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PROGRAMA DE PÓS-GRADUAÇÃO EM PSICOLOGIA COGNIÇÃO E COMPORTAMENTO

Mariuche Rodrigues de Almeida Gomides

How do children write Arabic numbers? Task complexity, cross-linguistic effects, and phonological processing skills contribute to success in number transcoding

Como as crianças escrevem os números arábicos? Complexidade da tarefa, efeitos interlinguísticos e processamento fonológico contribuem para o sucesso na transcodificação numérica

BELO HORIZONTE

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Orientador: Prof. Dr. Vitor Geraldi Haase

Co-orientadora: Prof. Dr. Júlia Beatriz Lopes-Silva

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PROGRAMA DE PÓS-GRADUAÇÃO EM PSICOLOGIA: COGNIÇÃO E COMPORTAMENTO

FOLHA DE APROVAÇÃO

Como as crianças escrevem os números arábicos? Complexidade da tarefa, efeitos interlinguísticos e processamento fonlógico contribuem para o sucesso na transcodificação numérica.

MARIUCHE RODRIGUES DE ALMEIDA GOMIDES

Tese submetida à Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação em PSICOLOGIA: COGNIÇÃO E COMPORTAMENTO, como requisito para obtenção do grau de Doutor em PSICOLOGIA: COGNIÇÃO E COMPORTAMENTO, área de concentração PSICOLOGIA: COGNIÇÃO E COMPORTAMENTO, linha de pesquisa Neuropsicologia do Desenvolvimento.

Aprovada em 27 de julho de 2021, pela banca constituída pelos membros:

Prof(a). Vitor Geraldi Haase - Orientador
UFMG

Prof(a). Julia Beatriz Lopes Silva
Universidade Federal de Minas Gerais

Prof(a). Patricia Martins de Freitas
Universidade Federal da Bahia

Prof(a). Izabel Augusta Hazin Pires
Universidade Federal do Rio Grande do Norte

Prof(a). Tatiana Cury Pollo
Universidade Federal de São João Del-Rei

Antonio Jaeger:89249763034 Assinado de forma digital por Antonio Jaeger:89249763034 Dados: 2021.08.18 11:32:51 -03'00'

Prof(a). Antônio Jaeger
UFMG

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Resumo

Gomides, M. R. A. (2021). Como as crianças escrevem os números arábicos? Complexidade da tarefa, efeitos interlinguísticos e processamento fonológico contribuem para o sucesso na transcodificação numérica. Tese de doutorado, Programa de Pós-graduação em Psicologia: Cognição e Comportamento, Faculdade de Filosofia e Ciências Humanas da Universidade Federal de Minas Gerais, Belo Horizonte.

A capacidade de converter diferentes notações numéricas (ou seja, numerais escritos e arábicos) entre si é chamada de transcodificação numérica. A aprendizagem da transcodificação numérica é uma etapa fundamental no desenvolvimento matemático das crianças, sendo um importante preditor da aquisição posterior de habilidades matemáticas mais complexas. O principal objetivo desta tese de doutorado foi investigar como diferentes mecanismos estão associados à transcodificação numérica. No capítulo 1, fornecemos evidências de como os mecanismos de nos níveis estruturais da tarefa e individuais afetam a aprendizagem da transcodificação numérica, apresentamos as suposições do modelo ADAPT e oferecemos uma breve visão geral das dificuldades com a aprendizagem da transcodificação numérica enfrentadas por crianças com dificuldades na matemática. Por fim, destacamos as lacunas na literatura e discutimos como planejamos abordá-las. No capítulo 2, investigamos a contribuição específica tanto da complexidade sintática, medida pelo número de regras de transcodificação segundo o modelo ADAPT, quanto do número de dígitos, para o desempenho na transcodificação numérica de crianças do segundo ao quinto ano do ensino fundamental. Os resultados demonstraram que o impacto da complexidade sintática na transcodificação numérica é relativamente independente do número de dígitos. Esse achado corrobora a suposição do modelo ADAPT de que os números são transcodificados por meio de regras de conversão baseadas em algoritmos. No capítulo 3, comparamos como as especificidades da formação de palavras numéricas em alemão e português afetam diferencialmente a transcodificação numérica (ou seja, a irregularidade sintática da inversão unidade-dezena em alemão e a irregularidade morfológica das palavras de centenas em português). Os resultados mostraram que as irregularidades sintáticas no sistema de palavras numéricas em alemão afetaram o desempenho geral e os padrões de erros das crianças de língua alemã. Comparativamente, as irregularidades morfológicas no sistema de palavras numéricas em português não afetaram o desempenho geral e os padrões de erros das crianças de língua portuguesa. Seguindo as suposições do modelo ADAPT, irregularidades ao nível sintático exigiriam a implementação de regras de transcodificação adicionais. No capítulo 4, investigamos a contribuição específica dos três componentes do processamento fonológico (ou seja, memória de trabalho fonológica, consciência fonêmica e acesso lexical), além da memória de trabalho visuoespacial, para o desempenho geral e os padrões de erro na transcodificação numérica. Os resultados demonstraram que o desempenho na transcodificação numérica foi seletivamente explicados pelos componentes do processamento fonológico, em particular a consciência fonêmica e o acesso lexical. Além disso, os erros lexicais foram melhor explicados pela consciência fonêmica e pelo acesso lexical, enquanto os erros sintáticos e combinados foram melhor explicados pela consciência fonêmica e pela memória de trabalho visuoespacial. No capítulo 5, resumimos e discutimos as contribuições da presente tese, reconhecemos suas principais limitações e delineamos os próximos passos de estudos futuros.

Palavras-chave: transcodificação numérica, escrita de números arábicos, discalculia, processamento fonológico.

Abstract

Gomides, M. R. A. (2021). How do children write Arabic numbers? Task complexity, cross-linguistic effects, and phonological processing skills contribute to success in number transcoding. Tese de doutorado, Programa de Pós-graduação em Psicologia: Cognição e Comportamento, Faculdade de Filosofia e Ciências Humanas da Universidade Federal de Minas Gerais, Belo Horizonte.

The ability to convert different numerical notations (i.e., number words and symbolic Arabic digits) into one another is referred to as number transcoding. The learning of number transcoding is a major building block in children's mathematical development, which predicts the later acquisition of more complex mathematical abilities. The main goal of this dissertation was to investigate how different mechanisms are associated with number transcoding. In chapter 1, we provide evidence of how structural task level and individual level mechanisms affect the learning of number transcoding, present the assumptions of the ADAPT model, and offer a brief overview of the difficulties with the learning of number transcoding faced by children with mathematical difficulties. At last, we highlight the gaps in the literature and discuss how we plan to address them. In chapter 2, we investigate the specific contribution of both syntactic complexity, as measured by the number of the ADAPT model's transcoding rules, and of number of digits, for the performance on number transcoding of children from second to fifth grades. Results demonstrated that the impact of syntactic complexity on number transcoding is relatively independent of the number of digits. This finding substantiated the assumption of the ADAPT model that numbers are transcoded via algorithm-based conversion rules. In chapter 3, we compare how the specificities of German and Portuguese number word formation differentially affect number transcoding (i.e. the syntactic irregularity of unit-decade inversion in German and morphological irregularity of hundred words in Portuguese). Results showed that syntactic irregularities in German number word system affected the overall performance and error patterns of German-speaking children. Comparatively, morphological irregularities in Portuguese number word system did not affect the overall performance and error patterns of Portuguese-speaking children. Following the assumptions of the ADAPT model, irregularities at the syntactic level would demand the implementation of additional transcoding rules. In chapter 4, we investigate the specific contribution of the three components of phonological processing (i.e., phonological working memory, phonemic awareness, and lexical access speed), in addition to visuo-spatial working memory, for number transcoding overall performance and error patterns. Results demonstrated that number transcoding performance was selectively predicted by phonological processing components, in particular phonemic awareness and lexical access speed. Furthermore, lexical errors were predicted best by phonemic awareness and lexical access, whereas syntactic and combined errors were predicted best by phonemic awareness and visuo-spatial working memory. In chapter 5, we summarize and discuss the original contributions of the present dissertation, acknowledge its main limitations, and outline the next steps of future studies.

Keywords: number transcoding, Arabic number writing, dyscalculia, phonological processing.

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CHAPTER 1

General introduction

Being able to represent magnitudes in different formats is a trivial task of our daily lives. However, the creation of the symbolic number representations (i.e., number words and the Arabic digit notation) is one of the most important cultural artefacts developed by humankind (Ifrah, 1997). In particular, the Arabic digit notation allows us to represent numbers in an economic and efficient way and to develop sophisticated forms of mathematical thinking such as arithmetic. From an ontogenetic perspective, mastering the correspondence of these two symbolic codes is a laborious task for young children, demanding formal instruction and practice throughout the first years of elementary school (Seron et al., 1992; Moura et al., 2015; Power & Dal Martello, 1990; Sullivan et al., 1996). In turn, children become able to convert number words into Arabic digit notation and vice-versa. This latter ability is referred to as number transcoding (Deloche & Seron, 1982).

Learning Arabic digit numbers and their verbal labels is a critical foundation for arithmetic development in young children, which could be considered analogous to the role of early letter knowledge as a critical longitudinal predictor of reading development (Göbel et al., 2014). Recent evidence suggested that tasks tapping number knowledge, such as Arabic digit identification, place-value understanding, and number transcoding, predict early arithmetic knowledge over and above domain-specific abilities (symbolic and nonsymbolic magnitude comparison and counting) and domain-general abilities (nonverbal reasoning and language; see Clayton et al, 2020; Göbel et al., 2014; Habermann et al., 2020; Moeller et al., 2011). Therefore, it is crucial to investigate the learning of number transcoding by children and the mechanisms involved in this process.

The main goal of the present dissertation was to investigate what makes the learning of number transcoding harder. To address this question, we investigate the role of different mechanism at the structural task level and at individual level on number transcoding. In particular, we investigated how the linguistic aspects on number word formation and the syntactic organization on the Arabic number system affect the performance on number transcoding at the structural task level. Moreover, we investigated how phonological processing abilities affect the performance on number transcoding at the individual level. We explain the specific contribution of each one of these mechanisms on number transcoding based on the assumptions of a prominent cognitive model of number transcoding, the ADAPT model (A Developmental Asemantic and Procedural model for Transcoding; Barrouillet et al., 2004).

In the following sections we provide evidence of how structural task level and individual level mechanisms affect the learning of number transcoding, present the assumptions of ADAPT model, and offer a brief overview of the difficulties with the learning of number transcoding faced by children with mathematical difficulties. At last, we highlight the gaps in the literature and discuss how we plan to address them.

The correspondence between number words and Arabic digit notation

The mastering of number transcoding relies upon the understanding of the correspondence between number words and Arabic digit notation. The number word system is composed of a lexicon of words (e.g., *five, thirty, hundred*) which are organized in different classes (e.g., units, decades, hundreds). Numbers words are generated by combining this lexicon of words following additive (e.g., “forty-two” is equal to forty plus two) and multiplicative (e.g., “three hundred” is equal to three times hundred) syntactic principles.

In turn, the Arabic digit system is composed of a smaller lexicon of symbols (digits from 0 to 9), which are combined and placed in different positions in the number chain to generate all possible combinations of Arabic digit numerals. This place-value principle is determined by a power of base ten that increases from the rightmost digit to the left (e.g., 635 is equal to $6 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$). The digit zero has a special role in the Arabic digit system. It is used as a place-holder, indicating that there is no value associated with the power of base ten.

The mastering of the correspondence between the two symbolic codes is challenging for young learners, given that verbal and Arabic numerals obey different syntaxes and the correspondence between number words and Arabic digit notation is not always one-to-one. For instance, the zero in the number "305" is not explicitly named in the number word format. Additionally, the correspondence between number words and Arabic digit notation is influenced by language, such that some languages present a more transparent correspondence than others.

The Arabic digit number system is the most common form of representing numbers in a symbolic-visual fashion. However, the morphological and syntactic characteristics of numbers' symbolic-verbal representations (i.e., the number word system), is determined by language (Comrie, 2005). Consequently, some languages show a more consistent one-to-one correspondence between number words and Arabic digit notation than others. For instance, the number words for 13 and 20 in Japanese could be literally translated as "ten-three" and "two-ten", respectively. In both cases, there is a one-to-one correspondence between the number words and the Arabic digit notation. Comparatively, the number words for 13 and 20 in English, thirteen and twenty, are less transparent.

Cross-linguistic studies observed that the transparency of number words affects place-value understanding. As such, Chinese-, Japanese-, and Korean-speaking children, whose languages have a more transparent number word system, presented a better performance in tasks which demand the representation of numbers using base-ten blocks in comparison to American-, French-, and Swedish-speaking children, whose languages have a less transparent number word system (Miura & Okamoto, 2003; Miura et al., 1994; Miura et al., 1988). These results suggested that, because the number word system of the former languages is built upon the reflection of groups of ten, it allows a more straightforward mapping of number words on the place-value structure of the Arabic digit system.

The development of number transcoding by children

As shown above, learning how to transcode numbers demands the acquisition of complex abilities, such as the consolidation of a number names' lexicon and the understanding of the place-value structure of the Arabic number system. Consequently, it takes three to four years for children to master the number transcoding of numbers with four digits (Moura et al., 2013, 2015). Despite this, children present a rudimentary knowledge of the place-value structure even long before they start school (Barrouillet et al., 2010; Byrge et al., 2014; Mix et al., 2014, Yuan et al., 2019). Yuan and colleagues (2019) investigated whether five- and six-year-old children were able to identify and compare multi-digit numbers that required knowledge about the place-value (e.g., 14 vs. 41, and 206 vs. 260). Results showed that five-year-old children were able to properly identify multi-digit numbers by their names and, by the age of six, children were able to properly make judgments about multi-digit numbers magnitude. The authors conclude that five-year-old children understand that number words are mapped onto multi-digits from left to right, and that each position's value decreases from left to right.

Error patterns in number transcoding

The analysis of error patterns in number transcoding provides insights into children's initial understanding of multi-digit numbers and into the possible lexical and syntactic mechanisms involved in number processing. Difficulties with number transcoding are associated with two main error types: lexical and syntactic errors (Deloche & Seron, 1982). In lexical errors, a lexical element of the number is replaced by another without changing the length of the number chain (e.g., "two hundred and seventy-three" written as "263"). In contrast, in syntactic errors, the lexical elements of the number are preserved, but the overall size of the number chain is incorrect (e.g., "two hundred and seventy-three" written as "20073").

The understanding of numbers' syntactical structure represents the greatest challenge for children during the learning of number transcoding, with syntactic errors representing almost all error rates (Barrouillet et al., 2004; Moura et al., 2013; Zuber et al., 2009). Usually, syntactic errors commonly involve violations of both additive (e.g., "two-hundred and forty-five" written as "20045") and multiplicative (e.g., "two-hundred and forty-five" written as "210045") principles (Moura et al., 2013; Power & Dal Martello, 1990; Seron & Fayol, 1994; Zuber et al., 2009). Additionally, syntactic errors might be literal (e.g., "two-hundred and forty-five" written as "20045") or partial literal (e.g., "two-hundred and forty-five" written as "2045") transcriptions of a number (Camos 2008; Moura et al., 2013; Seron & Fayol, 1994).

The ADAPT model

The ADAPT model (A Developmental Asemantic and Procedural model for Transcoding; Barrouillet et al., 2004) proposes that numbers are transcoded procedurally by implementing a series of rules without the mediation of the number's semantic representation. According to the model, in the first step of number transcoding, number words are phonologically encoded and temporarily maintained in a phonological buffer. In cases where a corresponding lexicalized form is available in long-term memory, the Arabic digit form is automatically retrieved (P1 rule). Otherwise, number transcoding occurs procedurally by the implementation of a set of rules. In this case, numbers are decomposed into smaller units. Then rules responsible for the creation of slots corresponding to the necessary place-value stacks (P2 and P3 rules for hundreds and thousands, respectively) are triggered by the identification of separators (i.e., the word hundred or thousand). After all slots are filled with the corresponding Arabic digit forms retrieved by P1 rule, a last rule (P4 rule) is responsible for filling any empty slot with zero and determining the end of the process.

Because number transcoding is implemented procedurally and is restricted by the constraints of our cognitive architecture (i.e., limited resources of working memory), the model predicts that the level of difficulty in transcoding a number is based on the number of procedures involved. Thus, the more transcoding rules required to transcode a number, the more error prone the number is. Previous studies have shown that the error rates increase with the number of transcoding rules (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2015).

The ADAPT model also accounts for developmental change. Throughout development, children would learn more complex rules but also expand their number lexicon. Consequently, they would abandon more primitive rules and retrieve large lexical units from long term memory. Already in kindergarten, children develop a number lexicon consisting of number words and Arabic digit form of one-digit numbers, which are referred to as lexical primitives. By second grade, children would incorporate to their number lexicon the Arabic digit form of two-digit numbers, which would be successfully transcoded by simply achieving direct memory retrieval (P1 rules only). The expansion of the number lexicon would also incorporate large but familiar numbers (the current year, year of birth, certain important dates in history, etc.), which would be also transcoded by direct retrieval without the application of rules.

Moreover, the authors of the ADAPT model, which was originally developed upon French number words, state that it can be easily adapted in order to incorporate specificities of number word formation of other languages by adding or removing transcoding rules.

In addition, the model explicitly distinguishes the different mechanisms associated with lexical and syntactic errors. Lexical errors are assumed to represent a difficulty in the lexical mechanisms due to problems in the retrieval of correct Arabic forms (i.e., P1 rules). In contrast, syntactic errors are more likely to be related to failures in the procedural mechanisms due to problems in the management of place-value stacks (i.e., P2, P3 and P4 rules).

Mechanisms involved number transcoding

In the next section, we present evidence about the mechanisms involved in number transcoding, which were separated into two categories. The first, which we will refer to by the term structural task level mechanisms, is related to the linguistic specificities of the number word system and the syntactic organization of the Arabic number system. The second, which we will refer to by the term individual level mechanisms, is related to general-domain cognitive abilities that explains the performance on number transcoding.

Structural task level mechanisms

Number word formation

Several studies demonstrated that the transparency of the number words system affects the performance on number transcoding tasks. Seron and Fayol (1994), for instance, investigated the impact of the irregularity of specific decades in the French number words system comparing French children to Belgian children. While some decades in the French spoken in France are irregular (e.g., 90 named as “quatre-vingt-dix”, corresponding to four times twenty plus ten), the same decades are more regular (e.g., 90 named as “nonante”, corresponding to ninety) in the French spoken in Belgium. The authors found that French children presented higher error rates in comparison to Belgian children. In addition, error patterns of French children suggested problems in the understanding of the syntax needed to transcode irregular decades. For example, the number 82 (i.e., “quatre-vingt-deux”, corresponding to four times twenty and two) was written as 4202 or 422, which are literal transcriptions of the respective number word.

Pixner and colleagues (2011) provided additional evidence of the impact of number word system transparency on number transcoding in a different language. Authors assessed number transcoding abilities of seven-year-old Czech-speaking children. In Czech, there are two different number word systems. Thus, two-digit numbers are spoken in a non-inverted order, in which tens are spoken before units (e.g., “dvadsetpat’’ corresponding to “twenty-five”) or in an inverted order, in which units are spoken before tens (e.g., “patdvadset’’ corresponding to “five-and-twenty”). Results demonstrated that when numbers were dictated using the inverted order, about half of all errors were inversion related (e.g., “patdvadset’’ written as 52 instead of 25). Compared to the inverted number word system, there were almost no inversion-related errors in the non-inverted.

In addition, Moeller and colleagues (2015) also investigated how the inversion property affects the performance on number transcoding. Authors compared the performance on a number transcoding task of seven-year-old children who speak German, a language with decade-unit inversion, and children who speak Japanese, a language with no decade-unit inversion. Results demonstrated that German-speaking children made more errors in general and more inversion errors, which were hardly frequent in Japanese-speaking children. Furthermore, error pattern analyses showed that German-speaking also made more syntactic errors unrelated to inversion (i.e., additive composition and multiplicative composition errors). This result suggested that irregularities in the number word system, such as the inversion property, may affect the understanding of place-value more broadly than it was initially proposed.

The Portuguese number word system also presents irregularities. Apart from German number word system, in which irregularities occur in the syntactic level, Portuguese number word system presents irregularities in the morphological level. The hundred number words constitute a specificity of the Portuguese number word system. In Portuguese, only the number word used for 100 is regular (i.e., “cem”), starting from 101 up to 199, a derived word is used (i.e., “cento”, e.g., 102 corresponds to "cento e dois"). From 200 up to 900, number words are formed by a specific radical derived from units names and the suffix "-zentos" (e.g., 200 corresponds to "duzentos") or “-centos” (e.g., 400 corresponds to "quatrocentos"). The number 500 is an exception as the morphology derives from Latin: "quinhentos" in Portuguese and "quingenti" in Latin. Consequently, the multiplicative principle in hundreds is not explicit in the Portuguese number words system (e.g., 200 is spoken as “duzentos” rather than “dois cem”).

Previous studies have investigated the impact of number word systems’ syntactic irregularities on number transcoding. As shown earlier, several studies investigated how the order of number words, whether it matches the Arabic digit notation or not, affects the performance on number transcoding (Clayton et al., 2020; Pixner et al., 2011; Moeller et al., 2015; Zuber et al., 2009). However, less is known about the impact of number words systems’ morphological irregularities on number transcoding. Therefore, investigating the morphological irregularities of Portuguese's hundreds number words in comparison to a transparent language is an opportunity to understand how the morphological irregularities affect the performance on number transcoding.

Syntactic complexity

According to the ADAPT model (Barrouillet et al., 2004), number transcoding, due to its procedural nature, is constrained by the limited resources available in working memory. Thus, the more procedures are involved in number transcoding, the more challenging it becomes. Barrouillet and colleagues (2004) proposed that the syntactic complexity, measured by the number of transcoding rules, would represent a parameter of difficulty in number transcoding. Previous studies have found a strong association between the number of transcoding rules and the performance on number transcoding (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2015). As presented before, the number of digits also affect performance on number transcoding. While young children are capable of transcoding small numbers containing two digits at the second grade, they need at least two more years to become capable of transcoding large numbers containing three and four digits (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2015; Power & Dal Martello, 1990; Seron et al., 1992). Although syntactic complexity and number of digits have been previously associated with number transcoding, to date, no study has simultaneously considered the role of both mechanisms to number transcoding in one comprehensive study.

Individual level mechanisms

Working memory

The ADAPT model explicitly recognizes the impact of working memory on number transcoding (Barrouillet et al., 2004). In the model, working memory plays a significant role during the transcoding process since it would be responsible for holding information from the parsing process, from long-term memory, and from the digital chain under construction. Previous studies have been consistently observed an association between working memory and number transcoding. However, evidence about which working memory modality, phonological or visuospatial, is more important for number transcoding is still controversial (see also Camos, 2008; Clayton et al, 2020; Moura et al., 2013; van der Ven et al., 2017; Zuber et al, 2009). For instance, Clayton and colleagues (2020) found that the central executive component of working memory (Baddeley, 2002; Baddeley & Hitch, 1974), as measured by backwards phonological and visuospatial span tasks, explained the overall number of errors in 6 to 7-year-old children from inverted (German) and non-inverted (English) number word systems.

Phonological processing

The input in number transcoding is verbal, requiring children to distinguish the speech sounds of number words in order to properly transcode them into the Arabic digit notation. The ADAPT model proposes that, in the first stage of transcoding, numbers are phonologically encoded and temporarily maintained in the phonological buffer. Thus, one can hypothesize that phonological skills would play a role in number transcoding.

Previous investigations have shown an association between mathematical skills and phonemic awareness (De Smedt & Boets, 2010; De Smedt, 2018; Hecht et al, 2001; Magalhães et al., 2020). The latter is important for perception and manipulation of phoneme sounds and is considered to be an index of strength of phonological representations (Wagner & Torgesen, 1987). Nevertheless, most studies have focused on the contribution of phonemic awareness more in other mathematical abilities than number transcoding, such as arithmetic fact retrieval.

To date, only few studies have investigated the specific contribution of phonemic awareness on number transcoding (Lopes-Silva et al., 2014, 2016). For instance, Lopes-

Silva and colleagues (2014) investigated how phonemic awareness, along with nonverbal reasoning, phonological and visuospatial working memory, and non-symbolic numerical processing, contribute to number transcoding. A hierarchical regression model showed that phonological working memory was the only significant predictor of number transcoding after the effects of age and nonverbal reasoning were controlled. However, when phonemic awareness was added to the model, phonological working memory was no longer significant. In addition, path analyses investigating mediation effects of all previous measures on number transcoding demonstrated that the model in which the effect of phonological working memory was partially mediated by phonemic awareness presented the best fit to the empirical data (see also, Lopes-Silva et al., 2016; Teixeira & Moura, 2020).

Lexical access speed

In addition to working memory and phonemic awareness, lexical access speed may also play a role in number transcoding. Often measured by tasks of rapid automatized naming (RAN), lexical access speed is associated with the retrieval of phonological information previously stored in long-term memory (Wagner & Torgesen, 1987). Similarity to phonemic awareness, lexical access speed has been previously associated with mathematical skills, more specifically arithmetic fact retrieval (De Smedt, 2018; Magalhães et al., 2020). One important assumption of the ADAPT model is that, after being phonologically encoded, numbers are decomposed in smaller units that match the Arabic digit forms stored in long-term memory. With development, larger units would be stored in long-term memory, allowing children to transcode numbers automatically via a lexical route (cf. the lexical route in dual-route model of single word reading; Coltheart et al., 2001). Thus, one can argue in favor of the involvement of lexical access speed on number transcoding.

Yet, the role of lexical access in number transcoding has received far less attention than the role of working memory and phonemic awareness. To the best of our knowledge, only one study investigated the contribution of lexical access on number transcoding. Teixeira and Moura (2020) assessed the number transcoding skills of 49 children with either typical achievement or reading disability, with age ranging from 7 to 12 years old. The authors found that lexical access speed, along with phonemic awareness, explained the ANW overall performance as well as the frequency of both lexical and syntactic errors. Although the study presented a first insight about the contribution of lexical access speed on number transcoding, it presented some limitations. The study assessed a small number of children and neither phonological nor visuospatial WM were included in the analyses.

In summary, one can argue that the three components of phonological processing (i.e., phonological working memory, phonemic awareness, and lexical access), in addition to visuospatial working memory, would play a role on number transcoding. However, to date, no study has simultaneously considered the role of the aforementioned cognitive mechanisms in one comprehensive study.

Difficulties with the learning of number transcoding abilities

As shown above, learning how to transcode numbers is a complex process that demands extensive practice and effort for children. Consequently, it takes three to four years for children to master the number transcoding of numbers with four digits (Moura et al., 2013, 2015). Additionally, children with mathematical difficulties were observed to be at risk of developing difficulties in transcoding numbers (Imbo et al., 2014; Moura et al., 2013, 2015). Comparing children with mathematical difficulties to those with typical achievement, Moura and colleagues (2013), found that younger children with mathematical difficulties (1st and 2nd grades) presented problems with mastering the syntactic principles, as well as with building up an appropriate number lexicon. In contrast, older children with mathematical difficulties (3rd and 4th grades) were able to overcome the initial syntactic and lexical problems, primarily presenting difficulties with the syntactic principles of more complex numbers.

Overview of the present dissertation

According to the guidelines of the "Programa de pós-graduação em Psicologia Cognição e Comportamento", from the Faculdade de Filosofia e Ciências Humanas of UFMG, this dissertation will be constituted by scientific papers. Therefore, the present dissertation has a literature review and three empirical papers.

In **CHAPTER 2**, we investigated the specific contribution of syntactic complexity, in terms of number of transcoding rules, and of number of digits, for the performance on number transcoding. We expected that both syntactic complexity and number digits would independently influence the performance on a transcoding task. We assessed, using an orthogonal design, the effects of number of digits (three and four-digit numbers) and number of transcoding rules (three, four and five rules) on number transcoding skills of second to fifth graders.

In **CHAPTER 3**, we compared how the specificities of Portuguese and German number word formation differentially affect number transcoding (i.e. the syntactic irregularity of unit-decade inversion in German and morphological irregularity of hundred words in Portuguese). We assessed the number transcoding skills of German- and Portuguese-speaking children in the early years of schooling. We performed mixed-ANOVAs in order to investigate the within-effect of errors in number types (i.e., two- and three-digit numbers) and of types of errors (e.g., additive composition and multiplicative composition inversion errors), as well as the between-effect of language (i.e., German and Portuguese).

In **CHAPTER 4**, we investigate the specific contribution of the three components of phonological processing (phonological working memory, phonemic awareness, and lexical access), in addition to visuospatial working memory, to number transcoding overall performance and error patterns. Following the assumptions of the ADAPT model, we expected that these cognitive mechanisms would explain the performance on number transcoding as they would be important during specific steps of number transcoding. We also expected to find a dissociation between the cognitive mechanisms investigated and the error patterns, as according to the ADAPT model, errors would occur due to difficulties in these specific steps of number transcoding. We performed regression models investigating the specific contribution of these cognitive mechanisms on number transcoding overall performance and error types.

Finally, in **CHAPTER 5**, we summarize and discuss the original contributions of the present dissertation, acknowledge the main limitations, and outline the next steps of future studies.

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CHAPTER 2

Beyond the number of digits: syntactic complexity and Arabic number writing in 2nd to 5th graders

Authors: Mariuche Rodrigues de Almeida Gomides[1], Fernanda Rocha de Freitas[1], Juliana Sbicigo[2], Giulia Moreira Paiva [3], Jerusa Fumagalli de Salles[2], Julia Bahmuller[4 5 6], Korbinian Moeller[4 5 6] , Ricardo Moura[7], Júlia Beatriz Lopes-Silva[1] & Vitor Geraldi Haase[1 2]

[1,]Graduate Program in Psychology; Cognition and Behavior, Universidade Federal de Minas Gerais; [2]Developmental and Personality Psychology Department, Universidade Federal do Rio Grande do Sul; 3Graduate Program in Neuroscience, Universidade Federal de Minas Gerais; [4]Centre for Mathematical Cognition, School of Science, Loughborough University; [5]Leibniz-Institut für Wissensmedien; [6]LEAD Graduate School & Research Network, University of Tübingen; [7]Institute of Psychology, Universidade de Brasília.

Abstract

Number transcoding, defined as writing the corresponding Arabic digit form of dictated verbal numbers, is an elementary numerical skill which demands formal instruction and effort in order to be mastered. Previous studies demonstrated that children need more time and effort to master the number transcoding of numbers with higher syntactic complexity and with more digits. In the current study, we investigated how the syntactic complexity, as measured by the number of transcoding rules, and the number of digits simultaneously affect the performance on number transcoding. In total, the number transcoding skills of 754 children aged from 6 to 12 years were assessed. Results demonstrated that effects of syntactic complexity and number of digits were significant for all assessed grades. No interaction between these two factors was found. In addition, the number of digits presented greater magnitude effects on number transcoding performance in comparison to syntactic complexity, but influence of both factors tended to decrease with development. In conclusion, the present findings have important implications for pedagogical practices and suggest the need to address these aspects more explicitly and systematically throughout the learning of number transcoding skills.

Keywords: number transcoding, Arabic number writing, mathematical learning difficulties, dyscalculia, number processing.

Introduction

The ability to convert different numerical notations (i.e., number words and symbolic Arabic digits) into one another is referred to as number transcoding. Despite being considered a basic numerical skill, learning how to transcode number words into their Arabic digit notation imposes considerable challenge to children during the first years of formal schooling. Mastery of number transcoding requires the acquisition of complex cognitive processes, such as the consolidation of a lexicon of number names and an understanding of the place-value structure of the Arabic number system. It has been previously shown that transcoding is impacted not only by the number of digits of the to-be-transcoded number, but also by the syntactic complexity, defined as the total number of algorithm-based conversion rules needed to procedurally transcode a number, of the respective number words (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2015). In the present study, we investigated how syntactic complexity impacts the performance on number transcoding, even when the effects of number of digits are accounted for.

The learning of number transcoding

Mastery of number transcoding relies upon the understanding of the correspondence between number words and Arabic digit numbers. This process is challenging for young learners, as multi-digit verbal number words and Arabic numbers are built obeying different syntaxes. The number word system is composed of a lexicon of words (e.g., *five, thirty, hundred*) organized in different classes (e.g., units, decades, hundreds). Number words representing multi-digit numbers are generated by combining specific entries of this lexicon using an additive (e.g., 42 is equal to forty plus two) and multiplicative (e.g., 300 is equal to three times hundred) syntactic composition principles. In contrast, the Arabic number system is composed of a small lexicon of symbols (digits from 0 to 9), which are combined complying with a place-value structuring principle, in which the value of a digit increases by powers of ten from the rightmost digit towards the left (e.g., 635 is equal to $6 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$, Deloche & Seron, 1982).

Recently, it has been proposed that understanding complex structural correspondences between number words and Arabic digit numbers represents a critical foundation for future arithmetic development in young children. For instance, Habermann and co-workers (2020) demonstrated that number transcoding, assessed in 4-year-old children, was the sole predictor of early arithmetic knowledge over and above domain-specific numerical (i.e., symbolic and nonsymbolic magnitude comparison and counting) and domain-general skills (i.e., nonverbal reasoning and language) two years later (see also Clayton et al, 2020; Gobel et al., 2014; Moeller et al., 2011). Thus, assessing number transcoding in children in the first years of schooling, which can be done quickly and with high reliability, should be a powerful predictor of children's later arithmetic skills.

In the following paragraphs, before we specify the goals of the present study, we will first introduce the ADAPT model (Barrouillet et al., 2004) from which we derived our predictions about the role of syntactic complexity on number transcoding. Then, we will briefly summarize previous studies that investigated the impact of syntactic complexity and number of digits on number transcoding.

The ADAPT model

The ADAPT model (A Developmental Asemantic and Procedural model for Transcoding; Barrouillet et al., 2004) was proposed to explain the transcoding of number words into Arabic digit notation. The ADAPT model distinguishes itself from semantic number transcoding models (e.g., McCloskey et al., 1985; Power & Dal Martello, 1990) by proposing that numbers can be transcoded via implementing a series of algorithms without mandatory access to the numbers' semantic representation of quantity.

The model is composed by a) a lexicon, which stores Arabic digit forms of numbers in long-term memory, b) a parsing mechanism, which segments numbers into smaller processing units, and c) a production system, which is responsible for the sequential processing of the number chain from the leftmost digit to the right. In addition to that, the production system activates a set of rules, which are responsible for building the digital chain. In particular, there are rules which retrieve the segmented units' Arabic digit form from long-term memory, namely P1 rules. Another set of rules, named P2 and P3, creates slots that match the number's place-value when the word hundred and thousand, respectively, are identified. Lastly, other rules, named P4, are responsible for filling any empty slot(s) with 0s and ending the process (Barrouillet et al., 2004; Camos, 2008).

Effect of number size and syntactic complexity on number transcoding

According to the ADAPT model (Barrouillet et al., 2004), number transcoding, due to its procedural nature, is constrained by the limited resources available in working memory. Thus, the more rules are involved in number transcoding, the more challenging it becomes. Barrouillet and colleagues (2004) proposed that the syntactic complexity, measured by the number of transcoding rules, would represent a parameter of difficulty in number transcoding. In addition to syntactic complexity, the number of digits would also explain the difficulty in number transcoding. There is a positive association between syntactic complexity and number of digits. However, there is no perfect association between syntactic complexity and number of digits. Therefore, numbers with the same number of digits may have different levels of syntactic complexity (e.g., three thousand vs. three thousand fifteen).

In favor of the impact of syntactic complexity on number transcoding, Barrouillet and colleagues (2004) observed that the number of transcoding rules was significantly associated with performance on number transcoding (*r*=.90). Camos (2008) replicated this finding and, in addition to that, demonstrated that children with lower working memory capacity were more susceptible to the effects of syntactic complexity on number transcoding than children with higher working memory capacity. This pattern indicates that the transcoding of more complex numbers relies on the availability of working memory resources, as assumed by the ADAPT model. Further evidence of the impact of syntactic complexity on number transcoding was provided by Moura and colleagues (2015), who observed that error rates increased with the number of transcoding rules even when the number of digits were controlled for.

Alternatively, another aspect that influences the performance of number transcoding is the number of digits. In a computation simulation mimicking the learning of number transcoding skills, Barrouillet and colleagues (2004) demonstrated that the fewer digits the numbers for transcoding contained, more rapidly and strongly the probability to use algorithm-based conversion rules decreased with practice. Thus, the probability to transcode two-digit numbers algorithmically reached less than one percent after twenty-five cycles of training. Comparatively, after fifty cycles of training, even though the probability of algorithm use decreased, numbers with three- and four-digits still presented a higher probability to be algorithmically transcoded. In turn, numbers with five- and six-digits were exclusively algorithmically transcoded.

The interplay between number of digits and practice has been empirically demonstrated. Previous studies investigating number transcoding in different languages, such as Italian, French, and Portuguese, found that children in second grade were highly accurate to transcode two-digit numbers, but presented problems when writing three- and four-digit numbers (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2015; Power & Dal Martello, 1990; Seron et al., 1992). Studies that investigated number transcoding abilities in Portuguese- and English-speaking children from third and fourth grades found that they were able to overcome the initial difficulties with the transcoding of three- and four-digit numbers, presenting a ceiling effect by fourth grade (Moura et al., 2015; Sullivan et al., 1996).

The present study

In the present study, we investigated the impact of syntactic complexity, as measured by the respective number of transcoding rules, and of the number digits, on number transcoding performance. Based on the ADAPT model assumptions, we were particularly interested in investigating if the number of transcoding rules influence the performance on number transcoding independently of the number of digits.

Methods

Sample

Seven hundred and fifty-four children participated in this cross-sectional study (52.7% girls). Participants were attending 2nd to 5th grade (Mean age = 8.87 years, SD=1.16 years, Age range= 6-12 years) and presented typical nonverbal reasoning abilities (as indicated by performance better than percentile 15 in Raven's Coloured Progressive Matrices test). The study was approved by the Ethics in Research Committee of the Universidade Federal de Minas Gerais (ETIC 42/08 and CAAE 15070013.1.0000.5149) and Universidade Federal do Rio Grande do Sul (Protocol number 1.023.371). All participants provided informed consent signed by their parents or primary caregivers. Additionally, oral assent was obtained from the children prior to testing.

Procedures

Children were tested in small groups of approximately five children in quiet separated rooms in their schools. The assessment lasted approximately 90 minutes. In the following, instruments used in the assessment will be described in more detail.

Instruments

Coloured progressive matrices of Raven (CPM-Raven): The CPM-Raven is a widely used task assessing nonverbal reasoning ability in children aged 6 to 11 years and 11 months. The task consists of 36 matrices or drawings with a missing part. Children are instructed to select the part that completes the figure appropriately from six possibilities. One point is given for each item answered correctly. The Brazilian validated version was used and Z-scores were calculated from the manual's norms (see Raven et al., 2018).

Arabic number writing task (ANW): The task requires children to convert verbally dictated numbers into their respective digital-Arabic notation. Children were instructed to write down the dictated number word in its corresponding digital-Arabic notation. There was no stop criterion applied and no time limit. One point was awarded for each correctly transcoded number word.

Syntactic complexity of each item was reflected by the number of transcoding rules required to transcode the respective number correctly as proposed by the ADAPT model (Barrouillet et al.,2004; Camos, 2008). The more transcoding rules are necessary to transcode a number, the more difficult an item is. The present task was developed by

Moura (2014) and comprises 81 items (split up into 2 one-digit numbers, 6 two-digit numbers, 19 three-digit numbers, and 54 four-digit numbers). There were 8 two-rule numbers, 11 three-rule numbers, 20 four-rule numbers, 24 five-rule numbers, 10 six-rule numbers, and 8 seven-rule numbers (see Table 1).

Analyses of the psychometric properties of the task (see Gomides et al., *in press*) indicated that the task presents a one factorial structure. The latent ability measured by this factor was labeled as “Number transcoding”. Factorial loadings were acceptable, varying from 0.47 to 0.98. Analyses based on the Item Response Theory (IRT) demonstrated that a two-parameter model was more adequate to the data. The estimates of the difficulty and discrimination of items indicated that items varied from very easy to average difficulty and from high to very high discrimination. The item-total correlations and reliability index were appropriate, providing evidence of the task’s adequacy for the assessment of number transcoding abilities in children from 2nd to 5th grades ($KR\text{-}20_{total}=0.98$; $KR\text{-}20_{2nd\ grade}=0.96$; $KR\text{-}20_{3rd\ grade}=0.98$; $KR\text{-}20_{4th\ grade}=0.96$; $KR\text{-}20_{5th\ grade}=0.95$).

Table 2.1. Items of the Arabic number writing task classified according to the number of transcoding rules (i.e., syntactic complexity) of the ADAPT model.

Items 1-41	**Number**	**Number of rules**	**Items 42-81**	**Number**	**Number of rules**
1	4	2	42	1114	4
2	7	2	43	1111	4
3	11	2	44	1140	4
4	13	2	45	1170	4
5	40	2	46	1135	5
6	80	2	47	1179	5
7	51	3	48	8000	3
8	68	3	49	9000	3
9	100	2	50	7003	5
10	109	4	51	2004	5
11	101	4	52	7013	5
12	112	3	53	2014	5
13	115	3	54	3070	5
14	150	3	55	8050	5
15	190	3	56	7063	6
16	174	4	57	8094	6
17	189	4	58	9800	4
18	200	3	59	5700	4
19	700	3	60	7105	6
20	902	5	61	2102	6
21	703	5	62	3112	5
22	215	4	63	5118	5
23	314	4	64	2140	5
24	450	4	65	5170	5
25	870	4	66	5147	6
26	643	5	67	9178	6
27	951	5	68	6400	5
28	1000	2	69	3900	5
29	1002	4	70	2609	7
30	1009	4	71	4701	7
31	1300	5	72	6713	6
32	1900	5	73	3815	6
33	1015	4	74	4870	6
34	1012	4	75	9740	6
35	1060	4	76	8844	7
36	1040	4	77	3791	7
37	1057	5	78	9745	7
38	1083	5	79	6289	7
39	1100	3	80	2785	7
40	1107	5	81	5748	7
41	1103	5			

Statistical Analysis

In the following paragraphs, we describe the analysis strategy in more detail. First, we performed descriptive analyses in order to the influence of grade on the overall number transcoding performance. Next, we performed more specific analyses in order to investigate the influence of syntactic complexity and number of digits on number transcoding for each grade. As shown in Table 2, the ANW task was not experimentally designed to control the effect of the number of transcoding rules and of digits simultaneously. Thus, there is no complete overlap among all levels of digits and rules. In order to overcome this limitation, we analyzed only the items in which the levels of the two variables are matched. Therefore, we analyzed three- and four-digit numbers that have three, four, and five rules. We performed repeated ANOVAs investigating the effects of number of digits (three- and four-digit numbers) and transcoding rules (three, four, and five rules) separated by grade. In the ANOVAs in which the sphericity assumption was violated, the original degrees of freedom together with the respective Greenhouse–Geisser coefficient (GGs) were reported.

Table 2.2. Number of items of the ANW task in each level of number of digits and of transcoding rules.

	2 rules	**3 rules**	**4 rules**	**5 rules**	**6 rules**	**7 rules**
1 digit	2					
2 digits	4	2				
3 digits	1	6	8	4		
4 digits	1	3	12	19	10	8

Results

The results are presented as follows: To begin with, we present descriptive analyses showing the impact of grade on number transcoding. Then, we present the results of the repeated ANOVAs investigating the influence of syntactic complexity and number of digits on number transcoding separated by grade.

Descriptive analyses

Seven hundred and fifty-four children responded to the ANW task (52.7% female, M_{age}=8.87 years, *SD*=1.16, Age range=6-12 years). The sample comprised 83 second-graders, 238 third-graders, 352 fourth graders, and 81 fifth graders. A one-way ANOVA showed differences in ANW task's performance across grades, $F(3, 750)=193.18$, $p<.001$, $\eta_p^2=.436$. Bonferroni corrected pairwise comparisons demonstrated that second graders (*M*=22.88, *SD*=19.23) presented a lower score than third graders (*M*=55.22, *SD*=24.80), fourth graders (*M*=73.72, *SD*=14.88), and fifth graders (*M*=76.69, *SD*=10.87). Also, third graders presented a lower score than fourth- and fifth graders. However, there was no difference between the scores of fourth- and fifth graders.

Impact of number of digits and transcoding rules on number transcoding

The results of the repeated ANOVAs investigating the impact of number of digits and rules on number transcoding demonstrated that, in second grade (see Figure 2.1A), the main effect of number of digits was significant, $F(1, 82)=61.53$, $p<.001$, $\eta_p^2=.43$. Bonferroni pairwise comparisons demonstrated that second graders presented higher scores in three-digit numbers in comparison to four-digit numbers. In addition, the main effect of the number of transcoding rules was also significant, $F(2, 164)=11.21$, $p<.001$, $\eta_p^2=.12$, GG=.82. Bonferroni pairwise comparisons indicated that second graders presented higher scores in three-rule numbers in comparison to four- and five-rule numbers. However, there was no difference in the scores of four- and five-rule numbers. There was no interaction between the number of digits and transcoding, $F(2, 164)=2.37$, $p=.11$, $\eta_p^2=.03$, GG=.84.

In third grade (see Figure 2.1B), the main effect of the number of digits was significant, $F(1, 237)=151.63$, $p<.001$, $\eta_p^2=.39$. Bonferroni pairwise comparisons demonstrated that third graders presented higher scores in three-digit numbers in comparison to four-digit numbers. Also, a significant main effect of the number of transcoding rules was found, $F(2, 474)=18.85$, $p<.001$, $\eta_p^2=.07$, GG=.87. Bonferroni pairwise comparisons indicated that scores decreased as the syntactic complexity increased: higher scores were observed in three-rule numbers in comparison to four- and five-rule numbers. Moreover, higher scores were observed in four-rule numbers in comparison to five-rule numbers. There was no interaction between the number of digits and transcoding rules, $F(2, 474)=1.28$, $p=.28$, $\eta_p^2=.01$, GG=.89.

In fourth grade (see Figure 2.1C), a main effect of the number of digits was observed, $F(1, 351)=46.49$, $p<.001$, $\eta_p^2=.12$. As demonstrated by Bonferroni pairwise comparisons, fourth graders presented higher scores in three-digit numbers in comparison to four-digit numbers. The main effect of the number of transcoding rules was also significant, $F(2, 702)=33.63$, $p<.001$, $\eta_p^2=.09$, GG=.80. Bonferroni pairwise comparisons indicated that fourth graders presented higher scores in three-rule numbers than four- and five-rule numbers. However, children presented a comparable performance in four- and five-rule numbers. Additionally, the interaction between number of digits and transcoding rules was significant, $F(2, 702)=6.14$, $p<.01$, $\eta_p^2=.02$, GG=.84.

Lastly, in fifth grade (see Figure 2.1D), there was a significant main effect of the number of digits, $F(1, 80)=8.54$, $p<.01$, $\eta_p^2=.09$. Fifth graders presented higher scores in three-digit numbers in comparison to four-digit numbers, as demonstrated by Bonferroni pairwise comparisons. There was also a significant main effect of the number of transcoding rules in fifth grade, $F(2, 160)=5.48$, $p<.01$, $\eta_p^2=.06$, GG=.73. Bonferroni pairwise comparisons indicated no significant differences between performance on three-and four-rule numbers, as well as between four- and five-rule numbers. However, the performance difference between three-rule numbers and in five-rule numbers was significant with an advantage for three-rule numbers. There was no interaction between the number of digits and transcoding rules, $F(2, 160)=0.14$, $p=.81$, $\eta_p^2=.01$, GG=.77.

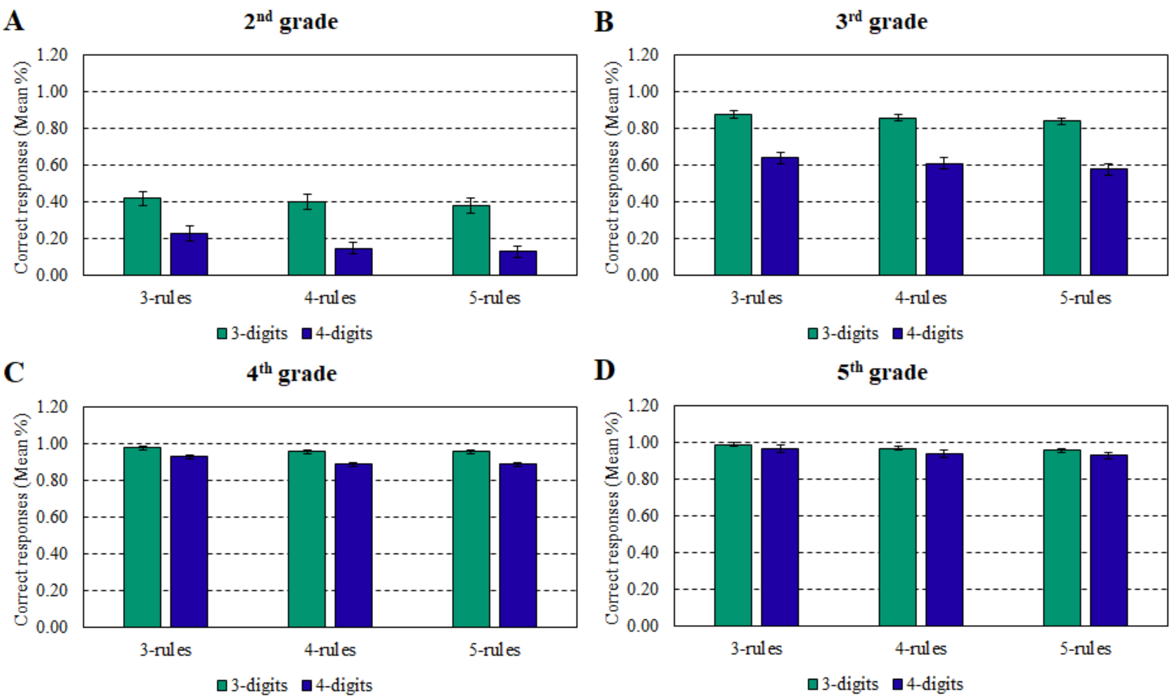

Figure 2.1. Mean percentage of correct responses as a function of number of transcoding rules and of digits for each grade. Error bars indicate standard errors.

Discussion

The present study aimed to investigate the influence of syntactic complexity, in terms of number of transcoding rules, as well as of number of digits on the performance on number transcoding. In particular, we investigated whether both factors independently influence the performance on number transcoding, even though they are correlated. We assessed, using an orthogonal design, the effects of number of digits (three and four-digit numbers) and number of transcoding rules (three, four and five rules) on number transcoding skills of second to fifth graders.

Results demonstrated that children were more accurate in transcoding numbers with three-digits than numbers with four-digits. In addition, children were more accurate in transcoding lower syntactically complex numbers, with less transcoding rules, than higher syntactically complex numbers. There was no interaction between these two factors, except for the fourth grade, suggesting that their influence on number transcoding is relatively independent. We also observed that the impact of both factors tends to weaken with development, as indicated by the decrease of the magnitude effect with grade.

In the following paragraphs, we interpret the influence of the number of transcoding rules and of the number of digits on number transcoding based on the assumptions of the ADAPT model (Barrouillet et al., 2004).

In line with the ADAPT model, the results of the present study indicated that the influence of syntactic complexity is relatively independent of the number of digits. The ADAPT model postulates that numbers are transcoded through the interplay of retrieving information from long term memory and of applying algorithm-based conversion rules. In particular, these algorithm-based conversion rules, referred to as transcoding rules, should reflect the syntactic structure of the number system, in such a way that more syntactically complex numbers need more steps to be transcoded. The significant effect of number of transcoding rules found in the present study substantiated the assumptions of the ADAPT model that numbers are transcoded via algorithm-based conversion rules.

Previous evidence from school-age children indicated that the performance on number transcoding is influenced by the structure of numbers, even when other linguistic and structural aspects were accounted for (van der Ven et al., 2017). As such, three-digit numbers composed by X00 and XX0 were significantly easier to transcode than numbers with three different non-zero digits (i.e., XXX). In contrast, three-digit numbers with a zero in the middle were the most difficult to be transcoded (i.e., X0X; see also Barrouillet et al., 2004; Power & Dal Martello, 1990; Zuber et al., 2009). This finding was replicated in studies with adults with brain injury, which found that the transcoding of numbers with an internal zero, which stands for a syntactic zero (e.g., 103), is more error prone than numbers with a lexical zero (e.g., 130; see Granà et al, 2003).

The digit zero plays an important role in the syntax organization of the Arabic number system since it is used as a place-holder, indicating that there is no value associated with the power of base ten. However, the syntactic zero is not explicitly named in the number word format (e.g., the number 305 is spoken as “three hundred and five” and not “three hundred and zero and five”), which might potentially affect the understanding of the syntactic zero. In the ADAPT model, numbers with a syntactic zero require the application of an additional type of rules, named P4, which are responsible for filling empty slots with the digit zero. Thus, the syntactic zero requires extra cognitive resources in order to be correctly transcoded in comparison to numbers with a lexical zero.

In the ADAPT model, developmental change would be explained by the expansion of the number lexicon and the abandonment of more primitive rules. With experience, rules for processing the positional value of the simplest and most frequent Arabic digit numbers, in this case two-digit numbers, would become obsolete as they would be processed as units and, consequently, be transcoded by direct retrieval. However, given that larger numbers are less frequent, the use of direct retrieval would be restricted for special cases (e.g., the current year, year of birth, certain important dates in history, etc.).

In the present study, we observed that children became highly precise at transcoding numbers with four-digits containing three to five transcoding rules by the fifth grade (i.e., overall accuracy above 90%). Even though the effects of number of digits and of transcoding rules decreased with development, they remained significant by fifth grade. Previous studies have observed similar developmental trends. Children from different languages were highly precise at transcoding two-digit numbers already in second grade and became able to transcode three- and four-digit numbers with high accuracy by fourth grade (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2015; Power & Dal Martello, 1990; Seron et al., 1992; Sullivan et al., 1996). In line with the present findings, Moura and colleagues (2015) have also observed that the effect of the number of transcoding rules decreased with development in children from first to fourth grade. Altogether, this evidence indicate that children become very efficient in transcoding numbers.

However, children with mathematical disability and with low working memory capacity are at risk to present difficulties in the mastering of number transcoding skills (Camos, 2008; Moura et al 2015). As such, Moura and colleagues (2015) observed a significant interaction between syntactic complexity and mathematical ability from first up to fourth grade. Children with mathematical difficulties presented persistent difficulties even in less syntactically complex numbers up to third grade, however they tend to reach the performance of typical achieving children by fourth grade. In addition, Moura and colleagues (2013) observed that children with mathematical difficulties were more likely to present a more immature error pattern, suggesting that they faced difficulties with the acquisition of more complex transcoding rules.

Some limitations of the present study need to be mentioned. The task used in the present study was not experimentally designed to control for the effect of the number of transcoding rules and of digits simultaneously. In order to overcome this limitation, we analyzed only the items in which the levels of the two variables are matched, which correspond to items with three- and four-digit numbers that have three, four, and five rules. Hence, items with two-digit numbers, as well as items with two, six and seven rules, were not included in the analyses. Future studies should improve our experimental design by creating a task containing a larger set of items balanced with the experimental conditions. In addition, the effects of more syntactically complex numbers could be assessed by including numbers with five-digits.

The influence of the number of transcoding rules on number transcoding was independent of the influence of the number of digits, except for the fourth grade. A possible explanation of why these two factors interact in the specific grade goes beyond the scope of the present study. This interaction might be driven by non-controlled contextual effects, such as specific curricular contents addressed in this grade, or rather a cohort effect.

The findings of the present study have important pedagogical implications. In the Brazilian national common core curriculum (Base Nacional Comum Curricular - BNCC; Brazil, 2018), the number of digits is systematically introduced in number transcoding lessons throughout the years of elementary school, starting from three-digit numbers in second grade up to six-digit numbers in fifth grade. In the lessons, the syntactic principles of the number system are taught using number composition and decomposition activities (e.g., the number 23 is composed of 2 tens and 3 units), as well as number comparison and ordering activities with the help of manipulatives and number lines. It is expected that children master these abilities by fifth grade. Notably, in the BNCC, the impact of the number of digits on the acquisition of number transcoding skills is hierarchically and systematically addressed.

In contrast, the influence of syntactic complexity on number transcoding is addressed in the curriculum in a less hierarchical and systematic manner. For instance, the BNCC suggests that the role of the number zero in place-value organization should be taught in second grade. However, there is evidence that the transcoding of numbers involving a syntactic zero is particularly challenging (Barrouillet et al., 2004; Granà et al, 2003; Power & Dal Martello, 1990; Zuber et al., 2009; van der Ven et al., 2017). Therefore, we propose that children might benefit from a curriculum organization in which the influence of syntactic complexity on number transcoding is addressed more hierarchically and systematically. A similar approach has been successfully used to train the correspondence between graphemes and phonemes in the promotion of reading skills (Becker, 2019; Scliar-Cabral, 2013). Easier phonemes are introduced first than difficult ones. This approach might particularly benefit children with mathematical disability and low working memory capacity, who are more error prone at transcoding numbers with high syntactic complexity (Camos, 2008; Moura et al, 2015).

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CHAPTER 3

Writing Arabic numbers by Portuguese- and German-speaking children: morphological and syntactic inconsistencies of number words

Authors: Mariuche Rodrigues de Almeida Gomides[1], Júlia Beatriz Lopes-Silva[1], Ricardo Moura[2], Julia Bahnmuller[3 4 5], Korbinian Moeller[3 4 5] & Vitor Geraldi Haase[1]

[1,]Graduate Program in Psychology; Cognition and Behavior, Universidade Federal de Minas Gerais; [2]Institute of Psychology, Universidade de Brasília; [3]Centre for Mathematical Cognition, School of Science, Loughborough University; [4]Leibniz-Institut für Wissensmedien; [5]LEAD Graduate School & Research Network, University of Tübingen.

Abstract

Number transcoding in languages with a less transparent number word formation is more difficult, reflected by both generally poorer performance and specific error patterns. In the present study, we compared how specificities of Portuguese and German number word formations differentially affect number transcoding (i.e. the syntactic irregularity of unit-decade inversion in German and morphological irregularity of hundred words in Portuguese). We assessed the number transcoding skills of Portuguese- and German-speaking children at the beginning of elementary school. Results corroborated reliable interactions between language and specific error types due to syntactic irregularities. In particular, German-speaking children made more errors in general, but also more language-specific inversion errors. However, there were no clear and consistent interactions between language and specific error types driven by morphological irregularities. The only consistent result indicated that Portuguese-speaking children made more wrong frame errors in the additive composition errors. The present study

substantiates the impact of linguistics aspects on number transcoding performance and further suggests that syntactic irregularities, such as the inversion property on German number word formation, have a greater impact on children's transcoding performance than morphological irregularities, such as the hundreds number words on Portuguese.

Keywords: Arabic number writing, number transcoding, inversion errors, inversion property, number word system.

Introduction

Number transcoding is defined as the capacity to establish relationships among distinct numerical representations and, thus, convert them into one another (Deloche & Seron, 1982). The ability to read and write multi-digit Arabic numbers is important in the early mathematical curriculum (McLean & Rusconi, 2014). In order to transcode multi-digit numbers successfully, children have to learn the correspondence between number words and the respective Arabic digit notation. However, learning this can impose considerable challenges for young children. Here, potential sources of difficulty may stem from syntactic and/or morphological irregularities regarding the way multi-digit number words are formed (i.e., the syntactic irregularity of unit-decade inversion in German and morphological irregularity of hundred words in Portuguese) which results in a lack of transparency between number word formation and the Arabic digit notation.

Mostly focusing on syntactic irregularities, previous findings demonstrated that number transcoding in languages with a less transparent number word formation is more difficult, reflected by both poorer overall performance but also by language-specific error patterns (e.g., Clayton et al., 2020; Imbo et al., 2014; Moeller et al., 2015; Pixner et al., 2011; Seron & Fayol, 1994). Going beyond the sole consideration of syntactic irregularities, the current study set out to investigate how both syntactic and morphological specificities of number word formations differentially affect transcoding performance in German- and Portuguese-speaking children.

In the following, we will first summarize evidence on how language specific multi-digit number word formations affect number transcoding and briefly outline how such irregularities might be considered in a prominent model of number transcoding (i.e., the ADAPT model; A Developmental Asemantic and Procedural model for Transcoding; Barrouillet et al., 2004). Subsequently, we will outline syntactic and morphological specificities in the German and Portuguese number word formation and elaborate how these specificities may affect number transcoding differentially (i.e. unit-decade inversion in German and morphological irregularity of hundreds words in Portuguese).

How number word formation affects number transcoding

To master the correspondence of number words and Arabic digit notation, children need to learn the rules of how multi-digit numbers are formed and map these number words successfully to the Arabic digit notation. In the Arabic notation, multi-digit numbers are generated by combining a small set of symbols (digits from 0 to 9) following the place-value structuring principle. The place-value structuring principle entails that the value of each digit in a digit string is determined by multiplying its face value with the respective base ten power. The base ten power is determined by the position of the digit in the digit string and increases from the rightmost digit to the leftmost digit (e.g., 635 reflects $6 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$, see McCloskey et al., 1985). In contrast, multi-digit number words usually involve a larger set of number words (e.g., *five, fourth, hundred*) that are organized in different classes (e.g., units, decades, hundreds, etc.) whereby classes may be indicated explicitly (hundred) or via a suffix (e.g., -teen, -ty). Number words are then formed by combining entries of the set of number words usually following additive (e.g., “forty-two” reflecting forty plus two) and multiplicative (e.g., "three hundred" reflecting three times one hundred) composition rules.

While the Arabic digit notation is used consistently in most Western countries, syntactic and morphological characteristics of multi-digit number words vary considerably across languages (Comrie, 2005). For instance, different from most Western languages, in east-Asian languages, such as Chinese and Japanese multi-digit number word formation is very transparent because it directly maps onto the Arabic digit notation (e.g., the number word for13 literally translates to "ten-three" and the number word for 20 translates to "two-ten"). Amongst other factors such as teaching and learning practices, the highly transparent number word formation in east-Asian languages may be one additional factor explaining better numerical and arithmetic achievement of East-Asian children in international comparison studies (Miura & Okamoto, 2003; Miura et al., 1994; Miura et al., 1988) because it allows for a more straightforward mapping of number words on the place-value structure of the Arabic number system.

However, different language-specific number word formations are not always as transparent as it is the case in east-Asian languages. Critically, many studies highlighted the negative impact of specific syntactic irregularities in number word formation on number transcoding performance and learning. One prominent example of such a syntactic irregularity is the number word inversion. In some languages, such as German, the order in which tens and units are named in the multi-digit number word is inverted with respect to the Arabic digit notation (e.g., in German, the number for 48 is "achtundvierzig" [literally eight and forty] and not "vierzigundacht" [literally forty-eight]). Previous studies suggested that the inversion property may represent an additional challenge for children when transcoding numbers from the number word to the Arabic digit notation. For instance, Zuber and co-workers (2009) demonstrated that about 50%

of all transcoding errors committed by first grade German-speaking children were inversion related (see also Moeller et al., 2015). However, compared to Japanese-speaking children, German-speaking children also committed more syntactic errors unrelated to inversion (i.e., “dreihundertvierundfünfzig” [literally three hundred four and fifty] written as “310054” or “30054”). This finding may suggest that the inversion property affects learning the syntactic principles of the number word system more broadly.

The impact intransparent number words on number transcoding have also been demonstrated in languages with irregularities other than number word inversion. For instance, Seron and Fayol (1994) showed that irregular decade words in France-spoken French (e.g., the number word for 90 is “quatre-vingt-dix” which literally translates to four twenty ten, meaning four times twenty plus ten) were more error prone in comparison to regular decade words in Belgium-spoken French (e.g., the number word for 90 is “nonante”, corresponding to ninety). Additionally, transcoding errors committed by French-speaking children mirrored the structure of the France-spoken number words. For example, the number 82 (i.e., “quatre-vingt-deux” which literally translates to “four twenty-two”, meaning four times twenty plus 2) was written as 4202 or 422, which are literal transcriptions of the respective number word.

In summary, previous cross-linguistic studies highlighted those irregularities in the number word formation negatively impact multi-digit number transcoding performance and learning, affecting both overall performance and specific error patterns (Clayton et al., 2020; Imbo et al., 2014; Moeller et al., 2015; Pixner et al., 2011; Seron & Fayol, 1994).

The processes of number transcoding: the ADAPT model

The ADAPT model (Barrouillet et al., 2004) was proposed to explain how number words are transcoded into Arabic digit notation. The main assumption of the ADAPT model is that numbers are transcoded via algorithm-based conversion rules, therefore, without mandatory access to the numbers' semantic representation of quantity. The difficulty of the process is determined by the number of transcoding rules needed to transcode a number. The more transcoding rules are needed, the higher are the demands on working memory and, consequently, the harder is the transcoding processing.

The model proposes that, in the first step of number transcoding processes, the verbal input is phonologically encoded and temporarily stored in a phonological buffer. Then, the number is segmented in smaller processing units and sequentially processed from the leftmost digit to the right. During the process, a set of rules responsible for the implementation of different steps are activated. P1 rules are activated when the smaller processing units temporarily stored in working memory match the content of long-term memory. As a result, the Arabic digit form is retrieved from long-term memory. P2 and P3 rules are activated when the words *hundred* and *thousand*, respectively, are identified in the number chain. The activation of P2 and P3 rules culminates in the creation of empty slots matching the corresponding place-value of three- and four-digit numbers (e.g., "three hundred" = 3_ _). Finally, P4 rules fill empty slot(s) with zero(s) and indicate the end of the transcoding process (Barrouillet et al., 2004; Camos, 2008).

The ADAPT model was originally developed based on French number words, however, the model can be easily adapted to account for the syntactic specificities of number word formations in other languages. Moreover, the model assumes that more transparent number word systems, such as Japanese and Chinese, would require the application of simpler rules, explaining why Japanese- and Chinese-speaking children perform better than Western children at the beginning of learning (Miura et al., 1993, 1994).

How number word formation in Portuguese and German may influence transcoding differentially

As in most Western languages, number word formation in German and Portuguese comprises a lexicon of number words organized in classes (e.g., units, teens, decades, hundreds, etc). In German, specific names are used for units and teens from 10 to 12 (i.e., "zehn", "elf", "zwölf"). The number words for the remaining teens (i.e., numbers 13 to 19) are expressed by a radical similar to unit names and the suffix “-zehn” (e.g., 14 is “vierzehn”). Likewise, the number words for the decades are expressed by a radical similar to unit names and the suffix “-zig” (e.g., 40 is “vierzig”). Both teens from 13 to 19 and two-digit numbers above twenty are inverted in German (except for round decades, such as 40), with the number word corresponding to the unit digit being named first followed by the number word of decade digit (e.g., 42 is named as two and forty). In contrast, hundred names are regular and are expressed by a radical similar to unit names and the suffix “-hundert”, which means hundred (e.g., 400 is “vierhundert”).

In Portuguese, specific number words are used for units and teens from 10 to 15 (e.g., "onze", "doze" etc.). In contrast, number words for teens from 16 to 19 are regular, thus, tens and unit digits are named in the same order as displayed in named Arabic digit notation (e.g., “dezoito” is equivalent to ten-eight). In turn, decade *and* hundred names are irregular. Decade names are expressed by a radical similar to unit names and the suffix “-enta” (e.g., 40 is “quarenta”). The only exception is the number 20 (i.e., “vinte”), which has neither connection with the radical two nor with the common suffix.

In hundreds, only the number word used for 100 is regular (i.e., “cem”). From 101 to 199, a derived word is used (i.e., “cento”, e.g., 102 corresponds to "cento e dois").

Similar to decade names, the remaining hundred names are expressed by a specific radical derived from units names and the suffix "-zentos" (e.g., 200 corresponds to "duzentos") or “-centos” (e.g., 400 corresponds to "quatrocentos"). The number 500 is an exception as the morphology derives from Latin: "quinhentos" in Portuguese and "quingenti" in Latin. The fact that - different from German - the multiplicative composition in for hundreds is not explicit in Portuguese (e.g., 200 is spoken as “duzentos” rather than “dois cem”) may make transcoding harder for Portuguese-speaking children learning the Arabic system.

The present study

As stated above, number word formation in German and Portuguese presents specific syntactic and morphological irregularities. While German-speaking children face difficulties to master two-digit number words due to the inversion property, mastery of hundreds number words may be specifically complicated for Portuguese-speaking children due to their morphological specificities. In the present study, we investigated whether and if so, how these irregularities of the respective number word formation affected performance on number transcoding differentially.

As overall error rates may be influenced by different factors such as general cognitive abilities, socioeconomic status, and quality of education, we focused on error rates reflecting specific difficulties in transcoding two-digit numbers and hundreds due to irregularities in number word formation, respectively. In particular, we expected that German-speaking children should present relatively higher error rates in numbers affected by inversion property, while Portuguese-speaking children should present relatively higher error rates in numbers involving hundreds.

In particular, we expected that error patterns should mirror the structure of number word formation in the respective language. Thus, inversion errors should be infrequent or even absent in Portuguese-speaking children because there is no inversion property in Portuguese number word formation. In contrast, inversion-related errors should be more frequent in German-speaking children. Finally, the irregularities of both languages' number word systems should reflect difficulties in understanding numbers' place-value, resulting in more additive and multiplicative errors. We were particularly interested in investigating if irregular hundred number words of Portuguese are associated with more syntactic errors.

In addition, we conducted more specific analyses investigating the number of intrusions of zero in additive and multiplicative errors. Previous studies demonstrated that children with transcoding difficulties made more additive and multiplicative errors in which the number of added zeros did not match the magnitude of the multiplicands (e.g., "three hundred and sixty-four" written as 3064 or 3164 instead of 30064 or 310064; see Camos, 2008; Moura et al., 2013). It has been suggested that these errors, namely wrong frame errors, should be the result of a delay in the acquisition of more complex transcoding rules (Camos, 2008; Moura et al., 2013). Thus, we investigated if the irregularity of hundreds in the Portuguese number word system would be associated with problems with the application of the ADAPT model transcoding rules (i.e., P2 rules) by examining the frequency of wrong frame errors.

Methods

Participants

One hundred and forty-eight Portuguese-speaking children (52.7% girls; mean age of 6 years and 6 months, *SD*=0.56 years, range= 6-8 years) from seven schools (i.e., one private and six public) in Belo Horizonte (Brazil) participated in the study. Children were enrolled in first and second grades (42.6% first graders). Data from 19 children with nonverbal reasoning skills, assessed with the Raven's Coloured Progressive Matrices

(Raven et al., 2018), more than one standard deviation below the average were excluded from analyses. First graders were assessed at the end of the first school year and second graders were assessed at the beginning of the second school year.

In addition, data of 130 German-speaking children (48.46% girls; mean age of 7 years and 4 months, *SD*=0.71 years, range= 6-8 years) previously reported by Zuber and colleagues (2009) were re-analyzed for the analysis. Children attended first grade in five Austrian elementary schools. Data from 2 children with nonverbal reasoning skills, measured by the Culture Fair Test 1 (Catell, Weiss, & Osterland, 1997), more than one standard deviation below the average were excluded from the analyses.

Portuguese-speaking children (*M*=84.92 months, *SD*=5.62 months) were significantly younger than German-speaking children (*M*=87.99 months, *SD*=7.15 months), $t(240.68)=3.83$ $p<.001$, $d=0.48$.

The study was approved by the local research ethics committees. Participation occurred only after informed consent was obtained in written form from parents or surrogates, and verbal assent from children prior to testing.

Instruments

Arabic Number Transcoding Task: The number transcoding task comprised 64 items varying in digit length. There were 4 one-digit numbers, 20 two-digit numbers, and 40 three-digit numbers. The items varied in their number structure (i.e., round numbers, numbers with internal zeros, and numbers without internal zeros; see Table 1). No tie numbers (e.g., 55) were included in the item set. Items of different digit lengths were presented in pseudo-randomized order, oversampling smaller numbers in the first half of the list. The very same items were used in both the German-speaking as well as the Portuguese-speaking samples.

Table 3.1. Total number of items and examples according to each number structure.

Number of items	Number structure	Example
One- and two-digit number		
4	X	7
4	Teens	12
4	X0	30
12	XX	56
Three-digit numbers		
4	X00	300
12	XX0	320
12	X0X	206
12	XXX	861

Transcoding errors were categorized following the taxonomy suggested by Zuber et al. (2009) differentiating lexical and syntactic errors in a first step (according to Deloche & Seron, 1982). Lexical errors reflect that a lexical element is substituted by another, however, the syntactic structure of the number is preserved (e.g., “four hundred and sixty-seven” written down as 457). In contrast, in syntactic errors the syntactic structure of the respective number is incorrect whereas its lexical elements are preserved (e.g., “four hundred and sixty-seven” written as 40067). Numbers with both lexical and syntactic errors were classified as combined errors (see Zuber et al., 2009 for a more detailed description).

Moreover, lexical and syntactic errors were further divided into subcategories following the taxonomy proposed by Zuber and colleagues (2009; also see Moeller and colleagues, 2015). Lexical errors were subdivided into (a) lexical errors involving 0 (e.g., “ninety” written down as 91), (b) lexical errors not involving 0 (e.g.,“twenty four” written down as 25), (c) lexical class errors (e.g., ”ninety” written down as 19), and (d) other lexical errors, representing lexical errors that could not be classified into one of the subcategories above.

Syntactic errors were broken down into (a) additive composition errors (e.g., "one hundred and twenty three" written down as 10023), (b) multiplicative composition errors (e.g., "four hundred" written down as 4100), and (c) inversion errors, which includes both errors involving decade-unit inversion errors (e.g., "twenty four" written down as 42) and errors which reflected a wrongly applied inversion (e.g.,"four hundred" written down as 104). Errors that could not be classified into those categories were coded as (d) other syntactic errors, representing syntactic errors that could not be classified into one of the subcategories above.

Combined errors were subdivided into combinations of (a) lexical and syntactic errors (e.g., "four hundred and sixty seven" written down as 40057), (b) lexical and inversion errors (e.g., "four hundred and sixty seven" written down as 475), (c) lexical, syntactic and inversion errors (e.g., "four hundred and sixty seven" written down as 40056), (d) two syntactic errors (e.g., "four hundred and sixty seven" written down as 410067), (e) syntactic and inversion errors (e.g., "four hundred and sixty seven" written down as 40076), and (f) two syntactic errors and inversion errors (e.g., "four hundred and sixty seven" written down as 410076).

In addition to that, additive composition and multiplicative composition errors that the added zeros did not match the magnitude of the multiplicand, less or more zeros than expected, were classified as wrong frame errors (e.g., "three hundred and fifty-two" written as 3052 or 300052).

Procedures

The procedures were the same as described by Zuber and colleagues (2009). Brazilian children completed the transcoding task in individual sessions of about 60 minutes in quiet separate rooms within their schools.

Statistical analysis

Analyses were performed using the proportion of errors relatively to the overall error rate. Prior to the analyses, an arcsine transformation was applied to approximate normal distribution.

We performed mixed model ANOVAs in order to investigate potential differences on the frequency of errors between languages and the within-factors of number characteristics and error types. In the cases in which within-factor had only one level, *t* tests were used instead. In the ANOVAs in which the sphericity assumption was violated, the original degrees of freedom together with the respective Greenhouse–Geisser coefficient (GGs) were reported. Bonferroni-corrected pairwise comparisons were used to test for differences in levels of both between- and within-factors when the main effects were significant.

In the cases in which the within-factor had only two levels, the interactions were analyzed in depth using Bonferroni-corrected pairwise comparisons. When the within-factor had three levels, it was decomposed in pairwise combinations which were entered in three separate two-ways ANOVAs (level 1 vs. level 2, level 1 vs. level 3, and level 3 vs. level 2). Thus, the interaction effects between language and each pairwise combination were reported. In order to account for influences of multiple testing we reduced the alpha level accordingly (significant when $p < 0.05/3 = 0.017$).

As shown in Figure 1, the analyses procedures were divided in three steps. In step 1, we conducted analyses of performance based on items' characteristics. First, we investigated the number of errors in two- and three-digit numbers. Then, we examined the number of errors in the number structures of two-digit numbers (i.e., Teens, X0 and XX numbers) and three-digit numbers (i.e., X00, XX0, X0X, and XXX).

In step 2, we performed an error type analysis. First, we examined potential differences between language and error types classified as lexical, syntactic, and combined. Importantly, since we calculated the proportion of errors dividing each error type for the overall error rate, in this specific case, summing up lexical, syntactic, and combined errors results in 100% of error rates. Therefore, including the three error types in the same analyses violates the assumption of independence of observations. In order to allow applicability of ANOVA methods, these error types were decomposed in pairwise combinations (lexical vs. syntactic, lexical vs. combined, and syntactic vs. combined) and were entered in three separate ANOVAs. Next, we investigated potential differences between language and the subtypes of syntactic errors (i.e., additive composition, multiplicative composition and inversion errors).

In step 3, we combined the former two steps and performed an analysis of error types based on the items' characteristics. In particular, in two-digit numbers, we analyzed potential differences in the frequency of inversion errors and additive composition errors between languages. Multiplicative composition errors were not included in the analyses because they were rather infrequent in two-digit numbers. In three-digit numbers, we analyzed potential differences in the frequency of additive composition and multiplicative composition errors between languages. Furthermore, we compared the frequency of wrong frame errors present in additive composition and multiplicative composition errors between the language groups.

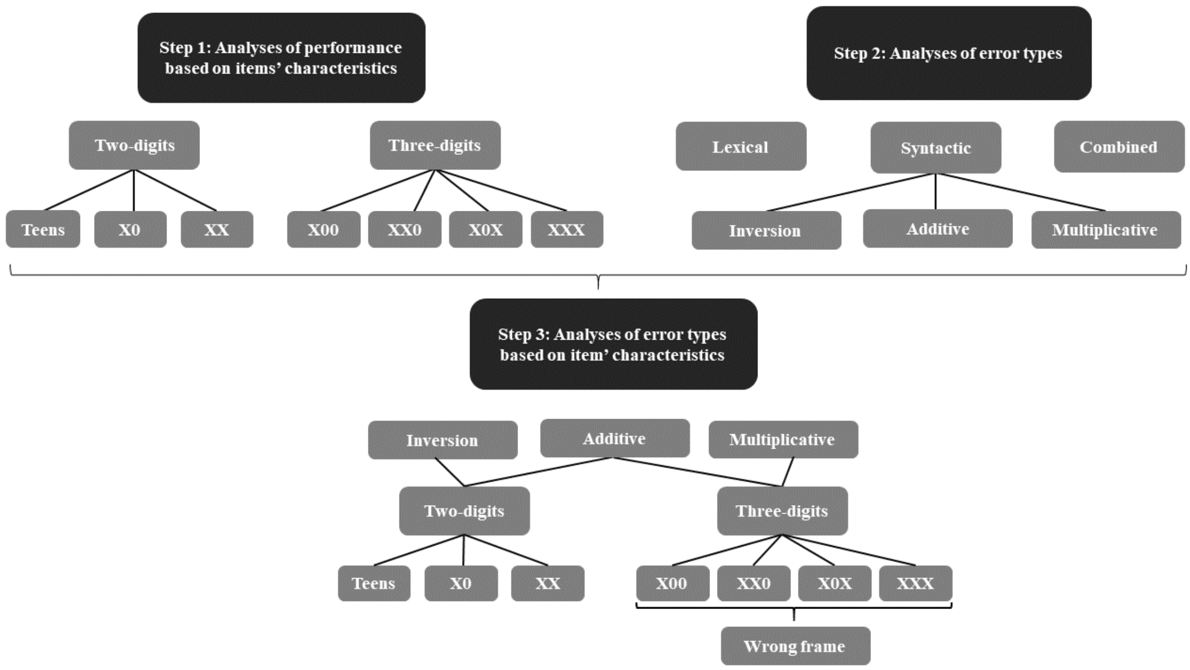

Figure 3.1. Schematic diagram of the analysis's procedures adopted in the present study.

Results

Results are presented in three sections reflecting the three steps described above.

Step 1 - Analyses of performance based on items' characteristics

Portuguese-speaking children produced a total of 3234 errors on the 64 items (39.17% of overall responses, *SD*=16.91) whereas German-speaking children produced a total of 3829 errors on the 64 items (46.74% of overall responses, *SD* = 10.49; see Zuber et al., 2009). Within the errors made by Portuguese-speaking children there were 9 non-responses. In contrast, there were no non-responses in the errors made by German-speaking children. Errors in one-digit numbers were rather infrequent, there were only three errors in this type of number which were made by German-speaking children. Thus, only errors in two- and three-digit numbers were analyzed in depth. The analyses of performance on two- and three-digit numbers and in their respective number structures are presented below.

A 2 x 2 mixed model ANOVA was performed on the frequency of errors with the within-subject factor number of digits (two- and three-digit numbers) as well as the between-subject factor language group (Portuguese and German, see Figure 3.2). Portuguese- and German-speaking children did not differ on the frequency of errors in general, as demonstrated by the non-significant main effect of language, $F(1, 255)=0.50$, $p>.05$, $\eta_p^2=.01$. However, a significant main effect of number of digits was found, $F(1, 255)=818.40$, $p<.001$, $\eta_p^2=.76$, indicating that the frequency of errors is not equally distributed in two- and three-digit numbers. Bonferroni pairwise comparisons demonstrated that children made significantly more errors in three-digit numbers than in two-digit numbers. Moreover, the interaction between language and number of digits was significant, $F(1, 255)=17.53$, $p<.001$, $\eta_p^2=.06$. Bonferroni pairwise comparisons decomposing this interaction demonstrated that Portuguese-speaking children made more errors in three-digit numbers whereas German-speaking children made more errors in two-digit numbers.

Next, we performed a 2 x 3 mixed model ANOVA on the frequency of errors in two-digit numbers with the within-subject factor number structures (Teens, XX and X0) as well as the between-subject factor language group (Portuguese and German, see Figure 3.2A). A significant main effect of number structures was found, $F(2, 510)=122.28$, $p<.001$, $\eta_p^2=.32$, GG=.77, indicating that errors were not equally distributed in two-digit numbers' structures. Bonferroni pairwise comparisons demonstrated that while errors were less frequent in teens in comparison to X0 and XX numbers, there is no difference between the latter two number structures. In addition, the main effect of language was also significant, $F(1, 255)=18.16$, $p<.001$, $\eta_p^2=.06$. Bonferroni pairwise comparisons indicated that German-speaking children made more errors in two-digit numbers in general than Portuguese-speaking children.

Furthermore, the interaction between language and number structures was significant, $F(2, 510)=33.85$, $p<.001$, $\eta_p^2=.12$. We performed three additional two-ways ANOVAs decomposing this interaction with the factors language and number structures. In the latter factor, pairwise combinations of error categories were entered in three separate two-way ANOVAs (Teens vs. X0, Teens vs. XX, and X0 vs. XX). The ANOVAs revealed reliable interaction between language and number structures for Teens vs. XX, $F(1, 255)=43.27$, $p<.001$, $\eta_p^2=.16$, as well as X0 vs. XX, $F(1, 255)=35.27$, $p<.001$, $\eta_p^2=.12$, but not for Teens vs. X0, $F(1, 255)=1.52$, $p=.22$, $\eta_p^2=.01$.

Afterwards, we performed a 2 x 4 mixed model ANOVA on the frequency of errors in three-digit numbers with the within-subject factor number structures (X00, XX0, X0X, and XXX) as well as the between-subject factor language group (Portuguese and German, see Figure 3.2B). A significant main effect of number structures was found, $F(3, 765)=126.64$, $p<.001$, $\eta_p^2=.33$, GG=.33, indicating that errors were not equally distributed in three-digit numbers' structures. Bonferroni pairwise comparisons demonstrated that errors were significantly less frequent in X00 numbers than in the other three number structures. While the frequency of errors was comparable in XX0 and X0X numbers, the errors in both number structures were less frequent in comparison to XXX numbers. However, neither the main effect of language, $F(1, 255)=3.29$, $p>.05$, $\eta_p^2=.01$, nor the interaction between language and number structures were significant, $F(3, 765)=2.16$, $p>.05$, $\eta_p^2=.01$.

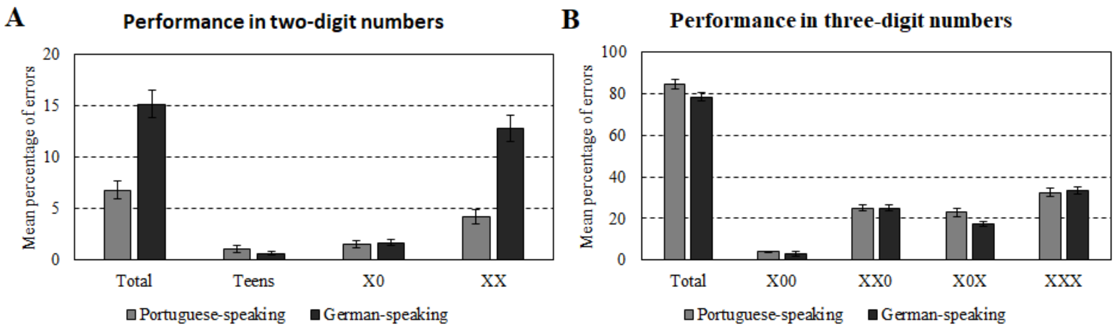

Figure 3.2. Mean percentage of errors for Portuguese- and German-speaking children in two- and three-digits numbers and their respective number structures. Error bars represent SEM.

Step 2 - Analyses of types of errors (lexical, syntactic, and combined) and subtypes of syntactic errors (additive composition, multiplicative composition, and inversion errors)

In this section we investigated potential differences between language groups and types of errors (lexical, syntactic, and combined) and subtypes of syntactic errors (additive composition, multiplicative composition, and inversion errors). To begin with, we performed three 2 x 2 mixed model ANOVAs investigating the impact of the error types and language group on the frequency of errors. In the first 2 (Error type: lexical and syntactic errors) x 2 (Language group: Portuguese and German) mixed model ANOVA a significant main effect of error type was found, $F(1, 232)=68.33$, $p<.001$, $\eta_p^2=.23$ (see Figure 3.3A). Bonferroni pairwise comparisons indicated that lexical errors were significantly less frequent than syntactic errors. A significant main effect of language was also found, $F(1, 232)=5.77$, $p<.05$, $\eta_p^2=.02$. Bonferroni pairwise comparisons indicated that Portuguese-speaking children made more errors than German-speaking children. Furthermore, the interaction between language group and error type was significant, $F(1, 232)=16.91$, $p>.001$, $\eta_p^2=.07$. Bonferroni pairwise comparisons decomposing this interaction indicated that Portuguese-speaking children made more lexical errors whereas German-speaking made more syntactic errors.

In the second 2 (Error type: lexical and combined errors) x 2 (Language group: Portuguese and German) mixed model ANOVA a significant main effect of error type was found, $F(1, 232)=9.67$, $p<.01$, $\eta_p^2=.04$ (see Figure 3.3A). Bonferroni pairwise comparisons indicated that lexical errors were significantly less frequent than combined errors. A significant main effect of language was also found, $F(1, 232)=8.09$, $p<.01$, $\eta_p^2=.03$. Bonferroni pairwise comparisons indicated that Portuguese-speaking children made more errors than German-speaking children. Furthermore, the interaction between language group and error type was significant, $F(1, 232)=17.91$, $p<.01$, $\eta_p^2=.07$. Bonferroni pairwise comparisons decomposing this interaction indicated that Portuguese-speaking children made more lexical errors whereas German-speaking made more combined errors.

At last, in the third 2 (Error type: syntactic and combined errors) x 2 (Language group: Portuguese and German) mixed model ANOVA a significant main effect of error type was found, $F(1, 232)=55.09$, $p<.001$, $\eta_p^2=.19$ (see Figure 3.3A). Bonferroni pairwise comparisons indicated that syntactic errors were significantly more frequent than combined errors. A significant main effect of language was also found, $F(1, 232)=18.48$, $p<.001$, $\eta_p^2=.07$. Bonferroni pairwise comparisons indicated that German-speaking children made more errors than Portuguese-speaking children. Furthermore, the interaction between language group and error type was not significant, $F(1, 232)=0.70$, $p>.05$, $\eta_p^2=.01$.

Afterwards, we first investigated whether the frequency of errors differed between language and subcategories of syntactic error types. The 2 (Language group: Portuguese and German) x 3 (Subtypes of syntactic errors: additive composition, multiplicative composition, and inversion errors) mixed model ANOVA revealed a significant main effect of error type, $F(2, 464)=107.14$, $p<.001$, $\eta_p^2=.32$, GG=.70, indicating that the frequency of syntactic errors subtypes were not equally distributed (see Figure 3.3B). Bonferroni pairwise comparisons demonstrated that additive composition errors were more frequent than multiplicative composition and inversion errors. In addition, inversion errors were more frequent than multiplicative composition. The main effect of language was also significant, $F(1, 232)=23.42$, $p<.001$, $\eta_p^2=.09$. Bonferroni pairwise comparisons indicated that German-speaking children made more syntactic errors in general than Portuguese-speaking children.

Furthermore, a significant interaction between language and error types was found, $F(2, 464)=27.20$, $p<.001$, $\eta_p^2=.11$, GG=.70. To break this interaction down and identify potential differential language differences for specific error types, we conducted three additional two-way ANOVAs with the factors language and error categories. In the latter factor, pairwise combinations of error categories were entered in separate ANOVAs (additive composition vs. multiplicative composition, additive composition vs. inversion, and multiplicative composition vs. inversion). To account for influences of multiple testing we reduced the alpha level accordingly (significant when $p < 0.05/3 = 0.017$). The ANOVAs revealed significant interactions of language group and error types for additive composition vs. inversion errors, $F(1, 232)=28.84$, $p<.001$, $\eta_p^2=.11$, and, multiplicative composition vs. inversion errors, $F(1, 232)=72.82$, $p<.001$, $\eta_p^2=.24$, but not for additive composition vs. multiplicative composition errors, $F(1, 232)=1.29$, $p>.05$, $\eta_p^2=.01$.

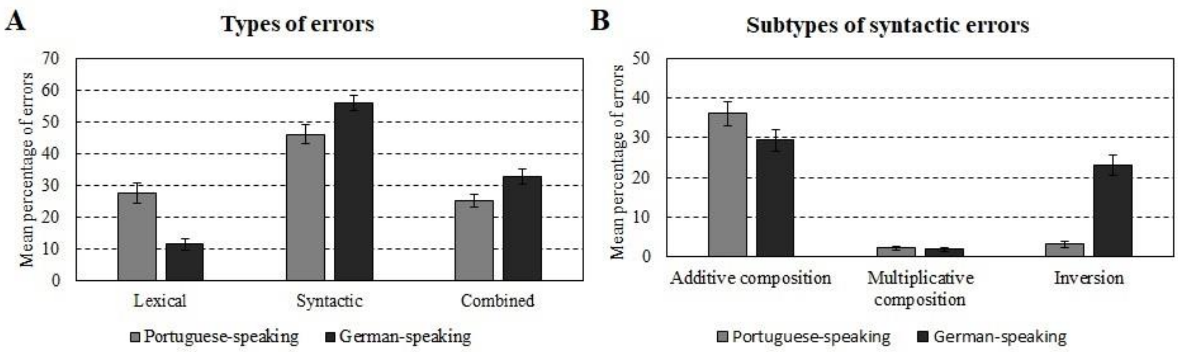

Figure 3.3. Mean percentage of errors for Portuguese- and German-speaking children in the error types (Lexical, Syntactic and Combined) and subtypes of syntactic errors (additive composition, multiplicative composition, and inversion errors). Error bars represent SEM.

Step 3 - Analyses of error types based on item' characteristics

In this section, we analyzed potential differences in the frequency of additive composition, multiplicative composition and inversion errors present in two- and three-digit numbers and in their respective number structures. In two-digit numbers, the 2 (Language group: Portuguese and German) x 2 (additive composition and inversion errors) mixed model ANOVA revealed a significant main effect of language, $F(1, 232)=52.75$, $p<.001$, $\eta_p^2=.19$. Bonferroni pairwise comparisons indicated that German-speaking made more errors than Portuguese-speaking children. The main effect of error type was also significant, $F(1, 232)=58.14$, $p<.001$, $\eta_p^2=.20$. Bonferroni pairwise comparisons indicated inversion errors were more frequent than additive composition errors. Furthermore, the interaction between language and error type was significant, $F(1, 232)=157.17$, $p<.001$, $\eta_p^2=.40$. Bonferroni pairwise comparisons decomposing this interaction demonstrated that while German-speaking made more inversion errors, Portuguese-speaking children made more additive composition errors.

We performed further analyses investigating language differences in the error types of two-digit numbers' structures. We observed that additive composition and inversion errors were rather infrequent in Teens and X0 numbers. Thus, only the error types present in the XX numbers were analyzed. In XX numbers, the 2 (Language group: Portuguese and German) x 2 (additive composition and inversion errors) mixed model ANOVA revealed a significant main effect of language, $F(1, 232)=45.65$, $p<.001$, $\eta_p^2=.16$. Bonferroni pairwise comparisons indicated that German-speaking made more errors in this number structure than Portuguese-speaking children. The main effect of error type was also significant, $F(1, 232)=44.16$, $p<.001$, $\eta_p^2=.16$. Bonferroni pairwise comparisons indicated that inversion errors were more frequent than additive composition errors. Furthermore, the interaction between language and error type was significant, $F(1, 232)=142.62$, $p<.001$, $\eta_p^2=.38$. Bonferroni pairwise comparisons decomposing this interaction demonstrated that while German-speaking made more inversion errors, Portuguese-speaking children made more additive composition errors.

In three-digit numbers, the 2 (Language group: Portuguese and German) x 2 (additive composition and multiplicative errors) mixed model ANOVA revealed a significant main effect of error type, $F(1, 232)=209.87$, $p<.001$, $\eta_p^2=.48$. Bonferroni pairwise comparisons demonstrated that additive composition errors were more frequent than multiplicative composition errors. However, neither the main effect of language, $F(1, 232)=1.65$, $p>.05$, $\eta_p^2=.01$, nor the interaction between error types and language were significant, $F(1, 232)=0.04$, $p>.05$, $\eta_p^2=.01$.

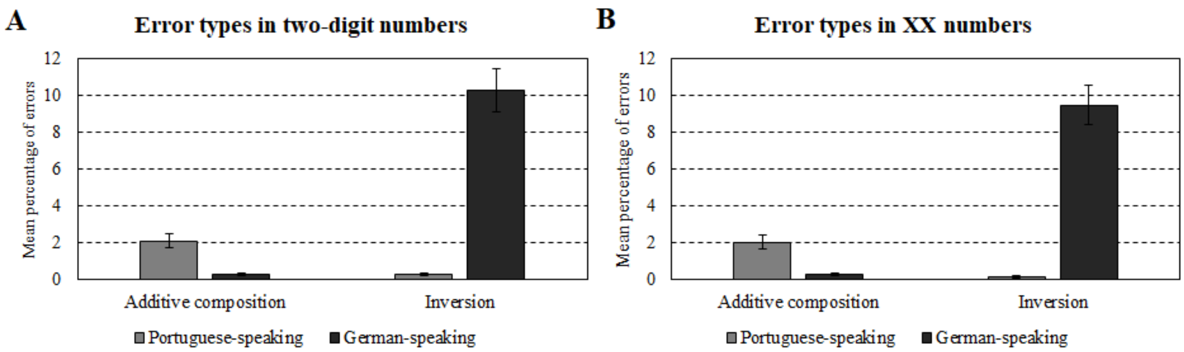

Figure 3.4. Mean percentage of errors for Portuguese- and German-speaking children in in the syntactic subtypes of errors made in two-digit numbers' structures. Error bars represent SEM.

We performed further analyses investigating language differences in the error types of number structures of three-digit numbers (i.e., X00, XX0, X0X, and XXX numbers). In X00, children made no additive composition errors. Thus, only multiplicative composition errors were analyzed. A t test indicated language differences in this type of error, $t(176.53)=3.28$, $p<.001$, $d=0.42$. Portuguese-speaking children made significantly more multiplicative composition errors in this number structure in comparison to German-speaking children.

In XX0, X0X, and XXX numbers, the ANOVAs evaluating language differences in the error types (additive composition and multiplicative composition errors) revealed a significant main effect of error type, $F(1, 232)=85.90$, $p<.001$, $\eta_p^2=.27$, $F(1, 232)=231.45$, $p<.001$, $\eta_p^2=.50$, and $F(1, 232)=231.45$, $p<.001$, $\eta_p^2=.50$, respectively. Bonferroni pairwise comparisons demonstrated that additive composition errors were more frequent than multiplicative composition errors. However, neither the main effect of language, $F(1, 232)=1.03$, $p>.05$, $\eta_p^2=.01$, $F(1, 232)=0.12$, $p>.05$, $\eta_p^2=.01$, and $F(1, 232)=2.30$, $p>.05$, $\eta_p^2=.01$, nor the interaction between error types and language were significant, $F(1, 232)=0.04$, $p>.05$, $\eta_p^2=.01$, $F(1, 232)=0.01$, $p>.05$, $\eta_p^2=.01$, and $F(1, 232)=0.31$, $p>.05$, $\eta_p^2=.01$.

At last, we investigated the frequency of wrong frame errors in additive composition and multiplicative composition errors between language groups. The 2 (Language group: Portuguese and German) x 2 mixed model (Wrong frame error type: additive composition and multiplicative composition) ANOVA revealed a significant main effect of error type, $F(1, 232)=85.06$, $p<.001$, $\eta_p^2=.27$. Bonferroni pairwise comparisons demonstrated that wrong frame errors in additive composition were more frequent than in multiplicative composition errors. The main effect of language was also significant, $F(1, 232)=22.83$, $p<.001$, $\eta_p^2=.09$. Bonferroni pairwise comparisons indicated that Portuguese-speaking children made more wrong frame errors in general than German-speaking children. Furthermore, the interaction between language and wrong frame error type was significant, $F(1, 232)=9.70$, $p<.01$, $\eta_p^2=.04$. Bonferroni pairwise comparisons decomposing this interaction demonstrated that while Portuguese-speaking children made more wrong frame errors in additive composition errors, there is no difference between language groups in wrong frame errors in multiplicative composition errors.

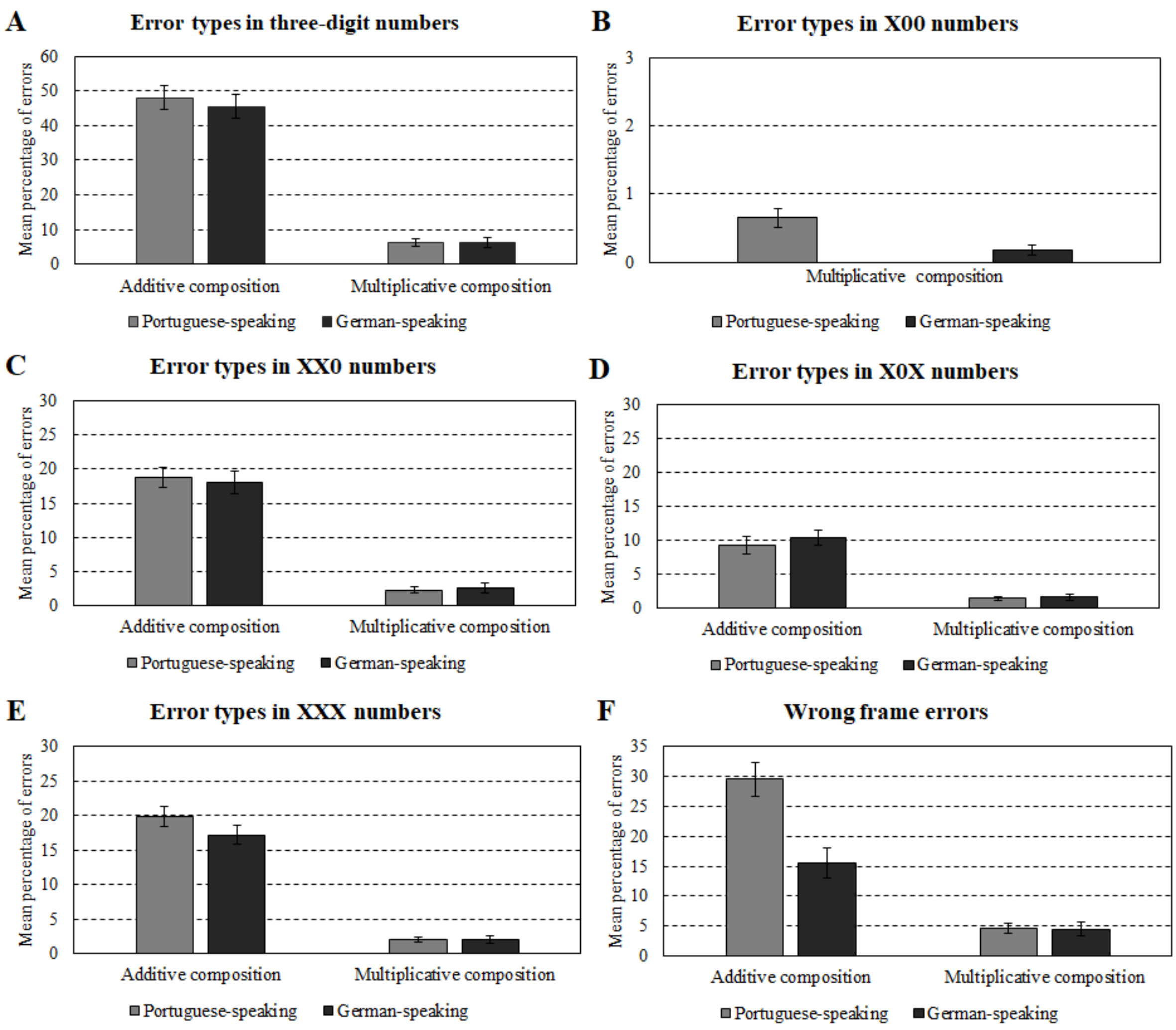

Figure 3.5. Mean percentage of errors for Portuguese- and German-speaking children in in the syntactic subtypes of errors made in three-digit numbers' structures. Error bars represent SEM.

In summary, the analyses of performance based on items' characteristics indicated a reliable interaction between language and number of digits. While German-speaking children made more errors in two-digit numbers, Portuguese-speaking children made more errors in three-digit numbers. However, when two- and three-digit numbers were entered in separate analyses, only the interaction between language and number structures in the two-digit numbers remained significant. Further analyses decomposing this interaction indicated that German-speaking children made errors in XX numbers.

The analyses of error types demonstrated reliable interactions between language and error types. In particular, further analyses decomposing these interactions indicated larger language differences for lexical vs. syntactic errors and lexical vs. combined errors, but not for syntactic vs. combined errors. We observed larger language differences for additive composition errors vs. inversion errors and multiplicative composition errors vs. inversion errors, but not for additive composition errors vs. multiplicative composition errors.

At last, the analyses of error types based on items' characteristics demonstrated reliable interactions between language and error types in two-digit numbers. Further analyses decomposing this interaction indicated that German-speaking children made more inversion errors, while Portuguese-speaking children made more additive composition errors. There were no reliable interactions between language and error types in three-digit numbers. However, there was a significant interaction between language and wrong frame errors present in additive composition and multiplicative composition errors. Portuguese-speaking children made more wrong frame errors in additive composition errors than German-speaking children. There was no difference between Portuguese-speaking and German-speaking children the frequency of wrong frame errors in multiplicative composition errors.

Discussion

In the present study, we investigated how the morphological and syntactic peculiarities of Portuguese and German number word systems affect the performance on number transcoding. We were particularly interested in investigating how the irregularity of decades and hundreds in German and Portuguese, respectively, affect the performance on number transcoding. We expected that both error rates and error types would reflect specific difficulties due to the irregularities of each number word system.

The results of the present study are partially in line with our expectations. Corroborating previous studies, we found that the inversion property represents an additional challenge for German-speaking children (Pixner et al., 2011; van der Ven et al., 2017; Zuber et al., 2009). We observed reliable interactions between language and frequency of errors in two-digit numbers. German-speaking children made more errors in two-digit numbers, mostly in XX numbers, which involve the application of the inversion property. Notably, in this type of number, inversion errors were reliably more frequent in German-speaking children in comparison to Portuguese-speaking, that in turn made more additive composition errors.

In contrast, we observed no reliable interactions between language and frequency of errors/types of errors in three-digit numbers. The irregularity of hundreds in Portuguese was neither associated with higher frequency of errors in general nor with higher frequency of syntactic errors (i.e., additive composition and multiplicative composition errors). However, we found that additive composition errors in Portuguese-speaking children, more frequently, did not match the magnitude of the multiplicand. Thus, Portuguese-speaking children added less or more zeros than expected (e.g., “three hundred and fifty-two” written as 3052 or 300052).

In the following paragraphs, we interpret the influence of linguistic specificities of both number word systems on number transcoding based on the assumptions of the ADAPT model.

The ADAPT model proposes that the transcoding of verbal numbers into their Arabic digit format occurs due to the application of algorithm-based conversion rules

(Barrouillet et al., 2004). The number is sequentially processed while a set of transcoding rules are responsible for the implementation of specific steps. Due to the procedural nature of number transcoding, the storage and manipulation of information recruit working memory resources. Therefore, the difficulty of the task is partially determined by its cost to working memory capacity. More complex numbers are more difficult given that they demand the implementation of more transcoding rules and, consequently, more working memory capacity.

The authors argue that the ADAPT model is a general number transcoding model, thus, it is relatively independent of linguistic specificity. Although they presented the application of the model to the French number word system, the ADAPT model can be easily adapted to other languages. In the case of Portuguese and German number word systems it would be necessary to delete the transcoding rules that process the complex decades in French (i.e., 70 [soixante-dix], 80 [quatre-vingts], and 90 [quatre-vingts-dix]). However, the German number word system would, in turn, need an additional rule responsible for the inversion property.

One can argue that number word systems with irregularities at the syntactic level, such as the one found in German and French number word systems, are more error prone because the application of additional rules would impose higher demands on working memory. In favor of this argument, several studies demonstrated that better performance on number transcoding is predicted by better performance on working memory tasks (see Barrouillet et al., 2014; Camos et al., 2008; Clayton et al., 2020; Moura et al., 2013). In particular, Zuber and colleagues (2009) demonstrated that central executive performance predicted inversion related errors but did not predict non-inversion related errors in 7-year-old German-speaking children (but see Imbo et al., 2014).

Alternatively, the application of an additional rule might delay the acquisition of more complex rules, hence, resulting in higher error rates. As such, Moeller and colleagues (2015) demonstrated that 7-year-old German-speaking children not only made more inversion errors, but also more syntactic errors non-related to the inversion property, such as additive composition errors, in comparison to Japanese-speaking children. In addition, the authors observed that some German-speaking children overgeneralized the inversion property for three-digit numbers, as they wrote the first dictated number on the last position as would be correct for two-digit numbers (e.g, "four hundred" written as 104). This finding substantiated the argument that the inversion property affects the learning of number transcoding more broadly than solely on two-digit numbers.

In contrast to the irregularities at the syntactic level, the ADAPT model does not discuss how irregularities at the morphological level, such as hundreds in Portuguese, would affect the implementation of transcoding rules. We hypothesized that, as an example of syntactic irregularities, morphological irregularities should harden the understanding of place-value at some level. In languages with regular number words, hundred number words should be more consistently reflected on the place-value structure of the Arabic system in comparison to hundreds in the Portuguese number word system. Based on the ADAPT model, we hypothesized that the irregularity of hundreds in Portuguese might affect the application of P2 rules, which are responsible for creating two slots when the word *hundred* is identified. Notwithstanding, we did not confirm this hypothesis since the results of the present study demonstrated that the specificities of Portuguese number word system did not differentially affect the performance of Portuguese-speaking children in terms of number of errors or types of errors. One can argue that different from morphological irregularities, syntactic irregularities would be more challenging as they would require the application of an additional rule. As we discussed, the application of an additional rule might be associated either to a working memory overload or a delay in the acquisition of more complex rules.

Even though the frequency of additive composition and multiplicative composition errors did not differ between Portuguese- and German-speaking children, an interesting pattern emerged when additive composition errors were analyzed in depth. In those errors, Portuguese-speaking children added less or more zeros than expected for the magnitude of the multiplicand when compared to German-speaking children. This type of error was referred to as wrong frame errors (see Moura et al., 2013). Wrong frames errors were previously observed in children presenting difficulties with number transcoding. Camos (2008) found that wrong frame errors (e.g., “two hundred and forty-two” written as 200042) were more frequent in children with low working memory capacity. In turn, in children with high working memory capacity, the added zeros matched the magnitude of the multiplicand (e.g., “two hundred and forty-two” written as 20042). The author interpreted the latter pattern to be a consequence of a cognitive overload due to a restricted capacity of working memory. In contrast, wrong frame errors were interpreted to arise from a delay in the acquisition of transcoding algorithms. In line with these findings, Moura and colleagues (2013) showed that children with mathematical difficulties presented more wrong frame errors than children with typical achievement. In line with this evidence, the higher frequency of wrong frame errors in Portuguese-speaking children might indicate that they use more primitive rules to transcode three-digit numbers.

Some considerations need to be addressed. We have not been able to perfectly match the ages of Brazilian and Austrian samples in the present study. In addition to that, previous studies have demonstrated that various contextual factors, such as parental support, curriculum differences, and socioeconomic status, might explain differences found in cross-linguistic comparison studies (Chen & Stevenson, 1989; Hess & Azuma, 1991; Stevenson et al., 1990; Stigler et al., 1987). Rather than comparing the overall performance, which would be more susceptible to the influence of this factors, in the present study, we investigate the error rates/error types reflecting specific difficulties in transcoding two- and three-digit numbers due to irregularities in number word formation. In fact, we observed that linguistic specificities affect the performance of German-speaking children, but in less extent the performance of Portuguese-speaking children.

To the best of our knowledge, this was the first study that investigated the impact of number words irregularities at the morphological level. Future studies should replicate our finding in other languages with morphological irregularities.

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CHAPTER 4

The impact of phonological processing on Arabic number writing

Authors: Luana Teixeira Batista[1]*, Mariuche Rodrigues de Almeida Gomides[2*], Gabriella Koltermann[3], Julia Bahnmueller[4 5 6], Korbinian Moeller[4 5 6], Jerusa Fumagalli de Salles[3], Vitor Geraldi Haase[1 2], Ricardo Moura[7], Júlia Beatriz Lopes-Silva[2]

[1]Graduate Program in Neuroscience, Universidade Federal de Minas Gerais; [2]Graduate Program in Psychology; Cognition and Behavior, Universidade Federal de Minas Gerais; [3]Institute of Psychology, Universidade Federal do Rio Grande do Sul; [4]Centre for Mathematical Cognition, School of Science, Loughborough University; [5]Leibniz-Institut für Wissensmedien; [6]LEAD Graduate School & Research Network, University of Tübingen; [7]Institute of Psychology, Universidade de Brasília.

*These authors contributed equally to this work.

Abstract

Number transcoding describes the ability to make conversions between verbal and Arabic digit numerical notations and was found to be a significant predictor of later math achievement. Therefore, it is important to better understand predictors of transcoding performance. Different aspects of working memory (including phonological working memory) have been repeatedly observed to drive transcoding performance in children. However, the relationship with others - specifically phonological processing abilities - remains less clear. As such, the present study investigated the predictive value of three abilities of phonological processing (i.e., phonological working memory, phonemic awareness, and lexical access speed) for children's transcoding performance. In particular, we were interested in potentially differential associations of phonological processing abilities with specific transcoding error types (i.e., lexical, syntactic, and combined errors). In total, 357 children aged 7 to 11 years were assessed. Results demonstrated that number transcoding performance was selectively predicted by phonological processing abilities, in particular phonemic awareness and lexical access speed. Furthermore, lexical errors were predicted best by phonemic awareness and lexical access speed, syntactic and combined errors were predicted by phonemic awareness. Taken together, these results corroborate and further specify the contribution of phonological processing abilities for number transcoding. We provided evidence of shared cognitive mechanisms in the learning of mathematical and spelling skills, which might explain the high comorbidity between dyslexia and dyscalculia.

Keywords: Arabic number writing, number transcoding, phonological processing, spelling.

Introduction

The ability to write Arabic digit numbers to dictation is one major building block in children's numerical development (McLean & Rusconi, 2014). In particular, it has been argued that mastering the correspondence between number words and the Arabic digit notation provides a pathway to understanding the Arabic number system (Habermann et al., 2020), which is considered to be an early predictor of later arithmetic performance (Habermann et al., 2020; Göbel et al., 2014, Moeller et al., 2011). Thus, it is crucial to identify and understand the cognitive mechanisms underlying successful Arabic number writing (ANW). While previous research already found a significant influence of both short-term and working memory (WM) on ANW (Camos, 2008; Moura et al., 2013; Zuber et al., 2009), less is known about influences of phonological abilities (see Lopes-

Silva et al., 2014; 2016). Addressing this research gap, the present study aims at systematically investigating the specific influences of different phonological processing abilities (phonological WM, phonemic awareness, and lexical access speed) on ANW performance within one comprehensive study.

In the following paragraphs, we will first introduce the ADAPT model of ANW (Barrouillet et al., 2004) from which we then derive our predictions. Second, we will briefly summarize previous studies that investigated the cognitive mechanisms underlying ANW performance before we specify the goals of the present study.

The ADAPT model

Different models were proposed to specify processes underlying number transcoding (Barrouillet et al. 2004; McCloskey, Caramazza & Basili, 1985; Power & Dal Martello, 1990). For the present study, we specifically considered the ADAPT model (A Developmental Asemantic and Procedural model for Transcoding; Barrouillet et al. 2004). The ADAPT model proposes that transcoding number words into the Arabic digit notation proceeds asemantically by implementing a set of rules. Thus, the model assumes that processing the magnitudes reflected by each digit of a given number, for instance, 362 (300, 60, 2) is not mandatory for transcoding it correctly.

According to the ADAPT model, in the first step of number transcoding, number words are phonologically encoded and temporarily maintained in a phonological buffer. In cases where a corresponding lexicalized form is available in long-term memory, the Arabic digit form is automatically retrieved (referred to as following P1 rules in the model). Otherwise, number transcoding occurs procedurally by the implementation of a set of other rules. In this case, numbers are decomposed into smaller units. Then, rules responsible for the creation of slots corresponding to the necessary place-value stacks (i.e., P2 and P3 rules for hundreds and thousands, respectively) are triggered by the identification of separators (i.e., the word hundred or thousand). After all slots are filled with the corresponding Arabic digits retrieved by P1 rules, a last set of rules (P4 rules) is responsible for filling any empty slot(s) with zero and ending the process.

Essentially, the ADAPT model proposes that numbers can be transcoded either by lexical retrieval or by implementing procedural rules. On the one hand, through practice and experience, phonological and Arabic forms of small and frequent numbers (e.g., two-digit numbers) but also familiar numbers (the current year, year of birth, certain important dates in history, etc.; see Moura et al., 2021 for a deeper discussion) should be lexicalized. These numbers are available in long term memory (LTM) and are successfully transcoded by direct memory retrieval (henceforth referred to as lexical route). On the other hand, larger and infrequent numbers are transcoded by the application of a set of rules (henceforth referred to as procedural route).

Moreover, according to the ADAPT model, difficulties with implementing both routes will result in different kinds of errors. In lexical errors, a lexical primitive is replaced by another one without affecting the length of the digit chain (e.g., writing "1952" instead of "1962" when dictated one thousand nine hundred sixty-two; Deloche & Seron, 1982). Those errors are assumed to occur due to problems while retrieving the Arabic forms, representing a difficulty along the lexical route (i.e, affecting P1 rules). In contrast, in syntactic errors, lexical primitives are retrieved correctly, but the composition of the digit chain is incorrect. For instance, additional zeros may be inserted into the digit chain (e.g., writing down "20018" instead of "218" when dictated two hundred and eighteen; Deloche & Seron, 1982). As such, syntactic errors reflect difficulties along the procedural transcoding route (i.e., P2, P3 and P4 rules).

Underlying cognitive mechanisms in ANW

The ADAPT model explicitly recognizes the relevance of different cognitive abilities, particularly working memory (WM), on number transcoding. It assumes that the intermediate storage and manipulation of information necessary in transcoding imposes considerable demands on both phonological and visuo-spatial WM (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2013; Lopes-Silva et al., 2014). On the one hand, phonological WM is assumed to be relevant for the representation and processing of phonological based information. On the other hand, visuo-spatial WM is assumed to be more specifically involved in managing place-value stacks (Zuber et al., 2009). However, previous studies have also shown results inconsistent with these assumptions.

Initially, Camos (2008) demonstrated that children with low phonological WM capacity were outperformed by children with high WM capacity in an ANW task, in particular when transcoding more complex numbers. Expanding these findings, Zuber and colleagues (2009) investigated the influence of the central executive, as measured by the combination of phonological and visuo-spatial backward span, on ANW performance of seven-year-old German-speaking children. The authors argued that demands on WM should be especially high for German-speaking children because of the inversion property of German number words (e.g., the number word for 24 is "vierundzwanzig" which translates to "four-and-twenty"). Results indicated that the central executive predicted overall transcoding performance and in particular the number of inversion-related errors.

Moura and colleagues (2013) found a different pattern when analyzing ANW performance in a sample of Portuguese-speaking Brazilian children from first to sixth grades (aged 7 from 12). Although both phonological and visuo-spatial WM were correlated with ANW performance only phonological WM was a significant predictor of overall transcoding performance in the regression models for the group of early elementary school children (1st and 2nd graders; 7 to 8-year-old). However, in the group of middle elementary, school children (3rd and 4th graders; 9 to 12-year-old) phonological and visuo-spatial WM did not significantly predict ANW anymore. Results from the studies by Zuber et al. (2009) and Moura et al. (2013) indicated that the influence of phonological and visuo-spatial WM components on ANW do not seem consistent in the literature. Inconsistencies in the contribution of WM modalities may be related to syntactic complexity of ANW tasks (low vs. high complex numbers), language specificities (inversion vs. non-inversion languages), and development (younger vs. older children).

In addition to the role of phonological WM, one may argue that other phonological processing abilities are involved in very early steps of ANW. In particular, both phonemic awareness and lexical access speed may contribute to ANW. Phonemic awareness is essential for the perception and manipulation of phoneme sounds and is considered to be an index of the strength of phonological representations (Simmons & Singleton, 2008). In line with this assumption, Lopes-Silva and colleagues (2014) demonstrated that phonemic awareness, assessed through a phoneme elision task, mediated the impact of phonological WM on ANW. In a further study, Lopes-Silva and colleagues (2016) observed that phonemic awareness significantly predicted ANW performance even when controlling for phonological and visuo-spatial WM.

Moreover, lexical access speed, usually assessed by tasks such as rapid automatized naming (RAN), is associated with the retrieval of previously stored phonological information (Wagner & Torgesen, 1987). One important assumption of the ADAPT model is that more frequent numbers (i.e., one- and two-digit numbers), would be transcoded primarily via the lexical route (Barrouillet et al., 2004). This seems to imply influences of lexical access speed on ANW. However, the role of lexical access speed in

ANW has received considerably less attention than influences of WM and phonemic awareness. More recently, Teixeira and Moura (2020) showed that lexical access speed, along with phonemic awareness, predicted ANW performance in general but also the frequency of both lexical and syntactic transcoding errors in particular in a small sample of typically developing and dyslexic children. Nonetheless, this study did not consider influences of phonological or visuo-spatial WM in the analyses.

The present study

Previous studies suggested that both visuo-spatial and phonological WM components are associated with ANW performance - even though results were inconsistent with respect to specific contributions (Barrouillet et al., 2004; Camos, 2008; Moura et al., 2013; Zuber et al., 2009). Importantly, however, contributions of other cognitive skills have received less research interest so far. Based on the ADAPT model, we hypothesized that cognitive skills associated with phonological encoding and retrieval, namely phonemic awareness and lexical access speed, should also be significant predictors of transcoding performance. Accordingly, the aim of the present study was to expand the findings of Lopes-Silva and colleagues (2016) by also considering lexical access speed as a predictor of ANW, controlling for the effects visuo-spatial and phonological WM and phonemic awareness. Thus, the present study is the first to comprehensively assess all three abilities of phonological processing (i.e., phonological WM, phonemic awareness, and lexical access speed), in addition to visuo-spatial WM, in one study.

Overall, we expected a result pattern similar to the one observed by Lopes-Silva and colleagues (2016). In particular, we expected to replicate the significant contribution of visuo-spatial working memory as well as phonological working memory and phonemic awareness to ANW performance. Additionally, because the ADAPT model predicts that numbers' lexical information is retrieved from long-term memory, we further expected lexical access speed to be a significant predictor of overall ANW performance.

The second aim of the current study was to evaluate whether the cognitive predictors considered in the present study (i.e., visuo-spatial and phonological WM, phonemic awareness, and lexical access speed) may be specifically predictive of lexical or syntactic aspects of transcoding as reflected by respective error types. In particular, syntactic errors reflecting problems within the procedural route of the ADAPT model should be predicted specifically by phonological and visuo-spatial WM. In contrast, because lexical errors reflect problems along the lexical route of the ADAPT model requiring lexical retrieval of phonological number representations, lexical errors should be predicted primarily by phonemic awareness and lexical access speed.

Methods

Participants

Four hundred and fifty-four 3rd and 4th graders from state-run schools in Belo Horizonte and Porto Alegre, Brazil, participated in the study. Data collection took place in schools. Data from 24 children had to be excluded from the analysis because they did not complete all tests. Additionally, data of 73 children were excluded because non-verbal reasoning scores were more than 1 SD below the mean using Raven's CPM test (Raven et al., 2018). The final sample consisted of 357 children (51% female) with an age range of 7 to 11 years (Mage=8.82 years, SD=0.77).

The study was approved by the local research ethics committees of the Universidade Federal de Minas Gerais and the Universidade Federal do Rio Grande do Sul, respectively (CAEE:15070013.1.0000.5149 and Protocol number 1.023.371). Children participated only after written informed consent was obtained from parents or surrogates. Additionally, oral assent was obtained from children prior to testing.

Instruments

Children completed an ANW task, as well as tests of non-verbal reasoning and word spelling which were administered in small groups of approximately five children. Subsequently, children were individually assessed for phonemic awareness, lexical access speed, and WM. Group assessment sessions and individual assessment sessions lasted approximately one hour each. In the following, instruments will be described in more detail.

Arabic Number Writing (ANW): To evaluate number transcoding, children were instructed to write down verbally dictated numbers in Arabic digit notation. The task comprised 81 items, with 1- to 4-digit numbers (i.e., 2 one-digit numbers, 6 two-digit numbers, 19 three-digit numbers, and 54 four-digit numbers). Items were controlled for syntactic complexity considering the number of transcoding rules required to transcode the respective number correctly as proposed by the ADAPT model (Barrouillet et al., 2004; Camos, 2008). There were 8 two-rule numbers, 11 three-rule numbers, 20 four-rule numbers, 24 five-rule numbers, 10 six-rule numbers, and 8 seven-rule numbers. The examiner dictated the numbers only once and children had to write it down on a sheet of paper. There was no stop criterion applied and no time limit. One point was awarded for each correct answer. The internal consistency was very high (Cronbach's α = .99; see Gomides et al., in press).

Word Spelling Task (Tarefa de escrita de palavras e pseudopalavras-TEPP; adapted from Rodrigues & Salles, 2013; Rodrigues et al., 2017): In the word spelling task, children had to write down a list of 24 words and 24 pseudowords that were dictated by an examiner. The examiner read aloud the word only once and children had to write it down on a sheet of paper. Words were chosen to reflect different levels of lexicality, word length, and word regularity, as well as effects of concreteness and grammatical class.

Pseudowords were created from words that exist in Brazilian Portuguese by exchanging or omitting letters and / or syllables. For example: “fopel” was derived from “papel” (i.e., paper), “veziona” is derived from “veneziana” (i.e., shutter). One point was awarded for each correctly written word. internal consistency of the test was very high (Cronbach’s α = .94; see Rodrigues et al., 2017).

Phoneme Elision Task (PET; Barbosa-Pereira et al., 2020): To measure phonemic awareness a phoneme elision task was used. Children hear 28 words and have to state the resulting word when a specific phoneme is omitted. All resulting words were real words from Brazilian Portuguese. For example: in Brazilian Portuguese "perua" without /u/ gives "pera", etc. Similar examples in English would be “farm” without /f/ giving “arm” and "cup" without /k/ giving "up". The length of the words ranged from two to three syllables. Eight words require the omission of a vowel and 20 words require the omission of a consonant. Also, omitted phonemes varied according to position within the words (i.e., initial: “filha” without /f/, middle: “atlas” without /l/ and, final: “cruz” without /z/). One point was awarded for each correct answer. Again, internal consistency was very high (Cronbach’s α = .91; see Barbosa-Pereira et al., 2020).

Rapid Automatized Naming (RAN; Van der Sluis et al., 2004): lexical access speed speed was assessed using a rapid automatized digit naming task. The digits one to four were randomly allocated in 5 rows and 8 columns, thus making 40 digits in total. The task starts with a training phase to ensure that children are able to correctly name the respective digits. Afterwards, children have to name all digits as fast and as accurately as possible. The time spent to complete the task was used as an independent variable in the analyses, with longer naming time indicating worse performance. The internal consistency was moderate in a sample of Brazilian children (Cronbach’s α = 0.76; see Lima et al., 2019).

Corsi Blocks (Kessels et al; 2000): The backward condition measures the visuo-spatial component of WM. In the backward condition, the experimenter taped a number of blocks on a board and the child was instructed to tap the blocks in reversed order. Sequence length increases from two to a maximum of nine blocks with two trials per sequence length. The task was interrupted when the child made two consecutive errors in the trials of the same sequence length. visuo-spatial WM span is determined by the longest sequence correctly repeated. The total score (correct trials x span) in backward order was used to reflect visuo-spatial WM capacity in the analyses (see Kessels et al., 2000). Internal consistency of the test was moderate (Cronbach's $\alpha = 0.69$; see Kessels et al., 2000).

Digit Span (Figueiredo, 2002; Figueiredo & Nascimento, 2007): The backward condition measures the phonological component of WM. The Digit span task was administered according to the Brazilian WISC-III subtest (see Figueiredo, 2002). The procedure is comparable to the Corsi blocks task. The experimenter named a sequence of digits and the child was instructed to repeat the sequence in the inverse order. Sequence length increases from two to a maximum of nine digits with two trials per sequence length. The task was interrupted when the child made two consecutive errors in the trials of the same sequence length. Backward digit span is determined by the longest sequence correctly repeated. The total score (i.e., correct trials x span) was used as an index of phonological WM in the analyses. The internal consistency was low (Cronbach's $\alpha = 0.50$; see Figueiredo & Nascimento, 2007).

Raven's Coloured Progressive Matrices (Raven's CPM; Raven et al., 2018): The Raven's CPM was used to assess non-verbal reasoning. The task consists of 36 matrices or drawings with a missing part. Children were instructed to choose the part that completes the figure appropriately from six possibilities. One point is given for each item answered correctly. The Brazilian validated version was used and the analyses were based on z-scores calculated from the manual's norms. Internal consistency of the test was high (Cronbach's α = .82; see Raven et al., 2018).

Statistical Analyses

Pearson correlations were used to explore the pattern of associations among variables of interest. Next, a hierarchical regression approach was chosen to investigate the predictive power of phonemic awareness and WM for both number transcoding and word spelling (cf. Lopes-Silva et al., 2016). For the regression model predicting number transcoding, non-verbal reasoning as measured by Raven's CPM and word spelling, as measured by TEPP, were included in the first block. Visuo-spatial WM as measured Corsi Blocks, phonological WM as measured Digit Span, phonemic awareness as measured by PET were included in the second block of the regression model. The regression model predicting word spelling was performed using the same approach, however, word spelling was replaced by number transcoding in the first block.

Afterwards, in a latter regression model, we investigated the impact of the lexical access speed in ANW. Lexical access speed, as measured by RAN, was included in the second block of predictors in this regression model together with visuo-spatial WM, phonological WM and phonemic awareness.

At last, we also aimed at investigating the influence of the components of phonological processing and visuo-spatial WM on different types of transcoding errors. Separate regression models were conducted for each error type (i.e., lexical, syntactic, and combined) including predictors in the same hierarchical manner using the stepwise method as described above. The absolute error rate (in percent) of each error type was used as the criterion in the regression models.

Results

Results are presented following the procedure steps presented above.

Correlation Analyses

Table 4.1 shows that performance in the ANW task was positively associated with non-verbal reasoning, word spelling, visuo-spatial WM, phonological WM, phonemic awareness, and lexical access speed was negatively associated with ANW. This indicated that better performance in the transcoding task was associated with better performance in all the other tasks. The correlation analyses with specific error types revealed that Raven's CPM, phonological WM, visuo-spatial WM, and phonemic awareness were negatively associated with all error types, indicating that better performance on these tests was associated with fewer errors of all error types. lexical access speed was positively correlated with lexical and combined errors, but not with syntactic errors. Moreover, there was no significant association between lexical and syntactic errors. Overall, correlations were small to moderate in size.

Table 4.1. Correlations between neuropsychological measures for all participants.

	2	3	4	5	6	7	8	9	10
Non-verbal reasoning (1)	.22**	.37**	.23**	.31**	-.09	.19**	-.15**	-.19**	-.13*
ANW (2)	1	.32**	.27**	.38**	-.33**	.49**	-†	-†	-†
Visuo-spatial WM (3)		1	.23**	.28**	-.16**	.26**	-.19**	-.20**	-.22**
Phonological WM (4)			1	.32**	-.23**	.31**	-.19**	-.16**	-.14*
Phonemic awareness (5)				1	-.27**	.57**	-.32**	-.26**	-.30**
Lexical access speed (6)					1	-.40**	.19**	-.06	.16**
Word spelling (7)						1	-.42**	-.31**	-.44**
Lexical errors (8)							1	-.09	.23**
Syntactic errors (9)								1	.58**
Combined errors (10)									1

Note: *ANW = Arabic Number Writing; WM = Working Memory.*
†*Correlations between error types and the overall performance in the task were omitted to avoid redundancy.*
**Correlation is significant at the .05 level (2-tailed); **Correlation is significant at the .01 level (2-tailed).*

Association between Arabic number writing and phonemic awareness

In an attempt to replicate results of the study by Lopes-Silva and colleagues (2016), the first regression model used ANW as the criterion variable and included non-verbal reasoning and word spelling in the first block as well as visuo-spatial WM, phonological WM and phonemic awareness in the second block. The final model explained about 32% of ANW variance, R^2=.33, R^2adjusted=.33, $F(1, 319)=3.88$, p=.05, with beta weights indicating that better performance on word spelling, visuo-spatial WM and phonemic awareness predicting better transcoding performance.

Next, we performed a regression model considering word spelling as the criterion variable. The model included non-verbal reasoning and ANW in the first block and visuo-spatial WM, phonological WM, and phonemic awareness in the second block. The model accounted for 46% of the variance, R^2=.47, R^2adjusted=.46, $F(1, 316)=4.65$, p=.03. Inspection of beta weights indicated that better performance on ANW, phonemic awareness, and phonological WM significantly predicted better performance on word spelling (see Table 4.2).

Table 4.2. Regression models showing the predictive power of WM and phonemic awareness on ANW (left chart) and word spelling (right chart).

Predictor	ANW ($R^2_{adj.}$=.34)			Word spelling ($R^2_{adj.}$=.45).		
	Beta	t	ΔR^2	Beta	t	ΔR^2
Intercept		5.59***			5.30***	
Non-verbal reasoning	.03	0.51		-.03	-0.79	
ANW	-	-	.30	.32	7.07***	.29
Word spelling	.42	7.35***		-	-	
Visuo-spatial WM	.17	3.54***	.03	.02	0.48	Excluded
Phonological WM	.07	1.48	Excluded	.10	2.16	.01
Phonemic awareness	.11	1.97*	.01	.46	9.67***	.18

Note: *p<.05; ** p<.01; **p<.001.

Incremental contribution of lexical access speed on Arabic number writing

Expanding the results presented above, the regression model included non-verbal reasoning, and word spelling in the first block, and visuo-spatial WM, phonological WM, phonemic awareness, and lexical access speed in the second block. The final model explained 34% of variance in the ANW, R^2=.35, R^2adjusted=.34, F(1, 319)=10.15, p<.001. Inspection of beta weights indicated that better performance on word spelling, visuo-spatial WM and lexical access speed predicted better transcoding performance (see Table 4.3). Importantly, the inclusion of lexical access speed in the model led to the exclusion of phonemic awareness.

Table 4.3. Regression models showing the predictive power of WM, phonemic awareness, and lexical access speed on ANW (left chart) and word spelling (right chart).

Predictor	**ANW ($R^2_{adj.}$=.34)**			**Word spelling ($R^2_{adj.}$=.45).**		
	Beta	**t**	**ΔR^2**	**Beta**	**t**	**ΔR^2**
Intercept		6.51***			-0.03	
Non-verbal reasoning	.05	1.07		.01	0.30	
ANW	-	-	.30	.25	5.31***	.26
Word spelling	.40	7.55***		-	-	
Visuo-spatial WM	.18	3.59***	.03	.05	0.97	Excluded
Phonological WM	.07	1.38	Excluded	.07	1.65	Excluded
Phonemic awareness	.11	1.87	Excluded	.43	9.13***	.17
Lexical access speed	-.16	-3.19**	.02	-.20	-4.38***	.03

Note: *p<.05;** p<.01; **p<.001.

When lexical access speed was added to the regression model, the influence of phonemic awareness was no longer significant. Rather than being an effect of collinearity between these two predictors (because they presented a weak correlation; VIF of phonological WM=1.15; VIF of phonemic awareness=1.64), this pattern of results seems to suggest that the influence of phonemic awareness on transcoding performance might be mediated by lexical access speed. To test this hypothesis directly, we conducted a mediation analysis using the PROCESS macro (Hayes, 2007), which is an extension of SPSS software (Statistics, 2013).

As shown in Figure 4.1, the mediation effect of lexical access speed on the contribution of phonemic awareness on ANW was significant, β=.07, BCa CI 95% [.03, .14]. However, the contribution of phonemic awareness on ANW remained significant after controlling for the effect of lexical access speed, b=.35, CI 95% [.25, .45], *p*<.001.

We estimated the percentage of variance mediated by lexical speed on the impact of phonemic awareness on ANW by dividing the standardized coefficients of the direct effect (i.e., c') by the total effect (i.e., C). Then, the resultant value was subtracted by one. The calculated estimative showed that lexical access speed mediated 12% of the variance of the relationship between phonemic awareness and ANW.

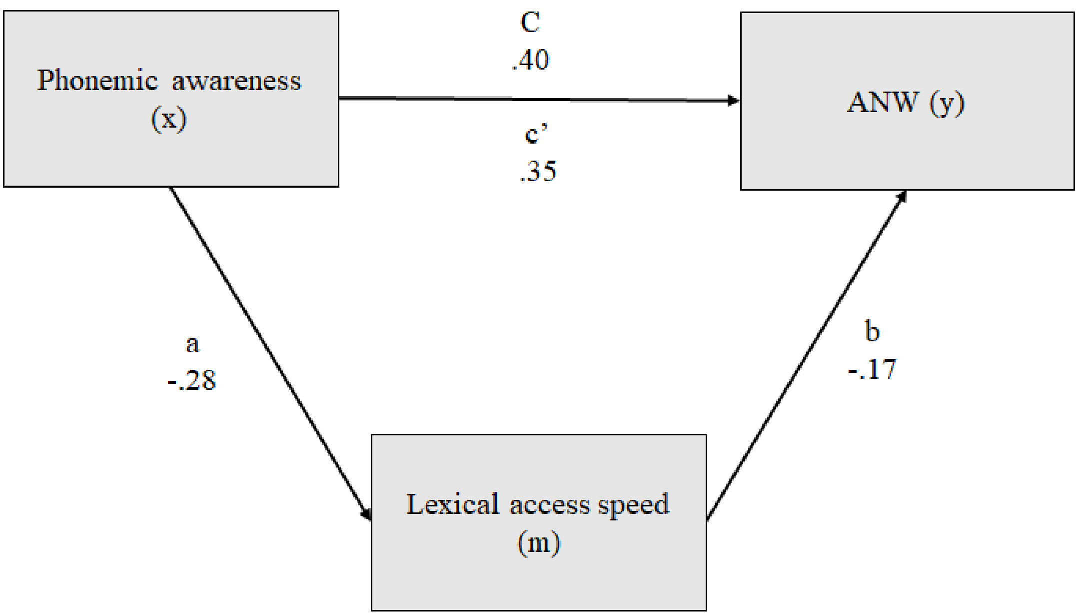

Figure 4.1. Mediation impact of lexical access speed on the relationship between phonemic awareness and ANW.

Legend: ANW = Arabic Number Writing; x = predictor variable; y = dependent variable; m = mediator variable; *C* = total effect; *c'*= direct effect; *a* = impact of phonemic awareness on lexical access speed; *b* = impact of lexical access speed on ANW.

We also included the contribution of lexical access speed for word spelling using the same approach of the previous regression model. The final model explained 45% of variance in the word spelling, R^2=.46, $R^2_{adjusted}$=.45, F(1, 316)=19.13, p<.001. Inspection of beta weights indicated that better performance on ANW, phonemic awareness, and lexical access speed predicted better word spelling performance (see Table 4.3). Importantly, the inclusion of lexical access speed in the model led to the exclusion of phonological working memory. Similar to the models for ANW, this pattern suggests that the influence of phonological WM on word spelling might be mediated by lexical access speed.

Figure 4.2 shows the analyses of mediation of lexical access speed on the contribution of phonological WM on word spelling. The mediation effect was significant, β=.07, BCa CI 95% [.02, .13]. However, the contribution of phonological WM on word spelling remained significant after controlling for the effect of lexical access speed, b=.69, CI 95% [.25, 1.13], *p*<.001. The lexical access speed mediated 31% of the variance of the relationship between phonological WM and word spelling.

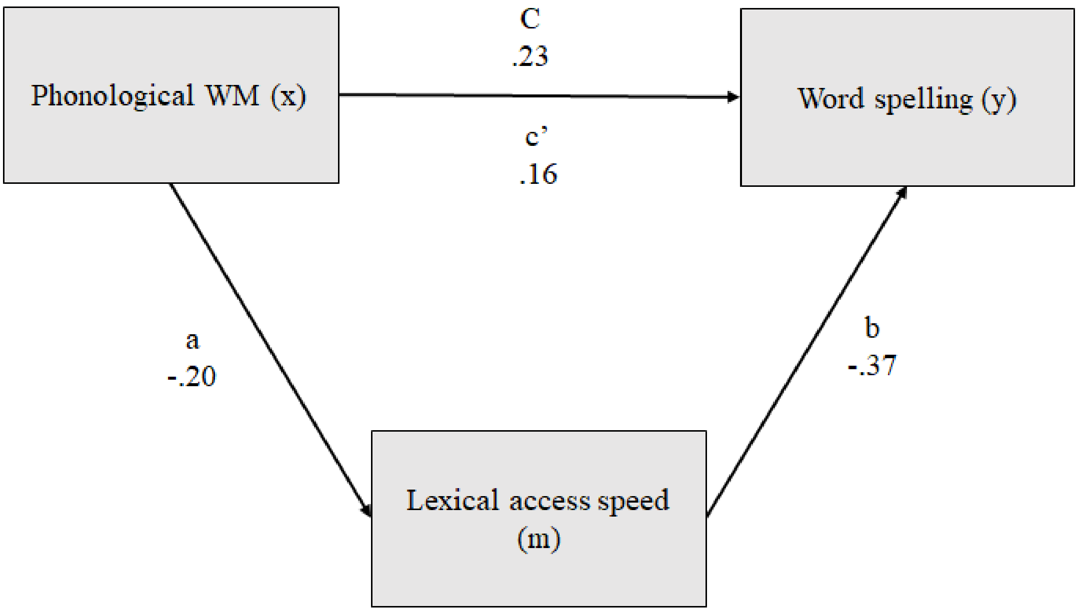

Figure 4.2. Mediation impact of lexical access speed on the relationship between phonological WM and word spelling.

Legend: x=predictor variable; m=mediator variable; y=dependent variable; *C*=total effect; *c'*=direct effect; *a*=impact of phonological WM on lexical access speed; *b*=impact of lexical access speed on ANW.

Predictors of lexical and syntactic errors in Arabic number writing

The same approach used in the former regression models was used to investigate the predictive power of visuo-spatial WM, phonological WM, phonemic awareness and lexical access speed on transcoding error types. The final regression model for *lexical errors* accounted for 10% of the variance, R^2=.11, $R^2_{adjusted}$=.10, $F(1, 353)$=4.02, p=.05, with phonemic awareness and lexical access speed being significant predictors of lexical errors. Inspection of beta weights indicated that worse performance on phonemic awareness and lexical access speed predicted higher error rates of lexical errors (see Table 4.4).

The final model for *syntactic errors* explained 7% of the variance, R^2=.08, $R^2_{adjusted}$=.07, $F(1, 353)$=3.89, p=.05. phonemic awareness and visuo-spatial WM were the only significant predictors. Consideration of beta weights indicated that worse performance on visuo-spatial WM and phonemic awareness predicted higher rates of syntactic errors (see Table 4.4).

The model for combined errors explained 9% of variance, R^2=.10, $R^2_{adjusted}$=.09, $F(1, 353)$=6.70, p=.01, with visuo-spatial WM and phonemic awareness being significant predictors of combined errors. Beta weights indicated that worse performance on visuo-spatial WM and phonemic awareness predicted higher rates of combined errors (see Table 4.4).

Table 4.4. Regression models showing the predictive power of WM, phonemic awareness, and lexical access speed on lexical, syntactic and combined errors.

Predictor	Lexical errors ($R^2_{adj.}$=.10)			Syntactic errors ($R^2_{adj.}$=.07)			Combined errors ($R^2_{adj.}$=.09)		
	Beta	**t**	**ΔR^2**	**Beta**	**t**	**ΔR^2**	**Beta**	**t**	**ΔR^2**
Intercept		4.08***			7.51***			8.01***	
Non-verbal reasoning	-.05	-0.91	.02	-.09	-1.55	.03	-.01	-0.09	.02
Visuo-spatial WM	-.10	-1.71	Excluded	-.11	-1.97*	.01	-.14	-2.59*	.02
Phonological WM	-.08	-1.53	Excluded	-.06	-1.12	Excluded	-.03	-0.46	Excluded
Phonemic awareness	-.27	-4.84***	.08	-.18	-3.23***	.04	-.24	-4.36***	.06
Lexical access speed	.11	2.00*	.01	-.03	-0.47	Excluded	.06	1.15	Excluded

Note: *p<.05; ** p<.01; **p<.001.

Discussion

The present study aimed to investigate the specific influence of phonological processing abilities (i.e., phonological WM, phonemic awareness, and lexical access speed) on overall transcoding performance as well as the frequency of specific error types. Based on the assumptions of the ADAPT model, we hypothesize that phonological processing abilities contribute selectively to specific steps of number transcoding. In particular, we hypothesize that i) phonemic awareness should be important in the phonological encoding of numbers in the initial step of transcoding; ii) phonological WM should be important for the storing and manipulation of information, and iii) lexical access speed should be important in the retrieval of Arabic numbers' lexical information.

Furthermore, we expected a selective pattern of contribution of these variables to lexical and syntactic errors, reflecting specific difficulties in the implementation of lexical and procedural routes of the ADAPT model. Thus, difficulties in the procedural route, resulting in syntactic errors, should be predicted by phonological and visuo-spatial WM. In turn, difficulties in the lexical route, resulting in lexical errors, should be predicted by phonemic awareness and lexical access speed.

Additionally, by replicating the same approach of Lopes-Silva and colleagues (2016), we compared the predictive power of phonological processing abilities in ANW and word spelling. We aimed to investigate if the contribution of phonological processing abilities is shared by ANW and word spelling.

Generally, our results are in line with, but also extend, the findings of Lopes-Silva and colleagues (2016) meaningfully. These authors showed that next to visuo-spatial WM, phonemic awareness is an important correlate of ANW. However, the current results indicate that when lexical access speed was considered as well, phonemic awareness was no longer a significant predictor of ANW. This suggested that the influence of phonemic awareness on ANW might be mediated by lexical access speed, which was substantiated by a subsequent mediation analysis. Moreover, specific analyses on separate error types revealed that lexical errors were indeed predicted best by phonemic awareness and lexical access speed selectively. In contrast, syntactic and combined errors were predicted best by phonemic awareness and visuo-spatial WM. The word spelling was predicted by phonemic awareness and phonological WM. However, when lexical access speed was entered in the regression model, phonological WM was no longer a significant predictor of word spelling.

In the following paragraphs, we discuss the relative role of different phonological processing abilities and visuo-spatial WM in the overall ANW performance and in the frequency of specific error types.

Previous studies investigated the association between phonological processing and numerical skills such as arithmetic fact retrieval (DeSmedt & Boets, 2010; Hecht et al, 2001). Simmons and Singleton (2008) argued that deficits in phonological WM and lexical access speed should impair the strength of phonological representations and, consequently, affect aspects of numerical cognition that involve the manipulation of a verbal code. Even though phonological WM and lexical access speed draw on phonological representations alongside with phonemic awareness, the latter may tap more on phonological representations (see Boada & Pennington, 2006; Elbro, 1996).

Accordingly, we proposed that phonemic awareness should be important for the initial phonological encoding of the verbal input on number transcoding prior to the implementation of any lexical and/or procedural mechanisms. Therefore, it is expected that difficulties in this initial stage should hinder the transcoding process. Although Barrouillet and colleagues (2004) did not assume a contribution for phonemic awareness on ANW, they argued that this phonological encoding stage would be affected by the degree of phonological similarity between number words. Thus, one can argue that phonemic awareness might be important to discriminate number words with similar phonemes such as "<u>se</u>ssenta" and "<u>se</u>tenta" (i.e., sixty and seventy) or "tr<u>ês</u>" and "se<u>is</u>" (i.e., three and six).

Only a few studies focused on investigating the specific association between phonological processing abilities and ANW (see Lopes-Silva et al., 2014, 2016). Lopes-Silva and colleagues (2016) investigated the contribution of phonemic awareness, phonological WM and visuo-spatial WM on the ANW skills of 7 to 11-year-old children. The regression model indicated that only phonemic awareness predicted the performance on ANW. Using a similar approach, the current study replicated this previous finding by demonstrating a significant contribution of phonemic awareness on ANW, even when the effects of working memory were considered. We expanded this previous study by also considering the contribution of lexical access speed. The results indicated that the effect of phonemic awareness on ANW is no longer significant when lexical access speed is simultaneously considered in the model. In addition, mediation analyses substantiated that lexical access speed partially mediated the influence of phonemic awareness on ANW. Given that phonemic awareness and lexical access speed tap the same latent variable, namely phonological processing, and that phonemic awareness tasks encompasses the influence of working memory and lexical retrieval, the specific contribution of each ability on ANW might be hard to disentangle (see Cunningham et al., 2015).

In addition to that, phonemic awareness was associated with all three types of errors. We proposed that initial difficulties in the phonological encoding of numbers, which should be associated with phonemic awareness, should be common to all types of errors. Alternatively, these results might be driven by the fact that the phonemic elision task used to measure phonemic awareness skills in the present study also involves manipulation of verbal information. It is possible to assume that this task also encompasses phonological WM skills, which could potentially explain why worse performance on the phoneme elision task was associated with higher error rates of all types.

Lexical errors were specifically predicted by lexical access speed (and phonemic awareness). The ADAPT model postulates that lexical errors would occur due to difficulties in retrieving lexical units from LTM, which would be implemented by P1 rules. In favor of this hypothesis, Barrouillet and colleagues (2004) found a positive association between the number of P1 rules that need to be applied when transcoding a number and the rate of lexical errors. The results of the present study suggest that children with less efficient lexical access speed made more lexical errors, which in turn reflects impairments in the application of P1 rules.

According to the ADAPT model, WM maintains the verbal units and organizes the output during number transcoding. In fact, significant associations between transcoding performance and phonological and visuo-spatial components of WM have been consistently reported in the literature. Here we further substantiated the association between ANW and both visuo-spatial and phonological WM by revealing how each WM component influences ANW performance alone and in addition to other variables. Specifically, we corroborated previous finding relating verbal and visuospatial WM to general performance in ANW (e.g., Camos, 2008; Imbo et al., 2014; Lopes-Silva et al., 2014, 2016; Moura et al., 2013; Zuber et al., 2009). More importantly, our results also indicated that phonemic awareness, as measured by a phoneme elision task, accounts for a shared part of variance also explained by phonological, but not visuo-spatial WM. This finding may be attributable to similarities between the tasks used to measure phonological

WM and phonological awareness (digit span and phoneme elision, respectively). As mentioned before, our phoneme elision task required the retention and manipulation of phonological information. Also, as suggested by Lopes-Silva et al. (2014) and discussed above, verbal processes in number transcoding can be assigned to, or mediated by, phonemic awareness.

More in-depth analyses on different error types also shed some light on the role of working memory in ANW. The specific association between visuo-spatial WM and syntactic errors, but also combined errors, supports the idea of a visuo-spatial processing of the place-value structure of the Arabic code (see Zuber et al., 2009). This result indicates that children with lower visuo-spatial WM would be more susceptible to face problems in the procedural route of the ADAPT model. However, it is worth noting that the same association was not found by Lopes-Silva and colleagues (2016) when investigating the influence of phonological processing and visuo-spatial WM on transcoding. We hypothesize that the use, in the present study, of a transcoding task with more syntactically complex numbers may have put higher demands on visuo-spatial WM resources and, thus, increased its association with number transcoding. This is, however, a post hoc explanation and should be tested further by directly manipulating number complexity in the same study.

The present findings showed that similar cognitive processes seem to be shared between word reading/writing and number writing. Regression models assessing the predictive power of phonological processing abilities on word spelling demonstrated that better performance on phonemic awareness and phonological WM predicted better performance on word spelling. However, the contribution of working memory is partially mediated by lexical access speed. Byrsbaert (2005) pointed out the parallels between word reading and single-digit numbers recognition. In both cases, processing words and numbers initially occurs in a sequential manner, demanding the implementation of algorithm-based conversion rules. Later on, more holistic/parallel and automatized processing takes place. Both forms of processing were operationalized as being different routes in the models of word reading/writing (Coltheart et al., 2001) and number writing (Barrouillet et al., 2004). The parallel route is primarily used to process frequent and familiar words and numbers, while the sequential route is most likely used to process infrequent and unfamiliar words and numbers. Additionally, less proficient students rely exclusively on the procedural route, whereas more proficient achievers may expand their word and number lexicon, being able to use both the procedural and lexical routes concurrently.

Conclusion

Altogether, the results of the present study suggest that different phonological processes play specific roles in ANW. In particular, we observed that lexical access speed mediated the previously reported influence of phonemic awareness on transcoding. Additionally, selective influences were found for phonological processing abilities and specific transcoding error types. While all error types were predicted significantly by phonemic awareness, lexical errors were specifically predicted by lexical access speed. In contrast, syntactic and combined errors were predicted significantly by visuo-spatial working memory and phonemic awareness.

The findings of this study have important theoretical implications. The ADAPT model explicitly assumes the involvement of working memory during ANW. However, it does not consider the involvement of other cognitive processes in transcoding. Here, we substantiated that phonemic awareness and visuo-spatial WM, and first demonstrated that lexical access speed significantly predicted transcoding performance. Based on the observed pattern of results, we hypothesize that these processes contribute selectively to specific steps of number transcoding, as follows: i) phonemic awareness should be important in the phonological encoding of numbers in the initial step of transcoding; ii) phonological WM should be important for the storing and manipulation of information, and iii) lexical access speed should be important in the retrieval of Arabic numbers' lexical information.

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CHAPTER 5

General discussion

In the present dissertation, we addressed how different mechanisms explain the challenges faced by children during the learning of number transcoding skills. In particular, we investigated how different mechanisms at the structural task level and individual level influence the performance on number transcoding. At the structural task level, we investigated the influence of the syntactic organization on the Arabic number system and of the linguistic aspects on number word formation. At the individual level, we examined the influence of phonological processing abilities on number transcoding. More importantly, we explain the specific contribution of each one of these mechanisms on number transcoding based on the predictions of the ADAPT.

In chapter 2 we demonstrated that the impact of syntactic complexity on number transcoding is relatively independent of the number of digits. Importantly, both factors were significantly associated with the performance on number transcoding, however, their influence tends to decrease with development. This finding substantiated the assumptions of the ADAPT model that numbers are transcoded via algorithm-based conversion rules (see also Camos, 2008; Barrouillet et al., 2004; Moura et al, 2015). We proposed that future studies should replicate and expand our results by design an experimental task containing more syntactically complex items including numbers with five-digits.

In chapter 3 we demonstrated significant language influences on the occurrence of specific error types on number transcoding. The inversion property of two-digit numbers of the German number word system was associated with higher error rates, mostly inversion-related errors. In contrast, the irregularity of hundred number words of the Portuguese number word system was neither associated with higher error rates in three-digit numbers nor with higher frequencies of syntactic errors. We hypothesized that number word systems with irregularities at the syntactic level might be more challenging than number word systems with irregularities at the morphological level. Following the assumptions of the ADAPT model, irregularities at the syntactic level would demand the implementation of additional transcoding rules.

Alternatively, it is possible to hypothesize that the irregularity of hundreds in Portuguese still might affect the understanding of place-value in a more specific level. Previous cross-linguistic studies showed that children from languages with more transparent number words presented a better performance in tasks which demand the representation of numbers using base-ten blocks than children from languages with less transparent number words (Miura & Okamoto, 2003; Miura et al., 1994; Miura et al., 1988). One can argue that base-ten blocks construction tasks assess place-value understanding more specifically than number transcoding tasks are affect by different mechanism, which we have been demonstrated here.

A limitation of the present study is that we were not able to perfectly match the ages of Brazilian and Austrian sample. In addition to that, as an example of others cross-linguistic comparison studies we did not control for contextual factors, such as the difference in the curriculum, that potentially could explain our results. A possible alternative to handle the influence of contextual factors would be investigate the impact of linguistic aspects within the same culture and educational system. For example, Pixner and colleagues (2011) observed the impact of linguistic specificities on number transcoding in Czech children, whose number words for two-digit numbers may be spoken/written in either an inverted or a non-inverted format. In a follow up study, would be possible to compare the performance in hundred and thousand words in Portuguese since the latter is regular. However, this would require the control of the syntactic complexity and number of digits affect the performance on number transcoding, as we demonstrated in the chapter 2, both factors affect the performance on number transcoding.

In chapter 4 we demonstrated that number transcoding performance was selectively predicted by phonological processing abilities, in particular phonemic awareness and lexical access speed. Furthermore, lexical errors were predicted best by phonemic awareness and lexical access speed, syntactic and combined errors were predicted by phonemic awareness.

The results of this study have important theorical and practical implications. First, we hypothesized that this phonological processing abilities would be important during the implementation of specific steps of the ADAPT model (Barrouillet et al., 2004). Although the lexical and syntactic errors were selectively predicted by phonological processing abilities (next to visuo-spatial working memory in the latter case) as we hypnotized based on the ADAPT model, future studies should address the specific contribution of these abilities in the steps of the ADAPT model with a more experimental designed.

Against our expectations, the three phonological abilities did not present a specific contribution when they were entered in the same regression model. A possible explanation is that the tasks used in the present study encompasses the three phonological abilities at some level, given that they tap the same latent variable, namely phonological processing. Previous studies have suggested that this might affect phoneme elision tasks in particular (Cunningham et al., 2015). Although there are other tasks that assess phonemic awareness, we choose to use a phoneme elision task because it is a classic measure of phonemic awareness and previous studies of our research team showed that it did not present neither a floor effect nor a ceiling effect (Barbosa-Pereira et al., 2020).

Second, we demonstrated that number transcoding and word spelling shares cognitive mechanism. We proposed that in both cases a verbal input is processed by the interplay of automatic/holistic and parallel/sequential mechanism (Barrouillet et al., 2004; Brysbaert, 2005; Coltheart et al., 2001; McCloskey & Rapp, 2017).

This finding might explain the high comorbidity rate between children with mathematical and reading/writing difficulties (Peng et al., 2020). Against the hypothesis that a double impairment in specific cognitive mechanism would explain the comorbidity between mathematical and reading/writing difficulties (Landerl et al., 2004), Simmons and Singleton (2008), argued in favor "weak phonological representation hypothesis". The authors assumes that phonological processing deficits, commonly present in children with reading/spelling learning difficulties, would affect the performance of mathematical abilities that involves manipulation of a verbal code. In favor of this hypothesis, previous studies showed that children with reading disabilities presented specific difficulties in the verbal numerical and arithmetic abilities (e.g., arithmetic fact retrieval and number transcoding), while nonverbal verbal numerical and arithmetic abilities are preserved (e.g., nonsymbolic comparison and estimation; De Clercq-Quaegebeur et al., 2018; De Smedt, & Boets, 2010; Teixeira & Moura, 2020).

In the cross-linguistic comparison study we observed that Portuguese-speaking made significantly more lexical errors than German-speaking children. It is possible to hypothesize that the contribution of phonological processing on ANW would be influenced by linguistic characteristics of the number word system, such as phonological similarities between number words. Future studies should investigate the impact of phonological processing on ANW in individuals of other languages.

In the present dissertation, we also discussed how the ADAPT model accounts for the influence of different mechanism at the structural task level and at the individual level mechanisms. The task used in the studies described in chapter 2 and 4 were designed manipulating the number of transcoding rules and of digits. However, our results demonstrated that other aspects might be controlled as well, such as the phonological similarity of items and the number words irregularities in future studies. In sum, we demonstrated that the ADAPT model offers theoretical insight about how different mechanisms at the structural task level and at the individual level.

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APPENDIX 1

The quandary of diagnosing mathematical difficulties in a generally low performing population

Mariuche Rodrigues de Almeida Gomides[1,2], Isabella Starling-Alves[3], Giulia Moreira Paiva[2,4], Leidiane da Silva Caldeira[2], Ana Luíza Pedrosa Neves Aichinger[2], Maria Raquel Santos Carvalho[5,6], Julia Bahnmueller[7,8,9], Korbinian Moeller[7,8,9,10], Júlia Beatriz Lopes-Silva[1,2], Vitor Geraldi Haase[1,2,4,11]

[1]Graduate Program in Psychology: Cognition and Behavior, Universidade Federal de Minas Gerais – Belo Horizonte, MG, Brazil. [2]Laboratory of Developmental Neuropsychology, Universidade Federal de Minas Gerais – Belo Horizonte, MG, Brazil. [3]Educational Psychology Department, University of Wisconsin-Madison – Madison, United States. [4]Graduate Program in Neuroscience, Universidade Federal de Minas Gerais – Belo Horizonte, MG, Brazil. [5]Graduate Program in Genetics, Universidade Federal de Minas Gerais – Belo Horizonte, MG, Brazil. [6]Department of Genetics, Ecology and Evolution, Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brazil. [7]Centre for Mathematical Cognition, School of Science, Loughborough University – Loughborough, United Kingdom. [8]Leibniz-Institut für Wissensmedien – Tübingen, Germany. [9]LEAD Graduate School & Research Network, University of Tübingen – Tübingen, Germany. [10]Department of Psychology, University of Tübingen – Tübingen, Germany. [11]Instituto Nacional de Ciência e Tecnologia sobre Comportamento, Cognição e Ensino – São Carlos, SP, Brazil.

Mariuche Rodrigues de Almeida Gomides. Developmental Neuropsychology Laboratory, Department of Psychology, Universidade Federal de Minas Gerais. Avenida Antônio Carlos, 6627 – Pampulha – 31270-901 Belo Horizonte MG – Brazil. E-mail: malu15gomides@gmail.com

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ABSTRACT. Brazilian students' mathematical achievement was repeatedly observed to fall below average levels of mathematical attainment in international studies such as PISA. **Objective:** In this article, we argue that this general low level of mathematical attainment may interfere with the diagnosis of developmental dyscalculia when a psychometric criterion is used establishing an arbitrary cut-off (e.g., performance<percentile 10) may result in misleading diagnoses. **Methods**: Therefore, the present study evaluated the performance of 706 Brazilian school children from 3^{rd} to 5^{th} grades on basic arithmetic operations addition, subtraction, and multiplication. **Results:** In line with PISA results, children presented difficulties in all arithmetic operations investigated. Even after five years of formal schooling, less than half of 5^{th} graders performed perfectly on simple addition, subtraction, or multiplication problems. **Conclusions:** As such, these data substantiate the argument that the sole use of a psychometric criterion might not be sensible to diagnose dyscalculia in the context of a generally low performing population, such as Brazilian children of our sample. When the majority of children perform poorly on the task at hand, it is hard to distinguish atypical from typical numerical development. As such, other diagnostic approaches, such as Response to Intervention, might be more suitable in such a context.

Keywords: diagnosis, dyscalculia, learning disabilities, mathematics.

RESUMO. O desempenho em matemática dos estudantes brasileiros mostra-se consistentemente abaixo da média mundial em estudos internacionais como o PISA. **Objetivo:** No presente artigo, argumenta-se que um baixo desempenho geral na matemática, a exemplo dos estudantes brasileiros, pode interferir no diagnóstico de discalculia do desenvolvimento quando um critério puramente psicométrico é usado para estabelecer um ponto de corte arbitrário (por exemplo, desempenho<percentil 10), o que pode resultar em falsos diagnósticos. **Métodos:** Para tanto, investigou-se o desempenho de 706 estudantes brasileiros do 3º ao 5º ano escolar em operações aritméticas básicas de

adição, subtração e multiplicação. **Resultados:** De forma consistente com os resultados do PISA, as crianças apresentaram dificuldades em todas as operações aritméticas investigadas. Mesmo após cinco anos de escolarização formal, menos da metade dos estudantes do 5º ano foi capaz de completar a tarefa envolvendo cálculos simples de adição, subtração ou multiplicação. **Conclusões:** Dessa forma, os resultados reforçam o argumento de que o uso exclusivo de um critério psicométrico pode não ser apropriado para o diagnóstico de discalculia no contexto de uma população com desempenho geral baixo, como no caso crianças brasileiras da presente amostra. Quando a maioria das crianças tem um desempenho aquém do esperado, torna-se difícil distinguir o desenvolvimento numérico atípico do típico. Portanto, outras abordagens diagnósticas, como Resposta à Intervenção, podem ser mais adequadas em tal contexto.

Palavras-chave: diagnóstico, dificuldades de aprendizagem, discalculia, matemática.

INTRODUCTION

Mathematics is an important predictor of scientific and technological development, which is important for success in competitive global economies.[1] For this reason, many countries have increased investments in basic mathematical education.[1-3] Despite increased international recognition and higher investments in mathematical education, mathematical achievement in several countries remains a cause for concern.[4] This is especially the case in Brazil.[5] According to Programme for International Student Assessment (PISA) scores, no significant improvement has been observed in mathematics achievement of Brazilian students from 2003 to 2018. Results of PISA 2018 indicated that performance of Brazilian students in mathematics was significantly below Organisation for Economic Co-operation and Development (OECD) average.[6] Moreover, the majority of students assessed scored below level 2 of math proficiency, which is considered the minimum necessary for young people to fully exercise their citizenship.[6] Finally, PISA results also showed another alarming result: the upper half of Brazilian students (i.e., performing above percentile 50) still performed worse than the lower half of students (i.e., performing below percentile 50) from countries scoring highest in PISA 2018 such as South Korea, Finland, and Canada.[7]

A cornerstone for developing more advanced mathematical abilities is the mastery of the basic arithmetic operations: addition, subtraction, and multiplication.[8,9] When children start learning basic arithmetic operations, they usually use rather effortful and error-prone procedural strategies, mostly based on (finger) counting.[10] With practice, children become able to use more sophisticated procedural strategies (e.g., based on mental calculation and using composition/decomposition of numbers, for example "16+7=16+4=20+3=23") and may even retrieve solutions from long-term memory for specific problems (e.g., tie problems such as "4+4") or operations such as multiplication. However, some children persistently struggle to learn arithmetic.

Difficulties in learning basic arithmetic operations have been associated with dyscalculia, which reflects a circumscribed disability in handling numbers and arithmetic operations.[11] A substantial number of school-aged children (i.e., between 3 and 6%, depending on the study)[12] suffer from this learning disability, characterized by severe and persistent difficulties in mathematical learning that cannot be explained by primary causes such as intellectual deficits, emotional/motivational problems, and/or lack of adequate schooling.[11] Dyscalculia is characterized by difficulties with the most basic aspects of mathematics, such as the ability to understand and discriminate quantities,[13-16] read and write numbers.[17,18] Additionally, difficulties with acquiring arithmetic facts knowledge are a cardinal symptom of dyscalculia.[19,20]

So far, there are no biological or cognitive markers sufficiently reliable to diagnose dyscalculia. Therefore, standardized tests of mathematical achievement are the most popular tool for diagnosing dyscalculia.[21] According to the Diagnostic and Statistical Manual of Mental Disorders (DSM-5),[11] dyscalculia can be diagnosed when: (a) performance in standardized tests of mathematical achievement falls below a specific cut-off point (i.e., psychometric criterion), (b) mathematical difficulties compromise the psychosocial adaptation of the individual (i.e., psychosocial impairment criterion), and (c) mathematical difficulties cannot be attributed to other primary causes as mentioned above (i.e., clinical exclusion criterion). Importantly, the clinical exclusion and the psychosocial impairment criteria have the downside of being subjective, and thus may well depend on the clinician's experience. However, the psychometric criterion is not less problematic.

The psychometric approach has important limitations.[22-24] So far, different cut-offs in standardized mathematical tests, ranging from the 5th to the 35th percentiles, have been employed across different studies (see[25] for a review). Furthermore, the use of standardized mathematical achievement tests, alone, does not provide information about potentially impaired neurocognitive processes underlying dyscalculia.[26] Instead, such tests usually only allow for the classification of a child's achievement as viewed against a comparison group (e.g., children of the same age or school grade).

Given the overall low mathematics achievement consistently observed among Brazilian children,[6] using a psychometric approach may lead to false-positive diagnoses of dyscalculia as performance below a specific percentile may not allow to differentiate between atypical and typical poor performance. As such, the main purpose of the present study was to assess the performance of Brazilian primary school children on basic arithmetic operations and evaluate how this information can be used to diagnose dyscalculia in the Brazilian context. Therefore, we assessed performance of 3rd, 4th, and 5th graders on basic arithmetic operations, including addition, subtraction, and multiplication, to evaluate the acquisition of these abilities across grades. With this approach, we aimed at finding out by which grade children achieve proficiency in basic arithmetic operations. In the following, we first present detailed information on the study before reporting and comparing results operations and grades. Finally, we discuss the challenge of diagnosing dyscalculia in Brazil, using the psychometric criterion, considering the present results.

METHOD

Participants

Participants were 706 children with typical general cognitive abilities (above percentile 15 in CPM-Raven)[27] attending third to fifth grade ($Mean_{age\ in\ years}$=9.11, ±1.01; 55.5% girls), selected from 13 public schools and one private school in Belo Horizonte, Minas Gerais, the state with the third highest income in Brazil.[28] All participants gave oral assent prior to testing and provided informed consent signed by their parents or primary caregivers. The study was approved by the local Research Ethics Committee.

Task and procedure

This study was part of a more comprehensive project investigating the development of mathematical abilities of school-age children in Brazil. In this project, children completed a battery of tasks measuring general cognitive abilities (e.g., executive functions), and numerical and mathematical abilities (e.g., nonsymbolic and symbolic magnitude processing and numerical transcoding). For the purpose of this article, we specifically focused on the results of the Basic Arithmetic Operations Task (BAOT), which was assessed individually in a quiet separate room at participants' school.

The BAOT consisted of 27 addition, 27 subtraction, and 28 multiplication problems. Problems of each operation were presented in fixed order of increasing difficulty on separate sheets of paper. Children were instructed to solve as many problems as possible within a 2-minute time limit per operation. The percentage of correctly solved items (i.e., the number of correctly solved problems divided by the total number of problems in the task) for each operation type was used as the dependent variable (for more information, see[29]).

The time limit in BAOT was established based on the performance of 16 college students ($Mean_{\textit{age in years}}$=22.93, ±2.56, 62.5% female), who mastered basic operations. Results showed that adults were well able to solve all addition, subtraction, and multiplication problems within 2 minutes (i.e., addition: $Mean_{seconds}$=59, ±9.83; subtraction: $Mean_{seconds}$=73, ±13.28; multiplication: $Mean_{seconds}$=83, ±13.60), with hardly any errors (i.e., percentage of correctly solved items for addition: $Mean_{corrects}$=0.99, ±0.01; subtraction: $Mean_{corrects}$=0.97, ±0.05; multiplication: $Mean_{corrects}$=0.92, ±0.08). Based on these estimates, we expected that children fairly fluent in solving basic arithmetic operations should be able to complete all problems within the 2-minute time limit per operation type.

RESULTS

In our analysis, we evaluated performance of 3rd, 4th and 5th graders in the BAOT operation types. First, we present descriptive analyses for each operation before the results of a mixed-model repeated measure ANOVA aiming to discern the influences of the independent between-participants variable grade level (i.e., 3rd *vs*. 4th *vs*. 5th grade) and the within-participants variable operation (i.e., addition vs. subtraction vs. multiplication) on the percentage of correctly solved items.

Addition

In 3rd grade, children were still learning basic addition, such that different scores were observed with similar frequencies in the task. In 4th grade, children started to master addition, with higher scores being observed more frequently than lower scores. Similarly, in 5th grade, higher scores were observed more frequently than lower scores, but children still did not reach perfect accuracy (Figure 1A). Tests of normal distribution (i.e., Kolmogorov-Smirnov, henceforth KS) indicated a non-normal distribution of addition scores for all three grades ($KS_{3rd\ grade}=1.51$, $p=0.02$; $KS_{4th\ grade}=2.08$, $p<0.001$; $KS_{5th\ grade}=2.01$, $p<0.01$). These results suggest that performance on addition problems seemed to improve from 3rd to 5th grade. However, less than 50% of children in 5th grade correctly solved more than 80% of the BAOT addition problems.

Subtraction

Third graders presented difficulties with subtraction operations, such that the most frequent percentage of correct responses was below 50% (Figure 1B). In 4th grade, a transition (i.e., similar frequencies for different scores) was observed, indicating that children were still learning subtraction. In 5th grade, children started to improve their performance in subtraction, with scores above 50% of correct responses becoming more frequent. However, most children still achieved less than 75% of correct responses. KS tests indicated non-normal distributions of subtraction scores in the 3rd ($KS=1.47$, $p=0.03$) and 4th ($KS=1.38$, $p=0.04$) grades, but not in the 5th grade ($KS=1.12$, $p=0.16$). Thus, similar to addition, results suggested an improvement in subtraction performance from 3rd to 5th grade. However, by 5th grade, less than 20% of students were able to solve all items correctly, even though these only involved minuends up to 20.

Multiplication

A floor effect was observed in 3rd grade for multiplication, with most children not being able to solve any of the problems correctly (Figure 1C). In 4th and 5th grades, children started to learn multiplication operations, such that different scores were observed with similar frequencies in the task, suggesting only limited improvement between these grades. KS tests revealed a non-normal distribution of multiplication scores for 3rd ($KS=3.64$, $p<0.001$) and 4th ($KS=1.62$, $p<0.01$) graders and a distribution closer to normal for 5th graders (KS=1.12, p=0.16). These results suggest that, despite some improvement in multiplication skills from 3rd to 5th grade, 5th graders still do not master multiplication tables for single-digit numbers, with less than 20% of children with a maximum score in multiplication.

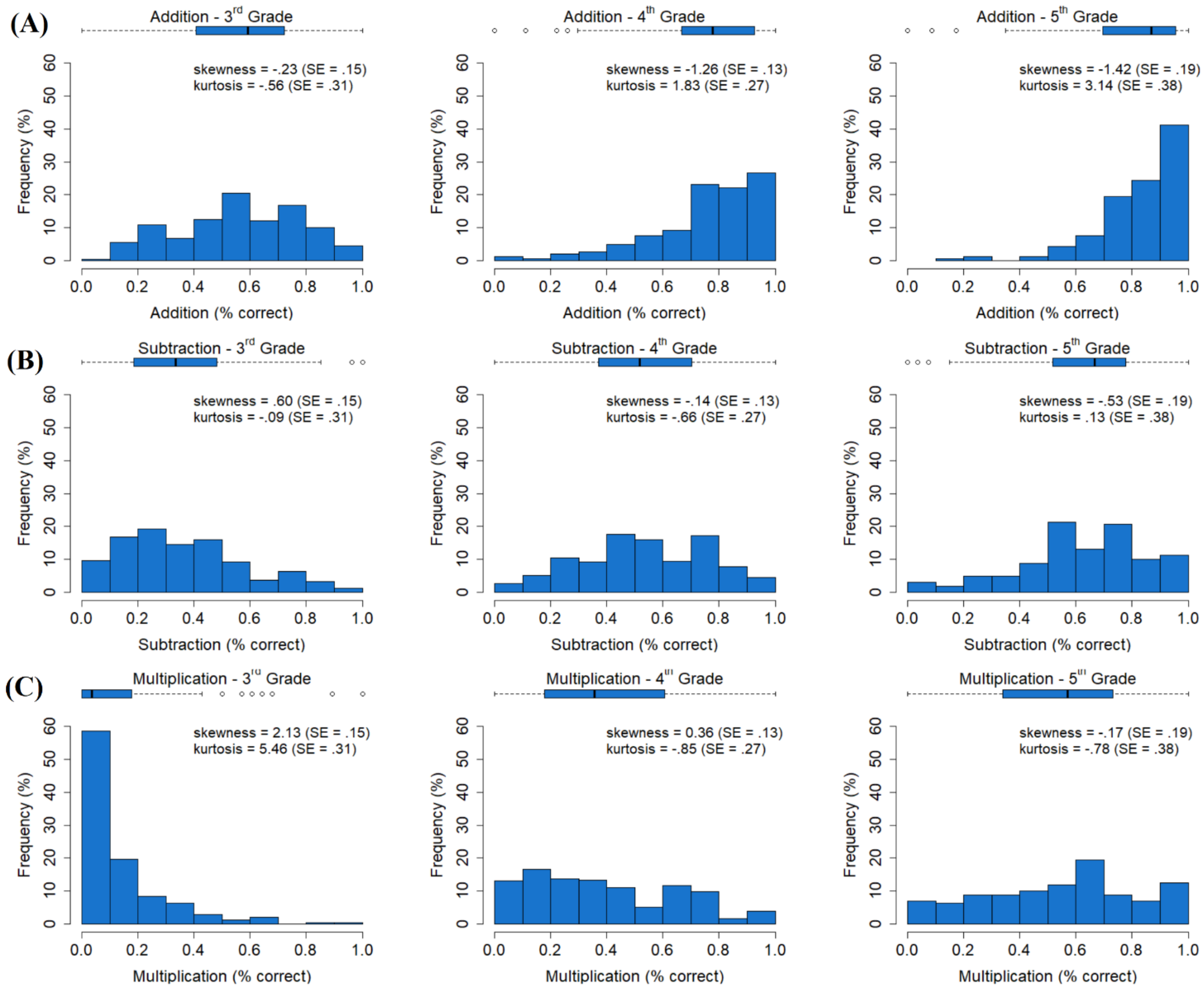

Figure 1. Children's performance on addition, subtraction, and multiplication operations across grades.

We considered the interval of 80 to 100% of correct responses as a criterion for fluency on BAOT operations. Then, we evaluated the percentage of children who met this criterion. Although this criterion was chosen more or less arbitrarily, we expected adults (i.e., as described above in the method section) and 5th graders to be able to fluently solve the BAOT operations based on the results of previous studies (cf.[30-33]). By choosing the interval of 80 to 100%, we do, however, leave room for occasional careless mistakes, or situational or motivational digressions.

The majority of 3rd graders (85.4%) did not master single-digit addition operations. This percentage drops considerably by 5th grade, in which only 34% of children had not yet mastered basic addition operations. A smaller improvement was observed for 5th graders with respect to subtraction and multiplication, in comparison to addition. For subtraction operations, 95.4% of 3rd graders failed to meet our criterion, dropping to 78.5% in 5th grade. For multiplication, 99.2% of 3rd graders failed to meet the criterion, dropping to 80.6% in 5th grade (Table 1).

Table 1. Percentage of children scoring above 80% of correct responses on addition, subtraction, and multiplication operations at each grade.

	Addition			**Subtraction**			**Multiplication**		
	3rd	4th	5th	3rd	4th	5th	3rd	4th	5th
80–90%	10.0	22.1	24.5	3.4	7.9	10.1	0.4	1.6	6.9
91–100%	4.6	26.6	41.5	1.2	4.6	11.4	0.4	3.9	12.5
80–100%	14.6	48.7	66.0	4.6	12.5	21.5	0.8	5.5	19.4

Finally, the mixed-model ANOVA indicated a significant main effect of grade, $F_{(2, 703)}=154.4$, $p<0.001$, $\eta_p^2=0.17$, with performance improving across grades. Pairwise comparisons indicated that 5th graders' scores were higher than those of 4th and 3rd graders, and 4th graders' scores were higher than those of 3rd graders. There also was a significant main effect of operation, $F_{(2, 1321)}=1085.34$, $p<0.001$, $\eta_p^2=0.26$. Pairwise comparisons indicated that addition operations were solved better than subtraction and multiplication operations and that subtraction operations were solved better than multiplication operations. Means and standard deviations are shown in Figure 2.

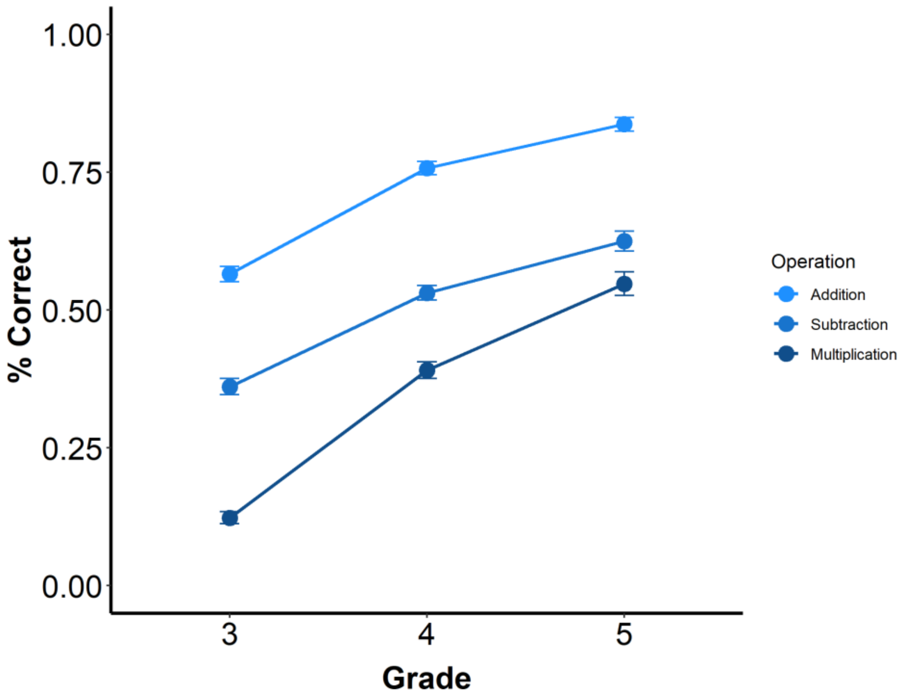

Figure 2. Mean percentage of correct responses for grades and operations. Error bars indicate standard erros.

The interaction of grades and operation types was also significant, $F_{(4,1321)}=20.36$, $p<0.01$, $\eta_p^2=0.01$. To evaluate where this interaction of two three-levelled factors originated from, we followed the procedure suggested by Kirk,[34] evaluating influences of grade level (i.e., 3rd *vs*. 4th *vs*. 5th grade) on differences between arithmetic operations using univariate ANOVAs. The first ANOVA indicated that performance differences between addition and subtraction was not significantly influenced by grade, $F_{(2,703)}=1.07$, $p=0.34$, $\eta_p^2=0.01$. Importantly, results were different for performance differences between addition and multiplication, $F_{(2,703)}=23.07$, $p<0.001$, $\eta_p^2=0.06$, as well as subtraction and multiplication, $F_{(2,703)}=31.61$, $p<0.001$, $\eta_p^2=0.08$, for which the ANOVAs indicated significant effects of grade. Pairwise comparisons indicated that, for both addition and subtraction, differences with multiplication decreased as grade increased, with all pairwise comparisons being significant ($p<0.05$). In summary, this means that the significant interaction between grade and arithmetic operation reflects a decrease in performance differences between addition and multiplication, as well as between subtraction and multiplication, as grade increases whereas differences between performance in addition and subtraction did not change significantly across grades.

DISCUSSION

In this study, we evaluated the performance of Brazilian children on basic arithmetic operations. Moreover, considering evidence showing that Brazilian students perform poorly in mathematics more generally, we aimed at evaluating the feasibility of diagnosing dyscalculia using the psychometric criterion. Our results indicated that a considerable percentage of primary school children did not master basic arithmetic operations by the end of fifth grade — even in a rather wealthy Brazilian region.[28]

As such, our findings are in line with the performance of Brazilian students on PISA, which repeatedly revealed average mathematical achievement to be below basic proficiency levels. However, rather than assessing specific mathematical abilities taught in school, the abilities measured by PISA are more generic and related to the use of mathematics in everyday life.[35] Given that our participants were not able to solve basic arithmetic operations flawlessly, it may be the case that applying this kind of arithmetic knowledge to everyday situations, such as required by PISA, is challenging for Brazilian students.

The difficulties observed with the basic arithmetic operations in the present sample also have implications for the diagnosis of dyscalculia using a psychometric criterion. When the psychometric criterion is used, a more conservative percentile cut-off (e.g., ≤percentile 10) might allow the identification of children with severe and persistent mathematical difficulties.[25] On the other hand, a more liberal criterion (e.g., ≤percentile 25) increases the chances of identifying children with less severe and persistent difficulties that are more likely associated with other causes.[25]

In the Brazilian context, with most children performing poorly on basic arithmetic operations, the psychometric criterion might become inappropriate for the diagnosis of dyscalculia. In these circumstances, the psychometric criterion can lead to both false-negative and false-positive diagnoses. False-negatives occur when children who have inherent difficulties are not distinguished from those classified as typical achievers. In contrast, false-positive occur when children whose difficulties are caused by factors such as poor education are diagnosed as having dyscalculia. Reducing both false-negatives and false-positives is important for providing services for children with more severe and persistent mathematical difficulties. Furthermore, under budget constraints, children formally diagnosed with developmental disorders are prioritized to participate in intervention programs.[36]

An alternative approach to the diagnosis of dyscalculia, increasingly adopted worldwide, is to base decisions not only on test scores but also consider children's response to intervention (RTI).[4] The RTI approach aims at identifying children at risk for mathematical learning difficulties as early as Kindergarten, to provide them with additional mathematical instruction in successive tiers of increasing intensity. This approach is both preventive and therapeutic. In this context, the diagnosis of dyscalculia is restricted to those children who do not respond to even the best and most intensive pedagogical efforts. RTI has the advantage of constraining the problem of learning difficulties to the school. However, its logistics are complex, expensive, and require personnel training and compliance from both teachers and children. Additionally, RTI has the drawback of potentially delaying recognition of serious health conditions possibly underlying mathematics learning difficulties (e.g., genetic syndromes), as children are usually referred to specialized services for their learning difficulties.

The low performance of our participants on basic arithmetic operations may be a result of external factors, such as socioeconomic status (SES)[37,38] and educational experiences.[39] However, specific evaluation of these was beyond the scope of this study. Nevertheless, SES was found to have a significant influence on a Brazilian national measure of mathematics achievement[40] such that children with a better SES background outperformed those with lower SES. In line with this, we also observed a significant, but small, correlation of children's performance in addition ($r=0.10$, $p<0.01$) and multiplication ($r=0.09$, $p<0.05$) with SES in our sample. This corroborates the interpretation that poor performance observed for basic arithmetic operations in the present study may not only indicate MLD but also reflect influences of external educational factors. In this sense, effects of SES are also reflected in performance gaps observed between public and private schools, with private schools achieving scores higher than public schools and higher than the national average.[41,42] Importantly, however, it should be noted that in addition to SES the gap between public and private schools may also be a result of different educational practices and school quality. Even though the use of a core curriculum is highly encouraged in Brazil,[43] private schools usually push students harder and provide them with better educational and emotional support.

Educational experiences may also influence the performance in arithmetic operations. The Brazilian Ministry of Education (*Ministério da Educação* [MEC]) recently suggested a core curriculum, the National Common Core (*Base Nacional Comum Curricular* [BNCC]), aiming to unify pedagogical principles and goals across the country.[43] According to the BNCC, basic arithmetic operations are gradually introduced with increasing grade level, starting with addition in 1st grade, subtraction in 2nd grade, and multiplication in 3rd grade. Formal strategies and procedures are recommended to be explicitly and systematically taught from 3rd grade. It is expected that 4th graders should be able to fluently implement formal algorithms in addition and subtraction. Conceptual aspects of arithmetic operations are explicitly and systematically taught only in 4th grade. Remarkably, BNCC emphasizes the learning of conceptual and procedural arithmetic knowledge, whereas less effort is dedicated to promoting automatization of arithmetic facts. Despite the importance of conceptual and procedural knowledge, direct retrieval-based solutions were argued to be more efficient than calculation.[44] Moreover, poor automatization of basic arithmetic operations has also been associated with difficulty in acquiring more complex mathematical abilities.[45,46] This evidence highlights the importance of pedagogical practices, such as repetitive exercises with feedback and cumulative review, that promote automatization of arithmetic operations.[47] As we used a speeded assessment, our results may be interpreted as reflecting difficulties with fluency or automatization, probably due to the lower emphasis on this in the Brazilian curriculum.

In this study, we evaluated the performance of Brazilian children on the basic arithmetic operations of addition, subtraction, and multiplication. Overall, most children presented difficulties in all arithmetic operations assessed. Children presented better scores in addition, compared to subtraction and multiplication, and 3rd and 4th graders were outperformed by 5th graders in all three operations. However, 5th graders still have not mastered these basic arithmetic operations fluently, with less than 50% of 5th graders performing at 80% or above on addition, subtraction, or multiplication. This alarming scenario discourages the sole use of a psychometric criterion to diagnose dyscalculia.

When the majority of children are performing poorly on a task, it is hard to differentiate those with dyscalculia from those whose poor performance is due to external factors, such as inadequate schooling. We question the use of the psychometric criterion as the only index of a developmental disability. Instead, RTI approaches might be better suited to the Brazilian context. In addition to contributing to clinical practice, these results might also inform educators and policy makers.

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Authors' contributions. MRAG: formal analysis, investigation, project administration, and writing-original draft. ISA: formal analysis, investigation, and writing-original draft. GMP: data curation, investigation, and project administration. LSC: investigation. ALPNA: investigation. MRSC: funding acquisition, project administration, and writing-review & editing. JB: writing-review & editing. KM: funding acquisition, supervision, and writing-review & editing. JBLS: supervision and writing-review & editing. VHG: conceptualization, funding acquisition, methodology, project administration, supervision, and writing-review & editing.

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APPENDIX 2

From one half to 12th: acquisition of fraction writing in adult education program students and children

Isabella Starling-Alves[1*], Mariuche R. A. Gomides[2,3], Denise O. Ribeiro[2,3], Vitor G. Haase[2,3,4,5], & Edward M. Hubbard[1,6]

1. Department of Educational Psychology, University of Wisconsin Madison

2. Graduate Program in Psychology: Cognition and Behavior, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil

3. Laboratory of Developmental Neuropsychology, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil

4. Graduate Program in Neuroscience, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil

5. Instituto Nacional de Ciência e Tecnologia sobre Comportamento, Cognição e Ensino (INCT -ECCE), São Carlos, São Paulo, Brazil

6. Waisman Center, University of Wisconsin Madison

*Corresponding authors: 1075C Educational Sciences Building, 1025 Johnson Street, Madison, WI 53706, USA. ialves@wisc.edu

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Word count: 10,496 (text + references)

Abstract

Number transcoding is an important skill, which indicates how numbers are represented and can function as a marker of learning disabilities and neurological disorders. Although whole number transcoding has been deeply explored, little is known about fraction transcoding. We addressed this gap by investigating how groups with limited formal fraction instruction transcode fractions. 2nd graders and students from an adult education program (AEP) completed a fraction writing task. Overall, AEP students outperformed 2nd graders. Participants who struggled with fraction writing made a high frequency of syntactic errors, while participants with average to high fraction writing skills made a high frequency of lexical errors. Many participants wrote fractions as either whole or ordinal numbers. These results suggest that informal experiences contribute to the acquisition of fraction writing skills and that students' error types shift across development. Finally, results also suggest the whole number bias and an ordinal number bias in fraction writing. We suggest that, in early phases of the acquisition of fraction writing, students can draw on their prior number knowledge, but lack knowledge of the fractions format, leading to a high frequency of syntactic errors. As students master the fraction format, they make fewer syntactic errors but may still make lexical errors due to higher working memory load and/or phonological interference. Finally, students master fraction writing skills and make fewer to no errors. Our study is the first systematic investigation of fraction writing and therefore contributes to our understanding of number transcoding generally and to how people learn fractions.

Keywords: Fraction writing, Transcoding, Error analyses, Fractions, Development of numerical cognition

From one half to 12th: acquisition of fraction writing in adult education program students and children

Fractions are very important to mathematics learning. Learning fractions may expand students' knowledge about numerical magnitudes and consolidate their reasoning about the relationship between different numbers (Empson et al., 2011; Wu, 2001). Despite their importance, fractions are challenging; both children and adults struggle with fraction tasks (Stigler et al., 2010; Bentley & Bossé, 2018). In particular, people who are still learning about fractions struggle with reading and writing fractions in the common notation (i.e., fractions in the format $\frac{\text{numerator}}{\text{denominator}}$; Gelman, 1991; Saxe et al., 2005).

Mastering the ability to transcode fractions (i.e., convert one fraction notation to another, such as from a verbal form "one half" to a written form "$\frac{1}{2}$", and vice versa) is an important step to developing basic fraction comprehension and advance in mathematics education. Reading and writing numbers is crucial for understanding their properties and magnitudes. Few studies have investigated how school-age children read and write fractions (Gelman, 1991; Saxe, 2005). However, to date, no study has conducted a systematic analysis of fraction writing skills across different phases of development. In this study, we addressed this gap by investigating how Brazilian students in an adult education program (AEP) and 2nd graders write common fractions. In addition to exploring participants' overall accuracy in a fraction-writing task, we have also conducted a qualitative analysis of their errors.

Difficulties in fraction transcoding

Students are typically introduced to fractions in elementary school, between third and fifth grade, after they have developed familiarity with whole numbers (Brasil, 2017; National Governors Association, 2010). Students' whole number skills may leverage fraction knowledge (Siegler et al., 2011; Sidney, 2020). However, interference from whole numbers on fractions has also been observed, which is known as the whole number bias (Ni & Zhou, 2005). The whole number bias occurs when people incorrectly assign whole number properties to fractions (Siegler et al., 2011).

To the best of our knowledge, only few studies have investigated how children transcode fractions (e.g., Hurst & Cordes, 2018; Gelman et al., 1989; Saxe et al., 2005). In general, previous studies that performed a qualitative analysis of fraction transcoding errors indicated the whole number bias. Gelman and colleagues (1989) have shown that, when reading fractions aloud, many children suppress or modify the vinculum (i.e., the bar). In this case, fractions are read as multi-digit whole numbers (e.g., "$\frac{1}{2}$" read as "twelve"), two separate single-digit whole numbers (e.g., "$\frac{1}{2}$" read as "one and two"), or an arithmetic operation with whole numbers (e.g., "$\frac{1}{2}$" read as "one plus two"). The whole number bias may also be present in fraction writing. Saxe and colleagues (2005) observed that school-age children apply an uncommon notation when writing fractions, such as using dashes, commas, or even just a space to separate numerator and denominator, forming whole numbers instead of fractions (e.g., writing "1 2" to represent "one half").

Development of Number Reading and Writing Skills

The development of fraction transcoding skills has still been underexplored. To the best of our knowledge, only one study has investigated developmental differences between children and adults in fraction transcoding, and that study only examined fraction reading. Hurst and Cordes (2018) explored, cross-sectionally, how American students (4^{th} to 12^{th} grade) and adults transcoded common fractions to a verbal code.

Younger participants frequently used formal names when reading fractions (e.g., "$\frac{2}{5}$" read as "two fifths"), but older children were more likely to use informal names that highlighted the fractions' relational/algebraic structure (e.g., "$\frac{2}{5}$" read as "two over five"). Overall, these results indicate developmental shifts in fraction reading, but the corresponding questions have not been explored in fraction writing.

Given the lack of studies investigating the development of fraction writing skills, we can take the considerable body of literature on whole number writing as a reference. Traditionally, the development of whole number writing skills has been investigated with an analysis of participants' accuracy and error types (e.g., Moura et al., 2013; Zuber et al., 2009). The accuracy analysis indicates a substantial improvement in whole number writing during elementary school, with mastery of multi-digit number transcoding by 4th grade in Brazilian Portuguese speakers (Moura et al., 2013). The analysis of error types also indicates developmental shifts: whole number writing errors become more systematic and tend to disappear by the end of elementary school.

Two main categories of errors have been observed in whole number writing: pure lexical and pure syntactic errors (Deloche & Seron, 1982a). Pure lexical errors occur when the structure of the written number is correct, but the number lexicon is incorrect (e.g., hearing "forty-eight", and writing "47"). Pure syntactic errors occur when the main components of the written number are correct, but the structure of the number is incorrect (e.g., hearing "forty-eight" and writing "408") or when the order of the digits is incorrect (e.g., hearing "forty-eight" and writing "84"). Finally, lexical and syntactic errors can also co-occur (e.g., hearing "forty-eight" and writing "407"), which is known as a combined error (Zuber et al., 2009). In whole number writing, pure lexical errors are less frequent than pure syntactic errors throughout development

(Barrouillet et al., 2004; Moura et al., 2013). By the end of elementary school, children tend to master number transcoding skills, and the frequency of lexical and syntactic transcoding errors becomes minimal (Moura et al., 2013).

An analysis of the frequency and type of transcoding errors is valuable because it indicates participants' underlying difficulties in numerical representation. Seminal studies that analyzed number transcoding errors in patients with brain injuries indicated that nonsymbolic, verbal, and Arabic representations of numbers are partially dissociated, as well as knowledge of numerical lexicon and syntax (e.g., Dehaene & Cohen, 1991; Delazer & Bartha, 2001; Deloche & Seron, 1982b). These studies were instrumental for the development of neurocognitive models of number processing, such as the abstract modular model of number processing (McCloskey et al., 1985) and the triple-code model (Dehaene & Cohen, 1995). Therefore, by investigating breakdowns in number processing, made explicit by number writing errors, it is possible to infer how numbers are represented. In general, a high frequency of syntactic errors indicates difficulties with the transcoding rules of a given language. In contrast, a high frequency of lexical errors indicates difficulties in understanding the lexicon of a given number (Barrouillet et al., 2004; Deloche & Seron, 1982a).

To date, no study has conducted a systematic analysis of fraction writing errors. Therefore, the main difficulties underlying fraction writing skills are still unknown. In the present study, we addressed this gap by analyzing fraction writing errors across different phases of development. We chose to investigate fraction writing in participants who had low experience with fractions to avoid ceiling effects in our measure, which is crucial to conducting an error analysis. We expected that highly educated adults and children in advanced elementary school grades would have already mastered fraction transcoding. We therefore investigated fraction writing in a unique adult population—

Brazilian students in an adult education program—and children in 2nd grade. Participants in these groups have an important commonality: they have not been formally introduced to fractions in schools.

Fraction Education in Brazil

In Brazil, children typically start elementary school when they are 6 years old and enter the first grade. The formal education system consists of 12 years of schooling, with 9 years of primary education and 3 years of high school (Brasil, 1996). Education in Brazil is currently mandatory, and children in poverty are encouraged to stay in school through financial assistance and free school meal programs (Simões & Sabates, 2014). Despite political efforts to improve education in Brazil, school dropout rates are still a reason for concern. Among 50 million Brazilian people with ages ranging from 14 to 29 years old, 20% have abandoned school (IBGE, 2019). Historically, school dropout rates in Brazil have been high, particularly before the 1990s (Barretto & Mitrulis, 2001). There are many reasons for the increased school dropout rates in Brazil; however, social inequality has been indicated as the most important reason (Neri, 2015). As a consequence of the school dropout rates, illiteracy is still a reality for many Brazilian adults. National demographic data from 2019 indicates that 11.1% of Brazilian people with age over 40 years old cannot read or write (IBGE, 2019).

To reduce illiteracy rates, the Brazilian government has encouraged unschooled adults to enroll in adult education programs (AEP). The Brazilian AEP is free and available for people above 15 years of age and allows the conclusion of schooling in a shorter time: a minimum of 2 years to complete elementary and middle school and 18 months to complete high school (Brasil, 2016). Students are assigned to the AEP grades according to their proficiency in basic reading and numerical skills, which are assessed during enrollment. Brazilian AEP students in the elementary and middle school levels have mandatory Portuguese, Science, Mathematics, English, Arts, History, Geography, and Sports classes (Ribeiro, 2001). Because there is a shorter time window to complete the program, the AEP classes focus on the practical application of knowledge to daily life activities (Ribeiro, 2001). In Brazilian AEP, schools can choose the curriculum they will adopt. However, the national AEP curriculum proposal suggests that fraction education should be focused on nonsymbolic representations—diagrams, charts, and area models—and representations typically used in calculators, such as decimals. The AEP curriculum proposal discourages schools from teaching common fractions in the initial grades (Ribeiro, 2001).

In regular school, the Brazilian common core indicates that children should learn the common fraction notation in 4th grade when they are approximately 9 years old (Brasil, 2017). According to the Brazilian common core, 4th-grade children should be introduced to the most frequent unit fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{100}$) as measurement units of magnitudes smaller than one, using the number line as a tool. Then, in 5th grade, children should be introduced to other common fractions, learn to transcode them, and identify their magnitudes using the number line. The common core is widely used among Brazilian schools. However, some schools push fractions education to as early as 3rd grade.

Fraction Names in Brazilian Portuguese

Similar to English, the fraction names in Brazilian Portuguese indicate the numerator first, followed by the denominator. In general, whole number names are used for numerators. However, the rules for denominators are more complex. When the denominator is either a single-digit number or a power of ten, specific words are used, as in English. In particular, ordinal number names are used for denominators. For example, "$\frac{1}{4}$" is read as "*um quarto* (one-fourth)". However, when a denominator is a multi-digit number and not a power of ten, the denominator is read as a whole number followed by the word "*avos*". For example, is "$\frac{1}{12}$" read as "*um-doze avos* (one-twelve "*avos*")".

Present study

In the present study, we conducted two experiments exploring how children and adults transcode fraction names to the common notation. Participants' success in a fraction writing task may indicate intact abilities in fraction comprehension and production. In contrast, the types and frequencies of participants' errors can inform us about the main difficulties in fraction writing. Furthermore, it can inform us how whole numbers and other number systems interfere with the ability to write common fractions. By understanding the main difficulties associated with fraction writing across development, we can have insights on how people mentally represent fractions and outline developmental models.

In Experiment 1, adults enrolled in the first year of a Brazilian AEP completed a fraction writing task. In addition to analyzing participants' overall performance, we conducted an in-depth analysis of their errors and propose an error categorization criterion. Since our participants had probably interacted with fractions in informal contexts, we expected that they would be able to accurately write at least some fractions. Furthermore, we expected our participants to commit more syntactic than lexical errors, analogous to patterns observed in whole number transcoding studies (Barrouillet et al., 2004; Moura et al., 2013). Finally, we expected that participants' knowledge of whole numbers could interfere with their ability to write fractions.

In Experiment 2, we investigated the effects of experience in fraction writing: Brazilian children in 2nd grade completed the same fraction writing task as the AEP students from Experiment 1. We analyzed children's performance in this task and contrasted it with the performance of AEP students from Experiment 1. Since the 2nd graders had less years of informal experience with fractions, we expected that they would commit more fraction writing errors than AEP students. Furthermore, similar to our predictions in Experiment 1, we expected that 2nd graders would commit more syntactic than lexical errors and that their error types would indicate a whole number bias in fraction transcoding.

Experiment 1: Fraction transcoding in AEP students

Material and methods

Participants

As part of a larger study, we recruited 40 students enrolled in the Brazilian adult education program (AEP). Three participants did not complete the fraction writing task and were excluded from our analysis. Thus, the final sample had 37 AEP students. Participants' mean age was 43.81 years (±8.53), and 59% of the sample self-identified as female (41% male). Participants received an average of 3.41 years (±1.24) of formal education when they were children. All participants were enrolled in the first year of the AEP program, given their proficiency in basic reading and numerical skills, according to their schools' assessment.

Procedures and materials

This study has been approved by the local Ethics Committee (CAAE 94116718.0.0000.5149). We recruited participants via oral advertisement in AEP schools from the metropolitan region of a large city in Minas Gerais-Brazil. There were two enrollment waves, one (n = 20) assessed in the second semester of 2018, and the other (n = 17) assessed in the first semester of 2019. Participants from the two different enrollment waves did not significantly differ in age, $t(35) = 1.34$, $p = .19$, $d = .45$, and years of schooling as children, $t(35) = 0.56$, $p = .58$, $d = .19$. All participants were individually assessed in quiet rooms in their schools, in two sessions of approximately one hour each. In the first session, they were introduced to the project, signed the consent form, and completed an intelligence measure. In the second session, participants completed mathematics tasks. Tasks are described below.

Intelligence. Participants' intelligence quotient (IQ) was estimated from two subtests of the Brazilian version of the Wechsler Abbreviated Scale of Intelligence (Wechsler et al., 2014): matrix reasoning and vocabulary. The Brazilian WASI has been normalized in Brazil with a diverse sample of adults, including adults with low literacy and numeracy.

Whole number writing task. AEP students from the first enrollment wave (n = 20) also completed a whole number writing task (see Moura et al., 2013). Participants heard whole number names and were asked to write them in the Arabic format. The task had 28 items (3 one-digit numbers, 9 two-digit numbers, 8 three-digit numbers, and 8 four-digit numbers). One point was given for each correct answer and we classified errors as syntactic, lexical, combined, or "others" (Deloche & Seron, 1982a; Zuber et al., 2009).

Fraction writing task. In the fraction writing task, the examiner read fraction names to the participants, who were asked to write them in the common format (e.g., hear "four sevenths" and write "$\frac{4}{7}$"). The examiners mentioned that participants were about to complete a fraction writing task in the instructions, but no example was given. The examiners repeated the item if requested by the participant by reading the full name of the fraction. Our team developed this task with 27 items generated from single-digit irreducible fractions (Table 1), which were identified from a previous study (Binzak et al., 2020). We decided to use single-digit irreducible fractions because multi-digit fractions could be inappropriate for our sample's expertise level. Furthermore, using irreducible fractions minimizes the possibility of highly diverse responses depending on how participants reduce the fractions (e.g., hear "ten twentieths" and write "$\frac{10}{20}$", "$\frac{5}{10}$", or "$\frac{1}{2}$"). One point was given for each correct answer, and we used the percent correct in our analyses.

Table 1. Items in the fraction writing task

Common fraction	**Fraction name in Portuguese**	**Common fraction**	**Fraction name in Portuguese**	**Common fraction**	**Fraction name in Portuguese**
$\frac{1}{9}$	*Um nono*	$\frac{1}{3}$	*Um terço*	$\frac{2}{9}$	*Dois nonos*
$\frac{1}{8}$	*Um oitavo*	$\frac{2}{7}$	*Dois sétimos*	$\frac{1}{5}$	*Um quinto*
$\frac{1}{7}$	*Um sétimo*	$\frac{1}{4}$	*Um quarto*	$\frac{1}{6}$	*Um sexto*
$\frac{3}{8}$	*Três oitavos*	$\frac{5}{8}$	*Cinco oitavos*	$\frac{5}{9}$	*Cinco nonos*
$\frac{2}{5}$	*Dois quintos*	$\frac{3}{5}$	*Três quintos*	$\frac{1}{2}$	*Um meio*
$\frac{3}{7}$	*Três sétimos*	$\frac{4}{7}$	*Quatro sétimos*	$\frac{4}{9}$	*Quatro nonos*
$\frac{2}{3}$	*Dois terços*	$\frac{8}{9}$	*Oito nonos*	$\frac{5}{6}$	*Cinco sextos*
$\frac{5}{7}$	*Cinco sétimos*	$\frac{7}{8}$	*Sete oitavos*	$\frac{4}{5}$	*Quatro quintos*

$\frac{3}{4}$	*Três quartos*	$\frac{6}{7}$	*Seis sétimos*	$\frac{7}{9}$	*Sete nonos*

Results

Intelligence

Participants' mean IQ indicated low but normal intelligence (*M* = 81.8 $\pm$ 10.9). There was no significant difference between participants' standardized scores in the Vocabulary (*M* = 39.9 $\pm$ 7.2) and the Matrix Reasoning subtests (*M* = 39.3 $\pm$ 6.9), *t(36)* = .52, *p* = .60, *d* = .09. Furthermore, participants' standardized scores in these subtests were significantly correlated, *r* = .56, *p*<.001.

Whole number writing task

The subset of AEP participants who completed the whole number writing task had high accuracy in this measure (*M* = 90% $\pm$ 8). Participants made a higher frequency of pure syntactic errors (80%) compared to pure lexical errors (11%) and errors classified as "others" (9%). Participants made no combined errors. In single-digit items and two-digit items, participants had perfect accuracy. They also had high accuracy in three-digit items (*M* = 97% $\pm$ 17) and four-digit items (*M* = 69% $\pm$ 46). In three-digit items, participants made a higher frequency of errors classified as "others" (60%) than syntactic (20%) and lexical errors (20%). In four-digit items, participants made a higher frequency of syntactic errors (86%) compared to lexical errors (10%) and errors classified as "others" (4%). Participants' performance in the whole number writing task was not significantly correlated to their performance in the fraction writing task (*r* = .38, *p* = .09).

Fraction writing task

Group-level analysis indicated that participants correctly wrote most items of the fraction writing task (M = 61% $\pm$ 45). However, the distribution was bimodal. Some participants had high performance while others struggled with this task. Thirteen participants correctly wrote less than 15% of items (11 participants had a score of 0, one participant had a score of 7%, and one participant had a score of 11%), one participant correctly wrote 55% of items, eighteen participants correctly wrote between 85% and 96% of items, and five participants had a perfect score.

The percentage of correct responses by item (i.e., sum of participants' scores in each item divided by the number of participants) is presented in Figure 1. The percentage of correct responses was similar across items, ranging between 51% to 68%. The item with the highest percentage of correct responses was $\frac{3}{8}$ (M = 68% $\pm$ 47), and the item with the lowest percentage of correct responses was $\frac{2}{9}$ (M = 51 % $\pm$ 51). Curiously, fractions that are frequently used in daily-life activities, such as $\frac{1}{2}$ (M = 54% $\pm$ 51) and $\frac{1}{3}$ (M = 59% $\pm$ 50), did not have a very high percentage of correct responses.

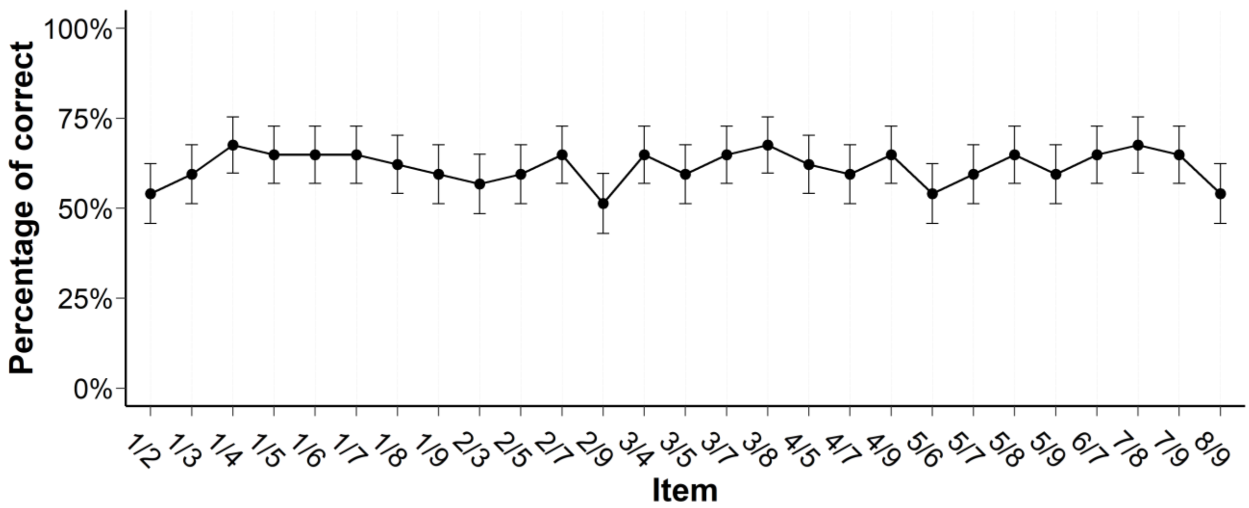


Figure 1. Percentage of correct responses by item.

We next conducted a qualitative analysis of participants' errors in the fraction writing task. We adapted a broad criterion extensively used in the whole number writing literature (Deloche & Seron, 1982a), and categorized participants' errors as:

1) **Lexical** when the roles of numerator and denominator were preserved, but the digits were incorrect.
2) **Syntactic** when the structure of the fraction was not preserved, or the numerator and the denominator were inverted
3) **Combined** when the roles of numerator and denominator were not preserved, and at least one digit was incorrect.
4) **Others** when participants' errors did not fit any of these categories (e.g., blank item) or the error only occurred once.

Using an iterative, data-driven approach, we developed a criterion of error subcategories that were specific to fraction writing. We considered the presence and format of the numerator, the denominator, and the vinculum (i.e., the bar). To validate our criterion, we invited two blind judges to categorize participants' errors according to our proposed error subcategories. Overall, there was moderate to high agreement between the judges, as indicated by the mean Cohen's kappa, $M = .90 \pm .05$. For the items on which the judges disagreed, a third judge was invited to help decide between the subcategorizations. Then, we investigated the frequency of each error subcategory.

We used the total number of errors committed in the task (384 errors out of 999 responses) to investigate the frequency of each error category. Corroborating our predictions, pure syntactic errors were the most frequent (86%), followed by pure lexical errors (8%), errors classified as "others" (3%), and combined errors (3%), as summarized in Figure 2. A chi-square test of goodness-of-fit, $\chi^2(3)$ =763.19, p<.001, confirmed that these frequencies were not equally distributed.

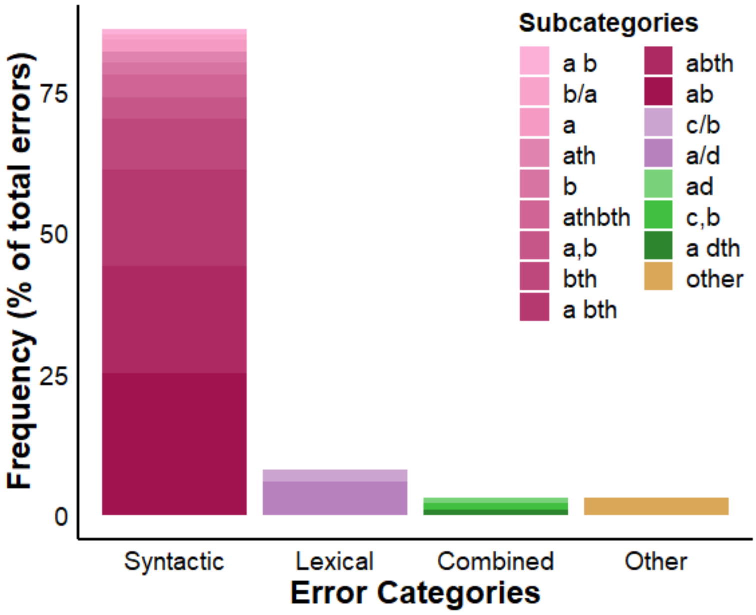
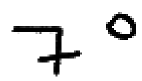
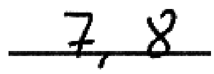


Figure 2. Frequency of the fraction writing error categories and subcategories in AEP students. Pure syntactic errors were more frequent than pure lexical errors, combined errors, and errors classified as "others".

Among the pure syntactic errors, we observed eleven error subcategories, as described in Table 2. As we predicted, many participants frequently wrote the fractions as whole numbers (e.g., "one half" written as "1", "2" or "12"), indicating whole number bias. Surprisingly, many participants wrote fractions as ordinal numbers (e.g., "one half" written as "1st", "2nd", or "12th"). Also, some participants separated the numerator and the denominator using the decimal mark instead of using the vinculum (e.g., "one half" written as "1.2"), and some participants inverted the position of numerator and denominator (e.g., "one half" written as "$\frac{2}{1}$").

Table 2. Syntactic error subcategories

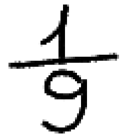
Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency*** **(% total errors)**
ab	Two-digit whole number composed of numerator and denominator	13 "One-third"	25
a b	Single-digit whole number composed of the numerator and single-digit whole number composed of the denominator	1 4 "One-fourth"	1
a	Single-digit whole number composed of the numerator	2 "Two-thirds"	2
b	Single-digit whole number composed of the denominator	$\underline{7}$ "One-seventh"	2
ab^{th}	Two-digit ordinal number composed of numerator and denominator	18° "One-eighth"	19
a^{th} b^{th}	Single-digit ordinal number composed of the numerator and single-digit ordinal number composed of the denominator	$\underline{2^{\circ}5^{\circ}}$ "Two-fifths"	4
a b^{th}	Single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator	5 8° "Five- eighths"	17
a^{th}	Single-digit ordinal number composed of the numerator	$\underline{1^{\circ}}$	2

Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency*** **(% total errors)**
		"One-half"	
b^th	Single-digit ordinal number composed of the denominator	 "One-seventh"	9
a.b	Decimal number with the numerator as the whole number part and the denominator as the decimal part	 "Seven- eighths"	4
b/a	Correct format with an inversion between the numerator and the denominator	 "Five-sevenths"	1

Note. *In Brazilian Portuguese 1) the symbol "º" indicates ordinal numbers, similar to "th" in English, 2) the decimal marker is a comma instead of a point. *Rounded values*

Overall, pure lexical errors were less frequent than pure syntactic errors. The subcategories of pure lexical errors are described in Table 3. We observed two subcategories of pure lexical errors: wrong digit in the numerator with a correct denominator (e.g., "one half" written as "$\frac{3}{2}$"), and correct numerator with a wrong digit in the denominator (e.g., "one half" written as "$\frac{1}{3}$").

Table 3. Lexical error subcategories

Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency*** **(% total errors)**
$\frac{c}{b}$	Correct format with lexical error in the numerator	 "Two-ninths"	2
$\frac{a}{d}$	Correct format with lexical error in the denominator	 "One-eighth"	6

Note. **Rounded values*

Finally, we observed three subcategories of combined errors, as described in Table 4: two-digit whole number composed of the numerator and an incorrect denominator (e.g., "one half" written as "14"), single-digit whole number composed of the numerator and an ordinal number as the denominator (e.g., "one half" written "1 4^{th}"), and a decimal number with incorrect numerator as the whole number part and the denominator as the decimal part (e.g., "one half" written as "3.2").

Table 4. Combined errors subcategories

Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency*** **(% total errors)**
ad	Two-digit whole number composed of numerator and denominator, with lexical error in the denominator	 "Six-sevenths"	1

Error **($\frac{a}{b}$ written as)**	**Error Specification**	**Example**	**Frequency*** **(% total errors)**
a dth	Single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator, with lexical error in the denominator	3 7º "Three-fourths"	1
c.b	Decimal number with the numerator as the whole number part and the denominator as the decimal part, with lexical error in the numerator	6,8 "Three-eighths"	1

Note. *In Brazilian Portuguese 1) the symbol "º" indicates ordinal numbers, similar to "th" in English, 2) the decimal marker is a comma instead of a point. *Rounded values*

We also compared the performance of participants who committed pure syntactic errors more frequently than the other error categories (N = 13) to the performance of participants who committed pure lexical errors more frequently than the other categories (N = 14). Overall, participants who committed predominantly pure lexical errors had a higher score in the fraction writing task (M = 91% $\pm$ 11) than participants who committed predominantly pure syntactic errors (M = 9% $\pm$ 27), U = 170, p<.001. These results suggest that people with very poor fraction writing skills may commit a high frequency of pure syntactic errors. On the other hand, people that have some familiarity with writing common fractions may commit a higher frequency of pure lexical than pure syntactic errors.

Discussion

In Experiment 1, we investigated the performance of AEP students in a fraction writing task. We observed that some participants had high to perfect accuracy while others had very low accuracy. This highly heterogeneous performance may not be explained by intellectual deficits, since participants' intelligence was normal. Furthermore, it may not be explained by poor whole number writing skills, since all participants who completed the whole number writing task had high to perfect accuracy, even those who struggled with fraction writing. Since some AEP students can write fractions even without receiving formal instructions on it, informal experiences may play an important role in the development of fraction transcoding. The extent of participants' informal experiences with fractions may explain their highly heterogeneous performance in fraction writing.

We also conducted a qualitative analysis of their errors. In general, we observed a high frequency of pure syntactic errors, followed by pure lexical errors, combined errors, and errors classified as "others". In particular, we observed that participants who struggled with fraction writing made a higher frequency of pure syntactic errors compared to pure lexical errors. In contrast, participants with higher scores in the fraction writing task made a higher frequency of pure lexical errors compared to pure syntactic errors, suggesting a shift in error type as participants start mastering this task. Finally, the higher frequency of pure syntactic than pure lexical error in fraction writing is similar to the pattern of error frequencies observed in the whole number writing task (Moura et al., 2013). Traditionally, syntactic errors reveal difficulties with the rules necessary for writing numbers, and lexical errors reveal poor lexical knowledge or are driven by executive functions (Barrouillet et al., 2004).

We have also analyzed specificities in participants' fraction writing errors. We observed that participants frequently wrote fractions as either whole numbers or ordinal numbers. Writing fractions as whole numbers is an error that has been previously observed in the literature, and indicates whole number bias (Gelman et al., 1991).

However, the fact that participants wrote fractions as ordinal numbers suggests that this number system may be an extra source of interference in fraction writing learning. These results suggest an ordinal number bias in fraction writing.

Experiment 2: Fraction writing in 2nd graders

To better investigate the role of informal experiences on fraction writing and characterize the fraction writing error types early in development, we conducted a second experiment. In Experiment 2, we investigated fraction writing in 2nd graders. Similar to AEP students, 2nd graders have not been formally introduced to fractions in schools. However, differently from the AEP students, 2nd graders have had less time interacting with fractions in informal contexts. If informal experiences explain AEP students' fraction knowledge, then 2nd graders should have lower performance than AEP students in the fraction writing task. We expected that 2nd graders would also commit a higher frequency of pure syntactic errors compared to pure lexical errors. Finally, we expected that their error types would indicate whole number bias. In addition to investigating 2nd graders performance in a fraction writing task, we have also investigated their intelligence and whole number writing skills. In Experiment 1, only a subset of participants completed a whole number writing task. In Experiment 2, we addressed this limitation by systematically investigating whole number writing skills of all participants.

Material and methods

Participants

As part of a larger study, 20 Brazilian 2nd graders were recruited. Participants' mean age was 7.27 years (± .46), and 75% of the sample self-identified as female (25% male). All participants gave oral assent, and their parents or legal guardians signed the consent form.

Procedures and Materials.

This study has been approved by the local Ethics Committee (CAAE 15070013.1.0000.5149). Participants were recruited via an advertisement in public schools in the metropolitan region of a large city in Minas Gerais - Brazil. Participants were assessed in groups of six children, in one-hour long sessions that took place in their schools. In addition to the fraction writing task, all participants completed measures of intelligence and whole number writing. We describe these tasks below.

Intelligence. We assessed participants' intelligence with the Raven's Coloured Progressive Matrices and calculated z-scores according to the Brazilian norms (Raven et al., 2018).

Whole number writing task. In the whole number writing task, children heard whole number names and were asked to write them in the Arabic format. This task was designed for children from 2nd to 5th grade and had 81 items (i.e., 2 one-digit numbers, 6 two-digit numbers, 19 three-digit numbers, and 54 four-digit numbers). One point was given for each correct response and z-scores were calculated according to Brazilian norms (Gomides et al., in press).

Fraction writing task. Children completed the same fraction writing task we used in Experiment 1.

Results

Intelligence

Results indicated that participants had normal intelligence, as measured with z-scores calculated based on local norms (M = -0.13 $\pm$ 0.77).

Whole number writing task

Participants' mean accuracy in the whole number writing task was 20% $\pm$ 10, which is expected for their grade according to z-scores computed based on local norms (M = -0.33 $\pm$ 0.40). In general, 2nd graders made a higher frequency of pure syntactic (60%) and combined errors (30%) than pure lexical errors (10%). Participants had perfect accuracy in the one-digit items (M = 100% $\pm$ 0), high accuracy in two-digit items (M = 88% $\pm$ 32), and low accuracy in three-digit (M = 40% $\pm$ 49) and four-digit items (M = 3% $\pm$ 17). In two-digit items, pure lexical errors (79%) were more frequent than pure syntactic errors (14%). However, in three-digit and four-digit items, pure syntactic errors (60%) were more frequent than pure lexical errors (10%). 2nd graders' scores were lower than AEP students' scores in whole number writing, even when just overlapping items across the different versions of the whole number writing task were considered, $t(38) = 10.48$, $p<.001$, $d = 3.38$ (see Supplementary Material).

Fraction writing task

The 2nd graders had a floor effect in the fraction writing task (all scores = 0), contrasting with AEP students' performance. This result indicates that 2nd graders still have not learned how to convert fraction names to the common fraction notation through informal experiences. Despite the floor effect, we still classified children's errors in the fraction writing task according to the criterion we developed in Experiment 1. We modified this criterion in an iterative process, including children's errors that we did not observe in the AEP students' data.

As in Experiment 1, we invited two judges to classify the children's errors according to the criterion we developed. Overall, there was moderate to high agreement between the judges, as indicated by the mean Cohen's kappa, M = .96 $\pm$ .07. In the items on which the judges disagreed, a third judge was invited to help decide between the subcategorizations. Then, we investigated the frequency of each error type.

We used the total number of errors committed in the task (540 errors) to investigate the frequency of each error category. As illustrated in Figure 3, pure syntactic errors were the most frequent (92%) followed by combined errors (4%) and errors classified as "others" (4%). The 2nd graders did not make pure lexical errors. The frequencies of pure syntactic, combined, and errors classified as "others" in children's data were not equally distributed, as indicated by a chi-square test of goodness-of-fit, $\chi^2(2) = 842.71$, $p<.001$. We conducted a chi-squared test of independence to compare the proportion of these broad error categories in 2nd graders and AEP students. Results indicated a significant association between group and proportion of errors, $\chi^2(3) = 46.05$, $p<.001$. Overall, 2nd graders were more likely to commit pure syntactic errors than AEP students.

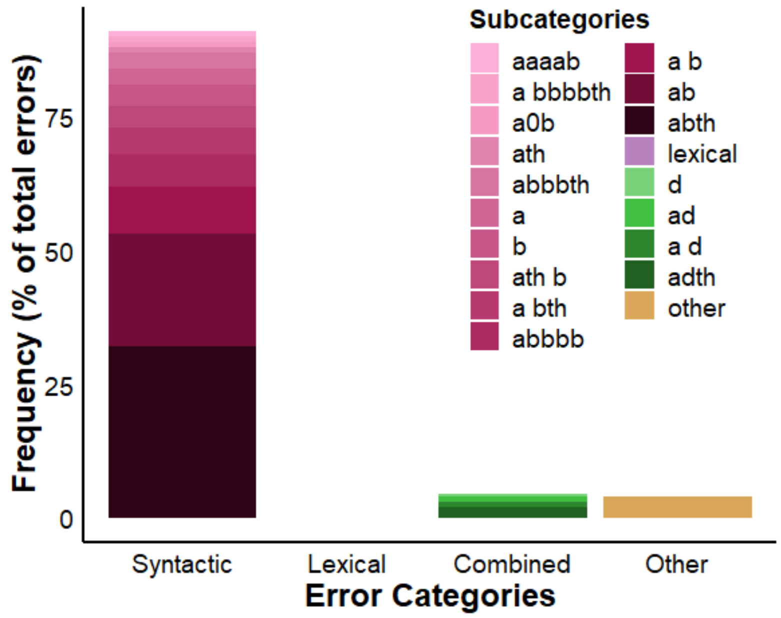
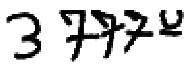
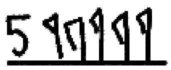


Figure 3. *Frequency of the fraction writing error categories and subcategories in 2nd graders. Pure syntactic errors were more frequent than combined errors and errors classified as "others". Participants made no lexical errors.*

Among the pure syntactic errors, we observed thirteen subcategories, as described in Table 5. Like AEP students, 2^{nd} graders frequently wrote fractions as either whole numbers (e.g., “one half” written as “12”, “1 2”, “2”) or ordinal numbers (e.g., “one half” written as “12^{th}”, “1 2^{nd}”, “1^{st}”, “1^{st} 2”). We also observed new error subcategories in 2^{nd} graders’ responses, which we had not observed in the AEP students’ data. Some participants wrote fractions as a multi-digit whole or ordinal number with repetition of either the denominator or the numerator (e.g., “one half” written as “1222^{nd}”, “1222”, or “1112”). Finally, some participants wrote a three-digits number with the number zero between the numerator and the denominator (e.g., “one half” written as “102”).

Table 5. Syntactic errors subcategories

Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency (% total errors)***
ab	Two-digit whole number composed of numerator and denominator	13 “One-third”	21
a b	Single-digit whole number composed of the numerator and single-digit whole number composed of the denominator	1 4 “One-fourth”	9
a	Single-digit whole number composed of the numerator	2 “Two-thirds”	3
b	Single-digit whole number composed of the denominator	7 “One-seventh”	4

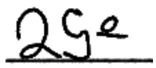
Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency (% total errors)***
abth	Two-digit ordinal number composed of numerator and denominator	 "One-eighth"	32
a bth	Single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator	 "Five-eighths"	5
ath	Single-digit ordinal number composed of the numerator	 "One-half"	1
ath b	Single-digit ordinal number composed of the numerator and single-digit whole number composed of the denominator	 "Three- eighths"	4
abbbth	Multi-digit ordinal number composed of numerator and denominator, with repetition of the denominator	 "Three-sevenths"	3
abbbb	Multi-digit whole number composed of numerator and denominator, with repetition of the denominator	 "Five-sevenths"	6
aaaab	Multi-digit whole number composed of numerator and denominator, with repetition of the numerator	 "One-sixth"	1
a bbbbth	Single-digit whole number composed of numerator and multi-digit ordinal number composed of the denominator, with repetition of denominator	 "Two-ninths"	1

Error ($\frac{a}{b}$ written as)	Error Specification	Example	Frequency (% total errors)*
a0b	Multi-digit whole number composed of numerator, zero, and denominator	 "One-fifth"	1

Note. *Please, note that, in Brazilian Portuguese, the symbol "º" indicates ordinal numbers, similar to "th" in English. *Rounded values*

We also observed four subcategories of combined errors, as described in Table 6. One type of combined error was also observed in Experiment 1: writing a two-digit whole number composed of numerator and denominator, with a lexical error in the denominator (e.g., "one half" written as "14"). However, 2nd graders committed three new errors, not observed in the AEP students' data: writing a two-digit ordinal number composed of numerator and denominator, with lexical error in the denominator (e.g., "one half" written as "13th"), writing a single-digit whole number composed of the denominator with lexical error (e.g., "one half" written as "3"), and writing a single-digit whole number composed of the numerator and a single-digit whole number composed of an incorrect denominator (e.g., "one half" written as "1 3"). Importantly, most children committed a combined error when writing "one half." Instead of using the digits 1 and 2, most children wrote "one half" using the digits 1 and 6. In Brazilian Portuguese, "six" is usually referred to as "half-dozen (*meia-dúzia*)." Therefore, participants may have associated the word "half" with the digit 6.

Table 6. Combined errors subcategories

Error ($\frac{a}{b}$ **written as)**	**Error Specification**	**Example**	**Frequency (% total errors)***
ad	Two-digit whole number composed of numerator and denominator, with lexical error in the denominator	 "Six-sevenths"	1
ad^{th}	Two-digit ordinal number composed of numerator and denominator, with lexical error in the denominator	 "Two-fifths"	2
d	Single-digit whole number composed of the denominator with lexical error (special case observed for ½)	 "Half"	0.4
a d	Single-digit whole number composed of numerator and single-digit whole number composed of incorrect denominator	 "Half"	1

Note. *Please, note that, in Brazilian Portuguese, the symbol "º" indicates ordinal numbers, similar to "th" in English. *Rounded values*

Discussion

In Experiment 2, we investigated how 2nd graders write fractions. Like the AEP students from Experiment 1, 2nd graders have not been formally introduced to fractions in schools. However, given their young age, 2nd graders have had less time to learn fractions in informal contexts than the AEP students. Our results indicated that 2nd graders could not correctly write any common fraction. This result suggests that younger children have not learned how to write fractions via informal experience. Our participants had normal intelligence, and their ability to write whole numbers was appropriate for their grade. More specifically, 2nd graders had high accuracy in single-digit and two-digit whole number writing. Therefore, their poor performance in fraction writing may not be explained by cognitive deficits or poor whole number writing skills.

A qualitative analysis of 2nd graders' errors indicated a high frequency of pure syntactic and combined errors. In particular, children frequently wrote fractions as either whole numbers or ordinal numbers. However, 2nd graders did not commit pure lexical errors. This result indicates that 2nd graders had poor knowledge of the rules necessary for writing common fractions. The high frequency of syntactic errors early in development, as observed in 2nd graders, is consistent with the error pattern we observed in Experiment 1: higher frequency of pure syntactic than pure lexical errors in participants who had not mastered fraction writing. However, these fraction writing errors contrast with children's error types in the whole number writing task. In whole number writing, children made both pure syntactic and pure lexical errors. Therefore, fraction and whole number writing may have distinct developmental trajectories, which should be investigated in future studies.

General Discussion

In this study, we thoroughly analyzed and classified error types in fraction writing. To the best of our knowledge, the present study is the very first to do so. Traditionally, the error types committed in number transcoding have been more informative than an accuracy analysis alone. The error types in whole number transcoding have been used to develop models of number processing (e.g., McCloskey et al., 1985; Dehaene & Cohen, 1995), and have been used as markers of learning disabilities and neurological disorders (e.g., Moura et al., 2013; Delazer & Bartha, 2001; Deloche & Seron, 1982b). We suggest that the analysis of fraction transcoding errors can similarly inform us about how fractions are represented and be used as a neuropsychological and educational tool.

We investigated the fraction writing skills of two groups: AEP students and 2nd graders. Both AEP students and 2nd graders have not received formal instruction on fractions in schools. However, the groups have had different amounts of informal experiences with fractions. We predicted that AEP students would accurately write at least some fractions, given their lifetime of informal experiences with fractions. Since 2nd graders had lower experience with fractions, we predicted that they would have lower performance than AEP students in a fraction writing task. Overall, results corroborated our predictions. However, AEP students' performance was bimodal. Some AEP students performed very well in the fraction writing task, and some performed poorly. Unlike AEP students, none of the 2nd graders could accurately write fractions, not even those that are more frequently used in daily life situations (e.g., "one half"). Our results indicated that the performance of participants who struggled with fraction writing could not be explained by intellectual deficits or poor whole number writing skills. Therefore, informal experiences with fractions could drive group differences in fraction writing skills.

We also analyzed the error types made by our participants in the fraction writing task. Overall, we expected that participants in both groups would commit a higher frequency of syntactic than lexical errors, aligned to what has been previously observed in whole number transcoding (Deloche & Seron, 1982a; Moura et al., 2013; Zuber et al., 2009). Corroborating our prediction, AEP students made a higher frequency of pure syntactic errors compared to pure lexical errors. The 2nd graders also made a high frequency of pure syntactic errors. However, they did not make pure lexical errors. The high frequency of syntactic errors in both groups, in particular among participants with low performance in fraction writing, indicates a poor knowledge of the rules necessary to write common fractions.

Finally, we predicted that the whole number bias would be observed in the qualitative analysis of fraction writing errors. Corroborating our prediction, we observed that many children and adults wrote fractions as whole numbers. In addition to the whole number bias, we also observed that many participants wrote fractions as ordinal numbers. Therefore, there may be an ordinal number bias in fraction transcoding.

Error types in fraction transcoding

Few studies have described children's errors in fraction transcoding. Gelman and colleagues (1989) asked children in different grades to read common fractions. Overall, children improved their fraction reading skills with grade. However, in all grades, some children read fractions as two single-digit whole numbers (e.g., "$\frac{1}{2}$" read as "one and two"), one multi-digit whole number (e.g., "$\frac{1}{2}$" read as "twelve"), or an arithmetic operation (e.g., "$\frac{1}{2}$" read as "one plus two"). Saxe and colleagues (2005) investigated children's ability to represent fraction area models (e.g., pies and squares) using common fractions. Some students wrote the fractions as single-digit whole numbers composed by either numerator or denominator (e.g., one half written as "1"), single-digit whole numbers separated by a dash, a comma, or space (e.g., one half written as "1-2"), or multi-digit whole numbers (e.g., one half written as "12"). Altogether, these results suggest that children have difficulties reading and writing fractions and that the whole number bias is present in fraction transcoding. However, the error types associated with transcoding fraction names to the common notation were still underexplored. In the present study, we have addressed this gap.

We observed that AEP students and 2nd graders made a higher frequency of pure syntactic errors compared to pure lexical errors in a fraction writing task. Furthermore, lexical errors were committed mostly by participants who had high scores in the fraction writing task. These results suggest that participants who struggle with fraction writing are those who have poor knowledge of fraction writing rules, which manifests as a high frequency of pure syntactic errors. These results are aligned with patterns observed in whole number transcoding studies, which have shown that pure syntactic errors occur more frequently than pure lexical and combined errors, particularly in early grades (Deloche & Seron, 1982a; Moura et al., 2013; Zuber et al., 2009).

We also used an iterative, data-driven approach to classify error subcategories specific to fraction writing. Our results corroborated our prediction that the whole number bias (Gelman, 1991; Ni & Zhou, 2005) would be observed in participants' errors. Since our participants have not mastered the common fraction notation yet, they may have written a code more familiar to them: whole numbers. Many participants wrote fractions as single-digit or multi-digit whole numbers, resembling errors previously observed by Saxe and colleagues (2005). Moreover, many participants wrote fractions as ordinal numbers. This error may be explained by the phonological similarity between the fraction names and the names used for ordinal numbers in Brazilian Portuguese. Writing fractions as ordinal numbers resembles the term-by-term correspondence error previously observed in the whole number literature, in which number components are literally transcoded, and the number transcoding rules are ignored (e.g., the French name for eighty, "quatre-vingts", literally "four-twenty", written as "420"; Deloche & Seron, 1982a). These results suggest an ordinal number bias in fraction transcoding: previous knowledge about ordinal numbers interferes with the ability to write common fractions. According to the Brazilian curriculum, ordinal numbers are taught in the first grade of regular school and the AEP program, before fractions are explored in the classrooms (Brazil, 2017; Ribeiro, 2001).

Effects of informal experiences on fraction knowledge

In the present study, both the quantitative and qualitative analyses indicated that some AEP students have learned how to write common fractions. Even without receiving formal instruction on fractions, the group of AEP students had accuracy above 60% in our fraction writing task. In contrast, 2nd graders could not accurately write any common fraction. These results indicate that informal experiences may contribute to the development of fraction writing skills.

The AEP students were fully functioning in society, working full-time jobs (e.g., cooks, drivers, housemaids). These participants may need to read and write fractions to complete tasks in informal contexts, such as measuring ingredients to cook a recipe, reading analog clocks, or reading and filling their cars' gas tanks. The 2nd graders may also have interacted with fractions in informal contexts. However, their years of informal experiences with fractions may not have been sufficient for them to learn to transcode fractions.

Although formal schooling is crucial for the development of mathematics knowledge, mathematics can also be learned via ecologically supported informal experiences (D'Ambrosio, 1985; Tunstall & Ferkany, 2017). In a classic study, Saxe (1988) showed that Brazilian children with little or no schooling successfully solved mathematics problems in informal contexts, such as selling candy in the streets. These results indicate that schooling is not the only path to learning mathematics (Carraher et al., 2013; Nunes et al., 1993). We suggest that informal experiences may play an analogous role in the development of fraction skills, including fraction transcoding.

The extent and quality of informal experiences with fractions may differ among people, depending on their background. D'Ambrosio (1985) suggests that the mathematics knowledge necessary for people to access their basic needs is context-dependent and should be analyzed through the lens of culture. The demands and resources of a specific environment may support the development of certain mathematics skills over others. These socio-cultural contexts may have varied among the AEP students composing our sample. Therefore, our sample's diverse backgrounds may explain the bimodal distribution of AEP students' fraction writing performance.

Development of Fraction Writing Skills

Traditionally, the analysis of error types in number transcoding has been an important tool to investigate the development of numerical representations (Moura et al., 2013; Seron & Fayol, 1994). In the present study, our analysis of fraction writing errors indicated that participants who struggled with fractions made a higher frequency of pure syntactic errors compared to pure lexical errors. In contrast, participants with higher performance in fraction writing made a higher frequency of pure lexical than pure syntactic errors. This shift in participants' error patterns at different stages of fraction writing acquisition reveals developmental features. To the best of our knowledge, no study has proposed a model accounting for the development of fraction writing skills. Based on our results, we propose a model of single-digit fraction writing (Figure 4), which should be empirically tested in future studies.

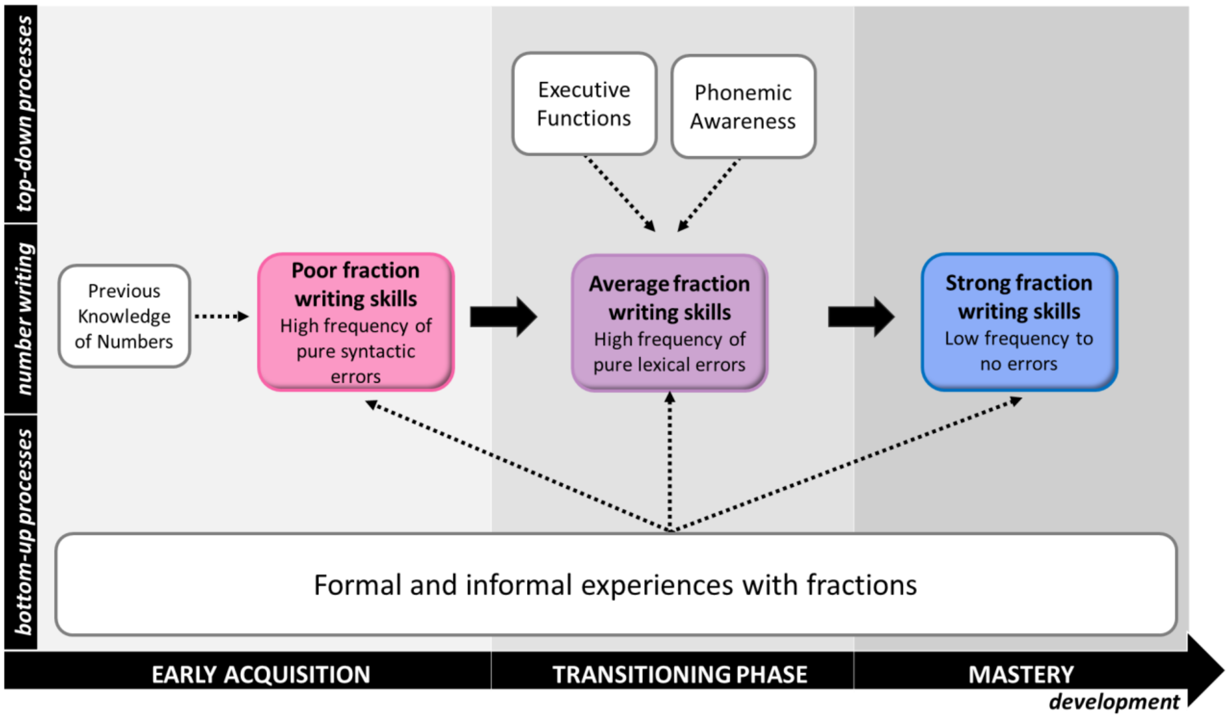


Figure 4. *Developmental model of single-digit fraction writing. In early phases, students have poor performance in fraction writing and make a high frequency of pure syntactic errors contrasted with a lower frequency of combined and lexical errors. The low frequency of lexical errors occurs due to previous knowledge about numbers in general. In a transitioning phase, students make few errors. Among their errors, pure lexical errors are the most frequent, which may be driven by executive functions and phonemic awareness. Finally, when students master fraction writing, a ceiling effect is observed, and participants make fewer to no errors.*

The first step in accurately transcoding a fraction from verbal to common notation is to identify its lexical components. When students are introduced to fractions in schools (e.g., around 3rd-5th grade in Brazil; Brasil, 2017) they have received years of formal instruction on whole numbers, in particular single-digit numbers, and ordinal numbers. Students may also have had informal experiences with numbers in general (e.g., LeFevre et al., 2009; Napoli & Purpura, 2018). These previous experiences with numbers in general may help build a strong knowledge of the number words used to compose fraction names. While previous experiences may be important throughout development, their effects may be more accentuated early on, particularly in languages such as Brazilian Portuguese, in which the fraction names are composed of the same words used for whole and ordinal numbers. In our data, participants' previous experience with numbers in general, as evidenced by their good performance in whole number writing, may have provided them a strong understanding of the fraction words: participants had a low frequency of pure lexical and combined errors.

However, knowledge of the number words is not sufficient to write fractions accurately. In addition to the lexicon, students need to learn the fractions syntax. More specifically, students need to learn that 1) the fraction name should be converted to a numerator separated from a denominator by the vinculum, and 2) the fraction name indicates which number is the numerator and which number is the denominator. Poor knowledge of these aspects is marked by very low performance in fraction writing and a predominance of syntactic errors. This error pattern occurs because students can identify the lexical components of fractions—based on their previous experiences with numbers in general—but fail to integrate digits into the correct format. Fractions written as whole and ordinal numbers (e.g., "half" written as 12 or 12^{th}) reveal poor knowledge of the common fraction notation, and inversion errors (e.g., "half" written as $\frac{2}{1}$) reveal difficulties in identifying the numerator and the denominator from the fraction name. In the early phases of fraction acquisition, participants commit a high frequency of pure syntactic errors and higher frequency of combined than pure lexical errors, as we observed in participants who had a low score in our fraction writing task.

In early phases of fraction transcoding acquisition, children could conceivably learn how to write some fractions as separate instances, in particular fractions frequently used in daily activities (e.g., $\frac{1}{2}$ and $\frac{1}{3}$). However, simply learning how to write some fraction as separate instances without a solid knowledge of the fraction syntax would not lead to the ability to productively write fractions. Although learning isolated fractions as special cases is conceivable, we did not find evidence that frequent fractions were transcoded more accurately than infrequent fractions: AEP students who struggled with fraction writing had poor accuracy even for the items $\frac{1}{2}$ and $\frac{1}{3}$. We predict that children in more advanced grades, whose performance we have not investigated, may learn how to write some fractions as separate instances, but they will not yet generalize their knowledge to writing other fractions. Learning isolated fractions without generalization may be analogous to how children first learn few isolated whole number names without generalization when acquiring counting skills (Le Corre, & Carey, 2008).

Via both formal and informal experiences, students learn the syntax of fractions, and their fraction writing skills improve significantly. However, during this transitioning phase, students still commit occasional pure lexical errors. Based on the literature on whole numbers, we argue that these occasional lexical errors may be mainly explained by executive functions and phonemic awareness (Lopes-Silva et al., 2014; Zuber, et al., 2009). To convert a fraction name to its common format, students need to pay attention to the fraction name, store it in their working memory, and write the numerator and the denominator. Given the bipartite structure of fractions, this may place high demands on working memory resources, particularly when students still have low experience with fraction writing. Furthermore, different fractions may have similar phonological structure, which can be an additional source of difficulty and lead to lexical errors. In our study, we observed that AEP students who had high scores in the fraction writing task still committed some occasional lexical errors.

Finally, after a few years of experience with fractions—which may occur in formal and informal contexts—students master the ability to write fractions, which is marked by a ceiling effect in fraction writing tasks. In this developmental phase, students automatize writing some fractions and very rarely, if ever, make either syntactic or lexical errors. In our study, we observed this pattern in some AEP students who had a perfect score in the fraction writing task.

In summary, our developmental model proposes that students learn how to write single-digit fractions both via top-down and bottom-up processes. Their previous knowledge about whole numbers, and their executive functions and phonemic awareness contribute to substantial improvements of their fraction writing skills. These improvements occur via formal and informal experiences with fractions. Finally, as students learn to write fractions, their error types shift from predominantly syntactic to predominantly lexical, and then, to no errors. This model should be tested in future studies and possibly expanded to multi-digit fractions.

Conclusion and Future Directions

In this study, we investigated fraction writing skills in groups who have not received formal fraction education in schools: Brazilian AEP students and 2nd graders. Overall, results indicated that informal experiences with fractions across the lifespan may promote fraction transcoding skills. Furthermore, results suggested a shift from higher frequency of pure syntactic errors to a higher frequency of pure lexical errors during fraction writing acquisition. Finally, error analysis indicated the whole number bias and a possible ordinal number bias in fraction writing. We integrated our results to propose a developmental model of single-digit fraction writing. In our model, we propose that students' previous knowledge about numbers and their executive functions and phonemic awareness contribute to the development of fraction writing skills, which occurs via both formal and informal experiences with fractions. We also propose that students make a high frequency of pure syntactic errors early in development, a high frequency of pure lexical errors in transitioning phases, and rare to no errors after mastering fraction writing skills.

This is the first systematic investigation of fraction writing skills in children and adults. Therefore, we open many unanswered questions that should be addressed in future studies. Our results suggested an ordinal number bias in fraction writing. However, it is still unclear how these results replicate to other populations. A cross-cultural comparison was beyond the present study's scope. Nevertheless, unpublished data from our lab suggest that American 2nd graders can accurately write some fractions and make a higher frequency of pure lexical than pure syntactic errors. Among the syntactic errors, American 2nd graders frequently wrote fractions as whole numbers, but they did not write fractions as ordinal numbers. These results from American 2nd graders contrast with the very poor performance and the types of errors observed in Brazilian 2nd graders. Therefore, accuracy and error types in fraction transcoding may differ across socio-cultural contexts, which should be investigated in future studies.

In this study, we had only investigated fraction transcoding from the verbal to the common notation. Participants' difficulties with fraction writing do not imply difficulties with other fraction representations. Furthermore, some of the error types we observed may be unique to fraction writing. A full understanding of fraction representations will only be possible with the investigation of other transcoding paths including nonsymbolic, verbal, and common fractions. Future studies should investigate, both quantitatively and qualitatively, fraction transcoding with other notations.

In addition to our specific contributions to fraction transcoding, our study also contributes to our knowledge about numerical cognition in unschooled adults. In most numerical cognition studies, participants were from Western, educated, industrialized, rich, and democratic cultures (Henrich et al., 2010). Because of this, some conclusions made by the numerical cognition literature may not apply to people coming from different backgrounds. Conducting studies with adults who have low schooling may expand our knowledge of numerical cognition in general, inform us about this population's cognitive profile, and have practical implications for adult education programs.

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Supplementary Material

Whole-Number Writing

Group (N)	***M (% correct)***	***SD***	***t (df = 38)***	***d***	***Error type frequency (%)***			
					Syntactic	***Lexical***	***Combined***	***Others***
28 (AEP students) vs. 81 items (2nd graders) whole number writing task								
AEP Students (20)	.90	.08	25.23***	7.93	80	11	0	9
2nd graders (20)	.20	.10			60	10	30	0
23 overlapping items across whole number writing task versions								
AEP Students (20)	.88	.10	10.48***	3.38	80	11	0	9
2nd graders (20)	.44	.16			47	21	31	0