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Shear and longitudinal modulus of elasticity in wood: relations based on static bending tests

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ABSTRACT. Improve quality of timber structures design is an aim that must be systematically sought by engineers in this area. An important topic that can contribute directly to be achieved in this subject is the more consistent knowledge related to structural properties of wood. Know values of longitudinal modulus of elasticity (E) and shear modulus (G) is essential for proper evaluation of plate structures performance, as example. It has been usual to adopt statistical equivalence for E and G values in plans longitudinal-radial and longitudinal-tangential, although experimental confirmation of this hypothesis is required. In this context, the aim of this work is to determine values of E_{LR}, E_{LT}, G_{LR} and G_{LT}, based on static bending tests, to five dicotyledonous species. Results showed statistical equivalence between the elastic properties in both plans, and the relation E = 35G was obtained for the five wood species here considered.

Keywords: static bending test, timber, shear modulus, longitudinal modulus of elasticity.

Módulo de elasticidade transversal e longitudinal da madeira: relações baseadas nos ensaios de flexão

RESUMO. Melhorias no projeto das estruturas de madeira são metas que devem ser sistematicamente almejadas por profissionais da área. Um tópico importante que pode contribuir diretamente para isso está relacionado a um conhecimento mais profundo a respeito das propriedades de resistência e de rigidez da madeira. O conhecimento dos módulos de elasticidade longitudinal (E) e transversal (G) é essencial para a avaliação do desempenho de placas entre outros elementos estruturais. No dimensionamento estrutural tem sido comum adotar equivalência estatística dos valores de E e G nas direções longitudinal-radial e longitudinais-transversal. Este trabalho objetivou determinar valores dos módulos de elasticidade E_{LR}, E_{LT}, G_{LR} e G_{LT} com base em testes de flexão estática para cinco espécies dicotiledôneas, possibilitando avaliar a equivalência ou não destas propriedades (E_{LR} e E_{LT}; G_{LR} e G_{LT}) assim como de estabelecer correlações adequadas entre E e G. Os resultados revelaram equivalência estatística entre as propriedades elásticas em ambas as direções, e a relação $E = 35 \cdot G$ foi obtida para espécies de madeira estudadas.

Palvras-chave: ensaio de flexão estática, madeira, módulo de elasticidade transversal, módulo de elasticidade longitudinal.

Introduction

Improve quality of timber structures design is a aim that must be systematically sought by professionals in this area. Among the important topics that can strongly contribute to be achieved this goal, a more consistent knowledge of structural properties of wood can be pointed out.

Some normative codes in this matter adopt arithmetic relations to describe wood properties in order to make simple and quick the evaluation of structural elements behavior. In the specific case of Brazilian Code NBR 7190 (Associação Brasileira de Normas Técnicas [ABNT], 1997), some relations between longitudinal modulus of elasticity (E) and shear modulus (G) are adopted, but without appropriate experimental basis. This can induce to doubts in structural design and someone can take calculation assumptions that lead to imprecise estimation of stresses, as asserted by Bodig and Jayne (1982), Calil Junior, Lahr, and Dias (2003), Karlsen (1967), Mateus (1961), and Ritter (1990). Know values of longitudinal modulus of elasticity (E) and shear modulus (G) is essential for proper evaluation of plate structures performance, as example, according to Christoforo, Panzera, Batista, Borges, and Lahr (2011), Herzog, Natterer, Schweitzer, Votz, and Winter (2000), among others.

Several studies have been conducted to optimize the theoretical basis aiming to determine shear modulus in wood, considering its features of orthotropy, being mentioned among them Gillis (1972), Holmberg, Persson, and Petersson (1999), Nairn (2007), Price (1929), and Schniewind (1959). These authors have contributed to better understanding of the problem, usually working with the well-known tests in clear specimens.

Researchers as Ballarin and Nogueira (2003) sought to obtain experimental values of G, although working mostly with small number of specimens, aspect that prevent generalization of the results obtained.

Moreover, it is noteworthy that the values of wood stiffness properties can be obtained from non-destructive testing techniques, according to Ballarin and Palma (2009). They point out that, although in some cases leading to high variability of results, non-destructive techniques are configured as an interesting alternative to characterize wood from planted forests, given the significant amount of defects present in them. Papers by Alves and Carrasco (2013), Bucur and Archer (1984), Gonçalves, Trinca, and Cerri (2011), Gonzales, Valle, and Costa (2001), Ross, Brashaw, and Pellerin (1998), Sandoz (1989), Stålne and Gustafsson (2002), Tallavo, Pandey, and Cascante (2013), Yang, Wang, Lin, and Tsai (2008) are other examples of the same propositions of the first mentioned authors.

Mascia and Lahr (2006), evaluating aspects of wood as an orthotropic material, calculated E and G values in the two longitudinal planes. Results published by these researchers were object of statistical analysis. In a more superficial approach, it could not be ruled out a possible difference between E_{LT} and E_{LR} and between G_{LR} and G_{LT} for the tropical species Jatobá (*Hymenaea stilbocarpa*). Also from data contained in the cited article, it's possible to infer that relation $E\ G^{-1}$ is close to 20, for Jatobá.

Christoforo, Ribeiro Filho, Panzera, and Lahr (2013) presented an analytical methodology for determination longitudinal and shear moduli for structural dimension lumber (proper to wood coming from planted forests), using three-point static bending tests; adapted from Brazilian Code NBR 7190 (ABNT, 1997). Wood species used in these trials were *Pinus elliottii* and *Corymbia citriodora*. The related equations were developed according to virtual forces method and the shear shape coefficient (f_s) to rectangular cross section was adopted as 1.20 (6/5). Results of coefficients (α) between moduli ($E = \alpha \cdot G$) for the referred wood species were, respectively, 18.70 and 21.20, very close to the coefficient (20) set by the aforementioned Brazilian Code.

Simplifying, it has been usual to adopt statistical equivalence for values of G in the longitudinal-radial (G_{LR}) and longitudinal-transversal (G_{LT}) directions, important parameters related to structural design requirements, as evidenced by Gillis (1972) and Kretschmann (2010), among others. Similar position is taken by the NBR 7190 (ABNT, 1997) that establishes a unique relationship between these properties, *i.e.*, $E = G\ 20^{-1}$.

Then, this work focuses on determining values of E_{LR}, E_{LT}, G_{LR} and G_{LT}, based on static bending tests, exclusively to some dicotyledonous species grown in Brazil, aiming to confirm its equivalence (E_{LR} and E_{LT}, G_{LR} and G_{LT}) or to establish proper correlations.

Material and methods

To achieve the proposed objective, five hardwood species were considered, each one representing a strength class, according to the prescriptions of Brazilian standard document NBR 7190 (ABNT, 1997):

- Cedrinho (*Erisma uncinatum*): class C20;
- Peroba rosa (*Aspisdosperma polyneuron*): class C30;
- Tereticornis (*Eucalyptus tereticornis*): class C40;
- Canafístula (*Cassia ferruginea*): class C50;
- Jatobá (*Hymenaea stilbocarpa*): class C60.

The specimens evaluated in experimental procedures were properly stored and tested in dependencies of Laboratory of Wood and Timber Structures (LaMEM), Department of Structural Engineering (SET), São Carlos Engineering School (EESC), University of São Paulo (USP).

Inclusion of these wood species in strength classes stipulated by NBR 7190 (ABNT, 1997) is based on the characteristic values of compression strength parallel to grain.

For each species, results of 12 tests species were considered, for specimens with nominal dimensions: 5×5×115 cm, with growth rings parallel to two opposite faces.

Each specimen was tested four times in static bending: two with force applied on LR plane (longitudinal-radial) and two in the LT plane (longitudinal-transversal). In all situations, the specimens were initially tested according to the four-point bending model (Figure 1a), used by

American Standard Code D 198 (American Society for Testing Materials [ASTM], 1997), with nominal span 105 cm (L_1), nominal height 5 cm, conforming the ratio L_1 hour ≥ 21^{-1} (ABNT, 1997) ensuring that shear stress contribution to vertical displacements is negligible. All tests were carried out in a non-destructive method, restricting displacement at the midpoint of specimens to $\delta_1 = L_1\ 200^{-1}$. It is ensured that proportionality limit was not exceeded, as it is prescribed by NBR 7190 (ABNT, 1997). Once determined force (F_1), responsible for displacement $L_1\ 200^{-1}$, and known specimens dimensions (height 'h' and width 'b' of cross section), these data are used in the equation of displacement for the above test structural model, derived from materials strength (Equation 1), consisting of one equation with two unknown terms (L and G). Thereafter, the supports were approximated, giving a useful second length (L_2) to the test piece (70 cm), and this was applied to a load (F_2) from the midpoint of the wood beam (Figure 1b). Maintaining the value of the scale and load displacement $\delta_2 = L_2\ 200^{-1}$ in the middle of the span of the structural model, these data were cast into Equation shifts from strength of materials (Equation 2) providing a second equation in variables E and G.

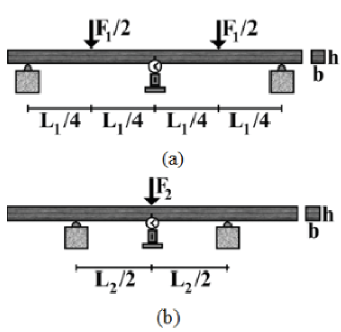

Figure 1. Static bending tests in (a) four-points and (b) three points.

$$\delta_1 = \frac{11 \cdot F_1 \cdot L_1^3}{768 \cdot I} \cdot \frac{1}{E} \quad (1)$$

$$\delta_2 = \frac{F_2 \cdot L_2^3}{48 \cdot I} \cdot \frac{1}{E} + \frac{f_s \cdot F_2 \cdot L_2}{4 \cdot A} \cdot \frac{1}{G} \quad (2)$$

Solving Equation 1 and 2, both derived from bending tests, it leads to values of longitudinal modulus of elasticity and shear modulus, Equation 3 and 4, respectively.

$$E = \frac{11 \cdot F_1 \cdot L_1^3}{768 \cdot \delta_1 \cdot I} \quad (3)$$

$$G = \frac{11 \cdot f_s \cdot F_2 \cdot L_2 \cdot F_1 \cdot L_1^3}{4 \cdot A \cdot (11 \cdot F_1 \cdot L_1^3 \cdot \delta_2 - 16 \cdot F_2 \cdot L_2^3 \cdot \delta_1)} \quad (4)$$

The cantilevers of the specimens in bending test do not influence deflections in region between supports to obtain E and G (Christoforo et al., 2013).

To evaluate equivalence between E_{LR} and E_{LT}, and between G_{LR} and G_{LT}, for each wood species, a hypothesis test at 5% significance level (α) was assumed. Null hypothesis (H_0) refers the equivalence of mean values of (E_{LR} and E_{LT}) and (G_{LR} and G_{LT}), and non-equivalence between means is the alternative hypothesis (H_1). P-value higher than level of significance or presence of zero in confidence interval (μ) leads to accept H_0, rejecting it otherwise. To investigate equivalence between E_{LT} and E_{LR}, regardless of wood species, the longitudinal elastic modulus achieved at LR were divided by the corresponding values of longitudinal modulus of elasticity obtained by in LT plane ($E_{LR}\ E_{LT}^{-1}$). The same procedure was taken to shear moduli ($G_{LR}\ G_{LT}^{-1}$). Therefore, null hypothesis formulated in hypothesis testing ($\alpha = 0.05$) consisted the means of values obtained, implying equivalence between E values, and different from as alternative hypothesis (values of E are not equivalent). P-value greater than 5% implies accepting H_0, rejecting it otherwise. To validate the hypothesis, Anderson-Darling normality test at the 5% level of significance was applied. Null hypothesis was to assume normality to E values, and non-normality as alternative hypothesis. P-value greater than 0.05 implies accepting H_0, rejecting it otherwise.

To estimate the shear modulus based on the values of longitudinal modulus of elasticity for any wood species, a linear model regression based on least squares method was used.

Significance and quality of data adjustment were evaluated by analysis of variance (ANOVA) of the regression (at level 5%). The null hypothesis was adopted as no significance for regression by adjusted coefficients, and significance of the regression as alternative hypothesis. P-values less than the significance level implies rejecting the null hypothesis, accepting it otherwise. Alternatively, similar to Christoforo et al. (2013), the relation between longitudinal and shear modulus ($E = \xi \cdot G$) was evaluated using of Equation 5 (least squares method), whose purpose is to better determine the coefficient (ξ).

$$f(\xi) = \frac{1}{2} \cdot \sum_{i=1}^{n} (E_i - \xi \cdot G_i)^2 \quad (5)$$

Results and discussion

Table 1 and 2 present the results of longitudinal modulus of elasticity and shear moduli, respectively, for the five wood species investigated.

Table 1. Results of E_{LR} and E_{LT}.

	Cedrinho		Peroba Rosa		Tereticornis		Canafístula		Jatobá	
	E_{LR} (MPa)	E_{LT} (MPa)	E_{LR} (MPa)	E_{LT} (MPa)	E_{LR} (MPa)	E_{LT} (MPa)	E_{LR} (MPa)	E_{LT} (MPa)	E_{LR} (MPa)	E_{LT} (MPa)
$\bar{x}$	8557	8434	12215	11879	11159	11691	14288	14466	18565	18298
Cv	15	16	18	13	14	12	15	18	13	14
Min	6647	5974	9626	9510	8308	9801	11980	11789	14494	14897
Max	10978	10821	15332	13878	13118	14310	17342	18573	22567	23050

$\bar{x}$: sample means, *Cv*: coefficient of variation, *Min*: smallest values, *Max*: largest values.

Table 2. Results of G_{LR} and G_{LT}.

	Cedrinho		Peroba rosa		Tereticornis		Canafístula		Jatobá	
	G_{LR} (MPa)	G_{LT} (MPa)	G_{LR} (MPa)	G_{LT} (MPa)	G_{LR} (MPa)	G_{LT} (MPa)	G_{LR} (MPa)	G_{LT} (MPa)	G_{LR} (MPa)	G_{LT} (MPa)
$\bar{x}$	210	213	248	249	367	399	425	414	458	466
Cv	15	18	17	17	17	16	18	20	27	28
Min	159	159	181	181	277	251	287	284	306	296
Max	263	284	308	303	509	481	521	542	639	691

$\bar{x}$: sample means, *Cv*: coefficient of variation, *Min*: smallest values, *Max*: largest values.

P-values of Anderson-Darling test for longitudinal modulus of elasticity ranged in interval 0.119 to 0.783. Distributions of all variables investigated ($p > 0.05$) are normal, validating the use of hypothesis test. Table 3 shows results of hypothesis testing between E and G for each wood species, with 21 degrees of freedom. Once P-values were superior to significance level 5% (or zero is present in the confidence intervals found), it is possible to admit there is equivalence between E_{LR} and E_{LT} for all species considered.

Table 3. Results of test hypothesis (E).

Wood Species	Relation	P-value	IC (μ)
Cedrinho	$E_{LT} \times E_{LR}$	0.822	$-996 \leq \mu \leq 1241$
Peroba Rosa	$E_{LT} \times E_{LR}$	0.688	$-1280 \leq \mu \leq 1953$
Tereticornis	$E_{LT} \times E_{LR}$	0.385	$-1777 \leq \mu \leq 714$
Canafístula	$E_{LT} \times E_{LR}$	0.856	$-2195 \leq \mu \leq 1838$
Jatobá	$E_{LT} \times E_{LR}$	0.794	$-1827 \leq \mu \leq 2360$
Cedrinho	$G_{LT} \times G_{LR}$	0.873	$-3{,}24 \leq \mu \leq 27{,}70$
Peroba Rosa	$G_{LT} \times G_{LR}$	0.948	$-37{,}90 \leq \mu \leq 35{,}60$
Tereticornis	$G_{LT} \times G_{LR}$	0.214	$-85{,}60 \leq \mu \leq 20{,}30$
Canafístula	$G_{LT} \times G_{LR}$	0.740	$-57{,}10 \leq \mu \leq 79{,}18$
Jatobá	$G_{LT} \times G_{LR}$	0.879	$-114{,}40 \leq \mu \leq 98{,}50$

For the ratio $E_L\ E_{LT}^{-1}$, P-value from Anderson-Darling normality test was 0.188, validating the adopted hypothesis. P-value and confidence interval for the investigated ratio were 0.752 and $0.9790 \leq \mu \leq 1.0289$, respectively. Thus, equivalence between values of E in directions LR and LT – regardless of wood species – can be assumed.

For the ratio $G_{LR}\ G_{LT}^{-1}$, P-value from Anderson-Darling normality test was 0.871, validating the adopted hypothesis. P-value and confidence interval for the investigated ratio were 0.930 and $0.9545 \leq \mu \leq 1.0416$, respectively. As the case of E, equivalence between values of G in directions LR and LT – regardless of wood species – can be assumed.

Figure 2 shows graph of the affine function ($G = 0.0182 \cdot E + 108.7$) obtained from the linear regression model by least squares method, with coefficient of determination R^2 adjusted and P-value equal to 31.70% and 0.483 (no significant model), respectively. So, it is showed inefficiency and no significance of the estimating shear modulus by longitudinal modulus of elasticity.

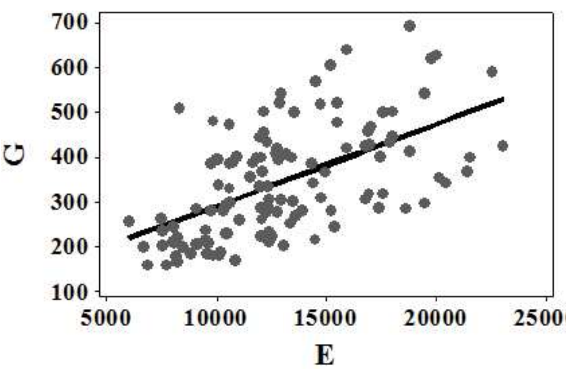

Figure 2. Linear regression: G as function of E.

From least squares model (Equation 7), the coefficient ξ for relation ($E = \xi \cdot G$) was 35. Therefore, $G = E\ 35^{-1}$ differs widely from the ratio proposed by NBR 7190 (ABNT, 1997), *i.e.*, G = /20.

However, as already mentioned, only to Jatobá (*Hymenaea stilbocarpa*) information in literature is available, to compare shear modulus with longitudinal modulus of elasticity, in wood. Mascia and Lahr (2006) estimated in 1/20 this relation. Its mention serves only as an illustrative example, once $G = E\ 35^{-1}$ refers to a mean value of five species and, in these cases, for a single essence differences as this can always be expected.

Conclusion

From foregoing, it concludes that:

- Results of hypothesis tests between modulus of elasticity (E_{LR} and E_{LT}) and shear modulus (G_{LR} and G_{LT}) obtained based on static bending tests showed equivalence by species and for all species simultaneously.
- Linear regression by least squares method shows that it is not possible to estimate shear modulus (G) by the longitudinal modulus of elasticity (E).
- The best coefficient, obtained by least squares model, for the cited relation was 35 ($G = E\ 35^{-1}$), *i.e.*, 75% different than coefficient ($G = E\ 20^{-1}$) prescribed by the Brazilian Code NBR 7190 (ABNT, 1997).

- This situation suggests the need of adjusting coefficient E/ G for adequate design of timber structures.

It's tempestive to signal that the conclusions here commented are only pertinent to tropical wood species.

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