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Luciana Sant'Ana Marques

Non-Cooperative Game Models for the Management of Distributed Energy Resources on the Residential Sector

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Luciana Sant'Ana Marques

NON-COOPERATIVE GAME MODELS FOR THE MANAGEMENT OF DISTRIBUTED ENERGY RESOURCES ON THE RESIDENTIAL SECTOR

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"Non-cooperative Game Models For The Management Of Distributed Energy Resources On The Residential Sector"

LUCIANA SANT´ANA MARQUES ARNOUX

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To people who fight for progress, especially in their personal lives.

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Many stories matter. Stories have been used to dispossess and to malign. But stories can also be used to empower and to humanize. Stories can break the dignity of a people. But stories can also repair that broken dignity.

Chimamanda Ngozi Adichie, 2009

Resumo

O controle transativo (TC) surgiu como uma forma de coordenar os múltiplos agentes dos sistemas de energia (consumidores, produtores, operadores de sistemas de distribuição, operadores de sistema de transmissão, agregadores etc.) considerando suas particularidades, prioridades, interesses e autonomia. A ideia é otimizar a alocação de recursos (por exemplo, geração, dispositivos controláveis e cargas) permitindo que os atores interajam entre si e troquem informações sobre consumo, geração, restrições e preferências até que um equilíbrio seja alcançado. Esse controle é naturalmente descentralizado e envolve um processo de tomada de decisão transparente. Essas características tornam o TC uma solução atrativa para o controle de recursos energéticos distribuídos (DERs), principalmente no setor residencial, onde a privacidade é uma das principais preocupações e existe um grande número de consumidores. Com o objetivo de contribuir para o desenvolvimento e avaliação de desempenho de algoritmos de teoria dos jogos no contexto de controle transativo, esta tese propõe um arcabouço teórico de jogos para incluir cargas inteiras e com energia variante ao conjunto de aparelhos gerenciados pela abordagem TC. Os impactos teóricos da inclusão de tais cargas são estudados, considerando diferentes modelos de jogo: 1) com função de custo total quadrático e faturamento por horário; 2) com função de custo total quadrática e faturamento proporcional ao consumo; e 3) com função de custo total de preço de pico e faturamento proporcional ao consumo. Múltiplos aspectos dos jogos propostos são analisados, como existência e multiplicidade de Equilíbrios de Nash, justiça e equidade dos diferentes faturamentos e comportamento de trapaça. Os jogos são simulados usando dados reais de uma comunidade de baixa tensão no sul da Espanha com 201 consumidores, e os resultados corroboram os desenvolvimentos teóricos da tese.

Palavras-chave: Comunidades de Energia, Escalonamento de Cargas, Jogos Não Cooperativos, Cargas Controladas Termostaticamente, Controle Transativo.

Abstract

Transactive control (TC) has emerged as a form of coordinating the multiple agents in power systems (consumers, producers, DSOs, TSOs, aggregators etc.) while considering their particularities, priorities, interests, and autonomy. The idea is to optimize the allocation of resources (e.g. generation, controllable devices and loads) by enabling actors to interact with each other and exchange information about consumption, generation, constraints and preferences until an equilibrium solution is reached. This market-based control is naturally decentralized and entails a transparent decision-making process. These characteristics make TC an attractive solution for controlling distributed energy resources (DERs), specially in the residential sector, in which privacy is a main concern and a large number of consumers exist. In order to contribute to the development and performance evaluation of game theoretic algorithms in the context of transactive control, this thesis proposes a game theoretic framework for including integer and energy variant loads to the set of appliances managed by the TC approach. The theoretical impacts of including such loads is studied, while considering different game models: 1) with quadratic total cost function and per-time-slot billing; 2) with quadratic total cost function and proportional-to-consumption billing; and 3) with peak pricing total cost function and proportional-to-consumption billing. Multiple aspects of the proposed games are analyzed, such as existence and multiplicity of Nash Equilibria, fairness and equity of the different billings, and cheating behavior. The games are simulated using real data from an LV community in the South of Spain with 201 consumers, and results corroborate the theoretical developments of the thesis.

Keywords: Energy Communities, Load Scheduling, Non-Cooperative Games, Thermostatically Controlled Loads, Transactive Control.

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List of abbreviations and acronyms

AC	Air Conditioner
BNE	Bayesian Nash Equilibrium
BR	Best Response
BRD	Best Response Dynamics
CPP	Critical Peak Pricing
DAP	Day-ahead Pricing
DER	Distributed Energy Resource
DG	Distributed Generation
DLC	Direct Load Control
DSM	Demand-side Management
DSO	Distribution System Operator
DR	Demand Response
ECC	Energy Consumption Controller
EMS	Energy Management System
EPRI	Electric Power Research Institute
ESS	Energy Storage System
HEMS	Home Energy Management System
IBR	Inclining Block Rates
LAN	Local Area Network
LV	Low Voltage

MV	Medium Voltage
MILP	Mixed-Integer Linear Problem
MIQP	Mixed-Integer Quadratic Problem
NE	Nash Equilibrium
ONS	National System Operator (Brazil)
PAR	Peak-to-Average Ratio
PoA	Price-of-Anarchy
PP	Peak-Pricing
P2P	Peer-to-Peer
PTC	Proportional-to-Consumption Billing
PTS	Per-Time-Slot Billing
PV	Photovoltaics
RTP	Real-Time Pricing
SCADA	Supervisory Control and Data Acquisition
SIN	National Interconnected System (Brazil)
TC	Transactive Control
TCL	Thermostatically-Controlled Load
TE	Transactive Energy
TOU	Time-of-Use
TSO	Transmission System Operator
VPP	Virtual Power Plants

List of symbols

n, m	Indexes for consumers
t	Indexes for time slots
k	Indexes for game stages
$\mathcal{N}$	Set of consumers (players)
N	Number of consumers (players)
$\mathcal{T}$	Set of time slots
T	Last slot of time horizon
$\mathcal{S}_n, \mathcal{X}_n$	Set of strategies of n
$\mathcal{S}, \mathcal{X}$	Set of strategy profiles of all consumers (players)
$\Delta_n(\mathcal{X}_n)$	Set of mixed strategy profiles of player n
$\Delta(\mathcal{X})$	Set of mixed strategy profiles of all player
$y_{n,t}$	Binary scheduling variable: is 1 if consumer n's AC is on at t and 0 otherwise
$l_{n,t}, q_{n,t}$	Total consumption of n at t
α_t	Total load at time slot t (auxiliary variables for computing the peak load)
$\mathbf{l}_n, \mathbf{x}_n$	Scheduling vector (or strategy profile) of player n
$\mathbf{l}_{-n}, \mathbf{x}_{-n}$	Scheduling vectors (or strategy profiles) of all players except n (n's opponents)
$\mathbf{l}, \mathbf{x}$	Scheduling vectors (or strategy profiles) of all players except n (n's opponents) of all players
$\sigma_n(\mathbf{x}_n)$	Mixed strategy profile of player n

$\sigma(\mathbf{x})$	Mixed strategy profile of all players
L_t	Community's total load at t
$C(\mathbf{L})$	Community's total cost when its load curve is $\mathbf{L}$
$C^Q(\mathbf{L})$	Community's total cost when its load curve is $\mathbf{L}$, using a quadratic function
$C^P(\mathbf{L})$	Community's total cost when its load curve is $\mathbf{L}$, using a peak pricing function
$u_n(\mathbf{l}_n, \mathbf{l}_{-n})$,$u_n(\mathbf{x}_n, \mathbf{x}_{-n})$	Utility of consumer (player) n when playing $\mathbf{l}_n$ ($\mathbf{x}_n$) while opponents are playing $\mathbf{l}_{-n}$ ($\mathbf{x}_{-n}$)
$u_n^C(\mathbf{l}_n, \mathbf{l}_{-n})$	Utility of consumer n using the proportional-to-consumption billing
$u_n^S(\mathbf{l}_n, \mathbf{l}_{-n})$	Utility of consumer n using the per-time-slot billing
$u_n(\sigma_n, \sigma_{-n})$	Utility of player n when playing a mixed strategy σ_n while opponents are playing σ_{-n}
δ	Size of time slots (in hours)
$w_{n,t}$	Inflexible load of n at t
$\theta_{n,t}$	Internal temperature of n's household at t
$\theta_{n,t}^{min}$	Minimum accepted internal temperature of n at t
$\theta_{n,t}^{max}$	Maximum accepted internal temperature of n at t
θ_t^{et}	External temperature at t
TC_n	Thermal capacity of n's AC
R_n	Thermal resistance of n's AC
η_n	Performance of n's AC
E_n	Power rate of n's AC
a_t	Quadratic parameter of the quadratic cost function
b_t	Linear parameter of the quadratic cost function
c_t	Volumetric energy component of the peak pricing cost function at time slot t
d	Peak load charge

f_n Energy share for the proportional-to-consumption billing of consumer n

$\phi(\mathbf{l}_n, \mathbf{l}_{-n})$,$\phi(\mathbf{x}_n, \mathbf{x}_{-n})$ Potential function when consumers (players) are playing $\mathbf{l}_n$ ($\mathbf{x}_n$) while opponents are playing $\mathbf{l}_{-n}$ ($\mathbf{x}_{-n}$)

$P_t(l_{n,t}, l_{-n,t})$ Energy prices at time slot t

F Fairness index

SV_n Shapley Value of consumer n

Contents

1. Introduction

1.1 Motivation

The introduction of the Smart Grid concept brought new solutions to electric grid planning, management and control. It employs advanced communication systems and modern computational technologies to increase grid's efficiency, autonomy, reliability and security [Energy Independence and Security Act, 2007, Farhangi, 2010, Yoldaş et al., 2017, Le Ray and Pinson, 2020, Butt et al., 2020]. For instance, two-way flows of electricity and information, distributed control, monitoring systems, and advanced technologies are added to the layers of the power grid—generation, transmission, distribution and consumption [Yoldaş et al., 2017]. This modernization intends to transform the grid into a self-monitoring, self-healing, and adaptive network [Butt et al., 2020], allowing the deployment and integration of renewable resources, distributed generation (DG), and demand-side management (DSM) programs.

In line with the Smart Grid concept, distributed energy resources (DERs) are the set of technologies and strategies to be connected or used at the distribution level with the potential to make energy use more efficient, accessible, and environmentally sustainable [LBNL, 2017]. They can include distributed generation (renewable or dispatchable); demand flexibility (demand management strategies and flexible loads); and storage systems [Kok, 2013]. Those emerging technologies and strategies provide opportunities to solve some traditional grid problems. For instance, distributed generation can reduce power losses in the transmission system, defer upgrades of the transmission and distribution grids, furnish ancillary services to the network and diminish the fossil fuel use [Lasseter, 2002, Allan et al., 2015]. Demand management strategies can reduce congestion and decrease the excess generation capacity to accommodate the peak demand [Hu et al., 2017]. Storage systems can help spreading renewable energy sources technologies, and support the operation and control of the smart grids [Lasseter, 2011].

However, a massive introduction of individual distributed energy resources without proper operation and control systems/strategies can cause as many problems as it may solve [Jiayi et al., 2008]. The integration of distributed generation units within the existing network can create issues related to the cost of energy, the price of electricity, the infrastructure requirements, the distribution system reliability and security (e.g. voltage control, power quality, protection system, fault level, grid losses) [Coster et al., 2011, Allan et al., 2015]. Moreover, many policies for installing DERs had focused on their connection, in a 'fit and forget' approach [Pudjianto et al., 2007], i.e. without considering those issues. In addition, a DER device primary reason of existence is not to provide services to the grid [Kok, 2013], which leads to the need of considering the DERs owner's constraints. Therefore, managing significant levels of DERs, each with its dynamics, resources, and specific control characteristics, can become overwhelming [Lasseter, 2002]. Hence, those technical and regulatory challenges must be overcome to guarantee the maintenance of network security and reliability standards, while implementing DERs strategies and taking the advantages of their potential [Olivares et al., 2014].

To address the distributed energy resources management and integration challenge, many innovative mechanisms and solutions have been proposed in the literature, which are named energy management systems (EMSs). They intend to coordinate the distributed resources, optimizing the techno-economical operation of the distribution networks, in multiple time scales [Ton and Reilly, 2017, Olivares et al., 2014]. Although the EMS concept is not limited to the distribution level, being employed to the management of the entire electrical systems, in this work we use it for approaches related to the distribution systems only. Therefore, there are four EMS categories of interest, i.e. top-down switching, price reaction, centralized optimization, and transactive control [Kok and Widergren, 2016]. They differ according to the entity responsible for taking decisions and the communication requirements between devices and agents (e.g. users, distribution system operators, aggregators, community manager, among others).

In top-down switching, decisions about the devices management are made by a central controller, generally the system operator or the utility [Kok and Widergren, 2016]. The controller broadcasts a signal to the distributed resources, which are turned on/off in response. Therefore, only a one-way communication system is required, for the controller to remotely manage end-users' devices [Kok and Widergren, 2016]. Even though this approach is simple and effective, it does not unleash the full DERs potential, because their status are not considered in the management decisions. As a result, the devices reaction to the control signals is uncertain, e.g. maybe they are already off when receiving a turning off signal, and vice versa [Kok and Widergren, 2016]. Moreover, this approach has some autonomy issues, because a third party controls the end-users' devices [Stenner et al., 2017].

In the centralized optimization approach, decisions on local issues are still taken centrally, but the communication is bidirectional [Kok and Widergren, 2016]. A complex optimization procedure coordinates the resources of the smart grid under analysis, e.g. a microgrid, a virtual power plant, a local community, or a group of end-users represented by an aggregator. Therefore, all relevant information for the decision making process must be broadcasted to the central controller, which calculates the global optimum according to the objectives of the system , for example the optimal distributed generation dispatch and flexible loads scheduling that minimizes a microgrid operating costs [Araújo and Uturbey, 2013, Karthikeyan and Parvathy, 2015, Tang and Zhong, 2016]. Therefore, a two-way communication system is necessary. This approach is capable of unlocking the DERs response potential and flexibility if the relevant local information is available [Kok and Widergren, 2016]. Moreover, the reaction of the system participants is known a priori, because the optimization procedure controls directly the devices. The problems related to agents' autonomy of the previous approach remain, since the resources are operated by a central program, and issues concerning information privacy are added, because local data about preferences and constraints are sent to the controller [Kok and Widergren, 2016]. Furthermore, communicating all local data to a central point limits the accuracy and scalability of this approach.

In the price reaction approach, dynamic price signals are sent to the final users. At certain time intervals, new electricity prices (or price profiles) for the next periods are communicated to the consumers, or to their automated systems. As a response, the users adjust their equipment (DERs) manually, or automatically and optimally via their automated EMSs [Kok and Widergren, 2016]. If compared to the centralized optimization approach, the general benefits of the price signals are: 1) only a simple one-way communication system is necessary, to receive the prices or demand-response events; 2) there is no autonomy issues for users, nor information privacy concerns, as long as the decisions are taken locally (manually or automatically); and 3) it is easily implementable in regions where a wholesale market exists and provides day-ahead and/or intraday prices profiles, specially the manual response [Kok and Widergren, 2016]. However, the reaction of all consumers and their devices is difficult to predict if the DERs status and the users' preferences are unknown by the utility or distribution system operator. Moreover, managing the load of each building separately, without looking to the neighbor's schedule, can generate load synchronization, create energy spikes or prevent the system to benefit from peak-to-average ratio reduction [Palensky and Dietrich, 2011, Pipattanasomporn et al., 2012]. Finally, consumers acting alone have less power in energy markets, being only pricing respondents, instead of real players. Thus, the transactive control proposition aims at overcoming those issues.

Transactive control has emerged as an alternative means of orchestrating the coordinated operation of the multiple intelligent devices being connected at the distribution systems, in a way to overcome the drawbacks of the preceding approaches [Kok and Widergren, 2016]. In this energy management system, decisions are made through an exchange of value-based information captured in transactions between participants, generating an efficient market without privacy issues [Kok and Widergren, 2016]. Moreover, given that coordinating a growing number of DERs poses a multi-objective control and optimization challenge, it embraces the economics and engineering of the power system [GWAC, 2015]. Indeed, some wholesale electricity markets around the world have already implemented this concept at the transmission level, giving decision autonomy to their agents (e.g. large power plants and consumers) [NORD POOL, 2020a]. At this level, the control of the large resources is done by auctions and bilateral tradings between participants, and a market operator is responsible for clearing the price and guaranteeing settlement and delivery. This trading and clearing mechanism involves a distributed and coordinated decision-making process, overcoming the preceding autonomy, privacy, and scalability issues [Kok and Widergren, 2016]. Still, at the retail market level, which is related to the distribution system operation, the transactive concept is lacking [Hu et al., 2014]. Therefore, there is a large potential for contribution in this field.

It is worth mentioning that controlling and operating the distribution systems have always being done by a central agent [Farhangi, 2010]. However, in a context where retail consumers start to take decisions about generating their own electricity locally, using electrical vehicles, increasing the number of big-watt home appliances, and negotiating their flexibility in the market, it becomes necessary to rethink the planning, controlling and operating model, to redesign the retail market and its rules, and to redefine how wholesale and retail markets relate to each other [Kok and Widergren, 2016]. Therefore, the only feasible solution to continue reliably delivering electricity and to facilitate those future developments at the end-users' side, is the active management of the distribution grids, and transactive control can give tools for this goals [Hu et al., 2017].

An effecting way of implementing TC is through non-cooperative game theory, because it allows modeling agents' preferences, priorities, conflicting interests, and complex interactions in a decentralized manner [Saad et al., 2012]. When applied to DERs control in the residential sector, game theoretic methods capture the load/generation scheduling interactions between mid- to small-size consumers that negotiate their load flexibility, excess generation, and storage services through their home management systems, using an electronic market algorithm [Hu et al., 2017]. In this local market, consumers exchange information and optimize their resources by controlling some flexible appliances, generation, and storage until an equilibrium is reached and all consumers are satisfied with the result.

However, implementing non-cooperative models for the optimization of distributed energy resources is not an easy task. Some challenges one can face are: designing effective games that can reach an energy efficient result for consumers and the distribution systems, while keeping end-users' engagement; installing advanced communication and resources scheduling technologies to establish the local market and guarantee the correct operation of the models; and putting in place new energy policies to allow the introduction of peer-to-peer mechanisms, as non-cooperative games [Sousa et al., 2019]. Research and literature studies are necessary to support decision-makers in technology, market, and policy sectors to advance, implement, and promote non-cooperative models for the management of distributed energy resources. Those compose the main motivation of this thesis.

1.2 Objective

In this framework, this research focuses on transactive control models for the management of distributed energy resources, and addresses the development of tools to be used in the context of smart grid operation planning. More specifically, non-cooperative methods for the transactive control of flexible loads are studied. The primary objective is to contribute on the development and performance evaluation of game theoretic algorithms to address the distributed energy resources coordination problem at the distribution level. The idea is to overcome some challenges related to the design of energy efficient models that can enhance end-users' engagement. To achieve such purpose, five secondary objectives were set:

1. propose a game theoretic framework for including integer and energy variant loads to the set of appliances managed by the transactive control approach;
2. study the theoretical impacts of including integer variables to the set of appliances;
3. study the theoretical impacts of including energy variant loads to the set of appliances;
4. compare different billings for sharing the total cost of the loads management;
5. verify if cheating behavior can occur in those models, and propose mechanisms to prevent it, if necessary/possible.

1.3 Contributions

To achieve the objectives aforementioned, this thesis advances the state-of-the-art of non-cooperative games applied to the day-ahead load scheduling of residential consumers in energy communities by including on/off and energy variant thermostatically controlled loads (TCLs) into the set of appliances considered in TC. Current technological solutions, developed around non-cooperative game theory, have shown promising results but they were unable to properly include TCLs, which are the largest source of flexibility among domestic loads. Besides, two aspects of TCLs control are not fully addressed by the current theory of non-cooperative games: 1) the on/off nature of the decisions, which makes the problem integer and changes the equilibria conditions; 2) the energy variant characteristic of the control, which contradicts the energy neutrality assumptions of the theory. Moreover, the different billing models to share the total cost of the integer and energy variant load scheduling game have not been fully studied. Therefore, it is important to understand the implications of these three theory gaps in real-world implementation of TC in the residential sector. The specific contributions are the following:

1. we model explicit TCL comfort constraints and formulate the problem with binary variables representing the real on/off control of this type of appliances, when defining the game. Two billing models are considered: per-time-slot and proportional-to-consumption;

2. since the integer nature of the control affects the theoretical foundations of the problem, we prove that multiple Nash Equilibria can exist and they can be sub-optimum;

3. we discuss the practical implications of having multiple NEs in real implementation of TC platforms, in terms of optimality of the total scheduling cost, variability in consumers' payments, and how the algorithm design defines the solution that will be effectively played;

4. we show that TCLs energy variant nature impacts the theoretic grounds of the game model, because the total energy in the scheduling horizon is not fixed. Thus, we discuss how this characteristic affects the equity among consumers when applying the best response dynamics with proportional-to-consumption billing to the non-cooperative game model;

5. in line with the previous contribution, we show that the game modeled with a proportional-to-consumption billing does not guarantee a potential game formulation, thus a Nash Equilibrium, in the presence of muti-period energy variant loads (such as thermal loads);

6. we propose a modified best response algorithm to solve the problem with the proportional-to-consumption billing and energy variant loads. We also show that the Nash Equilibrium can not be reached and cheating behavior can occur with the proportional-to-consumption;

7. we present a general formulation for the per-time-slot billing, including integer and energy variant TCLs;

8. we show that the general formulation for the per-time-slot can be applied to any type of loads, because its exact potential properties do not depend on loads constraints;

9. we propose an alternative solution to overcome the possibility of participants cheating in per-time-slot billing models, by showing theoretically that a simple adjustment

in the billing rules ex-ante instead of ex-post consumption is enough to discourage cheating behavior, which guarantees the strategy-proof of this mechanism.

As a minor contribution, we show that the game designed with a per-time-slot billing is fairer than the proportional-to-consumption model, when integer and energy variant loads are considered. We support these contributions with a case study involving an LV community in the South of Spain with 201 consumers.

1.4 Publications

- MARQUES L.; SILVA H. B.; THAKUR J.; UTURBEY, W. Shared PV business models in renewable markets: a categorization based on CANVAS and an analysis of transaction costs reduction of each category. Accepted for publication in: 3rd Latin American Conference on Sustainable Development of Energy, Water and Environment Systems SDEWES, 2022, São Paulo.
- MARQUES L.; UTURBEY, W.; HELENO, M. An Integer Non-Cooperative Game Approach for the Transactive Control of Thermal Appliances in Energy Communities. Energies. 14.21: 6971. 2021.
- MARQUES L.; HELENO, M.; UTURBEY, W. Transactive Control for residential demand-side management: lessons learned from non-cooperative game theory. In Decentralized Frameworks for Future Power Systems. 2022.
- AMANCIO, M. C. L. ; UTURBEY, W. ; MARQUES, L. S. . Desafios econômicos e regulatórios para inserção do armazenamento de energia no sistema brasileiro. In: XXV SNPTEE Seminário Nacional de Produção e Transmissão de Energia Elétrica, 2019, Belo Horizonte. XXV SNPTEE Seminário Nacional de Produção e Transmissão de Energia Elétrica, 2019.
- SILVEIRA, S. ; SILVA, H. B. ; UTURBEY, W. ; MARQUES, L. S. . A comparative analysis of PV markets in Brazil and Sweden. In: 8th Solar Integration Workshop, 2018, Stockholm. 8th Solar Integration Workshop, 2018.
- ARNOUX, L. S. M.; UTURBEY, W. A Cooperative Games Approach for Demand Side Management in Smart Grids. In: XII Latin American Congress on Electricity Generation and Transmission (CLAGTEE), 2017, Mar del Plata. Proceedings and Book of Abstracts of the 12th Latin-American Congress on Electricity Generation and Transmission: CLAGTEE 2017, 2017.

1.5 Thesis Structure

The rest of this thesis is organized as follows. In chapter 2 we present definitions related to game theory, specially non-cooperative models. This chapter is designed to give a brief overview of game theory to readers who are unfamiliar with it. Those who understand the fundamentals of game theory can skip this chapter. Chapter 3 describes basic concepts of smart grids, distributed energy resources, and energy management systems. The idea is to provide the framework in which transactive control models are applied. Again, readers who know those ideas can skip this chapter. In chapter 4, we detail the literature of non-cooperative games employed to the transactive control of integer and energy variant loads. We explain how our work differs from the literature models, paving the way to restating our contributions. In chapter 5, the proposed methodology is presented. We describe the system and load models, the cost functions, the game designed to schedule the loads, and the billing mechanisms used to share the total cost and to define consumers' utilities (bills). Moreover, we analyze theoretical aspects related to: existence of Nash Equilibrium; convergence of algorithms; multiplicity of Equilibria; fairness and equity of the billings; strategy-proof; price-of-anarchy; and general applicability of the game. In chapter 6, we show simulation results of the methodology applied to a Spanish LV community. Finally, a conclusion is given in chapter 7.

2. Game Theory

In this thesis, we study the transactive control of distributed energy resources. We focus on non-cooperative games to coordinate residential consumers' resources, because game theory offers an interesting analytical and conceptual framework to deal with the study of their complex interactions [Saad et al., 2012]. As a result, in this chapter we present basic concepts of game theory for readers who are unfamiliar with it. Those who understand the fundamentals of game theory can skip this chapter.

2.1 Game Definition

Game theory is a "bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact" [Osborne and Rubinstein, 1994, p. 1]. In those interactions, the participants' decisions affect each other's outcomes [Roughgarden, 2016], in a framework with conflict of interest [Lã et al., 2016]. The theory basic assumptions are: decision-makers seek well-defined objectives (they are rational); and they take into consideration the available information about other decision-makers' behavior (they act strategically) [Osborne and Rubinstein, 1994]. This is defined as the "rational decision-making" hypothesis in economics: decision-makers act rationally in the sense of choosing the option (an strategy) that gives them higher "payoffs"[1]. Those decision-makers are called "players" and are not limited to humans, being possible to include animals, devices, or machines [Lã et al., 2016].

Countless literature developing and detailing this theory exists. The studies can focus on non-cooperative or cooperative games, strategic or extensive form games, games with perfect or imperfect information, among others. In non-cooperative games, players act individually and rationally in order to get the best possible outcomes, while in cooperative games, players can join coalitions to create and capture value [Chatain, 2014]. In strategic games, players' decisions are taken simultaneously, while in extensive games, the possible order of events is specified, and players have to plan their actions [Osborne and Rubinstein,

[1] The term "payoff" does not necessary mean monetary payments, and although social and psychological factors are more difficult to estimate, they also influence payoffs.

1994]. In games with perfect information, all players have access to other players' strategy sets, while in games with imperfect information, players access to each others' information is limited [Osborne and Rubinstein, 1994]. In this thesis, we focus on non-cooperative games, represented by strategic form, and with perfect information.

2.2 Terminology and Notation

Representing a real-life situation using game theoretic models, as in any modeling method, means abstractly describing the situation. To do it formally, game theory uses mathematical formulations, because it allows defining concepts precisely, verifying results consistency, and exploring the implications of assumptions [Osborne and Rubinstein, 1994]. Therefore, we use the following terminology and notation:

- The set of real numbers is denoted by $\mathbb{R}$, the set of vectors of T real numbers by $\mathbb{R}^T$;
- Other sets are represented by uppercase letters using calligraphic font, e.g. $\mathcal{N}$, $\mathcal{T}$, $\mathcal{S}$, $\mathcal{X}$;
- Vectors are defined by bold lowercase letters, e.g. $\mathbf{x}$, $\mathbf{l}$, $\mathbf{w}$;
- A function $f : \mathbb{R} \mapsto \mathbb{R}$ is concave if $f(\alpha x + (1-\alpha)x') \geqslant \alpha f(x) + (1-\alpha)f(x')$ and it is strictly concave if $f(\alpha x + (1-\alpha)x') > \alpha f(x) + (1-\alpha)f(x')$ for all $x \in \mathbb{R}$, $x' \in \mathbb{R}$ and $\alpha \in [0,1]$;
- A function $f : \mathbb{R} \mapsto \mathbb{R}$ is convex if $f(\alpha x + (1-\alpha)x') \leqslant \alpha f(x) + (1-\alpha)f(x')$ and it is strictly convex if $f(\alpha x + (1-\alpha)x') < \alpha f(x) + (1-\alpha)f(x')$ for all $x \in \mathbb{R}$, $x' \in \mathbb{R}$ and $\alpha \in [0,1]$;
- Given a function $f : \mathcal{X} \mapsto \mathbb{R}$, the set of maximizers of f is denoted by $\operatorname{argmax}_{x \in \mathcal{X}} f(x)$;
- The set of players is denoted by $\mathcal{N}$;
- The set of strategies is denoted by $\mathcal{S}$ or $\mathcal{X}$;
- A strategy is defined by a vector $\mathbf{x}_n$ or $\mathbf{l}_n$, for each player $n \in \mathcal{N}$;
- The set of all possible strategies for a player is denoted by $\mathcal{S}_n$ or $\mathcal{X}_n$;
- For any player $n \in \mathcal{N}$, let $\mathbf{x}_{-n}$ (or $\mathbf{l}_{-n}$) be the matrix of all strategies $\mathbf{x}_m$ (or $\mathbf{l}_m$) of all players except n ($m \in \mathcal{N} \backslash n$);
- The joint strategies of all players defines a strategy profile, which is represented by $\mathbf{x} = (\mathbf{x}_n, \mathbf{x}_{-n})$ or $\mathbf{s} = (\mathbf{l}_n, \mathbf{l}_{-n})$;
- The joint set of all possible strategy profiles for the players is denoted by $\mathcal{X} = \bigtimes_{n=1}^{N} \mathcal{X}_n$ or $\mathcal{S} = \bigtimes_{n=1}^{N} \mathcal{S}_n$.

2.3 Game Aspects

Games describe the strategic interactions between players, including the constraints on their actions (which defines what decisions they can take) and their preferences (what drives players to take a specific action). Therefore, to mathematically characterize a non-cooperative game with perfect information and in strategic form, it is necessary to define three aspects: the set of players; the set of players' strategies (decisions they can take); and the set of utilities (their preferences over the decisions). The game in strategic form is thus represented by $\Gamma = \langle \mathcal{N}, (\mathcal{X}_n)_{n \in \mathcal{N}}, \{u_n\}_{n \in \mathcal{N}} \rangle$, with finite set of players $n \in \mathcal{N}$, finite or infinite set of actions/strategies $\mathbf{x}_n \in \mathcal{X}_n$, and utility functions $u_n(\mathbf{x}_n, \mathbf{x}_{-n})$.

2.3.1 Players

As stated by Osborne and Rubinstein [1994, p. 12], "a player may be an individual human being or any other decision-making entity like a government, a board of directors, the leadership of a revolutionary movement, or even a flower or an animal." Moreover, in the era of technology and internet, many game models are used by information technology companies to describe and model the interactions of computerized machines and tasks [Lã et al., 2016]. Those inanimate entities "fight" for the use of limited resources, as a computer memory or an advertisement spot on a well-known search engine. The defined players n of the game form the players' set $\mathcal{N}$

2.3.2 Strategies

Strategies are a complete description of how to play the game. In strategic form games, the set of pure strategies is simply the agents' action space. Therefore, when a consumer chooses an action $\mathbf{x}_n$ (or $\mathbf{l}_n$) to play, it is also a pure strategy. Therefore, we use the terms action and pure strategy interchangeably. In this chapter, we use a general notation for actions/strategies sets, denoted by $\mathbf{x}$ and $\mathcal{X}$. In the methodology section, the notation uses $\mathbf{l}$ and $\mathcal{S}$, which are the same as the ones adopted here.

Players can also randomize independently among actions (pure strategies), which means they play an action according to a probability distribution over the actions set $\mathcal{X}_n$. In these cases, agents play mixed strategies. We define σ_n to denote the probability distribution on the action space of player $n \in \mathcal{N}$, that is, $\sigma_n(\mathbf{x}_n)$ is the probability that the mixed strategy σ_n assigns to $\mathbf{x}_n$. We denote the space of n's possible mixed strategies as $\Delta_n(\mathcal{X}_n)$, defined as:

$$\Delta_n(\mathcal{X}_n) = \left\{ \sigma_n : \mathcal{X}_n \mapsto [0,1] : \sum_{\mathbf{x}_n \in \mathcal{X}_n} \sigma_n(\mathbf{x}_n) = 1 \right\} \tag{2.1}$$

We define support of a mixed strategy σ_n as the set of pure strategies to which σ_n assigns positive probability. One can observe that pure strategies are a subgroup of mixed strategies σ_n that assign probability one to a pure strategy $\mathbf{x}_n$—in other words, they are mixed strategies with support cardinality equals to one. It is interesting to notice that, in all games, differently from the set of pure strategies, the set of mixed strategies is always infinite.

When all agents play some strategy, it is defined as an strategy profile. Therefore, a pure strategy profile of a game is an outcome $\mathbf{x} = (\mathbf{x}_n, \mathbf{x}_{-n}) \in \mathcal{X}$, in which $\mathcal{X} = \times_{n \in \mathcal{N}} \mathcal{X}_n$ is the joint action space. Moreover, we also define $\sigma \in \Delta(\mathcal{X})$ as a mixed strategy profile, where $\Delta(\mathcal{X}) = \times_{n \in \mathcal{N}} \Delta_n(\mathcal{X}_n)$ is the joint probability distribution space over mixed strategies.

2.3.3 Utilities

Players' preferences over the strategies in the strategy profile set $\mathbf{x} \in \mathcal{X}$ define their utilities (also known as payoffs). Those preferences can reflect players' feelings about the possible outcomes, the consequences of their actions, their chances of successfully reproducing (in case of unconscious organisms), among others [Osborne and Rubinstein, 1994].

In general, players' preferences can be represented by a preference relation $\gtrsim_{n \in \mathcal{N}}$. This relation can be mathematically described by a utility function $u_n : \mathcal{X} \mapsto \mathbb{R}$. For any two strategies profiles $\mathbf{x}^i = (\mathbf{x}^i_n, \mathbf{x}^i_{-n})$ and $\mathbf{x}^j = (\mathbf{x}^j_n, \mathbf{x}^j_{-n})$, the utility function must be able to express the preference relation of those strategies for a player n, in the sense that $u_n(\mathbf{x}^i) \geqslant u_n(\mathbf{x}^j)$ if $\mathbf{x}^i \gtrsim_n \mathbf{x}^j$.

The aforementioned utility function is defined for pure strategies: an action profile $\mathbf{x}$ gives a direct payoff $u_n(\mathbf{x})$ to player n. It is also possible to define utility functions for mixed strategies. In this case, the payoffs are calculated as an expectation over actions in the support of the mixed strategies, resulting in an expected utility (for finite games):

$$u_n(\sigma_n, \sigma_{-n}) = \sum_{(\mathbf{x}_n, \mathbf{x}_{-n}) \in \mathcal{X}} u_n(\mathbf{x}_n, \mathbf{x}_{-n}) \Pr(\mathbf{x}_n, \mathbf{x}_{-n} / \sigma_n, \sigma_{-n}) \tag{2.2}$$

Where the first term is given by the utility of pure strategy $(\mathbf{x}_n, \mathbf{x}_{-n})$, and $\Pr(\mathbf{x}_n, \mathbf{x}_{-n} / \sigma_n, \sigma_{-n})$ is the conditioned probability of an action profile, given a mixed strategy profile. This probability is calculated by:

$$\Pr(\mathbf{x}_n, \mathbf{x}_{-n}/\sigma_n, \sigma_{-n}) = \prod_{m \in \mathcal{N}} \sigma_m(\mathbf{x}_m) \tag{2.3}$$

Where $\sigma_m(\mathbf{x}_m)$ is the probability that user m assigns to action $\mathbf{x}_m$ in the strategy profile $(\mathbf{x}_n, \mathbf{x}_{-n})$, when agents play the mixed strategy profile (σ_n, σ_{-n}).

2.4 Solution Concepts

The game described in last section, with players, strategies and utilities, does not specify the actions the participants actually take (or play). In fact, a solution of the game is the systematic description of the outcomes that may arrive. Reasonable solution concepts for classes of games are part of game theory, with different properties.

In games, agents' goal is to choose and play optimal strategies, a.k.a. those leading to larger payoffs. However, the best strategy for an agent depends on the choices of the other agents. Therefore, based on the information players have about opponents' actions and utilities, they choose some strategy. In this context, solution concepts are formal rules for predicting how the game will be played. They describe which strategies will be adopted by players and, thus, the outcome of the game. The most usual solution concepts are equilibrium concepts, i.e. Nash Equilibrium, Bayesian Nash Equilibrium, ϵ-equilibrium, and correlated equilibrium. Most part of them are based on the idea of Best Response (BR).

Best Response is the best choice of one player, given his beliefs about what the other players will do. If this player knew what everyone else was going to do, it would be easy to choose his own action, and it would be a best response. Formally, we have:

Definition 1. ***(Best Response)*** *A pure strategy $\mathbf{x}_n \in \mathcal{X}_n$ of player $n \in \mathcal{N}$ is a best response to a strategy profile of the other players $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$ if it maximizes his utility when opponents play $\mathbf{x}_{-n}$:*

$$\mathbf{x}_n = BR_n(\mathbf{x}_{-n}) = \operatorname*{argmax}_{\mathbf{x}'_n \in \mathcal{X}_n} u_n(\mathbf{x}'_n, \mathbf{x}_{-n}) \tag{2.4}$$

Best response is not a solution concept, because it does not identify an interesting set of outcomes. However, the idea of best response can be leveraged to define what is arguably the most important solution concept in non-cooperative game theory: the Nash Equilibrium (NE).

Definition 2. ***(pure-Nash Equilibrium)*** *A pure strategy profile* $\mathbf{x}^* = (\mathbf{x}_n^*, \mathbf{x}_{-n}^*) \in \mathcal{X}$ *is a pure Nash Equilibrium if all players* $n \in \mathcal{N}$ *choose a pure strategy* $\mathbf{x}_n^*$ *that is a best response to opponents' strategies* $\mathbf{x}_{-n}^*$*. In other words, for all players* $n \in \mathcal{N}$*:*

$$u_n(\mathbf{x}_n^*, \mathbf{x}_{-n}^*) \geqslant u_n(\mathbf{x}_n, \mathbf{x}_{-n}^*) \quad \forall \mathbf{x}_n \in \mathcal{X}_n \tag{2.5}$$

More generally, we can define the concept of mixed strategies Nash Equilibrium:

Definition 3. ***(mixed-Nash Equilibrium)*** *A mixed strategy profile* $\sigma^* = (\sigma_n^*, \sigma_{-n}^*) \in \Delta(\mathcal{X})$ *is a mixed Nash Equilibrium if, for all players* $n \in \mathcal{N}$*:*

$$u_n(\sigma_n^*, \sigma_{-n}^*) \geqslant u_n(\sigma_n, \sigma_{-n}^*) \quad \forall \sigma_n \in \Delta_n(\mathcal{X}_n) \tag{2.6}$$

Even though both pure and mixed strategy Nash Equilibrium exist, in this thesis, whenever the term Nash Equilibrium is used, it refers to pure strategy NE. When players choose strategies that are best responses to each other, no player has an incentive to deviate to an alternative strategy. Therefore, the Nash Equilibrium is a stable strategy profile, with no "force" pushing the system toward a different outcome. Consequently, NE is an interesting solution concept, because it predicts an steady-state strategy profile. Moreover, it has been proven that every finite game has at least one (mixed) Nash Equilibrium:

Theorem 1. ***Nash [1951]*** *Every finite game has a mixed strategy Nash Equilibrium.*

Other solution concepts exist, but they are not the focus of this thesis. Readers interested on the topic are referred to Osborne and Rubinstein [1994], Fudenberg and Tirole [1991], and Roughgarden [2016].

2.5 Popular Examples of Non-Cooperative Games

To illustrate the concepts presented in the last sections, we give some popular examples next. They are finite non-cooperative games with two players and two strategies, thus represented in matrix form.

2.5.1 Prisoners' Dilemma

In this popular game, two suspects of a crime are put in separate rooms, and they have a choice between confessing to the crime or remaining silent. The set of players is $\mathcal{N} = \{P1, P2\}$, and the set of strategies is equal for them $\mathcal{X}_{P1} = \mathcal{X}_{P2} = \{\text{Confess } (C), \text{Silent } (S)\}$. Their payoffs are defined as follows: if they both confess, they will be sentenced to four years in prison $u_{P1}(C,C) = u_{P2}(C,C) = -4$; if only one confess, his/her term will be reduced to one year and he/she will be used as a witness against the other, who will get a sentence of five years $u_{P1}(C,S) = u_{P2}(C,S) = -1$ and $u_{P1}(S,C) = u_{P2}(S,C) = -5$; if both remain silent, they will both be convicted of a minor offence and spend two years in prison $u_{P1}(S,S) = u_{P2}(S,S) = -2$. There are four possible outcomes depending on the prisoners' choices. They can be represented in matrix form as in table 1, in which the rows are suspect 1's choices (P1), the columns are suspect 2's choices (P2), the values represent their utilities (u_{P1}, u_{P2}), and the best responses of each player to the other player's strategies are underlined.

Table 1 – Prisoners' Dilemma payoff matrix.

P1\P2	Confess (C)	Silent (S)
Confess (C)	-4, -4	-1, -5
Silent (S)	-5, -1	-2, -2

This game is a dilemma because, even though there are gains from cooperation—the best outcome for the players is that both remain silent, there is an incentive for confessing. Therefore, the only Nash Equilibrium is that both confess, leading to a Pareto inefficient solution.

2.5.2 Battle of the Sexes

In the Battle of Sexes (BoS), another popular game, a couple is deciding on a date $\mathcal{N} = \{\text{Man } (M), \text{Woman } (W)\}$. They both consider two possibilities: going to a baseball game or going to the cinema $\mathcal{X}_M = \mathcal{X}_W = \{\text{Baseball } (B), \text{Cinema } (C)\}$. The man prefers going to the cinema, and the woman to a baseball game, but they both prefer spending time together rather than separately. Again, the outcomes of this game can be represented in matrix form as in table 2, in which the rows are the man's choices, the columns are the woman's choices, the values represent their utilities (u_M, u_W), and the best responses of each player to the other player's strategies are underlined.

Table 2 – Battle of Sexes payoff matrix.

Man\Woman	Baseball	Cinema
Baseball	1, 2	0, 0
Cinema	0, 0	2, 1

The game has two Nash Equilibria: (Baseball,Baseball) and (Cinema,Cinema). In general, BoS models situations in which players wish to coordinate, but have conflicting interests [Osborne and Rubinstein, 1994].

2.5.3 Matching Pennies

In matching pennies, two players choose either Head or Tail. If their choices are the same, player 1 pays player 2 a dollar; if they differ, player 2 pays player 1 a dollar. Each player cares only about how much money he/she receives. The matrix of this game outcomes is shown in table 3, in which the rows are player 1's choices (P1), the columns are player 2's choices (P2), the values represent their utilities (u_{P1}, u_{P2}), and the best responses of each player to the other player's strategies are underlined.

Table 3 – Matching Pennies payoff matrix.

P1\P2	Head	Tail
Head	-1, $\underline{1}$	$\underline{1}$, -1
Tail	$\underline{1}$, -1	-1, $\underline{1}$

This game has no pure-strategy Nash Equilibrium, because players' interests are diametrically opposed—it is a "strictly competitive" game. However, as stated by Nash [1951] in theorem 1, a mixed strategy NE exists. If both players randomize equally between actions, i.e. $\sigma_n(\text{Head}) = 0.5$ and $\sigma_n(\text{Tail}) = 0.5$, they both have expected payoffs of zero, i.e. $u_n(\sigma_n, \sigma_{-n}) = 0$, and none of them can be better of by playing a pure strategy.

2.6 Potential Games

Potential games are a category of games with an associated function—the *potential function*—that maps the game strategy space $\mathcal{X}$ to the set of real numbers $\mathbb{R}$. Mathematically, various types of potential games can exist, depending on the relationship between the potential function and the players' utility functions. Moreover, they have many interesting properties in game theory, that are useful for proving the existence of pure strategy Nash Equilibria, locating them, and analyzing the convergence/applicability of "myopic" learning dynamics.

The term was coined by Monderer and Shapley [1996], which investigated the characteristics/properties of those games and presented their fundamental results. It listed four types of potential games, i.e. ordinal, weighted, exact, and generalized. They are of interest for the development of this thesis and we are going to discuss them next.

Definition 4. ***(Exact Potential Games)*** *The game Γ is an exact potential game, if there exists a potential function $\phi : \mathcal{X} \mapsto \mathbb{R}$ such that, for every player $n \in \mathcal{N}$ and for every* *opponents' strategy* $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$*, it holds that:*

$$u_n(\mathbf{x}_n, \mathbf{x}_{-n}) - u_n(\mathbf{x}'_n, \mathbf{x}_{-n}) = \phi(\mathbf{x}_n, \mathbf{x}_{-n}) - \phi(\mathbf{x}'_n, \mathbf{x}_{-n}), \quad \forall \mathbf{x}_n, \mathbf{x}'_n \in \mathcal{X}_n \tag{2.7}$$

The function ϕ is called a potential for the game Γ, and is the same for every player. In such games, changing the utility of a single player by swapping his strategy implies the same amount of change in the potential function, and vice versa. If Γ is a continuous game—i.e. the strategy sets $\mathcal{X}_n$ of all player $n \in \mathcal{N}$ are continuous intervals of real numbers, and the utility functions $u_n(\cdot)$ of all players are continuous and differentiable—a similar definition can be stated. A continuous game Γ is an exact potential game if, for every $n \in \mathcal{N}$ and for all opponents' strategy $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$, it holds that:

$$\frac{\partial u_n(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n} = \frac{\partial \phi(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n}, \quad \forall \mathbf{x}_n \in \mathcal{X}_n \tag{2.8}$$

Exact potential games form the more strict type of potential games. Although other categories can be defined by relaxing the exact equality constraint (2.7), exact potential games have more interesting properties and have received more attention in both theoretical research and practical applications [Lã et al., 2016]. It is important to notice that more than one potential function can be defined for the game Γ.

Definition 5. ***(Weighted Potential Games)*** *The game* Γ *is a weighted potential game, if there exists a weighted potential function* $\phi : \mathcal{X} \mapsto \mathbb{R}$ *and a weight vector* $g = (g_n)_{n \in \mathcal{N}}$ *of positive numbers, such that, for every player* $n \in \mathcal{N}$ *and for every opponents' strategy* $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$*, it holds that:*

$$u_n(\mathbf{x}_n, \mathbf{x}_{-n}) - u_n(\mathbf{x}'_n, \mathbf{x}_{-n}) = g_n \left[\phi(\mathbf{x}_n, \mathbf{x}_{-n}) - \phi(\mathbf{x}'_n, \mathbf{x}_{-n}) \right] \quad \forall \mathbf{x}_n, \mathbf{x}'_n \in \mathcal{X}_n \tag{2.9}$$

Similar to the exact version, the function ϕ is called a weighted potential for the game Γ. Moreover, the change in player's payoff due to an strategy swap also equals a change in the potential function, but adjusted by a weight factor. Clearly, when $g_n = 1$ for every $n \in \mathcal{N}$, the game is an exact potential game. We can rewrite equation (2.9) for continuous games to be weighted potential as, for every $n \in \mathcal{N}$ and for all opponents' strategy $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$, it must hold that:

$$\frac{\partial u_n(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n} = g_n \frac{\partial \phi(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n}, \quad \forall \mathbf{x}_n \in \mathcal{X}_n \tag{2.10}$$

Weighted potential games can always be transformed in exact potential games by diving the players' utility functions by their weights. Therefore, they have the same properties.

Definition 6. ***(Ordinal Potential Games)*** *The game* Γ *is an ordinal potential game, if there exists an ordinal potential function* $\phi : \mathcal{X} \mapsto \mathbb{R}$*, such that, for every player* $n \in \mathcal{N}$ *and for every opponents' strategy* $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$*, it must hold that:*

$$u_n(\mathbf{x}_n, \mathbf{x}_{-n}) - u_n(\mathbf{x}'_n, \mathbf{x}_{-n}) > 0 \iff \phi(\mathbf{x}_n, \mathbf{x}_{-n}) - \phi(\mathbf{x}'_n, \mathbf{x}_{-n}) > 0 \quad \forall \mathbf{x}_n, \mathbf{x}'_n \in \mathcal{X}_n \quad (2.11)$$

Differently from exact potential games, ordinal potential games only require that both changes—of players' utility and ordinal potential function—have the same sign when switching some strategy. In other words, if the strategy swap leads to a larger (or smaller) payoff, then the ordinal potential function of the same strategy swap must lead to a larger (or smaller) value, and vice versa.

We can also define a rule for a continuous game to be an ordinal potential game as, for every $n \in \mathcal{N}$ and for all opponents' strategy $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$, it holds that:

$$\text{sign}\left(\frac{\partial u_n(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n}\right) = \text{sign}\left(\frac{\partial \phi(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n}\right), \quad \forall \mathbf{x}_n \in \mathcal{X}_n \quad (2.12)$$

Where $\text{sign}(\cdot)$ is the signum function.

Definition 7. ***(Generalized Ordinal Potential Games)*** *The game* Γ *is a generalized ordinal potential game, if there exists a generalized potential function* $\phi : \mathcal{X} \mapsto \mathbb{R}$*, such that, for every player* $n \in \mathcal{N}$ *and for every opponents' strategy* $\mathbf{x}_{-n} \in \mathcal{X}_{-n}$*, it holds that*

$$u_n(\mathbf{x}_n, \mathbf{x}_{-n}) - u_n(\mathbf{x}'_n, \mathbf{x}_{-n}) > 0 \Rightarrow \phi(\mathbf{x}_n, \mathbf{x}_{-n}) - \phi(\mathbf{x}'_n, \mathbf{x}_{-n}) > 0 \quad \forall \mathbf{x}_n, \mathbf{x}'_n \in \mathcal{X}_n \quad (2.13)$$

In this case, a change in a player's utility function due to an unilateral strategy deviation must imply a change in the generalized ordinal potential with the same sign, but the reverse must not be true. Therefore, ordinal potential games form a subset of the generalized version.

The relationship between the aforementioned classes of potential games can be established as follows. Consider E, W, O, and G the classes of finite exact, weighted, ordinal, and generalized potential games, respectively. Then $E \subset W \subset O \subset G$. The proof of this result can be found in Lã et al. [2016].

We now present some important properties of potential games, related to the existence of pure strategy Nash Equilibrium. Other valuable characteristics about convergence to these equilibria are going to be discussed in section 2.7.

Theorem 2. ***Monderer and Shapley [1996]*** *Every finite ordinal potential game has at least one pure strategy Nash Equilibrium.*

Proof. Suppose that a pure strategy profile $\mathbf{x}^*$ corresponds to the global maximum of an ordinal potential function ϕ of a game Γ. Then, for any $n \in \mathcal{N}$, by the definition of a global maximum, $\phi(\mathbf{x}_n^*, \mathbf{x}_{-n}^*) - \phi(\mathbf{x}_n, \mathbf{x}_{-n}^*) \geqslant 0$ for all $\mathbf{x}_n \in \mathcal{X}_n$. Since ϕ is an ordinal potential for Γ, then, by definition 6, it holds that $u_n(\mathbf{x}_n^*, \mathbf{x}_{-n}^*) - u_n(\mathbf{x}_n, \mathbf{x}_{-n}^*) \geqslant 0$ for all $n \in \mathcal{N}$ and for all $\mathbf{x}_n \in \mathcal{X}_n$. Therefore, $\mathbf{x}^*$ is a pure-Nash Equilibrium. Moreover, since the game is finite, $\mathcal{X}$ is bounded and a maximum for ϕ always exists. □

It is interesting to notice that other pure strategy Nash Equilibra may also exist, corresponding to local maxima. We can establish a similar result for continuous games:

Theorem 3. *Every continuous ordinal potential game whose strategy space $\mathcal{X}$ is compact—i.e. closed and bounded—has at least one pure strategy Nash Equilibrium. In addition, if the potential function is strictly concave, the NE is unique.*

The same argument for proving theorem 2 holds for continuous ordinal potential games with compact strategy space. Note that these results apply to (finite or continuous) exact and weighted potential games as well [Lã et al., 2016]. Moreover, it is possible to establish a general condition for a continuous game to be potential:

Theorem 4. ***Monderer and Shapley [1996]*** *Let Γ be a game in which the strategy sets are intervals of real numbers. Suppose the payoff functions are twice continuously differentiable. Then Γ is a potential game if and only if:*

$$\frac{\partial^2 u_n(\mathbf{x}_n, \mathbf{x}_{-n})}{\partial \mathbf{x}_n \partial \mathbf{x}_m} = \frac{\partial^2 u_m(\mathbf{x}_m, \mathbf{x}_{-m})}{\partial \mathbf{x}_n \partial \mathbf{x}_m} \quad \forall n, m \in \mathcal{N} \tag{2.14}$$

It is important to highlight that having a pure strategy Nash Equilibrium is more powerful than having a mixed strategy NE—as demonstrated by John Nash in theorem 1 for all finite games—because the result of the game is more predictable and stable in the case of a pure NE. Moreover, most practical applications would prefer their systems to operate at one stable point rather than oscillating among multiple states.

2.6.1 Examples of Non-Cooperative Potential Games

2.6.1.1 Prisoners' Dilemma

The prisoners' dilemma described in section 2.5.1 is an exact potential game. A potential function in matrix form can be written as in table 4.

Table 4 – Prisoners' Dilemma potential function.

P1\P2	Confess	Silent
Confess	y	$y-1$
Silent	$y-1$	$y-2$

If player 1 switches from confessing to remaining silent while prisoner 2 confesses, his/her payoff increases by one: $u_{P1}(C,C) - u_{P1}(S,C) = 1$. If he/she does the same swap while prisoner 2 remains silent, his/her payoff increases by one: $u_{P1}(C,S) - u_{P1}(S,S) = 1$. The same happens with prisoner 2. Those changes lead to the same difference on the potential function, i.e. $\phi(C,C) - \phi(S,C) = y - y + 1 = 1$, and $\phi(C,S) - \phi(S,S) = y - 1 - y + 2 = 1$. Therefore, table 4 is an exact potential function for this game for any $y \in \mathbb{R}$.

2.6.1.2 An Ordinal Potential Game

The ordinal potential game in table 5 is a variant of the prisoner's dilemma.

Table 5 – An ordinal potential game as a variant of the Prisoners' Dilemma.

P1\P2	Confess	Silent
Confess	<u>-5</u>, <u>-4</u>	<u>-1</u>, -8
Silent	-7, <u>-1</u>	-3, -2

This version of the prisoner's dilemma is an ordinal potential game with same potential function as described in table 4. If player 1 switches from confessing to remaining silent while prisoner 2 confesses, his/her payoff increases by two: $u_{P1}(C,C)-u_{P1}(S,C) = 2$. If he/she does the same swap while prisoner 2 remains silent, his/her payoff increases by two: $u_{P1}(C,S) - u_{P1}(S,S) = 2$. If prisoner 2 switches from confessing to remaining silent while prisoner 1 confesses, then $u_{P2}(C,C) - u_{P2}(C,S) = 4$. If he/she does the same swap while prisoner 1 remains silent, then $u_{P2}(S,C) - u_{P2}(S,S) = 1$. Therefore, all the changes have positive sign, as in the changes in the potential function.

2.6.1.3 A Generalized Ordinal Potential Game

The game in table 6 is a generalized potential game from Lã et al. [2016]. Its generalized ordinal potential function is written in table 7.

One can notice that $\phi(1A,2A) - \phi(1A,2B) > 0$ does not imply $u_2(1A,2A) - u_2(1A,2B) > 0$, but the opposite is true for all utility changes that are positive: $u_2(1B,2A)-$

Table 6 – A generalized ordinal potential game.

	2A	2B
1A	4, 3	3, 3
1B	3, 4	4, 3

Table 7 – Potential function of a generalized ordinal potential game.

	2A	2B
1A	3	0
1B	2	1

$u_2(1B, 2B) > 0$ implies $\phi(1B, 2A) - \phi(1B, 2B) > 0$, and $u_1(1A, 2A) - u_1(1B, 2A) > 0$ implies $\phi(1A, 2A) - \phi(1B, 2A) > 0$.

2.7 Learning in Games

So far, we have defined what a game is, introduced the idea of players, strategies and utilities, explained which are the outcomes of a game (solution concepts), and explained the idea of potential games. However, some questions remain open: how do we calculate the Nash Equilibria? If we let the users interact freely, do they play a Nash Equilibrium? Which decision rules and mechanisms do we need to implement to guarantee that players would play an NE?

To analyze the first two questions, we discuss the origin of the NE notion in game theory. Equilibrium as a solution concept arose from the idea that rational players, in a context of complete information[2], reason about the situation and play the best option as a response to the best opponents' options, and so on. However, in a practical context, identifying the equilibrium point can be arduous, and the complexity grows with the number of strategies and players of the game.

As an example, consider a game between 18 graduate students, whose rules are: each student writes an integer between 0 and 100 in a paper; the professor collects all numbers and calculate the average; the student whose guest is closest to two thirds of the average wins and takes the prize; the other players receive nothing; if there is a tie, the prize is divided equally among the winners; students are not allowed to communicate between themselves. In this case, all players know the rules, the available strategies, and the payoff functions. Moreover, graduate students can be considered rational. Therefore, the Nash Equilibrium is expected to be played in this case. However, although the unique

[2] Players' strategies and payoff functions are common knowledge.

pure strategy NE is everybody choosing 0, many experiments have shown that it fails to predict the behavior of the students [Nagel, 1995].

More technically, many algorithms have been proposed in the literature to solve the NE computation problem, e.g. linear programming models in the case of 2-person zero-sum games [Moulin and Vial, 1978], Lemke-Howson algorithm for 2-person general-sum games [Shapley, 1974], and iterative elimination of dominated strategies [Rapoport and Amaldoss, 2000]. However, they are specific for a restricted number of games (2-person in general) and/or are limited (for example, if the NE is composed by strictly dominant strategies). Moreover, Daskalakis et al. [2009] proved that the problem of finding a Nash Equilibrium is PPAD-complete, and, therefore, there is no polynomial algorithm to solve it. In addition, with those algorithms, each player would have to access others needs and preferences (complete information) to compute locally the NE and play it, which would demand a lot of computational processing. Finally, in the case of multiple Nash Equilibria, miscoordination can occur. In the last two cases, a central agent would be required to calculate the (multiple) NE and determine which one should be played.

With the graduate students example and the above discussion of NE computation complexity and implications, we can conclude that NE is not easily calculated and, if players interact freely, they do not necessarily play the NE at first. For the third question, we resume the example of the students. Nagel [1995]'s results show that, at first, some students rationalize and succeed to play the Nash Equilibrium 0, but not everybody. After some rounds of the game, however, the students learn the game behavior and it converges to the aforementioned equilibrium. Therefore, the NE calculation is made via iterative plays between the agents and, even without a control mechanism, the players succeed to play the equilibrium. This iterative gameplay is known as learning model, developed from an alternative idea that equilibrium actually arises as the long-run outcome of a repeated play between rational players with (in)complete information. Therefore, we use this notion to develop decision rules and mechanisms to guarantee that players, at the end of the learning process, will choose a Nash Equilibrium.

More specifically, we look at how the players can achieve an NE, particularly in potential games. We focus on myopic learning algorithms and present a popular sequential decision dynamics—best response dynamics (BRD). We discuss aspects of its convergence in potential games, as those games are going to be used as a modeling tool in this thesis.

2.7.1 Best Response Dynamics

The Best Response Dynamics (BRD) consists of an alternating decision process in which users take turn to play in sequence. At each stage of the learning process k, one player $n \in \mathcal{N}$ selects a new strategy $\mathbf{x}_n^k$ that corresponds to the best response to opponents' previous strategies $\mathbf{x}_n^k = BR(\mathbf{x}_{-n}^{k-1})$. Therefore, this learning rule is based on the concept of Best Response introduced in definition 1. We give a detailed explanation of this dynamics in section 5.8.

It is interesting to notice that this learning rule is myopic, because players only consider last actions in the decision process without taking into account future payoffs. Moreover, players do not need to know their opponents' payoffs and strategy sets. Therefore, this algorithm turns the complete information scenario into an incomplete one, in which players handle with less data and have more privacy.

Even though this learning approach is decentralized, one can observe that sequential decision making involves a method to define the order of the consumers to take turn in the optimization process, and to ensure that no two consumers update their consumption vectors at the same time. This alternating characteristic is essential for the Best Response Dynamics to converge, and its practical implications are outside of the scope of this thesis—interested readers are referred to Christodoulou et al. [2012], Engelberg et al. [2013], Durand and Gaujal [2016], Durand et al. [2018, 2019].

Best Response Dynamics is known to converge to a pure strategy NE for ordinal potential games, and, thus, for exact potential games. To prove this, we need to introduce other concepts and theorems from the literature.

Definition 8. ***(Finite Improvement Path)*** *A path is a sequence $\gamma = (\mathbf{x}^0, \mathbf{x}^1, \mathbf{x}^2, \dots)$ of strategy profiles such that, for every $k \geqslant 1$, there exists a unique player n, called the "deviator", such that $\mathbf{x}^k = (\mathbf{x}_n^k, \mathbf{x}_{-n}^{k-1})$ for some $\mathbf{x}_n^k \neq \mathbf{x}_n^{k-1}$, $\mathbf{x}_n^k \in \mathcal{X}_n$. The path γ is an improvement path if, for all $k \geqslant 1$, $u_n(\mathbf{x}_n^k, \mathbf{x}_{-n}^{k-1}) > u_n(\mathbf{x}_n^{k-1}, \mathbf{x}_{-n}^{k-1})$, where n is the deviator at stage k. γ is a finite improvement path if it has a terminal point.*

Theorem 5. *If a strategic game Γ has a finite improvement path, then its terminal point is a Nash Equilibrium.*

Proof. Let γ be a finite improvement path with terminal point $\mathbf{x}^K$. If $\mathbf{x}^K$ is not a Nash Equilibrium, then there exists a player n who can improve his utility by unilaterally deviating his current strategy $\mathbf{x}_n^K$ to a new one, say $\mathbf{x}_n^{K+1} \neq \mathbf{x}_n^K$. Therefore, we could add this new strategy to the improvement path γ, which contradicts the initial assumption that the path was a finite improvement path with terminal point $\mathbf{x}^K$. $\square$

Theorem 5 shows that any decision dynamic capable of generating a finite improvement path converges to a Nash Equilibrium. Therefore, the Best Response Dynamics converges to the NE as long as, at each stage k, one consumer improves his utility in response to opponents' previous strategies. We need now to prove that potential games have finite improvement paths.

Definition 9. ***Finite Improvement Property (FIP)*** *A game* Γ *has the Finite Improvement Property if every improvement path is finite.*

Theorem 6. *Monderer and Shapley [1996] Every finite ordinal potential game has the Finite Improvement Property.*

Proof. The definition of ordinal potential games (6) implies that an improvement in the utility function of a player results in an improvement in the ordinal potential function. Therefore, for every improvement path $\gamma = (\mathbf{x}^0, \mathbf{x}^1, \mathbf{x}^2, \ldots)$, it holds that

$$\phi(\mathbf{x}^0) < \phi(\mathbf{x}^1) < \phi(\mathbf{x}^2) < \ldots . \tag{2.15}$$

As $\mathcal{X}$ is a finite set (by the definition of finite games), the sequence γ must be finite. Moreover, by theorem 5, its terminal point is a Nash Equilibrium. □

With theorem 6 we assure that finite potential games have finite improvement paths, which implies that Best Response Dynamics converges to Nash Equilibria for those games.

3. Basic Concepts on Power Systems and Energy Markets

3.1 Smart Grid

The existing power grid was built in the 20th century as a hierarchical and centrally controlled network composed by several large synchronous generators, a transmission, and a distribution electrical network. It is unidirectional in nature and generates electricity in large power plants, located faraway from the consumption sites, which thereby has to be transmitted in long distances, and distributed to the final users. In this "dumb model", there is a big loss of electricity in the transmission process, and a meaningful part of generation and transmission capacity exists only to meet the peak demand [Farhangi, 2010, Le Ray and Pinson, 2020]. Besides, the hierarchical topology causes domino-effect failures [Farhangi, 2010]. Moreover, the key elements and principles of planning, operation, and control of this power system model were developed before the rise of computer and communication networks [Butt et al., 2020]. In addition, many components of the grids are reaching the end of their life cycle [Yoldaş et al., 2017]. Furthermore, this power grid model suffers with the fossil fuel depletion, which raises the operational costs, and are related to the climate changes. Finally, the existing infrastructure does not accommodate the growing demand for electricity [Farhangi, 2010].

To address these challenges, the next-generation electricity grid, known as Smart Grid, is expected to be more efficient, autonomous, reliable, secure, resilient, and flexible [Energy Independence and Security Act, 2007, Farhangi, 2010, Le Ray and Pinson, 2020, Yoldaş et al., 2017, Butt et al., 2020]. For this to happen, two-way flows of electricity and information, distributed control, monitoring systems and advanced technologies are added to the layers of the power grid—generation, transmission, distribution, and consumption. This modernization transforms the grid into a self-monitoring, self-healing and adaptive network. Moreover, it allows the deployment and integration of renewable resources, distributed generation (DG), and demand-side management (DSM) programs. The changes expected for moving the traditional "dumb" power system towards a smarter grid are shown in figure 1.

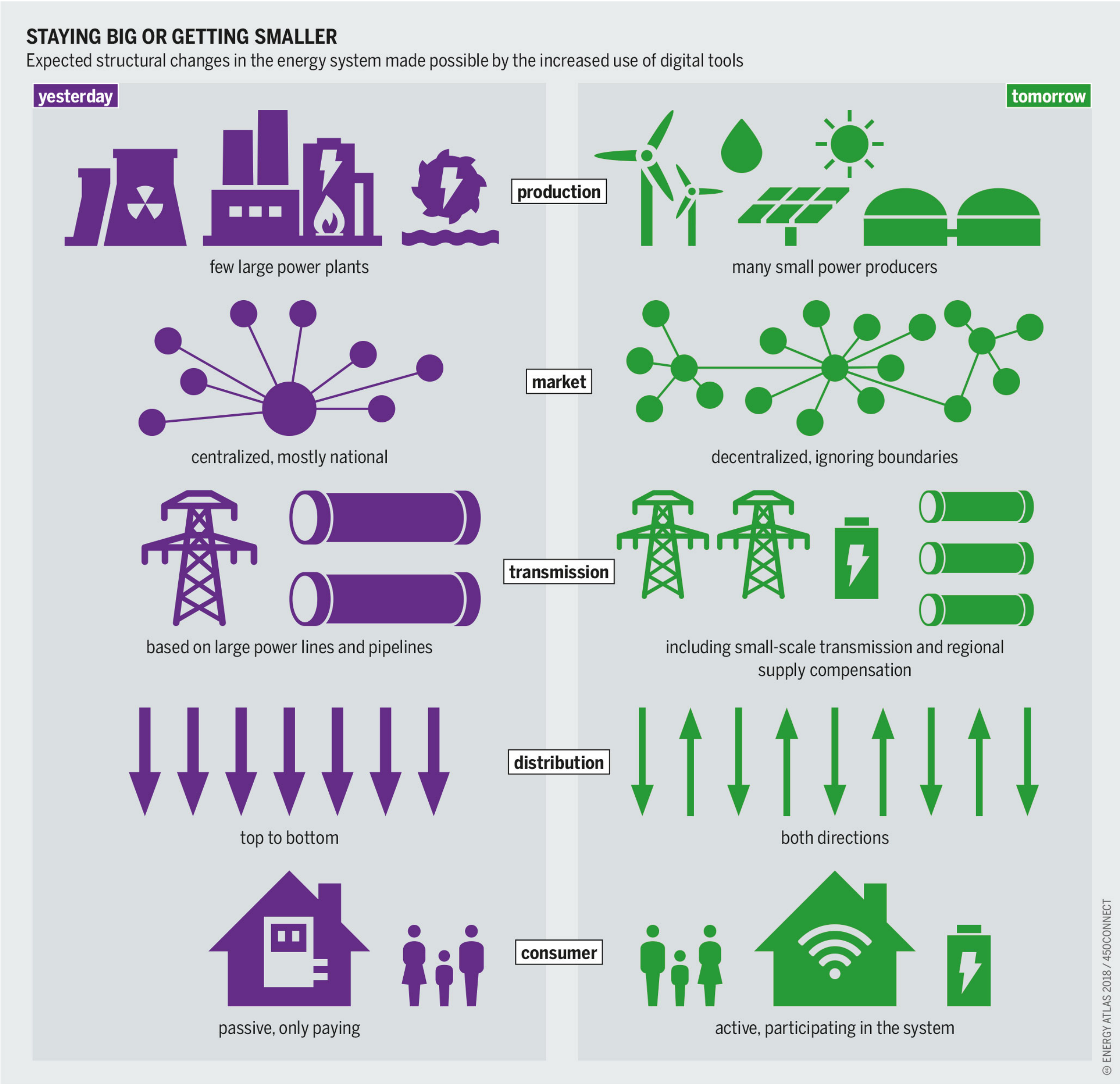


Figure 1 – Moving towards a smart grid.
Source: Atlas [2018, p. 33]

The origin of the Smart Grid concept is undetermined, because its developments happened alongside with the traditional grid [Tuballa and Abundo, 2016]. For example, the utilities' chronic concern about managing peak loads induced the design of automatic meter reading devices back in the 1970s [IEEE Global History Network, 2020]. They were the beginning of the smart meters, a basic requirement of any intelligent grid system. Moreover, other concerns related to congestion, atypical power flows, and large-scale cascade failures in the highly interconnected traditional grid brought computational solutions for protection, control, and operation of the systems components, substations, and power plants [Butt et al., 2020]. In line with this evolution, the US federal government released the Energy Independence and Security Act of 2007 with a section about Smart Grids, supporting the modernization of the grid, establishing an advisory committee and a task force, and creating a federal matching fund for Smart Grid investments [Energy Independence and Security Act, 2007]. Since then, a lot of research and development have been done on Smart Grids, and communities recognize the importance of transitioning to this new model.

Therefore, many solution trials and Smart Grid demonstration projects have been put in place by state programs and initiatives in the developing countries—see [Tuballa and Abundo, 2016, Yoldaş et al., 2017, Ault, 2017, Butt et al., 2020]. Even though they promote advanced information and communication technologies, propose solutions to enhance customer participation, develop low carbon appliances, and study methods to increase renewable energy participation, there are still many questions to be addressed to make smart grids business as usual. Ault [2017] discusses some of these key challenges as the need to replace old network assets, the adjustments in established electrical sector norms and legislation, the capital and operating high costs of the enabling Smart Grid infrastructure, the lack of organizational/institutional skills to plan, build, operate, and maintain the new and advanced equipment, and the risks related to these new systems. Moreover, another big challenge in the Smart Grid context is the coordination of an ever-growing number of intelligent devices that will become actively involved in the system operation and control, and will require innovative mechanisms and solutions [Kok and Widergren, 2016]. Finally, revolutionary changes are not expected in the transmission network, because it is already reliable and controllable, thus being the distribution system the one with major opportunities for Smart Grid concepts [Kok and Widergren, 2016]. In this context, this research focuses on the development of tools to be used in the context of Smart Grids operation planning and addresses a big actual challenge of its deployment: the coordination of the new assets introduced at the distribution level, known as Distributed Energy Resources (DERs).

3.2 Distributed Energy Resources

Distributed Energy Resources (DERs) are usually defined as a set of technologies and strategies to be connected or used at the distribution level with the potential to make energy use more efficient, accessible, and environmentally sustainable [LBNL, 2017]. They can be classified in three categories:

Distributed Generation (DG): includes the technologies for producing electricity that are connected directly to the medium or low voltage distribution grid, including at consumer's side [Kok, 2013]. They can be fuel-fired and dispatchable or operate with renewable sources [Olivares et al., 2014, LBNL, 2017]. Some examples are cogeneration, photovoltaics (PV), gas turbines, micro-turbines, fuel cells, and wind-power.

Demand-side Management (DSM): includes demand management strategies (e.g. load shifting, demand response, and peak shaving) [LBNL, 2017] and flexible loads (e.g. air conditioning, washing machine, heat pumps and electric vehicles) [Hu et al., 2017].

Energy Storage Systems (ESS): includes the devices capable of bi-directional exchange of power with the distribution grid [Kok, 2013]. Some examples are thermal storage tanks, vehicle-to-grid solutions, flywheels, energy capacitors, and pumped hydro [Jiayi et al., 2008, Olivares et al., 2014, LBNL, 2017].

In line with the smart grid concept, DERs emerging technologies and strategies provide opportunities to solve some traditional grid problems. For instance, DG can reduce power losses in the transmission system, defer upgrades of the transmission and distribution grids, furnish ancillary services to the network and diminish the fossil fuel use [Lasseter, 2011, Allan et al., 2015]. Demand management strategies can reduce congestion and decrease the excess generation capacity to accommodate the peak demand [Hu et al., 2017]. Storage systems can help spreading renewable energy sources technologies and support the operation and control of the smart grids [Lasseter, 2011].

However, a massive introduction of individual distributed resources without proper operation and control systems/strategies can cause as many problems as it may solve [Jiayi et al., 2008]. The integration of DG units within the existing network can create issues related to cost of energy, price of electricity, infrastructure requirements, distribution system reliability and security (e.g. voltage control, power quality, protection system, fault level, grid losses) [Coster et al., 2011, Allan et al., 2015]. Moreover, many policies for installing DERs had focused on their connection, in a 'fit and forget' approach [Pudjianto et al., 2007], i.e. without considering those issues. In addition, a DER device primary reason of existence is not to provide services to the grid [Kok, 2013], which leads to the need of considering their owner's constraints. Therefore, managing significant levels of distributed energy resources, each with its dynamics, resources, and specific control characteristics, can become overwhelming [Lasseter, 2011]. Hence, those technical and regulatory challenges must be overcome to guarantee the maintenance of network security and reliability standards, taking the advantages of DERs potential [Olivares et al., 2014].

One way to manage the massive integration of distributed energy resources is to break the distribution network down into small subsystems, denominated microgrids, with distributed optimizing controls to coordinate their operation [Yoldaş et al., 2017]. In these smaller systems, the technical problems are solved in a decentralized fashion, diminishing the requirement for a complex central coordination mechanism, extremely ramified within the power system, and facilitating the accomplishment of the Smart Grid concept [Olivares et al., 2014]. Another way to manage this massive DERs integration is with a technical and/or commercial entity, called virtual power plants (VPP), also responsible for the coordination of the resources in a distribution fashion. A final way is to organize consumers in smart communities [Nan et al., 2018], where they can coordinate their energy utilization and manage their distributed resources [Cornélusse et al., 2019]. Those entities are sometimes referred as "aggregators". In the following section, we explain the concepts of microgrids, VPPs, smart communities, and aggregators, along with their opportunities, challenges, and role for the management and integration of distributed energy resources.

3.3 Aggregation Platforms for Distributed Energy Resources

3.3.1 Microgrid

Although the microgrid concept is age-old—Thomas Edison's first power plant, built in 1882, was essentially a microgrid [Asmus, 2010]—small autonomous grids in the traditional ac network emerged as a solution to remote communities where the supply of energy from the main power grid was not feasible due to techno-economical issues [Olivares et al., 2014]. These isolated systems were designed as miniatures of the power grids, with the same generation sources, operation techniques, and control methods. However, with the development of the Smart Grid concept and the progress of greener generation technologies, the microgrids evolved to a manner for increasing grid reliability, security and resilience [Jiménez-Estévez et al., 2017], as well as a solution for integrating distributed energy resources (DERs) [Hatziargyriou et al., 2007].

Microgrids are defined as a cluster of loads and energy micro-generators, operated as a single system, delivering electricity and heat to its local area [Lasseter, 2002]. They are connected to the LV or MV systems, have their energy resources controlled locally, and are seen from the grid perspective and energy markets as a single entity, producing or consuming energy [Hatziargyriou et al., 2007]. It means that, in their most frequent configuration, DERs are coupled together on their own feeder, and then linked to the grid at a single point of interconnection [Asmus, 2010, Olivares et al., 2014]. More recent definitions add the storage systems and the demand flexibility to the cluster of energy resources and highlight the importance of microgrids local operation for DERs coordination and system reliability and resilience [Olivares et al., 2014, Jiménez-Estévez et al., 2017]. An essential characteristic of a microgrid is the capacity to operate in grid-connected or island-mode [Asmus, 2010, LBNL, 2017]. Early definitions of microgrids focused on the islanding capability as a solution concept for grid resilience, thus serving critical infrastructures in emergency circumstances, like hospitals and communication antennas [Ton and Reilly, 2017, Jiménez-Estévez et al., 2017, Vaahedi et al., 2017]. However, as mentioned before, the definitions have evolved to include the management of DERs, characterizing an advanced microgrid [Ton and Reilly, 2017]. In this new paradigm, the small grids require controllers, working within their boundaries, to balance their electrical demand with their sources, schedule the dispatch of their resources, and guarantee grid reliability and resilience [Ton and Reilly, 2017]. Moreover, this control system is responsible for ensuring the transition from/to island-mode under transient conditions [Joos et al., 2017]. With the microgrids control technology, the DERs benefits to the power network can be assessed and explored.

There are many microgrid projects and pilots around the world, designed for various purposes, with different sizes, components and operational solutions [Venkata and Shahidehpour, 2017, Ton and Reilly, 2017]. They can be categorized based on the type of the consumers (e.g. commercial, industrial, community, campus, institutional, military etc.), the type of application (e.g. costs reduction, resilience enhancement, reliability improvement, community participation, cyber and physical security, remote areas energy supply etc.), the type of connection (remote or grid-connected), the electric current (ac, dc or mixed), the voltage levels, and the owner (utility or non-utility) [Joos et al., 2017, Maitra et al., 2017].

For instance, Uluski et al. [2017] details the Philadelphia Navy Yard (TNY) microgrid planning and implementation. It is located in a commercial/industrial area composed by 150 companies and four Navy activity centers, representing almost 7.5 million ft^2 of buildings, where nearly 12,500 people work. The authors describe the GridSTAR microgrid, an operational part of the TNY project composed by critical loads, DERs, monitoring, and control facilities. Jiménez-Estévez et al. [2017] presents three isolated community microgrid projects, two in Chile (Huatacondo and Ollagüe), and one in Mexico (Puertecitos), developed for enhancing distribution system resiliency in the case of high-impact, low-probability power system events. Shahidehpour et al. [2017] describes the Bronzeville community microgrid (BCM) project, in the city of Chicago, a plan to integrate essential city services, residential housing, and educational institutions, and to transform the region in a sustainable environment. The authors also discuss the idea to prototype networked microgrids to test coordinated control strategies, given that the community is adjacent to an existing microgrid on the campus of the Illinois Institute of Technology (IIT). Finally, Vaahedi et al. [2017] illustrates the concept of transactive microgrids with the Open Access Technology International Inc. (OATI) Microgrid Technology Center (South Campus), located in Bloomington, Minnesota. The South Campus microgrid was built for three purposes: economics, reliability and green/renewable energy desire, and incorporates a control layer with transactive operations with the energy market/main grid.

Despite this diversity, an electrical system must have three distinct features to be considered a microgrid, videlicet: well-defined electrical boundaries, a controller to manage and dispatch DERs within the micro-system, and installed generation capacity that surpasses the critical load [Joos et al., 2017]. Therefore, some components are related to these features and are traditionally integrated to the microgrids. Joos et al. [2017] and Maitra et al. [2017] present some of them (see Fig. 2 for further comprehension):

1. an isolating device at the point of interconnection (e.g. a breaker or similar), to connect/disconnect the microgrid to/from the LV or MV distribution system;
2. the microgrid controller, i.e. the energy management system (EMS);
3. local distribution system equipment (e.g. transformers, capacitors etc.);
4. devices for connecting the microgrid components (breakers); and
5. physical devices within the microgrid boundaries (DERs), electrical and thermal loads).

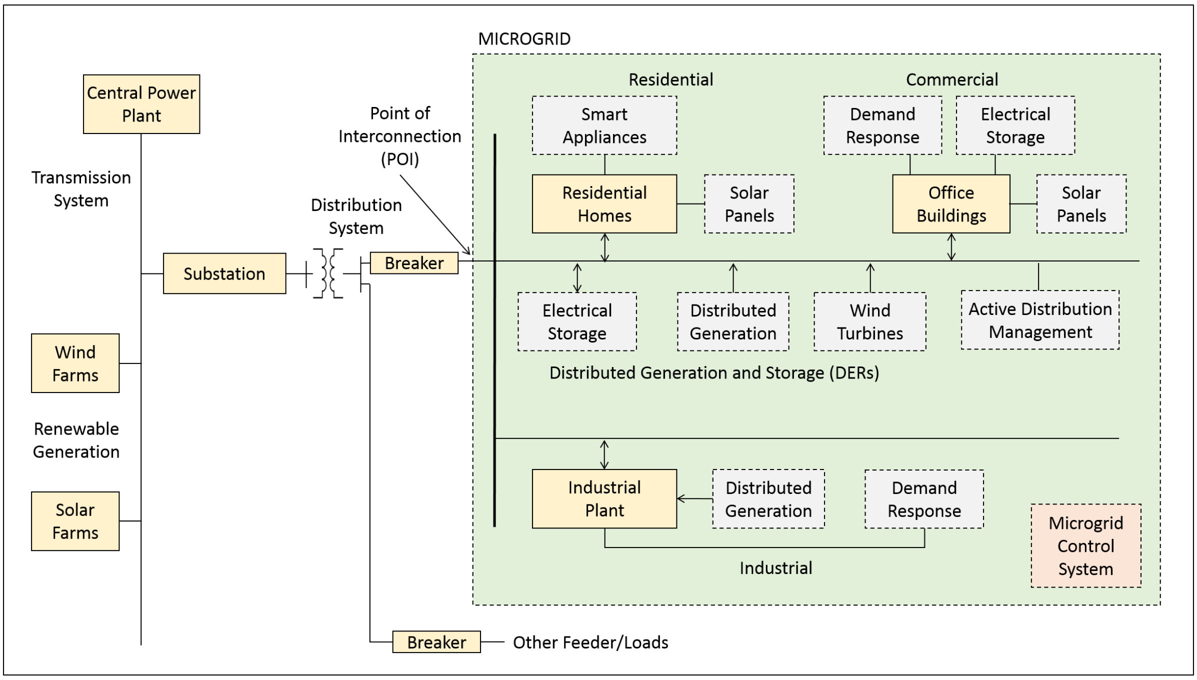


Figure 2 – A representation of a microgrid with its components and inner structure. Source: Adapted from Joos et al. [2017, p. 34]

Moreover, whatever is the configuration of the microgrid devices and DERs, its controller must perform some functions related to coordination of resources and loads, automatic transition from grid-connected to islanded-mode and back re-synchronization, energy management, and ancillary services provision [Joos et al., 2017]. In terms of the controller implementation, it can be composed by software and/or hardware, and can have a centralized or distributed control approach [Joos et al., 2017].

3.3.2 Virtual Power Plant

As discussed before, microgrids are responsible for the management, aggregation, and deployment of distributed energy resources (DERs), with the specific characteristic of islanding. Another choice to help introducing DERs in the smart grids is the concept of virtual power plants (VPPs). Asmus [2010, p. 75] describes VPPs as entities which "rely upon software systems to remotely and automatically dispatch and optimize generation or demand-side or storage resources in a single, secure web-connected system". Moreover, they not only aggregate the capacity of many types of DERs, they also create a single operating entity from a mix of the DERs parameters, and include spatial constraints into their energy management systems [Pudjianto et al., 2007]. For the last, given that many different resources can be included in a VPP, and they can be connected in various points of the networks, the network characteristics have to be frequently considered in its operation.

VPPs can be of many types and can aggregate diverse energy resources. In Europe, a typical VPP aggregates supply side resources, in general a diverse pool of renewable energy resources, at the distribution and/or transmission level. However, in the same region, there are VPP initiatives related to the aggregation of many consumers interested in negotiating their capacity purchase in the wholesale market. In the US, the term is more related to demand response and critical peak pricing programs [Asmus, 2010]. Therefore, the VPP concept can be used to promote the participation of DERs in the wholesale energy markets (e.g. day-ahead and real-time energy markets), including the provision of ancillary services to support the transmission system management (e.g. reserve market and frequency regulation) [Pasetti et al., 2018]. For that reason, VPPs can have a more commercial goal, related to market participation, or a more technical objective, related to the system management and support [Pudjianto et al., 2007]. In addition, a virtual power plant can also be characterized by a set of generation and/or controllable load parameters, depending on the types of DERs they have, e.g. generation or load scheduling, generation limits, stand-by capacity, ramp rates, load elasticity, among others [Pudjianto et al., 2007].

Even though VPPs and microgrids share some critical goals, features, and capabilities, they differ in some points [Asmus, 2010]:

- VPPs are always grid-tied, but microgrids can also be remote systems;
- VPPs do not have the capacity of islanding as microgrids have;
- VPPs may or may not have storage capacity, whereas microgrids, in general, need to have it;
- VPPs focus more on smart meters and software development, whereas microgrids depend upon hardware innovations, e.g. inverters and switches;

- VPPs can mix and match among many different resources over large geographic regions, whereas microgrids include a static set of resources in their fixed boundaries;
- VPPs can also negotiate in the wholesale market, whereas microgrids typically remain at the retail market;
- VPPs can be implemented under current regulatory structures and tariffs in many regions, whereas microgrids still face regulatory and political barriers.

It is worth mentioning that, regardless of the VPP and microgrid differences, an energy management system is also essential to the former operation.

3.3.3 Smart Energy Communities

Smart Energy Communities can be defined as a collective of prosumers managing their assets in a collaborative way [Moret and Pinson, 2019], in order to better fulfill the communities' needs through local decision processes [Koirala et al., 2016]. Those locally and collectively organized systems can integrate and coordinate a range of distributed energy resources as distributed generation, demand-side management strategies, flexible loads, and energy storage systems [Koirala et al., 2016]. Moreover, by adding smart grid technologies, the smart communities can increase the reliability and efficiency of the power system, as well as provide a range of ancillary services [Cornélusse et al., 2019].

Communities' participants, when organized in smart energy communities, coordinate their energy utilization and manage their distributed resources in a way to reduce their electricity bills and costs, increase their revenues, and use their assets more efficiently [Shaw-Williams and Susilawati, 2020, Cornélusse et al., 2019, Moret and Pinson, 2019]. Moreover, this architecture enables consumers to be more proactive and autonomous, giving them decision power to define how to manage their own assets [Koirala et al., 2016]. For this to happen, a market framework must be designed to allow local optimization of resources and general coordination of prosumers. They must be able to trade their resources with the community members, subjected to their needs and comfort constraints, and be rewarded (or billed) accordingly [Moret and Pinson, 2019]. Sousa et al. [2019] discusses two designs that applies to smart energy communities, which are depicted in figure 3:

- Full peer-to-peer (P2P) market: the trading/optimization process takes place in a peer-to-peer (P2P) environment, fully distributed, where consumers communicate with each other and exchange the necessary information, e.g. the Brooklyn microgrid project [Mengelkamp et al., 2018].

- Community-based market: a community manager (or aggregation platform) organizes the trading activities inside the community, and acts as an intermediary between the rest of the system and the community. This supervisory node is key to managing the interface between different markets, because the community members are joined in a single node. One example is the energy collectives model designed by Moret and Pinson [2019].

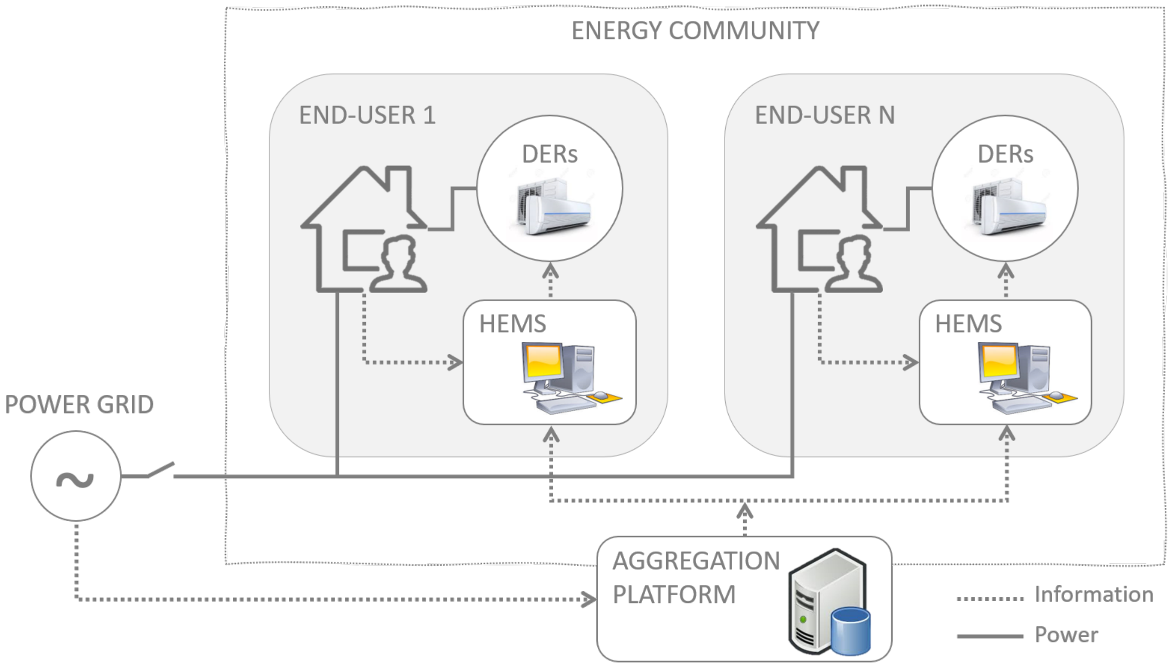


Figure 3 – A representation of a smart energy community. Consumers can communicate between themselves using a local network (full P2P market), or the trading process can be driven by a community manager (aggregation platform). In both situations, the DERs are locally managed by consumers' home energy management system (HEMS).

It is important to notice that smart community models are well aligned with the transactive control concept [Moret and Pinson, 2019], and allows the implementation of non-cooperative game methods studied in this work. Moreover, this concept is different from microgrids and virtual power plants, because it is based on the principle of collaborative economy [Sousa et al., 2019]. Therefore, VPPs and microgrids can be considered as smart communities, depending on the market architecture they implement, but the opposite is not necessarily true. Finally, having an energy management system is also key for the successful implementation and coordination of smart energy communities.

3.3.4 Aggregators

Another concept related to DERs aggregation, management, deployment, and operation is the "Aggregator", which has been used in the smart grid context to designate a new player of the electricity markets. Rahnama et al. [2014] defines aggregator as an entity responsible of a number of flexible consumers, with the objective of managing their energy/power demand. They are placed between a grid operator and the consumers, and their responsibilities can vary according to several factors, e.g. control strategies, provided services, demand types, among others. Generally speaking, the aggregator can be seen as a virtual power plant focused on the management of demand flexibility.

As the other three aggregation platforms, there are many types of aggregators, with multiple purposes and different activities. They can control directly the load of their consumers, being responsible for optimally operating a portfolio of distributed resources, or send control signal to the end-users, who respond locally. Moreover, the aggregators' intention may vary, which means they can offer multiple services when aggregating the resources. For instance, they can use their consumers flexibility for providing ancillary services to the grid, can manage user's loads for reducing their bills, can optimize user's consumption to follow a target power curve or for a valley-filling purpose, among others. In general, the aggregators work with demand flexibility, and many literature studies focus on electric vehicles coordination and control [Li et al., 2016a, Aghajani and Kalantar, 2017, Mediwaththe and Smith, 2018].

3.3.5 Final Remarks on Aggregation Platforms

In the framework of aggregation platforms for DERs integration, we focus on the energy management functionality for microgrids, virtual power plants, smart energy communities, and aggregators operation planning. All those platforms implement an energy management system (EMS) to coordinate energy generation and consumption, in other words, manage multiple distributed resources. EMSs develop optimal operating strategies in multiple time scales [Ton and Reilly, 2017]. The management system interfaces with the utilities and retail markets, and is an essential technology for the evolution of Smart Grids. Given the relevance of the controller for DERs implementation and progress in the electricity networks, we explain the energy management approaches, discussing their benefits and drawbacks in the following sections. To give a general overview of the subject, including the controlling techniques and methodologies, and analyze what can be applied in the context of microgrids, VPPs, and aggregators, we do not limit the discussion to EMSs for those four aggregation platform.

3.4 Energy Management Systems

Energy Management Systems (EMSs) are responsible for coordinating the supply and demand of energy at the transmission and distribution networks, as well as inside large consumption sites (e.g. industries with local generation). Their objectives are related to the development of strategies for the optimal techno-economical operation of the networks, in multiple time scales [Ton and Reilly, 2017, Olivares et al., 2014]. Optimizing the energy management, at planning and operation levels, results in preventing blackouts, enhancing security, reducing the impact on the environment, besides minimizing the electricity costs.

With the advent of the smart grids, many technologies, in terms of hardware and software, have being developed to improve the energy management process. In this work, we will use the EMS classification proposed by Kok and Widergren [2016], which focus on approaches related to smart grids at the distribution level and discusses their benefits and drawbacks. Fig. 4 depicts the four categories, their strengths (+) and negative points (-). The vertical axis represents whether the local issues (e.g. distributed generation operating and scheduling, load switching etc.) are decided locally or centrally; and the horizontal axis is related to the communication scheme—if it is unidirectional or bidirectional. Each category will be explained and discussed, with literature examples, on the following sections.

3.4.1 Top-Down Switching

This quadrant includes classical demand response programs, in which a group of devices (e.g. water heaters, air conditioners, lighting, pool pumps) are turned on/off simultaneously in response to a signal transmitted by the system operator or the utility [Kok and Widergren, 2016]. When related to consumers' load, this approach is called direct load control (DLC), and allows the distribution system operator to remotely manage end-user's appliances during peak periods and/or critical events. Consumers are generally rewarded for participating in DLC programs with financial incentives such as free hardware installation, electricity bill reductions, annual payment, among others [Stenner et al., 2017].

Even though this approach is simple and effective, it does not unleash the full potential of devices response, because it does not consider their status when taking the switching decision. As a result, the system reaction—of its agents and their devices—are not known a priori, thus being necessary to use statistics and worst-case scenario analysis for planning and operating the distribution network. Moreover, the method ignores the group of consumers and their preferences, interfering with their autonomy [Kok and Widergren, 2016]. Finally, motivating end-users to participate on top-down switching programs, as DLC, is not an easy task. Among the many barriers for consumers' engagement, Stenner et al. [2017] discusses the impact of users' trust and distrust in the utilities on their decision-making. With a field-based survey experiment, they found that respondents' self-professed distrust was related to a reduced willingness to register for DLC programs. Therefore, energy utilities, practitioners and policymakers must take into account those

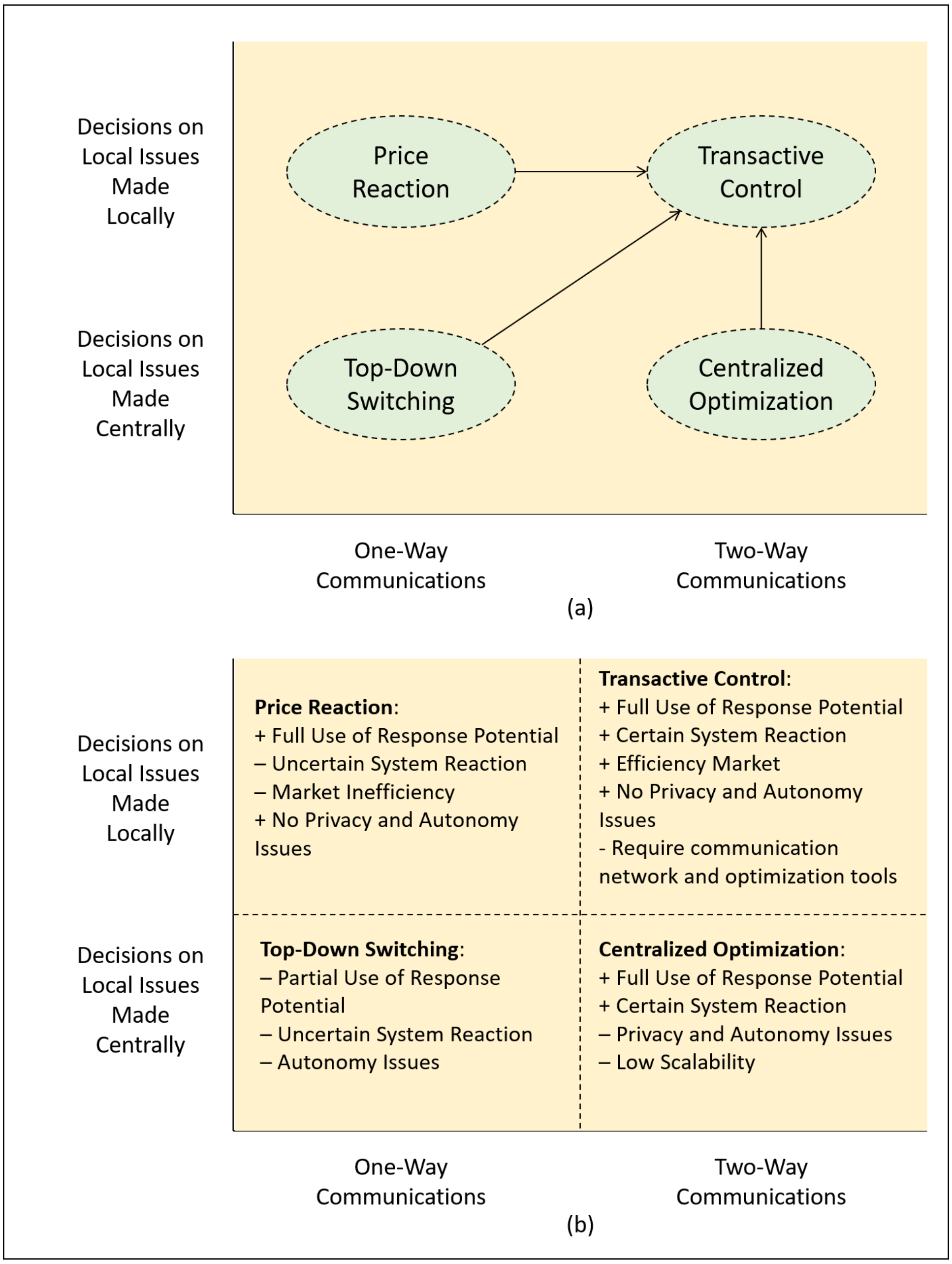


Figure 4 – Classification of energy management approaches for distribution systems.
Source: Adapted from Kok and Widergren [2016, p. 36]

barriers when deciding how to promote such approach.

This simple demand response model have been successfully employed around the world. For example, Brazil has an automatic procedure, called Regional Scheme for Load Relief (ERAC), responsible for the protection of its National Interconnected System (SIN). In sub-frequency situations, the National System Operator (ONS) switches off amounts of load [ONS, 2020]. The scheme does not focus on the smart grid operation at the distribution level, because it is a top-down approach for emergency control of the entire network frequency. However, it is possible to notice the response uncertainty in this example, since it is necessary to monitor constantly the load levels available for switching—they cannot deviate more then 10% from the established values—to guarantee the effectiveness of ERAC performance [ONS, 2020].

3.4.2 Centralized Optimization

In the centralized optimization approach, decisions on local issues are still taken centrally, but the communication is bidirectional. A complex optimization procedure coordinates the distributed energy resources (DERs) of the smart grid under analysis, e.g. a microgrid, a virtual power plant, a smart community, or a group of end-users represented by an aggregator. Therefore, all relevant information for the decision making process must be broadcasted to the central controller, which calculates the global optimum according to the objectives of the system, for example the optimal distributed generation dispatch and flexible loads scheduling that minimizes a microgrid operating costs [Kok and Widergren, 2016].

We discuss two types of central controller models: one focused on the management of users' appliances (e.g. heaters, air conditioners, charging of electric vehicles, washing machine etc.), also known as demand-side management (DSM); and a general DERs dispatch, which considers not only demand flexibility, but also distributed generation (e.g. PV panels, cogeneration etc.) and storage systems (e.g. batteries, vehicle-to-grid etc).

In the first, a centralized optimization for managing consumers' demand flexibility is designed and applied. Because a third party operated end-users' appliances, this model can be considered as a direct load control. An aggregator, the utility, or the distribution system operator (DSO) is responsible of a group of consumers and control their consumption pattern in order to reach some objective, like reduce their bills [Wu et al., 2012], bring the electricity demand curve as close as possible to an objective curve [Logenthiran et al., 2012] etc. This third party must receive the customers' information about consumption pattern, appliances, and preferences, as well as estimate correctly the market parameters (e.g. electricity prices). Moreover, it must charge their clients correctly.

In the second, the centralized optimization considers the three types of DERs. They are broadly applied in microgrid controllers, as a local EMS for the coordination of resources within the small network boundaries. In this case, typical dispatch models as unit commitment and economic dispatch can be adapted and applied to the smaller grid. The model's objective can be minimize the small grid's total energy costs [Araújo and Uturbey, 2013, Karthikeyan and Parvathy, 2015, Tang and Zhong, 2016], minimize consumers' bills [LBNL, 2017, Ghatikar et al., 2016, Heleno et al., 2017, Mashayekh et al., 2017, Armendariz et al., 2017, Nan et al., 2018], maximize renewable energy utilization [Karthikeyan and Parvathy, 2015], reduce CO_2 emissions [Karthikeyan and Parvathy, 2015], optimize the peak load factor [Karthikeyan and Parvathy, 2015, Nan et al., 2018], among others.

Some models have been applied in real microgrids. The Distributed Energy Resources Customer Adoption Model (DER-CAM), a physically-based economic optimization model for planning and controlling microgrids was developed by the Lawrence Berkeley National Laboratory (LBNL). It has been used in the University of New Mexico (UNM) [LBNL, 2016], the Fort Hunter Liggett [Ghatikar et al., 2016], the Santa Rita Jail Microgrid [LBNL, 2018], and in a real isolated microgrid in Alaska [Heleno et al., 2017]. Moreover, EPRI has developed the Integrated Grid Benefit-Cost Analysis Framework for Microgrids, a four-stage design evaluation model to help the technology selection and DERs assets sizing while accounting for the microgrid goals [Maitra et al., 2017]. It was applied to design the Buffalo Niagara Medical Center (BNMC) microgrid.

We now discuss the benefits and drawbacks of the centralized optimization approach. First, it is capable of unlocking the DERs response potential and flexibility if the relevant local information is available. Moreover, the reaction of the system participants is known a priori, because the optimization procedure controls directly the devices. The problems related to agents' autonomy of the previous approach remain, since the DERs are operated by a central program, and issues concerning information privacy are added, because local data about preferences and constraints are sent to the controller. Furthermore, communicating all local data to a central point, including equipment status changes, limits the accuracy and scalability of this approach. For the accuracy, detection of the communicating and optimizing failures is difficult to be implemented on the program. For the scalability, if the number of responsive homes, buildings and installations grows, the communication and optimization times rise exponentially [Kok and Widergren, 2016]. This is because these models are more complex and difficult to solve, considering that they have more variables to be coordinated. Generally, they are solved using heuristic methods [Logenthiran et al., 2012, Wu et al., 2012, Araújo and Uturbey, 2013, Karthikeyan and Parvathy, 2015, Heleno et al., 2017], linear programming [Tang and Zhong, 2016], or mixed-integer linear programming Ghatikar et al. [2016], Heleno et al. [2017], Mashayekh et al. [2017], Nan et al. [2018], which can be combinatorial.

It is worth mentioning that this approach aligns with the traditional operation and control method of the electrical systems. In general, the TSO controls the central power plants—using a SCADA—for balancing energy supply with demand, and the DSO manages, also in a centralized manner, the status of key devices, as breakers and capacitor banks [Hu et al., 2017]. We do not analyze those large scale models here, because our work focuses on the management and control of distributed generation, flexible loads and storage systems, in other words, on the coordination of DERs in smart grids, at the distribution level. It should be noted that the constructive model of the actual management and control power systems is not capable of integrating distributed resources directly because of its centralized nature [Hu et al., 2017].

It is interesting to notice that the most part of literature models applying centralized optimization focus on the coordination problem of multiple and different types of DERs (general dispatch). We believe that the presence of numerous and distinct resources imposes more difficulties on the coordination, and dealing with grouped DERs, for example in a microgrid or VPP, helps to overcome the problem. Those grouped situations claim for a centralized optimization model. However, new transactive control approaches come into sight as decentralized options to solve the multiple DERs coordination problem in large scale—as can be seen in section 3.4.4. In the case of only demand flexibility management, general decentralized price response approaches are more common, as can be seen in the next section, but transactive approaches can also be applied.

3.4.3 Price Reaction

In this approach, dynamic prices signals are sent to the final users. At certain time intervals, new electricity prices (or price profiles) for the next periods are communicated to the consumers, or to their automated systems. As a response, the users adjust their equipment (DERs) manually, or automatically and optimally via their automated EMS [Kok and Widergren, 2016]. Thus, we discuss three aspects of this model: pricing models and end-users' manual responses; automated controllers for the coordination of DERs when there are price signals; and benefits/drawbacks of this approach.

Manual adjustments of devices by users are typically called pricing models, and are the most widely used mechanisms for implementing demand participation—one of DERs categories. They penalize certain peak periods of time with higher prices, guiding consumers' load curves. Various pricing models have been proposed for the retail electricity markets, and they all have the same purpose: to pass along the "real" electricity cost/value to the end users, encouraging them to shift their high-load appliances to off-peak periods [Mohsenian-Rad and Leon-Garcia, 2010]. Some models used are:

Time-of-use (TOU) tariffs: penalize peak consumption periods of time (e.g. 18:00 to 21:00) with higher prices [Palensky and Dietrich, 2011]. It is possible to have multiple periods with different prices as on-peak, mid-peak, and off-peak, but they are generally fixed over a season [Gholian et al., 2016]. This model is very easy to implement and is commonly used around the world. Because the values are fixed over a long period of time, they can be communicated by internet, TV or other existing ways. In Brazil, TOU tariffs are available for all LV consumers connected to the SIN, as an option to the flat rate model. The proposed pricing scheme is called *Tarifa Branca* (white tariff) and has three different electricity prices according to the day of the week and the hour of the day [ANEEL, 2020a]. The prices vary depending on the utility's concession area. Fig. 5 presents an schematic example of the Brazilian TOU tariff model;

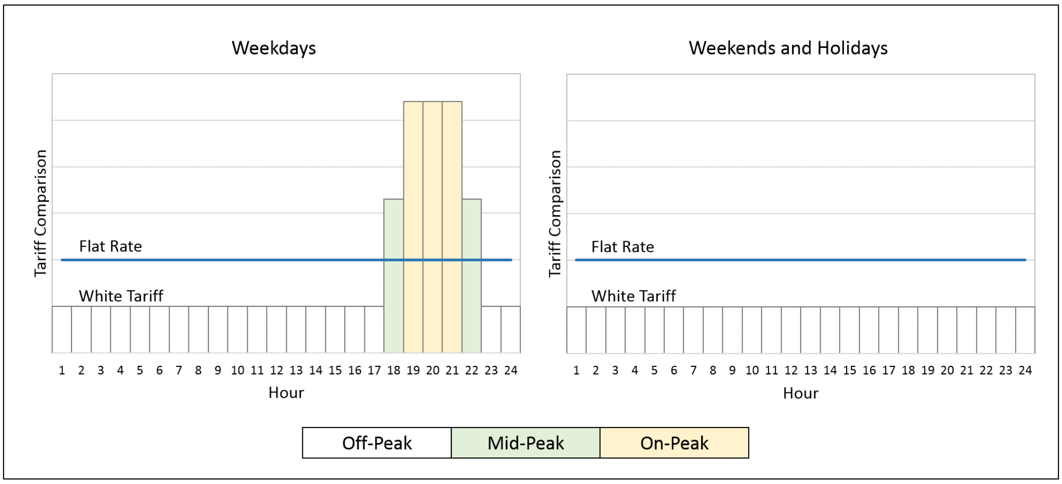


Figure 5 – Brazilian time-of-use tariff scheme.
Source: ANEEL [2020a]

Real-time pricing (RTP): the electricity price changes hourly (or more often), reflecting the utility's generation cost [Zhao et al., 2013]. As in the case of TOU tariffs, the prices can be available via telephone and/or internet. However, consumers have to monitor more constantly the values in this case, thus an automated system to respond to the prices is essential [Mohsenian-Rad and Leon-Garcia, 2010];

Day-ahead Pricing (DAP): this is a variation of the RTP model in which the hourly prices are released by the utility the day before the consumption [Gholian et al., 2016]. In Illinois/USA, Ameren Inc. offers a real-time pricing option to its customers. The market prices are higher during peak demand times, generally from 17:00 to 21:00 and during summer. Moreover, the utility releases day-ahead prices for the participants to plan their consumption [AMEREN INC., 2017]. Fig. 6 shows examples of values in four different days of 2017;

Peak-pricing (PP): generally, this pricing model is composed by two rates: usage charge and peak load charge [Gholian et al., 2016]. The former is based on other pricing models (e.g. flat rates or TOU tariffs) and the later depends on the consumers' daily/monthly highest load [Gholian et al., 2016]. The peak load charge is usually much higher in an attempt to flatten the load curve and reduce the peak-to-average ratio. Also in Brazil, utilities charge large consumers (industrial and commercial) using a PP model, with a TOU energy rate and a peak load charge. For the last, the user must contract his/her peak load in advance, and he/she cannot surpass the committed value [ANEEL, 2020b];

Inclining Block Rates (IBR): this is another pricing method that promotes more balanced load curves [Gholian et al., 2016]. In IBR, the energy marginal price

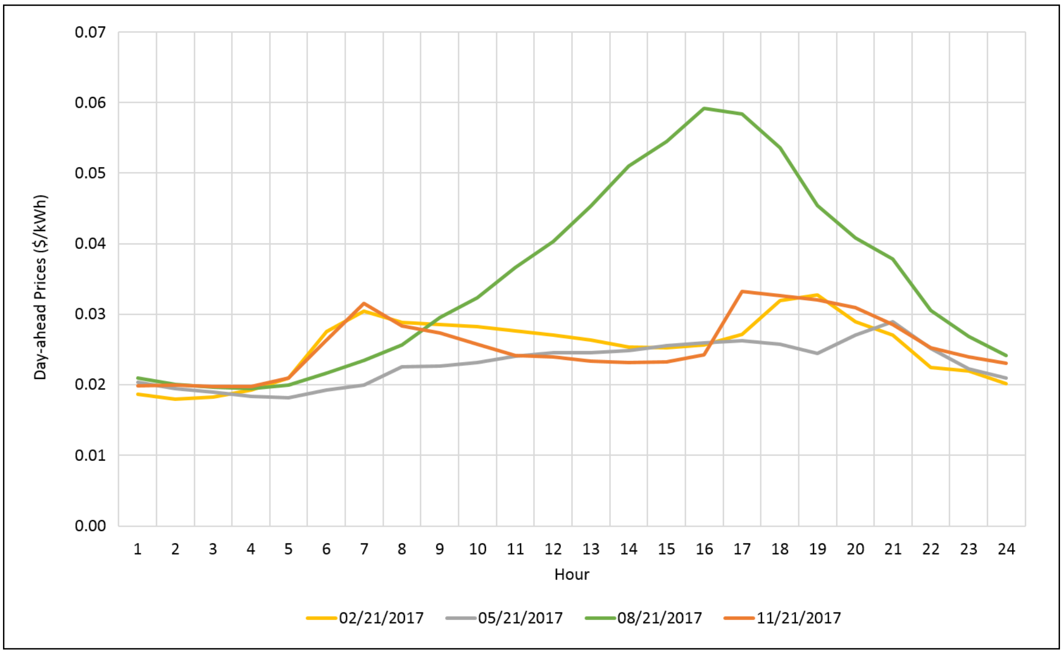


Figure 6 – Ameren's day-ahead pricing in four different days of 2017.
Source: AMEREN INC. [2017]

increases when total monthly/daily/hourly load exceeds a threshold, which gives incentives to consumers to distribute their load at different time slots and avoid paying for electricity at higher prices [Mohsenian-Rad and Leon-Garcia, 2010]. In Canada, the British Columbia Hydro Company uses an IBR model, called Residential Conservation Rate. Consumers are charged one rate for electricity up to a threshold, and a higher rate for the consumption beyond that limit. This "stepped" rate is designed to encourage energy conservation [BC HYDRO, 2020];

Critical Peak Pricing (CPP): in this pricing method, an additional charge is included in electricity prices when the utility experiences total load spikes [Gholian et al., 2016]. Users do not know when the higher-price moments are going to happen, because they depend on the combined behavior of the utility's consumers [Gholian et al., 2016]. For that reason, utilities have to send warnings to its consumers before the peak events (for 5 minutes to 24 hours in advance) [Gholian et al., 2016]. In Colorado/USA, Fort Collins Utilities charge customers more for consuming energy during the monthly peak hour, which depends generally on the season. Commonly, from June to September it happens in mid-to-late afternoon on a very hot day, from November to March in an early evening on a very cold day, and in April, May and October, in an early afternoon on a very warm day or early evening on a very cold day, as can be seen in Fig. 7 [FORT COLLINS UTILITIES, 2018].

To unleash all the DERs response potential with the pricing models, intelligent local controllers, owned by the consumers and/or under their control, can be applied.

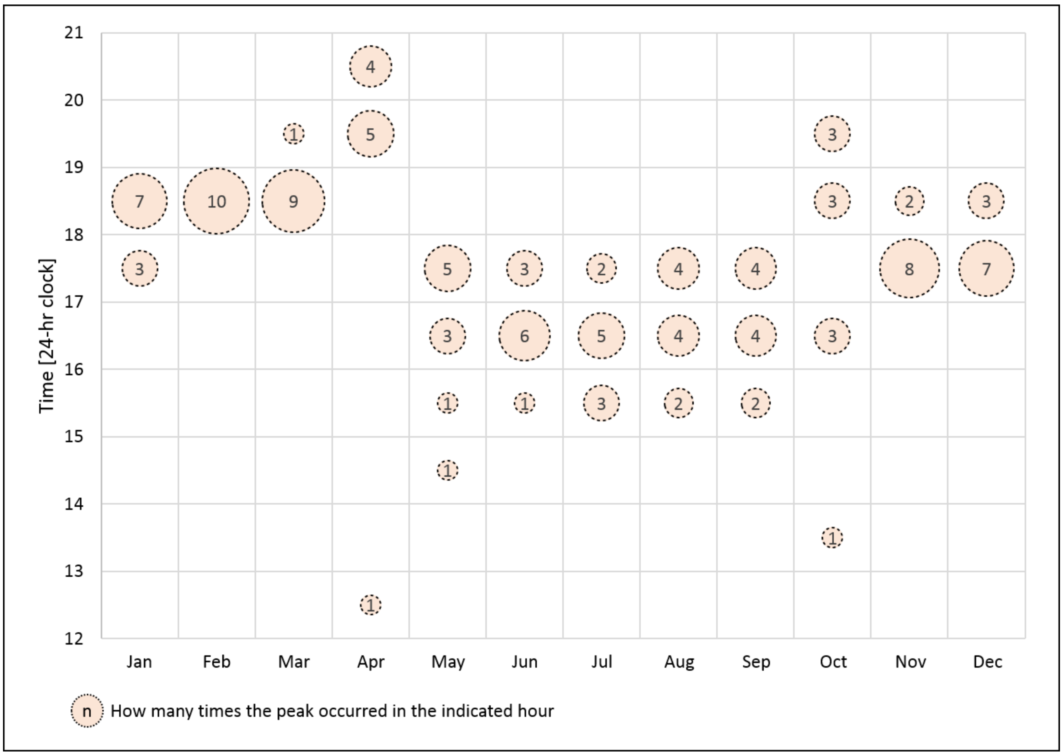


Figure 7 – Fort Collins critical peak pricing model: coincident peak data from 2008 to 2017.
Source: FORT COLLINS UTILITIES [2018]

The automated control of responsive devices would be able, for instance, to increase user's distributed generation and decrease his/her load during high-prices time slots, while considering the devices status and user's preferences [Kok and Widergren, 2016]. Moreover, the automation dispenses the consumers' direct management of DERs, motivating them to participate in smart grid programs, as demand responses and pricing schemes [Du and Lu, 2011].

The intelligent controllers can be separated in two categories: load control, which focuses on the management of user's appliances (e.g. heaters, air conditioners, charging of electric vehicles, washing machine etc.), and is classified as DSM; and general control for DERs coordination, which considers not only demand flexibility, but also distributed generation (e.g. PV panels, cogeneration etc.) and storage systems (e.g. batteries, vehicle-to-grid etc).

For the automatic control of flexible loads, the models add technology to the demand management process, helping consumers to benefit from DSM advantages without having to continuously analyze prices and consumption to decide whether to turn on the television or prepare dinner. Much research has focused on how to minimize energy costs under the different pricing structures and the scheduling of appliances has been studied extensively. There have been research developing local controllers for residential, commercial and industrial contexts. They design methods and algorithms to schedule and control user's appliances locally in order to minimize his/her costs, mainly based on the aforementioned pricing models gave by the market, utility or DSO. The control technology can be either real-time (online) [Barker et al., 2012, Pipattanasomporn et al., 2012] or solved for predefined time horizons (offline), mainly the day-ahead [Du and Lu, 2011, Wang et al., 2013]. Moreover, they can schedule all loads of a smart home [Mohsenian-Rad and Leon-Garcia, 2010, Zhao et al., 2013], a small office building [Manandhar and Cao, 2015], or an industry [Gholian et al., 2016].

Further studies consider the other two categories of DERs in the optimization process. They model storage system and small generators, including renewable resources, in the EMS. For instance, Rastegar et al. [2012] adds plug-in electric vehicles and storage systems to the load commitment model with TOU tariffs, and Roldán-Blay et al. [2017] proposes an algorithm for managing the DERs of an academic building where the loads are uncontrollable, and an RTP model is applied. Finally, Fridgen et al. [2018] studies 10 different tariff models and their effects on residential microgrid prosumers' bills and load/generation profiles. It concludes that volumetric tariffs (e.g. TOU tariffs) can increase consumers' bills and encourage load/generation peaks; tariffs with capacity rates (e.g. peak pricing) have little impact on electricity bills and system's peak; and tariffs that account for system and energy retail costs (e.g. critical peak pricing) lower consumers' bills, promote peak shaving, and allocate better the program costs.

The general benefits of the price signal approaches are: 1) only a simple one-way communication system is necessary, to receive the prices or demand response events; 2) there is no autonomy issues for users, nor information privacy concerns, as long as the decisions are taken locally (manually or automatically); and 3) it is easily implementable in regions where a wholesale market exists and provides day-ahead and/or intraday prices profiles, specially the manual response [Kok and Widergren, 2016].

In the case of manual responses to pricing models, even though they are easy to implement, they depend on consumers' disposal to change their household routines, which is a challenging task [Barker et al., 2012]. Moreover, it is difficult to consumers to respond manually to the time-varying prices [Mohsenian-Rad and Leon-Garcia, 2010, Rastegar et al., 2012]. Finally, they can create new peaks in cheaper hours [Palensky and Dietrich, 2011, Zhao et al., 2013].

The automated decentralized management of DERs overcomes some manual response issues, and add another advantages as: they require less computational time to be solved and the user's information is secured. Because the problems are smaller (fewer appliances to be committed means fewer decision variables), they can be solved simply using linear programming and commercial software [Mohsenian-Rad and Leon-Garcia, 2010, Rastegar et al., 2012, Manandhar and Cao, 2015, Gholian et al., 2016], or fast problem specific algorithms [Du and Lu, 2011, Barker et al., 2012, Pipattanasomporn et al., 2012, Zhao et al., 2013, Wang et al., 2013, Manandhar and Cao, 2015, Roldán-Blay et al., 2017].

Those characteristics represent benefits of the price reaction approach, if compared to the centralized optimization. However, the reaction of all consumers and their devices is difficult to predict if the DERs status and the users' preferences are unknown by the utility or DSO. Moreover, managing the load of each building separately, without looking to the neighbor's schedule, can generate load synchronization, create energy spikes or prevent the system to benefit from peak-to-average ratio reduction. For example, simulation results in Pipattanasomporn et al. [2012] show that a low limit level in household consumption management may result in a new peak during off-peak hours after a demand response event finishes. This adverse effect could damage a local transformer, and should be avoided. Finally, consumers acting alone have less power in energy markets, being only pricing respondents, instead of real players. Thus, the Transactive Control proposition aims at overcoming those issues.

It is important to notice that, regardless of the type of price signal model application (with or without an automated system, online or offline EMS, for a single device or for the entire home/building) an advanced meter, with the capacity to receive the prices signals and measure the energy consumption in an appropriate time resolution, is imperative for charging correctly the users. Recent developments on metering technologies provide solutions which mitigates the privacy risks [Kok and Widergren, 2016].

3.4.4 Transactive Control

The Transactive Energy (TE) concept has emerged as a means of orchestrating the coordinated operation of the multiple intelligent devices being connected at the distribution systems, a.k.a the Distributed Energy Resources (DERs) [Kok and Widergren, 2016]. The Gridwise Architecture Council (GWAC) defines transactive energy as "a system of economic and control mechanisms that allows the dynamic balance of supply and demand across the entire electrical infrastructure using value as a key operational parameter" [GWAC, 2015, p. 11]. Therefore, decisions are made through an exchange of value-based information captured in transactions between participants [Kok and Widergren, 2016], generating an efficient market without privacy issues. Moreover, given that coordinating a growing number of DERs poses a multi-objective control and optimization challenge, transactive energy embraces the economics and engineering of the power system [GWAC, 2015].

Indeed, some wholesale electricity markets around the world have already implemented this concept at the transmission level [Kok and Widergren, 2016], giving decision autonomy to their agents (e.g. large power plants and consumers). For example, the Nord Pool electricity market operator "offers trading, clearing, settlement and associated services in both day-ahead and intraday markets across nine European countries" [NORD POOL, 2020a]. Its customers (generators and consumers holding a balancing agreement with the TSO) participate in half hourly day-ahead auctions and/or intraday markets, sending bids to the Nord Pool platform, which is responsible for clearing the price and guaranteeing settlement and delivery [NORD POOL, 2020b]. This trading and clearing mechanism, which involves a distributed and coordinated decision-making process, is a conventional type of transactive control, and is already well understood at the transmission level [Kok and Widergren, 2016]. Is at the retail market level, which is related to the distribution system operation, that the transactive concept is lacking [Hu et al., 2017].

Transactive control (TC) is a TE attribute, related to the energy management process and coordinating mechanisms [GWAC, 2015]. Hu et al. [2017] classifies transactive control literature models according to its power system application, time scale, and implementing models. For the first, the authors present five applications: frequency regulation via secondary and tertiary control, congestion and voltage management, operation of balance responsible parties, aggregators, and DSO, development of new electricity spot market mechanisms, and optimal residential energy management considering the network.

For the time scale, the transactive control models can utilize forward or spot transactions [Hu et al., 2017]. The former is used to investment decision-making, operations planning, and risk management. The latter is important for operating decisions related to coordination, besides risk mitigation.

For the implementing methods, Hu et al. [2017] includes one-time and iterative information exchange-based methods. In the former, each DER generates a bid (a quantity and/or a price) for every period (e.g. an hour), and sends it to the auctioneer, which performs a price-discovery mechanism. The cleared price is used for controlling the devices. This is similar to the Nord Pool market, but with a focus on smaller customers. In the latter, the clearing price is found after a certain number of information exchanges between the participants, when an equilibrium is reached.

It is worth mentioning that the one-time information exchange-based method is less complex, which means that it requires lower communication and is more scalable. This can explain why this method is widely applied in demonstration projects in the US and Europe [Hu et al., 2017]. Even though the iterative method gives the actors more opportunities to declare their constraints and willingness, it needs more time to reach equilibria. Therefore, the iterative method is more suitable for the forward time scale, when planning and scheduling decisions are made, while the one-time method is more interesting for the spot time scale, when fast, real-time controlling decisions are fundamental.

As Transactive Control is the energy management system (EMS) focused in this thesis, we give a broadly presentation of research models that apply TC for the management of distributed energy resources. Their main aspects are summarized in table 8, following the aforementioned classification of Hu et al. [2017]. However, the non-cooperative game literature for DERs coordination, which is related to the proposition of this work, is discussed in detail in the next chapter.

In the group of models that apply a one-time information exchange-based method, Samadi et al. [2012] proposes a Vickrey-Clarke-Groves mechanism aiming at maximizing the social welfare of a group of consumers—defined as the sum of their consumption utility minus the total cost to deliver the electricity. Consumers with flexible loads have their preferences and electricity needs modeled as an utility function, which parameters are revealed to the electric utility. This electric utility then performs the Vickrey-Clarke-Groves allocation rule, and sends back the energy assignments and respective payments to each user. Vandael et al. [2013] presents a method for the demand-side management of plug-in hybrid electric vehicles. A market-based control is applied at the real-time management step, and an auctioneer agent sends incentive signals to the devices in order to split the energy schedule between the participants, considering the vehicles urgency to charge their batteries. De Craemer et al. [2014] extends the previous work, proposing a dual coordination mechanism for the auctioneer agent. Weckx et al. [2014] incorporates distribution transformer and voltage constraints in the electric vehicle charging problem. A "concentrator" agent, which sums up the bid functions of the electric vehicles and sends the aggregated function to the auctioneer, now checks if the equilibrium prices violate transformer and voltage limits when building the aggregated bid. Li et al. [2016b] proposes a market-based mechanism for coordinating thermostatically-controlled loads (TCLs) considering the feeder capacity, and Li et al. [2016c] extends the work by addressing unknown parameters of the model. Behboodi et al. [2018] applies a market-based control, using an agent-based modeling approach to coordinate the operation of distributed TCLs such as heat pumps and air-conditioners. The proposed model is able to offer demand response of TCLs in real-time retail electricity markets. All these models only coordinate flexible loads.

Kok [2013], on the other hand, develops a one-time information exchange-based coordination mechanism, fully decentralized, for any type of DERs (flexible loads, distributed generators, storage systems). It develops the PowerMatcher, which applies, again, an auction with "concentrator" agents. This tool has been successfully implemented in large-scale demonstration projects, e.g. the CRISP project, the Couperus Smart Grid project, and the PowerMatching City [Kok, 2013, Hu et al., 2017]. Moreover, Cornélusse et al. [2019] also develops a one-time coordination model, applied to the optimal operation of a community microgrid. It proposes a market-oriented pricing mechanism using a bi-level programming model.

In the group of iterative information exchange-based methods, Hu et al. [2014] proposes a two-step approach for the day-ahead congestion management of a distribution grid. The algorithm is based on shadow prices for the time slots where there is line congestion, which are calculated according to electric vehicles schedules. Those prices and schedules are exchanged between the fleet operators and DSO/market operator until an equilibrium is reached. The work is extended in Hu et al. [2015], in which transformer thermal capacity and voltage limitations are considered when calculating the shadow prices. Moradzadeh and Tomsovic [2013] also proposes a Lagrangian method, but for the decentralized optimization of residential DERs aiming at minimizing the cost to the utility company and its customers while respecting the users' preferences. Gan et al. [2013] introduces a different optimal decentralized algorithm to the electric vehicles charging scheduling problem. Its purpose is to fill the valleys in load profiles. As the aforementioned approaches, it consists of local scheduling of each electric vehicles considering their users' needs, communication of the charging profiles to the utility, and recalculation and broadcast of the control signal (e.g. the prices) to the devices. The difference is on the sub-gradient model used, and on the application of an asynchronous algorithm, guaranteeing the optimization even when the vehicles update their profiles based on outdated control signals. Gatsis and Giannakis [2012] also considers a sub-gradient method to schedule DERs aiming at maximizing the social welfare of an utility company and its residential customers. However, it considers multiple types of devices, including storage systems, and the economical operation of the company. Moret and Pinson [2019] also deals with different types of DERs, aiming at optimizing the management of a community. It uses the alternative direction method of multipliers (ADMM) in orther to plan the day-ahead operation of the community's assets.

Table 8 – Literature review about transactive control models.

Reference	Application	DER types	Implementing Method	Time Scale
Samadi et al. [2012]	Residential optimal energy management of electric utility's consumers	Flexible loads	One-time: Vickrey-Clarke-Groves mechanism	Day-ahead operations planning
Vandael et al. [2013] and De Craemer et al. [2014]	Plug-in hybrid electric vehicles optimal charging coordination	Flexible loads	One-time: market-based control	Real-time control
Weckx et al. [2014]	Plug-in hybrid electric vehicles optimal optimal charging coordination with distribution transformer and voltage constraints	Flexible loads	One-time: market-based control	Real-time control
Li et al. [2016b] and Li et al. [2016c]	Coordination of Thermostatically-Controlled Loads considering the feeder capacity	Flexible loads	One-time: dominant-strategy incentive compatible mechanism	Real-time control
Behboodi et al. [2018]	Coordination of Thermostatically-Controlled Loads	Flexible loads	One-time: market-based (agent-based modeling)	Real-time control
Hu et al. [2014] and Hu et al. [2015]	Congestion and voltage management using electric vehicles coordination	Flexible loads	Iterative: Lagrangian multiplier mechanism	Day-ahead scheduling, intra-day market, or real-time control

Moradzadeh and Tomsovic [2013]	Residential optimal energy management of utility's consumers considering network constraints	Flexible loads	Iterative: Lagrangian multiplier mechanism and optimal power flow	Day-ahead pricing with real-time adjustments
Gan et al. [2013]	Electric vehicles optimal charging coordination	Flexible loads	Iterative: sub-gradient method	Forward scheduling horizon and real-time control
Kok [2013]	System balance and congestion management	Flexible loads, distributed generators, storage systems	One-time: market-based control	Real-time control
Cornélusse et al. [2019]	Community microgrid operation and reserve provision	Flexible loads, distributed generators, storage systems	One-time: market-based control (bi-level programming)	Day-ahead operations planning
Gatsis and Giannakis [2012]	Residential optimal energy management of utility's consumers	Flexible loads, storage systems	Iterative: Lagrangian multiplier mechanism	Day-ahead scheduling
Moret and Pinson [2019]	Optimal energy management of community	Flexible loads, distributed generators, storage systems	Iterative: alternate direction method of multipliers (ADMM)	Day-ahead operations planning

As mentioned before, the TC models applying non-cooperative games will be detailed in the next chapter. Finally, we discuss the benefits and background of the TC approach. As can be seen in Fig. 4, transactive control encompasses all benefits of the other EMS approaches. Similarly to the price reaction quadrant, the DERs are operated optimally and locally by a controller under the supervision of the end-user. However, the calculation of the price signal is made considering the local information of the devices—preferences, flexibility, willingness to pay, and constraints—which results in a more effective price, depending on the system objective, e.g. congestion management, system balance, residential load coordination etc. Moreover, the system reaction can be predicted and controlled more efficiently, as in the centralized optimization quadrant. Therefore, this approach gets together the advantages of both methodologies: social welfare maximization without autonomy issues.

Furthermore, if the transactive control mechanism is designed carefully and is properly implemented, it can result in a highly scalable system, capable of including many types of DERs, and multiple market agents, e.g. aggregators, virtual power plants, energy communities, DSO, single devices, residential consumers, among others. In summary, transactive control approaches can unlock the full response potential of DERs, furnish more certainty about system reaction, accomplish an efficient market, and respect the privacy and autonomy of the end-users [Kok and Widergren, 2016]. Therefore, this approach is a good solution to the coordination problem of a growing number of flexible/responsive devices connected to the distribution networks.

The drawbacks of transactive control approaches are: they require a communication network, sometimes capable of many information exchange (for the iterative methods); the local controller must have an embedded optimization tool, connected to the devices; and a marketplace must be settled for the agents coordination.

4. Literature Review and Thesis Contributions

As discussed in the previous chapter, transactive control (TC) has emerged as a form of coordinating the multiple agents in power systems (consumers, producers, DSOs, TSOs, aggregators etc.) while considering their particularities, priorities, interests, and autonomy [Kok and Widergren, 2016]. The idea is to optimize the allocation of resources (e.g. generation, controllable devices and loads) by enabling actors to interact with each other and exchange information about consumption, generation, constraints and preferences until an equilibrium solution is reached [Hammerstrom et al., 2009]. This market-based control is naturally decentralized and entails a transparent decision-making process. These characteristics make TC an attractive solution for controlling distributed energy resources (DERs), specially in the residential sector, in which privacy is a main concern and a large number of consumers exist [GWAC, 2015].

An effecting way of implementing TC is through non-cooperative game theory, because it allows modeling agents' preferences, priorities, conflicting interests, and complex interactions in a decentralized manner [Saad et al., 2012]. When applied to DERs control in the residential sector, game theoretic methods capture the load/generation scheduling interactions between mid- to small-size consumers that negotiate their load flexibility, excess generation, and storage services through their home management systems, using an electronic market algorithm [Hu et al., 2017]. In this local market, consumers exchange information and optimize their resources by controlling some flexible appliances, generators, and storage, until an equilibrium is reached and all prosumers are satisfied with the result. Therefore, a detailed discussion about transactive control models applying non-cooperative game theory is done in this chapter. In table 9, we present literature references related to the topic, including important features of the models proposed by those studies:

- Application: for what reason the non-cooperative game was designed (e.g. optimal residential energy management). We follow Hu et al. [2017] classification, but we add more detailed information for some references;
- Implementing method: what algorithm is used to solve the game (e.g. best response

dynamics). This determines if the method is one-time or iterative information exchange-based [Hu et al., 2017];

- DERs model: what types of distributed resources are considered (flexible loads, distributed generators, storage systems); what variables are used to model the resources (continuous, integer), including if the study uses simple equations to represent the DERs operations; and if the DERs are energy variant (e.g. thermostatically controlled loads), or their total load/generation is fixed/constant (e.g. shiftable loads as washing machines);

- Utility function: what are the payoffs for participating prosumers. It is defined by the total cost of the transactive control (e.g. quadratic function), and a billing mechanism that divides this total cost among participants (e.g. per-time-slot billing). In some references, the utility for each prosumer is defined directly, without considering a total cost function with a billing method. For those, the total cost column is the utility function, and the billing is filled with "according to total cost";

- Aspects analyzed: what features of the games are studied on the reference. Eight aspects are possible: equilibrium existence; equilibrium uniqueness; equilibrium optimality; solution fairness; cheating behavior; price of anarchy (PoA); information losses; and prosumers' privacy. Equilibrium states for: Nash equilibrium, correlated equilibrium, ϵ-equilibrium, Bayesian Nash equilibrium, and equilibrium (in general).

The last three features allow the analysis of non-cooperative game models for the transactive control of DERs, and the statement of our contributions as an advancement of this state-of-the-art. In the next sections, those references are further detailed to pave the way for presenting those contributions.

We use the terms prosumer (when a consumer has distributed generation) and consumer interchangeably. To ease the reading, we present some abbreviations that are largely used in this chapter. The list of all abbreviations and acronyms is presented in the beginning of this thesis. Moreover, an explanation of game related nomenclature (e.g. best response, potential games, etc.) can be found in chapter 2, and the characterization of non-cooperative games for transactive control is done in chapter 5.

- BNE: Bayesian Nash equilibrium;
- BRD: best response dynamics;
- DER: distributed energy resource;
- NE: Nash equilibrium;
- PoA: price-of-anarchy;

- PTC: proportional-to-consumption billing;
- PTS: per-time-slot billing;
- TC: transactive control;
- TCL: thermostatically controlled load.

Table 9 – Literature review about transactive control models applying non-cooperative game approaches.

Reference	Application	Implementing Method	DERs Model			Utility Function		Aspects Analyzed
			DERs Types	Variables	Energy Variant	Total Cost	Billing	
Mohsenian-Rad et al. [2010]	Residential optimal energy management	Best response dynamics	Flexible loads	Continuous (simple)	No	Quadratic	PTC	• NE existence
Baharlouei et al. [2013]	Residential optimal energy management	Not specified	Flexible loads	Continuous (simple)	No	Quadratic	PTS vs PTC	• NE existence • Fairness
Baharlouei and Hashemi [2014]	Residential optimal energy management	Best response dynamics (modified)	Flexible loads	Continuous (simple)	No	Increasing strictly convex	PTS vs PTC vs Proposition	• NE existence • Fairness • Cheating
Chen et al. [2014]	Residential optimal energy management	Proximal-point, Synchronous Agreement, Asynchronous Gossip	Flexible loads	Continuous (simple)	No	Polynomial	PTS	• NE existence • NE uniqueness • NE optimality • Fairness • Cheating

Chakraborty and Khargonekar [2014]	Optimal management of demand response	None (theoretical demonstration only)	Flexible loads	Continuous (simple with limits)	No	Positive, concave, differentiable, monotonically increasing	PTS	• NE existence • PoA
Bahrami and Wong [2015]	Electric vehicles optimal charging coordination	Sub-gradient method	Flexible loads	Continuous (charging rates)	No	Quadratic	PTS and dissatisfaction	• NE existence • NE uniqueness
Rahman et al. [2017]	Residential optimal energy management	Best response dynamics (secure)	Flexible loads	Continuous (simple)	No	Quadratic	PTS vs PTC	• NE existence • Cheating • Info losses • Privacy
Liang et al. [2017]	Residential optimal energy management	Projected gradient	Flexible loads	Continuous (simple)	No	Quadratic	PTC	• NE existence • NE optimality • Info losses • Privacy

Karavas et al. [2017]	Optimal energy management of microgrids	Fuzzy logic	Distributed generators, storage systems	Continuous	No	Generation costs of DG/storage	Depending on source	• NE existence
Baharlouei et al. [2018]	Residential optimal energy management	Fast convergent BRD model	Flexible loads	Continuous (simple)	No	Increasing strictly convex	PTC	• NE existence • NE uniqueness • NE optimality
Fernandez et al. [2018]	Optimal energy management of neighbourhood	Round-robin	Flexible loads, distributed generators	Continuous (simple)	No	Quadratic	PTC	• NE existence • NE uniqueness • NE optimality
Noor et al. [2018]	Optimal energy management of microgrids	Best response dynamics	Flexible loads, storage systems	Continuous (simple with limits)	No	Quadratic cost with users' discomfort	PTC	• NE existence • Privacy
Wang et al. [2018b]	Social welfare optimization with electric vehicles charging response	Spatial adaptive play	Flexible loads, distributed generators	Continuous	No	Benefits of DR minus generation costs	According to total cost	• NE existence • NE optimality

Collins and Middleton [2019]	Peak demand response	Iterative synchronous best response dynamics	Flexible loads	Continuous (simple)	No	Compound function	According to total cost	• NE existence • PoA • Privacy
Zhou et al. [2019]	Electric vehicles online optimal charging coordination	Queues (Lyaponuv method)	Flexible Loads	Continuous (simple)	No	Convex functions	PTC	• ϵ-NE existence • ϵ-NE optimality • Privacy
Jacquot et al. [2019]	Offline/online demand response optimization	Cycling BRD, projected gradient descent	Flexible loads	Continuous	Yes	Affine, positive, increasing	PTS	• NE existence • NE uniqueness • NE optimality • PoA
Gong et al. [2019]	Coordination of flexible devices to offer energy reserve	Agent-based model (Lyaponuv method)	Flexible loads, storage systems	Continuous (simple with limits)	No	Electricity costs, reserve revenue, EVs' discomfort	Revenue minus costs	• NE existence • NE optimality

Bhatti and Broadwater [2020]	Optimal energy management of microgrids	Extremum seeking approach	Flexible loads, distributed generators, storage systems	Continuous (generic)	No	Non-quadratic	Revenue minus costs	• NE existence • NE uniqueness • NE optimality • Fairness
Eksin et al. [2014], Eksin et al. [2015], and Eksin et al. [2018]	Residential optimal energy management with renewables	Best response dynamics	Flexible loads, distributed generators	Continuous (preferences described by constant)	No	Quadratic	PTS	• BNE existence • BNE uniqueness
Wang et al. [2018a]	Optimal energy management of microgrids	Best response dynamics	Flexible loads, distributed generators, storage systems	Continuous and integer	Yes	Benefits of DR minus generation costs	According to total cost	• NE existence
Wang et al. [2020]	Social welfare optimization	Spatial adaptive play under network anomaly	Flexible loads, distributed generators	Continuous and integer	No	Benefits of DR minus generation costs	According to total cost	• NE existence • NE optimality • Info losses
Zhu et al. [2015]	Residential optimal energy management	Best response dynamics	Flexible loads	Integer (simple)	No	Quadratic	PTC	• NE existence • NE optimality

Barbato et al. [2015a]	Residential optimal energy management	Best response dynamics	Flexible loads	Integer (simple)	No	Regular functions	PTS	• NE existence
Barbato et al. [2015b]	Residential optimal energy management	Reinforcement learning	Flexible loads	Integer (simple)	No	Increasing functions	PTS	• NE existence • Fairness
Yaagoubi and Mouftah [2015]	Residential optimal energy management	Reinforcement learning	Flexible loads	Integer (simple)	No	Quadratic cost with users' discomfort	PTS	• Correlated Equilibrium existence
Rottondi et al. [2017]	Residential optimal energy management	Round-robin	Flexible loads	Integer (simple)	No	Regular functions	PTS	• NE existence • Privacy
Zeng et al. [2019]	Optimal energy management of microgrids	Best response dynamics	Flexible loads, distributed generators, storage systems	Integer	Yes	Sum of utilities	Depending on DER	• NE existence • NE optimality • Info losses

4.1 Non-Cooperative Games for the Transactive Control of Integer and Energy Variant DERs

The decentralized coordination features of transactive control models applying non-cooperative games, in particular the ability to optimize demand (DERs flexible loads category) while considering consumers' autonomy and preferences, have received some attention from the research community [Cheng and Yu, 2019]. One of the first studies in this field proposed a non-cooperative game approach to schedule residential appliances considering consumers' preferences [Mohsenian-Rad et al., 2010]. In this pioneering study, the authors adopted simple load models with continuous decision variables, constant daily energy to be scheduled and no operation constraints. Additionally, to divide the market results among participants, this study considered a billing mechanism that allocates the total quadratic cost according to the energy share of each individual consumer (herein called proportional-to-consumption billing). Later, other studies extended this discussion on billing mechanisms applied to transactive load control by analyzing the fairness of different methodologies [Baharlouei et al., 2013], studying potential cheating behaviors [Baharlouei and Hashemi, 2014], proposing alternatives to avoid untruthfulness of consumers [Rahman et al., 2017], introducing other billing mechanisms [Chakraborty and Khargonekar, 2014, Collins and Middleton, 2019, Jacquot et al., 2019], and analyzing other costs models [Chen et al., 2014].

At the same time, other works have focused on implementation issues, for example by proposing new coordination algorithms that require less communication among participants and consider communication losses [Liang et al., 2017], by analyzing the stability of the Nash Equilibria when non-quadratic payoffs are designed [Bhatti and Broadwater, 2020], and by introducing a market mechanism able to deal with a large population of devices [Gong et al., 2019]. These contributions kept using relatively simple models to describe controllable loads, in particular using continuous variables to describe the scheduling problem.

More recently, alternative forms of load have been introduced as resources of TC in the context of non-cooperative games. For example, Bahrami and Wong [2015] and Zhou et al. [2019] added electric vehicles (EV) to the continuous non-cooperative game model. The later one addressed the uncertainty related to the number of EVs in a system. On a separate path, the authors of Fernandez et al. [2018] replicated the study of Mohsenian-Rad et al. [2010] in a more realistic manner, including a simplified form of heating, ventilation and air conditioning (HVAC) systems modeled as continuous loads, but without explicitly representing their operation constraints. A more explicit form of consumers' discomfort associated with HVAC loads was added as part of the cost function in Noor et al. [2018] together with additional energy supply constraints.

Other works considered more types of distributed energy resources in the energy management problem. For instance, Eksin et al. [2014], Eksin et al. [2015], and Eksin et al. [2018] applied a Bayesian scheduling game for the management of prosumers' flexible loads, because of the uncertainty of distributed renewable generation. Wang et al. [2018b] solved the load management problem as a social welfare optimization game, thus distributed generation was considered. Karavas et al. [2017] even studied the management problem without considering flexible loads, solving the problem for distributed generators and storage systems only.

Although Mohsenian-Rad et al. [2010], Baharlouei et al. [2013], Baharlouei and Hashemi [2014], Rahman et al. [2017], Chakraborty and Khargonekar [2014], Collins and Middleton [2019], Jacquot et al. [2019], Chen et al. [2014], Zhou et al. [2019] provide important theoretical insights on the application of game theory to the management of residential loads, Liang et al. [2017], Bhatti and Broadwater [2020], Gong et al. [2019], Bahrami and Wong [2015], Fernandez et al. [2018], Noor et al. [2018] address some implementation challenges, and Eksin et al. [2014, 2015, 2018], Wang et al. [2018b], Karavas et al. [2017] consider other types of DERs, they rely on the assumption that the control is continuous. In fact, as demonstrated in Baharlouei et al. [2018] and Jacquot et al. [2019], the assumption of a continuous control allows some important guarantees in terms of uniqueness/optimality of the Nash Equilibria (NE) when applying game theory to these type of problems. However, most of the real world decisions related to load control, particularly in the domestic sector, are discrete. Indeed, the popular forms of load management in residential context rely on simple low-cost technologies, such as smart plugs, while the most representative loads (e.g. HVAC systems or water heaters) have often an on/off control activated by a thermostat, which, realistically, implies an integer (binary) representation. This means that, in practical terms, the equilibria guarantees of a continuous space might not fully apply to the TC in the residential sector, particularly in situations where the number of consumers is small and the controllable resources are highly discrete. Even though few studies have considered this discrete nature of residential control, e.g. in the context of potential games [Barbato et al., 2015b, Zhu et al., 2015, Wang et al., 2018a, Zeng et al., 2019, Wang et al., 2020], generalized ordinal potential games [Barbato et al., 2015a, Rottondi et al., 2017], and games with correlated equilibria [Yaagoubi and Mouftah, 2015], the equilibria conditions of the discrete space and their implications in real world TC remains unexplored.

Additionally, none of the studies referred above explicitly model thermostatically controlled loads (TCLs), especially the constraints related to consumers' comfort, such as room temperatures. This lack of attention to TCLs in the context of non-cooperative game models may sound surprising, given that these loads are a main source of flexibility in the residential sector [Pérez-Lombard et al., 2008], are widely explored in other forms of control, and are present in other TC models. Examples of studies that use TCLs in other forms of control include Nelson et al. [2019], which develops six cooling strategies for optimizing the sizing and control of HVAC systems in order to reduce energy costs; Iria et al. [2019], that proposes a two-stage stochastic optimization model to help aggregators of flexible loads (TCLs and EVs) formulate bids in the day-ahead energy and reserve markets; Heleno et al. [2015], which calculates the availability of TCLs to be aggregated for providing reserve, and Zhou et al. [2017], which proposes a two-level scheduling model for the optimal control of aggregated TCLs to arbitrage in the intraday electricity market. In the case of other TC models that control TCLs, Behboodi et al. [2018] proposes bidding strategies for controlling aggregated TCLs in real-time market; Tang et al. [2019b] uses a Nikaido-Isoda function to solve the TC of building clusters with thermal mass and energy storage; Tang et al. [2019a] models the problem as a Stackelberg game; De Paola et al. [2019] proposes a mean-field game approach to operate large populations of TCLs; and Kazmi et al. [2019] applies a multi-agent reinforcement learning to better operate and control a large amount of TCLs. All those studies consider TCLs in the context of load control, for different purposes and using other methods (including other game models) than non-cooperative games.

The problem is that TCLs, and comfort preferences in particular, raise new challenges to the game models, because they are not purely shiftable and entail the so called "energy payback", as shown by Bischke and Sella [1985], which studied the payback impact of water heaters on the load management, and by Wei and Chen [1995], which calculated the payback ratio of controlling air conditioners with a direct load control model. In other words, shifting TCLs in time while maintaining comfort standards implies overall energy losses/gains in relation to the baseline consumption. This energy variant characteristic of the thermal control is not aligned with the existing transactive billing methods [Mohsenian-Rad et al., 2010, Baharlouei et al., 2013, Baharlouei and Hashemi, 2014, Chen et al., 2014, Zhu et al., 2015, Rahman et al., 2017, Liang et al., 2017, Baharlouei et al., 2018, Fernandez et al., 2018, Noor et al., 2018, Zhou et al., 2019] that assume a constant energy consumption during load scheduling. Thus, understanding the practical implications of this energy neutrality assumption in the context of TCLs is key to allow extending transactive energy control to these loads.

In summary, transactive control offers important decentralized characteristics that are suitable to residential demand side management. Current technological solutions, developed around non-cooperative game theory, have shown promising results but they were unable to properly include TCLs, which is the largest source of flexibility among domestic loads. Besides, two aspects of TCLs control are not fully addressed by the current theory of non-cooperative games: 1) the on/off nature of the decisions, which makes the problem integer and changes the equilibria conditions; 2) the energy variant characteristic of the control, which contradicts the energy neutrality assumptions of the theory. Therefore, it is important to understand the implications of these two theory gaps in real-world implementation of TC in the residential sector.

To address these issues, this thesis provides a game-theoretic framework to include domestic on/off TCLs in transactive control based on non-cooperative games. It applies the proposed framework to a realistic energy community in Spain and evaluates the impacts of the integer and energy variant characteristics of the TCLs control in this context. Our work advances the state-of-the-art of non-cooperative games applied to the day-ahead load scheduling of residential consumers by including on/off TCLs into the set of loads considered in transactive control. The specific contributions are the following:

1. we model explicit TCL comfort constraints and formulate the problem with binary variables representing the real on/off control of this type of appliances, when defining the game;
2. since the integer nature of the control affects the theoretical foundations of the problem, we prove that multiple Nash Equilibria can exist and they can be sub-optimum;
3. we discuss the practical implications of having multiple NEs in real implementation of TC platforms, in terms of optimality of the total scheduling cost, variability in consumers' payments, and how the algorithm design defines the solution that will be effectively played;
4. we show that TCLs energy variant nature impacts the theoretic grounds of the game model, because the total energy in the scheduling horizon is not fixed. Thus, we discuss how this characteristic affects the equity among consumers when applying the proportional-to-consumption billing to the non-cooperative game model.

4.2 Billing Models for the Transactive Control of DERs

One of the most important aspects when applying a non-cooperative game model to coordinate consumers' demand is the billing mechanism, which defines how the scheduling costs are divided among participants. The chosen mechanism determines consumers' costs and their availability to change load and adopt specific consumption patterns that favour the system. Therefore, this thesis also discusses the implication of two billing mechanisms for the schedule of integer and energy variant loads (e.g. thermal loads) in terms of Nash Equilibria (NE) existence, fairness, and strategy-proof. The first billing considers participants' total daily consumption to define their shares (herein named proportional-to-consumption); and the second one bills consumers according to their consumption in each time slot (herein called per-time-slot).

Proportional-to-consumption (PTC) billing is one of the first mechanisms proposed to divide the total scheduling cost of residential appliances in the context of non-cooperative games. It appears in the work of Mohsenian-Rad et al. [2010], which uses a game model with continuous decision variables, simple load constraints to represent consumers' preferences, and a total quadratic cost divided with the PTC. Liang et al. [2017] includes concerns related to information losses and privacy to the model, while Fernandez et al. [2018] considers renewable generation, and Noor et al. [2018] adds storage systems. In addition, Zhou et al. [2019] considers electric vehicles modeled as continuous variables using the same framework. In fact, Baharlouei et al. [2018] and Fernandez et al. [2018] show that the PTC billing has important properties when applied to continuous loads and strictly convex cost functions, in terms of the uniqueness and optimality of Nash Equilibrium, and convergence of solution algorithms. In the context of integer games, Zhu et al. [2015] applies the PTC to divide a total quadratic cost and shows that the resulting game is exact potential.

On the other hand, Baharlouei et al. [2013] and Chen et al. [2014] show that the per-time-slot (PTS) billing is a more fair approach than PTC when applied to a continuous scheduling problem. Moreover, Chakraborty and Khargonekar [2014] adds more constraints related to the consumption limits of these continuous loads and proves that there is a lower bound of half for the game's price of anarchy (PoA)—the scheduling total cost of the game is at most two times the value when a centralized approach is used. Jacquot et al. [2019] extends the work of Chakraborty and Khargonekar [2014] by adding constraints for continuous thermal loads in a game with an hourly PTS, proving that the NE is unique in this game, and demonstrating that the game's PoA is numerically close to one. Eksin et al. [2014, 2015, 2018] consider renewable generation in the loads game, proving that a Bayesian Nash Equilibria exists and is unique. Bahrami and Wong [2015] model plug-in hybrid electric vehicles in this continuous setting with the PTS. Finally, Barbato et al. [2015a,b], Yaagoubi and Mouftah [2015], Rottondi et al. [2017] model shiftable loads using integer variables, and consider PTS with regular total cost functions, leading to generalized potential games.

Although PTS mechanisms are shown to be simple, intuitive, optimal, and more fair for continuous games, also guaranteeing equilibria existence for integer games, the work presented in Baharlouei and Hashemi [2014] shows that this billing mechanism is not strategy-proof and gives consumers the possibility of cheating. In response, it proposes another billing model, based on specific characteristics of consumers' preferences and flexibility, which narrows the types of appliances that can be scheduled, and impacts consumers' privacy. In addition, Rahman et al. [2017] presents mathematical proofs to the incentives for cheating in a PTS billing mechanism and proposes two procedures to discourage cheating behavior.

In conclusion, despite demonstrating important theoretical insights on the application of proportional-to-consumption billing mechanisms, Mohsenian-Rad et al. [2010], Liang et al. [2017], Fernandez et al. [2018], Noor et al. [2018], Zhou et al. [2019], Baharlouei et al. [2018], Zhu et al. [2015] do not consider the limitations of this billing mechanism when applied to a broader class of appliances, as integer and energy variant loads (e.g. thermal loads). Moreover, most part of the studies that apply the per-time-slot billing, which is demonstrated to be more fair than the PTC, only considers continuous variables [Baharlouei et al., 2013, Chen et al., 2014, Chakraborty and Khargonekar, 2014, Jacquot et al., 2019, Eksin et al., 2014, 2015, 2018, Bahrami and Wong, 2015, Baharlouei and Hashemi, 2014, Rahman et al., 2017]. Even though Barbato et al. [2015a,b], Yaagoubi and Mouftah [2015], Rottondi et al. [2017] consider shiftable appliances, many aspects of the impact of the PTS billing to integer loads remain unexplored, as Nash Equilibria existence, fairness, and cheating behavior.

Therefore, our thesis advances the state-of-the-art of billing mechanisms for dividing the total cost of non-cooperative game models for the transactive control of flexible loads by including on/off and energy variant thermostatically controlled loads (TCLs) into the set of appliances. The addition of these loads have the potential to impact the theoretical foundations of the game model, depending on the billing mechanism used. We compare PTC and PTS billings in terms of Nash Equilibria (NE) existence, fairness, and strategy-proof. It is important to stress that such discussion is critical for any practical implementation, it does not exist in the literature of integer games [Zhu et al., 2015, Barbato et al., 2015a,b, Yaagoubi and Mouftah, 2015, Rottondi et al., 2017], and it requires different theoretical elements than the ones used in the context of continuous games [Mohsenian-Rad et al., 2010, Liang et al., 2017, Fernandez et al., 2018, Noor et al., 2018, Zhou et al., 2019, Baharlouei et al., 2018, 2013, Chen et al., 2014, Chakraborty and Khargonekar, 2014, Jacquot et al., 2019, Eksin et al., 2014, 2015, 2018, Bahrami and Wong, 2015, Baharlouei and Hashemi, 2014, Rahman et al., 2017]. More specifically, other five contributions are added, and are stated as follows:

5. we show that the game modeled with a proportional-to-consumption billing does not guarantee a potential game formulation, thus a Nash Equilibrium, in the presence of muti-period energy variant loads (such as thermal loads);

6. we propose a modified best response algorithm to solve the problem with the proportional-to-consumption billing and energy variant loads. We also show that the Nash Equilibrium can not be reached and cheating behavior can occur with the PTC;

7. we present a general formulation for the per-time-slot billing, including integer and energy variant TCLs. Since the integer nature of the control affects the theoretical

foundations of the problem, we prove that this game is an exact potential game.

8. we show that the general formulation for the PTS can be applied to any type of loads, because its exact potential properties do not depend on loads constraints;
9. we propose an alternative solution to overcome the possibility of participants cheating in per-time-slot billing models, by showing theoretically that a simple adjustment in the billing rules ex-ante instead of ex-post consumption is enough to discourage cheating behavior, which guarantees the strategy-proof of this mechanism.

As a minor contribution, we show that the game designed with a per-time-slot billing is fairer than the proportional-to-consumption model, when integer and energy variant loads are considered. We support these contributions with a case study involving an LV network in the South of Spain with 201 consumers.

5. Methodology

5.1 System description

As discussed in section 3.3.3, when consumers are organized in smart residential communities [Nan et al., 2018], they can coordinate their energy utilization and manage their distributed resources in a way to reduce their electricity bills and costs [Cornélusse et al., 2019], increase their revenues [Moret and Pinson, 2019], and use their assets more efficiently [Shaw-Williams and Susilawati, 2020]. Moreover, this architecture enables consumers to be more proactive and autonomous, giving them decision power to define how to manage their own assets [Koirala et al., 2016]. This organization model is well aligned with the transactive control (TC) concept [Moret and Pinson, 2019], and allows the implementation and analysis of the non-cooperative game method we propose in this thesis.

Therefore, we consider a smart residential community of consumers connected to the same substation, with (or without) a market aggregation platform [Pasetti et al., 2018], as shown in figure 8. The consumers have flexibility from thermostatically controlled loads (TCLs) and are willing to manage/schedule them in a TC approach in order to reduce their bills. Their interest is to minimize their payments individually, dividing the total community cost in a fair way, while also minimizing this total cost, and respecting their preferences and their autonomy.

Consumers' appliances are controlled locally by their energy consumption controller (ECC), which is part of a home energy management system (HEMS). The HEMSs from different consumers communicate with each other exchanging aggregated load information with the neighbors' HEMS, and pricing parameters/billing mechanism with the market aggregation platform (or retail market if this platform does not exist). Consumers' management systems can be deployed inside their smart meters (or as an additional equipment that communicates with the energy meter), which are connected to the power line and to a local area network (LAN), for the information exchange. Moreover, a market platform can exist for negotiating prices with the retail market, trading ancillary services, and/or guiding the optimization process. However, it is no longer responsible for directly coordinating and controlling the users' devices, performing an indirect control [Heussen et al., 2012], because consumers take decisions locally.

In effect, the proposed transactive approach works as follows: the day before the operation, the retail market sends to consumers' management systems (or to the aggregation platform) the pricing function parameters—via the LAN. The HEMSs perform the scheduling optimization model locally, based on the informed preferences by their local consumer, the pricing parameters, and aggregated neighbors' consumption patterns. They broadcast their own optimized consumption curve together with neighbors' consumption to the other HEMSs, which re-optimize the schedules. The process continues until equilibrium is reached, after which the optimal schedules are set and passed to the ECC operation. Then, consumers' ECCs send control messages with turn-on times for the smart thermostatically controlled loads, according to HEMSs local decisions.

For the purpose of this thesis, TCLs considered are air conditioners (ACs), which are modeled as integer variables. We adopt the same AC model of Heleno et al. [2015], which uses physically-based load models to calculate the temperatures associated with the operation of the device. Consumers' preferences impose a temperature range to this operation, according to the comfort patterns. In fact, this is a general procedure applied to TCLs scheduling [Heleno et al., 2015], which means that the theoretical framework presented here for ACs can be easily extended to other forms of TCLs (e.g. water heaters, refrigerators, etc.).

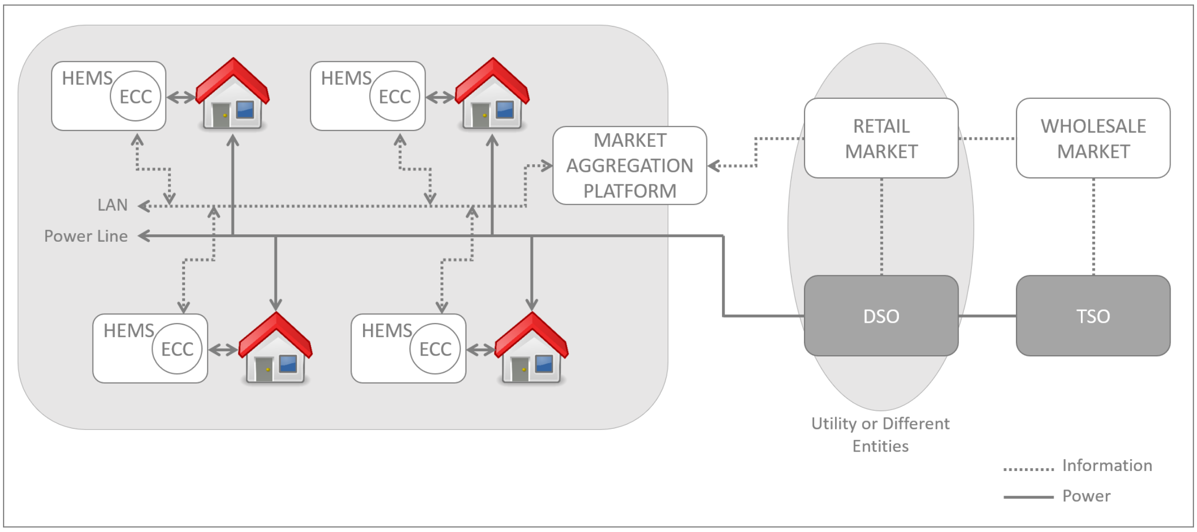


Figure 8 – System model considered in this study. Each consumer has TCLs controlled locally by his/her energy consumption controller (ECC), which is part of a home energy management system (HEMS) that communicates with the other HEMS (exchanging load information) and with the aggregation platform/retail market (exchanging pricing/billing information).

The system model presented here is a schematic application of TC. The real implementation of game theory to coordinate appliances locally is in its early stages.

Therefore, other questions related to, for example, the communication infrastructure and the specific technology, remain open. Even though this is not the focus of this work, we introduce two references that present real applications of TC: Liu et al. [2017] and Sousa et al. [2019].

5.2 Load Modeling

It is assumed that consumers in the community have ACs to be scheduled for the day-ahead with the objective of reducing the electricity costs. Moreover, it is also assumed that the remaining load is uncontrollable and deterministic: consumption is forecasted and summed in one inflexible load curve for each consumer ($\mathbf{w}_n$). Regarding the ACs, their electrical consumption can be scheduled in time as long as the consumers' preferences related to temperature comfort are respected, as defined in (5.1)-(5.2).

$$\theta_{n,t} \geqslant \theta_{n,t}^{min} \quad \forall n \in \mathcal{N} \quad \forall t \in \mathcal{T}, \tag{5.1}$$

$$\theta_{n,t} \leqslant \theta_{n,t}^{max} \quad \forall n \in \mathcal{N} \quad \forall t \in \mathcal{T}. \tag{5.2}$$

We use the ACs model presented in Heleno et al. [2015] to simulate the operation of these appliances. For all $n \in \mathcal{N}$ and for all $t \in \mathcal{T}$, the internal temperature $\theta_{n,t}$ evolves according to:

$$\theta_{n,t} = \theta_{n,t-1} - \frac{\delta}{TC_n R_n}\left(\theta_{n,t-1} - \theta_t^{et} + \eta_n R_n E_n y_{n,t}\right). \tag{5.3}$$

To simplify the notation, we use day-ahead energy scheduling vectors for each consumer $n \in \mathcal{N}$:

$$l_{n,t} = \delta\left(w_{n,t} + E_n y_{n,t}\right). \tag{5.4}$$

In summary, we can define a *feasible* energy consumption scheduling set $\mathcal{S}_n$ for each user n, which includes all possible scheduling vectors respecting their preferences:

$$\begin{aligned}\mathcal{S}_n = \big\{\mathbf{l}_n = [l_{n,1}, l_{n,2}, \ldots, l_{n,T}] \in \mathbb{R}^T : \\ \text{equations } (5.1), (5.2), (5.3), (5.4), \\ y_{n,t} = \{0,1\} \quad \forall t \in \mathcal{T}\ \big\}.\end{aligned} \tag{5.5}$$

We can define the energy consumption vector of the group of participants as:

$$L_t = \sum_{n \in \mathcal{N}} l_{n,t} \quad \forall t \in \mathcal{T}. \tag{5.6}$$

5.3 Community Costs

In this thesis, we study two total cost models: the common quadratic cost function, and a peak pricing function. The first one is widely applied in the literature, and we use it to study different billing mechanisms. The later is a more realistic scenario for a residential energy community

5.3.1 Quadratic Cost Function

We first focus on a family of quadratic functions, as in Mohsenian-Rad et al. [2010], Baharlouei et al. [2013], Baharlouei and Hashemi [2014], Chen et al. [2014], Eksin et al. [2014], Zhu et al. [2015], Eksin et al. [2015], Bahrami and Wong [2015], Yaagoubi and Mouftah [2015], Rahman et al. [2017], Liang et al. [2017], Fernandez et al. [2018], Eksin et al. [2018]. In addition to be extensively used in the literature of transactive control, it is strictly convex, allowing the application of potential games to analyze theoretical aspects of the proposed methodology.

In practice, equation (5.8) can represent real energy costs associated with thermal generation or power losses as well as specific tariffs contracted with aggregators or retailers. For example, two-step tariffs used to encourage consumers to reduce their energy load, as applied by British Columbia (BC) Hydro in Canada [BC HYDRO, 2020], are piecewise linear and can be approximated by quadratic functions. The total community cost in each scheduling time slot $t \in \mathcal{T}$ is defined by:

$$C_t^Q(L_t) = a_t(L_t)^2 + b_t L_t. \tag{5.7}$$

Thus the total community cost of the day-ahead operations planning is:

$$C^Q(\mathbf{L}) = \sum_{t \in \mathcal{T}} C_t^Q(L_t). \tag{5.8}$$

Where $a_t > 0$ and $b_t \geqslant 0$ are constants and can be time varying—e.g. have different values for different time slots of the day to represent better the power system state. It is worth mentioning that we do not consider the fixed component on the quadratic cost function (5.7) because the scheduling of controllable appliances influences the volumetric part of electricity costs [AF-Mercados, 2015]. The fixed costs and/or demand rates do not depend on the amount consumed, and can be charged to consumers directly, apart from this function.

The solution to the problem described above that minimizes the total system cost for a group of consumers $\mathcal{N}$ can be calculated as the following mixed-integer quadratic optimization model (MIQP), with the decision variables constrained to the scheduling set defined in (5.5):

$$C_{\mathcal{N}}^{Q*} = \min_{\substack{\mathbf{l}_n \in \mathcal{S}_n \\ n \in \mathcal{N}}} C^Q(\mathbf{L}) \tag{5.9}$$

5.3.2 Peak Pricing Function

In order to study a scenario with high potential for demand management, we also consider a peak pricing model in which the overall energy costs are composed by a retail energy price and a demand charge. Although the quadratic cost functions aforementioned are strictly convex and easy to optimize, a volumetric rate (applied to the energy consumption) together with a peak demand charge (applied to the peak load of the billing period) is a more realistic scenario for a residential energy community [Fridgen et al., 2018, Gholian et al., 2016]. Assuming the volumetric energy component as variant in time, e.g. a time-of-use (TOU) tariff, the total cost of the community can be written by:

$$C^P(\mathbf{L}) = \sum_{t \in \mathcal{T}} c_t L_t + d \max_{t \in \mathcal{T}} \frac{L_t}{\delta} \tag{5.10}$$

In which $c_t > 0 \ \forall t \in \mathcal{T}$ are the TOU tariffs for the energy consumption, and $d > 0$ is the peak demand charge for the community. It is important to note that this function is convex, as it results from the sum of two convex functions [Boyd et al., 2004]. The solution that minimizes the total system cost for the community composed by a group of consumers $\mathcal{N}$ can be calculated as the following mixed-integer linear program (MILP), in which α_t are auxiliary variables for computing the peak load:

$$\begin{aligned} C_{\mathcal{N}}^{P*} &= \min \sum_{t \in \mathcal{T}} c_t L_t + d\alpha_t \\ \alpha_t &\geqslant \frac{L_t}{\delta} \quad \forall t \in \mathcal{T} \\ L_t &= \sum_{n \in \mathcal{N}} l_{n,t} \quad \forall t \in \mathcal{T} \\ \mathbf{l}_n &\in \mathcal{S}_n \quad \forall n \in \mathcal{N} \end{aligned} \tag{5.11}$$

5.4 Integer Scheduling of Thermal Loads as a Game

In this section, we present the game model for scheduling integer and energy variant loads. As we use the equations of section 5.2 to explicitly model TCL comfort constraints, and the problem is formulated with binary variables representing the real on/off control of this type of appliances, the presented game is one of the contributions of this thesis.

We use the strategic form to represent the non-cooperative game for scheduling thermal loads of consumers, as explained in section 2.3, which is defined as a tuple $\Gamma = \langle \mathcal{N}, (\mathcal{S}_n)_{n\in\mathcal{N}}, \{u_n\}_{n\in\mathcal{N}} \rangle$, where:

$\mathcal{N} = \{1, 2, \ldots, N\}$ is the set of players;

$\mathcal{S}_n = \{\mathbf{l}_n\}_{\mathbf{l}_n \in \mathcal{S}_n}$ denotes the action space for consumer $n \in \mathcal{N}$. This set is composed by feasible scheduling vectors $\mathbf{l}_n$, that respect users' preferences—see (5.5). Moreover, $\mathcal{S} = \times_{n=1}^{N} \mathcal{S}_n$ is the joint action space;

$u_n : \mathcal{S} \mapsto \mathbb{R}$ is the utility function that defines user $n \in \mathcal{N}$ payoff. It can be written as a function of the actions chosen by all players $(\mathbf{l}_n, \mathbf{l}_{-n}) \in \mathcal{S}$, where $\mathbf{l}_n$ is the scheduling vector of player n, and $\mathbf{l}_{-n} = [\mathbf{l}_m]_{m\neq n}$ are the scheduling vectors of all players except n.

The utility function depends on the billing mechanism used to share the total cost of the load scheduling among consumers. We show their equations and implications to the game in the next sections.

5.5 Billing Functions for Defining Consumers' Utilities

In this thesis, we study two different billing mechanisms for the transactive control of integer and energy variant flexible loads: 1) proportional-to-consumption (PTC), and 2) per-time-slot (PTS).

5.5.1 Proportional-to-Consumption

Proportional-to-consumption (PTC) is a very popular billing mechanism used in the literature for sharing the cost among participants of the scheduling game [Mohsenian-Rad et al., 2010, Baharlouei et al., 2013, Baharlouei and Hashemi, 2014, Zhu et al., 2015, Rahman et al., 2017, Liang et al., 2017, Baharlouei et al., 2018, Fernandez et al., 2018, Noor et al., 2018, Zhou et al., 2019]. Consumers' utilities are defined by a constant share f_n multiplied by the total cost of the community:

$$u_n^C(\mathbf{l}_n, \mathbf{l}_{-n}) = -f_n C(\mathbf{l}_n, \mathbf{l}_{-n}) \tag{5.12}$$

$$f_n = \frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\sum_{t\in\mathcal{T}} L_t} \tag{5.13}$$

In this billing setting, if two participants n and m have total load $\sum_{t\in\mathcal{T}} l_{n,t} = \beta \sum_{t\in\mathcal{T}} l_{m,t}$ after the scheduling game is solved, then consumer n will pay β times consumer m's bill. Moreover, the sum of all consumers' payments will be equal to the community's total cost: note that $\sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}} l_{n,t} = \sum_{t\in\mathcal{T}} L_t$.

We study the application of this billing mechanism for sharing the total cost defined by both the quadratic function on equation (5.8) and the peak pricing on (5.10). Therefore, $C(\mathbf{l}_n, \mathbf{l}_{-n})$ in equation (5.12) can be either $C^Q(\mathbf{l}_n, \mathbf{l}_{-n})$ or $C^P(\mathbf{l}_n, \mathbf{l}_{-n})$.

5.5.2 Per-Time-Slot

This mechanism shares the total cost of the transactive solution according to consumers' energy use at each time slot. Therefore, their utility function is written as [Baharlouei et al., 2013, Baharlouei and Hashemi, 2014, Chen et al., 2014, Chakraborty and Khargonekar, 2014, Eksin et al., 2014, Barbato et al., 2015a,b, Eksin et al., 2015, Bahrami and Wong, 2015, Yaagoubi and Mouftah, 2015, Rahman et al., 2017, Rottondi et al., 2017, Eksin et al., 2018, Jacquot et al., 2019]:

$$u_n^S(\mathbf{l}_n, \mathbf{l}_{-n}) = -\sum_{t\in\mathcal{T}} \frac{l_{n,t}}{L_t} C_t(l_{n,t}, l_{-n,t}). \tag{5.14}$$

This equation depends on consumers' load in each time slot, thus a peak demand charge is more complicated to apply, because the peak is a function of the highest groups' consumption in one specific time slot. Therefore, we study the application of this mechanism for dividing total quadratic costs, and $C_t(l_{n,t}, l_{-n,t})$ equals $C_t^Q(l_{n,t}, l_{-n,t})$ of equation (5.7). In this case, we can rewrite the above equation as (5.15), where $P_t(l_{n,t}, l_{-n,t}) = a_t L_t + b_t$ are linear prices arising from the game.

$$u_n^S(\mathbf{l}_n, \mathbf{l}_{-n}) = -\sum_{t\in\mathcal{T}} l_{n,t} P_t\left(l_{n,t}, l_{-n,t}\right) \tag{5.15}$$

5.6 Game Types Analyzed

Considering the load model described in section 5.2, the total cost functions in 5.3, the integer game of 5.4, and the two billing mechanisms in 5.5, three types of games for scheduling integer and energy variant loads are analyzed in this thesis:

1. Game with the proportional-to-consumption billing and the quadratic total cost function;

2. Game with the proportional-to-consumption billing and the peak pricing total cost function;

3. Game with the per-time-slot billing and the quadratic total cost function;

Multiple aspects of those game types are studied in the next sections, as existence of Nash Equilibria (section 5.7), convergence properties (section 5.8), multiplicity of Nash Equilibria (section 5.9), fairness (section 5.10), equity (section 5.11), strategy-proof (section 5.12), general applicability of the models (section 5.14), and the price-of-anarchy (PoA) (section 5.13). We use simple examples to illustrate those properties and simulate the game types in the results chapter 6.

5.7 Existence of Nash Equilibria

In this section, we discuss the existence of Nash Equilibria for the integer scheduling of energy variant loads modeled as a non-cooperative game. Two contributions are given: 1) we show that a non-cooperative game with the proportional-to-consumption billing does not guarantee a potential game formulation, thus a Nash Equilibrium, in the presence of multi-period energy variant loads (such as thermal loads); 2) we prove that the game with the per-time-slot billing is an exact potential game.

Nash Equilibrium is a solution concept for non-cooperative games. For readers who are unfamiliar with it, an explanation is given in 2.4.

We reintroduce the concept of exact potential games presented in chapter 2 to develop the theoretical proofs. This concept is used to define the existence of Nash Equilibria for integer games, and its characteristics are described in section 2.6.

Definition 10. ***(Exact Potential Games)*** *Monderer and Shapley [1996] The game Γ is an exact potential game, if there exists a potential function $\phi : \mathcal{S} \mapsto \mathbb{R}$ such that, for every player $n \in \mathcal{N}$, for every opponents' strategy $\mathbf{l}_{-n} \in \mathcal{S}_{-n}$ and for every two strategies of player n, $\mathbf{l}_n, \mathbf{q}_n \in \mathcal{S}_n$, it holds that:*

$$u_n(\mathbf{l}_n, \mathbf{l}_{-n}) - u_n(\mathbf{q}_n, \mathbf{l}_{-n}) = \phi(\mathbf{l}_n, \mathbf{l}_{-n}) - \phi(\mathbf{q}_n, \mathbf{l}_{-n}). \tag{5.16}$$

5.7.1 Game with Proportional-to-Consumption Billing

5.7.1.1 Energy Invariant Loads

The proportional-to-consumption mechanism guarantees the existence of Nash Equilibrium when only energy invariant loads—appliances with a fixed amount of energy to be scheduled in a given day ($\sum_{t \in \mathcal{T}} l_{n,t}$ = constant, $\forall n \in \mathcal{N}$)—are considered. For instance, if the variables are continuous and the total cost function is strictly convex, the Nash Equilibrium is unique and minimizes the total system cost [Baharlouei et al., 2018, Zhou et al., 2019]. In addition, if the variables are integer, the game is an exact potential with potential function equal to the total system cost [Zhu et al., 2015], which means that consumers try to minimize the total system cost.

The proof is straightforward for all cases of variables types and total cost functions we consider in this thesis. Because f_n in equation (5.12) is a constant for energy invariant loads, consumers will try to optimize $C(\mathbf{l}_n, \mathbf{l}_{-n})$ to minimize their utilities. First, if the variables are continuous and the total cost function is strictly convex, as in the case of the quadratic total cost $C^Q(\mathbf{l}_n, \mathbf{l}_{-n})$ defined in (5.7), this function has a unique optimum value, thus the solution of the game will reach this minimum. Moreover, as f_n is fixed for all $n \in \mathcal{N}$, consumers' bills will be unique (and minimum)—see example 1. Second, if the total cost is convex, as in the peak pricing total cost function $C^P(\mathbf{l}_n, \mathbf{l}_{-n})$ defined in (5.10), it also has an attainable minimum (both global or local are optimum) that will be the solution of the game. Third, if the variables are integer, the total cost $C(\mathbf{l}_n, \mathbf{l}_{-n})$ will be the exact potential function ϕ of definition 10. However, in this last case, sub-optimal solutions of the game can be the Nash Equilibrium, as can be seen in example 2 and we further explain in section 5.9.

Example 1. Consider a scheduling game between three consumers $\mathcal{N} = \{1, 2, 3\}$. Each participant has one appliance to schedule in two time slots. The total energy to be scheduled is $H_1 = 1$ kWh, $H_2 = 2$ kWh, and $H_3 = 3$ kWh. The total cost of the resulting schedule is a quadratic function of the form $\phi = \sum_{t=1}^{2} a_t L_t^2$, in which $L_t = l_{1,t} + l_{2,t} + l_{3,t}$, $l_{n,t}$ is the energy consumer n places in time slot t, $a_1 = 1$ and $a_2 = 2$. The cost for n is proportional to his/her consumption: $u_n = \frac{H_n}{\sum_{n \in \mathcal{N}} H_n} \phi$. This is a potential game with potential function equal to ϕ. If the strategies $l_{n,t}$ are continuous, the set of possible scheduling vectors can be defined as $\mathcal{S}_n = \{[l_{n,1}, H_n - l_{n,1}]\}$. We can rewrite the total cost function as $\phi = (l_{1,1} + l_{2,1} + l_{3,1})^2 + 2(6 - l_{1,1} - l_{2,1} - l_{3,1})^2$. Each consumer seeks to minimize his/her utility. Thus, by taking the derivative $\frac{\partial u_n}{\partial l_{n,1}} = 0$, any solution $(l_{1,1}, l_{2,1}, l_{3,1})$ that satisfy $l_{1,1} + l_{2,1} + l_{3,1} = 4$ and $0 \leqslant l_{n,1} \leqslant H_n$, $n = \{1, 2, 3\}$, is a Nash Equilibrium. Even though there are infinity possible combinations, all of them have total cost equal to 24 and in all of them consumer 1 payoff is $u_1 = -4$, consumer 2 is $u_2 = -8$, and consumer 3 is $u_2 = -12$.

Example 2. Let's assume the game in example 1 is integer and the power rate of consumers' appliances is equal to H_n. Thus, the set of possible strategies for each consumer is reduced to turning it on at one of the two time slots: $\mathcal{S}_n = \{[H_n, 0]; [0, H_n]\}$. We represent this new version of the scheduling game in matrix form in table 10. All possible combinations of consumers' strategies and the resulting cost for each of them are shown in the table. The best responses of each player to the opponents' strategies are underlined. This game has 3 NEs: one equal to the continuous version (and with optimal total cost), which strategies are $sNE_1 = \{l_1 = [1,0], l_2 = [0,2], l_3 = [3,0]\}$; and two sub-optimal with total cost equal to 27, which strategies are $sNE_2 = \{l_1 = [0,1], l_2 = [2,0], l_3 = [3,0]\}$ and $sNE_3 = \{l_1 = [1,0], l_2 = [2,0], l_3 = [0,3]\}$, and utilities are $u_1 = -4.5$, $u_2 = -9$, and $u_3 = -13.5$.

Table 10 – Example 2 payoff matrix of the game with integer and energy invariant loads, considering a proportional-to-consumption billing.

$l_3 = [3,0]$	$l_2 = [2,0]$	$l_2 = [0,2]$
$l_1 = [1,0]$	-6.0, -12.0, -18.0	$\underline{-4.0}$, $\underline{-6.0}$, $\underline{-12.0}$
$l_1 = [0,1]$	$\underline{-4.5}$, $\underline{-9.0}$, $\underline{-13.5}$	-4.5, $\underline{-9.0}$, $\underline{-13.5}$

$l_3 = [0,3]$	$l_2 = [2,0]$	$l_2 = [0,2]$
$l_1 = [1,0]$	$\underline{-4.5}$, $\underline{-9.0}$, $\underline{-13.5}$	$\underline{-8.5}$, -17.0, -25.5
$l_1 = [0,1]$	-6.0, $\underline{-12.0}$, -18.0	-12.0, -24.0, -36.0

5.7.1.2 Energy Variant Loads

For energy variant loads with a multi-period scheduling characteristic (as TCLs), the total energy to be scheduled is no longer fixed. This energy variant nature results from the fact that thermal loads are not purely shiftable and entail the so called "energy payback" [Bischke and Sella, 1985, Wei and Chen, 1995]. In other words, shifting TCLs in time while maintaining comfort standards implies overall energy variations in relation to a base consumption. In fact, since the comfort constraints of TCLs' owners are related to temperature targets, different scheduling solutions result in a different overall energy consumption. Thus, looking at equation (5.12), the factor multiplying the total cost function is no longer constant, depending on the variables results, and consumers no longer seek to minimize the total cost. We show that, under this scenario, the game is no longer potential in theorem 7.

Theorem 7. *The non-cooperative scheduling game* $\Gamma = \langle \mathcal{N}, (\mathcal{S}_n)_{n\in\mathcal{N}}, \{u_n^C\}_{n\in\mathcal{N}} \rangle$ *with* $\mathcal{S}_n$ *defined in equation* (5.5)*, relaxing the integer constraints, i.e.* $0 \leqslant y_{n,t} \leqslant 1$*, and* u_n^C *equals to equation* (5.12) *with* $C(\mathbf{l}_n, \mathbf{l}_{-n}) = C^Q(\mathbf{l}_n, \mathbf{l}_{-n})$ *(proportional-to-consumption billing to divide a total quadratic cost) is not a potential game.*

Proof. Consider a two player game $\mathcal{N} = \{n, -n\}$ (one player against his/her opponents), the strategies of player n as $\mathbf{l}_n = \{l_{n,1}, l_{n,2}, \ldots, l_{n,t}\}$, and the coupled strategies of opponents $-n$ as $\mathbf{q}_{-n} = \{q_{-n,1}, q_{-n,2}, \ldots, q_{-n,t}\}$, i.e $q_{-n,t} = \sum_{n\in\mathcal{N}\setminus n} l_{n,t}$. To simplify the notation, we write $\sum_{t\in\mathcal{T}}\left(a_t(l_{n,t}+q_{-n,t})^2+b_t(l_{n,t}+q_{-n,t})\right)=C^Q(\mathbf{l}_n,\mathbf{q}_{-n})$. The first derivatives of player n's utility with respect to variables $\mathbf{q}_{-n}$ are:

$$\begin{aligned}\frac{\partial u_n^C(\mathbf{l}_n,\mathbf{q}_{-n})}{\partial \mathbf{q}_{-n}} &= \frac{\partial}{\partial \mathbf{q}_{-n}}\left[-\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})\right]\\ &= \left[\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^2}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})\right.\\ &\quad\left.-\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}\left(2a_v(l_{n,v}+q_{-n,v})+b_v\right)\right]_{v\in\mathcal{T}}. \end{aligned} \tag{5.17}$$

The first derivatives are a vector with values defined for each time slot $v\in\mathcal{T}$. Considering $c_v=2a_v(l_{n,v}+q_{-n,v})+b_v$ for all $v\in\mathcal{T}$, and $c_h=2a_h(l_{n,h}+q_{-n,h})+b_h$ for all $h\in\mathcal{T}, h\neq v$, the second derivatives of player n's utility with respect to variables $\mathbf{l}_n$ are:

$$\begin{aligned}\left.\frac{\partial^2 u_n^C(\mathbf{l}_n,\mathbf{q}_{-n})}{\partial \mathbf{l}_n\partial \mathbf{q}_{-n}}\right|_{h\neq v} &= \left[\frac{1}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^2}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})-2\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^3}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})\right.\\ &\quad\left.+\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^2}c_h-\frac{1}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}c_v+\frac{1}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}c_v\right], \end{aligned} \tag{5.18}$$

$$\begin{aligned}\left.\frac{\partial^2 u_n^C(\mathbf{l}_n,\mathbf{q}_{-n})}{\partial \mathbf{l}_n\partial \mathbf{q}_{-n}}\right|_{h=v} &= \left[\frac{1}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^2}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})-2\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^3}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})\right.\\ &\quad+\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^2}c_v-\frac{1}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}c_v+\frac{1}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}c_v\\ &\quad\left.-2\frac{\sum_{t\in\mathcal{T}} l_{n,t}}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}a_v\right]. \end{aligned} \tag{5.19}$$

Therefore, $\frac{\partial^2 u_n^C(\mathbf{l}_n,\mathbf{q}_{-n})}{\partial \mathbf{l}_n\partial \mathbf{q}_{-n}}$ is a $T\times T$ matrix with diagonal values equal to (5.19) and off-diagonal values equal to (5.18).

On the other hand, the first derivatives of player $-n$'s utility with respect to variables $\mathbf{q}_{-n}$ are:

$$\begin{aligned}\frac{\partial u_{-n}^C(\mathbf{l}_n,\mathbf{q}_{-n})}{\partial \mathbf{q}_{-n}} &= \frac{\partial}{\partial \mathbf{q}_{-n}}\left[-\frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})\right]\\ &= \left[-\frac{1}{\sum_{t\in\mathcal{T}}(l_{n,t}+q_{-n,t})}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})+\frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{\left(\sum_{t\in\mathcal{T}} l_{n,t}+q_{-n,t}\right)^2}C^Q(\mathbf{l}_n,\mathbf{q}_{-n})\right.\end{aligned}$$

$$- \frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{\sum_{t\in\mathcal{T}} (l_{n,t} + q_{-n,t})} (2a_v(l_{n,v} + q_{-n,v}) + b_v) \Bigg]_{v\in\mathcal{T}}, \tag{5.20}$$

and the second derivatives of player $-n$'s utility with respect to variables $\mathbf{l}_n$ are:

$$\begin{aligned} \left.\frac{\partial^2 u^C_{-n}(\mathbf{l}_n, \mathbf{q}_{-n})}{\partial \mathbf{l}_n \partial \mathbf{q}_{-n}}\right|_{h\neq v} = & \Bigg[\frac{1}{(\sum_{t\in\mathcal{T}} l_{n,t} + q_{-n,t})^2} C^Q(\mathbf{l}_n, \mathbf{q}_{-n}) - \frac{1}{\sum_{t\in\mathcal{T}} (l_{n,t} + q_{-n,t})} c_h \\ & - 2\frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{(\sum_{t\in\mathcal{T}} l_{n,t} + q_{-n,t})^3} C^Q(\mathbf{l}_n, \mathbf{q}_{-n}) + \frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{(\sum_{t\in\mathcal{T}} l_{n,t} + q_{-n,t})^2} c_h \\ & + \frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{\sum_{t\in\mathcal{T}} (l_{n,t} + q_{-n,t})} c_v \Bigg], \end{aligned} \tag{5.21}$$

$$\begin{aligned} \left.\frac{\partial^2 u^C_{-n}(\mathbf{l}_n, \mathbf{q}_{-n})}{\partial \mathbf{l}_n \partial \mathbf{q}_{-n}}\right|_{h=v} = & \Bigg[\frac{1}{(\sum_{t\in\mathcal{T}} l_{n,t} + q_{-n,t})^2} C^Q(\mathbf{l}_n, \mathbf{q}_{-n}) - \frac{1}{\sum_{t\in\mathcal{T}} (l_{n,t} + q_{-n,t})} c_v \\ & - 2\frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{(\sum_{t\in\mathcal{T}} l_{n,t} + q_{-n,t})^3} C^Q(\mathbf{l}_n, \mathbf{q}_{-n}) + \frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{(\sum_{t\in\mathcal{T}} l_{n,t} + q_{-n,t})^2} c_v \\ & + \frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{\sum_{t\in\mathcal{T}} (l_{n,t} + q_{-n,t})} c_v - 2\frac{\sum_{t\in\mathcal{T}} q_{-n,t}}{\sum_{t\in\mathcal{T}} (l_{n,t} + q_{-n,t})} a_v \Bigg]. \end{aligned} \tag{5.22}$$

Again, $\frac{\partial^2 u^C_{-n}(\mathbf{l}_n, \mathbf{q}_{-n})}{\partial \mathbf{l}_n \partial \mathbf{q}_{-n}}$ is a $T \times T$ matrix with diagonal values equal to (5.22) and off-diagonal values equal to (5.21). One can notice that the derivatives for player n are different from the derivatives of player $-n$, because consumers' own variables on the fraction numerator are different. Therefore, theorem 4.5 of Monderer and Shapley [1996]—see theorem 4 in section 2.6—is not respected for every $n, -n \in \mathcal{N}$, meaning the game with proportional-to-consumption billing, a quadratic cost function, and energy variant loads is not potential. □

Gopalakrishnan et al. [2014, Th. 1] proves that, in cost sharing games[1], a potential formulation is necessary to ensure the existence of a pure-strategy Nash Equilibrium. Therefore, as a potential game formulation is no longer guaranteed for the relaxed game (with continuous variables), the game does not have pure-strategy equilibria, and the solution algorithm does not converge when energy variant loads are included.

On the other hand, for the integer version, there is no theorem to generally demonstrate a game is not potential. Therefore, the integer load scheduling game with proportional-to-consumption billing and energy variant loads may or may not have Nash Equilibria. We discuss this notion with the following example.

[1] The scheduling game analyzed in this thesis can be rewritten as a cost sharing game, considering each time slot as a resource and the appliances power consumption as different demands of those resources.

Example 3. Consider a game with proportional-to-consumption billing, any convex total cost function (e.g. quadratic function, peak pricing function, or another one), and energy variant loads. A consumer n has the choice between two total consumption values: one equal to κ and the other equal to μ. The rest of the community has fixed total load equal to ζ, which is equally split among the consumers. Consider that two total costs are possible for this game: $C_1 > C_2$. As the rest of the community has a fixed consumption, it prefers the smaller total cost C_2. However, consumer n can have a better utility when choosing the strategy leading to the total cost C_1, say strategy κ, and a worst utility when choosing strategy μ leading to C_2. Thus:

$$\frac{\kappa}{\kappa+\zeta}C_1 < \frac{\mu}{\mu+\zeta}C_2 \tag{5.23}$$

And consumer n chooses κ, enhancing the cost to C_1. As a response, the rest of the community manages to bring the total cost back to C_2:

$$\begin{aligned} \frac{\zeta}{\kappa+\zeta}C_1 &> \frac{\zeta}{\kappa+\zeta}C_2 \\ C_1 &> C_2 \end{aligned} \tag{5.24}$$

For both equations (5.23) and (5.24) to be true, it is necessary that:

$$\begin{aligned} \kappa(\mu+\zeta)C_1 &< \mu(\kappa+\zeta)C_2 \\ C_1 &< \frac{\mu(\kappa+\zeta)}{\kappa(\mu+\zeta)}C_2 \\ \frac{\mu(\kappa+\zeta)}{\kappa(\mu+\zeta)} &> 1 \\ \mu &> \kappa \end{aligned} \tag{5.25}$$

Therefore, a Nash Equilibrium does not exist if consumer n has a strategy κ with total consumption smaller than μ, leading to a higher total cost to the community C_1 in a way that this higher total cost does not surpasses $\frac{\mu(\kappa+\zeta)}{\kappa(\mu+\zeta)}C_2$. It is important to notice that, for this NE to not exist, both community and consumer n must have the possibility to choose a strategy to bring the cost back to C_1 or C_2 when the opponent changed it. That is the reason why the game is not potential with an infinite possibility of actions to change the total cost (continuous variables). However, with integer loads, it depends on the strategies the players have access to define if a Nash Equilibrium exists.

5.7.2 Game with Per-Time-Slot Billing

For continuous games, the per-time-slot billing has been shown to have Nash Equilibria [Baharlouei and Hashemi, 2014, Jacquot et al., 2019]. However, our decision variables in the vectors $\mathbf{x}_n$ are binary. Furthermore, the set of agents has N elements. Thus, when adding integer variables, the game becomes a finite game, because it has a finite set of actions and players [Nash, 1951]. Therefore, it is necessary to prove that this integer and finite game is potential to guarantee the existence of Nash Equilibria and the convergence of the algorithm used to solve it. We introduce the function (5.26), and prove it is the potential function for the integer scheduling game with PTS to divide the total quadratic cost in theorem 8. With this proof, we guarantee that this finite game has at least one pure-strategy Nash Equilibria [Lã et al., 2016]. We also show a simple example in 4 to illustrate the application of the per-time-slot billing to divide a total quadratic cost with integer and energy variant loads.

$$\phi(\mathbf{s}) = -\sum_{t\in\mathcal{T}}\left[\sum_{j\in\mathcal{N}} b_t l_{j,t} + \sum_{j\in\mathcal{N}} a_t (l_{j,t})^2 + a_t \sum_{j\in\mathcal{N}} \sum_{\substack{i\in\mathcal{N}\\ i<j}} l_{j,t} l_{i,t}\right] \tag{5.26}$$

Theorem 8. *The non-cooperative scheduling game* $\Gamma = \langle \mathcal{N}, (\mathcal{S}_n)_{n\in\mathcal{N}}, \{u_n^S\}_{n\in\mathcal{N}}\rangle$ *with* $\mathcal{S}_n$ *defined in equation* (5.5)*, and* u_n^S *equals to equation* (5.15) *(per-time-slot billing to divide a total quadratic cost) is an exact potential game.*

Proof. To prove the theorem, we need to show that for any two strategies $\mathbf{l}_n$ and $\mathbf{q}_n$, and for every player $n \in \mathcal{N}$, (5.16) holds. Starting from the left-hand side of the equation, for a fixed player $n \in \mathcal{N}$, we have:

$$\begin{aligned}
u_n^S(\mathbf{l}_n, \mathbf{l}_{-n}) - u_n^S(\mathbf{q}_n, \mathbf{l}_{-n}) \\
&= -\sum_{t\in\mathcal{T}}\left(b_t + a_t \sum_{m\in\mathcal{N}} l_{m,t}\right) l_{n,t} \\
&\quad + \sum_{t\in\mathcal{T}}\left[b_t + a_t\left(q_{n,t} + \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t}\right)\right] q_{n,t} \\
&= -\sum_{t\in\mathcal{T}}\left[b_t l_{n,t} + a_t (l_{n,t})^2 + a_t l_{n,t} \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t}\right] \\
&\quad + \sum_{t\in\mathcal{T}}\left[b_t q_{n,t} + a_t (q_{n,t})^2 + a_t q_{n,t} \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t}\right].
\end{aligned} \tag{5.27}$$

Consider $\phi_t(\mathbf{s})$ as the potential of a strategy profile $\mathbf{s}$ for a specific time slot $t \in \mathcal{T}$ of (5.26). The difference in the potential function when this same player changes his/her strategy from $\mathbf{l}_n$ to $\mathbf{q}_n$, which means the game strategy profile changes from $(\mathbf{l}_n, \mathbf{l}_{-n})$ to $(\mathbf{q}_n, \mathbf{l}_{-n})$, is:

$$\begin{aligned}
\phi_t(\mathbf{l}_n, \mathbf{l}_{-n}) - \phi_t(\mathbf{q}_n, \mathbf{l}_{-n}) \\
&= -b_t l_{n,t} - b_t \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t} - a_t (l_{n,t})^2 - a_t \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} (l_{m,t})^2 \\
&\quad - a_t l_{n,t} \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t} - a_t \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} \sum_{\substack{j\in\mathcal{N}\\ j<m\\ j\neq n}} l_{j,t} l_{m,t} \\
&\quad + b_t q_{n,t} + b_t \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t} + a_t (q_{n,t})^2 + a_t \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} (l_{m,t})^2 \\
&\quad + a_t q_{n,t} \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t} + a_t \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} \sum_{\substack{j\in\mathcal{N}\\ j<m\\ j\neq n}} l_{j,t} l_{m,t} \\
&= -b_t l_{n,t} - a_t (l_{n,t})^2 - a_t l_{n,t} \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t} \\
&\quad + b_t q_{n,t} + a_t (q_{n,t})^2 + a_t q_{n,t} \sum_{\substack{m\in\mathcal{N}\\ m\neq n}} l_{m,t}. \qquad (5.28)
\end{aligned}$$

By summing up both sides of the above equation with respect to $t \in \mathcal{T}$, we show that the difference in the potential function $\phi(\mathbf{s})$ is equal to (5.27). Therefore, Definition 10 holds, which means that the game proposed is an exact potential game with potential function equal to (5.26) and the proof is complete. □

Example 4. Consider a scheduling game between two consumers $\mathcal{N} = \{1, 2\}$. Each participant has one air conditioner to schedule in four time slots. The total cost of the resulting group's schedule is a quadratic function of the form $\phi = \sum_{t=1}^{4} a_t L_t^2$, in which $L_t = l_{1,t} + l_{2,t}$, $l_{n,t}$ is the energy consumer n places in time slot t, $a_1 = 1$, $a_2 = 1$, $a_3 = 2$, and $a_4 = 2$. To respect their temperature preferences, each consumer has three possible scheduling strategies (in kWh), i.e. $\mathcal{S}_1 = \{[1, 1, 0, 1], [0, 1, 1, 0], [1, 1, 1, 0]\}$ and $\mathcal{S}_2 = \{[0, 0, 2, 2], [2, 2, 0, 2], [0, 2, 2, 0]\}$. One can notice that, by summing the consumption of the possible strategies, the flexible load is energy variant. Using a per-time-slot billing, the game in matrix form in presented table 11. All possible combinations of consumers' strategies and the resulting cost for each of them are shown in the table. The best responses of each player to the opponents' strategies are underlined. This game has two NEs: one with total cost equal to 20, in which consumers' utilities are $u_1 = -6$ and $u_2 = -14$; one with total cost equal to 23, in which consumers' utilities are $u_1 = -5$ and $u_2 = -18$.

Table 11 – Example 4 payoff matrix of the game with integer and energy variant loads, considering a per-time-slot billing.

	$l_2 = [0, 0, 2, 2]$	$l_2 = [2, 2, 0, 2]$	$l_2 = [0, 2, 2, 0]$
$l_1 = [1, 1, 0, 1]$	-8, -20	-12, -24	<u>-6</u>, <u>-14</u>
$l_1 = [0, 1, 1, 0]$	<u>-7</u>, -20	<u>-5</u>, <u>-18</u>	-9, <u>-18</u>
$l_1 = [1, 1, 1, 0]$	-8, -20	-8, -20	-10, <u>-18</u>

5.8 Solution Algorithm and its Convergence Properties

Exact potential games have the finite improvement property, which assures the convergence to the equilibrium set of any myopic learning method based on the one-sided better reply dynamic [Monderer and Shapley, 1996]—see section 2.7. Therefore, we apply the Best Response Dynamics (BRD) to solve the integer scheduling problem with different cost functions and billing mechanisms. This learning method consists of a sequential decision model in which consumers take turn and best respond to opponents' last strategies (TCLs schedules in our framework). We use a version in which consumers communicate the total group's load, instead of their individual profiles, which leads to less messages exchanges and reduces the privacy issue [Rahman et al., 2017]. We provide a brief overview of the algorithm, a pseudo code in 1, and its flowchart in figure 9.

Assuming that each consumer's home energy management system keeps track of opponents' total consumption vector ($\mathbf{L}^k_{-n}$), and the total load of all players is initialized with zeros $\mathbf{L}^0 = [\mathbf{0}]$, each iteration of the gameplay consists of 3 steps:

1. consumers receive the total load vector and calculate the load of opponents $\mathbf{L}^k_{-n}$ using their previous best strategies;

2. they update their strategies $\mathbf{l}^{k+1}_n$ as a response to $\mathbf{L}^k_{-n}$, by solving the local mixed-integer linear/quadratic[2] program (5.29);

$$\mathbf{l}^{k+1}_n = BR_n(\mathbf{L}^k_{-n}) = \underset{\mathbf{q}_n \in \mathcal{S}_n}{\operatorname{argmax}}\, u_n(\mathbf{q}_n, \mathbf{L}^k_{-n}) \tag{5.29}$$

3. consumers add their local strategy $\mathbf{l}^{k+1}_n$ to the opponents' total consumption vector $\mathbf{L}^k_{-n}$ and send this new aggregated consumption profile $\mathbf{L}^{k+1}$ to the next player.

As BRD converges to a Nash Equilibrium for all exact potential games [Monderer and Shapley, 1996], the process continues until an equilibrium is reached and consumers

[2] If the total cost function is the peak pricing, the model to be solved is a MILP, if it is quadratic, the model is a MIQP.

can no longer reduce their bills when changing their schedules. It is worth mentioning that this algorithm is decentralized and each consumer has autonomy to schedule locally his/her TCLs, which is one of the pillars of transactive control.

Algorithm 1 Best Response Dynamics (BRD)

Input: Local data about consumers' appliances and preferences $(\mathcal{N}, \mathcal{S})$; market data related to the pricing function which defines consumers' utility (u_n)
Output: load schedules $\mathbf{s} = (\mathbf{l}_1, \mathbf{l}_2, \ldots, \mathbf{l}_N)$ which forms the Nash Equilibrium

$k \leftarrow 0$;
Initialize $\mathbf{l}_n^k = [\mathbf{0}]\ \forall n \in \mathcal{N}$ and $\mathbf{L}^k = \sum_{n \in \mathcal{N}} \mathbf{l}_n^k$;
count $\leftarrow 1$;
while *count* > 0 **do**
  count $\leftarrow 0$;
  for all $n \in \mathcal{N}$ **do**
    Calculate $\mathbf{L}_{-n}^k$ using $\mathbf{L}^k$ received: $\mathbf{L}_{-n}^k = \mathbf{L}^k - \mathbf{l}_n^k$;
    Solve local problem (5.29): $\mathbf{l}_n^{k+1} = BR_n(\mathbf{L}_{-n}^k)$, to calculate the new schedule;
    Update and broadcast new $\mathbf{L}^{k+1} = \mathbf{l}_n^{k+1} + \mathbf{L}_{-n}^k$ to the next consumer $(n+1)$;
    if $u_n(\mathbf{l}_n^{k+1}, \mathbf{L}_{-n}^k) < u_n(\mathbf{l}_n^k, \mathbf{L}_{-n}^k)$ **then**
      count $\leftarrow$ *count* $+ 1$
    end if
    $k \leftarrow k + 1$;
  end for
end while
return $\mathbf{s}$;

The convergence of this algorithm to a Nash Equilibrium is guaranteed for potential games, thus for games applying the proportional-to-consumption billing to schedule energy invariant loads, and for games with the per-time-slot billing to schedule any type of loads. In the case of the PTC applied to energy variant loads, we proved that the game is not potential in theorem 7, which means the existence of NE is not ensured, and thus the algorithm may not converge. Moreover, equation (5.12) is a non-linear and non-quadratic function when energy variant loads are present, which can not be solved using commercial solvers. Thus, to analyze this billing with thermal loads, two approaches are used: 1) we develop an algorithm based on the BRD to evaluate the convergence of this game (m-BRD); and 2) we estimate the total energy consumption for each end-user and we run the BRD considering those flexible loads as invariant. The algorithm of the first option is shown in 2, and the equity of the payoffs distribution considering an estimation of consumers' total energy is analyzed in 5.11.

5.9 Multiplicity of Nash Equilibria

In this section, we discuss the multiplicity of Nash Equilibria for the integer scheduling of energy variant loads modeled as a non-cooperative game. Two contributions are given: 1) since the integer nature of the control affects the theoretical foundations of the

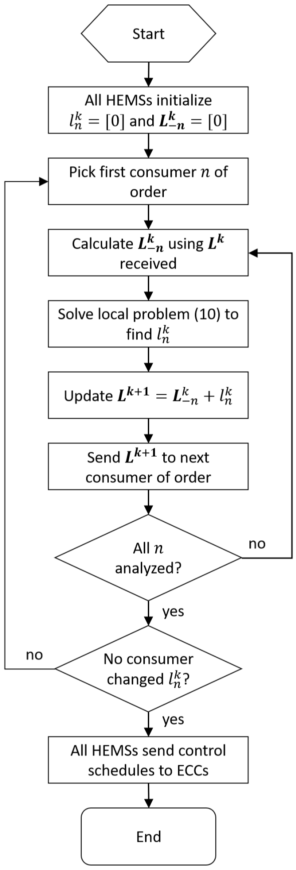


Figure 9 – Flowchart of the game solving process using Best Response Dynamics (BRD).

problem, we prove that multiple Nash Equilibria can exist and they can be sub-optimum; 2) we discuss the practical implications of having multiple NEs in real implementation of TC platforms, in terms of optimality of the total scheduling cost, variability in consumers' payments, and how the algorithm design defines the solution that will be effectively played.

The proof that the game with integer loads has multiple NEs is done by contradiction: we present the necessary characteristics of a game to guarantee a unique NE, and then we show our game (i.e. multiple TCL scheduling decisions in an energy community) does not have all these properties. We also discuss the impact on the problem of having many possible outcomes, and explore factors that determine the NE that is going to be reached at the end.

To have a unique NE, a game must respect two conditions: players' utility functions must be strictly concave and the strategy sets must be convex [Neyman, 1997, Rosen, 1965]. To explain why those conditions are necessary for a unique equilibrium, we use the exact potential games theory presented in chapter 2 and section 5.7, as it is easier to understand. As showed in definition 10, a game is exact potential if it has a function $\phi : \mathcal{S} \mapsto \mathbb{R}$ that maps the joint strategy set to the set of real numbers. When a game has an exact potential function, each consumer's change in strategy generates a change in his/her payoff that equals the change in the potential function, while opponents' strategies are kept fixed.

Algorithm 2 Procedure for scheduling thermal loads with the proportional-to-consumption billing (m-BRD).

Input: Local data about consumers' appliances and preferences $(\mathcal{N}, \mathcal{S})$; market data related to the pricing function which defines consumers' utility (u_n)
Output: load schedules $\mathbf{s} = (\mathbf{l}_1, \mathbf{l}_2, \ldots, \mathbf{l}_N)$ which forms the Nash Equilibrium

```
k ← 0;
Initialize l_n^k = [0] ∀n ∈ N and L^k = Σ_{n∈N} l_n^k;
count ← 1;
Initialize f_n^k = [0] ∀n ∈ N of equation (5.13)
while count > 0 do
    count ← 0;
    for all n ∈ N do
        Calculate L_{-n}^k using L^k received: L_{-n}^k = L^k − l_n^k;
        Solve local problem (5.29): l_n^{k+1} = BR_n(L_{-n}^k) = argmax_{q_n∈S_n} u_n^C(q_n, L_{-n}^k) =
        argmax_{q_n∈S_n} −f_n^k C^Q(q_n, L_{-n}^k), to calculate the new schedule;
        Update and broadcast new L^{k+1} = l_n^{k+1} + L_{-n}^k to the next consumer (n + 1);
        if u_n(l_n^{k+1}, L_{-n}^k) < u_n(l_n^k, L_{-n}^k) then
            count ← count + 1
        end if
        Update n's share f_n^{k+1} = (Σ_{t∈T} l_{n,t}^{k+1}) / (Σ_{t∈T} L_{n,t}^{k+1})
        k ← k + 1;
    end for
end while
return s;
```

For example, if a consumer n shifts from strategy $\mathbf{l}_n^1$ to $\mathbf{l}_n^2$, when opponents are playing a joint strategy profile $\mathbf{l}_{-n}$, his/her payoff change will be equal to $u_n(\mathbf{l}_n^2, \mathbf{l}_{-n}) - u_n(\mathbf{l}_n^1, \mathbf{l}_{-n})$, which will be the same as the change in the potential function $\phi(\mathbf{l}_n^2, \mathbf{l}_{-n}) - \phi(\mathbf{l}_n^1, \mathbf{l}_{-n})$. If the potential function is strictly concave, it has a unique global maximum. Moreover, if the strategy set is convex and bounded, the unique maximum will be always reached, as every consumer will try to unilaterally ameliorate his/her utility (thus the potential function), until its maximum is reached.

For example, in the game with the proportional-to-consumption billing and energy invariant loads, the total cost function will be the exact potential function of the game—see section 5.7.1.1. This function is strictly convex for the total quadratic cost function[3]. If the variables are continuous, each consumer seeks to minimize this convex total cost, reachable by the continuous control, the NEs have the same minimum total cost, and even if they are different in terms of schedules of appliances, they allocate the same bills to consumers, which makes the result unique—the reader is referred to Baharlouei et al. [2018] for the complete proof. Even with the per-time-slot billing, if the variables are continuous, those conditions (potential function strictly concave and convex strategy sets) are respected [Jacquot et al., 2019]. However, in our scheduling game, the strategy sets are no longer convex, because of the integer nature of the TCLs control [Jünger et al., 2009]. This means

[3] Remember that maximize a strictly concave function is the same as minimize a strictly convex function.

that pure strategy NEs with sub-optimal total cost may also exist, corresponding to "local minima" [Monderer and Shapley, 1996]. The consequences of this are twofold. First, the total cost to be shared can vary among equilibria, which means that the result of the play can be sub-optimal for the community (in terms of potential function). In example 2, two out of the three equilibria have total cost 27, instead of the minimum 24.

Second, multiple equilibria means dispersion of individual payments: in one equilibrium an specific consumer can pay more than in another. As seen in example 4, one equilibrium is better for consumer 1 ($l_1 = [0, 1, 1, 0]$ and $l_2 = [2, 2, 0, 2]$) and the other is better for consumer 2 ($l_1 = [1, 1, 0, 1]$ and $l_2 = [0, 2, 2, 0]$). Therefore, the integer control nature of the TCLs will open space for multiple and sub-optimal results, both for the community and for the consumers. Thus, when solving the load scheduling problem with integer potential games, choosing one equilibrium out of the possible ones is an important concern. If some participants are always benefited at the end of the gameplay, and this process is transparent, unfairness can be raised by the others participants.

One question remains: what determines which NE will be reached? Consumers' playing order and the initial solution of the BRD. This algorithm, described in the pseudo code 1, generates finite improvement paths—see chapter 2—for the game: at each iteration k, one consumer $n \in \mathcal{N}$ gathers the total load information of the past iteration $(k-1)$, and best responds to it. If $u_n(\mathbf{l}_n^k, \mathbf{l}_{-n}^{k-1}) > u_n(\mathbf{l}_n^{k-1}, \mathbf{l}_{-n}^{k-1})$, this consumer is the "deviator". The process continues until there's no more "deviators", which means that a terminal point (NE) is reached. Moreover, at each round of the game all consumers play once to verify if there is no more "deviators" in the community. Also, the process starts from load schedules $\mathbf{l}_n^k = [\mathbf{0}]\ \forall n$. Therefore, it is the consumers playing order defined in the set $\mathcal{N}$, and the algorithm's starting point that determines which finite improvement path will be generated, thus what terminal point will be reached. In example 4, if the BRD starts with strategies $l_1 = [1, 1, 0, 1]$ and $l_2 = [0, 0, 2, 2]$, and the playing order is first consumer 1 and then consumer 2, the Nash Equilibrium attained is the one better for consumer 1 ($l_1 = [0, 1, 1, 0]$ and $l_2 = [2, 2, 0, 2]$). However, if the starting point is $l_1 = [1, 1, 0, 1]$ and $l_2 = [2, 2, 0, 2]$, and the playing order is first consumer 2 and then consumer 1, the Nash Equilibrium reached is the one better for consumer 2 ($l_1 = [1, 1, 0, 1]$ and $l_2 = [0, 2, 2, 0]$).

In conclusion, the game types studied here, with integer variables, have multiples NEs, including sub-optimum ones. As a consequence, the community and its consumers may reach sub-optimal solutions when applying this TC approach to coordinate the TCLs. Even though the TC has advantages related to decentralization, coordination, and autonomy, the addition of integer controlled loads leads to the existence of sub-optimum NEs. Moreover, when solving the game with the BRD, the starting point of the algorithm and consumers' playing order define which NE will be reached. In the simulation results, we vary consumers' order when applying the BRD to solve the game. We identify what factors of the game, e.g. parameters of the pricing function, number of consumers participating in the scheduling game, and amount of consumers' flexibility, induce to higher/lower variability among solutions. In the results, we also measure how sub-optimum the multiple NEs are.

5.10 Fairness of Different Billings

In this section, we present the methodology used to evaluate the fairness of the two billing mechanisms studied in this thesis to divide the scheduling costs of integer and thermal appliances. As this method was proposed in the literature of continuous games [Baharlouei et al., 2013, Baharlouei and Hashemi, 2014], here we give a minor contribution: we show that the game designed with a per-time-slot billing is fairer than the proportional-to-consumption model, when integer and energy variant loads are considered.

The design of an utility function depends on the billing mechanism chosen, which should consider the contributions of each consumer to the total cost. As stated by Baharlouei et al. [2013], Baharlouei and Hashemi [2014], consumers with more flexibility to schedule their loads and who intend to consume less energy should have better payoffs. In the context of cooperative games, Shapley [Shapley, 1953] proposed a method to share the total cost of a game that is unique and fair. It is based on the contribution of each player n to all possible coalitions that he/she does not participate in $\mathcal{G} \subseteq \mathcal{N} \backslash \{n\}$. The utility for player n can be calculated by (5.30), in which $C^*_{\mathcal{G}}$ is the minimum cost of the scheduling problem for a coalition $\mathcal{G}$ and $C^*_{\mathcal{G} \cup \{n\}}$ is the minimum cost when consumer n is added to that coalition. Both costs are calculated by solving the total cost problem in (5.9) or (5.11)—i.e. $C^*_{\mathcal{G}}$ can be either $C^{Q*}_{\mathcal{G}}$ or $C^{P*}_{\mathcal{G}}$, depending on the game type. One should notice that the complexity of calculating the Shapley Value (SV) is $o(2^n)$, as long as it implies the total cost calculation of all possible subsets of $\mathcal{N}$ [Shehory and Kraus, 1993].

$$SV_n = \sum_{\mathcal{G} \subseteq \mathcal{N} \backslash \{n\}} \frac{|\mathcal{G}|! \, (|\mathcal{N}| - |\mathcal{G}| - 1)!}{|\mathcal{N}|!} \left[C^*_{\mathcal{G} \cup \{n\}} - C^*_{\mathcal{G}} \right] \tag{5.30}$$

The fairness index is calculated by the distance between the vector of the consumers' utilities after the game and the utilities they should have when applying the SV [Baharlouei et al., 2013, Baharlouei and Hashemi, 2014]. This can be done using (5.31), where u_n is the utility of consumer n at the NE of the game (with the per-time-slot or the proportional-to-consumption billing) and u^*_n is his/her utility calculated by the SV.

$$F = \sum_{n \in \mathcal{N}} \left| \frac{u_n}{\sum_{m \in \mathcal{N}} u_m} - \frac{SV_n}{\sum_{m \in \mathcal{N}} SV_m} \right| \tag{5.31}$$

We give an example to compare the fairness of both the per-time-slot and proportional-to-consumption mechanisms when applying the Shapley Value in 5. Moreover, in the results section, we use this index to compare the fairness of the integer game with both mechanisms in a real case. As the complexity of the SV calculation grows exponentially with the number of consumers, we use sub-cases with 10 consumers to make the comparison.

Example 5. To compare the fairness of both mechanisms studied in this thesis we use an example with energy invariant loads and a quadratic total cost function. Consider a scheduling game between two participants $\mathcal{N} = \{1, 2\}$ for the next four time slots $\mathcal{T} = \{1, 2, 3, 4\}$. Consumer 1 has base load $\mathbf{w}_1 = [6, 2, 1, 1]$ and consumer 2 has $\mathbf{w}_2 = [3, 1, 2, 2]$. Each consumer has 3 possible strategies for scheduling a flexible load: $n = 1$ has $\mathbf{x}_1^1 = [2, 2, 0, 0]$, $\mathbf{x}_1^2 = [0, 2, 2, 0]$ and $\mathbf{x}_1^3 = [0, 0, 2, 2]$; $n = 2$ has $\mathbf{x}_2^1 = [0, 6, 0, 0]$, $\mathbf{x}_2^2 = [0, 0, 6, 0]$ and $\mathbf{x}_2^3 = [0, 0, 0, 6]$. The final scheduling vectors for each consumer are $\mathbf{l}_n = \mathbf{w}_n + \mathbf{x}_n$ Considering $a_t = 1$ and $b_t = 0$ for all $t \in \mathcal{T}$, the payoff matrix of this game using the per-time-slot billing is shown in table 12, and with the proportional-to-consumption billing in table 13. In both matrices, u_1, u_2 is the utility of consumers 1 and 2, respectively. They are calculated using equations (5.15) for the per-time-slot billing and (5.12) for the proportional-to-consumption, considering the total cost as a quadratic function. The best responses of each player to the opponents' strategies are underlined.

Therefore, it is easily verified that this game has two NE in the per-time-slot game: $(\mathbf{l}_1^3, \mathbf{l}_2^1)$ and $(\mathbf{l}_1^2, \mathbf{l}_2^3)$. The first one is the global optimum because it maximizes the potential (equals to -168), and the second NE is a local optimum, having potential equal to -176. Both potentials where calculated using (5.26). Even though both equilibria minimize the total cost, the first NE is better for consumer 2 and the second for consumer 1. On the other hand, in the case of the proportional-to-consumption billing, there are three NE: $(\mathbf{l}_1^3, \mathbf{l}_2^1)$, $(\mathbf{l}_1^2, \mathbf{l}_2^3)$ and $(\mathbf{l}_1^1, \mathbf{l}_2^2)$. The first two are the same as the per-time-slot game and are Pareto-efficient. However, the third NE is a local optimum. Even though it harms both consumers, it can be attained depending on the BR sequence and the tie-break rule when two strategies lead to the same payoff.

Table 12 – Payoff matrix of the fairness example using the per-time-slot billing.

	$\mathbf{l}_2^1 = [3, 7, 2, 2]$	$\mathbf{l}_2^2 = [3, 1, 8, 2]$	$\mathbf{l}_2^3 = [6, 2, 3, 3]$
$\mathbf{l}_1^1 = [8, 4, 1, 1]$	-138, -122	-120, $\underline{-116}$	-120, $\underline{-116}$
$\mathbf{l}_1^2 = [6, 4, 3, 1]$	-116, -120	-110, -126	$\underline{-98}$, $\underline{-114}$
$\mathbf{l}_1^3 = [6, 2, 3, 3]$	$\underline{-102}$, $\underline{-110}$	$\underline{-108}$, -128	-108, -128

The total cost of the load scheduling for each coalition $\mathcal{G} \subseteq \mathcal{N}$ is calculated in table 14. Therefore, the SV for each consumer is $SV_1 = -102$ and $SV_2 = -110$. One should

Table 13 – Payoff matrix of the fairness example using the proportional-to-consumption billing.

	$\mathbf{l}_2^1 = [3, 7, 2, 2]$	$\mathbf{l}_2^2 = [3, 1, 8, 2]$	$\mathbf{l}_2^3 = [6, 2, 3, 3]$
$\mathbf{l}_1^1 = [8, 4, 1, 1]$	-130, -130	-118,-118	-118, -118
$\mathbf{l}_1^2 = [6, 4, 3, 1]$	-118, -118	-118, -118	-106, -106
$\mathbf{l}_1^3 = [6, 2, 3, 3]$	-106, -106	-118, -118	-118, -118

notice that consumer 1 has more flexibility than consumer 2 as long as he/she can split his/her load among more time slots. This explains why consumer 1 pay less using the SV than consumer 2, even though they have the same total load of 14. For each NE and billing mechanism, we can calculate the fairness index. When applying the per-time-slot billing, for the point $(\mathbf{l}_1^3, \mathbf{l}_2^1)$, the consumers' utilities are the same as the SV and the fairness index equals 0.00%. For the NE $(\mathbf{l}_1^2, \mathbf{l}_2^3)$, the consumers' utilities are $(-98, -114)$ and the fairness index equals 3.77%. In the case of the proportional-to-consumption, the utilities are $(-106, -106)$ or $(-118, -118)$. Both lead to a fairness index equals to 3.77%. Therefore, one of the solutions of the PTS is fairer than the NEs of the PTC.

Table 14 – Minimum cost of the possible coalitions of example 5.

$\mathcal{G}$	User 1's Strategy	User 2's Strategy	Value ($C_{\mathcal{G}}^*$)
{∅}	$\mathbf{l}_1^0 = [0, 0, 0, 0]$	$\mathbf{l}_2^0 = [0, 0, 0, 0]$	0
{1}	$\mathbf{l}_1^3 = [6, 2, 3, 3]$	$\mathbf{l}_2^0 = [0, 0, 0, 0]$	-58
{2}	$\mathbf{l}_1^0 = [0, 0, 0, 0]$	$\mathbf{l}_2^1 = [3, 7, 2, 2]$	-66
{1,2}	$\mathbf{l}_1^3 = [6, 2, 3, 3]$ $\mathbf{l}_1^2 = [6, 4, 3, 1]$	$\mathbf{l}_2^1 = [3, 7, 2, 2]$ $\mathbf{l}_2^3 = [3, 1, 2, 8]$	-212

5.11 Equity of the Proportional-to-Consumption

In this section, we discuss the equity of the proportional-to-consumption billing mechanism when energy variant loads are present in the scheduling process. One contribution is given: we show that TCLs energy variant nature impacts the theoretic grounds of the game model, because the total energy in the scheduling horizon is not fixed. Thus, we discuss how this characteristic affects the equity among consumers when applying the proportional-to-consumption billing to the non-cooperative game model.

As proved in section 5.7.1.2, when those loads are added to the game, it loses its potential features. To deal with energy variant loads in a PTC environment, we estimate the total energy consumption for each end-user and run the BRD considering those flexible loads as invariant. We then analyze the equity of the payoffs distribution using the estimation compared with the real total consumption after the game is solved.

With TCLs, the factor f_n in the utility function (5.12) is not fixed, resulting in one of the following issues. If the variables are kept in the factor, consumers' goal disconnects from the community's goal, and algorithm 1 may not converge—see sections 5.7.1.2 and 5.8. On the other hand, if an estimate is used to define consumers' load a priori, as we propose in this section, the game solution may lack equity.

TCLs' energy variant nature results from the fact that thermal loads are not purely shiftable and entail the so called "energy payback" [Bischke and Sella, 1985, Wei and Chen, 1995]. In other words, shifting TCLs in time while maintaining comfort standards implies overall energy increase in relation to the baseline consumption. For instance, a consumer may turn on his/her AC more times at a valley time interval to keep the temperature within the feasible range during a peak time interval, which would increase the daily energy consumption. Therefore, the total load $\sum_{t\in\mathcal{T}} l_{n,t}$ of each consumer $n \in \mathcal{N}$ is a result of the game, and the fraction f_n is not constant.

The problem of letting the variables in this fraction is that consumers' utility (5.12) is no longer equal to minus a constant f_n times the total cost. In this setting, the consumers will not aim at minimizing the total cost when playing the scheduling game, which affects the coordination aspect of the game. Moreover, the existence of an NE for this modified game is not guaranteed, as well as the BRD convergence, because the utility function ceases to be concave and equal for every consumer [Monderer and Shapley, 1996]—see section 5.7.1.2.

To solve this problem in a practical way, in this section we propose to do an a priori estimation of the consumption, and assume it constant during the game. This assumption would allow guaranteeing the same conditions applied in the literature studies that used similar billing methods [Mohsenian-Rad et al., 2010, Baharlouei et al., 2013, Baharlouei and Hashemi, 2014, Zhu et al., 2015, Rahman et al., 2017, Liang et al., 2017, Baharlouei et al., 2018, Fernandez et al., 2018, Noor et al., 2018, Zhou et al., 2019]. Moreover, keeping each consumer's energy share constant during the game play makes the optimization process transparent (consumers know at each iteration how much they are going to pay), guarantees the solution algorithm convergence, and ensures that consumers coordinate to minimize the total community cost. However, this practical assumption might impact the equity of the billing application when the a priori estimation is very different from the a posteriori consumption, i.e. after the scheduling decisions. Therefore, in the results section we analyze this difference and its consequences in real scenarios.

5.12 Strategy-proof of Different Billings

In this section, we discuss the strategy-proof of both billing mechanism studied here. Two contributions are given: 1) we propose an alternative solution to overcome the possibility of participants cheating in per-time-slot billing models, by showing theoretically that a simple adjustment in the billing rules ex-ante instead of ex-post consumption is enough to discourage cheating behavior, which guarantees the strategy-proof of this mechanism; 2) we show that the proportional-to-consumption can lead to untruthful behavior of consumers when the m-BRD is applied to solve it for energy variant loads.

5.12.1 Proportional-to-consumption

The proportional-to-consumption mechanism prevents cheating behaviors when only energy invariant loads are considered, as the potential function is equal to the total cost. This means that lying about consumption increases players own bills [Baharlouei and Hashemi, 2014]. However, for energy variant loads, the potential function is no longer the total cost, as shown in section 5.7.1.2. Therefore, the strategy-proof of this mechanisms is no longer guaranteed.

Moreover, in the results chapter 6, we show that the modified Best Response Dynamics presented in algorithm 2 leads to non-optimal solutions for consumers, who can change their strategies after the equilibrium was reached to get better payoffs. This simplified algorithm, created to deal with the energy variant nature of loads, implies fixing the fraction f_n at each iteration. However, consumers can have better payoffs when increasing the total cost of the community while reducing their energy consumption. Therefore, cheating is possible in this scenario. The following example illustrates that:

Example 6. Consider an integer game with proportional-to-consumption billing, any convex total cost function (e.g. quadratic function, peak pricing function, or another one), and energy variant loads. A consumer n has the choice between two total consumption values: one equal to κ and the other equal to μ. The rest of the community has fixed total load equal to ζ, which is equally split among the consumers. Consider that two total costs are possible for this game: $C_1 > C_2$. Consumer n can have a better utility when choosing the strategy κ leading to the total cost $C_1 = C(\kappa, \zeta)$, and a worst utility when choosing strategy μ leading to $C_2 = C(\mu, \zeta)$. When applying algorithm 2, and starting with strategy κ, the following will happen:

$$\frac{\kappa}{\mu + \zeta} C(\mu, \zeta) < \frac{\kappa}{\kappa + \zeta} C(\kappa, \zeta) \tag{5.32}$$

So he chooses the strategy μ, because it leads to a smaller total cost. In the following iteration, f_n is updated to μ, leading to:

$$\frac{\mu}{\mu+\zeta}C(\mu,\zeta) < \frac{\mu}{\mu+\zeta}C(\kappa,\zeta) \tag{5.33}$$

Therefore, consumer n keeps the strategy μ, because of the minimization of the total cost given by the m-BRD. Even though the algorithm converges to an equilibrium, it is not a Nash Equilibrium of the original game, because the following is true:

$$\frac{\mu}{\mu+\zeta}C(\mu,\zeta) > \frac{\kappa}{\kappa+\zeta}C(\kappa,\zeta) \tag{5.34}$$

Thus, consumer n can cheat after reaching the m-BRD equilibrium by changing strategy μ to κ, which increases the total community cost but decreases his own utility. One can notice that this behavior will harm the other consumers because the additional cost will be divided among them (according to their consumption).

5.12.2 Per-time-slot

The per-time-slot billing has been shown to lead to untruthful behavior of the players for continuous games [Baharlouei and Hashemi, 2014, Rahman et al., 2017]. Both studies claim that a consumer can take advantage of informing opponents a different consumption than his/her true value, which would force other users to schedule their loads apart from the cheater's preferred time interval.

We prove theoretically that the cheating behavior is more likely to arise when the per-time-slot billing is done over the total cost realized after real consumption (ex-post), as considered in Baharlouei and Hashemi [2014] and Rahman et al. [2017], because cheating can be a dominant strategy in this scenario. However, if prices $P_t(L_t)$ are obtained as a result of the game, as we propose using (5.15), consumers have no incentives to lie. In our proposal, the prices are defined the day before consumption (ex-ante), following the output of the BRD algorithm, and they are used to calculate consumers' payoffs. Therefore, informing opponents a higher load would increase the prices of the cheater's preferred time interval, which would increase his/her own cost. We use example 7 to illustrate this idea.

Theorem 9. *In a non-cooperative scheduling game* $\Gamma = \langle \mathcal{N}, (\mathcal{S}_n)_{n\in\mathcal{N}}, \{u_n\}_{n\in\mathcal{N}} \rangle$ *with per-time-slot billing* (5.15)*, a consumer has incentive to declare false information about his/her consumption if prices are defined ex-post consumption. On the other hand, if prices are defined ex-ante consumption, consumers have no incentive to lie.*

Proof. If prices are ex-ante consumption, the amount of energy a cheater would have to add to a time slot to make an opponent move his/her consumption away from it would increase its price, which would decrease his/her utility, resulting in no benefit for him/her. In the ex-post case, the prices do not increase with the cheater's lie. To prove that, consider a group of consumers participating in the scheduling game with equal preferences and AC parameters. Assume that $\delta = 1$ and $\mathbf{w}_i = [0]\ \forall i \in \mathcal{N}$. Assume that consumers only need to turn on their AC once during the time horizon to keep the temperature inside the feasible region. At a certain stage of the game k, a consumer n schedules his/her AC at time slot t. His/her payoff is:

$$u_n(l_n, l_{-n}) = -l_{n,t} P\left(l_{n,t} + l_{-n,t}\right). \tag{5.35}$$

Which means the energy price at time slot t is $p_t = a_t(l_{n,t} + l_{-n,t}) + b_t$. Now we consider that n is a dishonest consumer and he/she declares a consumption $l_{n,t} + \Delta l_{n,t}$ on the time slot t he/she wants to schedule his/her AC. As a response, opponents will play a strategy $l'_{-n,t} = l_{-n,t} - \Delta l_{-n,t}$, which means they will move $\Delta l_{-n,t}$ from time slot t. Therefore, if prices are ex-post consumption, they will be calculated without considering consumer n's lie ($\Delta l_{n,t}$), resulting in $p''_t = a_t(l_{n,t} + l_{-n,t} - \Delta l_{-n,t}) + b_t$. The utility of consumer n is then $u''_n(l_n, l'_{-n}) = -l_{n,t} p''_t$. Thus, $u''_n(l_n, l'_{-n}) \geqslant u_n(l_n, l_{-n})$ if:

$$\begin{aligned} -l_{n,t}[a_t(l_{n,t} + l_{-n,t} - \Delta l_{-n,t}) + b_t] \geqslant \\ - l_{n,t}\left[a_t\left(l_{n,t} + l_{-n,t}\right) + b_t\right]. \end{aligned} \tag{5.36}$$

As a result, it is always interesting to declare a higher consumption during the time slot player n intends to turn on the AC, because $p''_t \leqslant p_t$. One can see that if no opponent moves his/her consumption from t, then $\Delta l_{-n,t} = 0$ and $p''_t = p_t$, which means the cheater would pay at most the same amount as (5.35). Therefore, in this framework, declaring a higher consumption for the time slot t is a dominant strategy[4].

If prices are ex-ante consumption (as a result of the game), they can be calculated as $p'_t = a_t(l_{n,t} + \Delta l_{n,t} + l_{-n,t} - \Delta l_{-n,t}) + b_t$. The utility of consumer n is then $u'_n(l_n, l'_{-n}) = -l_{n,t} p'_t$. Thus, $u'_n(l_n, l'_{-n}) \geqslant u_n(l_n, l_{-n})$ if:

$$\begin{aligned} -l_{n,t}\left[a_t(l_{n,t} + \Delta l_{n,t} + l_{-n,t} - \Delta l_{-n,t}) + b_t\right] \geqslant \\ -l_{n,t}\left[a_t\left(l_{n,t} + l_{-n,t}\right) + b_t\right]. \end{aligned} \tag{5.37}$$

Thus, consumer n takes benefit from cheating only if:

$$\Delta l_{n,t} \leqslant \Delta l_{-n,t}. \tag{5.38}$$

[4] There are other frameworks where this would not be a dominant strategy and consumer n could in fact pay more. For instance, if he/she cheats in more time slots and opponents' constraints make them move consumption to time slots consumer n uses energy; or if there are more cheaters in the game. However, the discussed framework shows that there exists the possibility of benefiting from cheating in the ex-post scenario.

Now, we need to analyze how opponents respond ($\Delta l_{-n,t}$) to an increase of $\Delta l_{n,t}$ during a time slot they scheduled their AC. As long as consumer n only uses time slot t, this depends on the consumption of opponents on time slots $\{-t\}$. Considering $a_j = a$ and $b_j = b \ \forall j \in \mathcal{T}$, and given that the BRD algorithm is sequential, we analyze the response of the next player, say opponent i. He/she will change his/her AC from time slot t to $t' \in \{-t\}$, if:

$$L_t + \Delta l_{n,t} - \Delta l_{i,t} > L_{t'} + \Delta l_{i,t}. \tag{5.39}$$

Where $L_t = l_{n,t} + l_{-n,t}$. Therefore, there are three scenarios for the relation between L_t and $L_{t'}$ which would impact the behavior of opponent i. If $L_t = L_{t'}$, then $\Delta l_{n,t} > 2\Delta l_{i,t}$ to make consumer i change from t to t', which contradicts the necessary condition (5.38) for consumer n to take benefit from cheating.

If $L_t > L_{t'}$, then $\Delta l_{n,t} > 2\Delta l_{i,t} - f_1$, $f_1 = L_t - L_{t'} > 0$. Thus, if $f_1 > 2\Delta l_{i,t}$, condition (5.38) is respected. However, one can easily notice that this would never be true, because consumer i would have chosen to place $\Delta l_{i,t}$ at time slot t' before, at round $k-1$. On the other hand, if $f_1 \leqslant 2\Delta l_{i,t}$, condition (5.38) is not respected.

Finally, if $L_t < L_{t'}$, then $\Delta l_{n,t} > 2\Delta l_{i,t} - f_2$, $f_2 = L_t - L_{t'} < 0$. As a result, consumer n would have to declare a value greater than $2\Delta l_{i,t}$ plus a positive value to make i moves $\Delta l_{i,t}$ to time slot t', which again, contradicts (5.38). □

Example 7. Consider the example shown in Rahman et al. [2017, Figs. 3, 4] (necessary information is contained in table 15 and 16 herein), but with binary variables.[5] There are three users playing the scheduling game. User A plays first, followed by user B and user C. The per-time-slot billing mechanism to divide a total quadratic cost is applied, and $a_t = 1$ and $b_t = 0$ for all $t \in \mathcal{T}$. Therefore, in the first round, user A declares the consumption vector $\mathbf{l}_A = [5, 6, 4]$, for which user B best responds with $\mathbf{l}_B = [2, 4, 6]$ and user C with $\mathbf{l}_C = [5, 5, 5]$. After some rounds, the NE is reached, and the results are shown in table 15 herein. The final prices are $\mathbf{P} = [28, 28, 28]$ and consumers' utilities are calculated using (5.15). Suppose that user B decides to cheat in an attempt to have less opponents consuming energy in time slot 3, provided that he/she wants to consume more in it. Thus, at the first round, he/she declares the double of his/her real consumption, leading to the results in table 16. Therefore, if the billing is ex-post, and the prices are defined after the consumption, then $\mathbf{P} = [32, 32, 20]$ and user B benefits from lying, because he/she pays \$312 instead of \$336. His/her attitude also harms the other players, adding \$36 to their bills. However, if the prices are defined ex-ante, right after the end of the game, then $\mathbf{P} = [32, 32, 32]$ and user B pays more than he/she would have paid being honest (\$384 instead of \$336). The other users are also damaged (they pay \$480 instead of \$420).

[5] For the purpose of this example, it is not necessary to detail users' preferences. In Rahman et al. [2017], integer values are used and the comparison is possible.

Table 15 – Example 7 without cheating behavior—from Rahman et al. [2017].

Consumer	First Round			Nash Equilibrium			Billing
	S 1	S 2	S 3	S 1	S 2	S 3	Per-time-slot
A	5	6	4	6	4	5	420
B	2	4	6	3	3	6	336
C	5	5	5	5	7	3	420

Table 16 – Example 7 when user B cheats and the prices are defined ex-post or ex-ante.

Consumers	First Round			Nash Equilibrium			Real Consumption		
	S 1	S 2	S 3	S 1	S 2	S 3	S 1	S 2	S 3
A	5	6	4	7	6	2	7	6	2
B	2	4	12	3	3	12	3	3	6
C	5	5	5	6	7	2	6	7	2

5.13 Price-of-Anarchy

Price-of-anarchy (PoA) is a concept in algorithmic game theory related to the question: how inefficient is the equilibrium reached by selfish rational players in comparison to an idealized situation in which the agents would collaborate with a common goal? [Nisan et al., 2007]. It measures the amount of damage suffered by the agents due to the absence of a central authority [Lã et al., 2016]. More specifically, it is computed as a ratio between the socially Pareto optimal outcome and the equilibrium outcome from the distributed interaction among selfish players. In our transactive coordination problem, the PoA can be calculated as a ratio between the solution of the centralized optimization problem in equations (5.9) or (5.11) and the sum of the utilities of players in the decentralized propositions.

Mathematically, consider a *welfare function* $\Theta(\mathbf{s})$ measuring the efficiency of a strategy profile $\mathbf{s} = (\mathbf{l}_n, \mathbf{l}_{-n})$. We use the *utilitarian* welfare function, as defined in Lã et al. [2016].

Definition 11. ***Utilitarian welfare function*** *Given a strategy profile* $\mathbf{s} = (\mathbf{l}_n, \mathbf{l}_{-n}) \in \mathcal{S}$, *the utilitarian welfare function* $\Theta : \mathcal{S} \mapsto \mathbb{R}$ *is the sum of the utilities (or costs) of all players:*

$$\Theta(\mathbf{s}) = \sum_{n \in \mathcal{N}} u_n(\mathbf{s}) \tag{5.40}$$

Let $\mathbf{s}^*$ be the optimal socially point with respect to the welfare function, e.g. the solution of equation (5.9) or (5.11), depending on the game type. Let $\mathbf{s}'$ be a Nash Equilibrium of the coordination game types described section 5.6. We define the price-of-anarchy as:

$$PoA = \frac{\Theta(\mathbf{s}')}{\Theta(\mathbf{s}^*)} \tag{5.41}$$

Therefore, the optimal PoA is 1, the minimum value a game can achieve. Other results indicate how many times the decentralized approach is worst than the centralized one.

5.14 General Applicability of the Game Types

The first general advantage of using a non-cooperative game to coordinate consumers flexible loads is related to the decentralized nature of a game model. In a non-cooperative game, the interactions between consumers to reduce their individual energy bills can be mimicked by the iterative and decentralized learning algorithm described in section 5.8. The idea is to reach an equilibrium point, the Nash Equilibrium (NE) [Nash, 1951], which is a stable solution. In other words, if consumer n unilaterally deviates from his/her stable schedule when the neighbors are following the equilibrium schedules, he/she reduces his/her utility (increases his/her bill). This also means that the autonomy of consumers is respected during the process, because they decide their schedules individually and locally, according to their constraints and preferences. Moreover, non-cooperative games are based on the self-interest assumption, which means consumers take their decisions based on their preferences and needs, with the goal to minimize their bills. Deviating from the NE is, thus, against their self-interest.

In addition, when applying a proportional-to-consumption billing model to schedule energy invariant loads, consumers must coordinate their controllable load schedules in order to reduce the community peak consumption (in the case of a peak pricing total cost), or to flatten the load curve (in the case of the quadratic total cost function), otherwise they will pay more. Here, it is important to note that, despite the name, non-cooperative games can be intentionally created to achieve cooperation goals [Fudenberg and Tirole, 1991], as shown by the design of this utility function. However, this billing mechanism fails to include energy variant loads, as thermal loads, because it ceases to be potential. Therefore, the existence of a Nash Equilibrium is not guaranteed, the algorithm may not converge (or may converge to a non-equilibrium solution), and cheating behavior can occur.

On the other hand, the per-time-slot billing mechanism can be extended to other appliances, because the potential characteristic of this billing game only depends on the utility function. Lã et al. [2016] explains that the equality functions in definition 10, i.e. $u_i(s_i, s_{-i}) - u_i(s_i', s_{-i}) = \phi(s_i, s_{-i}) - \phi(s_i', s_{-i})$, hold for any $(s_i, s_{-i}), (s_i', s_{-i}) \in \mathcal{S}$ if the game is an exact potential game. Therefore, they remain valid for any subset of $\mathcal{S}$. This result constitute a useful property for modelling new potential games from existing ones, as long as, if it is known that the added constraints only modify the strategy space, then the new game is also an exact potential game. This is true for the PTS billing, thus different types of thermostatically controlled loads (TCLs) which have similar behavior to the ACs presented—see Heleno et al. [2015], and shiftable appliances, such as washing machines, which have also an on/off control, could be added. Even devices that can be modeled as continuous variables could be included to the PTS game without changing its properties. Therefore, all conclusions related to the existence of pure-strategy Nash Equilibria, convergence of BRD, fairness of the per-time-slot billing, and cheating behavior prevention are applicable and do not depend on the appliances controlled and their constraints. As the PTC depends on the appliances being energy invariant to be potential, we conclude that the PTS is more general than the PTC.

6. Results

6.1 Case Study

To simulate the proposed game types in a realistic context, verify their applicability, and analyze their advantages and drawbacks, we use real data collected from a Spanish community with 632 buses, depicted in figure 10. Hourly active power consumption of each bus was collected and averaged from June 2019, to build a daily consumption curve. For simplicity, we consider each bus as a consumer/player, but this assumption can be loosened according to the data gathered. In addition, we simulate scenarios in which 201 consumers in this network participate in the demand-side management program.

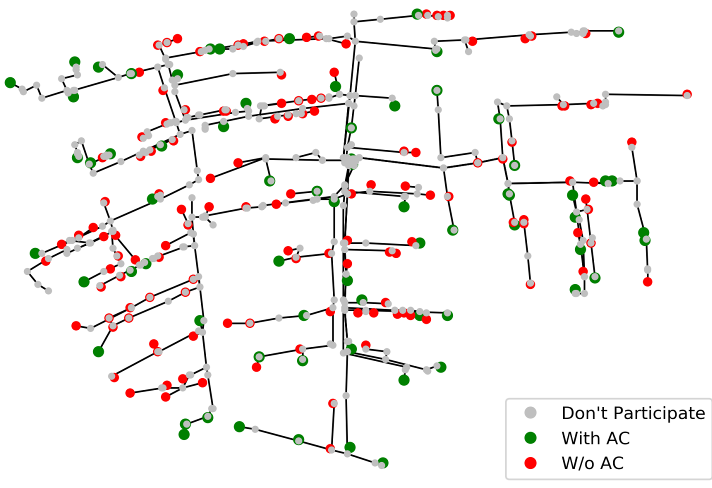


Figure 10 – Spanish LV Community considered in this thesis. Each bus is a consumer. Grey dots represent consumers of the community who do not participate in the transactive control program, thus their load curves do not impact the community's cost function. Green dots represent consumers with controllable loads, and red dots, those without flexible appliances but with inflexible load.

The day-ahead time horizon is divided in 96 time slots of 15 minutes, which means $\delta = .25$. Therefore, we split the hourly consumption among the 15 minutes time intervals, considering an equal power consumption in each of the four time slots within an hour. This is done in order to respect the original data, and the result is a daily power consumption vector for each consumer $\mathbf{H}_n = [H_{n,0}, H_{n,1}, \ldots, H_{n,T}]$.

Some preferences and AC's parameters are generated randomly and others are calculated in order to keep feasibility. The minimum acceptable interior temperature for each consumer $\theta_{n,t}^{min}$ is generated using a discrete uniform distribution between 18 and 22°C. To calculate the maximum acceptable temperature $\theta_{n,t}^{max}$, a value between 2 an 4°C is also generated randomly using a discrete distribution and added to $\theta_{n,t}^{min}$. Both temperature preferences are kept the same during the entire day, in the time slots the property is occupied. For feasibility purposes, the initial temperature $\theta_{n,0}$ is randomly selected between $\theta_{n,t}^{min}$ and $\theta_{n,t}^{max}$, for all consumers.

Based on Heleno et al. [2015], we generate the thermal resistance R_n and performance value η_n for each consumer's AC using an uniform distribution. For the first, we draw values from the range 5 to 8°C/kW, and for the second, 3 to 3.2. The external temperature is selected as 35°C the entire day. Power rates and thermal capacities are calculated in accordance to the aforementioned preferences and parameters generated. We explain the method used next.

First, we determine when the property is occupied. Considering H_n^{min} as the minimum power consumption of consumer n and H_n^{avg} as its average power consumption, we assume that the property is occupied if $H_{n,t} \geqslant H_n^{min} + 0.3H_n^{avg}$. To avoid strange behavior (property's occupation status changing fast and many times in small intervals), we adjust the occupation status considering a minimum time the consumer must be inside/outside the property (1 hour). With that information, we calculate the power rate of the ACs as a value randomly chosen, following a triangular distribution, between the minimum power rate the equipment could have without exceeding $H_{n,t}$, the maximum power rate of ACs (we choose 3.5 kW), and a mode in the point representing 75% of this difference. Consumers whose minimum power consumption in occupied time slots is less than 1.5 kW do not have ACs. The other consumers have one AC with power rate equal to the value drawn by the triangular distribution.

For the thermal capacity, we define the minimum and maximum C_n values that makes the temperature variation of equation (5.3) be between 0.5 and 1°C. We then draw randomly one value between those limits. For the time slots t when the consumer n's property is unoccupied, or for the consumers without AC, we defined $\theta_{n,t}^{\min} = 0°C$ and $\theta_{n,t}^{\max} = 100°C$. The final parameters generated are presented in table 17.

We then simulate the operation of the ACs using equation (5.3). When $\theta_{n,t}$ reaches the upper limit, the AC is turned on at that time slot. Moreover, for the moments right after consumers arrive, the restriction on $\theta_{n,t}^{max}$ is loosen, because the ACs need time to lower the temperature to the target temperature range. The result of this simulation gives the power load of the ACs. To calculate the base load $\mathbf{w}_n$ of each consumer n, we subtract the AC load from the initial $\mathbf{H}_n$. The final consumption of ACs and inflexible

Table 17 – System parameters generated or calculated to create the case study.

N	201	$\theta_{n,t}^{\min}$ (°C)	[18, 22]
T	96	$\theta_{n,t}^{\max}$ (°C)	[20, 26]
δ	0.25 (15 min)	$\theta_{n,0}$ (°C)	$[\theta_{n,t}^{\min}, \theta_{n,t}^{\max}]$
R_n (°C/kW)	[5.0, 8.0]	θ_t^{et} (°C)	35
E_n (kW)	[1.5, 3.5]	η_n	[3.0, 3.2]
C_n (kWh/°C)	[0.5, 4.2]		

loads construct a base scenario (a case without energy management), which is depicted in figure 11.

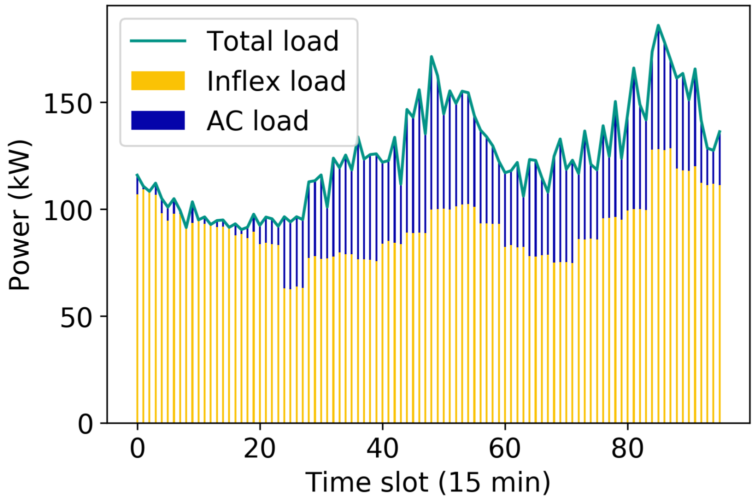


Figure 11 – Total energy consumption of the community with ACs in the base scenario (without energy management): the blue bars are the calculated AC loads, and the yellow bars are the remaining inflexible loads.

The parameters of the cost and utility functions were generated following two different methodologies depending on the type of function: quadratic or peak pricing. For the first, we use a three-step piecewise function to represent the cost function. The parameters are defined based on the following tiers prices: 5, 15, and 30 ¢/kWh, and the thresholds between tiers are adopted as 60% and 75% of the group's peak load on the base scenario. We adjust a quadratic curve without intercept to this piecewise function, resulting in parameters $a_t = 0.065$ ¢$/kWh^2$ and $b_t = -0.858$ ¢$/kWh$ for all $t \in \mathcal{T}$. The original piecewise function and the quadratic adjusted curve are depicted in figure 12.

For the peak pricing function, the chosen parameters are: 1 \$/kW for the peak charge, and a two-level TOU tariff for the volumetric rate, with values equal to 0.12 \$/kWh between 0h and 17h, and 0.20 \$/kWh between 17h and 24h.

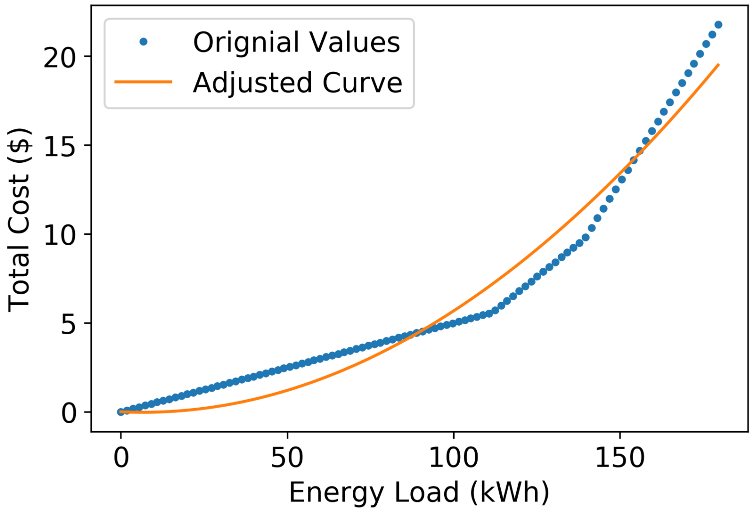


Figure 12 – Quadratic adjustment of a piecewise pricing function.

6.2 Games with Quadratic Total Cost

In this section, we show results for the game types applying a quadratic total cost to the community (with parameters a_t and b_t aforementioned). As this cost can be shared using both proportional-to-consumption (PTC) and per-time-slot (PTS) billings, we compare solutions of the two billings. To solve the PTS case, we apply the Best Response Dynamics (BRD)—presented in algorithm 1, as this game type was proved to be potential independently of the appliances scheduled. For the PTC case, we apply the modified Best Response Dynamics (m-BRD)—proposed in algorithm 2, because this game is not potential when energy variant loads are present. For both game types, consumers' preferences and AC parameters follow the data generated according to section 6.1. Additionally, we calculate the solution to problem (5.9) to have a quality measurement of the equilibria. We also analyze the ability of the TC models to flatten the community's load curve, if the equilibrium of the PTC using the m-BRD is a Nash Equilibrium of the game with energy variant loads, the convergence process of the algorithms, the fairness of the billings when dividing the total cost among consumers, the multiplicity equilibria aspects of the games, and their ability to prevent cheating.

It is important to notice that, in this section, we apply the m-BRD for the case with the proportional-to-consumption billing. This approach does not raise equity problems when sharing the total cost among consumers, but has concerns about the resulting equilibrium and cheating behavior. In section 6.3, another approach is analyzed, i.e. estimating participants' total consumption a priori and keeping their share fixed throughout the optimization process.

Table 18 – Solution of each scenario when a quadratic total cost is applied to the community

Scenario	Total Cost ($)	PAR	Total Energy (kWh)
Base scenario	37.387	1.492	2,991.062
Centralized	36.579	1.254	2996.737
PTS (BRD)	36.594	1.233	3,002.512
PTC (m-BRD)	36.577	1.260	2,996.287

6.2.1 Existence of Nash Equilibria

The Nash Equilibrium of the PTS (using the BRD) and the equilibrium of the PTC (applying the m-BRD) are shown in table 18 and figure 13. Both total cost and peak-to-average ratio (PAR)—which measures how flat the resulting load curve is [Zhou et al., 2019]—are optimized with the TC approaches. Moreover, the PTC with m-BRD is able to reach an optimal total cost, while the solution of the PTS with BRD is very close to optimal. This leads to a price-of-anarchy (PoA) close to 1.0 for both cases. As this parameter measures the amount of damage suffered by consumers due to the absence of a central authority [Lã et al., 2016], being close to 1.0 indicates that the game solution with PTC/PTS can reach the optimal value, and the community as a whole has no loss when giving consumers the autonomy to decide their own schedules. In addition, the PARs are also reduced, indicating that the load curves with TC are flattened. One can notice that, to reduce the total cost and PAR, the final schedules turn-on the ACs more times than the original BAS case. The sum of consumers' total load in the base scenario is 2,991.062 kWh. After the PTS with BRD, this value is 3,002.512 kWh, and the PTC with m-BRD, 2,996.287. The additional kWh in both cases confirms the energy variant nature of TCLs and their "payback" characteristic: there is a trade-off between energy consumption and total cost with the TC approaches, specially when the quadratic parameter of equation (5.7) is high, inducing consumers to flatten the load curve.

Finally, we demonstrate that the equilibrium reached by the m-BRD for the scheduling game with energy variant loads and the PTC does not reach a Nash Equilibrium. Even though the algorithm converges to the solution discussed above, it is not a Nash Equilibrium of the game with PTC: fixing consumers' energy share f_n at each iteration can lead participants to optimize the community's total cost, but they can have better payoffs if they change it. Table 19 shows that 7 consumers can have a smaller bill if they play the solution in the base case instead of the equilibrium solution reached by the m-BRD, while the other consumers are playing the m-BRD equilibrium. Therefore, this equilibrium solution is not a Nash Equilibrium of the game with the original utility function (5.12). Consumers can in fact have a better payoff when increasing the community's total cost, because their shares are a function of the results. Those 7 consumers, when changing their strategy to the BAS schedule, increase the total cost to values between $36.580 and

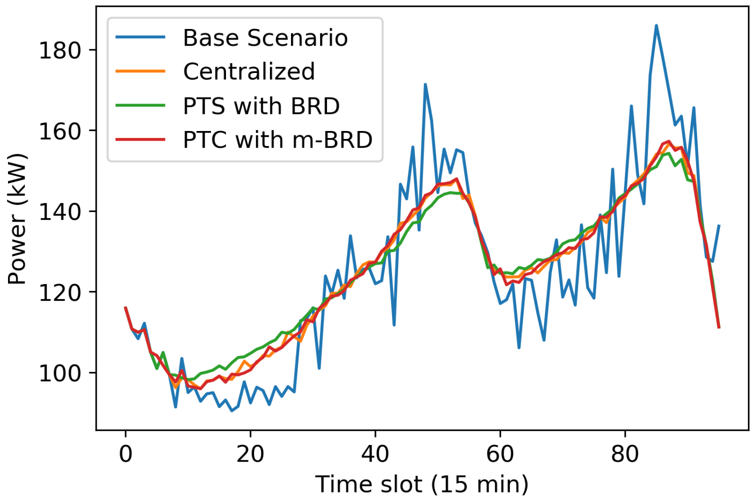


Figure 13 – Final load of the community in each scenario: base scenario (blue), centralized (orange), game with PTS solved with BRD (green), and game with PTC solved with m-BRD (red). One can notice that the load curves of CEN and TC scenarios are very close.

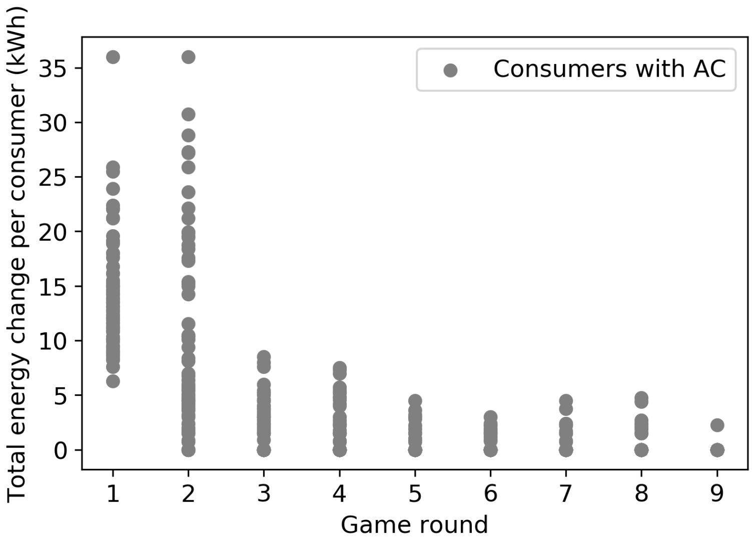
Table 19 – Bills some consumers can have if they choose the schedule of the base case instead of the equilibrium solution reached by the m-BRD for the PTC approach (in $).

$n =$	0	5	28	48	63	92	100
m-BRD	0.811	0.205	0.486	0.258	1.007	1.432	0.292
BAS	0.802	0.199	0.478	0.244	1.000	1.424	0.285

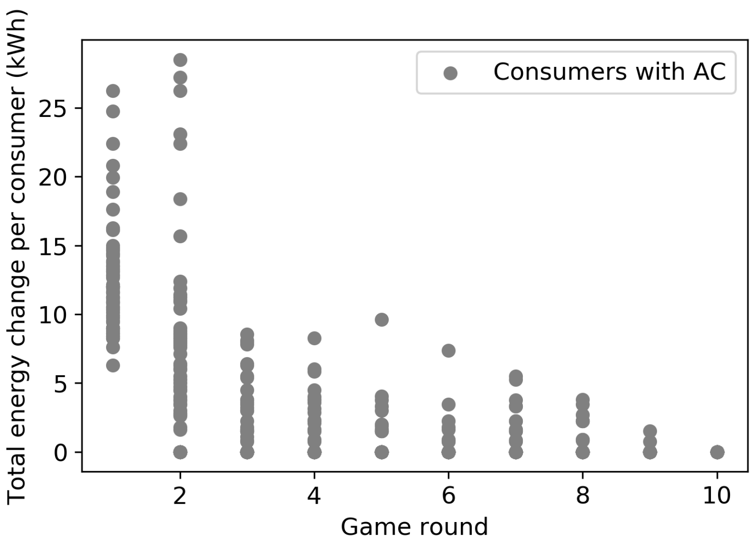
$36.607. This demonstrates that, when energy variant loads are present and the PTC is applied, consumers' goal disconnects with the community's goal, and the game is no longer potential.

6.2.2 Convergence of the Algorithms to Divide a Quadratic Total Cost

The green line in figure 13 represents a Nash Equilibrium: the solution attained by the BRD with quadratic total cost and per-time-slot billing. Moreover, the red line of the same figure represents the equilibrium reached by the m-BRD for the proportional-to-consumption version. In figure 14, we show the convergence path taken by the BRD and m-BRD to reach those solutions. The plots depict change in consumption per iteration of all consumers with AC. As seen in the plots, in both algorithms' first round all 70 consumers modify their schedules, as this round is designed to construct an optimized initial solution: the consumers start without load, best respond to the previous players at their turn, and the total community load is known after the end of this first round when all consumers had played once (including consumers without AC, who do not have flexibility, but add their inflexible load schedule to the game). Moreover, the number of consumers changing their schedules in each round reduces throughout the iterations of both algorithms until no modifications exist and the NE is reached for the BRD, and an equilibrium is attained for the m-BRD.



(a) Game with PTS and BRD



(b) Game with PTC and m-BRD

Figure 14 – Convergence of algorithms for the game types dividing a total quadratic cost. (a) shows that the BRD converges to a Nash Equilibrium of the game with PTS in 9 rounds; (b) depicts the convergence of the m-BRD to an equilibrium of the game with PTC after 10 rounds. In both plots, the 70 consumers with ACs modify their schedules until there is no more benefit on changing, which means an equilibrium is reached.

It is worth mentioning that this convergence of the m-BRD is possible because consumers' shares on the community's total cost are updated throughout iterations, allowing consumers to optimize the total community's cost. However, as discussed in the previous section, this solution is not a Nash Equilibrium of the original game with the proportional-to-consumption billing, and cheating behavior can occur after this equilibrium is reached—the strategy-proof of this approach is discussed in section 6.2.4.

6.2.3 Fairness of the billings with Quadratic Total Cost

In this section, we compare the fairness of both billing mechanisms to divide the total quadratic cost of the integer and energy variant loads scheduling. We use the methodology presented in section 5.10. As the PTC may not have a Nash Equilibrium when energy variant loads are considered, we use the equilibrium solution of the m-BRD.

We calculate consumers' savings in each TC solution by comparing their payoffs with the BAS solution[1], and we show results for a month. Only active participants, i.e. with AC loads, are analyzed. We classify them in 3 groups according to their preferences (day periods they want their AC to be operating): dawn for use between midnight and 7h; day from 7h to 17h; and night from 17h to midnight. Consumers can choose to use their AC in more than one period of the day. To illustrate the impact on consumers, we plot monthly individual savings against consumers' total energy consumption in figure 15. In the PTS case, consumers using their AC during peak periods (specially night) have less savings when comparing with those using the AC during valley periods (e.g. dawn). For instance, the consumers marked in red on the plot are located upper than consumers with the same amount of consumption but with AC usage during the day and/or night (brown, green and blue), which means they have more savings. This is explained by the billing being done according to the time slots consumers use energy. On the other hand, for the PTC, only the total amount consumed influences how much savings a participant will have, because this is the only factor considered in this billing, which explains the straight line of plot 15. Therefore, the PTS rewards consumers for choosing to use energy in valley periods, while PTC rewards them for using more energy, which is against energy efficiency. Finally, PTS can lead to negative savings for consumers with intense energy use during peak times (see the blue and green dots around total energy consumption of 20 kWh), whereas PTC gives positive savings for everybody, independently of their consumption pattern or flexibility, which can affect the engagement of flexible consumers.

To analyze the fairness of payoffs/savings distribution, we use the Shapley Value and fairness index on sub-cases of the data in 6.1. We constructed 6 sub-cases with 10 consumers: the first 3 by picking them randomly; one for those with the largest total energy consumption; one for those with the largest AC load as a percentage of their total energy use; and one for those with the smallest total energy consumption. We run the BRD applying the per-time-slot and the m-BRD for the proportional-to-consumption billing,

[1] BAS minus NE payoffs when applying PTS for both, and BAS minus equilibrium payoffs when applying PTC for both.

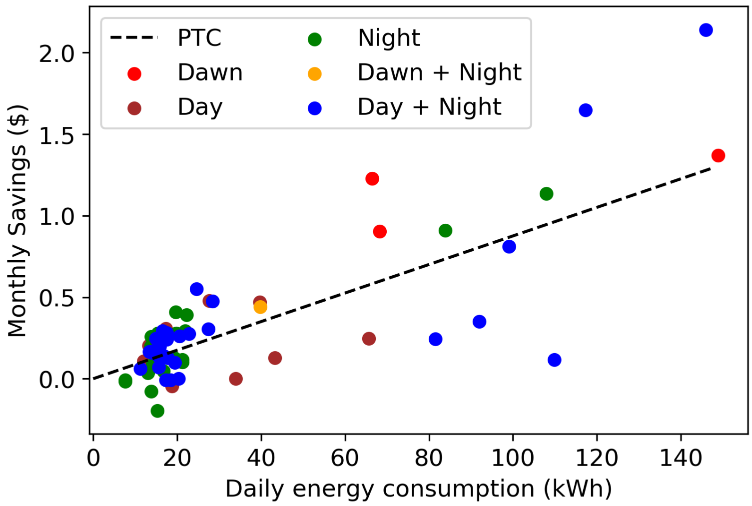


Figure 15 – Consumers' savings from BAS scenario with the TC approach and the PTS billing. The dashed line shows the same consumers' savings when the PTC approach is used. Consumers are classified according to the day period they want to use their AC. The PTS billing rewards better consumers according to their flexibility.

Table 20 – Fairness index comparison between per-time-slot and proportional-to-consumption billings.

Case	Random 01	Random 02	Random 03	Highest Load	Highest AC	Smallest Load
PTS	3.15%	1.86%	0.55%	0.53%	1.87%	3.38%
PTC	7.92%	6.78%	12.45%	7.26%	3.91%	4.76%

and calculate the SV for all 6 sub-cases. Results of the fairness index are shown in table 20. For all 6 sub-cases, this index is smaller when the PTS is applied, which means that its solution is closer to the SV value than when using the PTC, thus the PTS is fairer.

For the case "random 03", which has the biggest difference between PTS and PTC fairness index, we plot consumers payoffs in each TC approach and the SV to show how these differences impact consumers' payments. Values are shown in figure 16, and for all consumers, the payoff reached when applying the per-time-slot billing is almost equal to the SV, explaining the 0.55% fairness index. However, for the proportional-to-consumption, the values differ, leading to the 12.45% index. With those simulations and results, we can conclude that the PTS is fairer than the PTC for integer scheduling games.

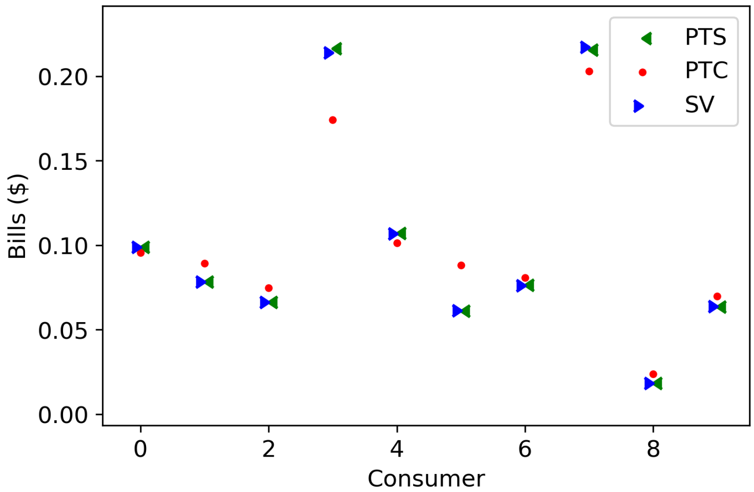


Figure 16 – Consumers' bills after the BRD with per-time-slot billing (PTS), and after the m-BRD with proportional-to-consumption billing (PTC), compared with Shapley Value (SV) calculation. PTS values are almost the same of the SV, which does not happen with the PTC, meaning that the PTS is fairer (sub-case random 03).

6.2.4 Strategy-proof of the Billings with Quadratic Total Cost

6.2.4.1 Per-time-slot

As this billing has the problem of cheating—see section 5.12.2, we test the proposed ex-ante prices to discourage this behavior. To verify its efficiency, we choose a large load consumer ($n = 14$) and run the best response algorithm with the per-time-slot billing 5 times, varying the amount of lie this consumer adds to his/her consumption vector. We consider that he/she declares to opponents, at the time slots he/she turns on the AC, a load value α times the real one ($l_{n,t} \times \alpha$), with $\alpha = [1.0, 1.1, 1.5, 2.0, 3.0]$. Results in table 21 show that if the prices are ex-post consumption, the cheater has a real incentive to lie, adding a large load to his/her preferred time interval, in an attempt to make opponents move as much load as possible away from it. For instance, if he/she declares to opponents 3.0 times the real AC load, opponents move their consumption away from the cheater's preferred time slots, reducing cheater's bill from the Nash Equilibrium \$1.883 to \$1.864. Moreover, the other participants are harmed because of this non-equilibrium solution, with a bill increase, e.g. consumer $n = 96$ has a bill of \$1.602 instead of \$ 1.578. However, if the prices are ex-ante, the more load the cheater adds to the consumption vector, the bigger the prices are at those time slots, decreasing his/her utility. In the $\alpha = 3.0$ scenario, his/her bill with an ex-ante price increases to \$1.977. Therefore, ex-ante prices are an effective way to discourage cheating behavior when applying the per-time-slot billing.

Table 21 – Cheater's and Opponent's Bills when a Big Consumer with AC Cheats (in $).

α	Cheater $n = 14$		Opponent with AC $n = 96$	
	ex-ante	ex-post	ex-ante	ex-post
1.0	1.883		1.578	
1.1	1.890	1.884	1.586	1.584
1.5	1.908	1.878	1.595	1.585
2.0	1.931	1.875	1.610	1.593
3.0	1.977	1.864	1.639	1.602

Table 22 – Cheater's and Opponents' Bills when a Consumer Changes the PTC equilibrium solution to the BAS schedule (in $).

Cheater plays:	Cheater $n = 0$	Opponent $n = 14$	Opponent $n = 68$	Opponent $n = 96$
Equilibrium	0.811	1.781	1.341	1.808
BAS solution	0.802	1.783	1.342	1.809

6.2.4.2 Proportional-to-consumption

This billing, when applied to energy variant loads, converts consumers' utility to a non-linear and non-quadratic function. To solve it with a best response approach, we linearize the fraction $f_n = \frac{\sum_{t \in \mathcal{T}} l_{n,t}}{\sum_{t \in \mathcal{T}} L_t}$ and change it at each iteration. This modified BRD is able to reach an equilibrium solution in which consumers seek to reduce the community's total cost. However, as shown in section 5.7.1.2, the final solution is not a Nash Equilibrium of the game with the original PTC utility. We also show that cheating is possible in this scenario. Table 22 shows that, when consumer $n = 0$ plays the BAS solution instead of the equilibrium reached by the m-BRD, while all other consumers play the equilibrium, his/her bill decreases (from \$0.811 to \$0.802). Moreover, 7 opponents are harmed with this change, seeing an slightly increase in their bills—we show 3 harmed participants in table 22. Even though the bill differences are small, we use this example as an illustration of the cheating possibility when applying the PTC to energy variant loads, using the BAS solution. Consumers could find even better strategies to play after the equilibrium was reached, which would increase more the community's total cost and cause greater harms to opponents.

6.3 Game with Peak Pricing Total Cost

The other game type we analyze in this thesis uses a proportional-to-consumption billing to divide a peak pricing total cost (with parameters c_t and d). To solve it and analyze its aspects, we apply the Best Response Dynamics (algorithm 1) with fixed f_n, calculated according to consumers' total load in the base scenario (figure 11), as proposed in section

Table 23 – Solution of each scenario: community's total cost and peak-to-average ratio of the group's final load curve.

Scenario	Total Cost ($)	PAR
Base	625.20	1.492
Centralized	580.84	1.141
Transactive Control	585.49	1.188

5.11. Consumers' preferences and AC parameters follow the data generated according to section 6.1. At first, we consider one playing order, and results are a Nash Equilibrium of the PTC with fixed total consumption approach, using the a priori estimation. We also calculate the optimal solution to problem (5.11) (CEN), to have a quality measurement of the NE attained. Considering that the centralized model is more complex to solve (more variables and constraints), we limit its solving time to the total time the BRD with same total cost function takes. Besides comparing the results of the game with the centralized solution, we also analyze its ability to flatten the community's load curve, how the consumers behave individually, what is the convergence process of the BRD, how savings are distributed among consumers, and the equity of using a fixed f_n to define consumers' bills. Then we run the BRD for different playing orders, and analyze what is the impact on consumers and the community to have multiple Nash Equilibria.

6.3.1 Existence of Nash Equilibria

Results of the BRD and CEN with the peak-pricing total cost and the PTC billing are shown in table 23 (including the base scenario). The solution of the TC approach is very close to the centralized one, in terms of community's total cost. The game solution is slightly more expensive ($4.65), a value that does not affect much consumers' payments given that they are 201. In addition, the price-of-anarchy (PoA)[2] of the game solution, which measures the amount of damage suffered by consumers due to the absence of a central authority [Lã et al., 2016], is 1.008. In other words, the game solution is 1.008 times worst than the optimal solution (CEN). This close to optimal result is a consequence of the utility function design that makes consumers' goal to be optimize the community's total cost. Moreover, in the TC setting, consumers benefit from a decentralized optimization process in which they have local autonomy to define their TCLs schedule.

In addition to optimizing the community's total cost, the TC approach also optimizes the group's peak-to-average ratio (PAR). This parameter measures how flat the resulting load curve is [Rastegar et al., 2012]. In figure 17, which depicts the load dispatch of the three scenarios, it can be seen that the load curve is flattened with the TC approach, almost as much as in the centralized case. Due to the peak charge in the community's cost

[2] Price-of-anarchy concept and formula are given in section 5.13.

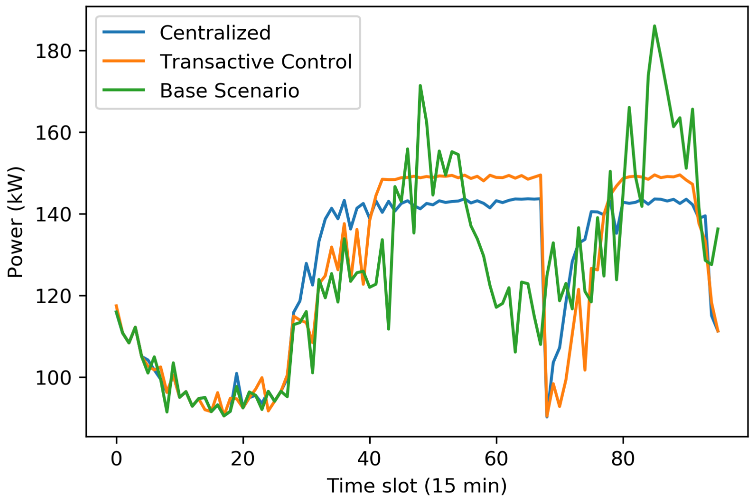


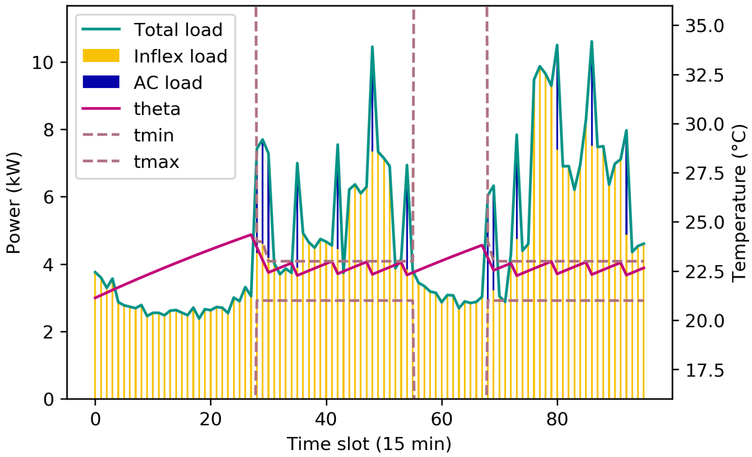
Figure 17 – Final load of the community in each scenario: centralized optimization (blue), game solved with BRD (orange), and base scenario (green). One can notice that the load curves of CEN and TC scenarios are very close, because consumers' utility is a fixed share of the total cost, which has the peak load as a major value to be minimized. Therefore, both approaches seek to flatten the load curve.

function (5.10), with the consumers' utility proportional to this total cost, consumers are motivated to reduce the group's peak to reduce their own bills.

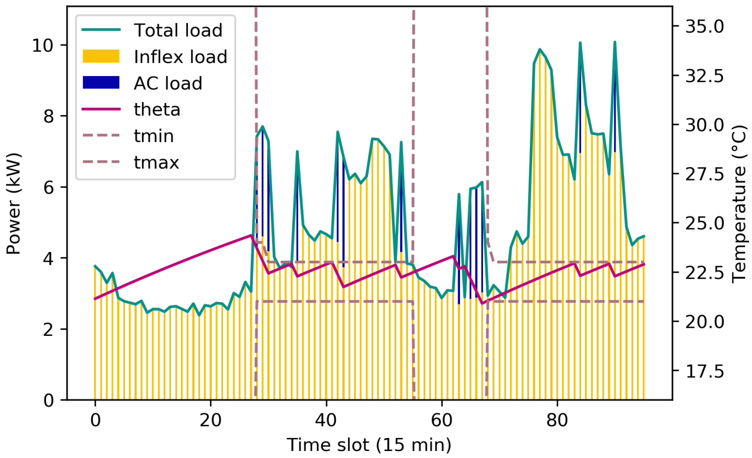
As we are discussing the impact of adding TCLs to scheduling games, we show one consumer's solution in figure 18. Consumer's load without energy management is shown in the upper plot, and the NE solution is presented on the lower one. The plots include the inflexible load (yellow bars), the AC dispatch (blue bars), the temperature inside the room (theta), and consumers' preferences. Between time slots 0 and 27 (6h45), and 56 (14h00) and 67 (16h45), this consumer is not at home or does not need the internal temperature to be from 21°C to 23°C (his/her comfort constraints). Therefore, preferences during these time slots are relaxed, i.e. are kept 0°C and 100°C. By comparing the figures it is possible to conclude that, in order to optimize the community's total cost, the equilibria solutions keep using AC power when the consumer is not at home to anticipate consumption during the community's peak periods. This flexibility is essential for the decentralized model to minimize the total cost: optimizing the solution depends on reducing the community's load peak and filling its valley.

6.3.2 Convergence of the Algorithm to Divide a Peak Pricing

The orange line in figure 17 represents a Nash Equilibrium: the solution attained by the TC approach with peak pricing and proportional-to-consumption billing (keeping f_n fixed). In figure 19, we show the convergence path taken by the BRD to reach this solution.



(a) Base Scenario



(b) Game with Peak Pricing and Proportional-to-Consumption

Figure 18 – Final load curve with the dispatch of one consumer's AC. Theta, tmin, and tmax are the temperature inside the room, the minimum, and the maximum comfort constraints, respectively. (a) represents the consumer's load in the base scenario (without energy management); (b) shows the resulting schedule of the same consumer in the TC approach. Optimizing the solution depends on the ability of the HEMS to control the AC autonomously during the moments the consumer does not need his/her AC on (e.g. is out of home or sleeping)—notice the solutions' difference in time slots between 56 and 67.

The plot depicts change in consumption per iteration of all consumers with AC. As seen in the figure, in the algorithm's first round all 70 consumers modify their schedules, as this round is designed to construct an optimized initial solution: the consumers start without load, best respond to the previous players at their turn, and the total community load is known after the end of this first round when all consumers had played once (including consumers without AC, who do not have flexibility, but add their inflexible load schedule to the game). Moreover, the number of consumers changing their schedules in each round reduces throughout the iterations until no modifications exist and the NE is reached.

It is worth mentioning that this convergence is possible because consumers' shares

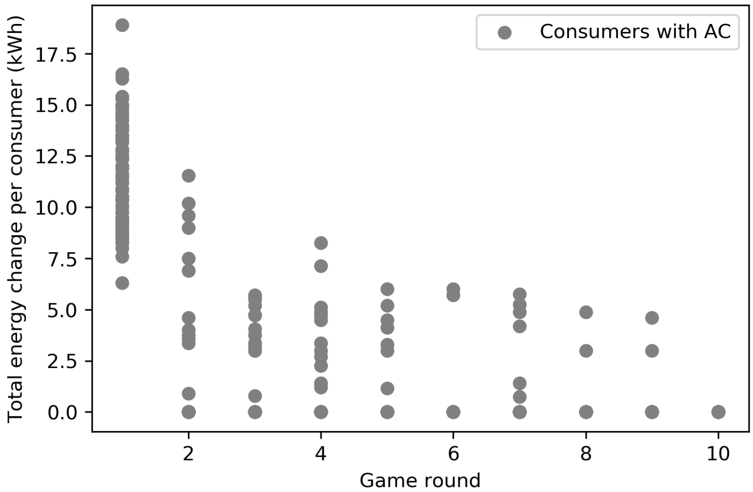


Figure 19 – Convergence of the Best Response Dynamics: it converges after 10 rounds. The 70 consumers with ACs modify their schedules until there is no more benefit on changing, which means the Nash Equilibrium is reached, and happens in the 10th round of the game.

on the community's total cost were defined a priori (using the base case), and kept fixed throughout the optimization process. This procedure is able to guarantee a potential formulation to this game with energy variant loads. However, equity problems arise from this procedure, and are discussed in section 6.3.4.

6.3.3 Fairness of the PTC with Peak Pricing

To analyze the fairness of the proportional-to-consumption billing when sharing the community's total cost among consumers, we present consumers' savings provided by the TC approach when compared to the base scenario. Savings are calculated as the difference between consumers' utility in the scenario without energy management (BAS), and their utility when they participate in the scheduling game. Both bills are calculated using the billing factor f_n defined in the base scenario. Daily consumers' savings vary from \$0.00 (very small consumers) to \$1.97 (very large consumers). The community saves around \$40 with the TC coordination, which are distributed among consumers according to the billing method—see equation (5.13). This popular billing mechanism in the literature of load scheduling games has some advantages as it is easy to implement, transparent to consumers, and simple to understand. In addition, it is able to align consumers' goal with the community's objectives, bringing a natural coordination to the game.

It is interesting to observe that all consumers have savings, even those without AC, which can be an incentive for consumers to stay in the community program regardless of the flexibility they have to offer. However, this can be also a disadvantage. As shown in figure 20, there is no relationship between savings and the availability of consumers to

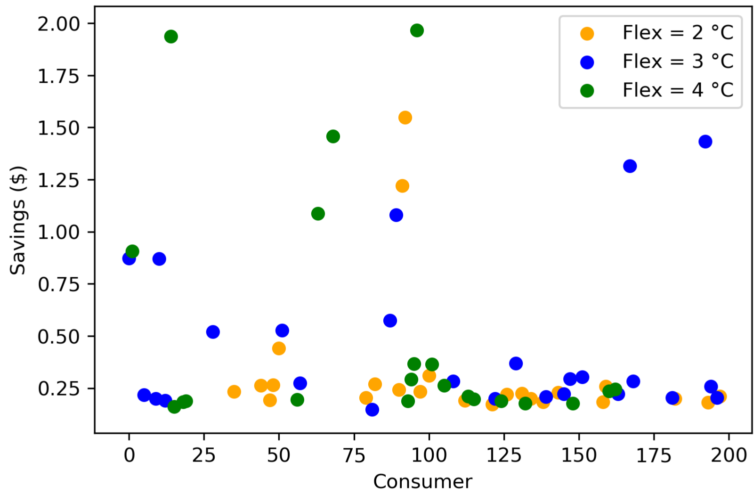


Figure 20 – Consumers' energy savings according to their comfort relaxation: the dots distribution is unrelated to the temperature range. Only consumers with AC are considered.

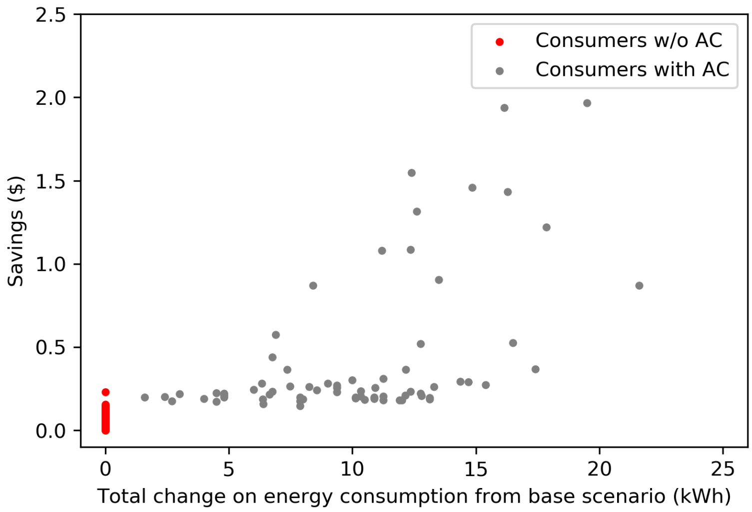
relax their comfort (the difference between $\theta_{n,t}^{max}$ and $\theta_{n,t}^{min}$). The figure compares consumers with different comfort relaxations (from 2°C to 4°C) and their savings are clearly unrelated with their comfort preferences. This means that two consumers that are equally available to modify their consumption to help the community will have different outcomes in terms of savings.

Even when looking at actual consumption changes, as a result of the scheduling, we can not find a clear relationship with the savings. As shown in figure 21, consumers with higher consumption modification (i.e. higher flexibility delivered), do not necessarily have higher savings. Thus, despite their higher contribution to reduce system costs, they do not always get the corresponding savings. This can be seen when all consumers participate in the coordination program (plot 21a), and also when only consumers with AC are considered (plot 21b).

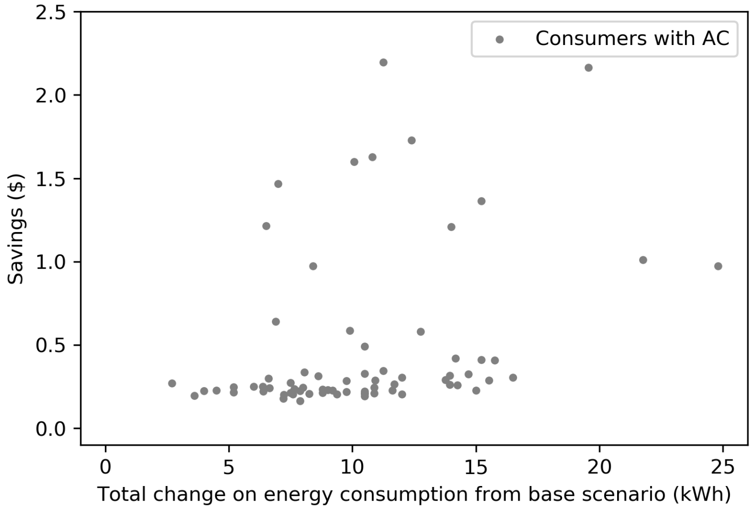
Therefore, considering only the total energy consumption in the billing mechanism, as done in the literature so far, can facilitate the coordination among consumers but it does not reward consumers according to the availability and the flexibility they bring to the system. This is considered as a fairness issue of this billing, as it does not compensate flexibility and energy efficiency [Baharlouei and Hashemi, 2014]. In addition, this mechanism also raises equity problems when applied to energy variant loads, which is discussed in next section.

6.3.4 Equity of the PTC with Peak Pricing

As discussed in section 5.11, the scheduling of TCLs comprises an energy variant characteristic, as the total energy consumption depends on the result of the scheduling



(a) All Consumers



(b) Only Consumers with AC

Figure 21 – Consumers' energy savings according to their "used flexibility". (a) all consumers participate on the program (with or without AC); (b) we solve the Best Response Dynamics with only those with AC. Consumers that made available more flexibility do not necessarily have higher savings in both cases.

game. We also discuss that finding an a priori estimation of this value (e.g. calculating f_n according to consumers' load in the base scenario) allows the application of a non-cooperative games framework to the transactive control of TCLs, as it guarantees the existence of an NE and the convergence of BRD. In this section we measure the difference between the a priori value f_n^{BAS} and the a posteriori one f_n^{TC}, realized after the play. We also calculate the impact on consumers payments generated by this difference. It is worth mentioning that, as shown in figures 20 and 21, all consumers have savings with this TC approach. However, in this section we demonstrate that their savings would be greater/smaller if an a posteriori billing was used instead of the a priori, which was applied because of the energy variant nature of the thermal loads.

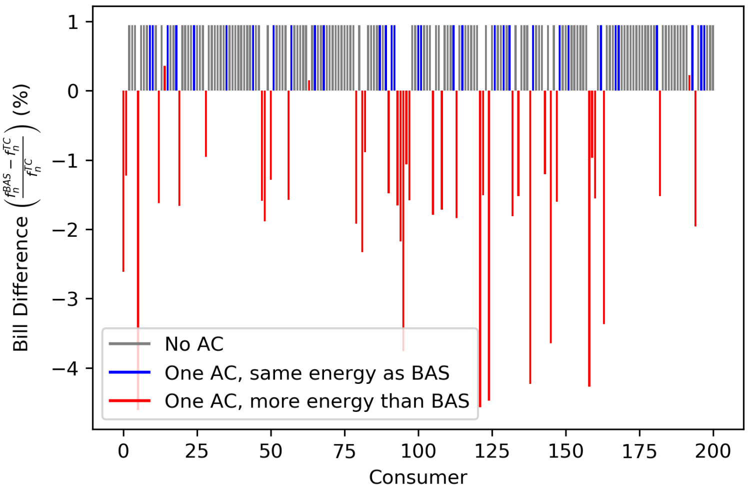


Figure 22 – Increase/decrease on consumers' bill when calculating the energy fraction a priori, using the base scenario (f_n^{BAS}), compared to the fraction a posteriori, calculated according to the real consumption after the play (f_n^{TC}). Consumers that do not change their total consumption have a bill approximately 1% higher than if it were calculated using the real consumption. Almost all consumers that increase their consumption decrease the bill.

In the NE presented in section 6.3.1, the sum of consumers' total load in the base scenario is 2,991.06 kWh. After the Best Response Dynamics, this value is 3,019.31 kWh, an increase of 0.94%. The additional 28.25 kWh from ACs confirms the energy variant nature of TCLs and their "payback" characteristic: there is a trade-off between energy consumption and total cost in this game. Moreover, those added kilowatt-hours are spread among 40 consumers with AC, going from 0.38 kWh to a maximum of 3.00 kWh added to one consumer's daily load.

Although the difference between using the a priori f_n^{BAS} and the a posteriori f_n^{TC} consumption is small in terms of the total community costs, it has an impact on the individual consumers' remuneration. In other words, consumers might pay more/less than they should have paid if the real (final) consumption was used to define their billing share. Those differences are shown in figure 22. For the 161 consumers that have an energy neutral control (equal to the base scenario), their bill increases according to the total cost of the community, 0.94%. In general, the 40 consumers that had an energy variant control of TCLs (i.e. they added energy consumption), see a reduction in their bill (from 0.89% to 4.61%), due to the fact that an a priori consumption estimation was considered. Only 3 of these consumers do not benefit from this estimation and see their bills increasing between 0.15% to 0.36%.

In conclusion, the a priori estimation of the consumption has an impact on the billing equity. As these values are unknown before the play and create a non-linear and non-quadratic optimization, one option is to estimated them to allow a transparent decentralized optimization process, in which consumers know in advance the impact of their individual control in the energy bill. Consumers that increase their load during the optimization may be benefited from this process. One could argue that this is a fair result, as these consumers activated their flexibility to reduce the community costs. However, other consumers that delivered the same amount of flexibility without increasing the consumption are not benefited, which impacts the equity when applying this billing. Therefore, in practical implementations of decentralized TCLs control with the proportional-to-consumption billing and an a priori estimation of consumers' shares, energy communities should consider these impacts and eventually introduce correction mechanisms in their billing policy to mitigate this problem.

6.3.5 Multiplicity of NE when Applying PTC with Peak Pricing

As shown in section 5.9, the scheduling game with TCLs has multiple NEs, including sub-optimal results, due to the integer nature of the control. We also showed that the NE of each instance of the game will depend on the consumers' playing order and on the initial point of the BRD. Therefore, in this section we vary the playing orders (using 60 different instances) when applying the BRD to the case study above (herein referred as original case), with the objective of assessing practical implications of this aspect in the transactive control with TCLs. One can notice that the amount of possible orders equals the factorial of the number of consumers. As all the orders would be impossible to evaluate, we select 60 values randomly, because this represents 50% of the orders for the case with $N = 5$, which is also studied here.

The 60-multiple NEs of the original case are depicted in histogram 23. In this simulation, the minimum total scheduling cost the BRD attains is \$584.77 and the maximum is \$587.69. This represents a variability of \$2.92, and a standard deviation of \$0.57. Even though the variability of solutions is very small, none of the simulated orders reached the optimal solution of the centralized approach—depicted in table 23 (\$580.84). Therefore, the consequence of having multiple NEs in scheduling games is clearly demonstrated in this case: the total cost to be shared by the community members vary among equilibria, the result of the coordinated scheduling depends on the playing order and is sub-optimal. In the worst case simulated, the total cost is \$6.85 more expensive than the optimal solution. When looking at individual consumers' savings, a similar dispersion can be found, which means that the consumers' outcome will depend on how the community will start/proceed the game.

Although the values of the dispersion are relatively small in the case we are analyzing, for example the minimum PoA is 1.007 and the maximum 1.012, it is clear that the problem exists and it might raise optimality issues among consumers, which can

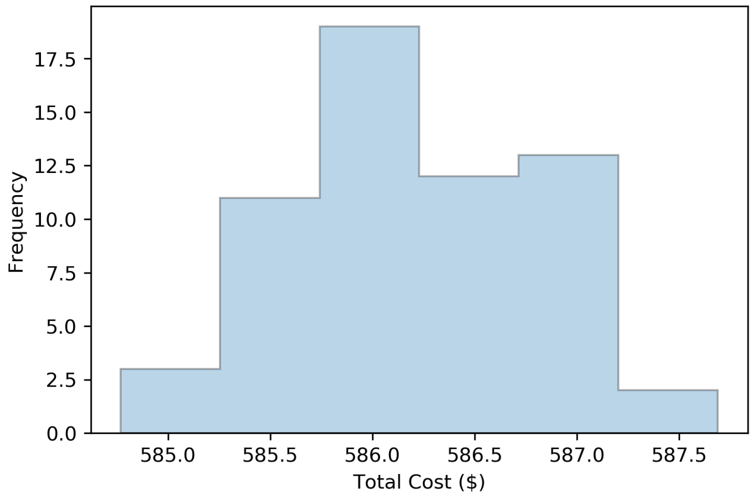


Figure 23 – Histogram of the total scheduling cost of the Nash Equilibria attained with different orders while running the BRD.

be a barrier for the implementation of TC approaches in energy communities using non-cooperative games with the proportional-to-consumption billing. Therefore, it is important to understand which factors affect this dispersion and in which conditions the playing order is relevant and can produce non-optimal results to the participants.

First, we analyze the impact of different components of the billing cost on the variability of the NE solution, when different initial playing orders are considered. In this sensitivity analysis, we take two cases of peak demand charge, PP 01 and PP 02, as well as two cases where the amplitude of the TOU rate (difference between peak and off-peak costs) is changed, TOU 01 and TOU 02. As shown in table 24, this change in the TOU tariffs has an impact on the results: when the price differentiation decreases, the dispersion of NE solutions increases. However, the more significant impact occurs when the peak demand charge varies, i.e. a higher peak cost (Case PP 02) leads to a higher amplitude and standard deviation of the NE solutions. This is illustrated in figure 24, which shows the normalized solutions (based on the average costs) of the 60 runs of the BRD for different values of demand charges. Both the dispersion of the solutions and the distance to the best solution (CEN) are increased with the higher peak prices.

Second, we analyze the impact on the dispersion of NE solutions when varying the size of the community. To analyze this effect, we create five scenarios with different numbers of consumers N = {5, 10, 50, 100, 150}, and considered the original case with N = 201. To neutralize the effect of the AC load percentage, we select consumers to construct instances with approximately 27% of AC on the total group's load. In addition, to neutralize the effect of individuals' flexibility and preferences, N = 5 is a subset of N = 10, which is a subset of N = 50, and so on. Cost parameters are kept the same as the original in all other cases. The results of the 60-orders simulations show that the dispersion of the NE solutions decreases with the size of the community, illustrated in figure 25.

Table 24 – Impact of the cost parameters on the dispersion of the solutions (Nash Equilibria) when running the Best Response Dynamics with 60 different orders.

Case	Original	PP 01	PP 02	TOU 01	TOU 02
TOU tariffs (\$/kWh)	[0.12, 0.20]	[0.12, 0.20]	[0.12, 0.20]	[0.16, 0.16]	[0.08, 0.24]
Peak Price (\$/kW)	1.00	0.50	2.00	1.00	1.00
Min $C(L)$ (\$)	584.77	510.37	732.39	628.09	537.43
Max $C(L)$ (\$)	587.69	511.52	740.58	631.99	540.60
Avg $C(L)$ (\$)	586.21	510.80	735.72	630.32	538.80
Std Dev (\$)	0.57	0.27	1.42	0.89	0.64
Amp. (\$)	2.92	1.14	8.19	3.89	2.63
Best Sol. (CEN) (\$)	580.84	508.79	723.61	624.35	534.99
Amp. ÷ Best (%)	0.50	0.22	1.13	0.62	0.49

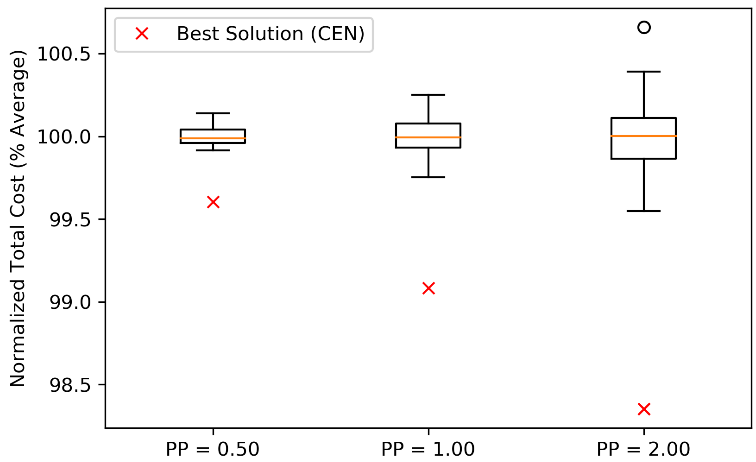


Figure 24 – Boxplot of the normalized total scheduling cost of the Nash Equilibria attained with different orders while running the BRD for different peak prices. The higher the peak price, the larger the variability of solutions, and the distance from the optimal value. The total cost is normalized by the average of each case.

However, the figure also shows that for very large communities the chance of reaching the theoretical optimum (from the centralized solution) is also lower.

Taken the case where the community size impacts more the variability of solutions ($N = 5$), we analyze a third aspect, which is the percentage of TCLs load in the overall community consumption. To study this aspect, we create games with different subset of consumers with AC appliances: 1) we take the five consumers with the lower AC load, which represents 9.37% of the total consumption 2) we take a random set of five consumers, and their AC load represents 27.21%; 3) we take the five consumers with the lower base load, and their AC load represents 62.75%; 4) finally, we take the five consumers with higher percentage of AC, and their load represents 83.39%. Again, the rest of the parameters remained constant and, for each case, we ran 60 instances of the game with different playing orders. As shown in figure 26, when the share of flexible load increases,

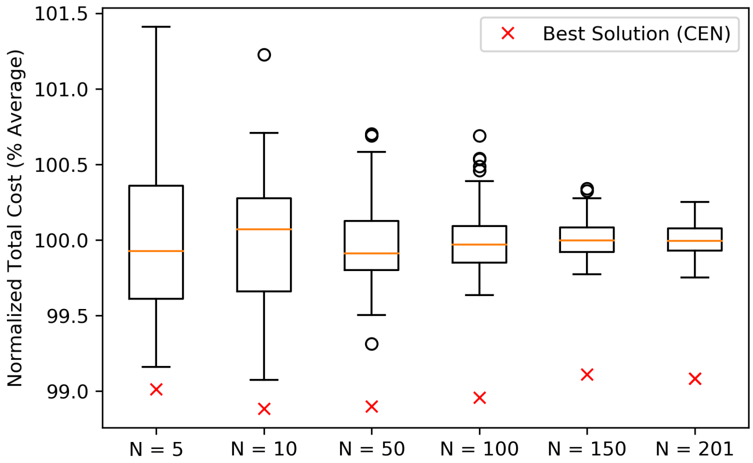


Figure 25 – Boxplot of the normalized total scheduling cost of the NEs attained with different orders while running the BRD for different number of consumers. The more consumers the community has, the lower the variability of solutions. The best solution is calculated by solving problem (5.11) for each case, with time limit equal to the time the BRD took to solve the first order. All solutions (TC and CEN) are normalized by the average of each case.

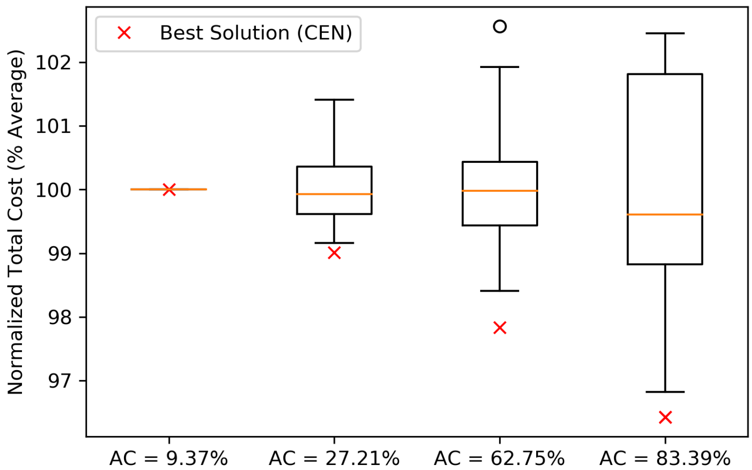


Figure 26 – Boxplot of the normalized total scheduling cost of the NEs attained with different orders while running the BRD for different percentages of AC load. The more AC the community has as a percentage of its total load, the larger the variability of solutions, especially when the community is small ($N = 5$). The best solution is calculated by solving problem (5.11) for each case, with time limit equal to the time the BRD took to solve the first order. All solutions (TC and CEN) are normalized by the average of each case.

the dispersion of NE also increases. When the share of AC load is high, the total cost variation observed is 5.63%. In practical terms, this means that, when the system has a high share of flexible resources that comprise a discrete control, the initial order considered in the TC can significantly impact the costs of the system, especially when the size of the community is small.

7. Final Discussion and Conclusion

As discussed in this thesis, non-cooperative game models can be used to optimally manage residential consumption. To model the coordination problem as a game, some basic blocks must be defined together with an information exchange structure. To conclude, we introduce a general framework (see figure 27) with those building blocks, including the communication notion behind transactive control. This framework is used here to make clear the design aspects of the game types proposed and analyzed in the thesis. In addition, we discuss the main differences between the games, from the perspective of analytical developments and simulation results.

First, thermostatically controlled loads were considered as load model, given that these loads are a main source of flexibility in the residential sector and are often left aside in non-cooperative game models for demand-side management (DSM). Moreover, two total cost functions were used: quadratic total cost and peak pricing total cost. Then, with two different billing—proportional-to-consumption (PTC) and per-time-slot (PTS)—applied to these total cost functions, three different utility functions (thus games) were proposed and analyzed: 1) game with quadratic total cost and per-time-slot; 2) game with quadratic total cost and proportional-to-consumption; and 3) game with peak pricing function and proportional-to-consumption. Those three games are compared in table 25.

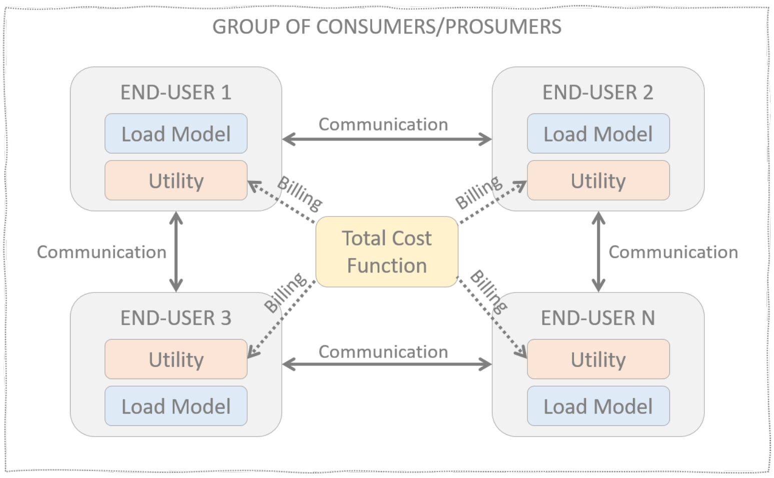


Figure 27 – General framework to model the demand-side management problem using non-cooperative games

Table 25 – Final discussion on the game types.

Aspects	Quadratic Total Cost		Peak Pricing
	PTS	PTC	PTC
Procedure used to solve and analyze	Best Response Dynamics (BRD)	modified Best Response Dynamics (m-BRD)	Best Response Dynamics (BRD) with fixed shares defined a priori
Existence of Nash Equilibrium	Potential game with NE	Not a potential game, but with equilibrium	Potential game with NE (only if TCL share is fixed)
Quality of Equilibrium	Optimize total cost and PAR	Optimize total cost and PAR	Optimize total cost and has best PAR
Convergence of algorithm	BRD converges to NE	m-BRD converges to equilibrium	BRD converges to NE
Fairness	Bills are close to Shapley Value and has valley-filling ability	Bills are distant from Shapley Value and is against energy efficiency	Create issues related to energy efficiency and end-users' engagement
Equity	Does not apply	Final shares equal to shares considered to define best responses	Final shares are different from shares considered in BRD
Multiplicity of Equilibria	Multiple NEs due to integer nature of TCLs	Multiple equilibria due to integer nature of TCLs	Variability of multiple NEs impacted by different factors
Strategy-proof	Ex-ante prices are able to discourage cheating	Cheating can occur when m-BRD is applied to find equilibrium	Does not apply

In addition, the communication aspect in figure 27 defines the solution algorithm used to find equilibrium points of the proposed games. Here, best-response dynamics (BRD) and modified best-response dynamics (m-BRD) were applied.

As can be seen in the table, the game with the per-time-slot billing is the only one guaranteed to be potential, thus having Nash Equilibria. When the proportional-to-consumption billing is applied, the final total consumption of each consumer is not fixed, thus equilibria can/cannot exist. In the case of the quadratic total cost function, a modified best response dynamics was used to find these equilibria. In the case of the peak pricing, the consumption shares of the TCLs were fixed before the application of the solution algorithm, assuring the existence of Nash Equilibria, but raising equity issues, because the final total consumption can be different than the initial one (used to define the shares).

Another aspect analyzed was the quality of the reached equilibria. Generally, all three games converged to an equilibrium able to optimize the total cost and reduce the peak-to-average ratio (PAR) of the consumers' total load. However, the peak pricing total cost (and resulting game) was the one with the smallest PAR, due to the ability of this cost function to penalize peaks.

In terms of fairness, it was demonstrated (theoretically and with the simulation results) that the per-time-slot billing was the one with the best fairness index. When this billing is applied, consumers final payments are close to the Shapley Value, indicating their bills reflect their flexibility provision. Moreover, the proportional-to-consumption billing is focused in consumers individually trying to reduce their total consumption, which can lead to global loss in energy efficiency and also to diminishing end-users' engagement in the DSM program. For instance, in the peak pricing with PTC game, a consumer with smaller final total consumption can pay more than a neighbor with higher final total consumption, given that the shares were defined prior to the run of the BRD. This equity aspect can discourage participation.

In terms of multiplicity of equilibria, all three games can have one or multiple equilibrium points, due to the integer nature of the thermal loads considered. Because the best-response dynamics was used to solve them (original or proposed modified version), two aspects were proven to impact the improvement path of the algorithm, thus what equilibrium is reached: 1) initial solution from which the algorithm starts; 2) playing order of the consumers (participants). This aspect was analyzed in detail for the peak pricing game with PTC, and simulation results have shown that the variability of the multiple NEs is impacted by different factors (as the number of consumers and the parameters of the cost function).

Finally, cheating behavior of the consumers was analyzed. It was shown that cheating behavior can happen in the games with quadratic total cost. However, a small change in the billing of the per-time-slot game is able to discourage this behavior, while in the proportional-to-consumption the cheating can occur given that thermal loads have an energy variant characteristics.

All those findings support the main objective of this thesis: to contribute on the development and performance evaluation of game theoretical algorithms to address the distributed energy resources coordination problem at the distribution level. By proposing different game models under the framework of figure 27 and analyzing their different aspects and characteristics we were able to overcome some challenges related to the design of energy efficient models that can enhance end-users' engagement.

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