IET Control Theory & Applications

Research Article

Constrained robust model predicted control of discrete-time Markov jump linear systems

ISSN 1751-8644
Received on 13th June 2018
Revised 27th October 2018
Accepted on 3rd January 2019
E-First on 18th February 2019
doi: 10.1049/iet-cta.2018.5543
www.ietdl.org

Rosileide O. Lopes[1,2], *Eduardo M. A. M. Mendes*[2,3], *Leonardo A. B. Torres*[2,3], *Reinaldo M. Palhares*[2,3] ✉

[1]*Federal University of Itajubá, Rua Irmã Ivone Drumond, 200 – Distrito Industrial II, Itabira - MG, 35903-087, Brazil*

[2]*Graduate Program in Electrical Engineering, Federal University of Minas Gerais, Av. Antônio Carlos 6627, Belo Horizonte – MG, 31270-901, Brazil*

[3]*Department of Electronics Engineering, Federal University of Minas Gerais, Av. Antônio Carlos 6627, Belo Horizonte – MG, 31270-901, Brazil*

✉ *E-mail: rpalhares@ufmg.br*

Abstract: This study is concerned with the problem of designing a robust model predictive control (MPC) for a class of uncertain discrete-time Markov jump linear systems. The main contribution is a set of linear matrix inequality (LMI) conditions obtained under new control policies for the unconstrained as well as the constrained MPC when uncertainties are present both in the system's matrices and in the transition probabilities of the modes. For the constrained MPC, hard constraints are considered over the input control and the states and results are extended to the so-called multi-step mode-dependent state-feedback control design. To illustrate the improvements obtained with the new set of LMI conditions, numerical simulations are carried out and compared with a recent reference in the literature.

1 Introduction

Model predictive control (MPC), also known as receding horizon control, is a popular technique used in different kinds of real-world control applications, mainly because of its inherent capacity of handling simultaneously constraints, optimising performance and, depending on the methodology, ensuring stability. The idea behind the MPC is to use a given model of the plant to predict the future output in the context that the input is tailored in such a way that the expected system response attains the desired outcome. See, for instance [1–6] for further discussions.

In the classical MPC formulation, the uncertainties of the parameters and constraints are not taken into account. As a matter of fact, in several real-world applications, exact models may be difficult to obtain either due to model uncertainty, error modelling or external disturbances, which can result in poor control performance. In this context, a robust MPC approach could be considered more appropriate since it deals with constraints and uncertainties in the model to guarantee stability and meet constraint specifications. For more details about robust MPC, see for instance [1, 7, 8]. However, when considering uncertainties on the deterministic MPC, the control inputs obtained may be conservative, reducing system performance. Furthermore, if the worst case uncertainty realisations considered are unlikely to occur, a stochastic MPC approach may be more appropriate because it takes into account probabilistic information about the uncertainty. In the stochastic version of the MPC, sometimes depending on the characteristics of the model, system constraints can be modelled as hard constraints such that no constraint violation is allowed and the constraints must be satisfied in a deterministic way or stochastic constraints may be considered and a partial violation of constraints is allowed (see, e.g. [9–11]). In [8], a MPC problem for a class of linear discrete-time systems subject to saturated inputs and randomly occurring uncertainties is investigated and a non-conservative stochastic MPC algorithm that considers uncertainty and stochastic disturbances is proposed in [12]. In the context of stochasticity, MPC applications can be seen in traffic management [13], data losses [14], in networked and distributed predictive control of non-linear systems [15]. For an overview of the developments in the area of stochastic MPC in the last few years for both linear and non-linear systems, the interested reader is referred to [10]. For a good discussion about the classic, robust and stochastic MPC, see [16].

Markov jump linear systems (MJLSs) constitute a popular class of stochastic systems that represent systems subject to abrupt changes in their structure (see [17, 18] and references therein for real world applications). Such changes can be, for instance, due to environmental disturbances, component failures or repairs, changes in subsystem interconnections or changes in the operation point. The interested reader can see a recent case study presented in [19], where the modelling of a wind turbine generator system driven by the switching wind speed is described as a class of linearised Markov jump controlled systems. MPC for discrete-time MJLSs with disturbances has been treated in [20, 21]. A method to solve the constrained MPC of MJLS problem with noisy inputs and unobservable Markov state is given in [22]. MPC for discrete-time MJLSs without uncertainty but taking into account constraints has been proposed, e.g. in [23, 24]. Considering a more challenging scenario, the constrained MPC for discrete-time MJLSs with uncertainty in both model parameters and transition probabilities has been analysed in [25].

In this study, new relaxed linear matrix inequality (LMI)-based conditions to MPC for discrete-time MJLSs with parameter and transition probability uncertainties for both mode-dependent hard constraints on input control and states as well as the unconstrained case are proposed. As the main discussion in this study is inspired from [25], some results are extended directly from that reference as well as most of the numerical comparisons. The main results proposed here are also extended to the context of multi-step mode-dependent state-feedback control as it was done in [25].

This paper is organised as follows. The constrained MPC problem for MJLSs with uncertainties is formulated in Section 2 following closely to what was laid out in [25]. The new LMIs and the main results are given in Sections 3 and 4. Section 5 presents a numerical example to illustrate the effectiveness of the proposed conditions together with comparisons with results in the literature. Finally, in Section 6, some conclusion remarks are presented.

The notation employed in this paper is the following: for a matrix A, A^{-1} is its inverse (if it exists), A^{T} is its transpose. The matrix inequality $A \succ 0\,(A \succeq 0)$ means that A is both square symmetric and positive definite (semi-definite). $\mathscr{I}$ and 0 represent, respectively, the identity and null matrices with appropriate dimensions. $x(k+n|k)$ and $u(k+n|k)$ are the predicted values of the vector x and u at a future time $k+n$, respectively, based on the information available at time k. $\mathbb{E}_{\bar{k}}[v]$ denotes the expected value of v conditioned on the information available at time $\bar{k}$, and $\mathbb{E}[v]$ is the expected value of v using all the available information.

2 Problem formulation

Consider the uncertain discrete-time MJLS described as

$$x(k+1) = A_{\xi(k)}(\theta(k))x(k) + B_{\xi(k)}(\theta(k))u(k) \tag{1}$$

in which $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$ and $\theta(k)$ are the system state, control input and system mode, respectively; the initial state is x_0, the initial mode is θ_0, and $A_{\xi(k)}(\theta(k))$ and $B_{\xi(k)}(\theta(k))$ are matrices of appropriate dimensions. It is assumed that the mode process $\{\theta(k); k = 0, 1, \dots\}$ is a discrete-time Markov chain taking values in the discrete finite integer set $\mathbb{M} = \{1, \dots, M\}$ with transition probabilities given by $p_k(i, j)$. The transition probability matrix $\mathscr{P}_k$

$$\mathscr{P}_k = [p_k(i, m)]_{i, m \in \mathbb{M}}$$

with

$$p_k(i, m) = \Pr(\theta(k+1) = m | \theta(k) = i) \tag{2}$$

is assumed to be not exactly known but belongs to a polytopic set given by

$$\mathscr{P}_k \in \Omega_{\mathscr{P}} = Co\{\mathscr{P}^1, \mathscr{P}^2, \dots, \mathscr{P}^\Gamma\}, \tag{3}$$

where Co denotes the convex hull, and for $\tau = 1, \dots, \Gamma$, $\mathscr{P}^\tau = [p(i, m; \tau)]_{i, m \in \mathbb{M}}$ are stochastic matrices, i.e. $0 \le p(i, m; \tau) \le 1$, and $\sum_{m=1}^{M} p(i, m; \tau) = 1$, $\forall i \in \{1, 2, \dots, M\}$ and $\forall \tau \in \{1, 2, \dots, \Gamma\}$.

It is also assumed that for $\theta(k) = i$, $i \in \mathbb{M}$, the matrices $A_{\xi(k)}(i)$ and $B_{\xi(k)}(i)$ are affine dependent upon the time-varying parameters $\xi(k) \in \mathbb{S}^L$, with $\mathbb{S}^L = \{\xi \in \mathbb{R}^L;\ \xi_\ell \ge 0, \sum_{\ell=1}^{L} \xi_\ell = 1, \ell = 1, 2, \dots, L\}$, such that

$$A_{\xi(k)}(i) = \sum_{\ell=1}^{L} \xi_\ell(k) A_\ell(i) \quad \text{and} \quad B_{\xi(k)}(i) = \sum_{\ell=1}^{L} \xi_\ell(k) B_\ell(i).$$

In this study, hard constraints on the control input and state are considered in the following way:

$$\left|[u(k+n)(i)]_j\right| \le [\bar{u}(i)]_j, \quad n \ge 0, i \in \mathbb{M}, j = 1 \dots, n_u, \tag{4}$$

$$\left|[\psi]_j x(k+n)\right| \le \bar{x}_j, \quad n \ge 1, i \in \mathbb{M}, j = 1, \dots, n_\psi, \tag{5}$$

where $\psi \in R^{n_\psi \times n_x}$ and $[\,.\,]_j$ denotes the jth row of a given matrix or the jth element of a given vector.

Considering (1)–(5) for the MPC design, the following minimisation problem can be formulated:

$$\begin{aligned} &\min_{u(k+n|k), n \ge 0} \quad \max_{\substack{[A_{\xi(k+n)}(i)\ B_{\xi(k+n)}(i)], \\ \mathscr{P}_{k+n} \in \Omega_{\mathscr{P}}, \xi(k+n) \in \mathbb{S}^L, \\ i = \theta(k+n|k), n \ge 0}} J_\infty(k), \\ &\text{s.t. } (1)-(5). \end{aligned} \tag{6}$$

in which the infinite horizon cost function is written as

$$J_\infty(k) = \mathbb{E}_k\left[\sum_{n=0}^{\infty} \| x(k+n|k) \|^2_{\mathscr{Q}(i)} + \| u(k+n|k) \|^2_{\mathscr{R}(i)}\right], \tag{7}$$

where $\mathscr{Q}(i)$ is the state weighting matrix and $\mathscr{R}(i)$ is the input weighting matrix, for the mode $i = \theta(k+n|k)$. In this MPC 'min–max' problem, the minimisation corresponds to choosing the time-varying plant $[A_{\xi(k+n)}(i)\ B_{\xi(k+n)}(i)]$, for all $i \in \mathbb{M}$, $n \ge 0$ using a prediction model to achieve the largest or 'worst-case' value of $J_\infty(k)$.

Following [25], the definition for stochastic stability in this study is given in terms of the following mean-square stability notion.

Definition 1: The system given in (1) is mean-square stable for a given initial state x_0 and initial mode θ_0 if $\mathbb{E}[x^{\mathrm{T}}(k)x(k)] \to 0$ whenever $k \to \infty$.

2.1 Two control policies

The general structure of the control policies to be adopted in the MPC strategy is similar to the one considered in [25]. At each time step, the control history is defined by the composition of an optimised value to be applied at the current step, followed by a state-feedback control law for the next steps in the prediction horizon

$$u(k+n|k) = \begin{cases} u(k), & n = 0; \\ F_{\xi(k+n)}(\theta(k+n|k))x(k+n|k), & n \ge 1; \end{cases} \tag{8}$$

where $u(k) \in \mathbb{R}^{n_u}$ is computed by solving (6) at each time step.

However, a key difference with respect to [25] is in the choice of feedback matrices, for each mode $i \in \mathbb{M}$, as convex combinations of a finite number L of matrices

$$F_{\xi(k+n)}(i) = \sum_{\ell=1}^{L} \xi_\ell(k+n) F_\ell(i), \quad i = \theta(k+n|k) \in \mathbb{M}; \tag{9}$$

such that the feedback gains $F_\ell(i)$ are optimised together with $u(k)$.

As an alternative – more similar to the one in [25] – it is possible to use a control policy at each time step k given by

$$u(k+n|k) = \begin{cases} u(k), & n = 0; \\ F_{\xi(k+n)}(\theta(k+n|k))x(k+n|k), & \\ \quad 1 \le n \le N-1; & \\ F_{\xi(k+N)}(\theta(k+n|k))x(k+n|k), & \\ \quad n \ge N; & \end{cases} \tag{10}$$

where N is the number of steps in the prediction horizon for which feedback matrices must be optimised to solve problem (6). In this particular case, considering $\theta(k+n|k) = i$, $i \in \mathbb{M}$, each feedback gain matrix $F_{\xi(k+N)}(i)$, for each mode i, optimised at $n = N$, is re-used for $n > N$.

It is noteworthy that, in [25], a control policy similar to (10) is employed, but based on optimising a feedback matrix for each time step during N steps without relying on the idea of having convex combinations of vertex matrices, such that the total number of matrices to be considered is proportional to N (see [25, expression (8)]). Here the number of matrices to be considered is proportional to L in the cases (8) and (9), and proportional to $L\ N$ in the case (10). Numerical results (Section 5) indicate that the two approaches proposed in this study can possibly enlarge the set of feasible solutions to problem (6).

3 New LMI for MPC unconstrained case

In this section, new LMI conditions for the design of an unconstrained MPC for MJLSs with uncertainties in both system matrices and transition probabilities are presented, considering (8)–(9).

The following affine parameter-dependent Lyapunov function [26] is selected:

$$\mathscr{V}(k+n) = \| x(k+n|k) \|^2_{P_{\xi(k+n)}(\theta(k+n|k))}, \quad n \ge 1. \tag{11}$$

For $\theta(k+n|k)=i,\ i\in\mathbb{M}$, the symmetric positive definite weighting matrix $P_{\xi(k+n)}(i)$ is defined by the convex combination

$$P_{\xi(k+n)}(i)=\sum_{\ell=1}^{L}\xi_{\ell}(k+n)P_{\ell}(i)\,. \tag{12}$$

Following [25], the stochastic contractivity constraints are given by

$$\begin{aligned}&\mathscr{V}(k+n)-\mathbb{E}_{k+n}[\mathscr{V}(k+n+1)]\\ &\quad\geq \left\| x(k+n|k)\right\|^{2}_{\mathbb{Q}(i)}+\left\| u(k+n|k)\right\|^{2}_{\mathscr{R}(i)},\ \ n\geq 1,\end{aligned} \tag{13}$$

where $i=\theta(k+n|k)$ will be enforced to obtain an upper bound on the cost function (7). The new LMI conditions to check if the stochastic contractivity constraints in (13) are satisfied are presented in the next Lemma.

Lemma 1: The stochastic contractivity constraints in (13) are satisfied if there exist matrices $G_{\ell}(i)$, $Y_{\ell}(i)$, $W_{h}(i)$, $Z_{h}(i)$, and symmetric matrices $Q_{\ell}(i)>0$; for all $\ell,h=1,\ldots,L$, $\tau=1,\ldots,\Gamma$ and $i\in\mathbb{M}_{+}^{j,\tau}$, with $\mathbb{M}_{+}^{j,\tau}=\{i\in\mathbb{M};\ \ p(i,j;\tau)>0\}$, such that the following LMIs are feasible:

$$\begin{bmatrix} G_{\ell}^{\mathrm{T}}(i)+G_{\ell}(i)-Q_{\ell}(i) & * & * & \cdots & * & * & * & * \\ A_{\ell}(i)G_{\ell}(i) & \hat{Q}_{\ell h}(1) & * & \cdots & * & * & * & * \\ A_{\ell}(i)G_{\ell}(i) & -R_{\ell h}(i) & \hat{Q}_{\ell h}(2) & \cdots & * & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ A_{\ell}(i)G_{\ell}(i) & -R_{\ell h}(i) & -R_{\ell h}(i) & \cdots & \hat{Q}_{\ell h}(M) & * & * & * \\ \mathbb{Q}^{\frac{1}{2}}(i)G_{\ell}(i) & 0 & 0 & \cdots & 0 & \mathscr{I} & * & * \\ \mathscr{R}^{\frac{1}{2}}(i)Y_{\ell}(i) & 0 & 0 & \cdots & 0 & 0 & \mathscr{I} & * \\ -Y_{\ell}(i) & \hat{Z}_{\ell h}(i) & \hat{Z}_{\ell h}(i) & \cdots & \hat{Z}_{\ell h}(i) & 0 & 0 & Z_{h}^{\mathrm{T}}(i)+Z_{h}(i) \end{bmatrix}\succ 0, \tag{14}$$

where

$$\hat{Q}_{\ell h}(j)=\frac{1}{p(i,j;\tau)}Q_{h}(j)-R_{\ell h}(i),\quad j=1,2,\ldots,M,$$

$$\hat{Z}_{\ell h}(i)=Z_{h}^{\mathrm{T}}(i)B_{\ell}^{\mathrm{T}}(i)-W_{h}(i),$$

$$R_{\ell h}(i)=B_{\ell}(i)W_{h}(i)+W_{h}^{\mathrm{T}}(i)B_{\ell}^{\mathrm{T}}(i)\,.$$

Proof: For $n\geq 1$, $\theta(k+n|k)=i$, suppose that the LMIs in (14) are feasible for some symmetric positive definite matrices $Q_{\ell}(i)$ and matrices $G_{\ell}(i)$, $Y_{\ell}(i)$ of appropriate dimensions. Therefore, it is possible to conclude that $G_{\ell}^{\mathrm{T}}(i)+G_{\ell}(i)-Q_{\ell}(i)>0$. Since

$$\begin{aligned}&[G_{\ell}(i)-Q_{\ell}(i)]^{\mathrm{T}}Q_{\ell}^{-1}(i)[G_{\ell}(i)-Q_{\ell}(i)]\\ &\quad=G_{\ell}^{\mathrm{T}}(i)Q_{\ell}^{-1}(i)G_{\ell}(i)-G_{\ell}^{\mathrm{T}}(i)-G_{\ell}(i)+Q_{\ell}(i)\geq 0,\end{aligned}$$

it can be concluded that $G_{\ell}^{\mathrm{T}}(i)Q_{\ell}^{-1}(i)G_{\ell}(i)\geq G_{\ell}^{\mathrm{T}}(i)+G_{\ell}(i)-Q_{\ell}(i)>0$ and the LMIs in (14) imply that

$$\begin{bmatrix} G_{\ell}^{\mathrm{T}}(i)Q_{\ell}^{-1}(i)G_{\ell}(i) & * & * & \cdots & * & * & * & * \\ A_{\ell}(i)G_{\ell}(i) & \hat{Q}_{\ell h}(1) & * & \cdots & * & * & * & * \\ A_{\ell}(i)G_{\ell}(i) & -R_{\ell h}(i) & \hat{Q}_{\ell h}(2) & \cdots & * & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ A_{\ell}(i)G_{\ell}(i) & -R_{\ell h}(i) & -R_{\ell h}(i) & \cdots & \hat{Q}_{\ell h}(M) & * & * & * \\ \mathbb{Q}^{\frac{1}{2}}(i)G_{\ell}(i) & 0 & 0 & \cdots & 0 & \mathscr{I} & * & * \\ \mathscr{R}^{\frac{1}{2}}(i)Y_{\ell}(i) & 0 & 0 & \cdots & 0 & 0 & \mathscr{I} & * \\ -Y_{\ell}(i) & \hat{Z}_{\ell h}(i) & \hat{Z}_{\ell h}(i) & \cdots & \hat{Z}_{\ell h}(i) & 0 & 0 & Z_{h}^{\mathrm{T}}(i)+Z_{h}(i) \end{bmatrix}\succ 0. \tag{15}$$

Now, multiplying (15) by

$$\begin{bmatrix} G_{\ell}^{-T}(i) & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & \mathscr{I} & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & \mathscr{I} & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \mathscr{I} & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & \mathscr{I} & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & \mathscr{I} \end{bmatrix}$$

on the left and by its transpose on the right yields

$$\mathscr{M}_{\ell h}(i,\tau)=\begin{bmatrix} Q_{\ell}^{-1}(i) & * & * & \cdots & * & * & * & * \\ A_{\ell}(i) & \hat{Q}_{\ell h}(1) & * & \cdots & * & * & * & * \\ A_{\ell}(i) & -R_{\ell h}(i) & \hat{Q}_{\ell h}(2) & \cdots & * & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ A_{\ell}(i) & -R_{\ell h}(i) & -R_{\ell h}(i) & \cdots & \hat{Q}_{\ell h}(M) & * & * & * \\ \mathbb{Q}^{\frac{1}{2}}(i) & 0 & 0 & \cdots & 0 & \mathscr{I} & * & * \\ \mathscr{R}^{\frac{1}{2}}(i)F_{\ell}(i) & 0 & 0 & \cdots & 0 & 0 & \mathscr{I} & * \\ -F_{\ell}(i) & \hat{Z}_{\ell h}(i) & \hat{Z}_{\ell h}(i) & \cdots & \hat{Z}_{\ell h}(i) & 0 & 0 & Z_{h}^{\mathrm{T}}(i)+Z_{h}(i) \end{bmatrix}\succ 0, \tag{16}$$

where $F_{\ell}(i)=Y_{\ell}(i)G_{\ell}^{-1}(i)$.

Taking the convex combination of (16) over ℓ associated with time $k+n$, and over h associated with time $k+n+1$; i.e. computing the matrix

$$\mathscr{M}(i,\tau)=\sum_{h=1}^{L}\xi_{h}(k+n+1)\left[\sum_{\ell=1}^{L}\xi_{\ell}(k+n)\mathscr{M}_{\ell h}(i,\tau)\right];$$

since both $\xi(k+n+1), \xi(k+n) \in \mathbb{S}^L$ and $M_{\ell h}(i,\tau) \succ 0$, one has that

$$\mathscr{M}(i,\tau) = \begin{bmatrix} Q^{-1}_{\xi(k+n)}(i) & * & * & \dots & * & * & * & * \\ A_{\xi(k+n)}(i) & \hat{Q}(i;1) & * & \dots & * & * & * & * \\ A_{\xi(k+n)}(i) & -R(i) & \hat{Q}(i;2) & \dots & * & * & * & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{\xi(k+n)}(i) & -R(i) & -R(i) & \dots & \hat{Q}(i;M) & * & * & * \\ \mathscr{Q}^{\frac{1}{2}}(i) & 0 & 0 & \dots & 0 & \mathscr{I} & * & * \\ \mathscr{R}^{\frac{1}{2}}(i)F_{\xi(k+n)}(i) & 0 & 0 & \dots & 0 & 0 & \mathscr{I} & * \\ -F_{\xi(k+n)}(i) & \hat{Z}(i) & \hat{Z}(i) & \dots & \hat{Z}(i) & 0 & 0 & Z^{\mathrm{T}}_{\xi(k+n+1)}(i) + Z_{\xi(k+n+1)}(i) \end{bmatrix} \succ 0, \tag{17}$$

where

$$\hat{Q}(i;j) = \frac{1}{p(i,j;\tau)} Q_{\xi(k+n+1)}(j) - R(i), \quad j = 1,2,\dots,M,$$

$$\hat{Z}(i) = Z^{\mathrm{T}}_{\xi(k+n+1)}(i) B^{\mathrm{T}}_{\xi(k+n)}(i) - W_{\xi(k+n+1)}(i),$$

and

$$R(i) = B_{\xi(k+n)}(i) W_{\xi(k+n+1)}(i) + W^{\mathrm{T}}_{\xi(k+n+1)}(i) B^{\mathrm{T}}_{\xi(k+n)}(i).$$

Multiplying (17) by

$$\begin{bmatrix} \mathscr{I} & 0 & \dots & 0 & 0 & 0 \\ 0 & \mathscr{I} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \mathscr{I} & 0 & 0 \\ 0 & 0 & \dots & 0 & \mathscr{I} & 0 \\ 0 & 0 & \dots & 0 & 0 & \mathscr{I} \\ 0 & -B^{\mathrm{T}}_{\xi(k+n)}(i) & \dots & -B^{\mathrm{T}}_{\xi(k+n)}(i) & 0 & 0 \end{bmatrix}$$

on the right and by its transpose on the left results in

$$\begin{bmatrix} Q^{-1}_{\xi(k+n)}(i) & * & \dots & * & * & * \\ A^e_{\xi(k+n)}(i) & p^{-1}(i,1;\tau) Q_{\xi(k+n+1)}(1) & \dots & * & * & * \\ A^e_{\xi(k+n)}(i) & 0 & \dots & * & * & * \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ A^e_{\xi(k+n)}(i) & 0 & \ddots & p^{-1}(i,M;\tau) Q_{\xi(k+n+1)}(M) & * & * \\ \mathscr{Q}^{\frac{1}{2}}(i) & 0 & \dots & 0 & \mathscr{I} & * \\ \mathscr{R}^{\frac{1}{2}}(i)F_{\xi(k+n)}(i) & 0 & \dots & 0 & 0 & \mathscr{I} \end{bmatrix} \succ 0, \tag{18}$$

where

$$A^e_{\xi(k+n)}(i) = A_{\xi(k+n)}(i) + B_{\xi(k+n)}(i) F_{\xi(k+n)}(i).$$

Let $P_{\xi(k+n)}(i) = Q^{-1}_{\xi(k+n)}(i)$, for all $n \ge 1$. By Schur complement, matrix inequality (18) is equivalent to the following quadratic Lyapunov-like inequality:

$$\begin{aligned} & P_{\xi(k+n)}(i) \\ & -\left(A^e_{\xi(k+n)}(i)\right)^{\mathrm{T}} \left[\sum_{j=1}^{M} p(i,j) P_{\xi(k+n+1)}(j)\right] A^e_{\xi(k+n)}(i) \\ & -\left[\mathscr{Q}(i) + F^{\mathrm{T}}_{\xi(k+n)}(i) \mathscr{R}(i) F_{\xi(k+n)}(i)\right] \ge 0, \end{aligned}$$

which is equivalent to

$$\begin{aligned} & P_{\xi(k+n)}(i) - \left(A^e_{\xi(k+n)}(i)\right)^{\mathrm{T}} \times \\ & \quad \mathbb{E}_{k+n}\left[P_{\xi(k+n+1)}(\theta(k+n+1|k))\right] A^e_{\xi(k+n)}(i) \\ & \quad -\left[\mathscr{Q}(i) + F^{\mathrm{T}}_{\xi(k+n)}(i) \mathscr{R}(i) F_{\xi(k+n)}(i)\right] \ge 0. \end{aligned}$$

Finally, by multiplying this last expression by $x^{\mathrm{T}}(k+n|k)$ on the left and by $x(k+n|k)$ on the right, one obtains (13). □

Now, following the same ideas as in [25, Section 3], an upper bound on the worst-case infinite horizon expected cost function in (7) can be obtained, namely $J_\infty(k) \le \| x(k) \|^2_{\mathscr{Q}(i)} + \| u(k) \|^2_{\mathscr{R}(i)} + \mathbb{E}_k[\mathscr{V}(k+1)] \le \gamma_1 + \gamma_2$, with $\| x(k) \|^2_{\mathscr{Q}(i)} + \| u(k) \|^2_{\mathscr{R}(i)} \le \gamma_1$ and $\mathbb{E}_k[\mathscr{V}(k+1)] \le \gamma_2$. In other words, an upper bound is found if the following minimisation problem is feasible at each time instant k

$$\begin{aligned} \min_{\gamma_1, \gamma_2, u(k), G_\ell(i), Q_\ell(i), Y_\ell(i)} \quad & \gamma_1 + \gamma_2 \\ \text{s.t.} \quad & (14), (20) \text{ and } (21) \end{aligned} \tag{19}$$

with γ_1 and γ_2 satisfying the following LMIs:

$$\begin{bmatrix} \gamma_1 & * & * \\ x(k) & \mathscr{Q}^{-1}(i) & * \\ u(k) & 0 & \mathscr{R}^{-1}(i) \end{bmatrix} \ge 0, \tag{20}$$

$$
\begin{bmatrix}
\gamma_2 & * & \cdots & * \\
A_\ell(i)x(k) + B_\ell(i)u(k) & p^{-1}(i,1;\tau)Q_\ell(1) & \cdots & * \\
\vdots & \vdots & \ddots & \vdots \\
A_\ell(i)x(k) + B_\ell(i)u(k) & 0 & \cdots & p^{-1}(i,M;\tau)Q_\ell(M)
\end{bmatrix} \geq 0, \tag{21}
$$

for all $\ell = 1, \ldots, L$, $\tau = 1, \ldots, \Gamma$ and for all . Constraints (20) and (21) are the same as those in (15) and (16) in [25], respectively. For $i \in \mathbb{M}_+^{j,\tau}$, the matrices $Q_1(i)$ in (16) in [25] were replaced in (21) by $Q_\ell(i)$ for all $\ell = 1, \ldots, L$.

The closed loop stability condition under the MPC controller design obtained via (19) is stated in the following theorem.

Theorem 1: Consider the unconstrained uncertain MJLS described in (1)–(3). If there exists a feasible solution to the optimisation problem (19) at instant k for initial state $x(k)$ and initial mode $i = \theta(k)$, there also exists a feasible solution at any instant $n + k \geq k$; and the MPC controller based on (19) guarantees the stability of the closed-loop system in the mean-square sense.

The proof of Theorem 1 follows in a similar way the one in [25, Theorem 1], and therefore it is omitted here.

4 New LMIs for MPC constrained case

In this section, a new set of LMIs with respect to the ones used in [25, Section 4] are proposed, in the context of the constrained MPC design for the MJLS (1)–(3), while considering (4) and (5). Specifically, significant improvements can be achieved, as is shown in Section 5, by considering (28) instead of inequalities [25, expression (21)]. Unless stated otherwise, the same steps from [25, Section 4] are followed below.

Consider the following Lyapunov function:

$$
\bar{\mathscr{V}}(k+n) = \| x(k+n|k) \|^2_{\mathscr{W}^{-1}_{\xi(k+n)}(\theta(k+n|k))}, \quad n \geq 1. \tag{22}
$$

For $\theta(k+n|k) = i$, $i \in \mathbb{M}$, $\mathscr{W}^{-1}_{\xi(k+n)}(i)$ are symmetric positive weighting matrices given by the convex combination $\mathscr{W}^{-1}_{\xi(k+n)}(i) = \sum_{\ell=1}^{L} \xi_\ell(k+n)\mathscr{W}^{-1}_\ell(i)$, such that $\mathscr{W}^{-1}_\ell(i)$ are determined by solving the optimisation problem associated with the MPC.

Furthermore, the following constraints on $\bar{\mathscr{V}}(k+n)$ are important to guarantee recursive feasibility as shown in [25]:

$$
\bar{\mathscr{V}}(k+1) \leq 1; \tag{23}
$$

$$
\bar{\mathscr{V}}(k+n+1) \leq \bar{\mathscr{V}}(k+n), \quad n \geq 1. \tag{24}
$$

Note also that inequality (5), for $n = 1$, is equivalent to

$$
\begin{gathered}
-\bar{x}_j \leq \psi_j[A_\ell(i)x(k) + B_\ell(i)u(k)] \leq \bar{x}_j, \\
j = 1, \ldots, n_\psi, \quad \ell = 1, \ldots, L, \quad i \in \mathbb{M};
\end{gathered} \tag{25}
$$

and (4), for $n = 0$, can be written as

$$
\begin{gathered}
-[\bar{u}(k)(i)]_j \leq [u(k)(i)]_j \leq [\bar{u}(k)(i)]_j, \\
j = 1, \ldots, n_u, \quad i \in \mathbb{M}.
\end{gathered} \tag{26}
$$

The next Lemma presents LMI-based conditions that ensure constraints (23) and (24) hold.

Lemma 2: The hard constraints on inputs given in (4) for $n \geq 1$, and on the states given in (5) for $n \geq 2$, together with inequalities (23) and (24) are satisfied if inequality (25) is true and there exist symmetric matrices $\mathscr{W}_\ell(i)$, $\mathscr{U}_\ell(i)$, $\mathscr{X}_\ell(i)$, $G_\ell(i)$, $Y_\ell(i)$, $Z_h(i)$ and $W_h(i)$, with $i \in \mathbb{M}_+^{j,\tau} = \{i \in \mathbb{M};\ p(i,j;\tau) > 0\}$, $j \in \mathbb{M}$ and $\ell, h = 1, \ldots, L$; such that the following LMIs are feasible:

$$
\begin{bmatrix}
1 & * \\
A_\ell(i)x(k) + B_\ell(i)u(k) & \mathscr{W}_\ell(i)
\end{bmatrix} \geq 0, \tag{27}
$$

$$
\begin{bmatrix}
G_\ell^{\mathrm{T}}(i) + G_\ell(i) - \mathscr{W}_\ell(i) & * & * \\
A_\ell(i)G_\ell(i) & \mathscr{W}_h(j) - R_{\ell h}(i) & * \\
-Y_\ell(i) & Z_h{}^{\mathrm{T}}B_\ell^{\mathrm{T}}(i) - W_h(i) & Z_h(i) + Z_h^{\mathrm{T}}(i)
\end{bmatrix} \succ 0, \tag{28}
$$

$$
\begin{cases}
\begin{bmatrix}
\mathscr{U}_\ell(i) & Y_\ell(i) \\
* & G_\ell^{\mathrm{T}}(i) + G_\ell(i) - \mathscr{W}_\ell(i)
\end{bmatrix} \succ 0, \\
[\mathscr{U}_\ell(i)]_{jj} \leq [\bar{u}(i)]_j^2,
\end{cases} \tag{29}
$$

$$
\begin{cases}
\begin{bmatrix}
\mathscr{X}_\ell(i) & \psi\mathscr{W}_\ell(i) \\
* & \mathscr{W}_\ell(i)
\end{bmatrix} \geq 0, \\
[\mathscr{X}_\ell(i)]_{jj} \leq \bar{x}_j^2,
\end{cases} \tag{30}
$$

where $[\,.\,]_{jj}$ denotes the jth diagonal element of the corresponding matrices and

$$
R_{\ell h}(i) = B_\ell(i)W_h(i) + W_h^{\mathrm{T}}(i)B_\ell^{\mathrm{T}}(i).
$$

Proof: By Schur complement and assuming that (5) is satisfied for $n = 1$; i.e. (25) is satisfied, then inequality (23) is a consequence of inequalities (27). Constraints (4) for $n \geq 1$ are guaranteed by (29), and constraints (5) for $n \geq 2$ are guaranteed by (30) (see [25] for further details). Thus, it remains to prove that the feasibility of the LMIs in (28) guarantees that the constraint (24) is verified.

Assume that (28) holds for some symmetric positive definite matrices $\mathscr{W}_\ell(i)$ and matrices $G_\ell(i)$, $Y_\ell(i)$ of appropriate dimensions. Similar to Lemma 1, it can be concluded that $G_\ell^{\mathrm{T}}(i)\mathscr{W}_\ell^{-1}(i)G_\ell(i) \geq G_\ell^{\mathrm{T}}(i) + G_\ell(i) - \mathscr{W}_\ell(i) > 0$ and (28) implies that

$$
\begin{bmatrix}
G_\ell(i)\mathscr{W}_\ell^{-1}(i)G_\ell^{\mathrm{T}}(i) & * & * \\
A_\ell(i)G_\ell(i) & \mathscr{W}_h(j) - R_{\ell h}(i) & * \\
-Y_\ell(i) & Z_h{}^{\mathrm{T}}B_\ell^{\mathrm{T}}(i) - \mathscr{W}_h(i) & Z_h(i) + Z_h^{\mathrm{T}}(i)
\end{bmatrix} \succ 0
$$

multiplying the last inequality by

$$
\begin{bmatrix}
G_\ell^{-T}(i) & 0 & 0 \\
0 & \mathscr{I} & 0 \\
0 & 0 & \mathscr{I}
\end{bmatrix}
$$

on the left and by its transpose on the right, one obtains

$$
\begin{bmatrix}
\mathscr{W}_\ell^{-1}(i) & * & * \\
A_\ell(i) & \mathscr{W}_h(j) - R_{\ell h}(i) & * \\
-F_\ell(i) & Z_h{}^{\mathrm{T}}B_\ell^{\mathrm{T}}(i) - W_h(i) & Z_h(i) + Z_h^{\mathrm{T}}(i)
\end{bmatrix} \succ 0, \tag{31}
$$

where $F_\ell(i) = G_\ell^{-1}(i)Y_\ell(i)$. Taking the convex combination of (31) over ℓ and h, and associating them with $\xi(k+n)$ and $\xi(k+n+1)$, respectively, as it was done in the proof of Lemma 1, gives

$$\begin{bmatrix} \mathscr{W}^{-1}_{\xi(k+n)}(i) & * & \cdots & * \\ A_{\xi(k+n)}(i) & \mathscr{W}_{\xi(k+n+1)}(j) - R(i) & \cdots & * \\ -F_{\xi(k+n)}(i) & \begin{pmatrix} Z^{\mathrm{T}}_{\xi(k+n+1)}(i)B^{\mathrm{T}}_{\xi(k+n)}(i) \\ -W_{\xi(k+n+1)}(i) \end{pmatrix} & \cdots & Z_{\xi(k+n+1)}(i) + Z^{\mathrm{T}}_{\xi(k+n+1)}(i) \end{bmatrix} \succ 0, \tag{32}$$

where

$$R(i) = B_{\xi(k+n)}(i)W_{\xi(k+n+1)}(i) + W^{\mathrm{T}}_{\xi(k+n+1)}(i)B^{\mathrm{T}}_{\xi(k+n)}(i).$$

Multiplying (32) by

$$\begin{bmatrix} \mathscr{I} & 0 \\ 0 & \mathscr{I} \\ 0 & -B^{\mathrm{T}}_{\xi(k+n)}(i) \end{bmatrix}$$

on the right and by its transpose on the left, it follows that

$$\begin{bmatrix} \mathscr{W}^{-1}_{\xi(k+n)}(i) & * \\ \begin{pmatrix} A_{\xi(k+n)}(i) \\ +B_{\xi(k+n)}(i)F_{\xi(k+n)}(i) \end{pmatrix} & \mathscr{W}_{\xi(k+n+1)}(j) \end{bmatrix} \succ 0.$$

Using the Schur complement, this last expression implies that

$$\begin{aligned} x^{\mathrm{T}}(k+n|k)\big\{&\mathscr{W}^{-1}_{\xi(k+n)}(i) - \big[A_{\xi(k+n)}(i) \\ &+B_{\xi(k+n)}(i)F_{\xi(k+n)}(i)\big]^{\mathrm{T}} \times \mathscr{W}^{-1}_{\xi(k+n+1)}(j) \\ &\times \big[A_{\xi(k+n)}(i) + B_{\xi(k+n)}(i)F_{\xi(k+n)}(i)\big]\big\}x(k+n|k) \geq 0, \end{aligned}$$

which corresponds to (24). □

The constrained MPC design can be solved as a minimisation problem and can be seen as an extension of (19) with the inclusion of Lemma 1 to ensure the stochastic contractivity constraints in (13). In addition, to guarantee the satisfaction of the input constraint at $n = 0$, and of the state constraint at $n = 1$, by properly choosing $u(k)$, it is necessary to explicitly consider (25) and (26). Finally, the MPC controller is then obtained if the following optimisation problem is feasible:

$$\begin{aligned} &\min_{\substack{\gamma_1, \gamma_2, u(k), G_\ell(i), Q_\ell(i), \\ Y_\ell(i), W_\ell(i), \mathscr{U}(i), \mathscr{X}(i)}} \gamma_1 + \gamma_2 \\ &\text{s.t. } (14), (20), (21), (25) - (30). \end{aligned} \tag{33}$$

The closed-loop stability condition under the constrained MPC controller obtained via (33) is stated in the following theorem.

Theorem 2: Consider the constrained uncertain MJLS described in (1)–(5). If there exists a feasible solution to the optimisation problem (33) at time instant k for initial state $x(k)$ and initial mode $i \in \mathbb{M}$, there also exists a feasible solution at any time instant $t \geq k$; and the MPC controller based on (33) guarantees the stability of the closed-loop system in the mean-square sense.

The proof of Theorem 2 follows similar steps as Theorem 2 in [25] and it is omitted.

4.1 Extension to the multi-step MPC case

Theorems 1 and 2 in this study can be adapted to encompass the multi-step mode-dependent state-feedback control law case (10) similar to the one investigated in [25].

A possible strategy to derive sufficient conditions to solve problem (6) in this case is simply to rewrite some variables in a proper way in Lemmas 1 and 2. Indeed, it is enough to replace, on these lemmas, $Y_\ell(i)$ by $Y^n_\ell(i)$, $Q_\ell(i)$ by $Q^n_\ell(i)$ and $G_\ell(i)$ by $G^n_\ell(i)$, for all $\ell = 1, \dots, L$, $n = 1, \dots, N$ and $i \in \mathbb{M}^{j,\tau}_+$. Furthermore, in LMI (14) $\hat{Q}_{\ell h}(\alpha)$ is replaced by $\hat{Q}^{n+1}_{\ell h}(\alpha) = p^{-1}(i, \alpha; \tau)Q^{n+1}_h(\alpha) - R_{\ell h}(i)$, and in LMI (27) $\mathscr{W}_\ell(i)$ is replaced by $\mathscr{W}^1_\ell(i)$.

Additionally, the Lyapunov matrices $P_{\xi(k+n)}(i)$ in (11) are replaced by

$$P^n_{\xi(k+n)}(i) = \sum_{\ell=1}^{L} \xi_\ell(k+n)P^n_\ell(i), \ \forall i \in \mathbb{M}, \ \forall n \geq 1,$$

where, for $n \geq N$, $P^n_{\xi(k+n)}(i) = P^N_{\xi(k+N)}(i)$. Also, the same is done for the Lyapunov matrices $\mathscr{W}_{\xi(k+n)}(i)$ in (22), which are replaced by

$$\mathscr{W}^n_{\xi(k+n)}(i) = \sum_{\ell=1}^{L} \xi_\ell(k+n)\mathscr{W}^n_\ell(i), \ \forall i \in \mathbb{M}, \ \forall n \geq 1,$$

where, for $n = N$, $j \in \mathbb{M}$, $\mathscr{W}^n_{\xi(k+n)}(i) = \mathscr{W}^N_{\xi(k+N)}(i)$.

Taking these modifications with respect to n into account, new possibly less conservative LMI based conditions for multi-step mode-dependent state-feedback control can be derived just in the same way as it was done in [25], with the possible disadvantage of having to optimise a number of feedback matrices L times greater.

5 Numerical examples

This section presents three examples to illustrate the effectiveness of the proposed results. In all the examples, the following discrete-time MJLS borrowed from [25] with three operation modes is considered.

For the mode $i = 1$, the system matrices are

$$A_1(1) = \begin{bmatrix} 0 & 1 \\ -2.6 & 3.3 \end{bmatrix}, \quad A_2(1) = \begin{bmatrix} 0 & 1 \\ -2.4 & 3.1 \end{bmatrix},$$

$$B_1(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

For the mode $i = 2$, the matrices are

$$A_1(2) = \begin{bmatrix} 0 & 1 \\ -4.4 & 4.6 \end{bmatrix}, \quad A_2(2) = \begin{bmatrix} 0 & 1 \\ -4.2 & 4.6 \end{bmatrix},$$

$$B_1(2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2(2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Also, finally for the mode $i = 3$, the system matrices are

$$A_1(3) = \begin{bmatrix} 0 & 1 \\ 5.4 & -5.3 \end{bmatrix}, \quad A_2(3) = \begin{bmatrix} 0 & 1 \\ 5.2 & -5.1 \end{bmatrix},$$

$$B_1(3) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2(3) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The vertices of the transition probability matrix are

$$\mathscr{P}^1 = \begin{bmatrix} 0.55 & 0.23 & 0.22 \\ 0.36 & 0.35 & 0.29 \\ 0.32 & 0.16 & 0.52 \end{bmatrix}, \quad \mathscr{P}^2 = \begin{bmatrix} 0.79 & 0.11 & 0.10 \\ 0.27 & 0.53 & 0.20 \\ 0.23 & 0.07 & 0.70 \end{bmatrix}.$$

For more details, the reader is referred to [25].

5.1 Example 1

The first example follows directly from the constrained one-step ($N = 1$) MPC controller (controller III) in the example given in

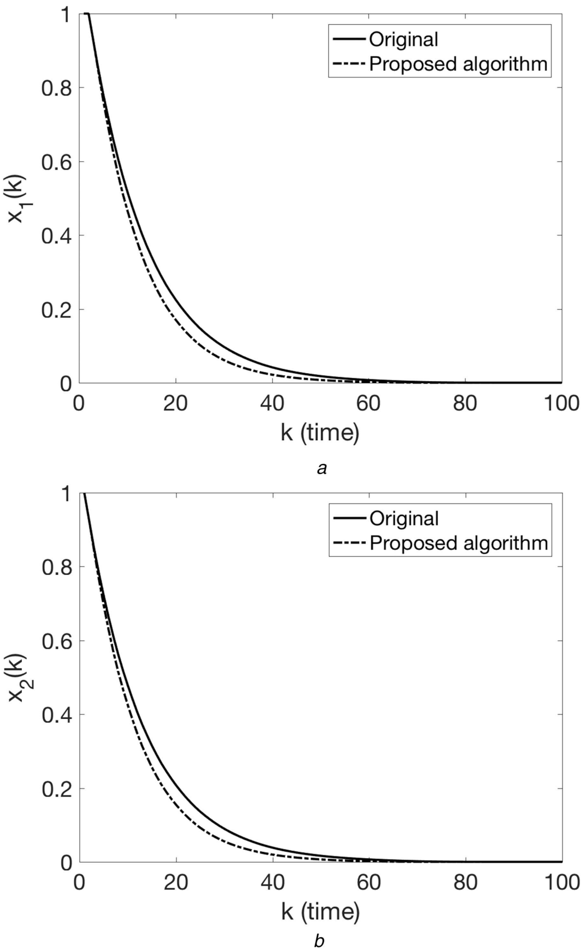

Fig. 1 *State response*
(a) $x_1(k)$ and, ***(b)*** $x_2(k)$. Solid line indicates the average state response using controller III in [25] and dash-dotted line indicates the average state response using (33)

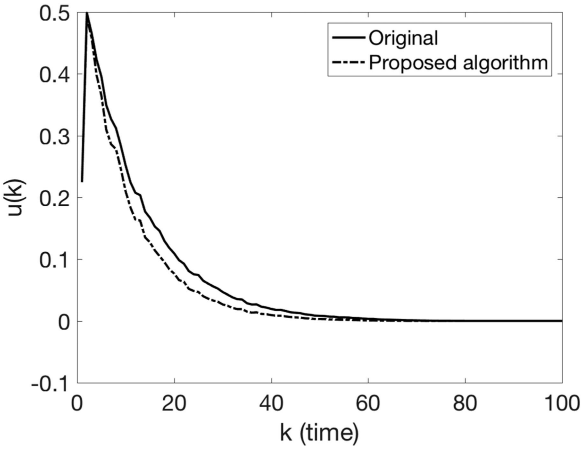

Fig. 2 *Control input: the solid line indicates the average control input using controller III in [25] and the dash-dotted line indicates the average control input using (33)*

Table 1 Average control cost over 250 realisations for one-step $N = 1$

[25] – Controller III	Proposed MPC controller
0.1744	0.1599

[25]. In this particular case, the maximum input is set to 1; i.e. $\bar{u}(i) = 1$, $i \in \mathbb{M}$; the initial state is $x_0 = [1 \quad 1]^{\mathrm{T}}$, the initial mode is $\theta_0 = 1$ and 250 possible realisations of the Markov chain were considered.

Fig. 1 depicts the average state response of the MJLS under controller III designed using the approach proposed in [25] (henceforth called *original*) and using the control sequence obtained via the optimisation problem in (33) in the MPC approach. Note that a faster state response was obtained using the proposed controller. The average control input is shown in Fig. 2.

To further illustrate that the new controller achieves better results, the control cost for both controllers was calculated and is shown in Table 1. [Note that the results presented here are slightly higher than the ones in [25] due to the longer simulation, 100 instead of 80, and to the larger number of realisations, 250 instead of 100.]

The proposed controller obtained by the minimisation problem (33) was tested in many other examples adapted from [25] and the performance was, at least, equal to the performance of the controller in [25]. For instance, if $N = 3$ is chosen the results using controller IV of [25] and the one obtained using the multi-step MPC extension to (33), as discussed in section 4.1, are quite similar with a greater advantage for the first as far as computational cost is concerned.

5.2 Example 2

In this example, only a small change in the control matrix, B, of one of the modes from the previous example is made, namely

$$B_1(2) = \begin{bmatrix} 0.4 \\ 1 \end{bmatrix}$$

is considered instead of

$$B_1(2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

for one step ($N = 1$). In this new situation, the maximal feasible region calculated using the proposed control obtained by (33) in the MPC approach is larger than the region calculated using controller III given in [25] (i.e. using the *original*) as can be seen in Fig. 3.

It is worth mentioning that using different solvers such as lmilab, mosek [28] and sdpt3 makes no difference as far as finding the feasible region is concerned.

In case the number of steps ahead is set to $N = 3$, the maximal region calculated using the proposed multi-step MPC extension to (33), as discussed in section 4.1, is again larger than the one using controller IV in [25] as it can been in Fig. 4. Note that the maximal feasible region becomes larger for both controllers as N increases.

5.3 Example 3

In this example, the input matrix is again modified to show that the proposed control strategy can still find a solution when the one in [25] cannot. Fig. 5 depicts the maximal feasible region for both methods in the case of one-step ($N = 1$). The region found using the method in [25] is a small region around the origin.

The modified input matrices used in this example are

$$B_1(1) = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, \quad B_2(1) = \begin{bmatrix} -0.1 \\ 0.9 \end{bmatrix},$$

$$B_1(2) = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}, \quad B_2(2) = \begin{bmatrix} -0.1 \\ 1 \end{bmatrix},$$

$$B_1(3) = \begin{bmatrix} 0.2 \\ 0.85 \end{bmatrix}, \quad B_2(3) = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}.$$

In order to check if the maximal feasible region becomes larger with the increase in the number of steps ahead, namely $N = 3$, using the proposed multi-step MPC extension to (33), as discussed in Section 4.1. Figure 6 shows an increase in the maximal feasible region for the proposed approach in this study. Also, although an increase in the region obtained for controller IV in [25] could be

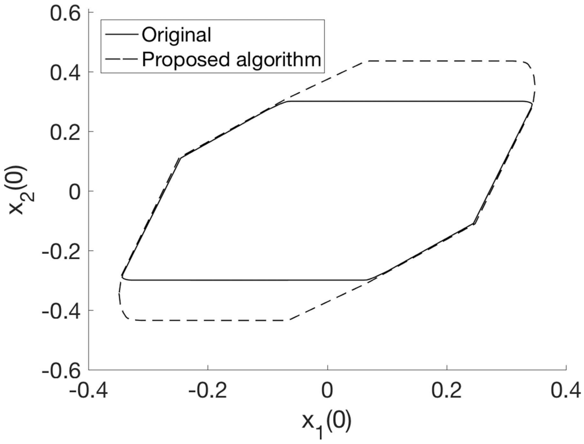

Fig. 3 *Maximal feasible region, input constrained problem:* $\bar{u}(i) = 1$, $\forall i \in \mathbb{M}$, *and* $N = 1$. *The parser Yalmip [27] and the solver sedumi were used to plot the region*

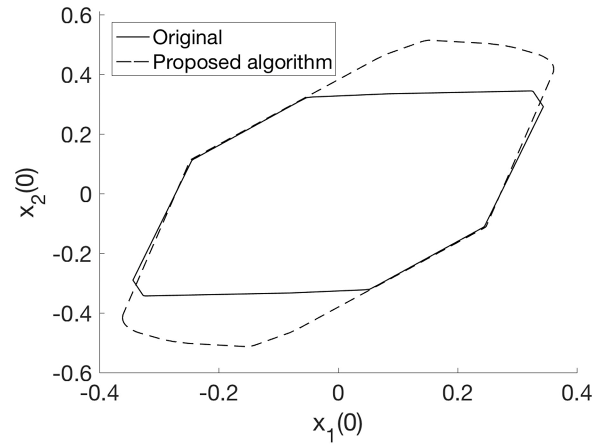

Fig. 4 *Maximal feasible region, input constrained problem:* $\bar{u}(i) = 1$, $\forall i \in \mathbb{M}$, *and the extension multi-step* $N = 3$ *in minimisation problem (33), as discussed in Section 4.1, and controller IV in [25]. The parser Yalmip [27] and solver sedumi were used to plot the region*

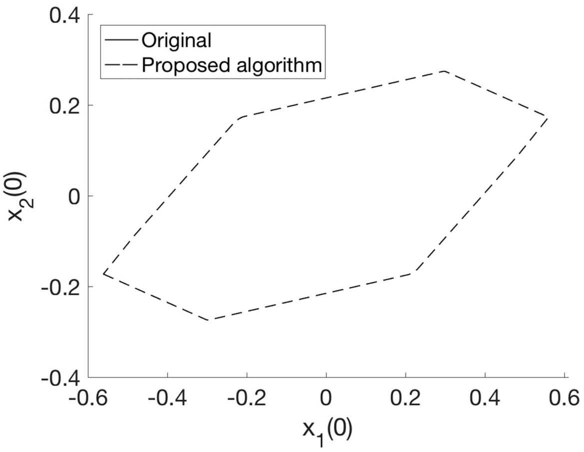

Fig. 5 *Maximal feasible region, input constrained problem:* $\bar{u}(i) = 1$, $\forall i \in \mathbb{M}$, *with* $N = 1$ *in the approach [25]. The parser Yalmip and solver lmilab were used to plot the region. The feasible region computed using the method proposed in [25] is probably only comprised by the origin*

detected, the maximal feasible region is still quite small and resides within a circular region with a radius equal to 1×10^{-4} around the origin.

6 Conclusions

In this study, the MPC problem for discrete-time MJLSs subject to polytopic uncertainties both in system matrices and in transition probabilities between modes was investigated. Results were proved for the unconstrained and constrained cases, ensuring mean square stability and satisfaction of hard constraints on system inputs and states. An extension to the multi-step case studied in [25] was also proposed. When compared to other numerical results available in [25], the simulations in this study, using the proposed methods illustrate the effectiveness of the new strategy in terms of cost, stability, and feasibility.

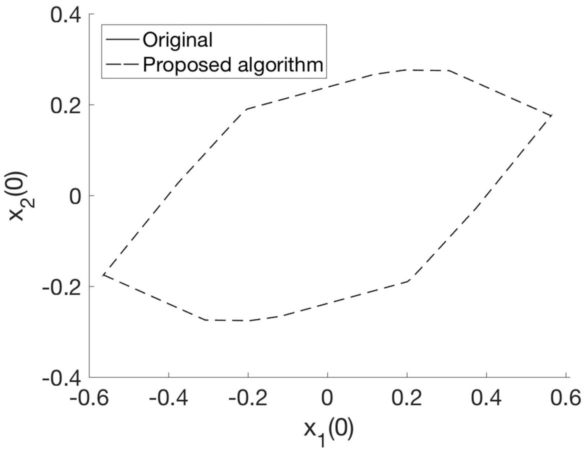

Fig. 6 *Maximal feasible region, input constrained problem:* $\bar{u}(i) = 1$, $\forall i \in \mathbb{M}$, *and* $N = 3$ *for both the approach in [25] and in extension multi-step (33), as discussed in Section 4.1. The parser Yalmip and solver lmilab were used to plot the region. The maximal feasible region of the algorithm proposed in [25] is a small region within a circular area with a radius equal to* 1×10^{-4} *around the origin*

7 Acknowledgments

This work was supported in part by the Brazilian agencies CNPq, FAPEMIG, and CAPES; and the 111 Project (no. B160). The authors would also like to show their gratitude to Dr Jianbo Lu, from Shanghai Jiao Tong University, for patiently answering all questions regarding his method. His valuable comments resulted in the implementation of his method that is now publicly available at https://github.com/emammendes/Constrained-model-predictive-control-synthesis.

8 References

[1] Kothare, M.V., Venkataramanan, V.B., Morari, M.: 'Robust constrained model predictive control using linear matrix inequalities', *Automatica*, 1996, **32**, (10), pp. 1361–1379

[2] Mayne, D.Q.: 'Model predictive control: recent developments and future promise', *Automatica*, 2014, **50**, (12), pp. 2967–2986

[3] Wan, Z., Kothare, M.V.: 'An efficient off-line formulation of robust model predictive control using linear matrix inequalities', *Automatica*, 2003, **39**, (5), pp. 837–846

[4] Campo, P.J., Morari, M.: 'Robust model predictive control'. IEEE American Control Conf., Minneapolis, MN, USA, 1987, pp. 1021–1026

[5] Rawlings, J.B., Mayne, D.Q.: '*Model predictive control: theory and design*' (Nob Hill Publishing, Madison, WI, USA, 2009)

[6] Mayne, D.Q., Rawlings, J.B., Rao, C.V., *et al.*: 'Constrained model predictive control: stability and optimality', *Automatica*, 2000, **36**, (6), pp. 789–814

[7] Mayne, D.Q.: 'Robust and stochastic model predictive control: are we going in the right direction?', *Annu. Rev. Control*, 2016, **41**, pp. 184–192

[8] Wang, J., Song, Y., Zhang, S., *et al.*: 'Robust model predictive control for linear discrete-time system with saturated inputs and randomly occurring uncertainties', *Asian J. Control*, 2018, **20**, (1), pp. 425–436

[9] Rawlings, J.B.: 'Tutorial overview of model predictive control', *IEEE Control Syst.*, 2000, **20**, (3), pp. 38–52

[10] Mesbah, A.: 'Stochastic model predictive control: an overview and perspectives for future research', *IEEE Control Syst.*, 2016, **36**, (6), pp. 30–44

[11] Farina, M., Giulioni, L., Scattolini, R.: 'Stochastic linear model predictive control with chance constraints–a review', *J. Process Control*, 2016, **44**, pp. 53–67

[12] Lorenzen, M., Tempo, R., Allgöwer, F.: 'Constraint-tightening and stability in stochastic model predictive control', *IEEE Trans. Autom. Control*, 2017, **62**, (7), pp. 3165–3177

[13] Liu, S., Hellendoorn, H., Schutter, B.: 'Model predictive control for freeway networks based on multi-class traffic flow and emission models', *IEEE Trans. Intell. Transp. Syst.*, 2017, **18**, (2), pp. 306–320

[14] Franzè, G., Tedesco, F., Famularo, D.: ‘Model predictive control for constrained networked systems subject to data losses’, *Automatica*, 2015, **54**, pp. 272–278

[15] Zhou, Y., Li, D., Lu, J., *et al.*: ‘Networked and distributed predictive control of non-linear systems subject to asynchronous communication’, *IET Control Theory Appl.*, 2018, **12**, (4), pp. 504–514

[16] Raković, V.S.: ‘Model predictive control: classical, robust, and stochastic [bookshelf]’, *IEEE Control Systems*, 2016, **36**, (6), pp. 102–105

[17] Costa, O.L.V., Fragoso, M.D., Marques, R.P.: ‘*Discrete-time Markov jump linear systems*’ (Springer-Verlag, London, UK, 2006)

[18] Shi, P., Li, F.: ‘A survey on Markovian jump systems: modeling and design’, *Int. J. Control, Autom. Syst.*, 2015, **13**, (1), pp. 1–16

[19] Lin, Z., Liu, J., Zhang, W., *et al.*: ‘Regional pole placement of wind turbine generator system via a Markovian approach’, *IET Control Theory Applic.*, 2016, **10**, (15), pp. 1771–1781

[20] Vargas, A.N., do Val, J.B., Costa, E.F.: ‘Receding horizon control of Markov jump linear systems subject to noise and unobserved state chain’. 2004 43rd IEEE Conf. on Decision and Control (CDC 2004), Nassau, Bahamas, 2004, vol. 4, pp. 4381–4386

[21] Yueyuan, Z., Yanyan, Y., Fei, L.: ‘Constrained model predictive control for Markov jump system with disturbances’. IEEE 2015 34th Chinese Control Conf. (CCC), Hangzhou, China, 2015, pp. 1816–1821

[22] Vargas, A.N., Furloni, W., do Val, J.B.: ‘Constrained model predictive control of jump linear systems with noise and non-observed Markov state’. IEEE American Control Conf., Minneapolis, MN, USA, 2006, p. 6

[23] Sala, A., Hernández-Mejías, M., Ariño, C.: ‘Stable receding-horizon scenario predictive control for Markov-jump linear systems’, *Automatica*, 2017, **86**, pp. 121–128

[24] Bernardini, D., Bemporad, A.: ‘Stabilizing model predictive control of stochastic constrained linear systems’, *IEEE Trans. Autom. Control*, 2012, **57**, (6), pp. 1468–1480

[25] Lu, J., Li, D., Xi, Y.: ‘Constrained model predictive control synthesis for uncertain discrete-time Markovian jump linear systems’, *IET Control Theory Applic.*, 2013, **7**, (5), pp. 707–719

[26] Gahinet, P., Apkarian, P., Chilali, M.: ‘Affine parameter-dependent Lyapunov functions and real parametric uncertainty’, *IEEE Trans. Autom. Control*, 1996, **41**, (3), pp. 436–442

[27] Löfberg, J.: ‘Yalmip: a toolbox for modeling and optimization in matlab’. Proc. of the CACSD Conf., Taipei, Taiwan, 2004

[28] ‘MOSEK ApS: The MOSEK optimization toolbox for MATLAB manual. Version 8.1’, 2017