Competition, Market Share and Technological Change in a Post-Keynesian Duopoly Model

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Abstract

The aim of this paper is to analyze the role of technological change in altering the intra-sector competitive landscape of an industry within a Post-Keynesian framework. In order to do so, we propose a duopoly model that extends the Post-Keynesian theory of the firm accounting for the relationship between market share, technological change and the growth of firms. With our formalization, we notice more than one possible scenario of changes in market domain driven by a technological shock, also affecting the firms' growth trajectories. By doing so, we are able to cast light upon possible market configurations and how these can be propelled or hindered with firm strategies and public policy. But this model can also be seen as a basis for other Post-Keynesian micro models and future more comprehensive macro models that aim to incorporate changes in the micro competitive structure.

Keywords: technological change, market share, competition, Post-Keynesian economics.

Resumo

O objetivo deste artigo é analisar o papel da mudança tecnológica em alterar o cenário competitivo intra-setorial de uma indústria dentro de um arcabouço Pós-Keynesiano. Para tanto, propõe-se um modelo de duopólio que estende a teoria Pós-Keynesiana da firma ao considerar a relação entre market share, mudança tecnológica e crescimento das firmas. Com esta formalização, nota-se mais de um cenário possível de mudanças no domínio do mercado impulsionadas por um choque tecnológico, afetando também as trajetórias de crescimento das firmas. A partir disso, o artigo ressalta possíveis configurações de mercado e como estas podem ser impulsionadas ou prejudicadas por estratégias das firmas e políticas públicas. No entanto, este modelo também pode ser visto como base para outros modelos micro pós-keynesianos e futuros modelos macro mais abrangentes que visam incorporar mudanças na estrutura de competição microeconômica.

Palavras-chave: mudança tecnológica, market share, competição, economia Pós-Keynesiana.

Área de Interesse de Submissão: Área 4: Microeconomia, economia industrial e estrutura produtiva.

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1 Introduction

In Schumpeter's (1934, 1942) notion of creative destruction, innovation has an important role in transforming economic activity. His concept, in simple terms, refers to the process in which a new product or new method displaces an old one, which becomes obsolete. Since innovation usually starts from one firm, this process generates a temporary monopoly for this innovative agent. Through innovation, firms obtain decisive cost, quality or variety advantages over their competitors that affect their profits and market shares, threatening the existence of other firms. The very nature of this process is thus uneven and asymmetric, with few firms leading the transformation of the economy, whereas the rest follows, trying to adapt (METCALFE, 1998). And as Dosi (1988) shows, there is empirical evidence of a causality from successful technological change to a higher degree of industrial concentration.

However, Schumpeter (1942) also highlights how the process of innovation is affected by features of market structure. Perfect competition would eliminate any incentives to innovate, since it allows for instant imitation of a firm's novelty. In this way, non-price competition, as with innovation, needs to be a possibility while remaining a threat. And also, for the author "the business man feels himself to be in a competitive situation even if he is alone in his field" (p. 85). In regard to the empirical support of this statement, Kamien & Schwartz (1975) highlight that most empirical investigations fail to deal with the simultaneity of the matter. Nevertheless, some studies suggest the hypothesis that an "intermediate" market structure, between monopoly and perfect competition, would promote higher inventive and innovative activity. More recently, Blundell, Griffith & Reenen (1999) investigate the relationship between market share and innovation in a sample of British manufacturing firms. Controlling for firm specific heterogeneity in the estimation of a dynamic count data model, they find evidence that market share influences innovation according to how competition is structured. Therefore, the characteristics of market competition could both influence and be influenced by technological change. In this sense, the conformation of those two aspects would define other firm related variables, like its rates of growth and profit (METCALFE, 2013).

The competitive features of an industry also have a relationship with the asymmetries between firms' technological capabilities. This is implied in the "Schumpeterian competition" process described, since as firms innovate, they create asymmetries between them and their competitors. Those asymmetries relate to competitive differences between firms, but also to process technologies and quality of the final output, creating an inter-firm technological gap. As Dosi (1984) argues, imitation and technological diffusion would then decrease this gap, through an adjustment mechanism related to changes in production method and in the product itself. These asymmetries imply that firms are heterogeneous agents in their respective industry. This industry, in turn, is conformed by the competitive process in which some firms survive given heterogeneous factors like their technological variety, their behaviour and strategies, and the industry's degree of imitation and diffusion. Thus, to understand the relationship between technological change and firm growth, the competitive process also needs to be analyzed.

Nevertheless, as summarized by Lavoie (2014), this view of the dynamics between competition and innovation is similar to the one that Post-Keynesian theory has in mind. This literature offers a more realistic view of the firm, as depicted for example in Eichner (1976) and Galbraith (1967) accounts of the current corporation. According to this theory, firms aim at power and growth instead of profits, in an environment with fundamental uncertainty. Markets are also seen as being imperfect, where a mark-up or full-cost pricing strategy prevails. In this case, the main competition mechanism between firms becomes non-price factors. Competition is then driven not by the adjustment of price, but by adjustments in the different branches of innovation, branding and advertisement.

Although this microeconomic perspective is assumed in most Post-Keynesian macroeconomic models, there are not many formal micro models of this competition dynamics, and how it changes intricately with technological progress. There is a recent literature that incorporates Schumpeterian microfoundations - mostly evolutionary neo-Schumpeterian ones (POSSAS et al., 2001; MELO; POSSAS; DWECK, 2016) following the seminal work of Nelson & Winter (1982) - into macrodynamic models with effective demand along Keynesian lines. These models have greatly contributed to the literature by incorporating important Keynesian and Schumpeterian features into the same theoretical framework with the aim to describe the competitive process among firms in an oligopolistic market. However, by assuming that firms essentially compete on prices, these models disregard one of the most salient features of the competitive process in modern oligopolistic markets, which is precisely non-price competition and its relation to the patterns of technological innovation and marketing strategies.

This paper proposes a theoretical duopoly model of how technological change, market share and the growth of firms are dynamically related, inspired by the Post-Keynesian theory of the firm. This model allows us to analyze the role of technological change in altering the intra-sector competitive landscape of an industry within this framework. With our formalization, we notice fundamental changes in market domain driven by innovation, much like what is proposed by the Schumpeterian literature. By doing so, we are able to cast light upon possible market configurations and how these can be propelled/hindered with firm strategies and public policy.

The paper proceeds as follows. Section 2 presents the basic assumptions describing the duopolist industry modelled. Section 3 looks at the medium-run behaviour of this economy, and how it is affected by an exogenous technology shock. Section 4 examines the long-run dynamics and the possible paths driven by technological change. Finally, section 5 concludes.

2 Framework of the Model

2.1 The duopoly setting

The model developed here involves two firms A and B competing in the same sector. Both firms belong to the same industry and produce goods with imperfect substitutability. Each firm uses two factors of production with a fixed coefficients technology, as the following production functions specify:

$$Y_A = \min\{a_A L_A, b_A K_A\} \tag{1}$$
$$Y_B = \min\{a_B L_B, b_B K_B\}, \tag{2}$$

where for each firm - as indicated in the subscripts throughout the paper - Y is output, L is employment, and K is capital stock, while a and b are labor and capital productivity, respectively. Here we assume that total employment L is divided between the two firms, formally, $L = L_A + L_B$. It is also assumed an infinitely elastic supply of labor, and employment is then determined only by the demand for labor. Assuming away capital depreciation, we have:

$$Y_A = b_A K_A \tag{3}$$
$$Y_B = b_B K_B. \tag{4}$$

Provided that the technical coefficients b_A and b_B remain constant, the growth of output of each firm is equal to the growth of its capital stock, as follows:

$$g_{Y_A} = g_{K_A} = g_A \quad (5)$$

$$g_{Y_B} = g_{K_B} = g_B, \quad (6)$$

where g_Y and g_K denote each firm's growth of output and growth of capital stock, respectively. For simplicity, from now on only g_A and g_B will be used.

Since there are only two firms in this sector, they are responsible for the total amount of value produced in it at a given time, so that:

$$PY^d \leq P_A Y_A + P_B Y_B \quad (7)$$

where P represents prices and Y^d is the sector's aggregate demand. The inequality states that the firms operate with excess capacity, while the equality defines the theoretical full capacity level of output. Here we are assuming that firms maintain a constant fraction of production as stock, in order to avoid a mismatch between potential and effective supply. According to Lavoie (2014), firms producing at a level in which there is reserve of capacity is one of the stylized facts of the Post-Keynesian firm. As we have already established with the production functions (1) and (2), in this model no substitution is acceptable, so there is a fixed combination of inputs that generates a certain level of output. And if this level is obtained with "adequate" production techniques, Steindl (1952) calls it "practical capacity". He defines such level as "...the output achieved with normal length of working time, with sufficient shut-downs to allow for repairs and maintenance, and without disturbance in the smooth running of the production process" (p. 7). However, it is standard to just name it "full capacity".

In the Post-Keynesian theory of the firm, marginal costs are constant up to full capacity, while average unit costs generally are decreasing (KALECKI, 1969; KALDOR, 1961). Yet, they start rising after this point, so the very shape of the cost curves corroborates that firms operate with reserves of capacity. There are also a number of technological reasons related to indivisibility and scale specifications of machines and plants that justify firms having planned excess capacity. Nevertheless, Steindl (1952) ultimately relates this fact to the presence of fundamental uncertainty in the real world. In this sense, a reserve of capacity would be necessary for firms to respond to demand shifts that cannot be foreseen.

This assumption also refers to the idea of real markets as imperfect markets. Since the market structure analyzed here is duopolist, prices in the short run will be cost-determined, possibly following some principle of mark-up pricing (EICHNER, 1976). In fact, as Kalecki (1971) points out, market imperfection is a fundamental condition for firms to be able to maintain reserves of capacity in the long-run. And this extra capacity, in turn, would make them able to fix prices. [3]

For simplicity, it is assumed that each firm's prices remain unchanged. In other words, firms in this model do not compete on prices, but only on non-price factors such as product quality and variety. We also assume that both firms set their prices at a sufficiently low level in order to prevent the entry of potential competitors in the market [4]

[3] However, as Lavoie (2014) argues, the market does not need to be oligopolistic to be imperfect. He claims that all markets could be considered imperfect markets, where prevails some sort of administered pricing mechanism, even with higher degrees of competition. From a different approach, Lee (2013) also criticizes Kalecki's assumption that the degree of competition has a direct effect on mark-ups. According to his studies and interpretation, there does not seem to exist empirical grounds to justify the shape of Kalecki's cost curves.

[4] See Sylos-Labini (1962) and Bain (1956) for a more detailed explanation on the relationship between prices and barriers to entry.

Since the two firms are responsible for the total output produced, they are sharing the market between themselves. In that way, each firm contributes to total output growth in an amount proportional to their market share. Thus, rewriting equation (7) in rates of change, we obtain:

$$\bar{g} = m_A g_A + m_B g_B, \tag{8}$$

where prevails the assumption that demand in this sector grows at a constant rate $\bar{g}$. Moreover, m_A and m_B are the market shares of firms A and B, respectively. From equation (8), their market shares can be thus defined as:

$$m_A = \frac{Y_A}{Y} \tag{9}$$

$$m_B = \frac{Y_B}{Y}, \tag{10}$$

where $m_A + m_B = 1$.

In equilibrium, the two firms would grow at the same rate, maintaining their respective market shares, such that the following relationship would be valid:

$$g_A = g_B. \tag{11}$$

In the neoclassical theory of the firm, firms decide to produce at the level that yields them the maximum rate of profit. Instead, in the Post-Keynesian theory firms maximize power, regardless of size and control scheme (GALBRAITH, 1973). Power, in turn, is related to growth. On the one hand, in a world with fundamental uncertainty, the bigger the firm, the bigger its control over unpredictable events. On the other hand, there is a scale effect related to size of the firm and the market it holds. Hence, it is not just a matter of ultimate control over the market, but how to survive in it. Power, growth and survival are inevitably intertwined.

Assume, without loss of generality, that firm A is the leading competitor in terms of technological efficiency and hence has some freedom to set its investment rate and output growth. That said, first we model the financial frontier and the investment function of firm A. In an economy with no financial sector, profits become the means that enable firms to grow, considering Kalecki's principle of increasing risk [5]. Firms can finance their expansion with the retention of part of their profits in order to invest. Therefore, following Sylos-Labini (1971) and Wood (1975), we can establish a function of the finance constraint faced by the firms, with growth as a positive function of profits:

$$g_A^F = s_A r_A \tag{12}$$

where g_A^F is the growth rate of firm A compatible with its financial constraint, r_A is firm A's rate of profit and s_A is its retention ratio, $0 < s_A < 1$. Equation (12) indicates that in order for the growth rate of the firm to increase, a higher rate of profit is necessary, given an admissible retention ratio [6]. Some empirical studies such as the one from Fazzari & Mott (1986) have also provided evidence of the role of internal finance for investment. With a large sample of U.S.

[5] If a financial sector were added to the model, it would be necessary to define interest rates on borrowed funds. However, given that Kalecki's principle of increasing risk holds (LAVOIE, 2014, p. 137), the firm is willing to borrow only a limited amount of their funds. This amount, in turn, is directly related to its internal and previously accumulated funds. For that reason, for the purposes of this paper we can simplify the model and not consider a financial sector.

[6] See Stockhammer (2005) for a discussion in how the financialization and an increase in shareholder's power have impacted the retention ratio and other related decisions of the firm

manufacturing firm data from 1970 to 1982, the authors find independent effects of internal finance on investment.

Since firms would obtain power through growth, the expansion frontier of firm A can be defined as an investment function. In the present model, the investment function of firm A has the following configuration:

$$g_A^E = \gamma_{0A} + \gamma_{1A} r_A - \gamma_{2A} m_A + \gamma_{3A} T \tag{13}$$

where g_A^E is the rate of expansion of the capital stock desired by the firm, T is the relative technology level of firm A in relation to firm B, γ_0 represents animal spirits and γ_i are positive parameters, $i = [0,3]$. The assumption that investment depends positively on the rate of profit is derived from Robinson (1956). According to her, an upward shift in expected profits leads firms to decide to undertake a greater amount of investment. For simplicity, current and expected rates of profit are taken as equal, with the former assumed to be a good enough approximation of the latter.

Equation (13) also states that the competitive features of the industry have an important role in determining the firm's investment decision. In general terms, the coefficient γ_{2A} is ambiguously signed. It means that, depending on the marketing strategy of the firm, a higher market share could either encourage or reduce capital accumulation. However, without loss of generality, we assume that γ_{2A} is positively signed, thus implying that the desired rate of capital accumulation, g_A^E, and the market share, m_A, are inversely related. This comes from the assumption that these firms have a strategy of maintaining their market share. This implies that if firm A obtains a higher market share, it holds a better competitive position in the industry, and does not need to keep expanding to increase its power. In this case, higher investment would lead only to a bad use of resources, going further from the firm's full capacity level. We also assume that this strategy does not change with increases or decreases in the concentration of the industry.

Gilbert & Lieberman (1987) provide empirical evidence of a pattern of investment consistent with market share maintenance for large firms, whereas for the smaller ones, the preemptive investment hypothesis would rule. Since the market here consists of two firms, they are considered to be large ones. Therefore, their strategy would be to increase capital accumulation if they encounter a significant reduction in their market shares unrelated to historical random variations. On a similar approach, Hay & Liu (1998) investigate whether investment in capacity could be used as a competition strategy in oligopolistic markets. For an oligopoly in which a group of firms dominate, similar to the one treated in this paper, they find that a cooperation strategy is preferred over an aggressive one.

Finally, in this model a higher level of technological content induces investment, following the Schumpeterian argument. Assuming that the firms are in different positions on the technological race, since they have heterogeneous artifacts and technological production attributes, this relative technological level can be called the "technology gap" between the two firms. As we are taking firm A as the one equipped with the most efficient technique of production within this sector, this gap can be defined as:

$$T = \frac{E_A}{E_B}, \tag{14}$$

where E_A and E_B are the stocks of technology - or knowledge - of firms A and B, respectively[7].

[7] In an oligopoly setting with three or more firms, T can be represented as $T = E_j/\bar{T}$, where E_j is the technological content level of each firm j and $\bar{T}$ would be the average technological content of the rest of the sector. Hence, firms with $T > 1$ would be more competitive than firms with $0 < T < 1$.

The assumption that technological progress affects investment is central to the theory of Schumpeter (1934), that focuses on the role of innovation in changing the dynamics of the economy. However, Kalecki (1971) also favors this channel. Technological progress would result in new machines and production routines more productive than the old ones. This causes the costs of the old methods to rise, which spurs new investment. In a different occasion, Kalecki (1941) proposes that innovation increases the expected rate of profit, generating indirectly a higher investment rate than in the case without innovation. Innovation would then change the expectations of the entrepreneur, who would become more optimistic, investing more.

Dutt (1986) proposes a macroeconomic model in Kaleckian lines that encompasses the effects of technological change in economic growth and income distribution. In his model, technological change does not impact investment directly. Instead, it affects indirectly the parameters of the investment function on a direction determined by the sort of constrains that the economy faces (from capital, labor or demand). In the model developed here, however, the technology gap influences the investment of firm A positively and directly. Similar to the creative destruction theory of Schumpeter (1942), innovation influences investment through the channel of competition. This implies that a higher level of technology of firm A would spur its investment, but that would also happen with a lower technological level of firm B. Supposing that firm A has a sufficiently more efficient technology relative to firm B, the costs of the old machinery would increase even more in relation to the ones of firm B, increasing investment of firm A.

Firm A would reach its highest growth and power position when the expansion frontier reaches the finance frontier, such that:

$$g_A^F = g_A^E, \tag{15}$$

which can be resolved for the profit rate. The equilibrium profit and growth rates of firm A are then:

$$r_A^* = \frac{\gamma_{0A} - \gamma_{2A} m_A + \gamma_{3A} T}{s_A - \gamma_{1A}} \tag{16}$$

$$g_A^* = \omega_A(\gamma_{0A} - \gamma_{2A} m_A + \gamma_{3A} T). \tag{17}$$

where $\omega_A = \frac{s_A}{s_A - \gamma_{1A}}$. We assume $\omega_A > 0$ as a stability condition for the model

In the case treated here, firm B would capture the residual demand of the market, the one that is not already with the leader firm A. Therefore, the equilibrium rate of growth for firm B is obtained substituting the equilibrium growth rate of firm A given in equation (17) in the specification of this market's demand in equation (8). Following this procedure and using the relationships between the two firms in expressions (9) and (10), the equilibrium rate of growth obtained for firm B is:

$$g_B^* = \frac{1}{1 - m_A}(\bar{g} - m_A \omega_A(\gamma_0 A - \gamma_{2A}(m_A) + \gamma_{3A} T) \tag{18}$$

2.2 Market Share

From (9), let us assume that the differential equation of the market share of firm A can be expressed as a positive function of the difference in growth rates between the two firms, favoring firm A:

$$\dot{m}_A = \Psi_A(g_A - g_B) \tag{19}$$

with $\Psi_A > 0$ and $m_A \in [0, 1)$. The latter assumption is expressing that the model developed here does not involve the case where firm A drives firm B completely out of the market and becomes monopolist. Evidently, for m_B a symmetric relationship is valid. This assumption is stated according to the already established view that firms aim at growing, and they do so to obtain a larger share of the market. As proposed by Robinson (1962) and Kaldor (1978), and highlighted by Lavoie (2014), firms expand to keep or increase their share of the market.

Substituting the growth rates of both firms, g_A and g_B, for their equilibrium expressions, (17) and (18), the differential equation of the market share of firm A can be rewritten as:

$$\dot{m}_A = \Psi_A \{ \frac{1}{1-m_A} [\omega_A(\gamma_{0A} - \gamma_{2A} m_A + \gamma_{3A} T) - \bar{g}] \}. \tag{20}$$

Equation (20) relates the rate of growth of firm A's market share with its level and the technology gap positively or negatively according to the values of the parameters of interest.

2.3 Technological Progress

We can also assume that the rate of technological change can be expressed by the following differential equation:

$$\dot{T} = \alpha_0 + \alpha_1(g_A - g_B) - \alpha_2 T \tag{21}$$

where $\alpha_i > 0$, $i = [0,2]$ and α_0 designates the autonomous component of technological progress of firm A relative to firm B. The technology gap, T, is assumed to have a negative relationship with the rate of change of technical progress. That happens because we are assuming that the higher the gap, more chances firm B would have to catch up, thus decreasing the rate of change of the technology gap (ABRAMOVITZ, 1986).

Besides, the rate of capital accumulation is taken as a driver of technical change, through a Kaldor-Verdoorn channel (MCCOMBIE; THIRLWALL, 1994). Supposing a given rate of capital accumulation of firm B, an increase in the one of firm A leads to a higher flow of technological innovations from this firm. As Palley (1996) argues, technical progress is an endogenous result of capital accumulation, since "(...) investment serves simultaneously as the means of (1) expanding the capital stock, (2) feeding technical innovations into the production process, and (3) uncovering further possibilities for innovation"(p. 124). This comes from the fact that investment is both an expansion of capital stock as well as a mean of introducing innovations into the capital stock. This introduction, in turn, occurs through the acquisition of newer or more productive capital goods, but also through the knowledge and research inherent to the firm, that allows further improvements in productivity and the final quality of production (POSSAS, 2008, p. 290).

Substituting the difference between the rates of capital accumulation given by (17) and (18) in (21) above, we obtain:

$$\dot{T} = \alpha_0 + \frac{\alpha_1}{1-m_A}[\omega_A(\gamma_{0A} - \gamma_{2A} m_A) - \bar{g}] - \left(\alpha_2 - \frac{\alpha_1 \omega_A \gamma_{3A}}{1-m_A}\right) T, \tag{22}$$

where we can restrain $\alpha_2 > \alpha_1 \omega_A \gamma_{3A}/(1-m_A)$ to keep the catching up mechanism running. This equation states that in this case market share and technological change have a non-linear relationship.

According to Metcalfe (1998), although small firms in competitive markets have larger incentives to innovate, since they need innovation to gain power in the market, most of the times they lack the resources to do so. Firms with large shares of the market, however, have little or no incentive to innovate, because they do not need to obtain any more competitive advantage. But they have the resources to go ahead and risk innovation, what could increase their productivity even further. At the same time, they could employ those resources in product variety, in order to conquer or create other markets. This sort of diversification could open new markets with greater opportunities to earn temporary monopoly profits. If the latter strategy rules, a higher market share would decrease the rate of technological change, the opposite for the former. It is reasonable, then, to consider that this relationship is in fact a non-linear one for this duopoly model.

3 Medium-Run Market Share and Technological Progress Dynamics

3.1 Equilibrium and stability

Equations (20) and (22) form a non-linear dynamical system that characterizes the medium-run dynamics of this industry. We are only considering the medium-run, since technological innovation involves a considerable gestation period to reach maturity. It is reasonable to assume that this system has at least one non-trivial equilibrium solution in the loci $\dot{m}_A = 0$ and $\dot{T} = 0$. The linear approximation of this system at a generic equilibrium point (m_A^e, T^e) is described by the following expression:

$$\begin{bmatrix} d\dot{m}_A/dt \\ d\dot{T}/dt \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} m_A - m_A^e \\ T - T^e \end{bmatrix}$$

The matrix formed by J_i elements on the right-hand side of the equation is the Jacobian matrix of the system, and its components are its partial derivatives, as follows:

$$J_{11} = \frac{\partial \dot{m}_A}{\partial m_A} = \frac{\Psi_A[\omega_A(\gamma_{0A} - \gamma_{2A} + \gamma_{3A}T^e) - \bar{g}]}{(1-m_A^e)^2} \lesseqgtr 0 \tag{23}$$

$$J_{12} = \frac{\partial \dot{m}_A}{\partial T} = \frac{\Psi_A \omega_A \gamma_{3A}}{1-m_A^e} > 0 \tag{24}$$

$$J_{21} = \frac{\partial \dot{T}}{\partial m_A} = \frac{\alpha_1[\omega_A(\gamma_{0A} - \gamma_{2A} + \gamma_{3A}T^e) - \bar{g}]}{(1-m_A^e)^2} \lesseqgtr 0 \tag{25}$$

$$J_{22} = \frac{\partial \dot{T}}{\partial T} = \frac{\alpha_1 \omega_A \gamma_{3A}}{1-m_A^e} - \alpha_2 < 0 \tag{26}$$

We must now analyze the stability of the linearized system around the equilibrium solution m_A^e and T^e, which is not formally presented here for simplicity. This property can be assessed with system's Jacobian matrix above. From this matrix, we see that different values of certain parameters will determine if the solution is stable or not. However, these parameters still need to fall within a relevant economic domain, validating further analysis of this system.

First, let us consider the case in which $J_{11} > 0$ and $J_{21} > 0$. This implies a downward-sloping straight line for $\dot{m}_A = 0$ and an upward-sloping one for $\dot{T} = 0$. In this scenario, the determinant of the Jacobian matrix (Det=$J_{11}J_{22} - J_{21}J_{12}$) is negative, suggesting that the equilibrium is a saddle point. This implies that any kind of changes in the system are amplified. Figure (1) illustrates this case.

In this configuration, if the equilibrium solution shifts to a point with a higher market share of firm A, this leads to a deepening of the technology gap. Firm A possibly would have considerable advantages in its overall innovative and learning process against firm B, that would fall behind in the technological race. However, by reaching a certain magnitude of the

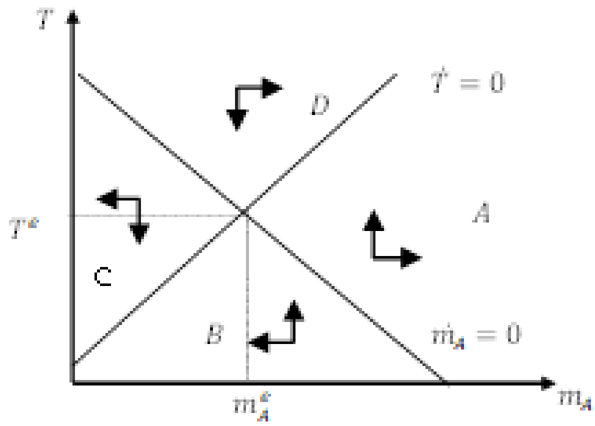

Figure 1 – Unstable loci $\dot{m}_A = 0$ and $\dot{T} = 0$

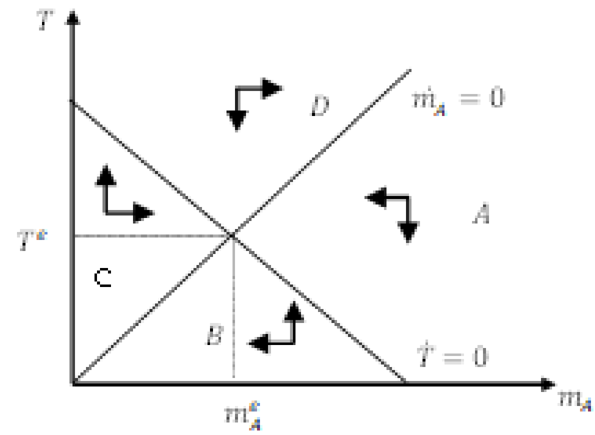

Figure 2 – Stable loci $\dot{m}_A = 0$ and $\dot{T} = 0$

technological gap, this advantage starts to get slimmer, because firm *B* eventually develops strategies to imitate firm *A*. From that moment on, firm *B* starts the catching up process, and the gap decreases again, while firm *A*'s market share increases. A vicious cycle is formed, which in this case would lead to a continuous concentration of the market by the dynamical forces of the system.

Another possible configuration of this duopoly is characterized by $J_{11} < 0$ and $J_{21} < 0$. This means that, on the contrary of the previous case, $\dot{m}_A = 0$ line is upward-sloping, while the $\dot{T} = 0$ line is downward-sloping. In this case, the determinant of the Jacobian matrix is positive, but the matrix's trace (Tr=$J_{11} + J_{22}$) is negative, configuring a stable equilibrium, as depicted in Figure (2).

The stability refers to the following dynamics: if again the equilibrium solution shifts to a point in the area A of Figure (2), an increase in market share of firm *A* leads to a decrease in the technology gap, because this firm now has less incentives to incorporate innovations. However, the smaller technology gap means that firm B is catching up, which would decrease firm A's market share again, reversing its incentives to innovate. In phase C, the technology gap starts to grow again, and with it, firm A's market share. The process continues until the equilibrium m_A^e and T^e is reached again.

It is also interesting to notice that a higher demand in the industry, represented here by $\bar{g}$, makes the stable case more likely. A higher demand could then decrease the intensity of the competition, allowing the sector to remain duopolistic. In the opposite case, when demand is lower, this model shows that the competition tends to be unstable and the market tends to rapidly become extremely concentrated with the leader, firm A.

3.2 Comparative statics of a technology shock

We can then analyze the effect of an exogenous shock in the autonomous component of technological progress. This exogenous technology shock is a type of technological change that is independent from the firms' resources. This could imply a technological innovation coming from the public sector, most likely from university R&D. In this case, however, firm A is able absorb it better than firm B, or even only firm A has the capabilities to absorb it at all. Another possibility would be an organizational innovation implemented by firm A, that did not have costs for this firm and improved, for example, its branding.

In terms of the model, this can be seen as a shock in α_0, which represents the exogenously determined level of firm A's technological content relative to firm B's technological content. This parameter affects both the intercept and the slope of both straight lines given by the linear approximation where $\dot{m}_A = 0$ and $\dot{T} = 0$. However, the direction and the magnitude of these movements are ambiguous. They depend on the values of the parameters, which reflect different possibilities for the different stability conditions discussed above.

In the unstable case, the most mathematically likely situation is the one where both straight lines move upwards, with the variation on their slopes as seen in Figure (3). In this case, an exogenous increase in the technological progress leads the dynamics of this system to start from a point with a higher technology gap and a higher market share of firm A. The instability of this dynamics suggests that this exogenous shock gives way to firm A to expand its advantage in the technology race, eventually being able to cast firm B out of the market completely.

This same shock could have different effects in the case where the equilibrium point is a stable one. Again, both lines $\dot{m}_A = 0$ and $\dot{T} = 0$ shift upwards, while both slopes become steeper. However, the ambiguity of the magnitude of these shifts makes two cases possible. Figures (4) and (5) illustrate these possibilities. In both cases the exogenous shock increases the technological gap of firm A before firm B. However, this could lead firm A to gain or lose market share.

The first case, of Figure (4), shows how firm A now has a higher technological content in its production, which confers to it larger market power. This is reflected in an increase in market share, with the system reaching a new equilibrium at point B. The second case is depicted in Figure (5). In this configuration, the higher technological gap leads to a higher rate of catching up by firm B than the rate of market share acquisition of firm A. By doing so, firm B becomes a more levelled competitor before firm A, which makes it able to increase its market share.

The latter situation, although mathematically possible, is not so theoretically intuitive. As was already discussed here, most of the theoretical work on innovation and competition suggests how the dynamics that concentrates the market with the leader firm is the typical one in a capitalist economy. Despite that, it is interesting to notice how this analytic formulation gives scope to different market configurations following a technology shock.

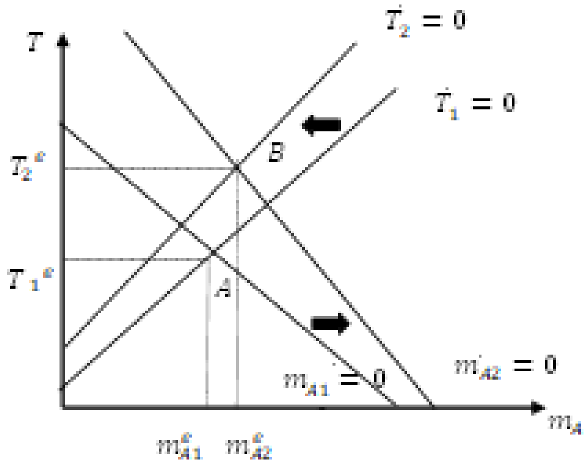

Figure 3 – Effects of an exogenous innovation shock - unstable case

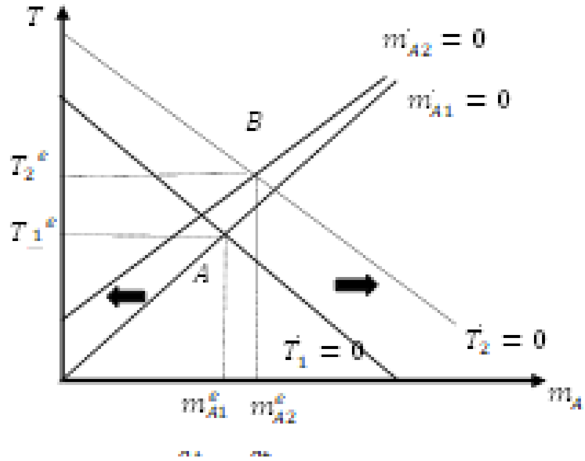

Figure 4 – Effects of an exogenous innovation shock - first stable case

4 Effects of Technological Change in the Long-Run: Growth and Market Concentration

4.1 Growth instability and market dominance

Schumpeter (1942)'s idea of creative destruction suggests that innovation would change the dynamics of the firm, increasing its rate of growth. However, it would also change the scenario of the competition in this sector in the long-run, what would have another round of effects over technology, competition and growth.

With that in mind, in the long-run, the rate of growth and the level of output of the firms are subject to innovation related changes. The previously described exogenous technological shock will have an effect on the firms' rate of growth given by how changes in the technology gap and the firms' market shares impact on growth. Formally, the total magnitude of these changes is

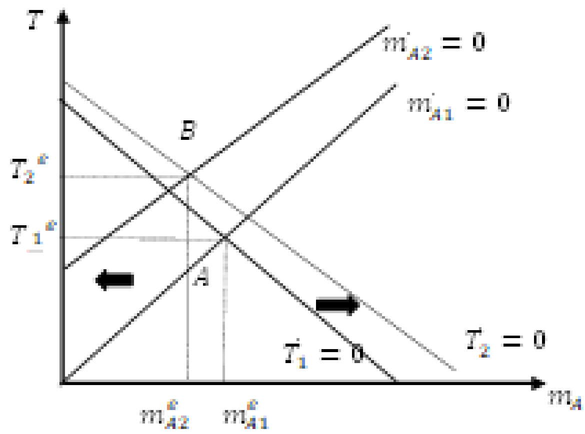

Figure 5 – Effects of an exogenous innovation shock - second stable case

given by the following total derivatives:

$$\Delta g_A = \frac{\partial g_A}{\partial T}\Delta T + \frac{\partial g_A}{\partial m_A}\Delta m_A \tag{27}$$

$$\Delta g_B = \frac{\partial g_B}{\partial T}\Delta T + \frac{\partial g_B}{\partial m_A}\Delta m_A, \tag{28}$$

so that the net change of this result will depend on the values and signs of the parameters of various expressions describing this industry and the magnitudes of the changes in the technology gap and the market share of firm A. Therefore, the effect of innovation on growth could be ambiguous in this model.

After an exogenous technological shock, a shock in α_0, there will be a disequilibrium between the growth rates of both firms in the long-run. In this case, when the market share of firm A increases, possibly $\Delta g_A > \Delta g_B$. If that is the case, an exogenous technological shock will lead to a greater growth rate of firm A, which in turn leads to a higher technological gap and market share with this firm.

In the unstable case, firm A could keep growing more than firm B, concentrating the market. This concentration, in turn, could lead to stagnation. When firm A dominates the market by maintaining $g_A > g_B$, if it keeps growing, at one point we will have $g_A > \bar{g}$, where $\bar{g}$ represents the rate of growth of the demand. Firm A will then change its strategy towards innovation, since the demand constraint will prevent it to increase its growth rate further. Therefore, demand will impose a restriction on growth leading to stagnation.

From equation (19), however, when we have medium-run stability, an increase in g_A will require that m_A and T adjust in order to keep $\dot{m}_A = 0$, so eventually the growth rates equalize again. Figure (6) illustrates this dynamics. After the shock, the growth rate of firm A, g_A, increases to g'_A. As this growth rate increases, the growth rate of firm B, g_B, necessarily decreases to g'_B. If demand continues to grow at a constant rate, the greater growth rate of firm A implies firm B growing at a smaller rate. However, since the system tends to an equilibrium, it shifts to a new equilibrium combination of T and m_A that equalizes both growth rates again. To reach this new point, the growth rate of firm A decreases, as the one for firm B increases, as the latter is able to catch up.

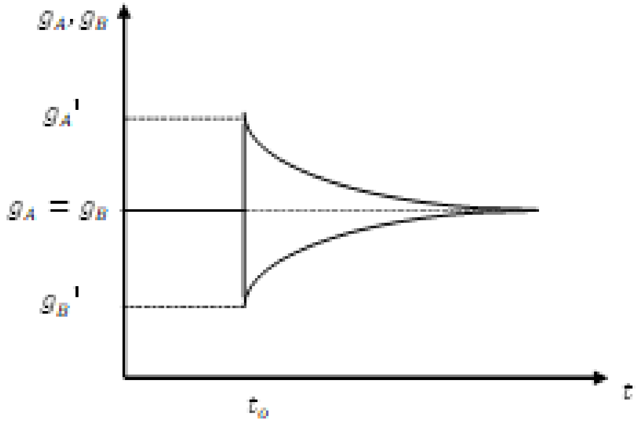

Figure 6 – Long-run dynamics of the firms' growth rates

4.2 Maintaining the duopoly market configuration

From now on, let us focus on the stable case when after a technology shock favoring firm *A*, it grows faster temporarily and obtains a higher market share. This course actually goes accordingly to the some of the predictions of neo-Schumpeterian evolutionary theory, as in the seminal model of Nelson & Winter (1982). In their model, the authors incorporate the idea that competition is a dynamic game, so that firm's growth will reflect the pattern of winner/loser in a specific game. Applying this reasoning to our model, if firm *A* has a greater technological advantage than firm *B*, it "wins" by obtaining a larger share of the market, a higher growth rate, and also by increasing the technology gap between its competitor. Nelson & Winter (1982) also predict that competition leads to market concentration, but they argue that this process could be avoided. In the scope of our model, knowing that firm *A* has a chance to dominate a larger share of the market, it is relevant to think about the strategies that firm *B* could employ in order to survive in it and how public policy could affect that.

Since in this model competition between the two firms is based on technological advantages, the firms not competing over price, the evident course for firm *B* to take is related to catching up strategies, like imitation. The strategy of firm *B* is then mainly focused on lowering the technology gap, T. In that way, as Metcalfe (1998) points out, if firm *B* could copy the new technologies fast enough, no competitive gains would come from innovation. Then, the almost-monopoly of firm *A* could last less. Firm *B* would not lose its share of market, even if it is a smaller one than the leader's, which in the case continues to be firm *A*.

In terms of the model, in equation (21), which sets the law of the dynamics of technological change, parameter α_2 indicates the amount by which the rate of the technological change would decrease with an increase in the level of the gap. Therefore, a higher absolute value of α_2 would mean a higher decrease in the rate of technological change, thus giving firm *B* a greater opportunity for catching up. In that way, firm *B* would increase its market share and would compete closer to firm *A*. This would lead to a more dynamically stable long-run path for this duopoly market, since new equilibrium would be less concentrated. The change in the value of parameter α_2 relates to procedures internal to both firms. Firm *B* strategy of increasing its value, trying to increase its imitation capacity, relates mainly to learning procedures, like its R&D expenditure and organization.

This result also refers to possible configurations of public policy that would affect the dynamics of this industry. Given that patenting policies are important to foster innovation, since they increase the incentives that firms have to innovate, it is important to consider the concentration that the innovation process entails. Antitrust policies, in turn, could benefit from a formal analysis of this matter. Therefore, technology progress considerations need to be included in order to shape a more parsimonious policy plan.

5 Concluding Remarks

In this paper we have developed a microeconomic model of a duopolist industry based on the Post-Keynesian theory of the firm. This model encompasses the dynamic relationship between market share and relative technological change, and how these factors influence the firms' growth rates in this general framework. The model implies a negative relation between market share and firm growth, but a positive one between a relative technological superiority and growth.

This paper has some theoretical implications. The mathematical analytic formulation presented is able to convey different scenarios for the industry following an exogenous technology shock. Nonetheless, the most intuitive one remains the shock increasing the competitive advantage of the industry leader, that gains a higher share of this market. This concentration is accompanied by a deepening of the technological gap between the two competitors. The model also indicates that the direction of the effect of this exogenous technology shock on the firms' growth rates is ambiguous; although all the scenarios considered indicate a tendency of technological change to lead to a higher concentration of the market with one of the two firms in the long-run. The long-run leader obtains a higher level of output, without having a permanently higher rate of growth.

These conclusions, however, apply only to this industry and cannot be extrapolated to the macroeconomic scenery with this formulation. Yet, as the macroeconomic models must be fundamented in the microeconomic theory, we argue that this model can serve as a basis for future more comprehensive Post-Keynesian macro models that incorporate changes in the micro competitive structure. Therefore, on the one hand, further works could rely on some flexibilizations and extensions of this simple model into the micro setting, such as incorporating other aspects of firms' decisions to innovate and taking firms' strategies as dynamically determined/altered by changes in the competitive status of the market. On the other hand, the effects of innovation on firms' competition and growth, as this model shows, can also be then better explored to analyze how they affect macroeconomic variables, such as the functional income distribution. Specially given the relative scarcity of the micro discussion in this literature, this attempt could be already a bridge between Post-Keynesian micro and macroeconomics.

Bibliography

ABRAMOVITZ, M. Catching up, forging ahead, and falling behind. *The Journal of Economic History*, v. 46, n. 2, p. 385–406, 1986.

BAIN, J. S. *Barriers to New Competition: Their Character and Consequences in Manufacturing Industries*. Cambridge, MA: Harvard University Press, 1956.

BLUNDELL, R.; GRIFFITH, R.; REENEN, J. V. Market share, market value and innovation in a panel of british manufacturing firms. *The Review of Economic Studies*, v. 66, n. 3, p. 529–554, 1999.

DOSI, G. *Technical Change and Industrial Transformation: The Theory and an Application to the Semiconductor Industry*. London: Macmillan, 1984.

DOSI, G. Sources, procedures, and microeconomic effects of innovation. *Journal of Economic Literature*, v. 26, n. 3, p. 1120–1171, 1988.

DUTT, A. K. Growth, distribution and technological change. *Metroeconomica*, v. 38, n. 2, p. 113–133, 1986.

EICHNER, A. *The Megacorp and Oligopoly: Micro Foundations of Macro Dynamics*. Cambridge: Cambridge University Press, 1976.

FAZZARI, S. M.; MOTT, T. L. The investment theories of kalecki and keynes: an empirical study of firm data, 1970–1982. *Journal of Post Keynesian Economics*, v. 9, n. 2, p. 171–187, 1986.

GALBRAITH, J. K. *The new industrial state*. Princeton: Princeton University Press, 1967.

GALBRAITH, J. K. *Economics and the Public Purpose*. Boston: Houghton Mifflin, 1973.

GILBERT, R. J.; LIEBERMAN, M. Investment and coordination in oligopolistic industries. *The Rand Journal of Economics*, v. 18, n. 1, p. 17–33, 1987.

HAY, D. A.; LIU, G. S. The investment behaviour of firms in an oligopolistic setting. *The Journal of Industrial Economics*, v. 46, n. 1, p. 79–99, 1998.

KALDOR, N. Capital accumulation and economic growth. In: LUTZ, F. A.; HAGUE, D. C. (Ed.). *The Theory of Capital*. New York: St. Martin's Press, 1961. p. 177–228.

KALDOR, N. *Further Essays on Economic Theory*. London: Duckworth, 1978.

KALECKI, M. A theorem on technical progress. *The Review of Economic Studies*, v. 8, n. 3, p. 178–184, 1941.

KALECKI, M. *Studies in the Theory of Business Cycles 1933-1939*. Oxford: Blackwell, 1969.

KALECKI, M. *Selected Essays on the Dynamics of the Capitalist Economy 1933-1970*. Cambridge: Cambridge University Press, 1971.

KAMIEN, M. I.; SCHWARTZ, N. L. Market structure and innovation: a survey. *Journal of Economic Literature*, v. 13, n. 1, p. 1–37, 1975.

LAVOIE, M. *Post-Keynesian Economics: New Foundations*. Cheltenham: Edward Elgar Publishing, 2014.

LEE, F. S. Competition, going enterprise, and economic activity. In: MOUDUD, J. K.; BINA, C.; MASON, P. L. (Ed.). *Alternative Theories of Competition*: Challenges to the orthodoxy. Abingdon, Oxon: Routhledge, 2013. cap. 7, p. 160–173.

MCCOMBIE, J. S.; THIRLWALL, A. *Economic Growth and the Balance-of-Payments Constraint*. London: Macmillan, 1994.

MELO, T. M.; POSSAS, M. L.; DWECK, E. Um modelo setorial baseado na abordagem kaleckiana da distribuição setorial funcional da renda e na teoria schumpeteriana da concorrência. *Economia e Sociedade*, v. 25, n. 1, p. 109–145, 2016.

METCALFE, J. S. *Evolutionary Economics and Creative Destruction*. London: Routhledge, 1998.

METCALFE, J. S. Schumpeterian competition. In: MOUDUD, J. K.; BINA, C.; MASON, P. L. (Ed.). *Alternative Theories of Competition*: Challenges to the orthodoxy. Abingdon, Oxon: Routhledge, 2013. cap. 5, p. 111–126.

NELSON, R. R.; WINTER, S. G. *An Evolutionary Theory of Economic Change*. Cambridge, MA: Harvard University Press, 1982.

PALLEY, T. I. Growth theory in a keynesian mode: some keynesian foundations for new endogenous growth theory. *Journal of Post Keynesian Economics*, v. 19, n. 1, p. 113–135, 1996.

POSSAS, M. L. Economia evolucionária neo-schumpeteriana: elementos para uma integração micro-macrodinâmica. *Estudos Avançados*, v. 22, n. 63, p. 281–305, 2008.

POSSAS, M. L. et al. Um modelo evolucionário setorial. *Revista Brasileira de Economia*, v. 55, n. 3, p. 333–377, 2001.

ROBINSON, J. *The Accumulation of Capital*. London: Macmillan, 1956.

ROBINSON, J. *Essays in the Theory of Economic Growth*. London: Macmillan, 1962.

SCHUMPETER, J. A. *The Theory of Economic Development*. Cambridge, MA: Harvard University Press, 1934.

SCHUMPETER, J. A. *Socialism, Capitalism and Democracy*. New York: Harper, 1942.

STEINDL, J. *Maturity and Stagnation in American Capitalism*. Oxford: Blackwell, 1952.

STOCKHAMMER, E. Shareholder value orientation and the investment-profit puzzle. *Journal of Post Keynesian Economics*, v. 28, n. 2, p. 193–215, 2005.

SYLOS-LABINI, P. *Oligopoly and Technical Progress*. Cambridge, MA: Harvard University Press, 1962.

SYLOS-LABINI, P. La theorie des prix en regime d'oligopole et la theorie du developpement. *Revue d'economie politique*, p. 244–272, 1971.

WOOD, A. *A Theory of Profits*. Cambridge: Cambridge University Press, 1975.