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DISTRIBUTED MATHEMATICAL MODEL AND EXPERIMENTAL VALIDATION FOR A CO2 HEAT PUMP ASSISTED BY SOLAR ENERGY

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Abstract. A use of CO2 operating in a transcritical cycle has been proven for heat pumps is a demonstrably viable and considerably interesting option due to the environmental advantages of CO2 over other refrigerant gases. In order to improve the energy performance of systems that use heat pumps, integrating a type of energy such as renewable geothermal, solar, wind and bio-fuels must be available. In this scenario, a mathematical model with experimental validation of the components that allows the modeling of the heat pump system to vary the input parameters and determine the outlet water temperature and the coefficient of performance (COP) of the heat pump. This article approaches the modeling of the DX-SAHP, in order to obtain the profile of temperature and pressure distribution along the gas cooler, and the values of heat exchange and pressure in collector solar/evaporator. The model was validated with experimental data from 88 tests performed under different operating conditions, even the DX-SAHP in question. In the experimental the radiation incidence range in the study environment was from 0 to 845 W/m² and at an ambient temperature of 21°C to 33°C. The maximum difference between the theoretical results and experimental results was 9.5%.

Keywords: DX-SAHP, Mathematical Mode, Evaporator

1. INTRODUCTION

Faced with the gradual increase in temperature over time, global warming is an important problem to consider to make the global economy sustainable. I In this scenario, researchers looking for ways to reduce greenhouse gas emissions and maximize equipment performance. To refrigeration area the Carbon dioxide (CO2 or R744) as a refrigerant fluid has zero net impact in climate change and it is a not toxic, corrosive fluid or flammable, it is inexpensive and readily available fluid.

The refrigerant fluid, in addition, technical solutions with better energy efficiency are used, for example for heating water, heat pumps show a COP (Coefficient of Performance) between 2 and 3, even more allied with the use of solar energy to maximize efficiency. In this case, this type of equipment is called Solar Assisted Heat Pump (SAHP).

The table 1, list different articles to study heat pump. to different types of collators UFP (Uncovered Flat Plate) CFP (Coverded Flat Plat) PVT (PhotoVoltaic Thermal). Only two articles of the table 1 use Heat Pumps with R744. The mathematical model presented by Duarte *et al.* (2019a) was validated using R134a although excellent values of relative errors the R744 is not used to experimental validation. Diniz *et al.* (2021) the model is complex, transient, and demand very cost computational and the experimental validation was made qualitatively, to exemplify the COP, which is one of the most important variables from the energy point of view of a heat pump, there is no experimental validation, in the study, only an analysis of the response is performed as a function of the variation in the area of the expansion device.

Authors	Collector Type	Collector size (m ²)	Refrigerant	Experimental Validation	Relative error (%)	Location	Tank size (L)	Water Temp. (°C)
Chaturvedi and Shen (1984)	UFP	3.4	R12	1	-	Norfolk	-	-
Chaturvedi et al. (1998)	UFP	3.5	R12	 ✓ 	-	Norfolk	-	-
Ito et al. (1999)	UFP	3.2	R22	1	-	Japan	-	30-60
Torres-Reyes and Gortari (2001)	UFP	4.5	R22	1	-	Guanajuato	-	-
Hawlader et al. (2001)	UFP	3.0	R134a	1	-	Singapore	250	55
Chyng et al. (2003)	UFP	1.9	R134a	1	10 (max.)	Taiwan	-	52-56
Kuang et al. (2003)	UFP	2.0	R22	1	5-30	Shanghai	150	50
Chata <i>et al.</i> (2005)	UFP	-	R404A, R407C, R410A R12, R22 and R134a	×	N.A.	-	-	-
Xu et al. (2009)	PVT	2.3	R22	X	N.A.	Naniing	150	50
Chow et al. (2010)	UFP	12	R134a	X	N.A.	Hong Kong	1500	50
Kong <i>et al.</i> (2011)	UFP	4.2	R22	1	1-4.6	Shanghai	150	50
Moreno-Rodríguez et al. (2012)	UFP	5.6	R134a	1	10 (max.)	Madri	300	51
Chaturvedi et al. (2014)	UFP	1-5	R134a	X	N.A.	Norfolk	-	50-70
Sun et al. (2015)	UFP	2.0	-	X	N.A.	Shanghai	150	55
Deng and Yu (2016)	CFP	2.5	R134a	X	N.A.	-	150	55
Kong et al. (2017)	UFP	4.2	R410A	X	N.A.	-	150	50
Mohamed <i>et al.</i> (2017)	UFP	4.2	R407C	1	4 (max.)	Nottingham	200	50
Duarte et al. (2019a)	UFP	1.6	R134a, R600a, R290,	1	≅5 (max.)	Belo Horizonte	200	65
			R1234yf and R744		1.6 (mean)			
Rabelo et al. (2019)	UFP	1.3-6	R134a and R290	1	-	Belo Horizonte	200	65
Kong et al. (2020)	UFP	2.1	R134a	1	10 (max.)	-	200	60
Diniz et al. (2021)	UFP	1.6	R744	1	-	Belo Horizonte	200	≅40
Wang <i>et al.</i> (2021)	UFP	2.1	R134a	1	-	-	200	30-50
Ma et al. (2021)	UFP	$\cong 2.3$	R22	1	13 (max.)	-	-	-

Table 1. Studies with complete mathematical models for DX-SAHP

The purpose of this study is present and validate a mathematical model to steady state with low computational cost with relative error estimated at 10 %.

2. MATHEMATICAL MODEL

To evaluate the performance of DX-SAHP for the production of hot water, a steady-state model was developed using the Equation Engineering Solver (EES) similar to the model described by Duarte *et al.* (2018). Losses in the pipes connecting the components were considered negligible. The evaporator/solar collector and gas cooler were assumed to be isobaric. A lumped model was used for the evaporator and a distributed model was used for the gas cooler.

For validation of the mathematical model, the data published by Duarte *et al.* (2021) was used, which has a total of 88 experiments. The model needs as input data the relative humidity, and for that, only 41 experiments had the information, thus, the validation was performed with these 41 experiments. To obtain the 10 coefficients of the compressor's electrical power equation AHRI (2020), a multiple linear regression was performed as describide by Chapra and Canale (2008), using 37 experimental points not contained in the set of experiments used to validate the model and the other 10 experiments remaining in the article were not used.

2.1 Compressor model

The hermetic compressor selected for this work is the model: SRcADB 6455, manufactured by SANDEN. In the literature, compressor models are described in detail (Yang *et al.*, 2013; Duarte *et al.*, 2019b; Bell *et al.*, 2020), but these models require many parameters and geometric details that are not provided by hermetic compressor manufacturers. Moreover, the compressor model used that integrates the complete model of the refrigeration system is simpler, assuming an isentropic compression process, as used in other studies (Minetto, 2011; Rabelo *et al.*, 2019; de Paula *et al.*, 2020). The compressor manufacturer does not provide the coefficients for AHRI (2020) equation to evaluate the electrical power

consumed by the compressor (\dot{W}_{cp}). These coefficients were found by linear regression using 37 experiments. The equation are a function of evaporator pressure (P_e) and gas cooler pressure (P_c):

$$\dot{W}_m = C_1 + C_2 P_e + C_3 P_c + C_4 P_e^2 + C_5 \left(P_e P_c \right) + C_6 P_c^2 + C_7 P_e^3 + C_8 \left(P_e^2 P_c \right) + C_9 \left(P_e P_c^2 \right) + C_{10} P_c^3 \tag{1}$$

Table 2. Coeficients of electrical power consumed by the compressor.

Coeficient	Value
C1	-3,049114E+04
C2	3,088764E+01
C3	1,115466E+03
C4	6,674906E+00
C5	-7,292516E+00
C6	-1,184790E+01
C7	-1,346469E+01
C8	-6,224440E-0,2
C9	7,600259E-02
C10	3,608758E-02

The refrigerant mass flow rate (\dot{m}_r) for a constant rotation speed reciprocating compressor is given by (Minetto, 2011):

$$\dot{m}_r = \rho_1 N V_s \eta_v \tag{2}$$

where ρ is the refrigerant density, N is the rotation speed, V_s is the compressor swept volume, η_v is the volumetric efficiency and the subscript 1 refers to compressor inlet or evaporator outlet.

The volumetric efficiency was determined by fitting equations proposed by Duarte *et al.* (2019a) to the compressor performance map provided by the manufacture. The volumetric and overall efficiencies is given by:

$$\eta_v = -0.0922 \left(\frac{P_c}{P_e}\right) + 0.9496\tag{3}$$

2.2 Direct expansion solar evaporator model

The heat transfer rate received by the refrigerant in the evaporator (\dot{Q}_{rev}) is given by:

$$\dot{Q}_{rev} = \dot{m}_r (i_1 - i_4) \tag{4}$$

where the subscript 4 refers to thermostatic valve outlet or evaporator inlet. To evaluate the energy gain from air and solar radiation in a flat plate collector in steady-state condition, Kong *et al.* (2011) suggest the following equation:

$$\dot{Q}_{air} = A_e F'[aI - U_L(\overline{T}_r - T_a)] \tag{5}$$

where A_e is the area of solar evaporator, F' is the collector effectiveness factor, I is the solar radiation, a is the solar absorptivity, U_L is overall heat loss coefficient, \overline{T}_r is the average temperature of the refrigerant fluid and T_a is the ambient air temperature.

The collector effectiveness factor proposed by Duffie and Beckman (2013), considering that the resistance to heat flow due the bond between the collector plate and tube can be neglected, is given by:

$$F' = \frac{1}{wU_{ev}} \left\{ \frac{1}{U_L[D_o + F(w - D_o)]} + \frac{1}{\pi D_i h_i} \right\}^{-1}$$
(6)

where the distance between the tubes in the evaporator is w, the fin efficiency is F, the outer diameter is D_o , the inner diameter is D_i , the internal convective coefficient is h_i , that is calculated by the correlation proposed by Shah (2017) for two phase flow, and by the correlation proposed by Gnielinski (1976), for single phase flow. Shah (2017) presented a new correlation and compared the results with another seven correlations, using 4852 experimental data points from 81 sources and 30 different fluids including R744.

The fin efficiency can be evaluated by:

$$F = \frac{\tanh\left[(w - D_o)/2\sqrt{U_L/(k\delta)}\right]}{(w - D_o)/2\sqrt{U_L/(k\delta)}}$$
(7)

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where δ is the fin thickness and k is the thermal conductivity.

The overall heat loss coefficient proposed by Duffie and Beckman (2013) is determined by:

$$U_L = (h_{conv} + h_{cond} + h_{rad})_{bot} + (h_{conv} + h_{cond} + h_{rad})_{top}$$

$$\tag{8}$$

where the subscripts *bot* and *top* represents the values at the bottom and top surface of the collector respectively. The convective coefficient (h_{conv}) is calculated by the correlation proposed by Kumar and Mullick (2010), the condensation convective coefficient (h_{cond}) from humid air is determined by the correlation proposed by Scarpa and Tagliafico (2016), the radiation heat transfer coefficient (h_{rad}) is given by:

$$h_{rad} = \varepsilon \sigma (\overline{T}_r + T_a) (\overline{T}_r^2 + T_a^2) \tag{9}$$

where the emissivity is ε , and the Stefan-Boltzmann constant is σ .

2.3 Gas cooler model

In the gas cooler model the following assumptions were made: (i) the thermal losses to environment and pressure drop are negligible; (ii) the properties of the walls, CO_2 and H_2O are equally distributed in the gas cooler section; (iii) thermal resistance of the wall tube due conduction is zero; and (iv) CO_2 and H_2O flow is one-dimensional. The energy balance for carbon dioxide, tube wall and water in any section of the gas cooler showed in Fig. 1, is given by:

$$\dot{m}_r \frac{\partial i_r}{\partial z} = h_r p_{ii} (T_s - T_r) \tag{10}$$

$$h_r p_{ii}(T_s - T_r) = h_w p_{oi}(T_w - T_s)$$
(11)

$$h_w p_{oi}(T_w - T_s) = -\dot{m}_w \frac{\partial i_w}{\partial z} \tag{12}$$

where p is the perimeter, z is the gas cooler position. Subscripts r, s and w refers to carbon dioxide, tube wall and water proprieties. Variables with subscripts ii, io and oi are geometrical parameter calculated with the diameters showed in Fig. 1. The convective coefficient h is calculated by the correlation proposed by Gnielinski (1976) for both, fluids and correct by Gosse (1981) factor for flow in annular region.



Figure 1. Gas cooler section geometry and control volume numbering

The boundary condition are the inlet temperature of water and refrigerant. To solve this system of differential equations, the finite volume technique and the central difference scheme descried by Versteeg and Malalasekera (2007) were used. This system of differential equations becomes the system of algebraic equations given by:

$$\dot{m}_r \frac{i_r(j) - i_r(j-1)}{\Delta z} = h_r p_{ii} [T_s(j) - T_r(j)]$$
(13)

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$$T_r(j) = \frac{h_w p_{oi} T_w(j) + h_r p_{ii} T_r(j)}{h_w p_{oi} + h_r p_{ii}}$$
(14)

$$h_w p_{oi}[T_w(j) - T_s(j)] = -\dot{m}_w \frac{i_w(j) - i_w(j+1)}{\Delta z}$$
(15)

The enthalpy variables represent values at the outlet edge and the temperatures variable represent the values at the center of control volume. The simultaneous solution of the nonlinear system of equations would result in huge arrays with difficult convergence. An iterative solution is described by Machado (1996) that allow solve the equations for refrigerant and water separately. This iterative solutions is described in Fig. 2, where E_{tol} is a tolerance error, E_{Tr} is the refrigerant temperature convergence error and E_{Ts} is the wall temperature convergence error. This errors are given by:

$$E_{Tr} = \frac{T_r^p - T_r}{T_r} \cdot 100 \tag{16}$$

$$E_{Ts} = \frac{1}{n} \sum_{j=1}^{n} \frac{T_s^p(j) - T_s(j)}{T_s(j)} \cdot 100$$
(17)

where the superscript p represents the value at previous iteration and n is the number of control volumes.

3. RESULTS AND DISCUSSION

3.1 Grid test

A spatial grid test was performed to determine the size of the control volumes in the gas cooler (Δz), following the directives described McHale and Friedman (2009). In this simulation was done considering the gas cooler pressure of 80 bar, ambient and inlet water temperature of 25°C, no wind, a solar radiation of 500 W/m² and a water flow rate of 28.8 L/h.For control volumes smaller than 110.5 mm, the changes in the values of *COP* and *T*_{wo} were smaller than the convergence tolerance error. Table 3 shows the results of the simulation done in the grid test.

$\Delta z \text{ (mm)}$	COP	ΔCOP (%)	$T_{wo} (^{o}\mathrm{C})$	ΔT_{wo} (%)
243	2.4198		63.258	
186.9	2.4790	-2,4	64.209	-1.5
143.8	2.5067	-1.1	64.712	-0.8
110.5	2.5217	-0.6	64.997	-0.4
85	2.5349	-0.5	65.246	-0.4

Table 3. Results of grid test for 80 bar, 25°C, 0 m/s, 500 W/m² and 28.8 L/h

Then, after the spatial grid test, the following results were based in the list of parameters presented in Tab. 4. The carbon dioxide, water, air and copper properties were calculated using internal EES libraries.

Parameter	Value	Parameter	Value
Atmospheric Pressure	905.0 kPa	Wind Speed	0 m/s
Emissivity	0.95	Solar absorptivity	0.95
Compressor displacement	1.75 cm ³ /rev	Compressor rotation speed	3500 rpm
Ambient temperature	25 °C	E_{tol}	±0.5 %
Δz	110.5 mm	Collector length	1563.4 mm
Collector width	1000.4 mm	Collector fin thickness	0.55 mm
Collector tube lenght	16.3 mm	Collector tube inner diameter	4.66 mm
Collector tube outer diameter	6.34 mm	Collector distance between the tubes	92.5 mm
Gas cooler tube lenght	24.3	Gas cooler tube outer dia. (flow Water)	12.5 mm
Gas cooler tube inner diameter (flow CO2)	4.66 mm	Gas cooler tube inner dia. (flow Water)	10.52 mm
Gas cooler tube outer diameter (flow CO2)	6.34 mm		



Figure 2. Flow chart for solving the gas cooler equations



Figure 3. Correlation between measured COP x theoretical COP



Figure 4. Correlation between measured electric power x theoretical electric power

3.2 Validation

The COP correlation graphics shows the distribution of the measured COP versus simulated and the distribution of points puts only 3 points out of 41 with the error above 5 %. The maximum error observed was 8.4%, and the mean absolute error was 2.7%. The simulation of the electrical power consumed by the compressor compared to the measurement also showed a satisfactory adjustment as observed in the graph, only 5 points out of 41 have an error above



Figure 5. Correlation between outlet water temperature x theoretical temperature

2%. The maximum error observed is 9.5% and the mean absolute error was 1.36%. The temperature graph shows that the errors are within a maximum error of 1.1%.

According to table 1, the maximum observed errors observed in the mathematical models presented are similar to the values found in several articles.

4. CONCLUSION

In this study, a set of experimental data from a CO2 Heat Pump Assisted by Solar Energy published by Duarte *et al.* (2021), was used to validate the mathematical model. To concentric counter flow gas cooler was used a distributed model, and to the components: evaporator, compressor and expansion valve was used a lumped model.

In the results analysis, the present mathematical model proved to be efficient to predict the COP and the outlet water temperature, and the maximum error less than 10% is in agreement with other models predicted in the literature. Its useful only steady state regime and simulations and has low computational cost for simulation.

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