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Ties of math and language: A cognitive developmental perspective

Because numerical and mathematical competencies play an important role in our everyday life (e.g., Butterworth et al., 2011), it is crucial to understand underlying cognitive processes and factors influencing the acquisition of these numerical and mathematical competences. In particular, a better understanding at the level of cognitive processes may help to develop targeted interventions, inform, and enhance the quality of mathematics teaching, which may raise student attainment (cf. The Royal Society & The British Academy, 2018).

To be able to deal with numbers and mathematical content in a competent and efficient way, a set of concepts, procedures, and (math) facts need to be acquired starting even before (formal) education in kindergarten, preschool, and (elementary) school years. Crucially, and probably more so than it is the case in many other school subjects, mathematics education is largely hierarchical in nature (e.g., Clements & Sarama, 2021). As such it is important and necessary to be able to draw on previously acquired competences and knowledge, because new numerical and mathematical content usually builds on these previously acquired competences, concepts, and procedures.

Besides considerable developmental variability on the individual level, international studies evaluating scholastic abilities have consistently reported large cross-cultural differences in mathematical achievement (e.g., OECD, 2019a). In addition to differences in schooling and cultural valuation (e.g., OECD, 2019b), it has been argued that influences of domain-general factors such as language also need to be considered as a potential source of but also resource for overcoming difficulties in the acquisition of numerical and mathematical competences. In particular, language may refer to a range of different linguistic aspects and/or specific aspects of language skills, each of which might interact with specific steps in the acquisition of numerical and mathematical competences.

So far, a wide range of studies investigated various language aspects critical for the acquisition of numerical and mathematical concepts. And indeed, findings of many of these studies are in line with a weak Whorfian hypothesis suggesting that different aspects of language seem to influence the way we acquire, think about, perceive, represent, and apply numerical and mathematical concepts, procedures, and (math) facts. In an attempt to classify and structure previously observed associations of language and mathematics as well as influences

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of language on mathematics, Dowker and Nuerk (2016) proposed a taxonomy differentiating linguistic categories that have previously been identified to influence numerical and mathematical processing in various ways (see also Bahnmueller et al., 2018; Berch et al., 2018). In particular, Dowker and Nuerk (2016) specified six categories: (1) lexical, (2) syntactic, (3) phonological, (4) visuo-spatial orthographic, (5) semantic, and (6) conceptual influences of language. Structuring associations of language and math from a linguistic point of view may thereby foster a broader, more theoretically guided approach to the investigation of how language and math are intertwined throughout development.

Drawing from this proposed taxonomy, this chapter will give an overview of a subset of specific linguistic categories covering those aspects we deem most influential with respect to the development of (early) numerical and mathematical competences. In particular, after a brief description of selected linguistic categories (i.e., lexical, syntactic, phonological, and semantic), we will discuss associations of language and numerical cognition along three consecutive content strands: (i) early numerical competences: number words, counting, and cardinality understanding; (ii) processing of multi-digit numbers; and (iii) basic arithmetic operations. Afterward, a summarizing paragraph will highlight differences, commonalities, and implications of the reported associations of language and numerical and mathematical development.

1 Linguistic influences in numerical/mathematical development

Linguistics is the objective study of natural languages addressing characteristics of language concerning the lexicon (e.g., words, morphemes, compound words), knowledge about language structures (phonology, morphology, syntax) as well as the creation and understanding of meaning of words and sentences in different contexts (semantics, pragmatics; Pickett et al., 2018). Drawing from linguistic categories, Dowker and Nuerk (2016) proposed above-mentioned taxonomy of different linguistic categories that were shown to influence numerical and mathematical processing. In the following, key aspects of (1) lexical, (2) syntactic, (3) phonological, and (4) semantic linguistic influences will be outlined briefly as they seem particularly relevant in the context of associations of language and mathematics from a developmental context.

Within the proposed taxonomy, *lexical* influences reflect the degree to which number words vary to obscure or emphasize features of a number system such as the most widely used Arabic number system. In this context, the transparency (i.e., the consistent reflection of the Arabic number system in a language's number word system) or rather the lack thereof poses a specific hurdle for children that needs to be overcome to master more sophisticated numerical and mathematical competences. For example, one of the most widely investigated intransparencies is the so-called inversion property of number words with respect to the digital-Arabic notation. Number word inversion reflects that in some languages (German, Dutch, Maltese, etc.) the unit digit is named first in two-digit number words which is inverted with respect to the order of tens and units in the digital-Arabic notation (e.g., the number word for 24 is "vierundzwanzig" – literally four and twenty in German; for an overview, see e.g., Klein et al., 2013). Overall, the *lexical* category seems the most widely investigated one in contexts of multi-digit number processing. There is now accumulating evidence suggesting that a lack of transparency has detrimental effects on different aspects of numerical processing (e.g., number transcoding: Imbo et al., 2014; number magnitude comparison: Pixner et al., 2011; addition: Göbel et al., 2014) as well as numerical and mathematical development (e.g., Moeller et al., 2011 for longitudinal influences of inversion-related difficulties on later arithmetic performance).

Syntactic influences usually result from (language-specific) grammatical rules and thus do not reflect influences on the word level but rather on the sentence level. For example, effects of grammatical number fall within this category. Effects of grammatical number on the early acquisition of cardinality knowledge result from differences in singular, dual, and plural marking between certain languages (e.g., Almoammer et al., 2013; Sarnecka et al., 2007; for an overview, see Sarnecka, 2014). In this context, Sarnecka and colleagues (2007) report, for instance, that children speaking Japanese (a language with hardly any marking of singular/plural) learned the meaning of the number word "one" later than English- as well as Russian-speaking children (with English and Russian having explicit plural marking). Thus, grammatical number seems to foster the very early acquisition of the meaning of small numbers.

Another important linguistic category reflects *phonological* influences, which cover effects of phonological language processes as well as effects related to verbal working memory.[1] As regards the former, one subcomponent of phonological processing, namely, phonemic awareness (i.e., the ability to perceive and manipulate phonemes that constitute words, Wagner & Torgesen, 1987), is of particular interest. Phonemic awareness has been argued to be associated with, for example, the early acquisition of number words (e.g., Koponen et al., 2013;

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[1] Dowker & Nuerk (2016) actually consider influences of verbal working memory in a 7th category ("other language-related skills").

Krajewski & Schneider, 2009; Soto-Calvo et al., 2015) as well as performance in multi-digit number transcoding (e.g., Lopes-Silva et al., 2014; see also Chapter 15 of this volume by Haase et al.) and arithmetic fact retrieval (e.g., De Smedt et al., 2010). As regards verbal working memory, it has been found that the ability to temporarily store and manipulate verbal information influences a multitude of different numerical and mathematical tasks (among other working memory components; for reviews, see e.g., Friso-Van den Bos et al., 2013; Peng et al., 2016) and was suggested to represent an integral part in numerical and mathematical development.

Besides Arabic numbers and number words, there are other words and symbols conveying numerical or mathematical meaning by means of their semantics (e.g., more, less, some many, buy, sell). As such, the proposed category of *semantic* influences shows considerable overlap with conceptualizations and investigations of domain-specific mathematical language (e.g., knowledge of terms such as more, less, near, and far; e.g., Purpura et al., 2017; Purpura & Reid, 2016). For example, results of a study by Purpura and Reid (2016) suggest that mathematical language might be a more important predictor of early numerical competences as compared to more general language-related predictors such as vocabulary. Furthermore, research on, for example, text problem solving nicely illustrates the context-dependency of certain (numerical/mathematical) words. For example, it was suggested that words such as “more,” “buy,” and “get” facilitate the processing of text problems requiring additions whereas words like “less” and “sell” interfere with solving addition problems (e.g., Verschaffel et al., 1992; see Daroczy et al., 2015 for a review on text problems).

Taken together, a variety of different linguistic influences seem to affect the acquisition of numerical and mathematical competencies. Crucially, some linguistic aspects seem to affect specific numerical and/or mathematical competences and concepts early on while others only follow later and might be critical for different competences and concepts. In the following, we will elaborate on selected linguistic influences (i.e., lexical, syntactic, phonological, and semantic) on three different consecutive content strands: (i) early numerical competencies including the acquisition of number words, counting principles, and cardinality understanding; (ii) multi-digit number processing; and (iii) basic arithmetic operations (see Fig. 1 for an overview). Please note, however, that providing a comprehensive and exhausting overview goes beyond the scope of this chapter. Therefore, and because not all linguistic influences seem to be of equal importance for each of the three content strands, we will give an overview of selected linguistic influences on above-named content strands of early numerical and mathematical development from a cognitive developmental perspective.

	LINGUISTIC LEVEL			
CONTENT STRAND	LEXICAL	SYNTACTIC	PHONOLOGICAL	SEMANTIC
NUMBER WORDS, COUNTING, CARDINALITY	lack of transparency of teen numbers (e.g., Miller et al., 1995)	grammatical number & cardinality knowledge (e.g., Sarnecka et al., 2007)	phonemic awareness & counting sequence (e.g., Krajewski & Schneider, 2009)	mathematical language (e.g., Purpura & Reid, 2016)
MULTI-DIGIT NUMBERS	lack of transparency of numbers >20 & transcoding/number comparison (e.g., Pixner et al., 2011 a,b)		verbal working memory & transcoding (e.g., Imbo et al., 2014)	
BASIC ARITHMETIC OPERATIONS	lack of transparency & arithmetic (e.g., Göbel et al., 2014; Van Rinsveld et al., 2015)		verbal working memory & arithmetic, phonemic awareness & fact retrieval (e.g., Friso-Van den Bos et al., 2013;)	consistency effect in word problems (e.g., van der Schoot et al., 2009)

Fig. 1: Examples for associations of selected linguistic categories (i.e., lexical, syntactic, phonological, and semantic; cf. Dowker & Nuerk, 2016) with three consecutive content strands discussed in the current chapter. Please note that empty cells do not necessarily indicate that there is no association but rather that these specific aspects are not covered in the current chapter.

2 Early numerical competences: Number words, counting, and cardinality understanding

In this first content strand, we will elaborate on the role of above-described language-related aspects in the context of early numerical competencies that build the foundation for further, more advanced numerical and arithmetic competences. In particular, we will address language-related benefits but also pitfalls that were observed to influence the acquisition of number words and the number word sequence as well as the cardinality of small numbers. Moreover, we will discuss the role of mathematical language for early numerical development.

The development of early numerical competencies is a complex process that begins well before formal mathematics instruction starts. In this context, the acquisition of number words as well as the counting sequence alongside specific counting principles (i.e., one-to-one principle, stable order, cardinality-principle; e.g., Gelman & Gallistel, 1978) represents an early milestone in numerical learning. As regards *phonological* influences, it has been demonstrated that, beyond the prominent relation of phonemic awareness with reading and writing skills (for a review, see e.g., Melby-Lervåg et al., 2012), phonemic awareness also seems to be associated with and predictive of the acquisition of number words and the number word sequence (e.g., Koponen et al., 2013; Krajewski & Schneider, 2009). For instance, in a longitudinal study with 5-year-old kindergarten children (at T1), Krajewski and Schneider (2009) investigated the association of phonemic awareness with future mastery of the counting sequence (e.g., counting forward and backward, identifying the successor and predecessor of a number). Results showed a substantial association of phonological awareness and mastery of the counting sequence. Thus, previous studies seem to be in line with the idea that phonemic awareness fosters the acquisition of number words by supporting the construction of sound-based representations in the same way as it facilitates the acquisition of other word categories (e.g., Gathercole, 2006).

Considering that number words are often embedded in sentences, it is not surprising to see language-specific grammatical rules to also shape the learning of the *semantic* meaning of (small) number words (i.e., number words “one,” “two,” and “three”). Substantiating this idea, several studies investigated influences of grammatical number on the acquisition of early cardinality knowledge typically assessed via the Give-N task (i.e., asking children to produce a set of a given size; e.g., Almoammer et al., 2013; Barner et al., 2009; Li et al., 2013; Sarnecka et al., 2007; for a review, see Sarnecka, 2014). The term “grammatical number” refers to singular, dual, and plural markings of certain languages like, for example, the morpheme "s" used as a suffix for plural marking in English. Research suggests that trajectories of number word learning and with this the acquisition of cardinality understanding are influenced by the presence or absence of explicit plural/dual markings in a certain language. As mentioned above, Sarnecka and colleagues (2007) report that English- as well as Russian-speaking children aged between 2 and 3 (with English and Russian having explicit singular/plural markings) learned the meaning of the number word "one" earlier than Japanese-speaking children (with Japanese being a language with hardly any marking of singular/plural). Moreover, 2- to 4-year-old children speaking a language with explicit dual marking such as Slovenian or Arabic (i.e., a specific form referring to exactly two things), appear to learn the meaning of "two" earlier than English-speaking children (Almoammer et al., 2013). Thus, not only the frequency of exposure to number words and the counting sequence but also numerical information expressed and made explicit through grammatical structures seems to influence learning the meaning of (small) number words (see also Barner et al., 2009 for syntactical aspects in quantifiers).

Next to grammatical structures, various studies suggest that the way number words are formed determines number word learning trajectories. As mentioned above, number word systems vary considerably with respect to their transparency by which the place-value structure of the Arabic number system is reflected in number word formation. From a *lexical* point of view, differences between language groups in number word acquisition should be comparably small for numbers up to ten as in most languages there are exactly ten arbitrary but ordered words (eleven when including zero) that need to be learned and mapped to the respective numerical magnitude. And indeed, cross-cultural studies investigating counting skills (i.e., correctly reciting the counting sequence) in children aged between 3 and 6 suggest that for numbers up to 10 average performance is fairly similar across different language groups (e.g., LeFevre et al., 2002; Miller et al., 1995; Miller & Stigler, 1987).

However, within the number range up to 20 (i.e., numbers "eleven" to "nineteen"; see Section 3 ("Processing multi-digit numbers") for effects for numbers >20), the transparency by which the place-value structure of the Arabic number system is reflected in number words starts to vary considerably between languages. Importantly, the teen number range seems to be special in that a lack of transparency for teen number words can be found in quite many languages (e.g., Arabic, English, Hindi, Italian, Polish, Russian, Spanish, Swedish) – even though number words for numbers larger than 20 are often quite transparent again in these languages. In particular, languages show a variety of peculiarities for teen number words such as, for example, (i) exceptional number words not indicating the teen range at all (e.g., English: "eleven" and "twelve"); (ii) inverted number words [e.g., English: “fourteen” instead of ten four; Polish: “jedenaście” (oneteen); German: “dreizehn” (three-ten)], or (iii) inconsistent construction of teen number words within a language [e.g., Italian: “undici” (one ten) but “diciotto” (ten eight); cf. Lewis et al. (2020)]. This may represent a source of considerable difficulty for children because rather than simply applying a consistent rule for the first two-digit number words children are confronted with, they have to deal with irregularities that may not facilitate the acquisition of numerical and place-value and concepts more broadly.

And indeed several studies investigated the acquisition of teen numbers in different languages and reported a delay in number word acquisition for numbers larger than 10 for languages with less transparent number words (e.g., Aunio et al., 2008; Cankaya et al., 2014; LeFevre et al., 2002; Lonnemann et al., 2019; Miller et al., 1995; Miller & Stiegler, 1987). For instance, Miller and colleagues (1995) compared early counting skills in three- to five-year-old English- and Mandarin-speaking preschoolers. Compared to perfectly transparent Mandarin number words [“shí yī” (ten one), “shí èr” (ten two), “shí sān” (ten three), etc.], English number words in the teens range are fairly in-transparent (“twelve” instead of ten-two, “fourteen” instead of ten-four, etc.). As mentioned before, this study found no differences in counting skills between language groups for numbers up to 10. However, the authors found significant language differences favoring Mandarin-speaking children starting in the teen range. When looking at individual teen numbers more closely, language differences in counting were most pronounced for numbers above 12 (Miller et al., 1995). Importantly, while approximately 75% of Mandarin-speaking children were able to correctly count up to 20, only 50% of English-speaking children were able to do so by the age of five. Based on these findings, the authors concluded that English-speaking children need more time to master number names for teen numbers and beyond because of the lack of transparency between their number word system and the place-value structure of the Arabic number system.

Notably, however, others have questioned specificities in number word systems as sole contributing factor to observed cross-cultural differences and argue that differences in, for instance, approaches to teaching and learning (e.g., Aunio et al., 2008) as well as differences in home experiences (e.g., LeFevre et al., 2002) need to be considered as plausible additional or even alternative explanations for the observed differences in counting. In this context, evaluating a counting intervention in three- and four-year-old Turkish- and English-speaking children, Cankaya and colleagues (2014) came to the conclusion that both the transparency of the number word system and prior experience with numeracy-related activities were crucial for the acquisition of counting skills. As in Mandarin, Turkish teen number words are very transparent [e.g., on üç (13) translates to ten three]. The authors report that in their intervention learning gains in counting were higher and more consistent for Turkish- compared to English-speaking children. The authors attributed this finding to the more transparent Turkish number words compared to the English ones. However, despite the transparent number word system, Turkish-speaking children showed poorer counting skills and poorer general numerical performance overall before the intervention. Thus, while the transparency of the number word system does seem to matter, additional culture-specific variables likely contribute to differences in developmental trajectories when learning to count.

Next to *lexical* influences concerning the (lack of) transparency of many number word systems (especially in the teen number range), both before and during formal schooling children need to learn further specific numerical and mathematical language. Such mathematical words may, for example, convey less precise numerical meaning than number words (e.g., many, fewer, less than) or may have a different meaning in a mathematical context (e.g., quarter, break apart; e.g., Powell et al., 2017; Powell & Nelson, 2017). Generally, Harmon et al. (2005) suggest that language used in numerical and mathematical contexts is highly content-specific and, thus, may require more explicit teaching of the meaning of specific words at times. Accumulating evidence suggests that mathematical language proficiency (e.g., Powell et al., 2017; Powell & Nelson, 2017; Purpura et al., 2017; Purpura & Reid, 2016; Schleppegrell, 2007; Toll & Van Luit, 2014a, 2014b) but also parent and teacher usage of this specific language (e.g., Boonen et al., 2011; Gunderson & Levine, 2011; see also Chapter 7 in this volume by Desoete et al.) is associated with and predicts future development of numerical and mathematical competences.

For young children, knowledge of two types of mathematical language terms seems to be critical: quantitative (e.g., more than, many, fewer; cf. quantifier knowledge, e.g., Hurewitz et al., 2006) and spatial terms (e.g., before, close to; e.g., Mix & Cheng, 2012; Pruden et al., 2011). In this early numerical context, results of a study by Purpura and Reid (2016) in 3- to 5-year-old preschool children suggest that mathematical language might be a more important predictor of early numerical competences (e.g., counting or relational knowledge) when compared to more general language-related skills (e.g., vocabulary, phonemic awareness). The relevance of mathematical language for early numerical skills was further substantiated by an intervention study conducted by Purpura et al. (2017) focusing on quantitative and spatial mathematical language in 3- to 5-year-old children. After the eight-week intervention, children in the intervention group not only outperformed the business-as-usual control group with respect to their knowledge of mathematical language terms but also with respect to early numerical skills (e.g., one-to-one correspondence, number order, set and numeral comparison, numeral identification, among others; Purpura et al., 2017).

Taken together, this evidence is in line with the idea that different aspects of language seem to shape the development of early numerical competences way before formal education starts. As such, very specific language aspects such as phonemic awareness and grammatical number but also formation of number words and use of unspecific quantifiers seem to influence the typical development of children’s early numerical development to a considerable degree - even without formal instruction but by simply living in the respective language environment. Importantly, however, while linguistic influences seem to reflect one important factor during the development of early numerical competences, other cultural and environmental factors (e.g., approaches to learning and teaching, home environment) ought not to be neglected when trying to evaluate the specific contribution of language to arrive at a comprehensive understanding of the driving factors in early numerical development.

3 Processing multi-digit numbers

One key concept of the Arabic, base-10 number system is its place-value structure. The place-value structure defines that overall magnitude of multi-digit numbers is represented by powers of ten increasing from right to left combined by specific multiplicative and additive composition rules (e.g., $342 = \{3\} \times 10^2 + \{4\} \times 10^1 + \{2\} \times 10^0$; McCloskey et al., 1985). In particular, deriving the magnitude of a specific number requires understanding that any digit in a multi-digit number informs about both the size (through the digit’s face value) and the power of ten it represents (through the digit’s position within the digit string). As such, mastery of the place-value structure of the Arabic number system is critical for understanding multi-digit numbers.

In this context, mastery of the place-value structure of the Arabic number system was indeed observed to be associated with current but also predictive of later arithmetic performance (e.g., Chan et al., 2014; Moeller et al., 2011). Moreover, deficient place-value knowledge has been discussed as a predictor or source of dyscalculia and mathematical learning difficulties (e.g., Cawley et al., 2007; Chan & Ho, 2010; Haase et al., 2014). For instance, Chan and Ho (2010) assessed 8- as well as 10-year-old children with and without mathematical difficulties and demonstrated that conceptual understanding of the place-value structure differentiated reliably between children with and without mathematical difficulties (see also Lambert & Moeller, 2019). Thus, mastering the place-value structure of the

Arabic number system represents an important milestone in numerical development (see also Herzog et al., 2017, 2019 for a developmental model of conceptual place-value understanding).

However, children often experience specific difficulties when learning the place-value structure and, thus, with many tasks involving multi-digit numbers. Moreover, as mentioned in the context of teen numbers before, number words do not always reflect the place-value structure properly, which further complicates the learning process. Such *lexical* influences are the most widely investigated linguistic aspects in the context of multi-digit numbers processing. Thus, this section will focus on linguistic influences on place-value processing resulting from specificities in the formation of number words.

Regarding multi-digit number processing, *lexical* influences cover both the (lack of) transparency of power (e.g., in Mandarin, power is expressed explicitly in both number symbols and words: 42 = 四十二 = sì shí èr = 4-10-2) and the (lack of) transparency of order (e.g., the inversion of number words in, e.g., German: the number word for 23 is “dreiundzwanzig,” literally three-and-twenty). Although many cultures share the Arabic number system, number word systems clearly vary with respect to the degree of transparency in which power and order are conveyed (see above for the case of teen numbers). While for many cultures using Arabic digits and for most numbers, power is expressed by different words for the same symbol depending on the position in the digit string (e.g., in English the number word for 4 is “four” and the number word for 42 is “*forty*-two”) or by adding a multiplier indicating the power (the number word for 342 is “three *hundred* forty-two”), many exceptions are found for specific number ranges in different number word systems.

In French number words, for example, most two-digit numbers are transparently composed of two words each reflecting the power of a digit in accordance with the place-value structure (e.g., the French number word for 42 is “quarante-deux,” literally forty-two). However, number words larger than 60 are constructed quite irregularly by drawing on a vigesimal system (i.e., a base-20 system), which is inconsistent with the base-10 structure of the Arabic number system (e.g., the number word for 72 is “soixante-douze,” literally sixty-twelve). Finally, the French number word for 80 is “quatre-vingt” (literally four-twenty) and larger numbers are constructed accordingly (e.g., the number word for 96 is “quatre-vingt seize,” literally four-twenty sixteen), which adds even more construction principles to the already complex number word system.

Lack of transparency with respect to order can also be found in English number words. As mentioned before, English number words for teens from 13 to 19 are inverted with respect to the digital-Arabic notation (e.g., 19 = “nineteen”), although English number words for two-digit numbers are otherwise fairly transparent (42 = "forty-two"). While in modern English the phenomenon of inversion is restricted to teen numbers, in old English and many other modern languages (e.g., Arabic, Danish, Dutch, German, Flemish, Malagasy, Maltese, and also partly in Czech and Norwegian; Comrie, 2005) number words of wider number ranges are inverted. For example, in German, all two-digit numbers are inverted (e.g., the number word for 42 is "zweiundvierzig," literally two and forty). Moreover, although hundreds and thousands are not inverted (e.g., the number word for 2342 is "zweitausenddreihundertzweiunsvierzig," literally two thousand three hundred two and forty), for thousands and ten thousands (e.g., powers 10^3 and 10^4) the inversion of numbers words occurs again (the word for 42,342 is "zweiundvierzigtausenddreihundertzweiundvierzig," literally two and forty thousand three hundred two and forty).

This exemplary illustration of some aspects of lack of transparency shows that there are number word systems that do not reflect basic principles of the Arabic number system such as its base-10 structure or/and the place coding scheme correctly (i.e., that value increases from right to left). Noteworthy, a long list of studies showed that lack of transparency of number word systems with respect to the place-value structure of the Arabic number system influences and, crucially, complicates multi-digit number processing (for an overview, see e.g., Klein et al., 2013).

As mentioned before in the context of teen numbers, in addition to, for instance, specific approaches to teaching and learning, Asian children seem to benefit from their highly transparent number word systems (i.e., power and order are transparently reflected in the number words themselves; e.g., in Mandarin the number word for 42 is sì shí èr (四十二), literally four-ten-two). When investigating the understanding of the place-value structure of the Arabic number system in children from various Asian and Western countries, several early studies demonstrated better place-value understanding in Asian compared to Western preschoolers and 1st graders (e.g., Miura et al., 1988; Miura & Okamoto, 2003; Miura et al., 1993). In particular, while Asian children preferred representing multi-digit numbers by using ten and one blocks (i.e., matching the place-value structure of multi-digit numbers), Western children preferred using collections of one blocks for longer suggesting delayed understanding of the place-value structure of the Arabic number system in Western children with less transparent number systems (e.g., Miura, et al., 1994; Towse & Saxton, 1998). Importantly, differences were already observed before the concept of the place-value system was explicitly taught (e.g., in school) questioning influences of differences in the teaching approaches as the sole determining factor (see Vasilyeva et al., 2015 for an opposing view).

Detrimental effects of the lack of transparency of certain number word systems were demonstrated in different numerical tasks. The probably most obvious effects can be observed in number transcoding (i.e., writing down numbers to dictation) in elementary school children. Typically, errors children commit suggest that they “simply write down what they hear”. More formally speaking, errors in transcoding often reflect insufficient knowledge of additive (e.g., “three hundred and forty-two” is written down as 30042) and/or multiplicative composition rules (e.g., “three hundred” is written down as 3100) or further language-specific (in)transparencies.

As regards the latter, it was shown, for example, that children speaking languages with inverted (e.g., German, Dutch), as compared to children speaking languages with non-inverted, number words (e.g., French, Italian) do not only commit more transcoding errors overall (e.g., Krinzinger et al., 2011; Pixner et al., 2011b; but see Imbo et al., 2014), but also commit up to 50% inversion-related errors (e.g., “vierundzwanzig” (24) – literally “four and twenty” – is written down as 42; Imbo et al., 2014; Krinzinger et al., 2011; see Pixner et al., 2011b for a within-culture approach in Czech).

Another example for the language specificity of transcoding errors is described in the study by Van Rinsveld and Schilz (2016) who investigated effects of the vigesimal structure in French number words larger than 60 (e.g., the number word for 72 is “soixante-douze,” literally sixty twelve; see also Seron & Fayol, 1994) in two computerized transcoding tasks (i.e., choosing the Arabic number with auditory verbal presentation and reading out loud the Arabic number presented on the screen). Results in both tasks indicated that performance in English-speaking fifth graders (aged 10) was comparable to French-speaking fifth graders for numbers up to 60. However, for numbers larger than 60 and, thus, the number range where number words in French follow the vigesimal and number words in English follow the decimal structure, English-speaking children were faster in both tasks and made fewer errors in the recognition task than French-speaking children. The fact that these results were observed in fifth graders is of particular interest because it illustrates that although the impact of specificities in number word formation might get smaller with age and experience, traces of in-transparent number word formation can still be detected in children way beyond the age of early numerical development.

The latter also applies to the processing of multi-digit number magnitude. Here, influences of lack of transparency in number word formation were shown for the unit-decade compatibility effect (Nuerk et al., 2001) in two-digit number magnitude comparison. When children (or adults) are asked to indicate the larger of two two-digit numbers, they usually respond faster to unit-decade compatible number pairs for which separate comparisons of tens and units bias the same decision (e.g., 32_57, 3 < 5 and 2 < 7). In contrast, in decade-unit incompatible number pairs comparing tens and units separately leads to opposing decision biases (37_62, 3 < 6 but 7 > 2), resulting in comparably slower responses due to the interference between comparisons of tens and units. The unit-decade compatibility effect (i.e., the performance difference between compatible and incompatible number pairs) was replicated for both adults and children (e.g., adults: Bahnmueller et al., 2019; Ganor-Stern et al., 2007; Macizo & Herrera, 2011; Nuerk et al., 2005; children: Landerl & Kölle, 2009; Pixner et al., 2011a; Van Rinsveld et al., 2016).

Importantly, the unit-decade compatibility effect is found in number pairs for which the comparison of the unit digit is actually completely irrelevant because identification of the larger number can be based solely on the comparison of the tens digits. This suggests two things: first, the unit digit is processed automatically although it is irrelevant for the task at hand, and second, that magnitudes of tens and units are processed in a decomposed way but in accordance with the place-value structure of the Arabic number system (i.e., tens are compared with tens and units are compared with units; see Wood et al. (2005) for expansions on this).

Although the compatibility effect is not a language-specific phenomenon (i.e., the effect was observed in many different languages), the effect was found to be modulated by language, and more specifically by the inversion property of number words. For example, Pixner, Moeller, and colleagues (2011) investigated the unit-decade compatibility effect in German-, Italian-, and Czech-speaking first graders. While German number words are inverted and Italian number words are not, in Czech there are two number word systems – one inverted and the other one not. Clear differences in the compatibility effect were observed with German-speaking children showing a significantly larger compatibility effect than the other two language groups and, for reaction times, Czech-speaking children showed a compatibility effect falling in between the German and the Italian group. This pattern of results suggests that number word formation influences the processing of two-digit number magnitude in an entirely symbolic number magnitude comparison task as neither the input nor the output in this task was verbal. In particular, in languages with inverted number words such as German, the unit digit is named first (e.g., the number word for 23 is “dreiundzwanzig,” literally three and twenty) and might, thus, lead to increased unit-based interference in incompatible trials, which in turn would increase the unit-decade compatibility effect for inverted languages. Thus, these results suggest that verbal number word information influences number processing even when it is not present in or necessary for the task at hand.

Research regarding transcoding competencies (i.e., writing numbers to dictation) in elementary school children further investigated possible *phonological* influences relating to verbal working memory capacities in multi-digit number processing. In this context, several studies in different language groups with and without inverted number words evaluated the idea that transcoding might be influenced by working memory capacity because incoming number word information needs to be manipulated and mapped onto the digital-Arabic notation. These studies observed that better working memory was associated with better transcoding performance in general (Imbo et al., 2014; Pixner et al., 2011b; Simmons et al., 2012; Zuber et al., 2009) but with a lower number of inversion-related errors in German-speaking first graders in particular (Zuber et al., 2009). However, while there is broad agreement that working memory is important for transcoding, findings are so far inconsistent with respect to specific working memory components (cf. Baddeley, 2000; Baddeley & Hitch, 1974). For instance, while some studies primarily reported associations of transcoding performance with verbal working memory capacities (e.g., Imbo et al., 2014), others highlight the relevance of visual-spatial working memory capacities (e.g., Simmons et al., 2012; van der Ven et al., 2017), or the central executive (Pixner et al., 2011b; Zuber et al., 2009). So far, findings suggest that for transcoding numbers from the verbal number word to the digital notation conveying the correct order of digits seems more relevant than solely being able to temporarily store verbal number word information – at least in children that are busy learning the place-value structure of the Arabic number system (cf. van der Ven et al., 2017).

Taken together, the presented studies provide further strong evidence that cognitive representations of multi-digit numbers are shaped by and differ between languages as indicated by significant linguistic influences in a variety of different tasks ranging from transcoding between the verbal and the digital-Arabic notation to tasks requiring the explicit processing of number magnitude information. Importantly, these observed language-related influences do not only foster our understanding of underlying principles of multi-digit number processing, but they may also be of diagnostic value. For instance, Moeller and colleagues (2011) showed that the number of inversion-related errors in transcoding as well as the size of the compatibility effect in the first grade predicted arithmetic performance in the third grade (including mathematics grades). Thus, better understanding language-specific aspects of place-value processing might help identifying children that may develop mathematical difficulties early on.

4 Basic arithmetic operations

Building on previously described numerical competencies, mastery of basic arithmetic operations – addition, subtraction, multiplication, and division – represents a further cornerstone in numerical and mathematical development. Not only is mastery of basic arithmetic operations a pervasive requirement in everyday life, it also represents a crucial basis for more advanced mathematical competencies (e.g., Geary & Hoard, 2005). Children use a variety of different strategies to solve arithmetic problems that may vary with the type of operation they are presented with. Moreover, strategies used by children become more efficient and adaptive with age and experience (Ashcraft, 1982; Carpenter & Moser, 1984; Geary & Hoard, 2005; Geary et al., 2004; Siegler, 1996; Siegler & Shrager, 1984). Usually, two major types of strategies are distinguished separating (i) procedural strategies including counting (cf. Fuson, 1982), mental computations and/or transformations, or keeping track of intermediate solutions; and (ii) retrieval strategies that allow direct retrieval of previously learned arithmetic facts from memory (Ashcraft, 1982). Strategy choices were reported to depend on a range of factors, including problem size (i.e., the numerical magnitude of the components of an arithmetic problem; e.g., De Smedt et al., 2010) and the respective arithmetic operation (e.g., Imbo & Vandierendock, 2007), as well as the presentation format or context in which a problem is presented (e.g., digital-Arabic format vs. embedded in word problem). Because heterogeneous solving strategies are involved in arithmetic problem solving, language may affect arithmetic processing differently depending on the strategy that is used when solving a particular problem. In this final section, we will therefore elaborate on *phonological* influences on arithmetic processing with respect to the use of both procedural and retrieval-based strategies. Moreover, we will describe *lexical* influences on multi-digit arithmetic problem solving as well as *semantic* influences in the context of word problems.

Regarding *phonological* influences on the development of arithmetic competence, a considerable body of research is concerned with the relation of working memory resources and arithmetic performance in both adults and children (for reviews, see DeStefano & LeFevre, 2004; Friso-van den Bos et al., 2013; Peng et al., 2016; Raghubar et al., 2010). While researchers seem to generally agree that working memory is crucial for arithmetic processing and learning, inconsistent findings also suggest that the relation between a specific working memory component (e.g., verbal and visual-spatial working memory, central executive, e.g., Baddely, 2000; Baddeley & Hitch, 1974) and arithmetic performance likely depends on several factors (age, mathematical outcome variable, working memory task, etc.; e.g., Raghubar et al., 2010). Concerning arithmetic processing in primary school, Friso-van den Bos and colleagues (2013) suggested verbal working memory to show the most pronounced association with arithmetic competencies.

Generally, working memory has been suggested to be of specific importance when procedural strategies including maintaining and manipulating intermediate results during calculation have to be used to solve a problem (e.g., DeStefano & LeFevre, 2004). This is, for example, the case for more complex problems with larger problem sizes (e.g., Barrouillet, Mignot & Thevenot, 2008; Imbo & Vandierendock, 2008), for addition problems requiring a carry procedure (e.g., Ashcraft & Kirk, 2001; Fürst & Hitch, 2000), or – more generally – for problems for which solutions cannot (yet) be retrieved from memory.

Critically, empirical evidence suggests that the respective contribution of verbal and visual-spatial working memory components changes with age (e.g., De Smedt et al., 2009; Rasmussen & Bisanz, 2005; Van de Weijer-bergsma et al., 2015). For example, van de Weijer-Bergsma and colleagues (2015) observed that while the importance of verbal working memory for all four arithmetic operations was shown to increase from the second to sixth grades, visual-spatial working memory influences decreased. A similar conclusion was drawn by McKenzie et al. (2003), who investigated influences of verbal and visual-spatial working memory on simple arithmetic competencies in 6- to 7- and 8- to 9-year-old children experimentally by using a dual task paradigm. Children were asked to solve simple, auditorily presented addition problems (e.g., 9 + 4, 4 + 3 + 7) in three conditions: a baseline condition without added interference, a verbal interference condition in which children heard an audiotaped story while solving the addition problems, and a visual-spatial interference condition in which children solved addition problems and at the same time saw a matrix of black and white squares that randomly changed on the screen. Results indicated that while performance of children in both age groups was affected by visual-spatial interference, verbal interference only decreased performance in the older group of children. Thus, this study seems to substantiate that younger children may rely more on visual-spatial working memory when acquiring arithmetic competences, whereas older children seem to draw from both verbal and visual-spatial working memory resources when solving arithmetic problems.

Studies that specifically address the involvement of verbal working memory resources when retrieving arithmetic facts provided somewhat mixed results (for a review, see DeStefano & LeFevre, 2004). For instance, some studies suggest that the retrieval of multiplication facts is interrupted by concurrent verbal processing (e.g., Lee & Kang, 2002; Lemaire et al., 1996); however, in other studies fact retrieval remained largely unaffected under verbal load (De Rammelaere et al., 2001; Seitz & Schumann-Hengsteler, 2000). Thus, the degree to which verbal working memory influences arithmetic problem solving seems to depend on the respective strategies available and used to solve a particular problem.

Interestingly, further studies have addressed an additional *phonological* language aspect in that they focused on the influence of phonemic awareness on arithmetic performance assessed by standardized tests (e.g., Fuchs et al., 2006; Hecht et al., 2001; Krajewski & Schneider, 2009; Leather & Henry, 1994; Rasmussen & Bisanz, 2005; Simmons et al., 2008) but also more specifically on arithmetic fact retrieval (De Smedt & Boets, 2010; De Smedt et al., 2010). While many studies provided quite substantial evidence for an association of phonemic awareness with general arithmetic skills, the precise mechanism driving this association seems less well understood. Following up on this, De Smedt et al. (2010) suggested that one mechanism driving the association of phonemic awareness with general arithmetic skills might lay in its functional role for the retrieval of arithmetic facts. To investigate this claim, 9- to 11-year-old children were asked to solve addition, subtraction, and multiplication problems of both small (<25) and large problem size. The idea was that problems with a small problem size are more likely to be solved via retrieval-based strategies and should, thus, show a more pronounced association with phonemic awareness than problems with a large problem size. And, indeed, results showed a significant association of phonemic awareness with performance on problems with a small but not with a large problem size. Interestingly, this was observed independent of the respective operation. Thereby, the results of De Smedt and colleagues (2010; see also De Smedt & Boets, 2010 for additional evidence in dyslexics) support the idea that phonemic awareness may play a critical role for the acquisition of arithmetic facts.

Beyond *phonological* influences and similar to previously reported tasks involving multi-digit numbers, *lexical* influences related to the lack of transparency of certain number word systems were also observed for basic arithmetic. Investigating inversion-related effects, Göbel and colleagues (2014) evaluated performance differences in mental addition between German- and Italian-speaking second graders (with German having inverted and Italian having non-inverted number words). The authors specifically focused on the so-called carry effect which describes the observation that it takes considerably longer and more errors are committed in addition problems that require a carry procedure compared to problems that do not contain a carry (e.g., Deschuyteneer et al., 2005; Fürst & Hitch, 2000; Imbo et al., 2007; Klein et al., 2010). For example, a carry procedure is needed for 18 + 35 = 53 because the units add up to a sum larger than 9 (i.e., 8 + 5 = 13) and, thus, the tens digit of the unit sum has to be carried to the sum of the tens digits. Göbel and colleagues (2014) observed a regular carry-effect for both language groups; however, the effect was more pronounced in

German-speaking as compared to the Italian-speaking children (for similar results in adults, see Lonnemann & Yan, 2015). In carry problems it is crucial to keep track of place-value information because a successful carry operation requires to carry the tens digit of the unit sum to the tens position of the result. The more pronounced carry effect in children speaking German – a language with inverted number words – was attributed to increased demands on the manipulation and the mapping of the digital-Arabic notation and number words due to the inversion-related lack of transparency in the German number word formation.

Next to inversion-related language effects, there are also effects of number word systems (partially) following vigesimal (i.e., base-20) structuring (e.g., in, e.g., French or Basque the number word for 35 literally means to twenty-fifteen). For instance, Van Rinsveld et al. (2015) investigated performance in addition problems in German-French bilinguals across grades 7 to 10. Results indicated that when problems had to be solved in French, it took participants longer and they made more errors for problems with sums larger than 70 as compared to when the same problems had to be solved in German. Similarly, Colomé, Laka and Sebastián-Gallés (2010) manipulated addition problems so that problems either did not (e.g., 25 + 10 =) or did match with a vigesimal number word structure (e.g., 20 + 15 =). While performance between conditions did not differ for Italian and Catalan speakers, Basque speakers were specifically faster when addends followed the same vigesimal structure as Basque number words.

A last important aspect in the context of *semantic* influences on basic arithmetic abilities concerns the fact that throughout formal education arithmetic (and other) problems are regularly presented as word problems. The difficulty of arithmetic word problems is influenced by many factors related to both linguistic and numerical aspects (e.g., single- vs. multi-digit numbers, type of operation; see Daroczy et al., 2015 for an overview). On the one hand, linguistic aspects of arithmetic word problems such as sentence structure and length (e.g., Abedi & Lord, 2001; Spanos et al., 1988) but also the presence or absence of additional irrelevant information (e.g., Muth, 1992) certainly affect arithmetic word problem difficulty. On the other hand, the role of mathematical language and, in particular, the role of explicit verbal cues has also been investigated (e.g., Boonen et al., 2016; Hegarty et al., 1992; Van der Schoot et al., 2009; Verschaffel et al., 1992). Explicit verbal cues include words and phrases whose semantic usually directly hints at a respective operation that needs to be performed to arrive at the solution of the problem (e.g., subtraction: “Henry has 9 books. He *sells* 4 books at the flea market. How many books does he have *left*?”; multiplication: “Henry has 5 friends that he will meet in the park later today. He wants to bring 3 gummy bears for *each* of his friends. How many gummy bears does he have to bring?”).

Unfortunately, verbal cues must not be used blindly because in some instances they are misleading. For example, the relational term “less” in the compare word problem “At the supermarket, a chocolate bar costs £1. This is 30 pence *less* than at the kiosk. How much do you have to you pay at the kiosk?” is inconsistent with the required operations (i.e., less would suggest a subtraction problem, however, to solve the problem correctly an addition needs to be performed). In this context, the consistency effect describes the finding that such inconsistent arithmetic word problems are more prone to errors compared to consistent problems (i.e., in which the term “less” indeed requires a subtraction; e.g., Hegarty et al., 1992; van der Schoot et al., 2009). Thus, while it is important to learn the semantic meaning of verbal cues and their associated arithmetic operations, it is also crucial to emphasize the integration of additional information across sentences to derive a proper mental model of the problem and with this a first step to a successful solution.

Taken together, as mentioned above, mathematics education is largely hierarchical in nature and, therefore, it is necessary to be able to draw on previously acquired competences, because new numerical and mathematical content usually builds on these previously acquired competences. Regarding some of the linguistic influences (i.e., *lexical*, *phonological*, *semantic*) there appears to be a similar pattern: some aspects that have been observed to already influence early numerical competences (i.e., counting and cardinality understanding) seem to persist or even increase their impact on more complex mathematical content strands such as arithmetic problem solving. This means that one may not assume linguistic influences on basic numerical competences to be overcome entirely with time. Instead, it seems that they exhibit a lasting influence on human numerical cognition.

5 Conclusion

In this chapter, we discussed (i) *lexical*, (ii) *syntactic*, (iii) *phonological*, and (iv) *semantic* aspects of language that seem to influence numerical and mathematical development and illustrated their relevance for selected numerical and mathematical content strands of (i) counting and cardinality understanding, (ii) multi-digit number processing, and (iii) basic arithmetic operations. In this last section we aim at discussing differences between but also commonalities across linguistic influences and content strands, before we elaborate on potential implications of the reported linguistic influences that arise for numerical and mathematical development.

First, it needs to be mentioned that linguistic influences seem to be most obvious, relevant, and detectable during specific time windows of numerical and mathematical development. On the one hand, some linguistic influences begin to affect numerical and mathematical development very early on even before formal education starts (e.g., effects of grammatical number, effects of phonemic awareness on the acquisition of the counting sequence). Others start to show their effect later when more advanced numerical and mathematical competences are acquired (e.g., lexical effects regarding the transparency of number words on multi-digit number processing and mental arithmetic). On the other hand, some linguistic influences seem to fade out rather quickly and, thus, can be observed only in a comparably small time window (e.g., effects of grammatical number), whereas others keep being relevant or become even more relevant throughout elementary school years when more and more complex mathematical competences are acquired (e.g., influences of verbal working memory, semantic influences regarding mathematical language). From this pattern of effects, it seems that linguistic influences occur in waves that peak for and when new numerical or mathematical concepts or procedures are learned. It seems that at these times of high external demands due to new to-be-learned content the cognitive system is more susceptible to influences of internal biases of numerical representations reflecting influences of *lexical*, *syntactic*, *phonological*, and *semantic* linguistic specificities of the respective language.

Second, because we aimed at summarizing linguistic influences on numerical and mathematical processing in children, we did not specifically consider evidence on adolescents or adults throughout this chapter. It is worth mentioning though that most of the reported linguistic influences can still be observed in highly skilled adults. For example, in a cross-cultural study, Moeller et al. (2015) realized a natural 2 by 2 design for the variables number word inversion (inverted vs. non-inverted) and reading direction (left-to-right vs. right-to-left) in a quadrilingual study with German-, English-, Hebrew-, and Arabic-speaking adults. Results were comparable to those observed for children by Pixner et al. (2011a) indicating *lexical* influences. In particular, Moeller et al. (2015) observed that unit-decade compatibility effects were larger when reading direction and order of tens and units as named in number words were in conflict (i.e., for German, left-to-right reading but units named before tens, and Hebrew, right-to-left reading but tens named before units) than for English- and Arabic-speaking participants for which reading direction and the order in which tens and units are named in number words match. Thus, even though linguistic effects might be more pronounced in children, traces of linguistic influences can also be found in adults.

Nevertheless, it needs to be noted that effects in adults are usually quantitatively smaller (a few dozen milliseconds) and, thus, are only detectable using more sensitive measures (e.g., reaction time measures). However, the consistent observation of linguistic influences in adults suggests that they are not a purely transient phenomenon but shape how we process numbers for good. Yet, studying linguistic influences in adults seems more relevant from a theoretical cognitive perspective aiming at understanding the underlying principles of numerical and mathematical cognition. Implications for numerical and mathematical learning or even educational practice may be limited because effects and differences in the millisecond range may not reflect practically relevant differences in numerical and mathematical competence in everyday life.

Finally, however, for a teaching practitioner, knowing that certain language aspects influence typical numerical and mathematical development in a certain time window might help identifying children that struggle or might struggle in the future. As mentioned earlier, Moeller and colleagues (2011) showed, for instance, that the number of inversion-related errors in transcoding as well as the size of the unit-decade compatibility effect in the first grade predicted arithmetic performance in the third grade. Thus, better understanding language-specific aspects of place-value processing might help identifying children that may develop mathematical difficulties later on. Moreover, while considering linguistic influences on numerical and mathematical development when developing interventional strategies is certainly necessary, it is also important to know that not all linguistic influences seem to cause lasting disadvantages for a particular language group or mathematical task.

In turn, this allows for a reconciliatory ending of this chapter. Although we presented explicit effects of linguistic aspects on numerical and mathematical development, it does not seem to be the case that any of the discussed linguistic aspects (alone) is a necessary predictor of numerical and mathematical development in an all or nothing manner. Instead, specific linguistic aspects may be detrimental to some aspects of numerical cognition while others may even facilitate numerical and mathematical learning (e.g., explicit plural markings or a transparent number word system). As such, it is important to be aware of the width of linguistic influences to be able to adapt teaching and learning approaches accordingly. These adaptations may then allow to compensate for disadvantageous influences and to foster beneficial linguistic aspects to help children to successfully develop sufficient numerical and mathematical competences to master everyday demands and needs.

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