to approximate nonlinear systems by different classes of systems. Figure 1.1 depicts the main approximations for nonlinear systems used in the context of robust control, namely Linear Time-Invariant (LTI) systems, uncertain LTI systems, Takagi-Sugeno (TS) fuzzy models, Linear Parameter-Varying (LPV) systems, and quasi-LPV systems. One of the simplest methods to represent nonlinear systems is to linearize the system around an operating point, in which case, a LTI system is obtained. However, the linearized system is only valid in the region of the operating point. On the other hand, if nonlinearities are modeled as uncertainties, an uncertain LTI system can be obtained.

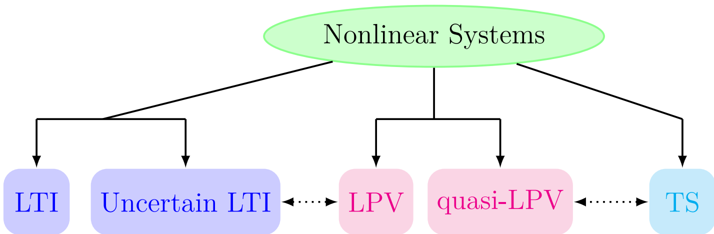

Figure 1.1 – Systems representation.

Among the different ways of representing nonlinear systems, there are TS fuzzy models with the aid of fuzzy sets, fuzzy rules, and a set of local linear models [2, 3, 4, 5, 6, 7]. The model is obtained by merging the local models through fuzzy membership functions that usually depend on the system states. For nonlinear TS fuzzy models, that is, TS fuzzy models with a nonlinear consequent, there are two main approaches to represent nonlinear systems, the first one consists of using TS fuzzy systems with polynomial consequent [8, 9], and the other one, sector-bounded functions are added to the TS fuzzy model to obtain the nonlinear consequent [10, 11, 12, 13, 14, 15]. In the case of Linear Parameter-Varying (LPV) systems, the nonlinearity is embedded in the time-varying parameters that depend on some endogenous signals [16, 17]. Often, uncertain LTI systems can be seen as a particular case of LPV systems. Besides, to make an additional distinction concerning pure LPV systems, if the time-varying parameters depend on some system states (similar to membership functions in TS fuzzy models), the system is referred to as a quasi-LPV system [18, 19, 20]. Notwithstanding, nonlinear parameter-varying systems can preserve a nonlinear structure instead of reducing it to a purely LPV system [21, 22, 23, 24].

The main advantage of considering LPV systems or TS fuzzy models is that, based on the Lyapunov stability theory [1], conditions for stability analysis and controller or observer design can be formulated as convex optimization problems given by Linear Matrix Inequalities (LMIs). Although many cases in the literature related to the control design of TS fuzzy models and LPV systems assume that the states of the systems are fully available, in numerous situations, some of the system states are not available for measurement, or the required sensors are too expensive, making the use of state-feedback controllers impractical. An alternative to cope with this inconvenience is considering only measured states in the control scheme. This is the foundation of output feedback control, which can be categorized into dynamic or static. In the first case, the controller has an associate (full-order) dynamics designed to stabilize the system using the measured outputs [25, 26, 27]. In contrast, static output feedback (SOF) controllers have a simpler structure that can be useful for implementations with lower costs. Based on that, techniques based on SOF control have received considerable attention. However, SOF control remains one of the most challenging topics in control theory, even for linear time-invariant systems [28]. One of the principal reasons for the static output-feedback design to be so difficult to deal with is due to its non-convex characterization [29]. Nonetheless, several methods have been developed intended to provide numerically tractable solutions for SOF designs.

In [30, 31], to deal with the SOF problem for linear continuous-time systems, a set of LMIs connected by the constraint that one Lyapunov matrix is the inverse of another one has been considered and a min/max algorithm has been employed to iterate between solving each of the Lyapunov inequalities until one of the resulting Lyapunov matrices is approximately the inverse of the other. A modified version of the min/max algorithm proposed in [31] to design SOF controllers bounded by a given linear quadratic performance index has been proposed in [32]. In [33], a SOF condition with matrix-equality constraint for linear discrete-time periodic systems has been proposed considering that the output matrix has full rank. In [34], the SOF problem for a class of linear continuous-time systems has been solved by applying a congruence transformation and by imposing a block-diagonal structure on the Lyapunov matrix. In [35], a sufficient SOF condition has been obtained considering that the Lyapunov matrix has a special structure. In [36], an algorithm that solves an optimal SOF problem for linear systems subject to convex gain constraints and ensures monotonic convergence to a local minimum has been provided. The cone complementary formulation has also been used to derive SOF control design conditions for linear continuous-time systems [37, 38], linear discrete-time systems [39, 40, 41, 42, 43], Takagi-Sugeno fuzzy models [44, 45, 46, 47]. Conditions for the existence of SOF solutions requiring a similarity transformation for the convexification procedure have been presented in [48, 49].

In [25], an LMI-based technique of designing robust SOF controllers for linear systems with time-invariant uncertainties has been proposed based on introducing a parameter-independent slack variable with a lower-triangular structure. However, the robust design requires the system output matrix be fixed, i.e., without uncertainties. In [50, 51, 52], these constraints have been relieved using the null space properties of output matrices and introducing slack variables with a lower-triangular structure. In [53], it is presented a condition for SOF control of discrete-time LPV systems with a prescribed bound on the rate of variation, and it depends on the constraint that the output matrix cannot be affected by the time-varying parameter. SOF conditions requiring matrix-equality constraints for the convexification procedure have been provided in [54, 55]. To reduce conservativeness, a descriptor redundancy to provide a SOF controller for Takagi-Sugeno fuzzy models has been considered in [56]. This strategy has been employed later in [57] for the discrete-time case to derive LMI conditions with extra decision variables introduced by the application of Finsler's lemma. In [58], a technique based on a line search for a robust SOF controller design has been proposed. In [59], a condition has been stated for the output-feedback $\mathcal{H}_\infty$ control design of linear discrete-time systems in which it is assumed that the system input matrix is of full column rank and an additional positive definite matrix is introduced. Robust SOF control $\mathcal{H}_\infty$ has been studied in [60] for uncertain linear systems and in [61] for TS fuzzy systems.

Numerical algorithms have also been used to design SOF controllers. A two-step method to deal with the SOF problem has been introduced in [62] for linear discrete-time systems. The approach is initiated by the state-feedback controller design, and that state-feedback gain is employed to determine the SOF control gain. Thereafter, the two-step approach has been employed to handle the SOF problem for linear continuous-time systems [63, 64], uncertain linear discrete-time systems [65], and LPV systems [66, 67, 68, 69]. A design procedure in terms of sequentially solving three parameter-dependent LMIs optimization problems has been presented in [70] for LPV systems. Other methods based on iterative algorithms have been proposed for linear systems [71, 72, 73, 74, 75], polytopic systems [76, 77, 78, 79], LPV systems [80, 81, 82], TS fuzzy models [83, 84], uncertain polynomial systems [85], and uncertain rational nonlinear systems [86].

Recently, in [87], a procedure has been proposed to adapt the conditions presented in [88] for state-feedback control design to deal with the output-feedback case. A given matrix is introduced to the problem to make dimension adjustments, enabling a linearization procedure of the inequalities associated with the output-feedback controller design. However, the output-feedback control design is based on a scalar parameter search, which is cited as a disadvantage by the authors since the computational burden increases. More recently, [89] proposed a linear SOF controller for discrete-time LPV systems based on augmenting the matrices related to the control input and control gain. In [90], a design method for gain-scheduled SOF controllers of saturated continuous-time LPV systems based on specific congruence transformations has been introduced. Another recent framework that has been studied for reducing design conservativeness in the SOF control design is the delayed approach. It is based on including past membership functions in both fuzzy controller and fuzzy Lyapunov function. This approach was considered by [91, 92] to design a delayed SOF controller for TS fuzzy models. Table 1.1 summarizes the main works presented above and the convexification procedure used to solve the SOF problem. Motivated by the aforementioned discussion, the first objective of this Thesis is:

(i) to propose a novel LMI-based SOF control conditions for discrete-time nonlinear systems represented by LPV systems and TS fuzzy models without requiring some kind of procedure to provide numerical tractable solutions.

Table 1.1 – Main constraints for the convexification procedure: I) full rank, II) matrix-equality constraint, III) Specific structure on Lyapunov matrix or slack variables, IV) Congruence or state coordinate transformation V) Two-stage approach, VI) Iterative algorithms, VII) cone complementary algorithm.

	LTI systems	Uncertain LTI systems	LPV systems	TS fuzzy model
I	[31, 32, 33, 34, 49, 59]	[35, 37, 50, 77]	[51, 55]	[44, 48]
II	[33]		[55]	[54]
III	[34]	[35, 25, 50]	[51, 53]	[52, 57]
IV	[49]	[58, 60]	-	[48]
V	[63, 64]	[62, 65]	[66, 67, 68, 69]	-
VI	[30, 31, 32, 71, 72] [73, 74, 75]	[77, 76, 78, 79]	[70, 80, 81, 82]	[83, 84]
VII	[38, 39, 42, 43]	[37]	-	[40, 41, 44] [45, 46, 47]

Based on the sector-nonlinearity approach, quasi-LPV systems and TS fuzzy models provide exact representations of nonlinear dynamical systems inside of a compact set $\mathcal{D} \subset \mathbb{R}^{n_x}$. Therefore, when a fuzzy or gain-scheduled controller is designed based on the TS or quasi-LPV representations, the closed-loop stability guarantees hold locally due to the validity of the model. For this reason, it is necessary to estimate the set of admissible initial conditions for which the state trajectories converge to the equilibrium point of the closed-loop system. It is worth mentioning that the methods [52, 44, 45, 48, 51, 53, 54, 57, 66, 67, 68, 69, 70, 81, 82, 83, 84, 87, 89, 92] which deal with the SOF control synthesis for LPV systems or TS fuzzy models do not consider the state constraints. That is, these approaches cannot deal with the case of local premise variables or scheduling parameters that depend on the systems' states. Therefore, the second motivation of this Thesis is:

(ii) to provide an estimate of the domain of attraction (DoA) of the closed-loop nonlinear system represented by an LPV system or a TS fuzzy model.

In the last decades, stability analysis and stabilization of linear time-delayed systems have been a very active research area. The interest relies on the fact that time delay is a phenomenon that is frequently encountered in control systems such as communication systems, vehicular traffic flows, networked control systems, engineering systems, population dynamics, and epidemics, among others [93]. In general, the existence of time delay can degrade the performance of systems, cause undesired oscillation, and even instability [94].

As a matter of fact, the Lyapunov-Krasovskii functional (LKF) approach is regarded as one of the most effective solutions, if not the most effective solution, to derive LMI conditions for analysis and control of time-delayed systems. Although selecting an appropriate LKF that provides nonconservative stability conditions is not an easy task, it has been noticed that the LKF approach for discrete time-delayed systems usually consists of two main steps. The first one is to construct the LKF per si and the second step is to derive sufficient conditions guaranteeing that the forward difference of an LKF is negative. Therefore, to improve stability and stabilization conditions many augmented LKFs and LKFs with multiple summation terms have been constructed [95, 96, 97, 98]. It has been shown in [99] that the following double summation term is one of the most relevant terms applied during the construction of LKF

$$V(x_k) = \sum_{i=-\overline{h}+1}^{-\underline{h}} \sum_{j=k+i}^{k} \eta_j^\top R \eta_j,$$

where $\underline{h}$ and $\overline{h}$ are, respectively, the lower and the upper bounds of a time-varying delay (i.e., $\underline{h} \leq h_k \leq \overline{h}$), R is a positive definite matrix, and $\eta_k = x_{k+1} - x_k$ with x_k being the system state. Computing the forward difference of $V(x_k)$ leads to

$$\Delta V(x_k) = (\overline{h} - \underline{h})\eta_{k+1}^\top R \eta_{k+1} - \sum_{i=k-h_k+1}^{k-\underline{h}} \eta_i^\top R \eta_i - \sum_{i=k-\overline{h}+1}^{k-h_k} \eta_i^\top R \eta_i. \tag{1.1}$$

During the development of stability criteria, a challenging problem related to Equation (1.1) arises regarding how to include these negative terms to derive LMI conditions. Initially, the trick was to apply the Jensen inequality or free-weighting matrix approach [42, 100]. However, these methods unavoidably introduce some conservativeness. By relaxing the Jensen-based inequality, Wirtinger-based inequalities have been provided in [99]. Following this idea, several stability and state-feedback synthesis conditions for discrete-time linear systems [98, 99, 101, 102, 103, 104, 105, 106, 107], uncertain linear time-delayed systems [108, 109, 110, 111], LPV systems [112, 113, 114, 115, 116], TS fuzzy systems [95, 117, 118, 119, 120] with time-varying delay have been derived.

On the other hand, it can be noticed that scant attention has been paid to SOF control problem of discrete-time delayed systems. In [41, 45], the cone complementary formulation has been used to derive SOF control design conditions, however, both techniques consider TS fuzzy models with constant time delay. The cone complementary linearization algorithm has also been used to derive SOF control design conditions for linear [39, 40, 42] and singular fuzzy [46] discrete-time systems with time-varying delay. However, to the best of the author's knowledge, the SOF problem has not been investigated so far for discrete-time LPV systems using the Lyapunov-Krasovskii theory. Besides that, even for TS fuzzy models with time-varying delay, there are no works in the literature that deal with the local SOF control problem. Therefore, the third motivation of this Thesis is:

(iii) to derive local delay-dependent SOF synthesis conditions for discrete-time nonlinear systems with time-varying delay represented by LPV and TS fuzzy models.

1.1 Objectives

This work is concerned with the static output-feedback stabilization of discrete-time nonlinear systems described by polytopic models. Hence, based on the forenamed motivations, the main objectives of this Thesis are:

(1) to use the sector-nonlinearity approach for obtaining an exact polytopic representation (LPV or TS fuzzy model) for nonlinear systems within the domain of validity $\mathcal{D}$.

(2) to propose novel SOF control conditions for discrete-time nonlinear systems represented by LPV systems and TS fuzzy models.

(3) to provide novel LMI-based *delay-dependent* conditions to design the SOF gain-scheduled controller for ensuring asymptotic stability of the polytopic system.

(4) to provide an enlarged estimate of the set of admissible initial conditions within the domain of validity $\mathcal{D}$ of the polytopic model.

1.2 Thesis outline and contributions

The Thesis's organization and the related contributions of each chapter are:

Chapter 2 presents an overview of Linear Parameter Varying (LPV) systems and Takagi-Sugeno (TS) fuzzy models. A comparison between these models is provided and it is shown how to represent nonlinear systems employing those kinds of polytopic models.

Chapter 3 provides local synthesis conditions for gain-scheduling static output-feedback control of discrete-time nonlinear systems represented by LPV and TS fuzzy models. The proposed approach is relatively simple, and the slack variables introduced along the formulation provide extra degrees of freedom to reduce design conservativeness. As extra advantages, no iterative algorithms are required and the output matrix can be parameter-dependent, without requiring that the output matrix has any specific structure or should admit particular similarity transformations (as previous works in the literature have done). Besides that, for the nonlinear case, the domain of attraction of the closed-loop polytopic system is estimated. The proposed conditions are presented in the form of LMIs. The results presented in this chapter are published in Peixoto, Coutinho & Palhares [121].

Chapter 4 introduces a novel delay-dependent condition for static output-feedback control of nonlinear systems with time-varying delay represented by quasi-LPV systems and TS fuzzy models. The appropriate choice of a Lyapunov-Krasovskii functional, Wirtinger-based inequality, delay-dependent Moon's inequality, and selection of a suitable augmented vector allow obtaining new stabilization conditions that depend on the minimum and maximum values of the time-varying delay. Additionally, Finsler's Lemma is employed to derive the stabilization conditions, which helps to obtain the controller gains. A distinctive feature of the proposed approach is the local asymptotic stability of the origin of the closed-loop system, unlike other works in the literature. This ensures the correct operation of the closed-loop system since its trajectories remain inside the guaranteed region of attraction estimation obtained within the validity region of the polytopic model. The results presented in this chapter are published in Peixoto *et al.* [122].

Chapter 5 briefly presents some other results that have been developed up to the qualifying exam, such as new LMI-based conditions for stability analysis and control design of LPV systems employing Lyapunov functions with nonmonotonic terms and using Lyapunov functions with dependence on delayed scheduling parameters. In addition, further works that have also been developed along the Ph.D. are shortly presented in this chapter, namely: results for the problem of periodic event-triggered control co-design for polytopic systems subject to stochastic deception attacks; a fault estimation method for a class of nonlinear parameter-varying systems subject to time-varying delay and unmeasured nonlinearities.

Finally, Chapter 6 presents concluding remarks and suggestions for future works.

2 SYSTEM DESCRIPTION: POLYTOPIC SYSTEMS

In this chapter, linear parameter varying (LPV) systems and Takagi-Sugeno (TS) fuzzy models are presented. It is illustrated how to represent nonlinear systems by means of quasi-LPV and TS fuzzy models. In addition, some connections between LPV systems and TS are provided.

2.1 Linear parameter-varying (LPV) systems

Linear parameter-varying (LPV) systems have been extensively studied by the control community in recent years. This kind of model allows describing the dynamics of linear systems affected by time-varying parameters as well as representing nonlinear systems in terms of a family of LPV models [123]. The first gain scheduling ideas in the context of LPV have been proposed by Shamma [124] in 1988. A great difficulty, at that time, was the lack of a general theory for analyzing the stability and for efficiently designing gain-scheduled control laws of LPV systems. The last thirty years have seen increasingly rapid advances in gain scheduling in both practical and theoretical outcomes. As a result, it has become widely employed in many engineering applications, such as robotic systems [125], fault-tolerant control [126], energy production systems [127], power systems [128, 129], and wind turbine systems [130].

Consider the following discrete-time LPV system

$$\begin{aligned} x_{k+1} &= A(\rho_k)x_k + B(\rho_k)u_k, \ \forall k \in \mathbb{Z}^+ \\ y_k &= C(\rho_k)x_k, \end{aligned} \tag{2.1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the input, $y_k \in \mathbb{R}^{n_y}$ is the output, $\rho = [\rho_{1,k}, \rho_{2,k}, \dots, \rho_{p,k}]^\top \in \Omega$ is a vector of time-varying parameters, which are functions of measured exogenous signals and/or the output, and Ω is a convex set. The parameters ρ are assumed to be bounded and are defined by the minimal $\underline{\rho}_i$, and maximal $\overline{\rho_i}$ values of ρ_i such that

$$\rho_{i,k} \in \left[\underline{\rho}_i, \ \overline{\rho}_i\right], \ i \in \mathbb{N}_{\leq p}.$$

There are several classes of LPV models, for instance: affine parameter dependence [131], polynomial parameter dependence [132, 133], rational parameter dependence [134], and polytopic models. Here the objective is to deal with polytopic models. Considering a polytopic representation, system (2.1) can be written as

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{N} \alpha_i(\rho_k) \left(A_i x_k + B_i u_k\right) \\ y_k &= \sum_{i=1}^{N} \alpha_i(\rho_k) C_i x_k \end{aligned} \tag{2.2}$$

being $N = 2^p$ the number of vertices of the polytopic domain, A_i, B_i, C_i are constant matrices, $\alpha(\rho) \in \Lambda$, and the unit simplex Λ is defined as

$$\Lambda = \left\{ \alpha(\cdot) \in \mathbb{R}^N : \sum_{i=1}^{N} \alpha_i(\cdot) = 1, \alpha_i(\cdot) \geq 0,\ i \in \mathbb{N}_{\leq N} \right\}. \tag{2.3}$$

In this work, the polytopic LPV system (2.2) is considered in the following general form

$$\begin{aligned} x_{k+1} &= A(\alpha_k)x_k + B(\alpha_k)u_k \\ y_k &= C(\alpha_k)x_k \end{aligned} \tag{2.4}$$

with

$$\begin{bmatrix} A(\alpha_k) & B(\alpha_k) \\ C(\alpha_k) & \end{bmatrix} = \sum_{i=1}^{N} \alpha_{i,k} \begin{bmatrix} A_i & B_i \\ C_i & \end{bmatrix}. \tag{2.5}$$

To illustrate, Figure 2.1 depicts the unit simplex considering the values for $N = 2$ and $N = 3$. For $N = 2$, the set takes the form of a segment on a line; for $N = 3$, the set is a triangular closed surface on a plane; and so on.

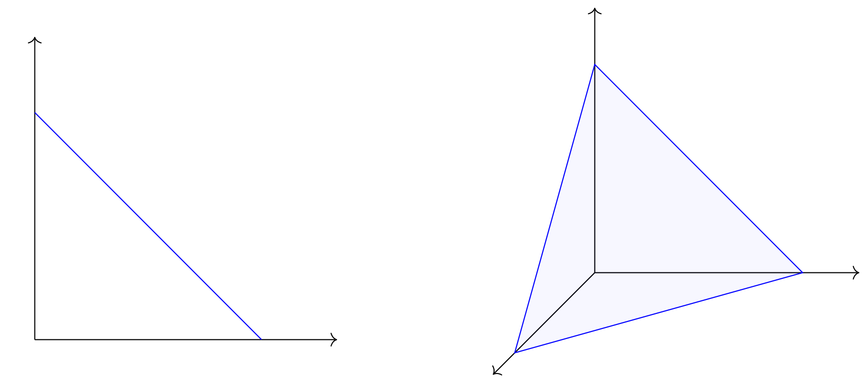

Figure 2.1 – Set Λ for $N = 2$ and $N = 3$.

For an LPV system with 2 bounded parameters, $\rho_{1,k} \in [\underline{\rho}_1, \overline{\rho}_1]$ and $\rho_{2,k} \in [\underline{\rho}_2, \overline{\rho}_2]$, the corresponding polytope has $N = 2^2 = 4$ vertices as:

$$\mathscr{P}_\rho = \left\{ (\underline{\rho}_1, \underline{\rho}_2), (\underline{\rho}_1, \overline{\rho}_2), (\overline{\rho}_1, \underline{\rho}_2), (\overline{\rho}_1, \overline{\rho}_2) \right\}.$$

The polytopic coordinates (α_i) are obtained as:

$$\begin{aligned}
\omega_1 &= \left(\underline{\rho}_1, \underline{\rho}_2\right), & \alpha_1 &= \left(\frac{\overline{\rho_1} - \rho_{1,k}}{\overline{\rho_1} - \underline{\rho_1}}\right) \times \left(\frac{\overline{\rho_2} - \rho_{2,k}}{\overline{\rho_2} - \underline{\rho_2}}\right) \\
\omega_2 &= \left(\underline{\rho}_1, \overline{\rho}_2\right), & \alpha_2 &= \left(\frac{\overline{\rho_1} - \rho_{1,k}}{\overline{\rho_1} - \underline{\rho_1}}\right) \times \left(\frac{\rho_{2,k} - \underline{\rho_2}}{\overline{\rho_2} - \underline{\rho_2}}\right) \\
\omega_3 &= \left(\overline{\rho_1}, \underline{\rho}_2\right), & \alpha_3 &= \left(\frac{\rho_{1,k} - \underline{\rho_1}}{\overline{\rho_1} - \underline{\rho_1}}\right) \times \left(\frac{\overline{\rho_2} - \rho_{2,k}}{\overline{\rho_2} - \underline{\rho_2}}\right) \\
\omega_4 &= (\overline{\rho}_1, \overline{\rho}_2), & \alpha_4 &= \left(\frac{\rho_{1,k} - \underline{\rho_1}}{\overline{\rho_1} - \underline{\rho_1}}\right) \times \left(\frac{\rho_{2,k} - \underline{\rho_2}}{\overline{\rho_2} - \underline{\rho_2}}\right)
\end{aligned}$$

where $\rho_{1,k}$ and $\rho_{2,k}$ are the instantaneous values of the parameters. Thus, the LPV system can be rewritten under the polytopic representation:

$$\begin{aligned}\begin{pmatrix} A(\rho) & B(\rho) \\ C(\rho) & \end{pmatrix} =& \alpha_1 \begin{pmatrix} A(\omega_1) & B(\omega_1) \\ C(\omega_1) & \end{pmatrix} + \alpha_2 \begin{pmatrix} A(\omega_2) & B(\omega_2) \\ C(\omega_2) & \end{pmatrix} \\ &+ \alpha_3 \begin{pmatrix} A(\omega_3) & B(\omega_3) \\ C(\omega_3) & \end{pmatrix} + \alpha_4 \begin{pmatrix} A(\omega_4) & B(\omega_4) \\ C(\omega_4) & \end{pmatrix}.\end{aligned}$$

Remark 2.1. *If the parameter ρ in the system* (3.1) *is constant (fixed), then a linear time-invariant (LTI) system is obtained. On the other hand, if ρ is fixed but unknown, system* (3.1) *is an uncertain linear time-invariant system. Table 2.1 presents these cases in more detail.*

Table 2.1 – Particular cases of LPV systems.

Parameter (α)	System	Description
Constant	LTI	$x_{k+1} = Ax_k + Bu_k$
Constant and unknown	Uncertain LTI	$x_{k+1}{=}A(\alpha)x_k{+}B(\alpha)u_k \quad [A(\alpha)\ B(\alpha)]{=}\sum_{i=1}^{N} \alpha_i[A_i\ B_i], \forall\alpha \in \Lambda$

Remark 2.2. *The quasi-LPV model is also a particular class of LPV systems [23] whose parameters depend only on endogenous signals, such as the state.*

In the sequel, an example illustrates how to represent a nonlinear system by a quasi-LPV model.

Example 2.1 (Discretized van der Pol equation: quasi-LPV model)**.** Consider the discretized van der Pol equation [135]

$$\begin{aligned} x_{1,k+1} &= x_{1,k} + Tx_{2,k} \\ x_{2,k+1} &= -9Tx_{1,k} + (1 + 2T(1 - x_{1,k}^2))x_{2,k} + Tu_k \end{aligned} \tag{2.6}$$

being $T = 0.05$s and $|x_{1,k}| \leq r_0$, with r_0 being a positive scalar. By considering the scheduling $\rho(x_k) = x_{1,k}^2$ which is bounded by $\rho(x_k) = [\underline{\rho}, \overline{\rho}] = [0, r_0^2]$, system (2.6) can be equivalently represented by the following polytopic quasi-LPV system

$$x_{k+1} = \sum_{i=1}^{2} \alpha_i(\rho_k) A_i x_k + Bu_k \tag{2.7}$$

with

$$A_1 = \begin{bmatrix} 1 & T \\ -9T & 1 + 2T \end{bmatrix}, \; A_2 = \begin{bmatrix} 1 & T \\ -9T & 1 + 2T(1 - r_0^2) \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ T \end{bmatrix},$$

and parameters

$$\alpha_1(\rho_k) = \frac{\overline{\rho} - x_{1,k}^2}{\overline{\rho} - \underline{\rho}} = \frac{r_0^2 - x_{1,k}^2}{r_0^2}, \; \alpha_2(\rho_k) = 1 - \alpha_1(\rho_k).$$

2.2 Takagi-Sugeno (TS) fuzzy models

Takagi–Sugeno (TS) fuzzy model-based techniques have been recognized as an effective alternative to developing constructive stability analysis and control synthesis conditions for nonlinear systems [3, 4, 5, 11, 13, 136]. The fuzzy model proposed by Takagi and Sugeno [137] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication rule by a linear system model. TS fuzzy models have been successfully used in the same fields where LPV systems demonstrated to be successful: robotic systems [138, 139], fault-tolerant control [140, 14], energy production systems [141], power systems [142], and wind turbine systems [143].

Consider a nonlinear system, whose dynamics can be described as

$$\begin{aligned} x_{k+1} &= A(\mu_k)x_k + B(\mu_k)u_k, \ \forall k \in \mathbb{Z}^+ \\ y_k &= C(\mu_k)x_k \end{aligned} \tag{2.8}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the input, $y_k \in \mathbb{R}^{n_y}$ is the system output, $\mu_k \in \mathbb{R}^{n_\mu}$ is the vector of measured premise variables. Using fuzzy modeling technique [144], the nonlinear system (2.8) can be reformulated as the following TS fuzzy model:

$$\text{Rule } \mathscr{R}_i: \ \text{IF } \mu_1 \text{ is } \mathcal{M}_1^i \text{ and } \ldots \text{ and } \mu_{n_\mu} \text{ is } \mathcal{M}_{n_\mu}^i. \text{ THEN}$$
$$\begin{cases} x_{k+1} = A_i x_k + B_i u_k \\ \quad y_k = C_i x_k \end{cases}$$

where the constant matrices (A_i, B_i, C_i) are known, $\mathscr{R}_i$ is the ith fuzzy rule, r denotes the number of fuzzy rules, and $\mathcal{M}_j^i$, with $i \in \mathbb{N}_{\leq r}$ and $j \in \mathbb{N}_{\leq n_\mu}$, are the fuzzy sets. The fuzzy membership functions are defined as

$$z_i(\mu_k) = \frac{\prod_{j=1}^{n_\mu} \lambda_j^i(\mu_j)}{\sum_{i=1}^{r} \prod_{j=1}^{n_\mu} \lambda_j^i(\mu_j)}, \quad i \in \mathbb{N}_{\leq r}, \tag{2.9}$$

where $\lambda_j^i(\mu_j)$ represents the membership grade of μ_j with respect to the fuzzy set $\mathcal{M}_j^i$. Notice that the membership functions defined in (2.9) belong to the unit simplex Λ with r vertices

$$\Lambda = \left\{ z \in \mathbb{R}^r : \ \sum_{i=1}^{r} z_i = 1, \ 0 \leq h_i \leq 1, \ i \in \mathbb{N}_{\leq r} \right\}, \tag{2.10}$$

with $z = \Big[z_1(\mu_k), z_2(\mu_k), \ldots, z_r(\mu_k)\Big]$. Applying the center-of-gravity method for defuzzification [144], the TS fuzzy system (2.9) can be rewritten as

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} z_i(\mu_k) \left(A_i x_k + B_i u_k\right) \\ y_k &= \sum_{i=1}^{r} z_i(\mu_k) C_i x_k. \end{aligned} \tag{2.11}$$

Example 2.2 (Discretized van der Pol equation: TS fuzzy model)**.** Consider again the discretized van der Pol equation described in (2.6). To represent system (2.6) as TS fuzzy model, the premise variable is given as $\mu_k = x_{1,k}^2 \in [0, r_0^2]$. Therefore, system (2.6) can be rewritten in the form (2.8) with

$$A(\mu_k) = \begin{bmatrix} 1 & T \\ -9T & 1 + 2T(1 - \mu_k) \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ T \end{bmatrix}.$$

Applying the sector nonlinearity approach [144, Chapter 2] with the measured premise variable $\mu_k = x_{1,k}^2$, system (2.6) can be represented by a two-rule TS fuzzy model (2.11) with

$$A_1 = \begin{bmatrix} 1 & T \\ -9T & 1 + 2T \end{bmatrix}, \; A_2 = \begin{bmatrix} 1 & T \\ -9T & 1 + 2T(1 - r_0^2) \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ T \end{bmatrix}.$$

The corresponding membership functions are given by

$$z_1(\mu_k) = \frac{r_0^2 - x_{1,k}^2}{r_0^2}, \quad z_2(\mu_k) = 1 - z_1(\mu_k).$$

Remark 2.3. *Notice that the LPV system* (2.2) *and the Takagi-Sugeno fuzzy model* (2.11) *are similar in their descriptions, that is, if the time-varying parameters depend on some systems' states (i.e.: quasi-LPV models), they are equivalent to membership functions in TS fuzzy models. The number of vertices N is related to the number of fuzzy rules r, and the premise variable μ_k is related to the scheduling function ρ_k. Table 2.2 shows the main analogies between LPV and TS fuzzy systems. It is worth mentioning that in this work the LPV model description is being used to construct the results, however, all content present here can be applied to both cases, LPV (quasi-LPV) systems and TS fuzzy models. Besides that, when the author of this Thesis refers to a system as a quasi-LPV model it can be understood as a Takagi-Sugeno fuzzy model.*

Table 2.2 – Analogies between LPV and TS fuzzy models.

quasi-LPV systems	Takagi-Sugeno fuzzy models
Number of vertices (N)	Number of fuzzy rules (r)
Scheduling function (ρ_k)	Premise variable (μ_k)
Time-varying parameters ($\alpha_i(\cdot),\ i \in \mathbb{N}_{\leq N}$)	Membership functions ($z_i(\cdot),\ i \in \mathbb{N}_{\leq r}$)

3 SOF STABILIZATION CONDITIONS OF POLYTOPIC SYSTEMS

This chapter introduces a novel local synthesis condition for gain-scheduling static output-feedback (SOF) control of discrete-time polytopic systems. The proposed condition is formulated as a set of parameter-dependent linear matrix inequalities that are incorporated into a convex optimization procedure to provide an enlarged estimate of the region of attraction of the closed-loop equilibrium. The results presented in this chapter are published in [121].

3.1 Problem Formulation

This section presents the system description as well as formulates the SOF control design problem of discrete-time polytopic systems. The description of the considered LPV system is presented below, however as mentioned in Chapter 2, all results presented here can also be applied to TS fuzzy models.

Consider the following discrete-time LPV system

$$\begin{aligned} x_{k+1} &= A(\alpha_k)x_k + B(\alpha_k)u_k \\ y_k &= C(\alpha_k)x_k, \end{aligned} \tag{3.1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the input, $y_k \in \mathbb{R}^{n_y}$ is the output. The parameter-dependent matrices $A(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $B(\alpha_k) \in \mathbb{R}^{n_x \times n_u}$ and $C(\alpha_k) \in \mathbb{R}^{n_y \times n_x}$ belong to a polytopic domain parameterized by the time-varying parameters $\alpha \in \Lambda$, defined as in (2.5).

Assumption 3.1. *The region where the LPV system* (3.1) *is valid is a polytope containing the origin described by*

$$\mathcal{D} = \{x \in \mathbb{R}^{n_x} : b_j x \leq 1,\ j \in \mathbb{N}_{\leq n_e}\}, \tag{3.2}$$

where n_e is the number of hyper-planes and $b_j \in \mathbb{R}^{n_x}$ defines the j-th hyper-plane.

Consider the following gain-scheduled SOF controller

$$u_k = X(\alpha_k)^{-1}L(\alpha_k)y_k, \tag{3.3}$$

being $L(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$ and $X(\alpha_k) \in \mathbb{R}^{n_u \times n_u}$ the control gains to be designed. The resulting closed-loop system is

$$x_{k+1} = \Big(A(\alpha_k) + B(\alpha_k)X(\alpha_k)^{-1}L(\alpha_k)C(\alpha_k)\Big)\, x_k. \tag{3.4}$$

For analyzing the local asymptotic stability of the closed-loop system (3.4), the following parameter-dependent Lyapunov function candidate is considered:

$$V(x_k) = x_k^\top P(\alpha_k)x_k, \quad \forall \alpha_k \in \Lambda, \tag{3.5}$$

being $P(\alpha_k) \in \mathbb{S}_+^{n_x}$ a parameter-dependent matrix.

Definition 3.1. *The region $\mathcal{R}_0$ associated with the unitary level set of the Lyapunov function candidate defined in* (3.5),

$$\mathcal{R}_0 = \{x \in \mathbb{R}^{n_x} : V(x_k) \leq 1, \quad \forall \alpha_k \in \Lambda\}, \tag{3.6}$$

is said to be positively invariant [1] if $\Delta V = V(x_{k+1}) - V(x_k) < 0$, $\forall x \in \mathcal{R}_0 \setminus \{0\}$, $\forall \alpha_k \in \Lambda$.

The following control problem is addressed in this chapter.

Problem 3.1. *Given the LPV system* (3.1)*, design a gain-scheduled static output-feedback (SOF) controller of the form* (3.3) *and compute the estimate of the domain of attraction of the system's equilibrium given by $\mathcal{R}_0$ in* (3.6)*, such that $\mathcal{R}_0 \subset \mathcal{D}$.*

3.2 Local static output-feedback (SOF) stabilization

This section presents a novel local stabilization condition for the discrete-time polytopic systems described in (3.1) using the gain-scheduled SOF controller in (3.3).

Theorem 3.1. *The origin of the closed-loop system* (3.4) *is asymptotically stable and the region $\mathcal{R}_0 \subset \mathcal{D}$ is an estimation of the DoA for the origin of* (3.4)*, if there exist matrices $P(\alpha_k) \in \mathbb{S}_+^{n_x}$, $X(\alpha_k) \in \mathbb{R}^{n_u \times n_u}$, $L(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $F(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $Y(\alpha_k) \in \mathbb{R}^{n_y \times n_y}$, $M(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $S(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $J(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $Z(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$ such that the following inequalities hold for a given $\eta \in \mathbb{R}_{>0}$:*

$$\begin{bmatrix} P(\alpha_k) & \star \\ b_j & 1 \end{bmatrix} \geq 0, \quad j \in \mathbb{N}_{\leq n_e} \tag{3.7}$$

$$\begin{bmatrix} -P(\alpha_k) + \eta \mathrm{He}\{(F(\alpha_k)A(\alpha_k) + B(\alpha_k)S(\alpha_k)C(\alpha_k)\} & \star & \star & \star \\ Z(\alpha_k)A(\alpha_k) + B(\alpha_k)J(\alpha_k)C(\alpha_k) - \eta F(\alpha_k)^\top & \Upsilon_{22} & \star & \star \\ \eta Y(\alpha_k)C(\alpha_k) + \eta\left(B(\alpha_k)L(\alpha_k) - B(\alpha_k)S(\alpha_k)\right)^\top & \Upsilon_{32} & \Upsilon_{33} & \star \\ \eta\left(M(\alpha_k)C(\alpha_k)\right) + \eta\left(F(\alpha_k)B(\alpha_k) - B(\alpha_k)X(\alpha_k)\right)^\top & \Upsilon_{42} & \Upsilon_{43} & \Upsilon_{44} \end{bmatrix} < 0, \tag{3.8}$$

with

$$\begin{aligned}
\Upsilon_{22} &= P(\alpha_{k+1}) - Z(\alpha_k) - Z(\alpha_k)^\top, \\
\Upsilon_{32} &= \left(B(\alpha_k)L(\alpha_k) - B(\alpha_k)J(\alpha_k)\right)^\top, \\
\Upsilon_{33} &= -\eta Y(\alpha_k) - \eta Y(\alpha_k)^\top, \\
\Upsilon_{42} &= \left(Z(\alpha_k)B(\alpha_k) - B(\alpha_k)X(\alpha_k)\right)^\top, \\
\Upsilon_{43} &= \eta\left(L(\alpha_k) - M(\alpha_k)\right), \\
\Upsilon_{44} &= -\eta X(\alpha_k) - \eta X(\alpha_k)^\top.
\end{aligned}$$

Proof. Consider that the inequalities (3.7)–(3.8) hold and that $x_k \in \mathcal{R}_0$. Firstly, multiplying (3.7) by $\begin{bmatrix} -x_k^\top & 1 \end{bmatrix}$ on the left and by its transpose on the right, it results in

$$1 + x_k^\top P(\alpha_k)x_k - x_k^\top b_j^\top - b_j x_k \geq 0. \tag{3.9}$$

Since for all $x_k \in \mathcal{R}_0$, one has $x_k^\top P(\alpha_k)x_k \leq 1$, it implies that $b_j x_k \leq 1$. This proves the inclusion $\mathcal{R}_0 \subset \mathcal{D}$. Finally, multiplying (3.8) by

$$\begin{bmatrix} \mathrm{I} & \mathcal{A}(\alpha_k)^\top & C(\alpha_k)^\top & C(\alpha_k)^\top L(\alpha_k)^\top X(\alpha_k)^{-\top} \end{bmatrix}$$

on the left and by its transpose on the right, it yields

$$\mathcal{A}(\alpha_k)^\top P(\alpha_{k+1})\mathcal{A}(\alpha_k) - P(\alpha_k) < 0, \tag{3.10}$$

with $\mathcal{A}(\alpha_k) = A(\alpha_k) + B(\alpha_k)X(\alpha_k)^{-1}L(\alpha_k)C(\alpha_k)$. Multiplying (3.10) by $x_k^\top$ on the left and by x_k on the right results in

$$\Delta V = V(x_{k+1}) - V(x_k) < 0,$$

for $V(x_k) = x_k^\top P(\alpha_k)x_k$, $\forall \alpha_k \in \Lambda$. This proves that if the condition on Theorem 3.1 is feasible, then the controller ensures that the origin of the closed-loop system (3.4) is asymptotically stable and $\mathcal{R}_0$ is an invariant set for (3.4), therefore it is an estimate of the DoA for the origin of (3.4). It is worth mentioning that the SOF controller does not depend on future instants, i.e., the control law can be computed in real-time.

Notice that the block Υ_{44} of (3.8) guarantees that $X(\alpha_k) + X(\alpha_k)^\top > 0$, ensuring the existence of the inverse of matrix $X(\alpha_k)$, $\forall \alpha_k \in \Lambda$. □

Remark 3.1. *Differently from existing results in the literature on SOF control design, the condition proposed in Theorem 3.1 does not require any structural constraint in the output matrix $C(\alpha_k)$ neither requires any similarity transformations. Moreover, no iterative algorithms are necessary for solving the proposed conditions. Furthermore, the given scalar η has only the role of introducing an extra degree of freedom in the proposed condition.*

Remark 3.2. *It is important to point out that for the case where the time-varying parameters depend only on measured exogenous signals, that is, a purely LPV system is considered, the LMI* (3.7) *in Theorem 3.1 can be removed.*

3.2.1 Enlarging the estimation of the DoA

Based on the result stated in Theorem 3.1, it is important to enlarge the estimation of the DoA. Therefore, the following optimization is considered to attain this goal:

$$\begin{aligned} &\min \ \mu \\ &\text{s.t.}: \begin{cases} \text{LMIs in (3.7) and (3.8)} \\ \operatorname{trace}(P(\alpha_k)) \leq \mu, \quad \forall \alpha_k \in \Lambda. \end{cases} \end{aligned} \tag{3.11}$$

The optimization problem in (3.11) intends to minimize the largest trace of $P(\alpha_k), \forall \alpha_k \in \Lambda$, which tends to enlarge the region $\mathcal{R}_0$ in (3.6). Notice that since the conditions in Theorem 3.1 are written as parameter-dependent inequalities, in order to computationally solve them, Lemma A.1 in Appendix A is employed to obtain finite sets of LMIs. Thus, (3.11) turns into a convex optimization problem.

3.3 Numerical Examples

In this section, numerical experiments are presented to illustrate the effectiveness of the conditions proposed in this chapter. The routines were implemented in Matlab using the parser Yalmip, and the semidefinite programming solver MOSEK.

Example 3.1. Consider the following discrete-time LPV system described in [51]:

$$A_1 = \begin{bmatrix} 1.3+\delta & -1 \\ 0.2 & 0.6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & -1.5 \\ 0.1 & 0.3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 1.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

The problem to be solved is to determine the largest $\delta > 0$, denoted as δ^*, such that the system can be static output feedback stabilized. This value is considered to evaluate the conservativeness of different approaches in the literature. In this example, the proposed condition in Theorem 3.1 is compared with the conditions in [51, 87, 16, 89, 69]. It is important to mention that for this example, the LMI constraint (3.7) can be removed since this is a purely LPV system. The results are presented in Table 3.1 where is shown the value of δ^*, the number of scalar decision variables (S_v), and the number of LMI rows (L_R). In particular, two scenarios are considered for comparisons: parameter-independent and gain-scheduling.

Table 3.1 – Maximum δ values obtained by several methods with number of LMI rows L_R and scalar variables S_v – Example 3.1.

Method	Parameter-independent				Gain-Scheduling		
	δ^*	Gain	S_v	L_R	δ^*	S_v	L_R
[51, Theorem 5]	0.33	-1.1161	16	32	-	-	-
[51, Theorem 1]	0.40	-1.3336	16	32	-	-	-
[16, Theorem 2] ($N=2$)	0.60	-1.0146	22	148	0.69	24	148
[16, Theorem 2] ($N=3$)	0.62	-1.0212	28	602	0.69	30	602
[89, Theorem 1]	0.86	-1.7581	36	48	-	-	-
[69, Theorem 2] ($\rho=1$)	0.60	-1.2069	14/24[1]	24/44	0.88	14/24	25/44
Theorem 3.1	**0.87**	-1.8447	20	40	**0.92**	22	40

Regarding the parameter-independent case in Table 3.1, for [51, Theorem 5], the maximum value of δ for feasibility is 0.33, whereas $\delta^* = 0.40$ is achieved with [51, Theorem 1]. For [16, Theorem 2], using $N = 2$ and a given matrix $\mathscr{Y} = B_1$, the maximum δ is 0.60. If one uses $N = 3$ and $\mathscr{Y} = B_1$, the maximum value is $\delta^* = 0.62$. The maximum value reached by [89, Theorem 1] is $\delta^* = 0.86$ using $\delta = 0.2$ and $\beta = 0.9$. Applying the two-stages method proposed in [69] the maximum value of δ^* for feasibility is 0.60. To solve the parameter-independent

[1] The approach in [69] is based on two-stages procedure, therefore, the first value is related to the first step while the second one is obtained in the second step.

condition using Theorem 3.1 proposed in this chapter, it is considered $Z(\alpha_k) = Z$, $F(\alpha_k) = F$, $Y(\alpha_k) = Y$, $M(\alpha_k) = M$, $J(\alpha_k) = J$, $L(\alpha_k) = L$ and $X(\alpha_k) = X$, the scalar search is performed considering $\eta \in [0, 5]$ and the best result, $\delta^* = 0.87$, is obtained for η assuming values in the interval $[1.6, 2.4]$.

On the other hand, for the parameter-dependent case in Table 3.1, it is considered Theorem 3.1 with parameter-dependent structures for $L(\alpha_k)$ and $X(\alpha_k)$ as in (2.3), and constant matrices $Z(\alpha_k) = Z$, $F(\alpha_k) = F$, $Y(\alpha_k) = Y$, $M(\alpha_k) = M$, $J(\alpha_k) = J$. For the scalar parameter assuming values in the interval $\eta = [0.2, 1.9]$, it follows that $\delta^* = 0.92$, which is clearly an improvement in comparison with the parameter-dependent conditions presented in [16, 69]. The approaches proposed in [51, 89] are not able to deal with the parameter-dependent case. It is worth mentioning that the condition presented in [87, Theorem 1] fails to find a solution for this example, we have used values suggested in [87] to perform the scalar searches, namely: $\xi \in [-0.9, 0.9]$ with a grid of 0.01 in ξ, $\gamma = -10^5$, $\rho = 1$, and a given matrix $Q(\alpha_k) = C(\alpha_k)$.

From Table 3.1, it can be verified that the proposed method can achieve less conservative results than the methods in [51, 87, 16, 89, 69]. Notice also that for the proposed condition in this chapter, there is a commitment between conservativeness reduction and numerical complexity, since the proposed Theorem 3.1 has provided the largest value of δ requiring less number of decision variables and the number of LMI rows in comparison to [16, 89, 69], in both parameter-independent and dependent cases.

Example 3.2. Consider the discrete-time LPV system with vertices:

$$A_1 = \begin{bmatrix} 0.7445 & -0.1398 & 0.5880 & -0.1402 \\ 0.7190 & -0.3427 & -0.6993 & 0.0017 \\ 1.1088 & 0.3664 & -0.7344 & 0.4628 \\ -0.0921 & -0.4727 & 0.5423 & -0.5607 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.3344 \\ 0.2966 \\ -2.1774 \\ 0.5941 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.0688 & -0.8214 & -0.6759 & 0.1621 \\ -0.2066 & -0.6346 & -0.3925 & -0.5422 \\ -0.2521 & -0.1341 & 0.4933 & -0.1415 \\ 0.1943 & -0.4248 & 0.1209 & 0.6465 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.9477 \\ -0.1021 \\ 0.1003 \\ 0.0466 \end{bmatrix},$$
$$C_1 = C_2 = C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

For this model the methods reported in [51, 53, 87, 16, 89, 69] fail to find a solution. For the condition presented in [87], the scalar searches were performed using $\xi \in \{-0.9, -0.8 \ldots, 0.8, 0.9\}$, $\gamma = -10^5$, $\rho = 1$, and a given matrix $Q(\alpha_k) = C(\alpha_k) = C$. The condition presented in [16] was tested with $N = 1, \ldots, 3$ and $\mathscr{Y} = B_1$. The condition of [89] was simulated considering a scalar search over the set $[\delta, \beta] \in \mathbb{U} \times \mathbb{U}$ with $\mathbb{U} = \{-1, -0.9, \ldots, 0.9, 1\}$. Notice that if one considers $Z(\alpha_k) = Z$, $F(\alpha_k) = F$, $Y(\alpha_k) = Y$, $M(\alpha_k) = M$, $J(\alpha_k) = J$, $X(\alpha_k) = X$ and

$L(\alpha_k) = L$, in Theorem 3.1 proposed in this chapter, no solution is found. On the other hand, the condition proposed in Theorem 3.1 employing parameter-dependent matrices is able to find a solution for a given $\eta = 0.1$ (other values for η can be selected). This example illustrates clearly the necessity of making use of parameter-dependent matrices to have a feasible solution. The gain-scheduling SOF control gains can be obtained for:

$$X_1 = 1.0354, \; X_2 = 0.9684,$$
$$L_1 = \begin{bmatrix} 0.8202 & 0.0325 \end{bmatrix}, \; L_2 = \begin{bmatrix} 0.1094 & -0.5138 \end{bmatrix}.$$

Figure 3.1 depicts the (convergent) state trajectories for the closed-loop system described in (3.4) (with the gain-scheduled controller designed using Theorem 3.1), the control input sequence, and the output trajectories for an initial condition $x_0 = \begin{bmatrix} -1.5 & 1 & 2 & -1 \end{bmatrix}^\top$. It can be noticed that the trajectories converge asymptotically to the origin. For this simulation, the time-varying parameter has been considered as $\alpha_{1,k} = 0.5 + 0.5\sin(0.85\pi k)$ and $\alpha_{2,k} = 1 - \alpha_{1,k}$, as shown in Figure 3.1d.

Remark 3.3. *It is noteworthy that even though the results presented here seem less conservative when compared to other works in the literature, the papers [89, 69] were published after our work in [121] that presents part of the results of this chapter. Furthermore, our approach [121] has inspired the method presented in [145], which provides results for the SOF problem in the context of difference-algebraic representations.*

Example 3.3. In this example, considering a TS fuzzy model, the goal is to evaluate the conservativeness of the approach presented in Theorem 3.1 when compared to fuzzy SOF controller design conditions. However, it is possible to notice that the system used can also be described as an LPV system. To this end, consider the following TS fuzzy model borrowed from [92]:

$$A_1 = \begin{bmatrix} 0.7 & 1.2 + 0.3a \\ -0.6 & -0.3 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ -1.8 \end{bmatrix}, \; C_1 = \begin{bmatrix} 0.3 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.1 & 1.6 \\ -1.4 & 0.1 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ -2.8 - 0.5b \end{bmatrix}, \; C_2 = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$

where $a, b \in \mathbb{R}$. The design conditions are constructed by evaluating their feasibility for several values of $a \in [0, 10]$ and $b \in [-5, 0]$. It should be pointed out that conditions of [48, 83, 146] cannot be applied here because this model has different output matrices. On the other hand, the method proposed in [92] can be applied in this case. The feasible sets obtained by Theorem 3.1 with $\eta = 0.1$ and by [92, Theorem 3] are depicted in Figure 3.2. Notice that the proposed condition provides a feasible set that contains the one obtained using [92, Theorem 3], illustrating less conservative outcomes. It is worth mentioning that to obtain more relaxed results, the technique presented in [92] has used the information of past states in the control law, as well as in the Lyapunov function, which is much more demanding than the approach provided in this chapter.

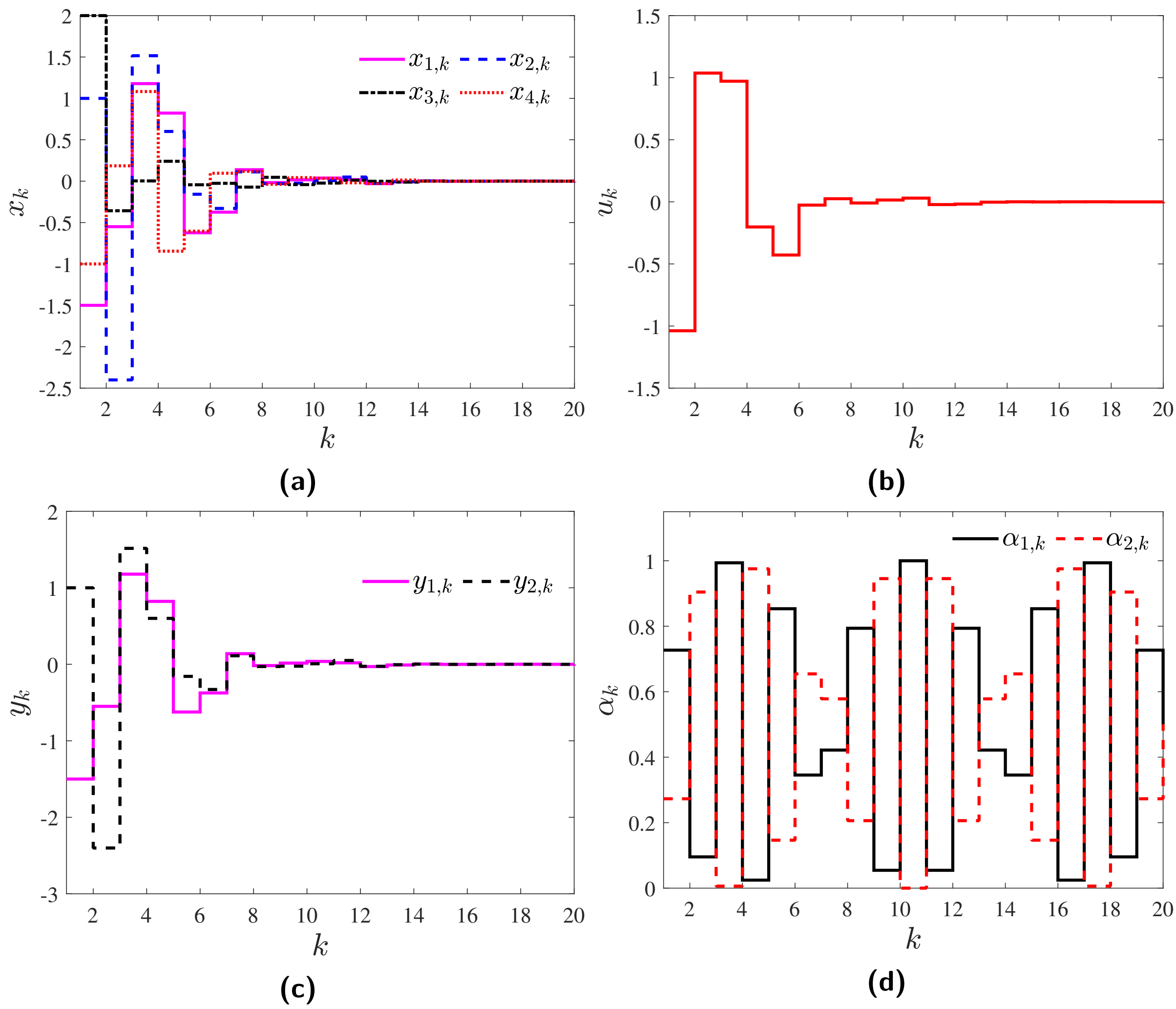

Figure 3.1 – (a) State trajectories for the closed-loop system, $x_{1,k}$ (straight magenta line), $x_{2,k}$ (blue dashed line), $x_{3,k}$ (dashed-dotted black line), $x_{4,k}$ (dotted red line); (b) trajectory of the control input u_k; (c) trajectory of the output y_k; (d) temporal evolution of the time-varying parameters – Example 3.2.

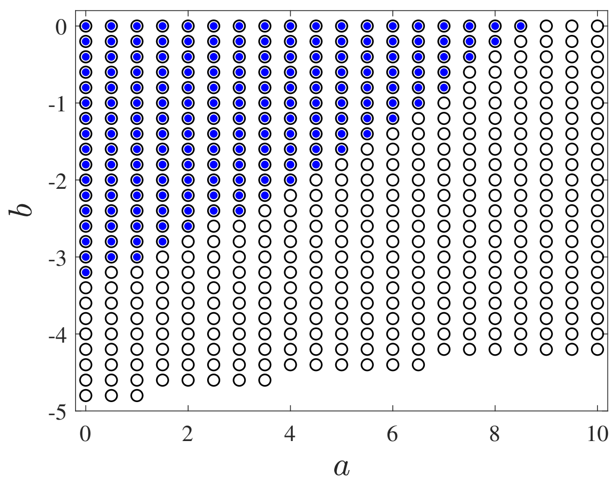

Figure 3.2 – Feasible sets for Theorem 3.1 with $\eta = 0.1$ ('$\circ$') and [92, Theorem 3] ('$\bullet$') - Example 3.3.

Example 3.4. Consider the following nonlinear system:

$$\begin{aligned} x_{1,k+1} &= 1.15x_{1,k}^3 + x_{2,k} \\ x_{2,k+1} &= -2x_{1,k} + 0.7x_{1,k}^2 x_{2,k} + (1 - 0.1x_{1,k}^2)u_k \\ y_k &= x_{1,k} \end{aligned} \tag{3.12}$$

where $\mathcal{D} = \{x \in \mathbb{R}^2 : |x_1| \leq 1, |x_2| \leq 0.5\}$. Considering $\rho(x_k) = x_{1,k}^2$, such that $\rho(x_k) \in [0, 1]$, as the scheduling parameter or premise variables, the nonlinear system given by (3.12) can be represented by means of a quasi-LPV system as in (3.1) or as TS fuzzy model with the following two vertices

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1.15 & 1 \\ -2 & 0.7 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ 0.9 \end{bmatrix}, \; C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The time-varying parameters or membership functions can be calculated as

$$\alpha_{1,k} = 1 - x_{1,k}^2 \text{ and } \alpha_{2,k} = 1 - \alpha_{1,k}.$$

In this example, the purpose is to design the SOF controller (3.3) obtaining an enlarged DoA estimate such that the origin of the closed-loop system (3.12) is asymptotically stable. Since a quasi-LPV (or TS fuzzy model) is obtained, in this case, to design a SOF controller for local stabilization of the nonlinear system (3.12), it is essential to obtain an estimate of the DoA inside the region in which both the polytopic model and the control law remain valid. However, the SOF design approaches for LPV [51, 87, 16, 89, 69] and TS fuzzy [48, 83, 146, 92] systems do not provide such a characterization, and therefore, these approaches will be not used to comparison in this case. To solve the optimization problem (3.11), a parameter-independent structure has been considered, that is, $Z(\alpha_k) = Z$, $F(\alpha_k) = F$, $Y(\alpha_k) = Y$, $M(\alpha_k) = M$, $J(\alpha_k) = J$, $L(\alpha_k) = L$ and $X(\alpha_k) = X$. Figure 3.3 depicts the objective function μ with respect to the scalar parameter η. Recall that for larger values of μ, the DoA estimation $\mathcal{R}_0$ tends to reduce. As can be seen in Figure 3.3, the results are improved with the scalar parameter search. In this case, the minimum $\mu = 8.2829$ is attained with $\eta = 0.11$.

Considering the solution of the optimization problem (3.11) with $\eta = 0.11$, the Lyapunov function matrices in (3.5) and the constant SOF controller are:

$$P_1 = \begin{bmatrix} 1.3104 & 1.3791 \\ 1.3791 & 6.1275 \end{bmatrix}, \; P_2 = \begin{bmatrix} 1.3511 & 1.5601 \\ 1.5601 & 6.9318 \end{bmatrix}, \; X = 7.9166, \; L = 15.8784.$$

For this case, several closed-loop trajectories starting within the region $\mathcal{R}_0$ are depicted in Figure 3.4. It can be noticed that all trajectories converge to the origin without leaving the region $\mathcal{R}_0 \subset \mathcal{D}$. Some divergent closed-loop trajectories initiating outside the region $\mathcal{R}_0$ are also provided to illustrate the effectiveness of the proposed local SOF control synthesis condition and the relevance of the estimation of $\mathcal{R}_0$. Figure 2.1 also depicts the estimated

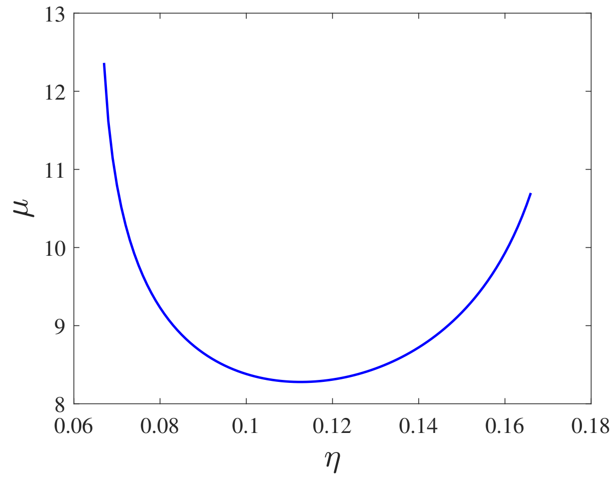

Figure 3.3 – The objective function μ obtained by solving the optimization problem (3.11) for different values of the parameter η - Example 3.4.

region $\overline{\mathcal{R}}_0$ obtained from [91, Theorem 1] with $\alpha = 0.1177$. As suggested by the authors in [91], a line grid search for α has been done with 100 points distributed over a logarithmic scale in $[10^{-4},\ 10^4]$ and we have chosen the value of α that provided the larger estimate of the DoA. As one can see, the proposed approach provides an estimate of the DoA for the origin of the closed-loop system (4.34) that contains the estimate obtained by [91]. It is worth mentioning that, unlike our approach, the technique presented in [91] has used a parameter-dependent structure in the control law that uses the information of past states. Besides that, Table 3.2 shows the computational efforts in terms of the number of scalar variables (S_v) and the number of LMI rows (L_R) required by the proposed approach and the method in [91]. Notice that the proposed technique provides a less conservative result with fewer number of LMI rows and scalar decision variables than the method in [91].

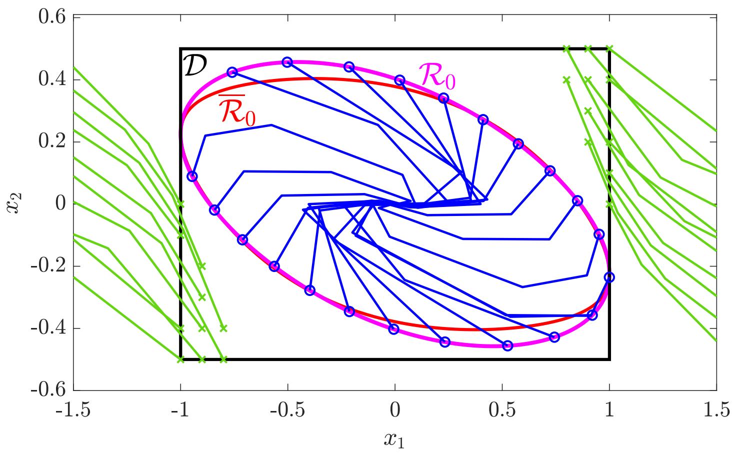

Figure 3.4 – Estimate of the domain of attraction $\mathcal{R}_0$ (magenta) obtained from the optimization problem (3.11) with $\eta = 0.11$; estimate of DoA $\overline{\mathcal{R}}_0$ (red line) obtained from [91], convergent trajectories (blue line) with initial conditions at the border of $\mathcal{R}_0$, and trajectories (green line) leaving the set $\mathcal{D}$ - Example 3.4.

Table 3.2 – Computational complexity given by the number of scalar decision variables (S_V) and the number of LMI rows (L_R) - Example 3.4.

	μ	S_V	L_R
[91, Theorem 1]	8.3275	55	81
Theorem 3.1	**8.2829**	**21**	**67**

3.4 Final remarks

This chapter has introduced new less conservative conditions to design gain-scheduled static output-feedback controllers for discrete-time nonlinear systems represented by LPV systems and TS fuzzy models. The synthesis conditions have been formulated in terms of linear matrix inequalities. One of the novelties in this Thesis is the possibility of the output matrix being parameter-dependent, without requiring any specific structure or admitting particular similarity transformations, unlike most of the approaches in the related literature. Another distinctive feature of the proposed approach is to ensure the local asymptotic stability of the origin of the closed-loop system. The proposed conditions sound to be less conservative when compared to other conditions in the literature, as illustrated by numerical examples.

4 SOF CONTROL FOR TIME-DELAYED POLYTOPIC SYSTEMS

This chapter introduces a novel delay-dependent condition for static output-feedback (SOF) control of quasi-linear parameter varying (LPV) systems and Takagi-Sugeno (TS) fuzzy models with time-varying delay. Unlike other works in the related literature, another distinctive feature of the proposed approaches is to ensure the local asymptotic stability of the origin of the closed-loop system, which is necessary due to the validity of the polytopic model obtained. This guarantees the correct operation of the closed-loop system since the set of admissible initial states is obtained inside of the domain of validity $\mathcal{D}$. The results presented in this chapter are published in [122].

4.1 Problem Statement

After a system description, this section formulates the SOF control design problem of nonlinear systems with time-varying delay represented by quasi-LPV models. However, once more as mentioned in Chapter 2, all results presented here can also be devoted to TS fuzzy models.

Consider the discrete-time linear parameter varying (LPV) system with time-varying delay:

$$\begin{aligned} x_{k+1} &= A(\alpha_k)x_k + A_d(\alpha_k)x_{k-h_k} + B(\alpha_k)u_k, \quad \forall k \in \mathbb{Z}^+ \\ y_k &= C(\alpha_k)x_k, \\ x_k &= \phi_k, \quad k \in \{-\overline{h}, \dots, 0\}, \end{aligned} \tag{4.1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the input, $y_k \in \mathbb{R}^{n_y}$ is the output, and ϕ_k is the initial condition. The parameter-dependent matrices $A(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $A_d(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $B(\alpha_k) \in \mathbb{R}^{n_x \times n_u}$, and $C(\alpha_k) \in \mathbb{R}^{n_y \times n_x}$ belong to a polytopic domain parameterized by the time-varying parameters $\alpha \in \Lambda$, defined as in (2.5).

The following assumptions are considered for system (4.1).

Assumption 4.1. *The time-varying delay h_k satisfies $\underline{h} \le h_k \le \overline{h}$, for $\forall k \in \mathbb{N}$, where the lower bound $\underline{h}$ and the upper bound $\overline{h}$ are given.*

Assumption 4.2. *The region where the LPV system* (4.1) *is valid is a polytope containing the origin described by*

$$\mathcal{D} = \{x \in \mathbb{R}^{n_x} : b_j x \le 1, \ j \in \mathbb{N}_{\le n_e}\}, \tag{4.2}$$

where n_e is the number of hyper-planes and $b_j \in \mathbb{R}^{n_x}$ defines the j-th hyper-plane.

The following gain-scheduled SOF controller is considered

$$u_k = X(\alpha_k)^{-1}L(\alpha_k)y_k \tag{4.3}$$

where $X(\alpha_k)$ and $L(\alpha_k)$ are gains to be designed. From (4.1) and (4.3), the closed-loop system is

$$x_{k+1}=\mathcal{A}(\alpha_k)x_k + A_d(\alpha_k)x_{k-h_k} \tag{4.4}$$

where $\mathcal{A}(\alpha_k) = A(\alpha_k)+B(\alpha_k)X(\alpha_k)^{-1}L(\alpha_k)C(\alpha_k)$.

Problem 4.1. *Design the gain-scheduled SOF controller* (4.3) *such that the origin of* (4.4) *is locally asymptotically stable. Moreover, determine the set of admissible initial conditions for which the state trajectories of the closed-loop system* (4.4) *converge towards the origin remaining constrained into the validity domain* $\mathcal{D}$.

4.2 Useful Lemmas

The following technical lemmas are useful to develop the design SOF controller as stated in Problem 4.1.

Lemma 4.1 (Wirtinger-based inequality [99]). *For a given symmetric positive definite matrix* $R \in S_+^n$*, integers* $b > a$*, any sequence of discrete-time variable* $x : \mathbb{Z}[a,b] \to \mathbb{R}^n$*, the following inequalities hold*

$$\sum_{i=a}^{b-1} \eta_i^\top R \eta_i \geq \frac{1}{b-a} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix}^\top \begin{bmatrix} R & 0 \\ 0 & 3\left(\frac{b-a+1}{b-a-1}\right) R \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} \tag{4.5}$$

$$\geq \frac{1}{b-a} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix}^\top \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} \tag{4.6}$$

where $\eta_i = x_i - x_{i-1}$*,* $\Theta_1 = x_b - x_a$*, and* $\Theta_2 = x_b + x_a - \frac{2}{b-a+1}\sum_{i=a}^{b} x_i$.

Lemma 4.2 (Moon's inequality [147]). *For any matrices* $R_1 \in \mathbb{S}_+^n, R_2 \in \mathbb{S}_+^n, Y_1 \in \mathbb{R}^{2n\times n}$*, and* $Y_2 \in \mathbb{R}^{2n\times n}$*, the following inequality holds*

$$\begin{bmatrix} \frac{1}{\vartheta} R_1 & 0 \\ 0 & \frac{1}{1-\vartheta} R_2 \end{bmatrix} \geq \Theta_M(\vartheta), \quad \forall \vartheta \in (0,1)$$

where

$$\Theta_M(\vartheta) = \mathrm{He}\left\{Y_1 \left[\mathrm{I}_n \quad 0_n\right] + Y_2 \left[0_n \quad \mathrm{I}_n\right]\right\} - \vartheta Y_1 R_1^{-1} Y_1^\top - (1-\vartheta) Y_2 R_2^{-1} Y_2^\top.$$

4.3 Guaranteed DoA estimation for time-delayed nonlinear systems

This section presents the main results of this chapter. First, based on an augmented Lyapunov-Krasovskii functional, a new delay-dependent LMI condition for the SOF control design is provided. In the sequel, the local asymptotic stability of the origin of the closed-loop system is presented, which ensures the correct operation of the closed-loop system since its trajectories remain enclosed in the guaranteed domain of attraction estimation obtained inside of the validity region of the LPV system.

Lemma 4.3. *For given positive integers $\underline{h}$ and $\overline{h}$, where $\underline{h} < \overline{h}$, and a scalar $\epsilon \in \mathbb{R}_{>0}$, if there exist parameter-dependent matrices $X(\alpha_k) \in \mathbb{R}^{n_u \times n_u}$, $L(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $H(\alpha_k) \in \mathbb{R}^{n_y \times n_y}$, $M_1(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $M_2(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $M_3(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $M_4(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $M_5(\alpha_k) \in \mathbb{R}^{n_x \times n_x}$, $M_6(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, $M_7(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$, positive definite matrices P, S, $D \in \mathbb{S}_+^{n_x}$, R_1, R_2, Z_1, $Z_2 \in \mathbb{S}_+^{n_x}$, and matrices T, E, $Q \in \mathbb{R}^{n_x \times n_x}$, $Y_1, Y_2 \in \mathbb{R}^{(8n_x+n_u+n_y)\times 2n_x}$, such that following inequalities hold*

$$\begin{aligned} &\mathcal{P} > 0, \\ &\begin{bmatrix} \Phi_{\alpha_k}(\underline{h}) & Y_2 \\ Y_2^\top & -\widetilde{Z}_2 \end{bmatrix} < 0, \quad \begin{bmatrix} \Phi_{\alpha_k}(\overline{h}) & Y_1 \\ Y_1^\top & -\widetilde{Z}_2 \end{bmatrix} < 0, \end{aligned} \tag{4.7}$$

where

$$\begin{aligned} \Phi_{\alpha_k}(\underline{h}) = {} & F_1^\top (\mathcal{P} + \Pi_1) F_1 - F_2^\top \mathcal{P} F_2 + \mathrm{He}\left\{\Gamma^\top(\underline{h})\mathcal{P}F_1\right\} - \mathrm{He}\left\{\Gamma^\top(\underline{h})\mathcal{P}F_2\right\} \\ & + \widetilde{R} - G_1^\top \widetilde{Z}_1 G_1 - \mathrm{He}\{Y_1 G_2 + Y_2 G_3\} + \mathrm{He}\{\mathcal{U}(\alpha_k)\mathcal{B}(\alpha_k)\}, \end{aligned} \tag{4.8}$$

$$\begin{aligned} \Phi_{\alpha_k}(\overline{h}) = {} & F_1^\top (\mathcal{P} + \Pi_1) F_1 - F_2^\top \mathcal{P} F_2 + \mathrm{He}\left\{\Gamma^\top(\overline{h})\mathcal{P}F_1\right\} - \mathrm{He}\left\{\Gamma^\top(\overline{h})\mathcal{P}F_2\right\} \\ & + \widetilde{R} - G_1^\top \widetilde{Z}_1 G_1 - \mathrm{He}\{Y_1 G_2 + Y_2 G_3\} + \mathrm{He}\{\mathcal{U}(\alpha_k)\mathcal{B}(\alpha_k)\}, \end{aligned} \tag{4.9}$$

$$\begin{aligned} \widetilde{Z}_1 &= \mathrm{diag}\left(Z_1, 3\sigma\left(\underline{h}\right) Z_1\right), \quad \widetilde{Z}_2 = \mathrm{diag}\left(Z_2, 3Z_2\right), \\ \Pi_1 &= \mathrm{diag}\left(\underline{h}^2 Z_1 + \tilde{h}^2 Z_2, 0_{2n_x}\right), \\ \widetilde{R} &= \mathrm{diag}\left(0_{n_x}, R_1, -R_1 + R_2, 0_{n_x}, -R_2, 0_{3n_x+n_u+n_y}\right), \\ v_i &= \begin{bmatrix} 0_{n_x \times (i-1)n_x} & \mathrm{I}_{n_x} & 0_{n_x \times (8-i)n_x} & 0_{n_x \times (n_u+n_y)} \end{bmatrix}, \\ F_1 &= \begin{bmatrix} v_1 - v_2 \\ v_6 - v_3 \\ v_7 + v_8 - v_4 - v_5 \end{bmatrix}, \ F_2 = \begin{bmatrix} 0_{n_x \times (8n_x+n_u+n_y)} \\ v_6 - v_2 \\ v_7 + v_8 - v_3 - v_4 \end{bmatrix}, \\ G_1 &= \begin{bmatrix} v_2 - v_3 \\ v_2 + v_3 - 2v_6 \end{bmatrix}, \ G_2 = \begin{bmatrix} v_3 - v_4 \\ v_3 + v_4 - 2v_7 \end{bmatrix}, \ G_3 = \begin{bmatrix} v_4 - v_5 \\ v_4 + v_5 - 2v_8 \end{bmatrix}, \\ \mathcal{P} &= \begin{bmatrix} P & Q & T \\ Q^\top & S & E \\ T^\top & E^\top & D \end{bmatrix}, \ \Gamma(\overline{h}) = \begin{bmatrix} v_2 \\ \underline{h}v_6 \\ (\overline{h} - \underline{h})v_7 \end{bmatrix}, \ \Gamma(\underline{h}) = \begin{bmatrix} v_2 \\ \underline{h}v_6 \\ (\overline{h} - \underline{h})v_8 \end{bmatrix}, \\ \mathcal{B}(\alpha_k) &= \begin{bmatrix} -\mathrm{I}_{n_x} & A(\alpha_k) & 0_{n_x \times n_x} & A_d(\alpha_k) & 0_{n_x \times 4n_x} & 0_{n_x \times n_x} & B(\alpha_k) \\ 0_{n_y \times n_x} & C(\alpha_k) & 0_{n_y \times n_x} & 0_{n_y \times n_x} & 0_{n_y \times 4n_x} & -\mathrm{I}_{n_y} & 0_{n_y \times n_u} \\ 0_{n_u \times n_x} & 0_{n_u \times n_x} & 0_{n_u \times n_x} & 0_{n_u \times n_x} & 0_{n_u \times 4n_x} & L(\alpha_k) & -X(\alpha_k) \end{bmatrix}, \end{aligned} \tag{4.10}$$

$$\mathcal{U}(\alpha_k) = \begin{bmatrix} M_1(\alpha_k) & B(\alpha_k)M_2(\alpha_k) & B(\alpha_k) \\ M_3(\alpha_k) & B(\alpha_k)M_4(\alpha_k) & \epsilon B(\alpha_k) \\ 0_{n_x\times n_x} & 0_{n_x\times n_y} & 0_{n_x\times n_u} \\ M_5(\alpha_k) & B(\alpha_k)M_6(\alpha_k) & \epsilon B(\alpha_k) \\ 0_{4n_x\times n_x} & 0_{4n_x\times n_y} & 0_{4n_x\times n_u} \\ 0_{n_y\times n_x} & \epsilon H(\alpha_k) & 0_{n_y\times n_u} \\ 0_{n_u\times n_x} & M_7(\alpha_k) & \mathrm{I}_{n_u} \end{bmatrix},$$

with $\tilde{h} \triangleq \overline{h} - \underline{h}$*, such that* $\sigma(\underline{h}) = 1$ *(if* $\underline{h} = 1$*) or* $\sigma(\underline{h}) = (\underline{h}+1)/(\underline{h}-1)$ *(if* $\underline{h} > 1$*). Then, the origin of the closed-loop system* (4.4) *is asymptotically stable for any integer time-varying delay* $h_k \in \{\underline{h}, \dots, \overline{h}\}$ *via the static output-feedback control law in* (4.3)*.*

Proof. Consider the Lyapunov–Krasovskii functional (LKF) candidate given by

$$W(\phi_k) = V(x_k) + U(\phi_k) \tag{4.11}$$

where

$$V(x_k) = x_k^\top P x_k,$$
$$U(\phi_k) = \sum_{i=1}^{3} U_i(\phi_k),$$

with

$$U_1 = \varpi_k^\top \widetilde{\mathcal{P}} \varpi_k,$$
$$U_2 = \sum_{i=k-\underline{h}}^{k-1} x_i^\top R_1 x_i + \sum_{i=k-\overline{h}}^{k-\underline{h}-1} x_i^\top R_2 x_i,$$
$$U_3 = \underline{h} \sum_{i=-\underline{h}+1}^{0} \sum_{j=k+i}^{k} \eta_j^\top Z_1 \eta_j + \tilde{h} \sum_{i=-\overline{h}+1}^{-\underline{h}} \sum_{j=k+i}^{k} \eta_j^\top Z_2 \eta_j,$$

being

$$\varpi_k = \begin{bmatrix} x_k \\ \sum\limits_{i=k-\underline{h}}^{k-1} x_i \\ \sum\limits_{i=k-\overline{h}}^{k-\underline{h}-1} x_i \end{bmatrix}, \quad \widetilde{\mathcal{P}} = \begin{bmatrix} 0_{n_x} & Q & T \\ Q^\top & S & E \\ T^\top & E^\top & D \end{bmatrix},$$

and $\eta_i = x_i - x_{i-1}$. Notice that the LKF in (4.11) is positive definite, since $\mathcal{P} > 0$, $R_1 > 0$, $R_2 > 0$, $Z_1 > 0$, and $Z_2 > 0$. Next, it is intended to derive an upper bound to $\Delta W(x_k)$ by employing the following augmented vector

$$\zeta_k = \begin{bmatrix} x_{k+1}^\top & x_k^\top & x_{k-\underline{h}}^\top & x_{k-h_k}^\top & x_{k-\overline{h}}^\top & \nu_{1_k}^\top & \nu_{2_k}^\top & \nu_{3_k}^\top & y_k^\top & u_k^\top \end{bmatrix}^\top \tag{4.12}$$

where

$$\nu_{1_k} = \frac{1}{\underline{h}+1} \sum_{i=k-\underline{h}}^{k} x_i, \quad \nu_{2_k} = \frac{1}{h_k - \underline{h}+1} \sum_{i=k-h_k}^{k-\underline{h}} x_i, \quad \nu_{3_k} = \frac{1}{\overline{h} - h_k + 1} \sum_{i=k-\overline{h}}^{k-h_k} x_i.$$

Given that

$$\begin{aligned}\varpi_k = & \begin{bmatrix} 0 \\ -x_k + \nu_{1_k} \\ \nu_{2_k} + \nu_{3_k} - x_{k-\underline{h}} - x_{k-h_k} \end{bmatrix} + \begin{bmatrix} x_k \\ \underline{h}\nu_{1_k} \\ (h_k - \underline{h})\,\nu_{2_k} + \left(\overline{h} - h_k\right)\nu_{3_k} \end{bmatrix} \\ = & \ (F_2 + \Gamma(h_k))\,\zeta_k,\end{aligned}$$

and

$$\begin{aligned}\varpi_{k+1} = & \begin{bmatrix} x_{k+1} - x_k \\ -x_{k-\underline{h}} + \nu_{1_k} \\ \nu_{2_k} + \nu_{3_k} - x_{k-h_k} - x_{k-\overline{h}} \end{bmatrix} + \begin{bmatrix} x_k \\ \underline{h}\nu_{1_k} \\ (h_k - \underline{h})\,\nu_{2_k} + \left(\overline{h} - h_k\right)\nu_{3_k} \end{bmatrix} \\ = & \ (F_1 + \Gamma(h_k))\,\zeta_k,\end{aligned}$$

where

$$\Gamma(h_k) = \begin{bmatrix} v_2 \\ \underline{h}v_6 \\ (h_k - \underline{h})v_7 + (\overline{h} - h_k)v_8 \end{bmatrix}.$$

Then $\Delta(V(x_k) + U_1(\phi_k))$ can be written as

$$\begin{aligned}\Delta(V(x_k) + U_1(\phi_k)) = \zeta_k^\top \Big[& (F_1 + \Gamma(h_k))^\top \mathcal{P}\,(F_1 + \Gamma(h_k)) \\ & - (F_2 + \Gamma(h_k))^\top \mathcal{P}\,(F_2 + \Gamma(h_k))\Big]\,\zeta_k.\end{aligned} \tag{4.13}$$

Moreover, the computation of $\Delta U_2(\phi_k)$ leads to

$$\Delta U_2(\phi_k) = x_k^\top R_1 x_k + x_{k-\underline{h}}^\top (R_2 - R_1)\, x_{k-\underline{h}} - x_{k-\overline{h}}^\top R_2 x_{k-\overline{h}}$$

which can be written as

$$\Delta U_2(\phi_k) = \zeta_k^\top \tilde{R} \zeta_k.$$

Finally, the computation of $\Delta U_3(\phi_k)$ results in

$$\Delta U_3(\phi_k) = \eta_{k+1}^\top \left(\underline{h}^2 Z_1 + \tilde{h}^2 Z_2\right)\eta_{k+1} - \underline{h}\sum_{i=k-\underline{h}+1}^{k} \eta_i^\top Z_1 \eta_i - \tilde{h}\sum_{i=k-\overline{h}+1}^{k-\underline{h}} \eta_i^\top Z_2 \eta_i, \tag{4.14}$$

that can be rewritten as

$$\begin{aligned}\Delta U_3(\phi_k)) = & \ \eta_{k+1}^\top \left(\underline{h}^2 Z_1 + \tilde{h}^2 Z_2\right)\eta_{k+1} - \underline{h}\sum_{i=k-\underline{h}+1}^{k} \eta_i^\top Z_1 \eta_i \\ & - \tilde{h}\sum_{i=k-\overline{h}+1}^{k-h_k} \eta_i^\top Z_2 \eta_i - \tilde{h}\sum_{i=k-h_k+1}^{k-\underline{h}} \eta_i^\top Z_2 \eta_i.\end{aligned} \tag{4.15}$$

Now, based on Lemma 4.1, an upper-bound can be derived to the second term of (4.15) regarding (4.5), and upper-bounds for the two last terms of (4.15) with (4.6). Thus, we have

$$
\begin{aligned}
-\underline{h}\sum_{i=k-\underline{h}+1}^{k}\eta_i^\top Z_1\eta_i &\leq -\begin{bmatrix}\Theta_1\\ \Theta_2\end{bmatrix}^\top \begin{bmatrix}Z_1 & 0\\ 0 & 3\left(\frac{\underline{h}+1}{\underline{h}-1}\right)Z_1\end{bmatrix}\begin{bmatrix}\Theta_1\\ \Theta_2\end{bmatrix},\\
-\tilde{h}\sum_{i=k-h_k+1}^{k-\underline{h}}\eta_i^\top Z_2\eta_i &\leq \frac{-\tilde{h}}{h_k-\underline{h}}\begin{bmatrix}\bar{\Theta}_1\\ \bar{\Theta}_2\end{bmatrix}^\top \begin{bmatrix}Z_2 & 0\\ 0 & 3Z_2\end{bmatrix}\begin{bmatrix}\bar{\Theta}_1\\ \bar{\Theta}_2\end{bmatrix},\\
-\tilde{h}\sum_{i=k-\overline{h}+1}^{k-h_k}\eta_i^\top Z_2\eta_i &\leq \frac{-\tilde{h}}{\overline{h}-h_k}\begin{bmatrix}\hat{\Theta}_1\\ \hat{\Theta}_2\end{bmatrix}^\top \begin{bmatrix}Z_2 & 0\\ 0 & 3Z_2\end{bmatrix}\begin{bmatrix}\hat{\Theta}_1\\ \hat{\Theta}_2\end{bmatrix},
\end{aligned}
$$

with

$$
\begin{aligned}
&\Theta_1 = x_k - x_{k-\underline{h}},\\
&\Theta_2 = x_k + x_{k-\underline{h}} - \frac{2}{\underline{h}+1}\sum_{i=k-\underline{h}}^{k} x_i,\\
&\bar{\Theta}_{1=} = x_{k-\underline{h}} - x_{k-h_k},\\
&\bar{\Theta}_2 = x_{k-\underline{h}} + x_{k-h_k} - \frac{2}{h_k-\underline{h}+1}\sum_{i=k-h_k}^{k-\underline{h}} x_i,\\
&\hat{\Theta}_1 = x_{k-h_k} - x_{k-\overline{h}},\\
&\hat{\Theta}_2 = x_{k-h_k} + x_{k-\overline{h}} - \frac{2}{\overline{h}-h_k+1}\sum_{i=k-\overline{h}}^{k-h_k} x_i.
\end{aligned}
$$

Then, with the definitions of matrices F_1, G_1, G_2, and G_3 in (4.10), the following upper-bound is derived for $\Delta U_3(\phi_k))$:

$$
\Delta U_3(\phi_k)) \leq \zeta_k^\top \left\{F_1^\top \Pi_1 F_1 - G_1^\top \widetilde{Z}_1 G_1 - \begin{bmatrix}G_2\\ G_3\end{bmatrix}^\top \begin{bmatrix}\frac{\tilde{h}}{h_k-\underline{h}}\widetilde{Z}_2 & 0_{2n_x}\\ 0_{2n_x} & \frac{\tilde{h}}{\overline{h}-h_k}\widetilde{Z}_2\end{bmatrix}\begin{bmatrix}G_2\\ G_3\end{bmatrix}\right\}\zeta_k. \tag{4.16}
$$

Re-injecting the expressions of $\Delta V(x_k)$, $\Delta U_1(\phi_k)$, $\Delta U_2(\phi_k)$, and the upper bound of $\Delta U_3(\phi_k)$, into the expression of $W(\phi_k)$, it leads to

$$
\begin{aligned}
\Delta W(\phi_k) \leq \zeta_k^\top &\Big[(F_1+\Gamma(h_k))^\top \mathcal{P}(F_1+\Gamma(h_k))\\
&- (F_2+\Gamma(h_k))^\top \mathcal{P}(F_2+\Gamma(h_k)) + \widetilde{R} + F_1^\top \Pi_1 F_1\Big]\zeta_k\\
&+ \zeta_k^\top\left[-G_1^\top \widetilde{Z}_1 G_1 - \begin{bmatrix}G_2\\ G_3\end{bmatrix}^\top \begin{bmatrix}\frac{\tilde{h}}{h_k-\underline{h}}\widetilde{Z}_2 & 0_{2n_x}\\ 0_{2n_x} & \frac{\tilde{h}}{\overline{h}-h_k}\widetilde{Z}_2\end{bmatrix}\begin{bmatrix}G_2\\ G_3\end{bmatrix}\right]\zeta_k.
\end{aligned} \tag{4.17}
$$

Hence, it follows from Lemma 4.2, with $\vartheta = \frac{(h_k-\underline{h})}{\tilde{h}}$, that

$$
\Delta W(\phi_k) \leq \zeta_k^\top \Xi(h_k)\zeta_k \tag{4.18}
$$

where

$$\begin{aligned}\Xi(h_k) = {} & F_1^\top(\mathcal{P}+\Pi_1)F_1 + \mathrm{He}\left\{\Gamma^\top(h_k)\mathcal{P}F_1\right\} - F_2^\top\mathcal{P}F_2 - \mathrm{He}\left\{\Gamma^\top(h_k)\mathcal{P}F_2\right\} \\ & + \widetilde{R} - G_1^\top\widetilde{Z}_1G_1 - \mathrm{He}\{Y_1G_2+Y_2G_3\} + \vartheta Y_1\widetilde{Z}_2^{-1}Y_1^\top + (1-\vartheta)Y_2\widetilde{Z}_2^{-1}Y_2^\top.\end{aligned} \tag{4.19}$$

Consider

$$\widetilde{\mathcal{B}}(\alpha_k)=\begin{bmatrix} -\mathrm{I}_{n_x} & A(\alpha_k) & 0_{n_x\times n_x} & A_d(\alpha_k) & 0_{n_x\times 4n_x} & 0_{n_x\times n_y} & B(\alpha_k) \\ 0_{n_y\times n_x} & C(\alpha_k) & 0_{n_y\times n_x} & 0_{n_y\times n_x} & 0_{n_y\times 4n_x} & -\mathrm{I}_{n_y} & 0_{n_y\times n_u} \\ 0_{n_u\times n_x} & 0_{n_u\times n_x} & 0_{n_u\times n_x} & 0_{n_u\times n_x} & 0_{n_u\times 4n_x} & X(\alpha_k)^{-1}L(\alpha_k) & -\mathrm{I}_{n_u} \end{bmatrix} \tag{4.20}$$

such that $\widetilde{\mathcal{B}}(\alpha_k)\zeta_k = 0$. Considering

$$\widetilde{\mathcal{U}}(\alpha_k) = \begin{bmatrix} M_1(\alpha_k) & B(\alpha_k)M_2(\alpha_k) & B(\alpha_k)X(\alpha_k) \\ M_3(\alpha_k) & B(\alpha_k)M_4(\alpha_k) & \epsilon B(\alpha_k)X(\alpha_k) \\ 0_{n_x\times n_x} & 0_{n_x\times n_y} & 0_{n_x\times n_u} \\ M_5(\alpha_k) & B(\alpha_k)M_6(\alpha_k) & \epsilon B(\alpha_k)X(\alpha_k) \\ 0_{4n_x\times n_x} & 0_{4n_x\times n_y} & 0_{4n_x\times n_u} \\ 0_{n_y\times n_x} & \epsilon H(\alpha_k) & 0_{n_y\times n_u} \\ 0_{n_u\times n_x} & M_7(\alpha_k) & X(\alpha_k) \end{bmatrix} \tag{4.21}$$

and using Finsler's lemma arguments (see [148, Lemma 2]), the negative definiteness of (4.18) is ensured if

$$\Xi(h_k) + \widetilde{\mathcal{U}}(\alpha_k)\widetilde{\mathcal{B}}(\alpha_k) + \widetilde{\mathcal{B}}(\alpha_k)^\top\widetilde{\mathcal{U}}(\alpha_k)^\top < 0. \tag{4.22}$$

Since the condition (4.22) is affine with respect to h_k, it can be satisfied for all $h_k \in \{\underline{h},\ldots,\overline{h}\}$ if it satisfied at the vertices of the interval $h_k \in \{\underline{h},\ldots,\overline{h}\}$. Therefore, we have

$$\Xi(\underline{h}) + \widetilde{\mathcal{U}}(\alpha_k)\widetilde{\mathcal{B}}(\alpha_k) + \widetilde{\mathcal{B}}(\alpha_k)^\top\widetilde{\mathcal{U}}(\alpha_k)^\top < 0, \tag{4.23}$$

$$\Xi(\overline{h}) + \widetilde{\mathcal{U}}(\alpha_k)\widetilde{\mathcal{B}}(\alpha_k) + \widetilde{\mathcal{B}}(\alpha_k)^\top\widetilde{\mathcal{U}}(\alpha_k)^\top < 0, \tag{4.24}$$

with

$$\begin{aligned}\Xi(\underline{h}) = {} & F_1^\top(\mathcal{P}+\Pi_1)F_1 + \mathrm{He}\left\{\Gamma^\top(h_k)\mathcal{P}F_1\right\} - F_2^\top\mathcal{P}F_2 - \mathrm{He}\left\{\Gamma^\top(h_k)\mathcal{P}F_2\right\} \\ & + \widetilde{R} - G_1^\top\widetilde{Z}_1G_1 - \mathrm{He}\{Y_1G_2+Y_2G_3\} + Y_2\widetilde{Z}_2^{-1}Y_2^\top, \\ \Xi(\overline{h}) = {} & F_1^\top(\mathcal{P}+\Pi_1)F_1 + \mathrm{He}\left\{\Gamma^\top(h_k)\mathcal{P}F_1\right\} - F_2^\top\mathcal{P}F_2 - \mathrm{He}\left\{\Gamma^\top(h_k)\mathcal{P}F_2\right\} \\ & + \widetilde{R} - G_1^\top\widetilde{Z}_1G_1 - \mathrm{He}\{Y_1G_2+Y_2G_3\} + Y_1\widetilde{Z}_2^{-1}Y_1^\top.\end{aligned}$$

Finally, it follows from Schur Complement in (4.23) and (4.24) that

$$\begin{bmatrix} \Phi_{\alpha_k}(\underline{h}) & Y_2 \\ Y_2^\top & -\widetilde{Z}_2 \end{bmatrix} < 0, \quad \begin{bmatrix} \Phi_{\alpha_k}(\overline{h}) & Y_1 \\ Y_1^\top & -\widetilde{Z}_2 \end{bmatrix} < 0, \tag{4.25}$$

with $\Phi_{\alpha_k}(\underline{h})$ and $\Phi_{\alpha_k}(\overline{h})$ defined in (4.8) and (4.9), respectively. Notice that (4.25) is exactly (4.7), which is affine and, consequently, convex concerning h_k, implying that (4.7) is negative definite for all $h_k \in \{\underline{h}, \ldots, \overline{h}\}$. Therefore, if LMIs (4.7) are feasible, the origin of the closed-loop system (4.4) is asymptotically stable for any integer delay $h_k \in \{\underline{h}, \ldots, \overline{h}\}$. This concludes the proof.

□

Remark 4.1. *Notice that the condition proposed in Lemma 4.3 does not require any structural constraint in the output matrix $C(\alpha_k)$ nor any similarity transformations. Moreover, no iterative algorithms are necessary for solving the proposed conditions.*

Remark 4.2. *Theorem 4.3 provides a delay-dependent condition to ensure the asymptotic stability of the origin of the closed-loop system* (4.4) *by employing the Wirtinger-based integral inequality and the Moon's inequality lemma. As pointed out by [147], the Moon's inequality inequality given in Lemma 4.2 allows deriving delay-dependent conditions which are related to the time-varying delay and usually lead to less conservative results.*

The following theorem characterizes the regions $\mathcal{R}_0$ and $\mathcal{R}$, guaranteeing that the trajectories of the closed-loop system, emanating from $\mathcal{R}_0$ converge to the origin without leaving $\mathcal{R}$.

Theorem 4.1. *Consider the closed-loop system* (4.4). *Assume that conditions in* (4.7) *are satisfied and* (4.11) *is a Lyapunov-Krasovskii functional that certifies the asymptotic stability of the origin of the closed-loop system* (4.4). *Let the sets*[1]

$$\mathcal{R}_0 \triangleq \{\phi_k \in \mathcal{C}^{n_x} : \|\phi_k\| \leq \gamma_1,\ \|\Delta\phi_k\| \leq \gamma_2\} \tag{4.26}$$

and

$$\mathcal{R} \triangleq \{x_k \in \mathbb{R}^{n_x} : V(x_k) \leq 1\}, \tag{4.27}$$

where γ_1 and γ_2 are scalars satisfying

$$\kappa_1\gamma_1^2 + \kappa_2\gamma_2^2 = 1$$

with

$$\begin{aligned} \kappa_1 =& \left(1 + \underline{h}^2 + \tilde{h}^2\right)(\lambda_{\max}(\mathcal{P})) + \underline{h}\lambda_{\max}(R_1) + \tilde{h}\lambda_{\max}(R_2), \\ \kappa_2 =& \frac{1}{2}\left[\underline{h}^2(1+\underline{h})\,(\lambda_{\max}(Z_1)) + \tilde{h}^2(1+\underline{h}+\overline{h})\,(\lambda_{\max}(Z_2))\right]. \end{aligned} \tag{4.28}$$

Then, for every initial condition, $\phi_k \in \mathcal{R}_0$, the state trajectory x_k, for all $k > 0$, converges asymptotically to the origin and remains confined in the region $\mathcal{R}$.

[1] For any $\phi \in \mathcal{C}^{n_x}$, $\|\phi_k\| = \max_{k\in\{-\overline{h},\ldots,0\}} \|\phi_k\|_2$ and $\|\Delta\phi_k\| = \max_{k\in\{-\overline{h},\ldots,0\}} \|\Delta\phi_k\|_2$, being $\Delta\phi_k = \phi_{k+1} - \phi_k$.

Proof. Consider that (4.11) is a Lyapunov-Krasovskii functional for the closed-loop system (4.4). At $k = 0$, one has

$$\begin{aligned} W(\phi_0) &\leq (1 + \underline{h}^2 + \tilde{h}^2)\left(\lambda_{\max}(\mathcal{P})\right)\|\phi_0\|_2^2 + \underline{h}\lambda_{\max}(R_1)\|\phi_0\|_2^2 + \tilde{h}\lambda_{\max}(R_2)\|\phi_0\|_2^2 \\ &\quad + 0.5\underline{h}^2(1+\underline{h})\left(\lambda_{\max}(Z_1)\right)\|\Delta\phi_0\|_2^2 + 0.5\tilde{h}^2(1+\underline{h}+\overline{h})\left(\lambda_{\max}(Z_2)\right)\|\Delta\phi_0\|_2^2 \\ &\leq \kappa_1\|\phi_0\|_2^2 + \kappa_2\|\Delta\phi_0\|_2^2 \end{aligned} \tag{4.29}$$

with κ_1 and κ_2 defined as in (4.28). Therefore, one has

$$W(\phi_0) \leq \kappa_1\gamma_1^2 + \kappa_2\gamma_2^2. \tag{4.30}$$

From Lemma 4.3, one has that $W(\phi_{k+1}) < W(\phi_k)$ and, consequently,

$$W(\phi_k) < W(\phi_0) \leq \kappa_1\gamma_1^2 + \kappa_2\gamma_2^2.$$

Given that $W(\phi_k) = V(x_k) + U(\phi_k)$ and $\kappa_1\gamma_1^2 + \kappa_2\gamma_2^2 = 1$, then

$$V(x_k) \leq W(\phi_k) < W(\phi_0) \leq 1.$$

Therefore,

$$V(x_k) \leq 1.$$

Thus, for every initial condition $\phi_k \in \mathcal{R}_0$, $k \in \{-\overline{h}, \ldots, 0\}$, the trajectories converge to the origin without leaving the region $\mathcal{R}$. □

4.4 Enlargement of the admissible initial condition set

Although the result in Theorem 4.1 ensures the local asymptotic stability of the origin of the closed-loop system (4.4), it is of interest to obtain an enlarged set of admissible initial conditions. Similar to [149], the following optimization problem is considered to enlarge the regions $\mathcal{R}_0$ and $\mathcal{R}$:

$$\begin{aligned} &\min\ \varrho \\ &\text{s.t.}: \begin{cases} \text{LMIs in (4.7)} \\ \begin{bmatrix} P & \star \\ b_j & 1 \end{bmatrix} \geq 0, \quad j \in \mathbb{N}_{\leq n_e} \\ \mathcal{P} \leq \operatorname{diag}(J_1, J_2, J_3) \\ J_1 \leq p_1\mathrm{I}_{n_x},\ J_2 \leq p_2\mathrm{I}_{n_x},\ J_3 \leq p_3\mathrm{I}_{n_x} \\ R_1 \leq r_1\mathrm{I}_{n_x},\ R_2 \leq r_2\mathrm{I}_{n_x} \\ Z_1 \leq \beta_1\mathrm{I}_{n_x},\ Z_2 \leq \beta_2\mathrm{I}_{n_x} \end{cases} \end{aligned} \tag{4.31}$$

where $J_1, J_2, J_3 \in \mathbb{S}_+^{n_x}$, $p_1, p_2, p_3, r_1, r_2, \beta_1, \beta_2 \in \mathbb{R}_{\geq 0}$, and

$$\varrho = p_1 + \underline{h}^2 p_2 + \tilde{h}^2 p_3 + \underline{h} r_1 + \tilde{h} r_2 + 0.5(\underline{h}^2)(1+\underline{h})\beta_1 + 0.5(\tilde{h}^2)(1+\underline{h}+\overline{h})\beta_2. \tag{4.32}$$

If the optimization problem is feasible, κ_1 and κ_2 are determined by

$$\begin{aligned}\kappa_1 &= \lambda_{\max}(J_1) + \underline{h}^2\lambda_{\max}(J_2) + \tilde{h}^2\lambda_{\max}(J_3) + \underline{h}\lambda_{\max}(R_1) + \tilde{h}\lambda_{\max}(R_2)\\ \kappa_2 &= \frac{1}{2}\underline{h}^2(1+\underline{h})\left(\lambda_{\max}(Z_1)\right) + \frac{1}{2}\tilde{h}^2(1+\underline{h}+\overline{h})\left(\lambda_{\max}(Z_2)\right).\end{aligned} \tag{4.33}$$

The second constraint of the optimization problem ensures that $\mathcal{R} \subset \mathcal{D}$, with $\mathcal{D}$ given as in (3.2) (see Chapter 3 for details). In addition, the admissible bounds of $\mathcal{R}_0$, γ_1 and γ_2, are determined by $\kappa_1\gamma_1^2 + \kappa_2\gamma_2^2 = 1$. In particular, if $\gamma_1 = \gamma_2 = \gamma$, then $\gamma = 1/\sqrt{\kappa_1 + \kappa_2}$. Finally, as the LMIs in (4.7) are in infinite-dimensional form, the relaxation considered in Appendix A is employed to obtain a finite set of solvable LMIs. Thus, (4.31) turns into a convex optimization problem. The optimization problem is solved in Matlab using the parser Yalmip and the solver MOSEK.

4.5 Numerical Examples

Two examples are presented in this section to illustrate the effectiveness of the proposed local delay-dependent conditions for SOF control design of nonlinear systems represented by quasi-LPV models.

Example 4.1. Consider the following nonlinear system with time-varying delay:

$$\begin{aligned}x_{1,k+1} &= -0.1x_{1,k} + 0.5x_{2,k} + 0.1x_{1,k-h_k} + 0.1x_{1,k}^2 u_k\\ x_{2,k+1} &= x_{1,k} + 0.5x_{1,k}^3 + x_{2,k} + 0.1x_{1,k}^2 x_{2,k-h_k} + u_k\\ y_{1,k} &= x_{1,k},\end{aligned} \tag{4.34}$$

where $\mathcal{D} = \{x \in \mathbb{R}^2 : |x_1| \leq 1, |x_2| \leq 1.5\}$. Applying the sector nonlinearity approach [144, Chapter 2] with $\rho(x_k) = x_1^2 \in [0,1]$, the quasi-LPV or TS fuzzy model with 2 vertices is obtained with the matrices:

$$\begin{aligned}&A_1 = \begin{bmatrix}-0.1 & 0.5\\ 1 & 1\end{bmatrix},\ A_2 = \begin{bmatrix}-0.1 & 0.5\\ 1.5 & 1\end{bmatrix},\ A_{d,1} = \begin{bmatrix}0.1 & 0\\ 0 & 0\end{bmatrix},\\ &A_{d,2} = \begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ B_1 = \begin{bmatrix}0\\ 1\end{bmatrix},\ B_2 = \begin{bmatrix}0.1\\ 1\end{bmatrix}, C_1 = C_2 = \begin{bmatrix}1 & 0\end{bmatrix}.\end{aligned} \tag{4.35}$$

The time-varying parameters or membership functions can be calculated as

$$\alpha_{1,k} = 1 - x_{1,k}^2, \text{ and } \alpha_{2,k} = 1 - \alpha_{1,k}.$$

The aim here is to design the SOF controller (4.3) obtaining an enlarged set of admissible initial conditions such that the origin of the closed-loop system (4.34) with (4.3) is asymptotically stable. The first experiment is to evaluate the influence of the maximum delay bound $\overline{h}$ over the estimated set of admissible initial conditions, which is related to the minimization of ϱ. The values of ϱ obtained by solving the optimization problem (4.31) for different values of $\overline{h}$,

Table 4.1 – Minimum ϱ for different values of $\overline{h}$ with $\underline{h} = 1$ and $\epsilon = 0.1$ such that the optimization problem (4.31) is feasible.

$\overline{h}$	2	3	4	5	6	7
ϱ	1.572	1.667	1.791	1.956	2.208	2.681

with $\underline{h} = 1$ and $\epsilon = 0.1$, are presented in Table 4.1. The maximum delay for feasibility is $\overline{h} = 7$. As the value of ϱ increases as $\overline{h}$ increases, the size of $\mathcal{R}_0$ is reduced for larger delays $\overline{h}$.

For $\underline{h} = 1$, $\overline{h} = 3$, and $\epsilon = 0.1$, the SOF control gains obtained with the optimization problem (4.31) are

$$L_1 = -0.3680, L_2 = -0.6523, X_1 = 0.2582, X_2 = 0.2764.$$

From (4.33), $\kappa_1 = 1.5667$ and $\kappa_2 = 0.1007$ are obtained, and for $\gamma_1 = \gamma_2 = \gamma$, one obtains $\gamma = 0.7744$. The set of admissible initial conditions for the closed-loop system (4.34) is shown in Figure 4.1 with several closed-loop trajectories initiating inside of $\mathcal{R}_0$.

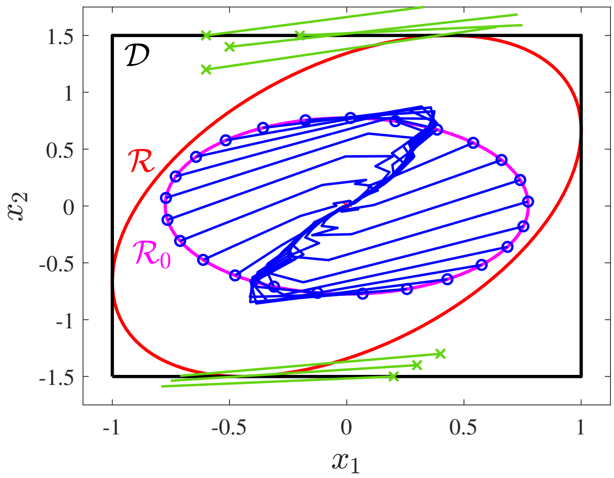

Figure 4.1 – Set of admissible initial conditions $\mathcal{R}_0 \subset \mathcal{R} \subset \mathcal{D}$ for $\underline{h} = 1$ and $\overline{h} = 3$ with convergent trajectories (in blue) initiating at the border of $\mathcal{R}_0$ and divergent trajectories (in green) leaving the set $\mathcal{D}$ with initial conditions denoted by "$\times$".

Notice that all these trajectories converge to the origin without leaving the region $\mathcal{R} \subset \mathcal{D}$. Also, some divergent closed-loop trajectories initiating outside the region $\mathcal{R}$, but inside of the modeling region $\mathcal{D}$, are also provided to illustrate the importance of performing a local analysis as proposed here. All simulations were performed considering the time-varying delay $h_k = \text{round}\,(2 + \sin(0.5k\pi))$ and the following initial condition $\phi_0 = x_0$,

$$\phi_{-3} = \begin{bmatrix} 0.2702 \\ 0.4207 \end{bmatrix}, \phi_{-2} = \begin{bmatrix} -0.2081 \\ 0.4546 \end{bmatrix}, \phi_{-1} = \begin{bmatrix} -0.4950 \\ 0.0706 \end{bmatrix}. \tag{4.36}$$

For further illustrations, consider the initial condition $x_0 = [0.7743\ 0.0130]^\top \in \mathcal{R}_0$, ϕ_0 as in (4.36), and time-varying delay $h_k = \text{round}\,(2 + \sin(0.5k\pi))$. The (convergent) state trajectory of the closed-loop system, the control input signal, the output trajectory, and the time-varying delay evolution are depicted in Figure 4.2. It illustrates the effectiveness of the proposed local SOF control design condition.

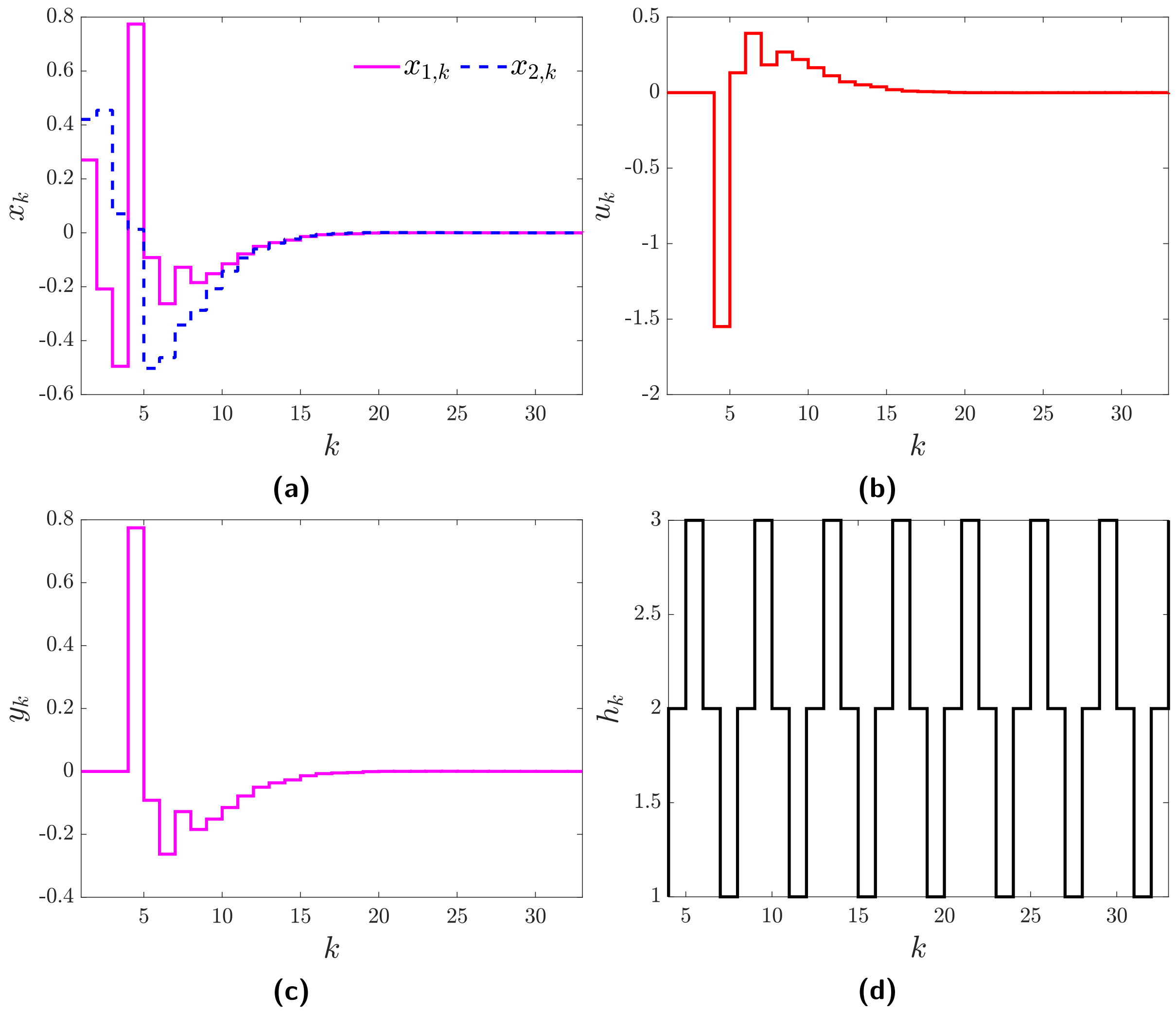

Figure 4.2 – (a) State trajectories for the closed-loop system with initial condition $x_0 = [0.7743\ 0.0130]^\top$, $x_{1,k}$ (straight magenta line), $x_{2,k}$ (blue dashed line); (b) trajectory of the control input u_k; (c) trajectory of the output y_k; (d) temporal evolution of the time-varying delay – Example 4.1.

Example 4.2. Consider the electronic circuit system depicted in Figure 4.3 which can be described for the following dynamical equations [150]:

$$
\begin{aligned}
x_{1,k+1} &= x_{k+1} - \frac{T_s}{R_1\overline{C}_1}x_{1,k-\tau_k} + \frac{T_s}{10R_2\overline{C}_1}x_{1,k}x_{2,k} - \frac{T_s}{R_3\overline{C}_1}u_k \\
x_{2,k+1} &= x_{2,k} - \frac{T_s}{R_5\overline{C}_2}x_{2,k} + \frac{T_s}{10R_4\overline{C}_2}x_{1,k-\tau_k} \\
x_{3,k+1} &= x_{3,k} - \frac{T_s}{R_7\overline{C}_3}x_{1,k} - \frac{T_s}{R_6\overline{C}_3}x_{3,k} + \frac{T_s}{10R_8\overline{C}_3}x_{2,k}^2 \\
y_k &= x_{2,k},
\end{aligned}
\tag{4.37}
$$

where x_j, $j \in \mathbb{N}_{\leq 3}$, are the voltages on the capacitors with $\mathcal{D} = \{x \in \mathbb{R}^2 : |x_1| \leq 0.8, |x_2| \leq 0.7, |x_3| \leq 1.5\}$, u is the voltage source, and T_s is the sampling period. The parameters are $R_1 = 30\Omega$, $R_2 = 66.667\Omega$, $R_3 = 100\Omega$, $R_4 = 5\Omega$, $R_5 = 1000\Omega$, $R_6 = 1100\Omega$, $R_7 = R_8 = 100\Omega$, $\overline{C}_1 = \overline{C}_2 = \overline{C}_3 = 10$mF.

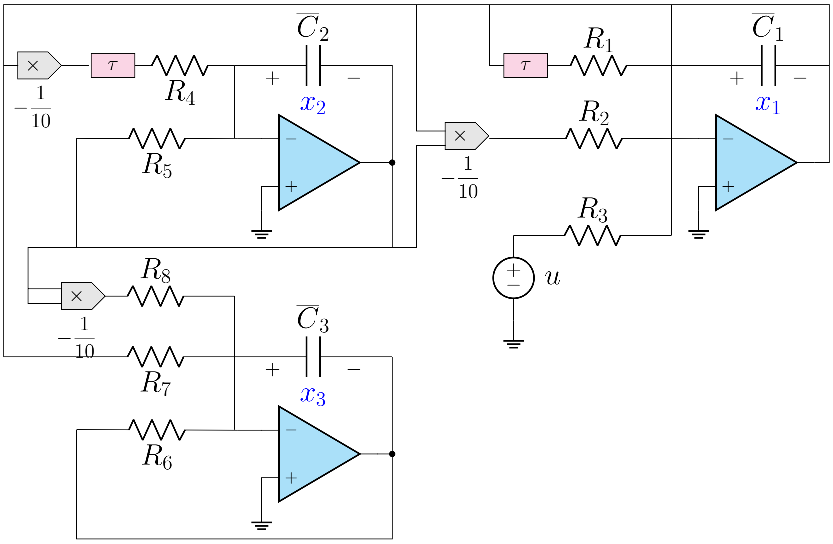

Figure 4.3 – Diagram of the electronic circuit system - Example 4.2.

Defining $\rho(x_k) = x_{2,k} \in [-0.7, 0.7]$ as the scheduling parameter, the system (4.37) can be rewritten as in (4.1) with

$$A(\alpha_k) = \begin{bmatrix} 1 + \frac{T_s}{10R_2\overline{C}_1}\rho(x_k) & 0 & 0 \\ 0 & 1 - \frac{T_s}{R_5\overline{C}_2} & 0 \\ \frac{-T_s}{R_7\overline{C}_3} & \frac{T_s}{10R_8\overline{C}_3}\rho(x_k) & 1 - \frac{T_s}{R_6\overline{C}_3} \end{bmatrix}, \ B = \begin{bmatrix} \frac{-T_s}{R_3\overline{C}_1} \\ 0 \\ 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} \frac{-T_s}{R_1\overline{C}_1} & 0 & 0 \\ \frac{T_s}{10R_4\overline{C}_2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

Considering $T_s = 0.1$ and applying the sector nonlinearity approach [144, Chapter 2], the quasi-LPV model with 2 vertices is obtained with the matrices:

$$A_1 = \begin{bmatrix} 0.9895 & 0 & 0 \\ 0 & 0.99 & 0 \\ -0.1 & -0.07 & 0.9909 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.0105 & 0 & 0 \\ 0 & 0.99 & 0 \\ -0.1 & 0.07 & 0.9909 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -0.3333 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 \\ 0 \\ 0 \end{bmatrix}.$$

The corresponding time-varying parameters are given by

$$\alpha_{1,k} = \frac{0.7 - x_{2,k}}{1.4}, \quad \alpha_{2,k} = 1 - \alpha_{1,k}.$$

In this example, the goal is to find a gain-scheduled SOF controller obtaining an enlarged set of admissible initial conditions such that the origin of the closed-loop system (4.37) is asymptotically stable. Considering $\underline{h} = 1$ and $\overline{h} = 2$, Figure 3.3 depicts the objective function ϱ concerning the scalar parameter ϵ obtained solving the optimization (4.31). Recall that for larger values of ϱ, the set of admissible initial conditions estimation $\mathcal{R}_0$ tends to reduce. As can be noticed in Figure 4.4, the results are enhanced with the scalar parameter search. In this case, the minimum $\varrho = 4.0291$ is reached with $\epsilon = 0.9$.

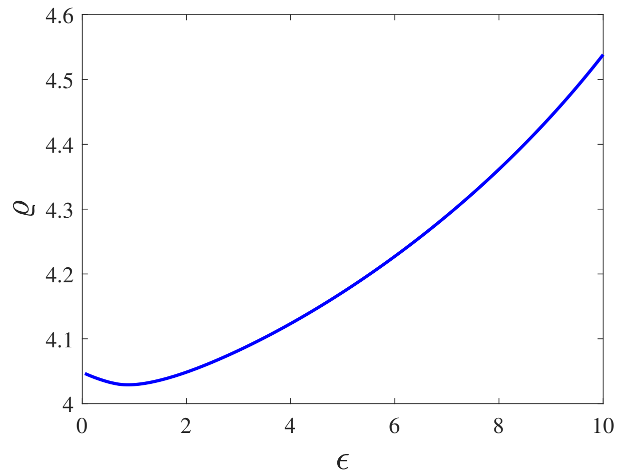

Figure 4.4 – The objective function ϱ obtained by solving the optimization problem (4.31) for different values of the scalar parameter ϵ with $\underline{h} = 1$ and $\overline{h} = 2$ - Example 4.2.

Therefore, for $\underline{h} = 1$ and $\overline{h} = 2$, and $\epsilon = 0.9$, the SOF control gains obtained with the optimization problem (4.31) are

$$X_1 = 0.0816,\ X_2 = 0.0926,\ L_1 = 0.0727,\ L_2 = 0.0838.$$

From (4.33), $\kappa_1 = 3.2286$ and $\kappa_2 = 0.8003$ are obtained, and for $\gamma_1 = \gamma_2 = \rho$, one obtains $\rho = 0.4982$. The set of admissible initial conditions for the closed-loop system (4.37) is shown in Figure 4.5 with several closed-loop trajectories initiating inside of $\mathcal{R}_0$.

Notice that all these trajectories converge to the origin without leaving the region $\mathcal{R} \subset \mathcal{D}$. In this example, all simulations were performed considering the time-varying delay $h_k = \mathrm{round}\,(1.5 + 0.5\cos(0.02k\pi))$ and the following initial condition $\phi_0 = x_0$,

$$\phi_{-2} = \begin{bmatrix} 0.2702 \\ 0.1683 \\ 0.2161 \end{bmatrix}, \phi_{-1} = \begin{bmatrix} -0.2081 \\ 0.1819 \\ -0.1665 \end{bmatrix}. \tag{4.38}$$

4.6 Final remarks

This chapter has addressed the synthesis of SOF controllers for discrete-time nonlinear systems with time-varying delays represented by quasi-LPV models and Takagi-Sugeno fuzzy

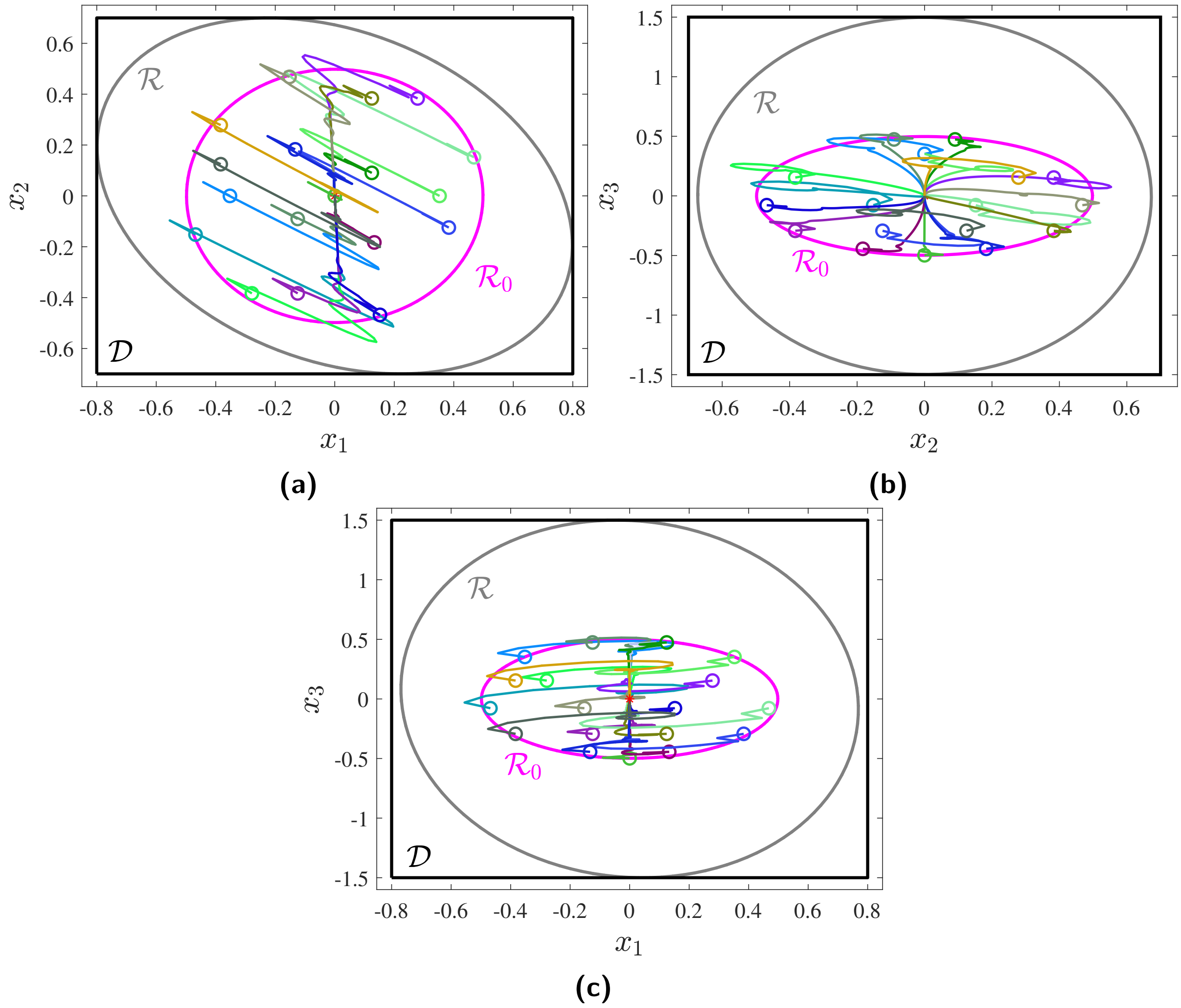

Figure 4.5 – Set of admissible initial conditions $\mathcal{R}_0 \subset \mathcal{R} \subset \mathcal{D}$ for $\underline{h} = 1$ and $\overline{h} = 2$ with convergent trajectories initiating at $\mathcal{R}_0$ - Example 4.2.

models. The proposed delay-dependent condition has been developed based on an augmented Lyapunov-Krasovskii functional together with the Wirtinger-based summation inequality and the Moon's inequality and formulated as parameter-dependent linear matrix inequalities. The main novelty of the proposed approach is the local asymptotic stability analysis performed to guarantee the correct operation of the closed-loop system since it is ensured that its trajectories remain inside of the guaranteed region of attraction estimation obtained inside of the validity region of the LPV system. In addition, another feature of the proposed approach is the possibility of considering a time-varying output matrix, namely it is not necessary to impose any structure on the system matrices or resort to iterative procedures. Finally, two numerical examples have been provided to illustrate the effectiveness of the proposed methods.

5 ADDITIONAL RESEARCH SUBJECTS DEVELOPED ALONG THE PH.D.

Additional works have been developed during the Ph.D. period, and the main ideas related to these works are shortly presented in this chapter. These works are presented apart in this chapter for the sake of brevity and cohesion.

5.1 Choosing different kinds of Lyapunov functional

Oftentimes, the choice of the Lyapunov function is a key point in the stabilization and stability analysis of LPV systems. In this sense, in the works initially developed in the Ph.D., the idea was to search for Lyapunov function candidates that would lead to less conservative results in the stability analysis and control design of LPV systems. In the sequel, the main ideas of these works are presented.

5.1.1 Lyapunov functions with nonmonotonic terms

Novel conditions for stability analysis, static output-feedback control, and state-feedback control have been presented employing Lyapunov functions with nonmonotonic terms. The proposed methodology is based on the combination of quadratic "Lyapunov-like" terms such that individually each one is not necessarily monotonically decreasing along the state trajectories. Besides that, unlike existing conditions for stability and control design of discrete-time LPV with polytopic time-varying parameters, the proposed approach makes use of the dynamics of the system to construct the Lyapunov function. The Lyapunov function candidate with nonmonotonic terms is described in the following lemma.

Lemma 5.1. *If there exist continuous functions $V_i : \mathbb{R}^{n_x} \mapsto \mathbb{R}$, $i \in \mathbb{N}_{\leq Z}$, such that*

$$\sum_{i=1}^{Z} iV_i(0) = 0, \tag{5.1}$$

$$\sum_{i=j}^{Z} V_i(x_k) > 0,\ \forall x \neq 0,\ j \in \mathbb{N}_{\leq Z} \tag{5.2}$$

$$\sum_{i=1}^{Z} \Big(V_i(x_{k+i}) - V_i(x_k)\Big) < 0 \tag{5.3}$$

hold along trajectories of the LPV system (2.4) *with $u_k \equiv 0$, then, its origin is asymptotically stable and*

$$W(x_k) = \sum_{j=1}^{Z} \sum_{i=j}^{Z} V_i(x_{k+j-1}) \tag{5.4}$$

is a Lyapunov function.

The function $W(x(k))$ fulfilling the previous Lemma is a Lyapunov function [1], since $W(x(0)) = 0$, $W(x_k) > 0$, $\forall x \neq 0$, and the variation of (5.4) is monotonically decreasing, *i.e.*,

$$\Delta W \triangleq W(x_{k+1}) - W(x_k) < 0, \; \forall x \neq 0.$$

However, it can be noticed from (5.3) that the individual variations $V_i(x_{k+i}) - V_i(x_k)$, $i \in \mathbb{N}_{\leq Z}$, are not necessarily monotonically decreasing, that is, they can be increasing or decreasing on different intervals of their domain. For this reason, they are referred to as non-monotonic terms.

We have considered $V_i(x_k) = x_k^\top P_i(\alpha_k) x_k$, being $P(\alpha_k) \in \mathbb{S}_+^{n_x}$ a parameter-dependent matrix. Therefore, with $Z = 1$, the parameter-dependent Lyapunov function with nonmonotonic terms (5.4) returns to the standard parameter-dependent Lyapunov function described as in (3.5). The results obtained using this approach were less conservative when compared with other approaches in the literature for stability analysis [151, 152, 153], state-feedback control design [154, 155, 156, 151], and output-feedback control problem [51, 87] of discrete-time LPV systems. These results are published in:

[16] **PEIXOTO, M. L. C.**; PESSIM, P. S. P.; LACERDA, M. J.; PALHARES, R. M. Stability and stabilization for LPV systems based on Lyapunov functions with non-monotonic terms. **Journal of the Franklin Institute**, v. 357, n. 11, p. 6595–6614, 2020.

A similar discussion to TS fuzzy models has been published in [5].

5.1.2 Delayed Lyapunov functions

New state-feedback design conditions for discrete-time LPV systems using Lyapunov functions with dependence on delayed scheduling parameters have been introduced. This class of Lyapunov functions candidate can be described as follows:

$$V(x_k) = x_k^\top \mathcal{P}_0 x_k, \tag{5.5}$$

with

$$\mathcal{P}_0 = \sum_{i_1=1}^{N} \alpha_{i_1,k+d_1} \sum_{i_2=1}^{N} \alpha_{i_2,k+d_2} \cdots \sum_{i_{n_P}=1}^{N} \alpha_{i_{n_P},k+d_{n_P}} P_{i_1 i_2 \dots i_{n_P}},$$

being $\{d_1, d_2, \dots, d_{n_P}\}$ delays related to the time-varying parameter. To illustrate this case, consider that the time-varying parameter of the Lyapunov function has the following delays $d_1 = -2$, $d_2 = -1$ e $d_3 = 0$. Therefore, the Lyapunov matrix can be rewritten as

$$\mathcal{P}_0 = \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} \sum_{i_3=1}^{N} \alpha_{i_1,k-2} \alpha_{i_2,k-1} \alpha_{i_3,k} P_{i_1 i_2 i_3}.$$

In addition, a lifted condition has been presented based on a Lyapunov function with dependence on delayed scheduling parameters, constructed in terms of an augmented state vector that takes into account a generic number of higher-order shifted states in the following form

$$\mathcal{V}(\tilde{x}) = \tilde{x}^\top \underbrace{\begin{bmatrix} \mathcal{P}_0^1 & 0_{n_x} & \cdots & 0_{n_x} \\ 0_{n_x} & \mathcal{P}_1^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{n_x} \\ 0_{n_x} & \cdots & 0_{n_x} & \mathcal{P}_{Z-1}^Z \end{bmatrix}}_{\mathscr{P}} \tilde{x}, \tag{5.6}$$

with

$$\tilde{x} = \begin{bmatrix} x_k & x_{k+1} & \ldots & x_{k+Z-1} \end{bmatrix},$$

equivalently

$$\mathcal{V}(\tilde{x}) = \underbrace{x_k^\top \mathcal{P}_0^1 x_k}_{\mathcal{V}_1} + \underbrace{x_{k+1}^\top \mathcal{P}_1^2 x_{k+1}}_{\mathcal{V}_2} + \underbrace{x_{k+2}^\top \mathcal{P}_2^3 x_{k+2}}_{\mathcal{V}_3} + \ldots + \underbrace{x_{k+Z-1}^\top \mathcal{P}_{Z-1}^Z x_{k+Z-1}}_{\mathcal{V}_Z}, \tag{5.7}$$

where Z depends on the size of the augmented state vector. In addition, $\mathcal{P}_j^i \in \mathbb{S}_+^{n_x}$, for all $i \in \mathbb{N}_{\leq Z}$ and $i \in \mathbb{N}_{\leq Z-1}$ with

$$\begin{aligned}
\mathcal{P}_0^1 &= \sum_{i_1=1}^{N} \alpha_{i_1,k+d_1} \sum_{i_2=1}^{N} \alpha_{i_2,k+d_2} \cdots \sum_{i_{n_P}=1}^{N} \alpha_{i_{n_P},k+d_{n_P}} P^1_{i_1 i_2 \ldots i_{n_P}}, \\
\mathcal{P}_1^2 &= \sum_{i_1=1}^{N} \alpha_{i_1,k+d_1+1} \sum_{i_2=1}^{N} \alpha_{i_2,k+d_2+1} \cdots \sum_{i_{n_P}=1}^{N} \alpha_{i_{n_P},k+d_{n_P}+1} P^2_{i_1 i_2 \ldots i_{n_P}}, \\
&\vdots \\
\mathcal{P}_{Z-1}^Z &= \sum_{i_1=1}^{N} \alpha_{i_1,k+d_1+Z-1} \sum_{i_2=1}^{N} \alpha_{i_2,k+d_2+Z-1} \cdots \sum_{i_{n_P}=1}^{N} \alpha_{i_{n_P},k+d_{n_P}+Z-1} P^Z_{i_1 i_2 \ldots i_{n_P}}.
\end{aligned}$$

Notice that the Lyapunov function candidate (5.6) with $Z = 1$ recovers the Lyapunov function candidate described in (5.5). The proposed condition for the state-feedback design of LPV systems has presented less conservative results when compared with the works presented by [155, 156, 87, 151, 154]. These results are published in:

[17] **PEIXOTO, M. L. C.**; LACERDA, M. J.; PALHARES, R. M. On discrete-time LPV control using delayed Lyapunov functions. **Asian Journal of Control**, v. 23, n. 5, p. 2359–2369, 2021.

5.1.3 Exploring alternative Lyapunov-Krasovskii functional

In the context of time-delayed systems, a parameter-dependent Lyapunov-Krasovskii functional with augmented vector and triple summation terms has been constructed to reduce the conservativeness in stabilization problems of LPV systems. To obtain the stabilization conditions, the following Lyapunov–Krasovskii functional candidate has been proposed:

$$V(x_k) = V_1(x_k) + V_2(x_k) + V_3(x_k) + V_4(x_k) + V_5(x_k) \tag{5.8}$$

where

$$\begin{aligned}
V_1(x_k) &= \begin{bmatrix} x_k \\ \sum_{i=k-\underline{h}}^{k-1} x_i \\ \sum_{i=k-\overline{h}}^{k-\underline{h}-1} x_i \end{bmatrix}^\top P(\alpha_k) \begin{bmatrix} x_k \\ \sum_{i=k-\underline{h}}^{k-1} x_i \\ \sum_{i=k-\overline{h}}^{k-\underline{h}-1} x_i \end{bmatrix}, \\
V_2(x_k) &= \sum_{i=k-\underline{h}}^{k-1} x_i^\top Q_1 x_i + \sum_{i=k-\overline{h}}^{k-\underline{h}-1} x_i^\top Q_2 x_i, \\
V_3(x_k) &= \underline{h} \sum_{i=-\underline{h}+1}^{0} \sum_{j=k+i}^{k} \eta_j^\top Z_1 \eta_j + (\overline{h} - \underline{h}) \sum_{i=-\overline{h}+1}^{-\underline{h}} \sum_{j=k+i}^{k} \eta_j^\top Z_2 \eta_j, \\
V_4(x_k) &= \sum_{i=-\overline{h}+1}^{-\underline{h}} \sum_{l=-\overline{h}+1}^{i} \sum_{j=k+l}^{k} \eta_j^\top S_1 \eta_j + \sum_{i=-\overline{h}+1}^{-\underline{h}} \sum_{l=i}^{-\underline{h}} \sum_{j=k+l}^{k} \eta_j^\top S_2 \eta_j, \\
V_5(x_k) &= \sum_{i=-\underline{h}+1}^{0} \sum_{l=i}^{0} \sum_{j=k+l}^{k} \eta_j^\top R_1 \eta_j + \sum_{i=-\underline{h}+1}^{0} \sum_{l=-\underline{h}+1}^{i} \sum_{j=k+l}^{k} \eta_j^\top R_2 \eta_j,
\end{aligned}$$

where $\eta_i = x_i - x_{i-1}$ and $P(\alpha_k) \in \mathbb{S}_+^{n_x}$ is a parameter-dependent matrix.

It should be pointed out that the part of Lyapunov-Krasovskii functional (5.8) is adapted from [99] for the parameter-dependent case, and the terms V_4 and V_5 are added to obtain less conservative results. The main contributions related to this part of the work are summarized as follows:

- a parameter-dependent Lyapunov-Krasovskii functional with augmented vector and triple summation terms has been constructed to reduce the conservativeness in stabilization problems;
- the appropriate choice of an LKF, multiple auxiliary functions, delay-dependent reciprocally convex inequality, and selection of a suitable augmented vector allow obtaining new stabilization conditions that depend on the minimum and maximum values of the time-varying delay. Additionally, Finsler's Lemma is employed to derive the stabilization conditions, which helps to obtain the controller gains;
- new delay-dependent LMI conditions for the state-feedback and SOF control design for time-delayed LPV systems and nonlinear parameter-varying systems where the nonlinearity is subject to cone-bounded sector constraints have been presented.

These results are published in:

[23] **PEIXOTO, M. L. C.**; BRAGA, M. F.; PALHARES, R. M. Gain-scheduled control for discrete-time non-linear parameter-varying systems with time-varying delays. **IET Control Theory & Applications**, v. 14, n. 19, p. 3217–3229, 2020.

5.2 SOF stabilization of discrete-time LPV systems under actuator saturation

The control of systems subject to actuator saturation is also a relevant topic since saturation occurs in several applications due to physical and technical constraints of the actuators or even for safety reasons. If it is neglected during the control design, the closed-loop performance may be degraded or it may even lead to instability [157]. Consequently, the local stability analysis must be considered to estimate the domain of attraction (DoA) of the closed-loop equilibrium. Despite its theoretical and practical significance, scant attention has been paid to the static output-feedback control design of LPV systems under actuator saturation. Therefore, the results proposed in Chapter 3 have been extended to deal with local stabilization of discrete-time LPV systems under actuator saturation and state constraint.

Consider the following discrete-time LPV system under actuator saturation

$$\begin{aligned} x_{k+1} &= A(\alpha_k)x_k + B(\alpha_k)\mathrm{sat}(u_k) \\ y_k &= C(\alpha_k)x_k, \end{aligned} \tag{5.9}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the input, $y_k \in \mathbb{R}^{n_y}$ is the output. The parameter-dependent matrices $A(\alpha_k)$, $B(\alpha_k)$, and $C(\alpha_k)$ belong to a polytopic domain parameterized by the time-varying parameters $\alpha \in \Lambda$, defined as in (2.5). Moreover, the control input is subject to the component-wise saturation map $\mathrm{sat}(\cdot) : \mathbb{R}^{n_u} \to \mathbb{R}^{n_u}$ defined as

$$\mathrm{sat}\left(u_{l,k}\right) = \mathrm{sign}\left(u_{l,k}\right) \min\left(\left|u_{l,k}\right|, \bar{u}_l\right), \ \ \forall l \in \mathbb{N}_{\leq n_u},$$

where $\bar{u}_l \in \mathbb{R}$, with $\bar{u}_l > 0$, is the maximum allowed bound of the l-th control input component due to the actuator saturation. Consider the following gain-scheduled SOF controller

$$u_k = X(\alpha_k)^{-1}L(\alpha_k)y_k, \tag{5.10}$$

being $L(\alpha_k) \in \mathbb{R}^{n_u \times n_y}$ and $X(\alpha_k) \in \mathbb{R}^{n_u \times n_u}$ the control gains to be designed. The resulting closed-loop system is

$$x_{k+1} = \left(A(\alpha_k) + B(\alpha_k)X(\alpha_k)^{-1}L(\alpha_k)C(\alpha_k)\right)x_k - B(\alpha_k)\psi(u_k),$$

where $\psi(u_k) = u_k - \mathrm{sat}(u_k)$ is the dead-zone nonlinearity, which can be handled by using the following lemma.

Lemma 5.2 (Adapted from [91]). *Consider the parameter-dependent diagonal matrix $S(\alpha_k) \in \mathbb{S}_+^{n_u}$ and the parameter-dependent matrix $U(\alpha_k) \in \mathbb{R}^{n_u \times n_x}$, $\forall \alpha_k \in \Lambda_N$. Let the set*

$$\mathcal{D}_u = \left\{x_k \in \mathbb{R}^{n_x} : \left|\left(S(\alpha_k)^{-1}U(\alpha_k)\right)_l x_k\right| \leq \bar{u}_l\right\}.$$

If $x_k \in \mathcal{D}_u$, then

$$\psi(u_k)^\top S(\alpha_k)\left[u_k - \psi(u_k) - S(\alpha_k)^{-1}U(\alpha_k)x_k\right] \geq 0, \tag{5.11}$$

for any $u_k \in \mathbb{R}^{n_u}$.

To demonstrate the effectiveness of the results obtained, consider the following nonlinear system

$$\begin{aligned} x_{1,k+1} &= x_{2,k} \\ x_{2,k+1} &= x_{1,k} + \frac{1}{3}x_{1,k}^3 + x_{2,k} + \mathrm{sat}(u_k) \\ y_{1,k} &= x_{1,k}, \\ y_{2,k)} &= x_{1,k}^3 + 3x_{1,k} \end{aligned} \tag{5.12}$$

with $\mathcal{D} = \{x \in \mathbb{R}^2 : |x_1| \leq 1, |x_2| \leq 1.5\}$, and $\bar{u} = 1$. Considering $\rho(x_k) = x_{1,k}^2$, such that $\rho(x_k) \in [0, 1]$, as scheduling parameter, the nonlinear system given by (5.12) can be represented by means of a quasi-LPV system as in (5.9) with the following two vertices

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \frac{4}{3} & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.13}$$

The proposed approach has been used to design the gain-scheduled SOF controller (5.10) which stabilizes (5.12) and maximizes the DoA estimate. Figure 5.1 depicts the estimated DoA for the origin of the closed-loop system (5.12) with (5.10) obtained by the proposed condition. Several closed-loop trajectories starting within the region $\mathcal{R}_0$ are depicted in Figure 5.1. It can be noticed that all trajectories converge to the origin without leaving the region $\mathcal{R}_0 \subset (\mathcal{D} \cap \mathcal{D}_u)$. Some divergent closed-loop trajectories initiating outside the region $\mathcal{R}_0$ are also provided to illustrate the effectiveness of the proposed local SOF control synthesis condition and the relevance of the estimation of $\mathcal{R}_0$. Figure 5.1 also depicts the estimated region $\overline{\mathcal{R}}_0$ obtained from [91]. As one can see, the proposed approach has provided a larger estimate of the DoA for the origin of the closed-loop system (5.12) than the method proposed in [91].

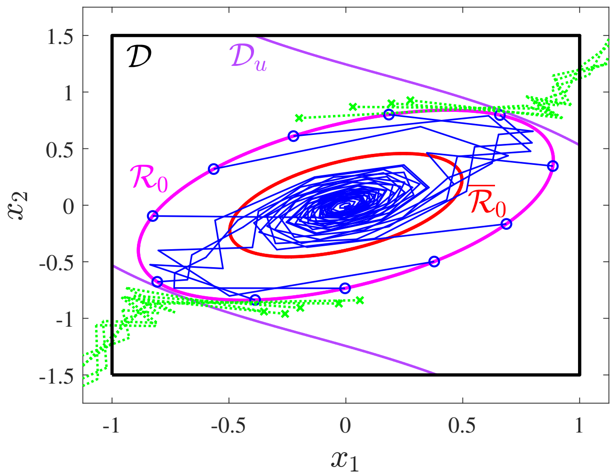

Figure 5.1 – Estimate of DoA $\mathcal{R}_0$ (magenta) obtained from the proposed optimization problem, Estimate of DoA $\overline{\mathcal{R}}_0$ (red line) obtained from [91], convergent trajectories (blue line) with initial conditions at the border of $\mathcal{R}_0$, and trajectories (dotted green line) leaving the set $\mathcal{D}$.

The results related to this part are published in:

[158] **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; BESSA, I.; PALHARES, R. M. Static output-feedback stabilization of discrete-time linear parameter-varying systems under actuator saturation. **International Journal of Robust and Nonlinear Control**, p. 5799–5809, 2022.

5.3 ETC of systems subject to cyber-attacks and network-induced time-delays

Networked control systems (NCS) are systems where a communication network is used in the data exchange between their elements, namely the actuators, sensors, and controllers. While the NCS are becoming popular due to the advances in internet resources and the convenience of the operation and maintenance, it raises new challenges and threats which are inherent to those systems, such as network-induced delays, data packet dropouts, limited bandwidth of communication networks, disorder, and the vulnerability to cyber-attacks. All those issues are discussed in [159] and references therein.

The event-triggered control (ETC) arises as a solution for reducing network communication burden while ensuring the desired performance of NCS in opposition to the traditional time-triggering mechanisms [160]. The ETC design strategies are classified into emulation [161] or co-design [162, 163] approaches. The emulation approaches consider a previously designed controller, and it only designs the event-triggering mechanism (ETM) which provides the desired guarantees (e.g., stability and performance). Meanwhile, in the co-design strategy, the controller is designed along with the ETM, which usually ensures better performance in terms of network resources saving. However, in the context of nonlinear systems, the ETC co-design is more challenging [20].

Some ETC strategies presented in the literature can deal with the possibility of cyber-attacks. The cyber-attacks can be classified into two main classes: deception attacks and denial-of-service. Denial-of-service attacks affect the transmission channels blocking the communication between the controller, sensors, and actuators [164, 165, 166]. Otherwise, the deception attacks affect the data integrity by injecting false data into some components, such as the actuators or controllers [167]. In particular, control strategies concerning deception attacks are still incipient due to the difficulties in accurately modeling those attacks, which may effectively be any type of signal injection. However, a common approach is to adopt a stochastic model for the attack success and assume some norm-bounded signal injection [168].

Along the Ph.D. term, the problem of periodic event-triggered control co-design for the stabilization of LPV systems and Takagi-Sugeno fuzzy models subject to stochastic deception attacks whose occurrence follows a given Bernoulli distribution has been addressed. A novel delay-dependent condition is presented to simultaneously design the event-triggering mechanism and the state-feedback controller to ensure the local mean-square asymptotic stability of the closed-loop system. The co-design condition is derived as linear matrix inequalities by considering Lyapunov-Krasovskii stability arguments. In the case of nonlinear systems, an optimization

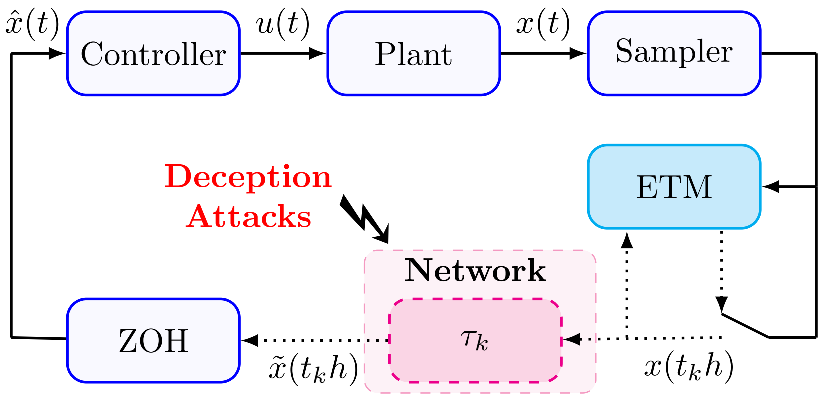

Figure 5.2 – A periodic ETC system under deception attacks.

procedure has been provided to estimate an enlarged set of admissible initial conditions and the corresponding ultimate bound within the validity of the domain of the fuzzy model.

Figure 5.2 presents the networked control configuration that has been used in this work. In this scenario, the communication through the network is packet-based. The plant information is periodically available with a sampling period h for the ETM that compares the signal from $x(jh)$, $j \in \mathbb{N}_0$, with the last signal sent over the communication network, $x(t_k h)$. In this way, the ETM determines the next transmission instant from an appropriate triggering function. In this context, the sequence of transmissions is given by $\{t_k h\}_{k \in \mathbb{N}_0}$, with $t_k \in \mathbb{N}_0$ satisfying $t_k < t_{k+1}$, $\forall k \in \mathbb{N}_0$, and $t_0 \geq 0$. At each transmission instant $t_k h$, $k \in \mathbb{N}$, the signal $x(t_k h)$ is sent to the controller node over the communication over the network subject to a bounded time-varying delay $\tau_k \in \mathbb{R}_{\geq 0}$. Considering the implementation of a zero-order hold, the signal available to the controller is held constant in the time interval between two consecutive transmissions. Furthermore, the network is subject to deception attacks, which are characterized by the injection of false information in order to generate a corrupted state signal, $\tilde{x}(t_k h)$, to be made available to the controller in place of the correct state information $x(t_k h)$. A stochastic deception attack is characterized by the injection of false information by a malicious attacker to corrupt the state measurement to be available to the controller. When the transmission of $x(t_k h)$ is attempted at $t_k h$, the attacker can replace it. In this work, stochastic deception attacks have been considered as in [169]. Notice that in the case where the network is not subject to deception attacks, one simply has $\tilde{x}(t_k h) \equiv x(t_k h)$.

Considering the configuration presented above, results where the plant is a nonlinear system represented by Takagi-Sugeno fuzzy models are submitted, and results where the plant is described for linear parameter-varying systems are published in:

[170] **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; PESSIM, P. S. P.; BESSA, I.; PALHARES, R. M. Controle em rede com acionamento por eventos para sistemas sujeitos a ataques cibernéticos. In: **Anais do XXIV Congresso Brasileiro de Automática.**, Fortaleza, CE, Brasil: [s.n.], 2022. p. 1–6.

5.4 Observer design approaches

In collaboration with Prof. Thierry-Marie Guerra and Prof. Anh-Tu Nguyen from the Université Polytechnique Hauts-de-France and LAMIH-CNRS laboratory, some observer-based approaches have been developed and these methods are presented shortly in the sequel.

5.4.1 Unknown input observers for time-delayed nonlinear systems

State estimation of nonlinear systems in the presence of unknown inputs has received increasing research attention. The main reason is that, within different application contexts, unknown inputs can be seen as unmodeled dynamics, faults in engineering systems, uncertain disturbances, attack signals in secure communication or cyber-physical systems [171, 172, 173, 174, 175]. Consequently, simultaneous estimation of the state of a dynamical system and its unknown input has become a key aspect in several practical applications [176, 177, 178].

A great deal of research effort has been devoted to designing observers represented by TS fuzzy and quasi-LPV models in the presence of unknown inputs [175]. It is noteworthy that when the premise variables or scheduling functions can be measured, many results existing for linear observer design can be extended to LPV and TS fuzzy systems [179]. However, the corresponding results can only be applied to a restrictive class of nonlinear systems. Hence, observer design with unmeasured scheduling functions or premise variables may be considered, which leads to a challenging observer design problem due to mismatching nonlinear terms involved in the estimation error dynamics [176, 180]. To avoid this major drawback, the mean value theorem has been exploited for TS fuzzy observer design with unmeasured nonlinearities [180, 173, 181]. Unknown input observers have also been developed for TS fuzzy systems with unmeasured premise variables [173, 181]. Despite great advances, there is a lack of literature on unknown input observer design for delayed nonlinear systems with unmeasured nonlinearities.

This work has given a new contribution to the design of unknown input observers for time-varying delay nonlinear systems with unmeasured nonlinearities. To this end, nonlinear systems are represented by a polytopic form, where all unmeasured nonlinearities are regrouped in the nonlinear consequents. By means of the mean value theorem, this enables an effective way to deal with the major issue related to unmeasured nonlinearities in gain-scheduling observer design. The effect of time-varying delay on the estimation error dynamics is explicitly taken into account in the observer design procedure via a parameter-dependent Lyapunov-Krasovskii functional with a suitable augmented vector to reduce the design conservativeness. The Wirtinger-based summation inequality has been used jointly with Moon's inequality as well as the well-known Finsler's lemma to derive numerically tractable unknown input observer design conditions. It is worth mentioning that the result of polytopic unknown input observer design for time-varying delayed systems with unmeasured nonlinearities has not been observed in any previous work in the open literature. Therefore, the main contributions related to this subject can be summarized as follows:

a) Using LKF stability tools jointly with various relaxation techniques, a new set of *delay-dependent* LMI conditions have been derived to design unknown input observers for time-varying delayed nonlinear systems with unmeasured nonlinearities.

b) The designed observer allows to simultaneously and asymptotically estimate both the system state and the unknown inputs without requiring any *a priori* information on unknown inputs.

This results have been consolidated in the following work currently in revision:

> **PEIXOTO, M. L. C.**; NGUYEN, A.-T.; GUERRA, T-M.; PALHARES, R. M. Unknown input observers for time-varying delay Takagi-Sugeno fuzzy systems with unmeasured nonlinear consequents. **Submitted**.

5.4.2 Fault estimation for nonlinear time-delayed systems

Modern control systems demand more sophisticated design requirements concerning safety, reliability, and maintainability. Possible occurrences of sensor and actuator faults can lead to closed-loop performance degradation or even instability [182]. To this end, Fault-tolerant control (FTC) techniques have been proposed to ensure desirable closed-loop requirements despite the presence of faults [14]. Generally speaking, FTC techniques are classified into passive schemes [183] and active schemes [182]. Passive FTC deals with the fault effects as system disturbances or uncertainties. In this case, specific information on the location or the severity of the faults is not required. On the contrary, active FTC makes use of these specific information provided by fault detection and isolation schemes [184] to act on the system by actively modifying the control law to mitigate the fault effects. The main advantage of active FTC over passive FTC is the ability to avoid unnecessary performance loss in fault-free conditions since passive FTC design is often performed considering worst-case fault estimations [185]. However, a key point for the correct operation of active FTC is a well-designed fault detection and isolation scheme, which is able to provide precise and correct information about the faults. Although fault detection and isolation schemes are designed to provide residual signals to indicate the fault occurrence and the information of its type and location, the exact information about the magnitude and the shape of the fault cannot be obtained, which has motivated the development of fault estimation techniques to provide more precise information about the fault [186].

Observer-based fault estimation techniques have been widely studied, including sliding mode observers [187], adaptive observers [188], and unknown input observers [189]. However, these techniques normally require the so-called matching condition to be satisfied, which may be restrictive, especially for nonlinear systems. To overcome this issue, Zhu *et al.* [186] have proposed the use of intermediate estimators to estimate both the states and the faults of nonlinear systems with Lipschitzian nonlinearities. Other results concerning intermediate estimators have been presented [190, 191, 192, 193]. However, the issue of intermediate estimator-based fault estimation has not been addressed for nonlinear time-delayed systems in the open literature. Accordingly, the first motivation of this work has been to address the fault estimation problem for a class of nonlinear time-delayed systems using observers without requiring matching conditions.

This problem has been handled here regarding the polytopic embedding of nonlinear systems. However, although these representations are useful to derive constructive and numerically implementable conditions for designing observers [175], there is an important issue that should be accounted into the observer design, i.e., the necessity to deal with unmeasured scheduling functions or premise variables in TS fuzzy systems. The results available for fault estimation and diagnosis for LPV systems [194] and TS fuzzy models [195] assume that the scheduling functions are measured or dependent on the output variables, which makes the design easier but limited to specific classes of nonlinear dynamical systems. To avoid this restriction, the second motivation of this work has been to provide constructive observer design conditions considering a polytopic representation of nonlinear time-delayed systems with unmeasured nonlinearities.

This line of work has addressed the fault estimation problem for a class of nonlinear time-delayed systems considering intermediate observers. The main contributions can be summarized as follows.

a) A new class of gain-scheduling intermediate observers has been proposed to simultaneously estimate the state and fault, without requiring matching conditions. Note that results on intermediate observers are only available for linear time-delayed systems [196, 197].

b) Time-delayed nonlinear systems have been represented by a specific nonlinear parameter-varying form, which allows circumventing the assumption of measured scheduling functions or premise variables in [194, 195].

c) Constructive and numerically implementable conditions have been derived in the form of linear matrix inequalities for gain-scheduling intermediate observer design such that the error dynamics is input-to-state stable with respect to the fault time-derivative. Results for nonlinear parameter-varying systems without time-varying delays have also been provided.

The main results of this discussion is under revision in:

PEIXOTO, M. L. C.; COUTINHO, P. H. S.; NGUYEN, A.-T.; GUERRA, T-M.; PALHARES, R. M. Fault estimation for nonlinear parameter-varying time-delay systems. **Submitted**.

6 CONCLUDING REMARKS

This Thesis has addressed the static output feedback control problem for nonlinear systems represented by LPV and TS fuzzy models.

Chapter 3 has introduced new less conservative conditions to design gain-scheduled static output-feedback controllers for discrete-time LPV systems and nonlinear systems represented by quasi-LPV and TS fuzzy models. Slack variables have been introduced along the formulation providing extra degrees of freedom to reduce design conservativeness. The novel proposed conditions are relatively simple and sound to be less conservative when compared to other conditions in the literature, as illustrated by numerical examples.

Chapter 4 has addressed the synthesis of SOF controllers for discrete-time nonlinear systems with time-varying delays. This problem has been handled regarding the polytopic embedding of nonlinear systems. The conditions have been developed based on an augmented Lyapunov-Krasovskii functional combined with Wirtinger-based summation inequality and Moon's inequality and formulated as delay-dependent linear matrix inequalities. The Finsler's Lemma has been employed to derive the stabilization conditions, which has helped to obtain the controller gains. Numerical examples have been provided to illustrate the effectiveness of the proposed methods.

In Chapter 3 and Chapter 4, one feature of the proposed approaches is the possibility of the output matrix being parameter-dependent without requiring any specific structure or admitting particular similarity transformations, unlike most of the methods in the related literature. Another novelty of the proposed approaches is the local asymptotic stability analysis performed to guarantee the correct operation of the closed-loop system since it is ensured that its trajectories remain inside of the guaranteed region of attraction estimation obtained within the validity region of the quasi-LPV model (TS fuzzy model).

Finally, Chapter 5 has briefly presented other works that have been developed during the doctoral term. Novel conditions to certificate stability and to compute scheduling output-feedback and state-feedback control gains for discrete-time LPV systems employing a Lyapunov function with non-monotonic terms have been obtained. New conditions to compute state-feedback control gains for discrete-time LPV using Lyapunov functions with dependence on delayed scheduling parameters have been proposed. The results of Chapter 3 have been extended to deal with local stabilization of discrete-time SOF systems under actuator saturation and state constraint. Additionally, approaches that have been obtained for unknown input observers and fault estimation of nonlinear time-delayed systems have been shortly presented.

6.1 Future research

In this section, some suggestions for potential next steps for this doctoral research are presented. They are based on the further development of the main objective of this Thesis: to propose novel stabilization conditions for nonlinear systems represented by polytopic models. The additional steps are listed as follows:

a) Other types of Lyapunov function candidates, such as Lyapunov with nonmonotonic terms and Lyapunov functions with dependence on delayed scheduling parameters, could be employed to reduce the conservativeness of the proposed approach in Chapter 3.

b) To reduce conservativeness, it is possible to include the use of parameter-dependent Lyapunov-Krasovskii functional and to extend the proposed methodology in Chapter 4 to employ Bessel-Legendre inequalities in an arbitrary order which is less conservative than the Jensen and Wirtinger-based inequalities (zero and first order Bessel–Legendre inequalities, respectively) [147].

c) The results presented in Chapters 3 and 4 can be extended to handle the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance.

d) It is possible to adapt the method proposed in Section 5.3 to deal with multiple cyber attacks [198]. In this case, the network may be subject to deception attacks as well as denial of service attacks.

6.2 Publications

During the period in which this doctoral research was developed, contributions concerned with the topics presented in Chapters 3, 4 and 5 have been attained. The publications related to this topics are listed below:

a) **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; LACERDA, M. J.; PALHARES, R. M. Guaranteed region of attraction estimation for time-delayed fuzzy systems via static output- feedback control. **Automatica**, p. 110438, jun 2022. <https://doi.org/10.1016/j.automatica.2022.110438>

b) **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; PALHARES, R. M. Improved robust gain-scheduling static output-feedback control for discrete-time LPV systems. **European Journal of Control**, Elsevier, v. 58, p. 11–16, 2021. <https://doi.org/10.1016/j.ejcon.2020.12.006>

c) **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; BESSA, I.; PALHARES, R. M. Static output-feedback stabilization of discrete-time linear parameter-varying systems under actuator saturation. **International Journal of Robust and Nonlinear Control**, p. 5799–5809, 2022. <https://doi.org/10.1002/rnc.6106>

d) **PEIXOTO, M. L. C.**; BRAGA, M. F.; PALHARES, R. M. Gain-scheduled control for discrete-time non-linear parameter-varying systems with time-varying delays. **IET Control Theory & Applications**, IET, v. 14, n. 19, p. 3217–3229, 2020. <https://doi.org/10.1049/iet-cta.2020.0900>

e) **PEIXOTO, M. L. C.**; PESSIM, P. S. P.; LACERDA, M. J.; PALHARES, R. M. Stability and stabilization for LPV systems based on Lyapunov functions with non-monotonic terms. **Journal of the Franklin Institute**, v. 357, n. 11, p. 6595–6614, 2020. <https://doi.org/10.1016/j.jfranklin.2020.04.019>

f) **PEIXOTO, M. L. C.**; LACERDA, M. J.; PALHARES, R. M. On discrete-time LPV control using delayed Lyapunov functions. **Asian Journal of Control**, v. 23, n. 5, p. 2359–2369, sep 2021. <https://doi.org/10.1002/asjc.2362>

g) **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; PESSIM, P. S. P.; BESSA, I.; PALHARES, R. M. Controle em rede com acionamento por eventos para sistemas sujeitos a ataques cibernéticos. In: **Anais do XXIV Congresso Brasileiro de Automática**., Fortaleza, CE, Brasil: [s.n.], 2022. p. 1–6.

During the doctoral term to which this Thesis refers, additional related topics have been researched and new results have also been obtained in collaboration with other members of the D!FCOM. The works published related to other topics are listed below:

a) **PEIXOTO, M. L. C.**; REIS, G. L.; COUTINHO, P. H. S.; TORRES, L.A .B.; PALHARES, R. M. Stability analysis of uncertain discrete-time systems with time-varying delays using difference-algebraic representation. In: **Proceedings of the 2022 European Control Conference.** London, United Kingdom, 2022. p. 2069–2074 <https://doi.org/10.23919/ECC55457.2022.9838021>

b) COUTINHO, P. H. S.; **PEIXOTO, M. L. C**.; BESSA, I.; PALHARES, R. M. Dynamic Event-triggered gain-scheduling control of discrete-time quasi-LPV systems. **Automatica**, v. 141, p. 110292, 2022. <https://doi.org/10.1016/j.automatica.2022.110292>

c) de SOUZA, L. T. F.; **PEIXOTO, M. L. C**.; PALHARES, R. M. New gain-scheduling control conditions for time-varying delayed LPV systems. **Journal of the Franklin Institute**, v. 359, n. 2, p. 719–742, 2022. <https://doi.org/10.1016/j.jfranklin.2021.04.029>

d) PESSIM, P. S. P.; **PEIXOTO, M. L. C**.; PALHARES, R. M.; LACERDA, M. J. Static output-feedback control for cyber-physical LPV systems under DoS attacks. **Information Sciences**, Elsevier BV, v. 563, p. 241–255, 2021. <https://doi.org/10.1016/j.ins.2021.02.023>

e) COUTINHO, P. H. S.; **PEIXOTO, M. L. C**.; LACERDA, M. J.; BERNAL, M.; PALHARES, R. M. Generalized non-monotonic Lyapunov functions for analysis and synthesis of Takagi- Sugeno fuzzy systems. **Journal of Intelligent & Fuzzy Systems**, v. 39, n. 3, p. 4147–4158, 2020. <https://doi.org/10.3233/JIFS-200262>

f) COUTINHO, P. H. S.; **PEIXOTO, M. L. C**.; BERNAL, M.; NGUYEN, A.-T.; PALHARES, R. M. Local sampled-data gain-scheduling control of quasi-LPV systems. In: **4th IFAC Conference on Embedded Systems, Computational Intelligence and Telematics in Control (CESCIT 2021)**, Valenciennes, France, 2021, p. 86–91. <https://doi.org/10.1016/j.ifacol.2021.10.015>

g) COUTINHO, P. H. S.; BESSA, I.; **PEIXOTO, M. L. C**.; PESSIM, P. S. P.; PALHARES, R. M. Controle com acionamento por eventos resiliente a ataques de negação de serviço. **In: Anais do XXIV Congresso Brasileiro de Automática**. Fortaleza, CE, Brasil, 2022. p. 1–6.

h) CORDOVIL, L. A. Q.; COUTINHO, P. H. S.; BESSA, I.; **PEIXOTO, M. L. C.**; PALHARES, R. M. Learning event-triggered control based on evolving data-driven fuzzy granular models. **International Journal of Robust and Nonlinear Control**, v. 32, n. 5, p. 2805–2827, 2022. <https://doi.org/10.1002/rnc.6024>

i) PESSIM, P. S. P.; COUTINHO, P. H. S.; BESSA, I.; **PEIXOTO, M. L. C.**; LACERDA, M. J.; PALHARES, R. M. Controle distribuıdo para sistemas não lineares interconectados sujeitos a retardos variantes no tempo nas interconexões. In: **Anais do XXIV Congresso Brasileiro de Automática**. Fortaleza, CE, Brasil, 2022. p. 1–6.

Finally, the following papers are under revision.

a) **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; BESSA, I.; PESSIM, P. S. P.; PALHARES, R. M. Event-Triggered Control of Takagi-Sugeno Fuzzy Systems under Deception Attacks. **Submitted**.

b) **PEIXOTO, M. L. C.**; COUTINHO, P. H. S.; NGUYEN, A.-T.; GUERRA, T-M.; PALHARES, R. M. Fault estimation for nonlinear parameter-varying time-delay systems. **Submitted**.

c) **PEIXOTO, M. L. C.**; NGUYEN, A.-T.; GUERRA, T-M.; PALHARES, R. M. Unknown input observers for time-varying delay Takagi-Sugeno fuzzy systems with unmeasured nonlinear consequents. **Submitted**.

d) COUTINHO, P. H. S.; BESSA, I.; **PEIXOTO, M. L. C.**; PALHARES, R. M. A co-design condition for dynamic event-triggered feedback linearization control. **Submitted**.

e) NGUYEN, A.-T.; COUTINHO, P. H. S.; **PEIXOTO, M. L. C.**; GUERRA, T-M.; PALHARES, R. M. Output Feedback Control of Takagi-Sugeno Fuzzy Systems with Unmeasured Nonlinearities via a Local Separation Principle. **Submitted**.

6.3 Awards

The author of this Thesis received the award for best paper [170] in the Ph.D. category in the *Congresso Brasileiro de Automática 2022*, which took place in the Fortaleza, Brazil.

Figure 6.1 – Certificate of the awarded paper at CBA 2022.

BIBLIOGRAPHY

[1] KHALIL, H. K. **Nonlinear Systems**. 3rd. ed. Upper Saddle River, NJ: Prentice Hall, 2002. Pages 14, 27, and 52.

[2] FIGUEREDO, L. F. C.; ISHIHARA, J. Y.; BORGES, G. A.; BAUCHSPIESS, A. Delay-dependent robust stability analysis for time-delay T-S fuzzy systems with nonlinear local models. **Journal of Control, Automation and Electrical Systems**, v. 24, n. 1-2, p. 11–21, 2013. Page 14.

[3] NGUYEN, A-T; TANIGUCHI, T.; ECIOLAZA, L.; CAMPOS, V. C. S.; PALHARES, R. M.; SUGENO, M. Fuzzy control systems: Past, present and future. **IEEE Computational Intelligence Magazine**, v. 14, n. 1, p. 56–68, 2019. Pages 14 and 24.

[4] COUTINHO, P. H. S.; LAUBER, J.; BERNAL, M.; PALHARES, R. Efficient LMI conditions for enhanced stabilization of discrete-time Takagi–Sugeno models via delayed nonquadratic Lyapunov functions. **IEEE Transactions on Fuzzy Systems**, v. 27, n. 9, p. 1833–1843, 2019. Pages 14 and 24.

[5] COUTINHO, P. H. S.; PEIXOTO, M. L. C.; LACERDA, M. J.; BERNAL, M.; PALHARES, R. M. Generalized non-monotonic Lyapunov functions for analysis and synthesis of Takagi-Sugeno fuzzy systems. **Journal of Intelligent & Fuzzy Systems**, v. 39, n. 3, p. 4147–4158, 2020. Pages 14, 24, and 52.

[6] ELIAS, L. J.; FARIA, F. A.; ARAUJO, R.; OLIVEIRA, V. A. Stability analysis of Takagi–Sugeno systems using a switched fuzzy Lyapunov function. **Information Sciences**, Elsevier Inc., v. 543, p. 43–57, 2021. Page 14.

[7] CORDOVIL, L. A. Q.; COUTINHO, P. H. S.; BESSA, I.; PEIXOTO, M. L. C.; PALHARES, R. M. Learning event-triggered control based on evolving data-driven fuzzy granular models. **International Journal of Robust and Nonlinear Control**, v. 32, n. 5, p. 2805–2827, 2022. Page 14.

[8] TANAKA, K.; YOSHIDA, H.; OHTAKE, H.; WANG, H. O. A sum-of-squares approach to modeling and control of nonlinear dynamical systems with polynomial fuzzy systems. **IEEE Transactions on Fuzzy Systems**, v. 17, n. 4, p. 911–922, 2009. Page 14.

[9] SALA, A. On the conservativeness of fuzzy and fuzzy-polynomial control of nonlinear systems. **Annual Reviews in Control**, v. 33, n. 1, p. 48–58, apr 2009. Page 14.

[10] DONG, J.; WANG, Y.; YANG, G. H. $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control of discrete-time T-S fuzzy systems with local nonlinear models. **Fuzzy Sets and Systems**, Elsevier, v. 164, n. 1, p. 1–24, 2011. Page 14.

[11] ARAUJO, R. F.; TORRES, L. A. B.; PALHARES, R. M. Distributed control of networked nonlinear systems via interconnected Takagi–Sugeno fuzzy systems with nonlinear consequent. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 8, p. 4858–4867, 2021. Pages 14 and 24.

[12] MOODI, H.; KAZEMY, A. Robust Controller Design for Takagi–Sugeno Systems with Nonlinear Consequent Part and Time Delay. **International Journal of Fuzzy Systems**, Springer Berlin Heidelberg, v. 21, n. 3, p. 745–754, 2019. Page 14.

[13] COUTINHO, P. H. S.; ARAÚJO, R. F.; NGUYEN, A.-T.; PALHARES, R. M. A multiple-parameterization approach for local stabilization of constrained Takagi-Sugeno fuzzy systems with nonlinear consequents. **Information Sciences**, Elsevier Inc., v. 506, p. 295–307, 2020. Pages 14 and 24.

[14] BESSA, I.; PUIG, V.; PALHARES, R. M. TS fuzzy reconfiguration blocks for fault tolerant control of nonlinear systems. **Journal of the Franklin Institute**, Elsevier Ltd, v. 357, n. 8, p. 4592–4623, 2020. Pages 14, 24, and 60.

[15] NGUYEN, A-T; COUTINHO, P. H. S.; GUERRA, T-M; PALHARES, R. M.; PAN, J. Constrained output-feedback control for discrete-time fuzzy systems with local nonlinear models subject to state and input constraints. **IEEE Transactions on Cybernetics**, v. 51, n. 9, p. 4673–4684, sep 2021. Page 14.

[16] PEIXOTO, M. L. C.; PESSIM, P. S. P.; LACERDA, M. J.; PALHARES, R. M. Stability and stabilization for LPV systems based on Lyapunov functions with non-monotonic terms. **Journal of the Franklin Institute**, v. 357, n. 11, p. 6595–6614, 2020. Pages 14, 29, 30, 33, and 52.

[17] PEIXOTO, M. L. C.; LACERDA, M. J.; PALHARES, R. M. On discrete-time LPV control using delayed Lyapunov functions. **Asian Journal of Control**, v. 23, n. 5, p. 2359–2369, sep 2021. Pages 14 and 53.

[18] ROTONDO, D.; PUIG, V.; NEJJARI, F.; WITCZAK, M. Automated generation and comparison of Takagi–Sugeno and polytopic quasi-LPV models. **Fuzzy Sets and Systems**, Elsevier B.V., v. 277, p. 44–64, oct 2015. Page 14.

[19] ROBLES, R.; SALA, A.; BERNAL, M. Performance-oriented quasi-LPV modeling of nonlinear systems. **International Journal of Robust and Nonlinear Control**, v. 29, n. 5, p. 1230–1248, 2019. Page 14.

[20] COUTINHO, P. H. S.; PEIXOTO, M. L. C.; BESSA, I.; PALHARES, R. M. Dynamic event-triggered gain-scheduling control of discrete-time quasi-LPV systems. **Automatica**, v. 141, p. 110292, 2022. Pages 14 and 57.

[21] CAI, X.; LIU, Y.; ZHANG, W. Control design for a class of nonlinear parameter varying systems. **International Journal of Systems Science**, Taylor & Francis, v. 46, n. 9, p. 1638–1647, 2015. Page 14.

[22] ZHU, P.; WANG, J.; ZENG, J. The guaranteed cost controller for nonlinear systems with time-varying parameters and input saturation. **Transactions of the Institute of Measurement and Control**, v. 42, n. 3, p. 565–575, 2019. Page 14.

[23] PEIXOTO, M. L. C.; BRAGA, M. F.; PALHARES, R. M. Gain-scheduled control for discrete-time non-linear parameter-varying systems with time-varying delays. **IET Control Theory & Applications**, IET, v. 14, n. 19, p. 3217–3229, 2020. Pages 14, 23, and 54.

[24] LU, L.; FU, R.; ZENG, J.; DUAN, Z. On the domain of attraction and local stabilization of nonlinear parameter-varying systems. **International Journal of Robust and Nonlinear Control**, v. 30, n. 1, p. 17–32, jan 2020. Page 14.

[25] DE OLIVEIRA, M. C.; GEROMEL, J. C.; BERNUSSOU, J. Extended $\mathcal{H}_2$ and $\mathcal{H}_\infty$ characterization and controller parametrizations for discrete-time systems. **International Journal of Control**, v. 75, n. 9, p. 666–679, June 2002. Pages 15 and 17.

[26] DEAECTO, G. S.; GEROMEL, J. C.; DAAFOUZ, J. Dynamic output feedback $\mathcal{H}_\infty$ control of switched linear systems. **Automatica**, Elsevier Ltd, v. 47, n. 8, p. 1713–1720, 2011. Page 15.

[27] DEAECTO, G. S.; DAIHA, H. R. LMI conditions for output feedback control of switched systems based on a time-varying convex Lyapunov function. **Journal of the Franklin Institute**, Elsevier Ltd, v. 357, n. 15, p. 10513–10528, 2020. Page 15.

[28] SADABADI, M. S.; PEAUCELLE, D. From static output feedback to structured robust static output feedback: A survey. **Annual Reviews in Control**, v. 42, p. 11–26, 2016. Page 15.

[29] CRUSIUS, C. A. R.; TROFINO, A. Sufficient LMI conditions for output feedback control problems. **IEEE Transactions on Automatic Control**, v. 44, n. 5, p. 1053–1057, 1999. Page 15.

[30] IWASAKI, T.; SKELTON, R.E.; GEROMEL, J.C. Linear quadratic suboptimal control with static output feedback. **Systems & Control Letters**, v. 23, n. 6, p. 421–430, dec 1994. Pages 15 and 17.

[31] GEROMEL, J. C.; de SOUZA, C. C.; SKELTON, R. E. LMI numerical solution for output feedback stabilization. In: **Proceedings of the 1994 American Control Conference**. Baltimore, MD, USA: [s.n.], 1994. p. 1608–1612. Pages 15 and 17.

[32] GEROMEL, J. C.; de SOUZA, C. C.; SKELTON, R. E. Static output feedback controllers: Stability and convexity. **IEEE Transactions on Automatic Control**, v. 43, n. 1, p. 120–125, 1998. Pages 15 and 17.

[33] de SOUZA, C. E.; TROFINO, A. An LMI approach to stabilization of linear discrete-time periodic systems. **International Journal of Control**, v. 73, p. 696–703, 2000. Pages 15 and 17.

[34] PREMPAIN, E.; POSTLETHWAITE, I. Static output feedback stabilisation with $\mathcal{H}_\infty$ performance for a class of plants. **Systems & Control Letters**, v. 43, n. 3, p. 159–166, 2001. Pages 15 and 17.

[35] HO, D. W. C.; LU, G. Robust stabilization for a class of discrete-time non-linear systems via output feedback: The unified LMI approach. **International Journal of Control**, v. 76, n. 2, p. 105–115, 2003. Pages 15 and 17.

[36] GEROMEL, J. C.; DEAECTO, G.S. Generalized Kleinman-Newton method. **Optimal Control Applications and Methods**, v. 39, n. 2, p. 1130–1140, 2018. Page 15.

[37] EL-GHAOUI, L.; OUSTRY, F.; AIT-RAMI, M. A cone complementarity linearization algorithm for static output feedback and related problems. **IEEE Transactions on Automatic Control**, v. 42, n. 8, p. 1171–1176, August 1997. Pages 15 and 17.

[38] BHATTACHARYYA, S.; PATRA, S. Static output-feedback stabilization for MIMO LTI positive systems using LMI-based iterative algorithms. **IEEE Control Systems Letters**, IEEE, v. 2, n. 2, p. 242–247, 2018. Pages 15 and 17.

[39] GAO, H.; LAM, J.; WANG, C.; WANG, Y. Delay-dependent robust output feedback stabilisation of discrete-time systems with time-varying state delay. **IEE Proceedings — Control Theory and Applications**, v. 151, n. 6, p. 691–698, November 2004. Pages 15, 17, and 18.

[40] ZHANG, L.; SHI, P.; BOUKAS, E. K. $\mathcal{H}_\infty$ output-feedback control for switched linear discrete-time systems with time-varying delays. **International Journal of Control**, v. 80, n. 8, p. 1354–1365, 2007. Pages 15, 17, and 18.

[41] NACHIDI, M.; TADEO, F.; HMAMED, A.; BENZAOUIA, A. Static output-feedback stabilization for time-delay Takagi-Sugeno fuzzy systems. In: **Proceedings of the 46th IEEE Conference on Decision and Control**. New Orleans, LA, USA: [s.n.], 2007. p. 1634–1639. Pages 15, 17, and 18.

[42] HE, Y.; WU, M.; LIU, G-P; SHE, J-H. Output feedback stabilization for a discrete-time system with a time-varying delay. **IEEE Transactions on Automatic Control**, v. 53, n. 10, p. 2372–2377, 2008. Pages 15, 17, and 18.

[43] SCHULER, S.; GRUHLER, M.; MÜNZ, U.; ALLGÖWER, F. Design of structured static output feedback controllers. In: **Proceedings of the 18th IFAC World Congress**. Milano, Italy: [s.n.], 2011. v. 44, p. 271–276. Pages 15 and 17.

[44] NACHIDI, M.; BENZAOUIA, A.; TADEO, F.; Ait-Rami, M. LMI-based approach for output-feedback stabilization for discrete-time Takagi-Sugeno systems. **IEEE Transactions on Fuzzy Systems**, v. 16, n. 5, p. 1188–1196, 2008. Pages 15 and 17.

[45] NACHIDI, M.; TADEO, F.; BENZAOUIA, A.; Ait Rami, M. Static output-feedback for Takagi-Sugeno systems with delays. **International Journal of Adaptive Control and Signal Processing**, v. 25, n. 4, p. 295–312, 2011. Pages 15, 17, and 18.

[46] CHEN, J.; LIN, C.; CHEN, B.; WANG, Q. G. Fuzzy-model-based admissibility analysis and output feedback control for nonlinear discrete-time systems with time-varying delay. **Information Sciences**, Elsevier Inc., v. 412-413, p. 116–131, 2017. Pages 15, 17, and 18.

[47] GONZÁLEZ, A.; CUENCA, A.; BALAGUER, V.; GARCÍA, P. Event-triggered predictor-based control with gain-Scheduling and extended state observer for networked control systems. **Information Sciences**, v. 491, p. 90–108, 2019. Pages 15 and 17.

[48] LO, J.-C.; LIN, M.-L. Robust $\mathcal{H}_\infty$ nonlinear control via fuzzy static output feedback. **IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications**, v. 50, n. 11, p. 1494–1502, 2003. Pages 15, 17, 31, and 33.

[49] BARA, G. I.; BOUTAYEB, M. Static output feedback stabilization with $\mathcal{H}_\infty$ performance for linear discrete-time systems. **IEEE Transactions on Automatic Control**, v. 50, n. 2, p. 250–254, feb 2005. Pages 15 and 17.

[50] DONG, J.; YANG, G.-H. Static output feedback control synthesis for linear systems with time-invariant parametric uncertainties. **IEEE Transactions on Automatic Control**, v. 52, n. 10, p. 1930–1936, October 2007. Pages 15 and 17.

[51] DONG, J.; YANG, G.-H. Robust static output feedback control for linear discrete-time systems with time-varying uncertainties. **Systems & Control Letters**, v. 57, n. 2, p. 123–131, 2008. Pages 15, 17, 29, 30, 33, and 52.

[52] DONG, J.; YANG, G.-H. Static output feedback $\mathcal{H}_\infty$ control of a class of nonlinear discrete-time systems. **Fuzzy Sets and Systems**, Elsevier, v. 160, n. 19, p. 2844–2859, 2009. Pages 15 and 17.

[53] DE CAIGNY, J.; CAMINO, J. F.; OLIVEIRA, R. C. L. F.; PERES, P. L. D.; SWEVERS, J. Gain-scheduled $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control of discrete-time polytopic time-varying systems. **IET Control Theory & Applications**, v. 4, n. 3, p. 362–380, March 2010. Pages 16, 17, and 30.

[54] KAU, S.-W.; LEE, H.-J.; YANG, C.-M.; LEE, C.-H.; HONG, L.; FANG, C.-H. Robust $\mathcal{H}_\infty$ fuzzy static output feedback control of T-S fuzzy systems with parametric uncertainties. **Fuzzy Sets and Systems**, v. 158, n. 2, p. 135–146, 2007. Pages 16 and 17.

[55] JUNGERS, M.; CASTELAN, E. B. Gain-scheduled output control design for a class of discrete-time nonlinear systems with saturating actuators. **Systems and Control Letters**, v. 60, n. 3, p. 169–173, 2011. Pages 16 and 17.

[56] BOUARAR, T.; GUELTON, K.; MANAMANNI, N. Static output feedback controller design for takagi-sugeno systems - a fuzzy lyapunov lmi approach. In: **Proceedings of the 48th IEEE Conference on Decision and Control — 28th Chinese Control Conference**. Shanghai, P.R. China: [s.n.], 2009. p. 4150–4155. Page 16.

[57] CHADLI, M.; GUERRA, T. M. LMI solution for robust static output feedback control of discrete Takagi-Sugeno fuzzy models. **IEEE Transactions on Fuzzy Systems**, IEEE, v. 20, n. 6, p. 1160–1165, 2012. Pages 16 and 17.

[58] DONG, J.; YANG, G.-H. Robust static output feedback control synthesis for linear continuous systems with polytopic uncertainties. **Automatica**, v. 49, n. 6, p. 1821–1829, 2013. Pages 16 and 17.

[59] CHANG, X. H.; YANG, G. H. New results on output feedback $\mathcal{H}_\infty$ control for linear discrete-time systems. **IEEE Transactions on Automatic Control**, v. 59, n. 5, p. 1355–1359, 2014. Pages 16 and 17.

[60] CHANG, X.-H.; PARK, J. H.; ZHOU, J. Robust static output feedback $\mathcal{H}_\infty$ control design for linear systems with polytopic uncertainties. **Systems & Control Letters**, v. 85, p. 23–32, 2015. Pages 16 and 17.

[61] ELIAS, L. J.; FARIA, F. A.; ARAUJO, R.; MAGOSSI, R. F.Q.; OLIVEIRA, V. A. Robust static output feedback $\mathcal{H}_\infty$ control for uncertain Takagi-Sugeno fuzzy systems. **IEEE Transactions on Fuzzy Systems**, v. 30, n. 10, p. 4434–4446, 2022. Page 16.

[62] MEHDI, D.; BOUKAS, E. K.; BACHELIER, O. Static output feedback design for uncertain linear discrete time systems. **IMA Journal of Mathematical Control and Information**, v. 21, n. 1, p. 1–13, March 2004. Pages 16 and 17.

[63] AGULHARI, C. M.; OLIVEIRA, R. C. L. F.; PERES, P. L. D. Static output feedback control of polytopic systems using polynomial Lyapunov functions. In: **Proceedings of the 49th IEEE Conference on Decision and Control**. Atlanta, GA, USA: [s.n.], 2010. p. 6894–6901. Pages 16 and 17.

[64] RUBIÓ-MASSEGÚ, J.; PALACIOS-QUIÑONERO, F.; ROSSELL, J. M.; KARIMI, H. R. Static output-feedback controller design for vehicle suspensions: an effective two-step computational approach. **IET Control Theory & Applications**, v. 8, n. 15, p. 1566–1574, oct 2014. Pages 16 and 17.

[65] AGULHARI, C. M.; OLIVEIRA, R. C. L. F.; PERES, P. L. D. Robust $\mathcal{H}_\infty$ static output-feedback design for time-invariant discrete-time polytopic systems from parameter-dependent state-feedback gains. In: **Proceedings of the 2010 American Control Conference**. Baltimore, MD: [s.n.], 2010. p. 4677–4682. Pages 16 and 17.

[66] BRAGA, M. F.; MORAIS, C. F.; TOGNETTI, E. S.; OLIVEIRA, R. C.L.F.; PERES, P. L.D. Discretization and event triggered digital output feedback control of LPV systems. **Systems & Control Letters**, v. 86, p. 54–65, 2015. Pages 16 and 17.

[67] AL-JIBOORY, A. K.; ZHU, G. Static output-feedback robust gain-scheduling control with guaranteed $\mathcal{H}_2$ performance. **Journal of the Franklin Institute**, Elsevier Ltd, v. 355, n. 5, p. 2221–2242, 2018. Pages 16 and 17.

[68] SERENI, B.; ASSUNÇÃO, E.; TEIXEIRA, M. C. M. New gain-scheduled static output feedback controller design strategy for stability and transient performance of LPV systems. **IET Control Theory & Applications**, v. 14, n. 5, p. 717–725, mar 2020. Pages 16 and 17.

[69] SERENI, B.; ASSUNÇÃO, E.; TEIXEIRA, M. C. M. Stabilization and disturbance rejection with decay rate bounding in discrete-time linear parameter-varying systems via $\mathcal{H}_\infty$ gain-scheduling static output feedback control. **International Journal of Robust and Nonlinear Control**, v. 32, n. 14, p. 7920–7945, sep 2022. Pages 16, 17, 29, 30, 31, and 33.

[70] SADEGHZADEH, A. Gain-scheduled static output feedback controller synthesis for discrete-time LPV systems. **International Journal of Systems Science**, Taylor & Francis, v. 48, n. 14, p. 2936–2947, 2017. Pages 16 and 17.

[71] BENTON, R. E.; SMITH, Dirk. Static output feedback stabilization with prescribed degree of stability. **IEEE Transactions on Automatic Control**, v. 43, n. 10, p. 1493–1496, 1998. Pages 16 and 17.

[72] CAO, Y.-Y; LAM, J.; SUN, Y.-X. Static output feedback stabilization: An ILMI approach. **Automatica**, v. 34, n. 12, p. 1641–1645, December 1998. Pages 16 and 17.

[73] HAIDAR, A.; BOUKAS, E. K.; XU, S.; LAM, J. Exponential stability and static output feedback stabilisation of singular time-delay systems with saturating actuators. **IET Control Theory and Applications**, v. 3, n. 9, p. 1293–1305, 2009. Pages 16 and 17.

[74] DU, B.; LAM, J.; SHU, Z. Stabilization for state/input delay systems via static and integral output feedback. **Automatica**, Elsevier Ltd, v. 46, n. 12, p. 2000–2007, 2010. Pages 16 and 17.

[75] DINH, Q. T.; GUMUSSOY, S.; MICHIELS, W.; DIEHL, M. Combining convex-concave decompositions and linearization approaches for solving BMIs, with application to static output feedback. **IEEE Transactions on Automatic Control**, v. 57, n. 6, p. 1377–1390, June 2012. Pages 16 and 17.

[76] BENTON, R. E.; SMITH, Dirk. Non-iterative LMI-based algorithm for robust static-output-feedback stabilization. **International Journal of Control**, v. 72, n. 14, p. 1322–1330, 1999. Pages 16 and 17.

[77] PEAUCELLE, D.; ARZELIER, D. An efficient numerical solution for $\mathcal{H}_2$ static output feedback synthesis. In: **Proceedings of the 2001 European Control Conference**. Porto, Portugal: [s.n.], 2001. p. 3800–3805. Pages 16 and 17.

[78] ARZELIER, D.; PEAUCELLE, D.; SALHI, S. Robust static output feedback stabilization for polytopic uncertain systems: Improving the guaranteed performance bound. In: **Proceedings of the 4th IFAC Symposium on Robust Control Design (ROCOND 2003)**. Milan, Italy: [s.n.], 2003. p. 425–430. Pages 16 and 17.

[79] CARNIATO, L. A.; CARNIATO, A. A.; TEIXEIRA, M. C. M.; CARDIM, R.; MAINARDI, E. I.; ASSUNÇÃO, E. Output control of continuous-time uncertain switched linear systems via switched static output feedback. **International Journal of Control**, v. 93, n. 5, p. 1127–1146, 2020. Pages 16 and 17.

[80] SATO, M. Discrete-time gain-scheduled output-feedback controllers exploiting inexact scheduling parameters via parameter-dependent Lyapunov functions. In: **Proceedings of the 50th IEEE Conference on Decision and Control — European Control Conference ECC 2011**. Orlando, FL, USA: [s.n.], 2011. p. 1938–1943. Pages 16 and 17.

[81] EMEDI, Z.; KARIMI, A. Fixed-structure LPV discrete-time controller design with induced l_2-norm and $\mathcal{H}_2$ performance. **International Journal of Control**, Taylor & Francis, v. 89, n. 3, p. 494–505, 2016. Pages 16 and 17.

[82] PALMA, J. M.; MORAIS, C. F.; OLIVEIRA, R. C. L. F. Linear matrix inequality-based solution for memory static output-feedback control of discrete-time linear systems affected by time-varying parameters. **International Journal of Robust and Nonlinear Control**, v. 31, n. 9, p. 4324–4336, jun 2021. Pages 16 and 17.

[83] WU, H-N. An ILMI approach to robust $\mathcal{H}_2$ static output feedback fuzzy control for uncertain discrete-time nonlinear systems. **Automatica**, v. 44, n. 9, p. 2333–2339, 2008. Pages 16, 17, 31, and 33.

[84] FENG, Z.; ZHANG, H.; LI, Rui B. State and static output feedback control of singular Takagi–Sugeno fuzzy systems with time-varying delay via proportional plus derivative feedback. **Information Sciences**, Elsevier Inc., v. 608, p. 1334–1351, 2022. Pages 16 and 17.

[85] JENNAWASIN, T.; BANJERDPONGCHAI, D. Iterative LMI approach to robust static output feedback control of uncertain polynomial systems with bounded actuators. **Automatica**, Elsevier Ltd, v. 123, p. 109292, 2021. Page 16.

[86] LIMA, T. A.; MADEIRA, D. S.; VIANA, V. V.; OLIVEIRA, R. C. L. F. Static output feedback stabilization of uncertain rational nonlinear systems with input saturation. **Systems & Control Letters**, Elsevier B.V., v. 168, p. 105359, 2022. Page 16.

[87] ROSA, T. E.; MORAIS, C. F.; OLIVEIRA, R. C. L. F. New robust LMI synthesis conditions for mixed $\mathcal{H}_2/\mathcal{H}_\infty$ gain-scheduled reduced-order DOF control of discrete-time LPV systems. **International Journal of Robust and Nonlinear Control**, v. 28, n. 18, p. 6122–6145, 2018. Pages 16, 17, 29, 30, 33, 52, and 53.

[88] GEROMEL, J. C.; KOROGUI, R. H. Analysis and synthesis of robust control systems using linear parameter dependent Lyapunov functions. **IEEE Transactions on Automatic Control**, v. 51, n. 12, p. 1984–1989, 2006. Page 16.

[89] SILVA, Rafael N; FREZZATTO, Luciano. A new parametrization for static output feedback control of LPV discrete-time systems. **Automatica**, Elsevier Ltd, v. 128, p. 109566, jun 2021. Pages 16, 17, 29, 30, 31, and 33.

[90] NGUYEN, A.-T.; CHEVREL, P.; CLAVEAU, F. Gain-scheduled static output feedback control for saturated LPV systems with bounded parameter variations. **Automatica**, Elsevier Ltd, v. 89, p. 420–424, mar 2018. Page 16.

[91] NGUYEN, A-T.; TANAKA, K.; DEQUIDT, A.; DAMBRINE, M. Static output feedback design for a class of constrained Takagi–Sugeno fuzzy systems. **Journal of the Franklin Institute**, Elsevier Ltd, v. 354, n. 7, p. 2856–2870, 2017. Pages 8, 16, 34, 35, 55, and 56.

[92] ESTRADA-MANZO, V.; LENDEK, Zs; GUERRA, T. M. An alternative LMI static output feedback control design for discrete-time nonlinear systems represented by Takagi–Sugeno models. **ISA Transactions**, Elsevier, v. 84, p. 104–110, 2019. Pages 8, 16, 17, 31, 32, and 33.

[93] FRIDMAN, Emilia. **Introduction to Time-Delay Systems**. Cham: Springer International Publishing, 2014. (Systems & Control: Foundations & Applications). Page 17.

[94] de SOUZA, L. T. F.; PEIXOTO, M. L. C.; PALHARES, R. M. New gain-scheduling control conditions for time-varying delayed LPV systems. **Journal of the Franklin Institute**, v. 359, n. 2, p. 719–742, jan 2022. Page 17.

[95] KIM, S. H. Improved approach to robust $\mathcal{H}_\infty$ stabilization of discrete-time T- S fuzzy systems with time-varying delays. **IEEE Transactions on Fuzzy Systems**, v. 18, n. 5, p. 1008–1015, 2010. Page 18.

[96] KWON, O. M.; PARK, M. J.; PARK, Ju H.; LEE, S. M.; CHA, E. J. Stability and stabilization for discrete-time systems with time-varying delays via augmented Lyapunov-Krasovskii functional. **Journal of the Franklin Institute**, v. 350, n. 3, p. 521–540, 2013. Page 18.

[97] NAM, P. T.; TRINH, H.; PATHIRANA, P. N. Discrete inequalities based on multiple auxiliary functions and their applications to stability analysis of time-delay systems. **Journal of the Franklin Institute**, Elsevier, v. 352, n. 12, p. 5810–5831, dec 2015. Page 18.

[98] ZHAO, T.; LIANG, W.; DIAN, S.; XIAO, J.; WEI, Z. Improved stability and stabilisation criteria for discrete time-delay systems via a novel double summation inequality. **IET Control Theory & Applications**, v. 12, n. 3, p. 327–337, 2018. Page 18.

[99] SEURET, A.; GOUAISBAUT, F.; FRIDMAN, E. Stability of discrete-time systems with time-varying delays via a novel summation inequality. **IEEE Transactions on Automatic Control**, v. 60, n. 10, p. 2740–2745, 2015. Pages 18, 37, and 54.

[100] ZHU, Xun-Lin; YANG, Guang-Hong. Jensen inequality approach to stability analysis of discrete-time systems with time-varying delay. In: **Proceedings of the 2008 American Control Conference**. Seattle, WA, USA: [s.n.], 2008. p. 1644–1649. Page 18.

[101] CHEN, J.; XU, S.; JIA, X.; ZHANG, B. Novel Summation Inequalities and Their Applications to Stability Analysis for Systems with Time-Varying Delay. **IEEE Transactions on Automatic Control**, IEEE, v. 62, n. 5, p. 2470–2475, 2017. Page 18.

[102] LEE, S. Y.; LEE, W. I.; PARK, P. Stability analysis of discrete-time systems with time-varying delays: generalized zero equalities approach. **International Journal of Robust and Nonlinear Control**, v. 27, n. 6, p. 981–999, 2017. Page 18.

[103] GYURKOVICS, É.; KISS, K.; NAGY, I.; TAKÁCS, T. Multiple summation inequalities and their application to stability analysis of discrete-time delay systems. **Journal of the Franklin Institute**, Elsevier, v. 354, n. 1, p. 123–144, 2017. Page 18.

[104] LEE, S. Y.; PARK, J. M.; PARK, P. G. Bessel summation inequalities for stability analysis of discrete-time systems with time-varying delays. **International Journal of Robust and Nonlinear Control**, v. 29, n. 2, p. 473–491, 2019. Page 18.

[105] LIMA, T. A.; de ALMEIDA FILHO, M. P.; TORRICO, B. C.; NOGUEIRA, F. G.; CORREIA, W. B. A practical solution for the control of time-delayed and delay-free systems with saturating actuators. **European Journal of Control**, Elsevier Ltd, v. 51, p. 53–64, 2020. Page 18.

[106] NAM, P. T.; LUU, T. H. A new delay-variation-dependent stability criterion for delayed discrete-time systems. **Journal of the Franklin Institute**, Elsevier Ltd, v. 357, n. 11, p. 6951–6967, jul 2020. Page 18.

[107] LI, X.; WANG, R.; DU, S.; LI, T. An improved exponential stability analysis method for discrete-time systems with a time-varying delay. **International Journal of Robust and Nonlinear Control**, v. 32, n. 2, p. 669–681, jan 2022. Page 18.

[108] PALHARES, R. M.; DE SOUZA, C. E.; PERES, P. L. D. Robust $\mathcal{H}_\infty$ filtering for uncertain discrete-time state-delayed systems. **IEEE Transactions on Signal Processing**, v. 49, n. 8, p. 1096–1703, August 2001. Page 18.

[109] CHEN, W.-H.; GUAN, Z.-H.; LU, X. Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay. **IEE Proceedings — Control Theory and Applications**, v. 150, n. 4, p. 412–416, 2003. Page 18.

[110] LACERDA, M. J.; LEITE, V. J. S.; OLIVEIRA, R. C. L. F.; PERES, P. L. D. Delay-dependent robust $\mathcal{H}_\infty$ filter design for state-delayed discrete-time linear systems via homogeneous polynomial matrices. **IET Control Theory & Applications**, v. 7, n. 1, p. 125–135, January 2013. Page 18.

[111] PEIXOTO, M. L. C.; REIS, G. L.; COUTINHO, P. H. S.; TORRES, L.A .B.; PALHARES, R. M. Stability analysis of uncertain discrete-time systems with time-varying delays using difference-algebraic representation. In: **Proceedings of the 2022 European Control Conference**. London, United Kingdom: [s.n.], 2022. p. 2069–2074. Page 18.

[112] WANG, C.; GAO, H.; WU, L.; SHI, P. Delay-dependent robust $\mathcal{H}_\infty$ and $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering for LPV systems with both discrete and distributed delays. **IEE Proceedings — Control Theory and Applications**, v. 153, n. 4, p. 483–492, 2006. Page 18.

[113] ZHOU, S.; ZHENG, W. X. A parameter-dependent Lyapunov function based approach to $\mathcal{H}_\infty$-control of LPV discrete-time systems with delays. In: **Proceedings of the 47th IEEE Conference on Decision and Control**. Cancun, Mexico: [s.n.], 2008. p. 4669–4674. Page 18.

[114] ZHANG, X.; ZHU, H. Robust stability and stabilization criteria for discrete singular time-delay LPV systems. **Asian Journal of Control**, v. 14, n. 4, p. 1084–1094, 2012. Page 18.

[115] KU, Cheung Chieh; CHEN, Guan Wei. Delay-Dependent Robust Control for Discrete-Time Uncertain Stochastic Systems with Time-Varying Delays. **Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME**, v. 139, n. 10, p. 1–11, 2017. Page 18.

[116] PESSIM, P. S. P.; COUTINHO, P. H. S.; BESSA, I.; PEIXOTO, M. L. C.; LACERDA, M. J.; PALHARES, R. M. Controle distribuıdo para sistemas não lineares interconectados sujeitos a retardos variantes no tempo nas interconexões. In: **Anais do XXIV Congresso Brasileiro de Automática**. Fortaleza, CE, Brasil: [s.n.], 2022. p. 1–6. Page 18.

[117] de Oliveira, T. G.; PALHARES, R. M.; Campos, V. C. S.; Queiroz, P. S.; Gonçalves, E. N. Improved Takagi-Sugeno fuzzy output tracking control for nonlinear networked control systems. **Journal of the Franklin Institute**, v. 354, n. 16, p. 7280–7305, 2017. Page 18.

[118] CHERIDI, D. E.; MANSOURI, N. Robust $\mathcal{H}_\infty$ Fault-Tolerant Control for Discrete-Time Nonlinear System with Actuator Faults and Time-Varying Delays Using Nonlinear T–S Fuzzy Models. **Circuits, Systems, and Signal Processing**, v. 39, n. 1, p. 175–198, jan 2020. Page 18.

[119] SUN, S.; ZHANG, H.; SU, H.; LIANG, Y. Robust reliable $\mathcal{H}_\infty$ optimization control for uncertain discrete-time Takagi–Sugeno fuzzy systems with time-varying delay. **Optimal Control Applications and Methods**, p. 848–876, 2021. ISSN 10991514. Page 18.

[120] CHEN, W.-H.; ZHANG, C.-K.; XIE, K.-Y.; ZHU, C.; HE, Y. Delay-Variation-Dependent Criteria on Stability and Stabilization for Discrete-Time T-S Fuzzy Systems With Time-Varying Delays. **IEEE Transactions on Fuzzy Systems**, IEEE, v. 6706, n. c, p. 1–1, 2022. Page 18.

[121] PEIXOTO, M. L. C.; COUTINHO, P. H. S.; PALHARES, R. M. Improved robust gain-scheduling static output-feedback control for discrete-time LPV systems. **European Journal of Control**, Elsevier, v. 58, p. 11–16, 2021. Pages 19, 26, and 31.

[122] PEIXOTO, M. L. C.; COUTINHO, P. H. S.; LACERDA, M. J.; PALHARES, R. M. Guaranteed region of attraction estimation for time-delayed fuzzy systems via static output-feedback control. **Automatica**, p. 110438, jun 2022. Pages 20 and 36.

[123] RUGH, W. J.; SHAMMA, J. S. Research on gain scheduling. **Automatica**, v. 36, n. 10, p. 1401–1425, 2000. Page 21.

[124] SHAMMA, J. S. **Analysis and design of gain scheduled control systems**. Tese (Doutorado) — Massachusetts Institute of Technology, 1988. Page 21.

[125] YU, Z.; CHEN, H.; WOO, P. Gain scheduled LPV $\mathcal{H}_\infty$ control based on LMI approach for a robotic manipulator. **Journal of Robotic Systems**, 19, n. 12, p. 585–593, 2002. Page 21.

[126] RABAOUI, B.; RODRIGUES, M.; HAMDI, H.; BRAIEK, N. B. A model reference tracking based on an active fault tolerant control for LPV systems. **International Journal of Adaptive Control and Signal Processing**, 32, n. 6, p. 839–857, 2018. Page 21.

[127] BIANCHI, R. J. M. F. D.; CHRISTIANSEN, C. F. Gain scheduling control of variable-speed wind energy conversion systems using quasi-LPV models. **Control Engineering Practice**, 13, n. 2, p. 247–255, 2005. Page 21.

[128] NOGUEIRA, F. G.; BARRA JUNIOR, W.; da COSTA JUNIOR, C. T.; LANA, J. J. LPV-based power system stabilizer: Identification, control and field tests. **Control Engineering Practice**, v. 72, n. December 2017, p. 53–67, 2018. Page 21.

[129] SCHAAB, K.; HAHN, J.; WOLKOV, M.; STURSBERG, O. Robust control for voltage and transient stability of power grids relying on wind power. **Control Engineering Practice**, Elsevier Ltd, v. 60, n. December 2016, p. 7–17, 2017. Page 21.

[130] IBÁÑEZ, B.; INTHAMOUSSOU, F. A.; de Battista, H. Wind turbine load analysis of a full range LPV controller. **Renewable Energy**, 145, p. 2741–2753, 2020. Page 21.

[131] COX, P. B.; WEILAND, S.; TÓTH, R. Affine parameter-dependent Lyapunov functions for LPV systems with affine dependence. **IEEE Transactions on Automatic Control**, v. 63, n. 11, p. 3865–3872, 2018. Page 21.

[132] WU, F.; PRAJNA, S. A new solution approach to polynomial LPV system analysis and synthesis. In: **Proceedings of the 2004 American Control Conference**. Boston, MA, USA: [s.n.], 2004. p. 1362–1367. Page 21.

[133] LACERDA, M. J.; FERREIRA, G. 0.; LEITE, V. J. S. Rational state-feedback control for discrete-time state-polynomial LPV systems. In: **Anais do XXIV Congresso Brasileiro de Automática**. Fortaleza, CE, Brasil: [s.n.], 2022. p. 1–6. Page 21.

[134] POLCZ, P.; PÉNI, T.; KULCSÁR, B.; SZEDERKÉNYI, G. Induced $\mathcal{L}_2$-gain computation for rational LPV systems using Finsler's lemma and minimal generators. **Systems and Control Letters**, Elsevier, v. 142, p. 104738, 2020. Page 21.

[135] SHAMMA, J. S.; TU, K-Y. Approximate set-valued observers for nonlinear systems. **IEEE Transactions on Automatic Control**, v. 42, n. 5, p. 648–658, 1997. Page 23.

[136] ARAÚJO, R. F.; COUTINHO, P. H.S.; NGUYEN, A.-T.; PALHARES, R. M. Delayed nonquadratic $\mathcal{L}_2$-stabilization of continuous-time nonlinear Takagi–Sugeno fuzzy models. **Information Sciences**, v. 563, p. 59–69, 2021. Page 24.

[137] TAKAGI, T.; SUGENO, M. Fuzzy identification of systems and its applications to modeling and control. **IEEE Transactions on Systems, Man, and Cybernetics**, SMC-15, n. 1, p. 116–132, January 1985. Page 24.

[138] BEGOVICH, O.; SANCHEZ, E. N.; MALDONADO, M. Takagi-Sugeno fuzzy scheme for real-time trajectory tracking of an underactuated robot. **IEEE Transactions on Control Systems Technology**, v. 10, n. 1, p. 14–20, 2002. ISSN 1063-6536. Page 24.

[139] LIANG, Y.-W.; XU, S.-D.; LIAW, D.-C.; CHEN, C.-C. A Study of T–S Model-Based SMC Scheme With Application to Robot Control. **IEEE Transactions on Industrial Electronics**, v. 55, n. 11, p. 3964–3971, 2008. Page 24.

[140] ICHALAL, D.; MARX, B.; RAGOT, J.; MAQUIN, D. New fault tolerant control strategies for nonlinear Takagi-Sugeno systems. **International Journal of Applied Mathematics and Computer Science**, v. 22, n. 1, p. 197–210, 2012. Page 24.

[141] KO, H.; JATSKEVICH, J. Power Quality Control of Wind-Hybrid Power Generation System Using Fuzzy-LQR Controller. **IEEE Transactions on Energy Conversion**, v. 22, n. 2, p. 516–527, 2007. Page 24.

[142] SHANMUGAM, L.; JOO, Y. H. Stability and stabilization for t–s fuzzy large-scale interconnected power system with wind farm via sampled-data control. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 4, p. 2134–2144, 2021. Page 24.

[143] GEORG, S.; SCHULTE, H.; ASCHEMANN, H. Control-oriented modelling of wind turbines using a takagi-sugeno model structure. In: **Proceedings of the 2011 IEEE International Conference on Fuzzy Systems**. Brisbane, QLD, Australia: [s.n.], 2012. p. 1–8. Page 24.

[144] TANAKA, K.; WANG, H.O. **Fuzzy Control Systems Design and Analysis: a Linear Matrix Inequality Approach**. [S.l.]: John Wiley & Sons, 2004. Pages 24, 25, 45, and 48.

[145] REIS, G. L.; ARAÚJO, R. F.; TORRES, L. A.B.; PALHARES, R. M. Stabilization of rational nonlinear discrete-time systems by state feedback and static output feedback. **European Journal of Control**, Elsevier Ltd, v. 67, 2022. Page 31.

[146] DONG, J.; YANG, G.-H. Static output feedback $\mathcal{H}_\infty$ control of a class of nonlinear discrete-time systems. **Fuzzy Sets and Systems**, Elsevier, v. 160, n. 19, p. 2844–2859, 2009. Pages 31 and 33.

[147] LIU, K.; SEURET, A.; XIA, Y. Stability analysis of systems with time-varying delays via the second-order Bessel–Legendre inequality. **Automatica**, Elsevier Ltd, v. 76, p. 138–142, feb 2017. Pages 37, 43, and 63.

[148] DE OLIVEIRA, M. C.; SKELTON, R. E. Stability tests for constrained linear systems. In: Reza Moheimani, S. O. (Ed.). **Perspectives in Robust Control**. New York, NY: Springer-Verlag, 2001, (Lecture Notes in Control and Information Science, v. 268). p. 241–257. Page 42.

[149] CHEN, Y.; WANG, Z.; FEI, S.; HAN, Q. L. Regional stabilization for discrete time-delay systems with actuator saturations via a delay-dependent polytopic approach. **IEEE Transactions on Automatic Control**, IEEE, v. 64, n. 3, p. 1257–1264, 2019. ISSN 15582523. Page 44.

[150] YANG, X.; LI, T.; WU, Y.; WANG, Y.; LONG, Y. Fault estimation and fault tolerant control for discrete-time nonlinear systems with perturbation by a mixed design scheme. **J. Franklin Inst.**, Elsevier, v. 358, n. 3, p. 1860–1887, 2021. Page 47.

[151] LEE, J.-W. On uniform stabilization of discrete-time linear parameter-varying control systems. **IEEE Transactions on Automatic Control**, v. 51, n. 10, p. 1714–1721, October 2006. Pages 52 and 53.

[152] XIANG, W. Parameter-memorized Lyapunov functions for discrete-time systems with time-varying parametric uncertainties. **Automatica**, v. 87, p. 450–454, 2018. Page 52.

[153] SALA, A. Stability analysis of LPV systems: Scenario approach. **Automatica**, Elsevier Ltd, v. 104, p. 233–237, 2019. Page 52.

[154] DAAFOUZ, J.; BERNUSSOU, J. Parameter dependent Lyapunov functions for discrete-time systems with time-varying parametric uncertainties. **Systems & Control Letters**, v. 43, n. 5, p. 355–359, 2001. Pages 52 and 53.

[155] MONTAGNER, V. F.; OLIVEIRA, R. C. L. F.; LEITE, V. J. S.; PERES, P. L. D. Gain scheduled state feedback control of discrete systems with time-varying uncertainties: An LMI approach. In: **Proceedings of the 44th IEEE Conference on Decision and Control — European Control Conference ECC 2005**. Seville, Spain: [s.n.], 2005. p. 4305–4310. Pages 52 and 53.

[156] PANDEY, A. P.; OLIVEIRA, M. C. de. A new discrete-time stabilizability condition for Linear Parameter-Varying systems. **Automatica**, Elsevier Ltd, v. 79, p. 214–217, 2017. Pages 52 and 53.

[157] TARBOURIECH, S.; GARCIA, G.; Gomes da Silva Jr., J. M.; QUEINNEC, I. **Stability and Stabilization of Linear Systems with Saturating Actuators**. London, UK: Springer, 2011. Page 55.

[158] PEIXOTO, M. L. C.; COUTINHO, P. H. S.; BESSA, I.; PALHARES, R. M. Static output-feedback stabilization of discrete-time linear parameter-varying systems under actuator saturation. **International Journal of Robust and Nonlinear Control**, n. January, p. 5799–5809, 2022. Page 57.

[159] PENG, C.; LI, F. A survey on recent advances in event-triggered communication and control. **Information Sciences**, Elsevier BV, v. 457-458, p. 113–125, 2018. Page 57.

[160] ABDELRAHIM, M.; POSTOYAN, R.; DAAFOUZ, J.; NEŠIĆ, D.; HEEMELS, W. P. M. H. Co-design of output feedback laws and event-triggering conditions for the $\mathcal{L}_2$-stabilization of linear systems. **Automatica**, Elsevier, v. 87, p. 337–344, 2018. Page 57.

[161] GIRARD, A. Dynamic triggering mechanisms for event-triggered control. **IEEE Transactions on Automatic Control**, IEEE, v. 60, n. 7, p. 1992–1997, 2015. Page 57.

[162] COUTINHO, P. H. S.; PALHARES, R. M. Dynamic periodic event-triggered gain-scheduling control co-design for quasi-LPV systems. **Nonlinear Analysis: Hybrid Systems**, v. 41, p. 101044, 2021. Page 57.

[163] COUTINHO, P. H. S.; PALHARES, R. M. Co-design of dynamic event-triggered gain-scheduling control for a class of nonlinear systems. **IEEE Transactions on Automatic Control**, v. 67, n. 8, p. 4186 – 4193, 2022. Page 57.

[164] PESSIM, P. S. P.; PEIXOTO, M. L. C.; PALHARES, R. M.; LACERDA, M. J. Static output-feedback control for cyber-physical LPV systems under DoS attacks. **Information Sciences**, Elsevier BV, v. 563, p. 241–255, 2021. Page 57.

[165] COUTINHO, P. H. S.; BESSA, I.; PEIXOTO, M. L. C.; PESSIM, P. S. P.; PALHARES, R. M. Controle com acionamento por eventos resiliente a ataques de negação de serviço. In: **Anais do XXIV Congresso Brasileiro de Automática**. Fortaleza, CE, Brasil: [s.n.], 2022. p. 1–6. Page 57.

[166] PESSIM, P. S.P.; LACERDA, M. J. On the robustness of cyber-physical LPV systems under DoS attacks. **Journal of the Franklin Institute**, Elsevier Ltd, v. 359, n. 2, p. 677–696, 2022. Page 57.

[167] WU, Z.; XIONG, J.; XIE, M. Dynamic event-triggered $\mathcal{L}_\infty$ control for networked control systems under deception attacks: a switching method. **Information Sciences**, v. 561, p. 168–180, 2021. Page 57.

[168] LIU, J.; DONG, Y.; ZHA, L.; TIAN, E.; XIE, X. Event-based security tracking control for networked control systems against stochastic cyber-attacks. **Information Sciences**, v. 612, p. 306–321, oct 2022. Page 57.

[169] LIU, J.; WU, Zheng G.; YUE, D.; PARK, J. H. Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, IEEE, v. 51, n. 2, p. 943–953, 2021. Page 58.

[170] PEIXOTO, M. L. C.; COUTINHO, P. H. S.; PESSIM, P. S. P.; BESSA, I.; PALHARES, R. M. Controle em rede com acionamento por eventos para sistemas sujeitos a ataques cibernéticos. In: **Anais do XXIV Congresso Brasileiro de Automática**. Fortaleza, CE, Brasil: [s.n.], 2022. p. 1–6. Pages 58 and 66.

[171] ROTONDO, D.; WITCZAK, M.; PUIG, V.; NEJJARI, F.; PAZERA, M. Robust unknown input observer for state and fault estimation in discrete-time Takagi-Sugeno systems. **International Journal of Systems Science**, v. 47, n. 14, p. 3409–3424, 2016. Page 59.

[172] XU, F.; TAN, J.; WANG, X.; PUIG, V.; LIANG, B.; YUAN, B.; LIU, H. Generalized set-theoretic unknown input observer for lpv systems with application to state estimation and robust fault detection. **International Journal of Robust and Nonlinear Control**, v. 27, n. 17, p. 3812–3832, 2017. Page 59.

[173] NGUYEN, A.-T.; PAN, J.; GUERRA, T.-M.; WANG, Z. Avoiding unmeasured premise variables in designing unknown input observers for Takagi–Sugeno fuzzy systems. **IEEE Control Systems Letters**, v. 5, n. 1, p. 79–84, 2021. Page 59.

[174] DING, D.; HAN, Q.-L.; GE, X.; WANG, J. Secure state estimation and control of cyber-physical systems: A survey. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 1, p. 176–190, 2020. Page 59.

[175] COUTINHO, P. H. S.; BESSA, I.; XIE, W.-B.; NGUYEN, A.-T.; PALHARES, R. M. A sufficient condition to design unknown input observers for nonlinear systems with arbitrary relative degree. **International Journal of Robust and Nonlinear Control**, Wiley Online Library, v. 32, p. 8331–8348, 2022. Pages 59 and 61.

[176] LÓPEZ-ESTRADA, F. R.; PONSART, J. C.; ASTORGA-ZARAGOZA, C. M.; CAMAS-ANZUETO, J. L.; THEILLIOL, D. Robust sensor fault estimation for descriptor-LPV systems with unmeasurable gain scheduling functions: Application to an anaerobic bioreactor. **International Journal of Applied Mathematics and Computer Science**, v. 25, n. 2, p. 233–244, 2015. Page 59.

[177] NGUYEN, A.-T.; DINH, T.-Q.; GUERRA, T.-M.; PAN, J. Takagi–Sugeno fuzzy unknown input observers to estimate nonlinear dynamics of autonomous ground vehicles: Theory and real-time verification. **IEEE/ASME Transactions on Mechatronics**, v. 26, n. 3, p. 1328–1338, 2021. Page 59.

[178] KORDESTANI, M.; SAIF, M. Observer-based attack detection and mitigation for cyber physical systems: A review. **IEEE Systems, Man, and Cybernetics Magazine**, v. 7, n. 2, p. 35–60, 2021. Page 59.

[179] PAN, J.; NGUYEN, A.-T.; GUERRA, T.-M.; ICHALAL, D. A unified framework for asymptotic observer design of fuzzy systems with unmeasurable premise variables. **IEEE Transactions on Fuzzy Systems**, v. 29, n. 10, p. 2938–2948, 2021. Page 59.

[180] GUERRA, T.-M.; MARQUEZ, R.; KRUSZEWSKI, A.; BERNAL, M. $\mathscr{H}_\infty$ LMI-based observer design for nonlinear systems via Takagi–Sugeno models with unmeasured premise variables. **IEEE Transactions on Fuzzy Systems**, v. 26, n. 3, p. 1498–1509, jun 2018. Page 59.

[181] NGUYEN, A.-T.; CAMPOS, V.; GUERRA, T.-M.; PAN, J.; XIE, W. Takagi–Sugeno fuzzy observer design for nonlinear descriptor systems with unmeasured premise variables and unknown inputs. **Int. J. Robust Nonlinear Control**, v. 31, n. 17, p. 8353–8372, 2021. Page 59.

[182] LAN, J.; PATTON, R. J. A new strategy for integration of fault estimation within fault-tolerant control. **Automatica**, Elsevier, v. 69, p. 48–59, 2016. Page 60.

[183] STEFANOVSKI, J. D. Passive fault tolerant perfect tracking with additive faults. **Automatica**, Elsevier, v. 87, p. 432–436, 2018. Page 60.

[184] BESSA, I.; PALHARES, R. M.; D'Angelo, M. F. S. V.; Chaves Filho, J. E. Data-driven fault detection and isolation scheme for a wind turbine benchmark. **Renewable Energy**, Elsevier, v. 87, p. 634–645, 2016. Page 60.

[185] BESSA, I.; PUIG, V.; PALHARES, R. M. Passivation blocks for fault tolerant control of nonlinear systems. **Automatica**, Elsevier, v. 125, p. 109450, 2021. Page 60.

[186] ZHU, J.-W.; YANG, G.-H.; WANG, H.; WANG, F. Fault estimation for a class of nonlinear systems based on intermediate estimator. **IEEE Transactions on Automatic Control**, IEEE, v. 61, n. 9, p. 2518–2524, sep 2016. Page 60.

[187] LIU, M.; ZHANG, L.; SHI, P.; ZHAO, Y. Fault estimation sliding-mode observer with digital communication constraints. **IEEE Transactions on Automatic Control**, IEEE, v. 63, n. 10, p. 3434–3441, 2018. Page 60.

[188] YAN, J.-J.; YANG, G.-H. Adaptive fault estimation for cyber-physical systems with intermittent DoS attacks. **Information Sciences**, Elsevier, v. 547, p. 746–762, 2021. Page 60.

[189] DU, D.; COCQUEMPOT, V.; JIANG, B. Robust fault estimation observer design for switched systems with unknown input. **Applied Mathematics and Computation**, Elsevier, v. 348, p. 70–83, 2019. Page 60.

[190] ZHU, J.-W.; YANG, G.-H. Fault-tolerant control for linear systems with multiple faults and disturbances based on augmented intermediate estimator. **IET Control Theory & Applications**, Wiley Online Library, v. 11, n. 2, p. 164–172, 2017. Page 60.

[191] WANG, G.; YI, C. Fault estimation for nonlinear systems by an intermediate estimator with stochastic failure. **Nonlinear Dynamics**, Springer, v. 89, n. 2, p. 1195–1204, 2017. Page 60.

[192] ZHU, J.-W.; YANG, G.-H.; ZHANG, W.-A.; YU, L. Cooperative fault tolerant tracking control for multiagent systems: An intermediate estimator-based approach. **IEEE Transactions on Cybernetics**, IEEE, v. 48, n. 10, p. 2972–2980, 2017. Page 60.

[193] ZHU, J.-W.; WANG, Q.; DING, S. X.; ZHANG, W.-A.; YU, L. Performance guaranteed attack reconstruction for cyber-physical systems: A new intermediate estimator-based framework. **Automatica**, Elsevier, v. 142, p. 110388, 2022. Page 60.

[194] HAMDI, H.; RODRIGUES, M.; MECHMECHE, C.; BRAIEK, N. B. Observer-based fault diagnosis for time-delay LPV descriptor systems. **IFAC-PapersOnLine**, Elsevier, v. 51, n. 24, p. 1179–1184, 2018. Page 61.

[195] YOU, F.; CHENG, S.; ZHANG, X.; CHEN, N. Robust fault estimation for Takagi-Sugeno fuzzy systems with state time-varying delay. **International Journal of Adaptive Control and Signal Processing**, Wiley Online Library, v. 34, n. 2, p. 141–150, 2020. Page 61.

[196] ZHU, J.-W.; YANG, G.-H. Fault accommodation for linear systems with time-varying delay. **International Journal of Systems Science**, Taylor & Francis, v. 48, n. 2, p. 316–323, 2017. Page 61.

[197] WEI, Y.; TONG, D.; CHEN, Q.; SUN, Y.; ZHOU, W. Simultaneous actuator and sensor fault estimation for neutral-type systems via intermediate observer. **Transactions of the Institute of Measurement and Control**, v. 44, n. 7, p. 1505–1517, 2022. Page 61.

[198] CAO, J.; DING, D.; LIU, J.; TIAN, E.; HU, S.; XIE, X. Hybrid-triggered-based security controller design for networked control system under multiple cyber attacks. **Information Sciences**, Elsevier Inc., v. 548, p. 69–84, 2021. Page 63.

Appendix

APPENDIX A – FINITE-DIMENSIONAL LMI RELAXATION

The LMIs proposed in this Thesis are parameter-dependent, that is, the problems are of infinite dimensions. These problems can be numerically solved using the following finite-dimensional LMI conditions.

Lemma A.1. Let $\Gamma_{ijql} = \Gamma_{ijql}^{\top}$, $i, j, q, l = 1, \ldots, N$, be matrices of appropriate dimensions. The polynomially parameter-dependent condition

$$\Gamma(\alpha_k, \alpha_{k+1}) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{q=1}^{N}\sum_{l=1}^{N} \alpha_{i,k}\alpha_{j,k}\alpha_{q,k}\alpha_{l,k+1}\Gamma_{ijql} < 0 \tag{A.1}$$

is certified if the following LMIs hold for all $i, j, q, l = 1, \ldots, N$:

$$\begin{aligned}
&\Gamma_{iiil} < 0, \quad i = j = q\\
&\Gamma_{iiql} + \Gamma_{iqil} + \Gamma_{qiil} < 0, \quad i = j,\ j < q\\
&\Gamma_{iqql} + \Gamma_{qiql} + \Gamma_{qqil} < 0, \quad i < j,\ j = q\\
&\Gamma_{ijql} + \Gamma_{iqjl} + \Gamma_{jiql} + \Gamma_{jqil} + \Gamma_{qijl} + \Gamma_{qjil} < 0, \quad i < j,\ j < q.
\end{aligned} \tag{A.2}$$

Proof. The parameter-dependent matrix $\Gamma(\alpha_k, \alpha_{k+1})$ in (A.1) can be equivalently rewritten as follows:

$$\begin{aligned}
\Gamma(\alpha_k, \alpha_{k+1}) &= \sum_{i=1}^{N}\sum_{l=1}^{N} \alpha_{i,k}^3 \alpha_{l,k+1}\Gamma_{iiil}\\
&+ \sum_{i=1}^{N-1}\sum_{q=i+1}^{N}\sum_{l=1}^{N} \alpha_{i,k}^2\alpha_{q,k}\alpha_{l,k+1}\left(\Gamma_{iiql} + \Gamma_{iqil} + \Gamma_{qiil}\right)\\
&+ \sum_{i=1}^{N-1}\sum_{q=i+1}^{N}\sum_{l=1}^{N} \alpha_{i,k}\alpha_{q,k}^2\alpha_{l,k+1}\left(\Gamma_{iqql} + \Gamma_{qiql} + \Gamma_{qqil}\right)\\
&+ \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{q=j+1}^{N}\sum_{l=1}^{N} \alpha_{i,k}\alpha_{j,k}\alpha_{q,k}\alpha_{l,k+1}\left(\Gamma_{ijql} + \Gamma_{iqjl} + \Gamma_{jiql} + \Gamma_{jqil} + \Gamma_{qijl} + \Gamma_{qjil}\right).
\end{aligned}$$

Therefore, LMIs (A.2) are sufficient to ensure that (A.1) holds. □