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ENHANCING THE SUSTAINABILITY OF THE MINAS GERAIS SOCIAL SECURITY SYSTEM: A COMPARATIVE ANALYSIS OF PENSION MODELS

Belo Horizonte
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Orientador: Prof. Dr. Juan Pablo Gama Torres

Coorientador: Prof. Dr. Bruno Perez Ferreira

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FELIPE NALESSO PEDERZINI

ENHANCING THE SUSTAINABILITY OF THE MINAS GERAIS SOCIAL SECURITY SYSTEM: A COMPARATIVE ANALYSIS OF PENSION MODELS

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RESUMO

Esta dissertação examina os desafios do sistema previdenciário do governo de Minas Gerais, destacando a necessidade de reforma devido aos desequilíbrios fiscais e ao aumento de aposentados. Explora o esquema de Contribuição Definida Nocional (NDC), como na Suécia, para incentivar períodos de trabalho mais longos e criar um ambiente econômico sustentável. Propõe a transição para um sistema Nocional e a implementação do seguro de solvência para resolver déficits e aumentar a integridade do sistema de seguridade social. Analisa as mudanças no sistema previdenciário de Minas Gerais resultantes da transição do sistema PAYG para o Nocional, com cálculos e discussões sobre o seguro de solvência para lidar com os déficits do fundo de previdência social administrado pelo IPSEMG. As simulações de 2023 a 2039 visam reduzir as perdas financeiras, começando com a criação de um fundo nocional para novos participantes em 2023 e explorando o seguro de solvência para funcionários públicos, pensionistas e aposentados atuais. A análise identifica lacunas e oportunidades para uma melhor gestão do sistema previdenciário, destacando a redução na relação entre os passivos atuariais. O estudo ressalta o papel central de um sistema previdenciário sustentável e robusto para indivíduos e a economia em geral, enfatizando a importância dos esquemas de capitalização e nocionais no incentivo à permanência prolongada no trabalho e na promoção da sustentabilidade econômica.

Palavras-chave: Regime NDC; Gestão do Equilíbrio das Contas Públicas; Desafios da Previdência Social.

JEL: H55; I38; J26; C61

ABSTRACT

This dissertation examines the challenges facing the Minas Gerais government's pension system, emphasising the necessity for reform due to fiscal imbalances and the rise in the number of retirees. The dissertation examines the potential of the Notional Defined Contribution (NDC) scheme, as exemplified by Sweden, to encourage longer working periods and foster a sustainable economic environment. The proposal put forth is the transition to a notional system and the implementation of solvency insurance, to resolve deficits and increase the integrity of the social security system. The dissertation analyses the changes in Minas Gerais' social security system resulting from the transition from the PAYG system to the NDC system. It presents calculations and discussions on solvency insurance to address the deficits of the social security fund administered by IPSEMG. The objective of the simulations from 2023 to 2039 is to reduce financial losses, beginning with the establishment of a notional fund for new participants in 2023 and subsequently examining the potential of solvency insurance for current civil servants, pensioners and retirees. The analysis identifies deficiencies and potential avenues for enhanced management of the pension system, emphasizing a reduction in the ratio of actuarial liabilities. The study underscores the pivotal role of a sustainable and robust pension system for individuals and the economy at large, emphasizing the importance of capitalization and notional schemes in encouraging prolonged permanence in work and promoting economic sustainability.

Keywords: NDC scheme; Public Account Balance Management; Social Security Challenges.
JEL: H55; I38; J26; C61

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SUMMARY

1 INTRODUCTION

The pension system in Brazil is confronted with considerable financial imbalances and structural challenges. These imbalances are the result of both cyclical factors, such as inadequate economic development, and fundamental structural aspects, such as declining fertility rates and an aging population (GIAMBIAGI et al., 2007). The reform discourse has primarily centered on parametric adjustments, including the reduction of benefits, the increase of contribution rates, and the elevation of the retirement age. In Minas Gerais, the number of retirees and pensioners has increased in comparison to the number of active workers. Consequently, fiscal imbalances have emerged, giving rise to concerns about the sustainability of the public pension system and the social burden it imposes (Bertucci et al., 2004). The demographic shifts and tepid GDP growth have prompted a heightened focus on social security systems in numerous countries, including Brazil and its constituent subnational entities. Notwithstanding recent reforms, these systems continue to confront significant challenges in achieving equilibrium in their actuarial accounts.

The initial chapter addresses the implementation of solvency insurance, a policy designed to address the persistent deficits resulting from the discrepancy between income and expenditure in the fund managed by IPSEMG. The analysis is based on projections for the PAYG fund from 2023 to 2039, as prepared by IPSEMG. These projections entail simulations designed to gradually reduce financial losses through incremental changes. The initial step is to establish a fund with nominal characteristics for new entrants, commencing in 2023. This will be followed by an assessment of the feasibility of introducing solvency insurance to cover the current cohort of civil servants, retirees, and pensioners.

The second chapter is concerned with the analysis of the impact of the transition from the PAYG system to the Notional system on the pension system of the state of Minas Gerais. The utility of each individual will be calculated based on their disposable income for consumption and the working time invested during the transition to both systems. The objective of this analysis is to evaluate the impact of changes in individual utility on retirement decisions and the government's strategic choice to improve the management of the balance of public accounts.

2 SOCIAL SECURITY ANALYSIS OF MINAS GERAIS

2.1 INTRODUCTION

The social security issue has become increasingly important in academic studies around the world (HOLZMANN; HINZ, 2005; GIAMBIAGI et al, 2007). In many parts of the world, the social security systems are part of the mechanisms that make it possible to protect society against financial problems, with four objectives: to function as an insurance device, to smooth consumption intertemporally, to help reduce poverty and, finally, to serve as an income redistribution mechanism (BARR; DIAMOND, 2006).

To ensure the societal benefits, Brazil was one of 34 The Organisation for Economic Co-operation and Development (OECD) countries to undergo social security reforms from 2009 to 2015. These reforms aimed to adapt to new economic and demographic characteristics, ultimately enhancing financial stability (AFONSO; CARVALHO, 2022).

The pension system in Brazil is facing considerable financial imbalances and structural challenges. These imbalances are due to cyclical factors, such as inadequate economic development, and fundamental structural aspects, such as declining fertility rates and an aging population (GIAMBIAGI et al., 2007). The reform discourse has primarily concentrated on parametric adjustments, including reducing benefits, increasing contribution rates, and raising the retirement age. In Minas Gerais, a state in Brazil, there has been a noTable rise in the number of retired and pensioned individuals in contrast to active workers. As a result, fiscal imbalances have arisen, prompting concerns about the sustainability of the public pension system and the societal burden it imposes (BERTUCCI et al., 2004).

Demographic changes have been approved to address these issues, but their implementation is happening gradually, implying that forthcoming pensioners will bear the brunt of the impact. Immediate reforms are imperative to mend the fiscal disparities of the Brazilian pension system and curtail societal expenses as a whole (NULLE; MOREIRA, 2019).

Furthermore, it is widely accepted in the literature, as documented by authors such as Bloom and Mckinnon (2010), Coleman (2006), Acosta-Ormaechea et al. (2017) and Amaglobeli et al. (2019), that these demographic changes have an adverse impact on pay-as-you-go (PAYG) systems, resulting in a tendency for expenses to increase in relation to future revenues.

According to Bispo (2004), the realization that reforms were necessary, incorporating all or part of capitalization systems, led to the implementation of reforms in the pension systems of Latin American countries during the 1990s. Peru (1993), Colombia (1994), Argentina (1994), Uruguay (1996), Bolivia (1997), El Salvador (1997), and Mexico (1997) all underwent either complete or partial replacement of their public social security system, which was previously financed on a PAYG basis, with individual capitalization systems. However, introducing funded pension systems can create significant challenges for countries, primarily in terms of financing the transition deficit between the proposed systems.

Taking these challenges into account, Sweden pioneered a reform concept in 1994 (Boado-Penas, 2021) that structurally modified its old pension system. The reform introduced a plan that had never been adopted in the world: Notional Defined Contribution (NDC), which was subsequently implemented in other countries such as Italy (Franco and Sartor, 2006), Latvia (Palmer et al., 2006), Poland (Chłoń-Domińczak and Góra, 2006) and Norway (Christensen et al., 2012).

Under NDC schemes, pension benefits are funded by current contributions, similar to a conventional system. However, the connection between contributions and benefits is determined on an individual basis through the notional accounting mechanism. This is similar to a typical defined contribution (DC) system, where individuals contribute to their pension accounts and accumulate credits. However, the account balance is considered fictitious or notional since no real capital is accumulated. It is worth noting that the calculation is objective and follows a pre-set formula. The income for the accounts depends on an internal rate of return that is influenced by factors related to the country's economic progress, rather than reference interest rates or inflation. Retirees receive lifetime annuities when accessing their benefits. The amount they receive is contingent upon the total value in their individual accounts, as well as certain factors such as average life expectancy and the internal rate of return (Auerbach & Lee, 2006; Vidal-Melía et al., 2006).

Therefore, this study aims to provide decision-makers in the Minas Gerais' own social security system (RPPS), as well as in other states of Brazil and in the General Social Security System (RGPS), with relevant information. The objective is to enhance the public debate on this subject by offering a sound basis for evaluating the costs and benefits of government measures, both presently and in the future.

This first chapter discusses the implementation of solvency insurance, a policy designed to address the persistent deficits resulting from the mismatch between revenues and expenditures in the fund managed by IPSEMG. The analysis is based on the IPSEMG's projections for the PAYG fund from 2023 to 2039. Simulations are conducted to gradually reduce the financial losses by introducing incremental changes at each stage. The first step is to create a fund with notional characteristics for new entrants starting in 2023. This will be followed by an assessment of the possibility of introducing solvency insurance to cover current civil servants, retirees and pensioners in varying proportions depending on their entry into the system. This comprehensive analysis will provide a modern diagnosis of the impact of the pension systems, identifying areas for improvement and opportunities for better management.

2.2 LITERATURE REVIEW

This section will evaluate all relevant information found in national and international studies that will enrich the discussions on the feasibility and importance of the research, as well as provide a basis for the discussion of the results generated from the proposed modelling. Therefore, we will begin by introducing the types of pension models that exist in Brazil, followed by the history of the pension system in MG, the focus of this study, and its current situation. Finally, we will discuss the theory of insurance from an economic perspective. This will be useful in the discussions of proposed reforms, as well as actuarial goals and the NDC system. We will highlight the positive and negative aspects of adopting this system in the pension system.

2.2.1 EXISTING TYPES OF PUBLIC SOCIAL SECURITY IN BRAZIL

According to Iyer (2002), Social Security systems are institutional structures created to protect the elderly, people with disabilities, and their dependents. They are generally established at the national level through government initiatives and are regulated by laws that define the rights and obligations of the people and institutions involved in the welfare system. This includes compulsory contributions and the benefits offered.

As a compulsory mechanism for specific groups of the population, they are managed directly by government agencies or by parastatal organizations subject to government supervision. In many cases, the ability to meet future commitments, known as actuarial

solvency, is guaranteed implicitly or explicitly by the government, with costs shared by society.

In Brazil, the social security system is made up of a tripartite system, consisting of three types of pension plans. This division is crucial to maintain economic equilibrium. The financing techniques employed are of paramount significance in providing social security services to taxpayers, and they can be deemed as significant in sustaining the fiscal well-being of social security systems (RAMOS, 2016). Two components of the social security system are mandatory for workers and operated publicly: the General Social Security Regime (RGPS) and the Private Social Security Regime (RPPS). The third component, the Complementary Social Security Regime (RPC), is optional and has a private, contractual nature (GUIMARÃES, 2013; DOGNINI, 2020).

2.2.1.1 RGPS

The RGPS, provided for in articles 201 et seq. of the 1988 Federal Constitution, as well as in Federal Laws 8,212/1991 and 8,213/1991. Law No. 8,213, of July 24, 1991, provides for social security benefit plans, including, among other aspects, beneficiaries (insured persons and dependents), i.e. not everyone who works for a public administration entity that has its own social security system will be insured under the respective system. Under the terms of art. 11, item I, points "g" and "h" of Law No. 8,213/1991, public employees occupying commissioned positions, without permanent ties, and those holding federal, state or municipal elective office, as long as they are not linked to their own social security system, are compulsory RGPS insured persons. Thus, this regime applies to private sector employees, employees of the Direct and Indirect Administration, public employees hired on a temporary basis and public employees occupying exclusively commissioned positions (article 40, § 13, of the 1988 Federal Constitution).

Benefits (benefits and services) according to art. 18 of Law No. 8,213/1991 historically include benefits or services such as disability retirement; retirement for age; retirement for length of service; retirement for length of contribution; special retirement; sickness benefit; family allowance. The law also sets out the waiting periods for access to benefits, the rules for calculating the value of benefits and their readjustment.

However, despite the 1988 Constitution having established the principle of financial and actuarial balance for social security systems, both this Constitution and subsequent

regulations failed, at the time, to establish the necessary conditions for reviewing the parameters with a view to making social security funding more sustainable. This deficiency was already evident in previous decades.

On this subject, Santos (2008) points out that the financial crisis of social security, which included the reduction in social security collection due to the decrease in the wage bill, financial commitments related to the debt assumed by the Central Bank and the need for fiscal resources to finance social policies, was already underway before the 1988 Constitution was enacted. This clearly indicated the need for reforms, including the elimination of the length-of-service pension and changes to the funding model.

It is therefore not surprising that the regulations for the General Social Security System (RGPS), established by Law No. 8,213/1991, have undergone annual modifications since its enactment, due to the urgent need for adjustments to deal with the financial crisis in social security. However, it is important to point out that these changes, especially the more substantial ones that can be characterized as reforms, require a favorable political and social environment for their promotion.

Despite this, it is noTable that Brazil still has room to advance in relation to OECD countries when it comes to the minimum age for retirement by contribution time, as shown in Graph 1. What's more, given the life expectancy for the male population of 73.6 years, and for women 80.5 years in 2021, there is still a large amount of time left to receive benefits compared to international peers.

From a theoretical perspective, the analysis of the RGPS makes a significant contribution to the research, since it allows the problem of social security to be contextualized within a broader framework. This is justified by the fact that, at the federal level, most public spending is directed towards social security benefits. However, the aim of this study is not an exhaustive analysis of the RGPS. On the contrary, it seeks to establish a comparison with the regime applicable to public employees, limited to the state of Minas Gerais.

2.2.1.2 RPPS

The social protection and retirement system for public employees in Brazil underwent significant changes with the enactment of the 1988 Federal Constitution. Prior to this, retirement for public employees was seen as a reward for years of service, without the need for them to contribute. However, this approach lacked solid funding strategies, leading to the

need for reforms over the years to ensure the sustainability of the system. These reforms have been widely discussed and covered by the media.

Public employees have historically received different treatment, which was maintained in the 1988 Constitution. Article 40 of the original text ensured that retirement benefits were calculated based on the full amount of the last remuneration, and pension remuneration was equivalent to that of active public employees. However, this equality is not applied to the general social security system.

The retirement and social protection system for public employees, known as the RPPS, applies to permanent and life position holders. Each federal entity has the autonomy to establish its own pension scheme, as long as it adheres to the general rules set by the central government. These schemes are regulated by Administrative Law and are subject to Law 9,717/1998, which establishes the criteria for their organization and operation.

Despite efforts to organize the public pension systems, challenges such as the historically generous retirement benefits, an increasing number of inactive public employees compared to active ones, and insufficient contribution revenues have led to the need for system reform. The aim is to make the system financially and demographically sustainable.

In summary, the retirement and social protection system for public employees in Brazil has undergone significant changes since the enactment of the 1988 Federal Constitution. The historical approach of rewarding years of service without contribution has been replaced by the need for funding strategies and sustainability. Public employees receive different treatment compared to the general social security system, and each federal entity has autonomy in establishing its own pension scheme within the parameters set by the central government. Reforms aimed at ensuring financial and demographic sustainability are necessary due to the challenges posed by generous retirement benefits, an increasing number of inactive public employees, and insufficient contribution revenues.

2.2.1.3 RPC

The RPC, on the other hand, is provided for in Article 202 of the 1988 Federal Constitution and regulated by Supplementary Laws 108/2001 and 109/2001. It is an alternative means of increasing the amount received during workers' retirement, of an optional nature. These entities are divided into two categories: Closed Supplementary Pension Funds and Open Supplementary Pension Funds.

According to Porto and Caetano (2015), Open Supplementary Pension Entities are set up in the form of public limited companies, for profit and operate individual or collective plans, available to any individual who wishes to make payment contributions. Commercial banks or insurance companies are generally responsible for operating these entities (BRASIL, 2011). These entities are authorized to operate and supervised by the Superintendence of Private Insurance (SUSEP), which is subordinate to the Brazilian Ministry of Finance. As in the case of the RPPS, its autonomy was established with the enactment of the 1988 Federal Constitution (GUIMARÃES, 2012). Finally, the RPC, also known as pension funds, are formed by private law foundations or non-profit civil society organizations, with all the funds invested going to the fund itself (BRASIL, 2011).

That said, one of the main points that distinguishes the different pension schemes is the financial regime to which they are subject. Financial regimes are methods designed to guarantee compliance with the obligations assumed by pension benefit plans, establishing only the way in which the resources needed to pay the benefits will be obtained (GUSHIKEN et al, 2002).

As Boulier and Dupré (2003) point out, the management of pension funds involves a combination of traditional financial management with some particularities and the insertion of typically financial problems into complex social systems. In both funded and PAYG systems, the main objective of the financial management of funds is to satisfy the needs of a population that includes governments, companies, salaried workers, retirees and other indirect participants. This approach creates the need to establish an appropriate "marriage" between assets and liabilities, similar to what happens in financial institutions.

With regard to the system present in the state of Minas Gerais, in 2013, the Escola de Administração Fazendária - ESAF published a feasibility study that explored the possibility of creating a supplementary pension system in the state of Minas Gerais, taking into account legal and actuarial aspects. The authors, Fagundes et al. (2013), based their analysis on data from the end of 2011 for Minas Gerais and were inspired by the examples of the supplementary schemes adopted by the Federal Government and the states of São Paulo and Rio de Janeiro.

According to the authors, the FUNFIP, operating on a simple PAYG system, was planned to be abolished in the future, as it would not allow the admission of new public employees. On the other hand, the FUNPEMG, which follows the capitalization system,

would be prepared to receive future public employees who would join the state public service, should the complementary pension system be created. This would imply that the burden of funding benefits above the RGPS limit for new public employees would fall on FUNPEMG.

In their conclusion, the authors Fagundes et al. (2013) stated that, given the beginning of Minas Gerais' transition to a capitalized pension system, the implementation of complementary pensions could result in fiscal benefits for the state.

After this study, in 2014, just over a month after the enactment of Complementary Law 131, Complementary Law 132 was published. This legislation was responsible for establishing the Complementary Pension Scheme for public employees who hold permanent positions in the state of Minas Gerais. This included members of the judiciary, the Public Prosecutor's Office, the Public Defender's Office and the Council of the Court of Auditors, as provided for in paragraphs 14 and 15 of article 40 of the 1988 Federal Constitution.

2.2.1.4 PAYG, CAPITALIZATION AND TRANSITION COSTS

When considering structural changes to social security systems, it is essential to examine the effects of transitioning from a PAYG to a capitalization system, either wholly or partially. Such changes result in an initial reduction in total social security contributions aimed at financing benefits as these payments are deposited into the individual accounts of the insured. The extent of the reduction may vary based on the pace of transitioning between the two systems.

If the transition occurs immediately, there will be a shortfall in financing benefits that is equal to the total contributions. However, this shortfall only reflects a portion of the social security obligations that become apparent as a result of the shift. Another issue concerns enhancing the value of current employees' individual accounts who previously contributed to the old PAYG system but intend to retire under the new capitalization system. On the other hand, during a gradual transition where only new participants are included in the capitalized system, the pension liability will be extended over time. In the second stage of the process, the deficit in the old system is reduced by eliminating benefits until the transition is completed (DOGNINI, 2020).

It is noteworthy that transitioning from a basic PAYG to a capitalization plan could alleviate the pension system's deficit, in case the former becomes imbalanced in the long term.

At that stage, the benefits' worth exceeds that of contributions, which determines a "transition cost" (ORSZAG; STIGLITZ, 1999).

2.2.2 HISTORY OF SOCIAL SECURITY IN MINAS GERAIS

The history of welfare and social protection in Minas Gerais dates back to the beginning of the 20th century, when Law No. 588, of September 6, 1912, established the Caixa Beneficente dos Funcionários Públicos de Minas Gerais. This pioneering law laid the foundations for an institution dedicated to protecting public employees and their families, offering financial support in situations of disability and death resulting from misfortune. This historic milestone marked the beginning of a trajectory of evolution and transformation of the state's welfare and social security systems (CALAZANS, 2010; DUARTE, 2017).

As Pinheiro (2000) points out, as early as the 1930s, various welfare institutions were created to serve categories of civil servants in different states and municipalities. These institutions operated in a heterogeneous and fragmented system, with the conception that social security was an extension of public employees' personal policies. In this context, social security was seen as a benefit inherent to civil servants' careers. This expansion of social security was influenced by the Retirement and Pension Fund models, introduced by the Eloy Chaves Law, which is attributed as the 1st Social Security Law, by Legislative Decree No. 4,682/1923, in which welfare benefits coexisted with traditional social security benefits, resulting in a comprehensive and diversified conception of the social security system (RIBEIRO, 2020; DUARTE, 2017).

Subsequently, another reform impacting the structure of public employees' pensions in the state of Minas Gerais underwent a significant overhaul by means of Decree-Law No. 1,416, dated November 24, 1945, which brought about a change in the institution's name, transforming it into what is currently known as the Minas Gerais State Servants' Pensions Institute - IPSEMG. With this reformulation, all civilian public employees under the age of 50 were obliged to contribute 4% or 5% of their salaries, depending on their salary bracket. In addition, a contribution equivalent to 50% of the total payroll by the state was established, thus guaranteeing the payment of pensions and strengthening the state pension system (IPSEMG, 2016). These changes represented an important step in consolidating social protection for public employees in Minas Gerais (CALAZANS, 2010; DUARTE, 2017).

The Basic Welfare Hospital, which was inaugurated in 1971, led to the expansion of health services. Contributions were adjusted over the years, with the most significant change being the segregation of contributions in 1986. This change earmarked a specific portion of contributions for healthcare, which was aimed only at assistance and pensions, not retirement. This reflects the idea that the state is responsible for supporting inactive civil servants. In 1990, the Minas Gerais State Military Servants Pension Institute (IPSM) was established to provide pension structures for the military. It is important to note that this model did not include pensions, which were fully maintained by the state. The discussion on social security evolved after the 1988 Constitution, with emphasis on aspects related to public employees and their relationship with the rest of the working population (DUARTE, 2017).

On March 26, 2002, Complementary Law No. 64, in line with Article 36 of the 1989 State Constitution and based on the concurrent competence of the state, established the Social Security and Social Assistance System for public employees in Minas Gerais, in addition to dealing with other relevant provisions. Later, in 2013, Complementary Law 131 brought changes, determining the extinction of the Minas Gerais State Welfare Fund - FUNPEMG and the creation of the Minas Gerais Welfare Fund - FUNPREV-MG. This new fund's main objective is to provide the necessary resources to guarantee the payment of pension benefits to public employees in the state of Minas Gerais (CALAZANS, 2010).

To guarantee the solvency of this system, two funds were created: the Financial Pension Fund (FUNFIP) and the Minas Gerais Pension Fund (FUNPEMG). FUNFIP, which is managed by the state, is the recipient of contributions from public employees hired up to 31/12/2001, and is responsible for paying the actual benefits and obligations related to public employees hired by public examination from 2002 onwards, provided that these benefits take effect by 31/12/2009. It is important to note that this fund is, by its nature, a deficit account that operates on a cash basis, with the aim of managing its members' social security income and expenses.

In turn, FUNPEMG, managed by IPSEMG, is intended to provide the necessary resources to guarantee the payment of benefits to its insured members and will only be extinguished if it is decided in a plebiscite to be held among all IPSEMG taxpayers. FUNPEMG, which operates on a capitalization basis, is responsible for guaranteeing the payment of social security benefits to public employees hired after 2002 and their dependents, whose benefits begin after 31/12/2009. With the creation of this eight-year grace period, the

legislator intended this fund to capitalize resources in order to guarantee the long-term sustainability of its benefits plan.

Supplementary Law No. 131, of December 6, 2013, amended Supplementary Law No. 64 of 2002, determining the extinction of FUNPEMG, the succession of all its rights by FUNFIP and the establishment of the Minas Gerais Social Security Fund - FUNPREV-MG. This new fund would operate, according to the law, on a capitalization system and would be responsible for guaranteeing the payment of social security benefits for public employees joining the state of Minas Gerais from the date the complementary social security system was authorized to operate. In addition, a new complementary law was to be submitted within 365 days containing its rules and structure.

Finally, Complementary Law No. 132/2014 introduced several changes, including the establishment of the Supplementary Pension Scheme for public employees holding permanent positions in the state of Minas Gerais, including members of the Judiciary, the Public Prosecutor's Office, the Public Defender's Office and the Counselor of the Court of Auditors. Within this context, article 3 of this law established a maximum limit for the benefits granted by the Minas Gerais Social Security System (RPPS/MG), corresponding to the ceiling established by the General Social Security System (RGPS), applicable to public employees who joined the state public service after Constitutional Amendment 132/2014 came into force. This measure aimed to ensure the sustainability of the state's pension system.

2.2.3 MINAS GERAIS' SOCIAL SECURITY SITUATION

Even with the reforms introduced with changes to rates and deadlines for starting to receive pensions, the Minas Gerais state still has successive deficits, according to the Budget Guidelines Laws of recent years.

FUNFIP's actuarial assessment found that 42,800 pensioners, out of a total of 278,000, receive more than the RGPS ceiling, a percentage of 15%. What the data shows is that with fewer contributors, the state pension system ends up increasing its weight in the state budget. With contributions of 11% from public employees and 22% from employers, in 2017, the Executive Branch had to contribute more than R$827 million on average per month to cover social security shortfalls (TESOURO NACIONAL, 2019).

In 2019, the National Treasury Secretariat (STN) published a diagnostic report titled "The Fiscal Situation of Minas Gerais," providing details on the state's revenues, expenses,

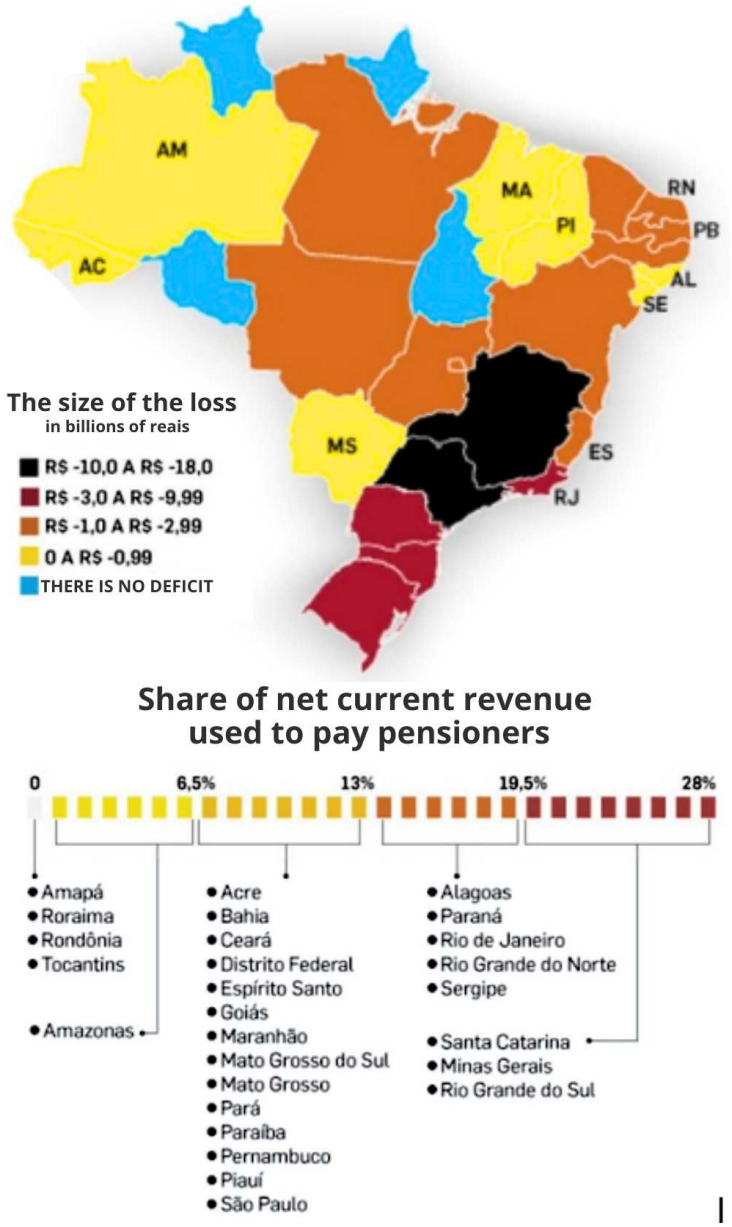
social security, and assets. In December 2018, the Executive Branch of the state employed over 313,000 individuals. The average salary for civilian pensioners was R$3,941.00, while inactive military personnel received R$8,532.00, indicating a significant disparity between the two categories. It was also noted that retirees and pensioners consume more than half of the payroll, which highlights the seriousness of the social security issue. However, the public employees' pension contributions to the RPPS increased by 43%, rising from R$557 million in 2011 to R$800 million in 2018. The military's earnings also witnessed a 98% growth in the same period, escalating from R$243 million to R$483 million.

However, it should be noted that pension supplements escalated from R$3.8 billion in 2011 to R$11.1 billion in 2018 despite the favorable data. This indicates that the regulations governing the RPPS system do not fully account for the social security disparity, as the challenges of developing and administering personnel policies also impact the identified fiscal condition.

Figure 01, based on Santos et al (2017) published in an article in the newspaper O Estado de São Paulo on March 27, 2017, provides insight into the pension deficit situation of Brazilian states. The study indicates that only four states do not have a pension deficit, meaning that a vast majority of states are facing financial challenges in terms of pension funding.

Minas Gerais, along with São Paulo, ranks as one of the states with the highest financial requirements in absolute terms. Furthermore, Minas Gerais is among the states that provide a major portion of Net Current Revenue (RCL) towards the payment of social security benefits. The presented data suggests a complex and challenging situation regarding the sustainability of the social security system in the state of Minas Gerais. Thus, effective measures must be implemented to ensure the uninterrupted payment of benefits to recipients.

Figure 01: Financial deficit of the RPPS in Brazilian states



Source: based by O Estado de São Paulo newspaper[1].

The analysis also reveals that the state has exceeded the limit on personnel expenditure set by the Fiscal Responsibility Law, which is due to the growth in salaries resulting from the

[1] Pension funds in 22 states and the Federal District are operating at a deficit. Alexa Salomão, O Estado de S. Paulo. March 27, 2017. <http://economia.estadao.com.br/noticias/geral,previdencias-de-22-estados-e-do-df-operam-com-deficit,70001715143>.

salary increase, which has contributed to an incremental increase in the benefits sheet, both for inactive and pensioners (53%) and for active deductions (32%).

In this way, the STN (2019) suggests that in order to improve the fiscal situation, in which the social security component prevails, an in-depth study is needed to assess the social security impacts related to the allocation of public resources aimed at remunerating public employees (GONÇALVES, 2019).

As in Freitas (2020), the initial hypothesis to be addressed is that the social security system in Minas Gerais has a structure in need of reform in order to solve the problem of the large and continuous deficit. Thus, the object of this study is the Minas Gerais RPPS in the face of the deficit presented by the state's pension system, related to the technical characteristics that hinder its restructuring. It is also associated with value, time, income and demographics.

2.2.4 INSURANCE THEORY IN ECONOMICS

The definition of the concept of risk is an inherently subjective aspect that varies according to the perspective of each individual and his or her field of activity. This diversity of interpretation stems from different disciplines, such as economics, statistics and actuarial science, where each professional develops his or her own understanding of the concept. A milestone in the conceptualization of risk is the contribution of Knight (1921), during a period marked by fundamental debates on probabilities involving Figures such as Keynes (1921), Mises (1928) and Kolmogorov (1933).

Since then, the literature has highlighted the distinction proposed by contemporary authors, such as Rejda (2010), between two types of risk: objective and subjective. Objective risk, also called "degree of risk," refers to the relative deviation between the actual loss and the expected loss. As in Costa (2013), let's consider, for example, an insurance company that insures 10,000 houses and expects 100 of them (1%) to suffer damage each year. However, the variation in the actual occurrence, which could be 90 or 110 homes, represents objective risk, which is the difference between the actual loss and the expected loss.

Subjective risk, on the other hand, is intrinsically linked to uncertainty based on an individual's state of mind or disposition. The perception of this risk varies from person to person, and a high subjective risk often results in more conservative and prudent behavior. For

example, the mental uncertainty of a drunk person deciding to drive home illustrates the concept of subjective risk.

In the context of insurance, risk can be defined as the possibility of a future and uncertain event occurring that may cause loss. This definition excludes inevitability and highlights the importance of the insurance contract in allocating assets to the insurer in the face of losses suffered by the insured, as argued by Brown (1980).

Moreover, risk is also present in contexts such as credit insurance, where the creditor's confidence in the debtor is tempered by the risk of total or partial default by the debtor. Mendes (1972) emphasizes that credit insurance, in the strict sense, is protection against default and is a real risk from the creditor's point of view.

The principles of the insurance contract are based on mutualism, which means that the insured come together to cooperate in mitigating individual losses. This principle allows for the social sharing of risks, which enables a collective to organize itself for joint protection against the occurrence of specific claims (COSTA, 2013).

With regard to good faith in insurance, the legislation emphasizes the importance of the ethical conduct of the parties, not only in relation to the immediate interests of the insured and the insurer, but also in consideration of the interests of other policyholders. The principle of the social function of the contract reinforces the need to respect good faith and the primacy of public order, limiting the application of the principles of autonomy of will and the binding nature of contracts (COSTA, 2013).

In relation to these characteristics, there is a strong apparatus to avoid systemic risks as much as possible. According to Newlands Jr. (2011), this risk involves the possibility of accumulated losses resulting from specific events affecting a number of interconnected financial institutions. It is further exacerbated by interconnectedness through the interbank market and the payment system, highlighting the importance of understanding and managing risk in a broader context.

The insurance contract is therefore an agreement in which the insurer undertakes to guarantee the legitimate interests of the insured against adverse risks, in return for which the insured is obliged to pay a premium. This contract is bilateral, creating obligations for both parties, and onerous, involving reciprocal benefits and sacrifices.

In a broader perspective, the discussion extends to Credit Default Swaps (CDS), which function as insurance contracts allowing investors to buy protection against events that lead a

country to default or restructure its debt (HULL; WHITE, 2003). Consiglio, Lofti, and Zenios (2017) point out that these contracts emerged in the 1990s as credit derivatives in the sovereign debt market, providing opportunities for portfolio protection or speculation.

Nevertheless, it is important to note that the insurance function described here will serve as a basis for the subsequent development of a government policy proposal to offset public pension accounts in order to reduce the permanent deficits. In this way, insurance will be created that will guarantee the solvency of the social security system and will make it possible to pay all public employees participating in the Minas Gerais RPPS in the future.

2.2.5 ACTUARIAL TARGET

According to the State of Paraná's Court of Auditors (TCE/PR) in 2004, maintaining social security system sustainability in the short and long term requires financial and actuarial balance. Achieving financial balance is essential, as it ensures that revenue earned each year meets the obligations of the RPPS to prevent immediate deficits and ensure timely benefit payouts (FERREIRA, 2006).

It involves the equivalence, at present value, between the flow of estimated revenues and projected long-term obligations, alongside demographic changes, contribution and benefit levels, and varying periods of contribution. Achieving actuarial balance requires a comprehensive strategic study. It involves the equivalence, at present value, between the flow of estimated revenues and projected long-term obligations, alongside demographic changes, contribution and benefit levels, and varying periods of contribution. An extensive range of elements influences the pension system, making a long-term assessment crucial. The primary aim of this system is to maintain the stability of social security and to ensure the payment of legal benefits, both currently and in the future. To maintain equilibrium, the system utilizes financial and actuarial methods that consider various factors, such as changes in population demographics, the current sum of contributions and benefits, the necessary contribution periods for benefit maintenance, and other pertinent factors that impact the system's longevity (VAZ, 2009).

The constitution of capitalization funds, such as the Minas Gerais Social Security Fund (FUNPREV-MG), plays a fundamental role in financing social security commitments. These funds invest in the financial market, seeking returns that exceed the actuarial target. According to Iyer (2002), this target represents the minimum rate of return needed to ensure

that the fund can cover its obligations to participants, taking into account factors such as life expectancy, contributions and the financial discount rate. It also serves as a reference in determining the maximum accepTable risk in the allocation of the fund's investments. In the case of social security funds, the maximum stipulated target, as determined by MPS Ordinance No. 4,992/1999, is a 6% real return, which results in the composite rate of this real return plus an inflation index in compound capitalization. As described by Ferreira (2006):

$$AT = [(1 + d)(1 + \pi)] - 1 \qquad (1)$$

where AT is the actuarial target, d is the financial discount rate and $d \leq 6\%$. π is the base inflation rate used.

2.2.6 CONTEXT OF REFORMS

Considering the actions of the interest groups mentioned previously, it is apparent that introducing modifications to the social security system, which may have an impact on the presumed rights already acknowledged by these groups, is a complex task. According to Mendes (2014), parliamentarians tend to approach these groups in a benevolent manner rather than a restrictive one. This approach is due to their representation of specific sectors, including rural farmers, hospitals, soccer clubs, workers in particular professions, pensioners, ethnic minorities, and others. These specific groups are capable of generating political capital for parliamentarians.

According to the author, parliamentarians lack motivation to prioritize the search for fiscal balance, which is the crux of pension reforms. Instead, they favor measures that yield short-term gains such as augmenting benefits beyond inflation and generating exclusive retirement plans for groups that perceive themselves as having unique identities from other employees.

This dynamic highlights the political and economic challenges involved in implementing pension reforms, since political interests often override the need to guarantee the sustainability of the pension system. The group that feels most affected by changes to the pension system often has a remarkable capacity for mobilization, exemplified by corporations of public employees, such as magistrates and the military. These groups often enjoy greater social prestige and economic benefits, which gives them considerable influence both in resisting pension reforms and in seeking advantages that are clearly perceived by those who

receive them. This can be illustrated by professional categories obtaining special pensions or population groups obtaining benefits without the need to prove a contribution.

As stated by Mendes (2014), the population bears the costs of these advantages through repercussions such as increased inflation and taxes. Consequently, there exist greater incentives to maintain and claim these privileges rather than to oppose them as these groups perceive greater direct benefits that outweigh the costs incurred by society at large. This dynamic can create substantial challenges for the promotion of pension reforms aimed at balancing systems and ensuring their sustainability.

Furthermore, Tafner and Giambiagi (2010) argue that nearly all pension systems were created under the PAYG system, resulting in distributive conflicts within the same generation featuring groups with different social characteristics and inter-generational conflicts that lead to higher costs for future generations. They stress that the transfer mechanism for income, with its contrasting elements, plays a key role in shaping social security systems.

Faced with this complex scenario involving changes in social security, it is clear that there is a reluctance to promote changes that provide greater sustainability to social security systems. This reluctance stems from the very concept of social security, society's lack of support for fiscal issues and traditional political interests. However, despite these challenges, some significant reforms have been implemented, albeit belatedly.

Thus, the possibility of applying a model that is capable of proposing a resolution, as has been adopted in countries such as the social security model adopted in Sweden, as described by Boado-Penas (2021), Italy, as referenced by Franco and Sartor (2006), Latvia, mentioned by Palmer et al. (2006), Poland, as highlighted by Chłoń-Domińczak and Góra (2006), and Norway, according to Christensen et al. (2012). This pension model is the NDC, which, as mentioned above, has a part-capitalized approach with individual accounts.

2.2.7 NDC SCHEMES

According to Holzmann (2017), the need for reforms in pension systems arose in OECD countries after the pause in economic growth in the aftermath of World War II. This was triggered by the primary oil price shock in the 1970s and demographic changes, following the implementation and expansion of these systems in the 1950s and 1960s.

Many nations with well-established pay-as-you-go benefit programs are grappling with funding challenges, caused by various factors, including program maturity, an aging

population, excessive promises of benefits, shifts in employment patterns (such as early retirement), and, in some instances, fiscal issues stemming from the switch to a market-oriented economy. The shift from a defined benefit (DB) scheme to a fixed defined contribution (FDC) or a multi-pillar scheme that integrates an FDC component is deemed by numerous public pension experts as a viable long-term resolution for the expected funding difficulties that afflict most of these systems (WILLIAMSON; WILLIAMS, 2003; FOX; PALMER, 2001; HOLZMANN; STIGLITZ, 2001; WORLD BANK, 1994).

The concept of notional accounts originates from Buchanan (1968), who proposed replacing payroll taxes with the purchase of Social Security bonds to finance retirement. The bonds would yield an interest rate based on the long-term US Treasury bond rate and GDP. The accumulated bonds would constitute the Notional Capital, used to fund retirement. At retirement age, individuals would receive an annuity adjusted by life expectancy and GDP growth. Valdés-Prieto (2000) notes that Buchanan's proposal aimed to ensure pension funding and protect it from political interference. Disney (1999) suggests indexing bonds and annuities to the contribution pool growth would have been a more natural approach. Boskin et al. (1988) proposed an unfunded individual account scheme in the US, with a guaranteed annuity based on long-term projections and an interest rate determined by an independent commission. Valdés-Prieto (2000) also mentions the adoption of points-based systems in France and Germany, where taxpayers accumulate points for retirement benefits based on salary ratios.

Notional Defined Contribution (NDC) account systems, also called Non Financial Accounts schemes, are funded on a PAYG basis and do not have actual financial materialization, unlike FDC systems which are fully or partially funded. In NDC schemes, the individual accounts are virtual in nature, meaning that the contributions made are subject to an interest rate based on economic and demographic factors. On the other hand, in FDC systems, the accounts are typically capitalized at the rates of return of investment portfolios. These two types of schemes differ in terms of their funding and the way the accounts are handled.

Among the specific benefits of NDC proposed by the aforementioned authors are: greater flexibility, savings in administrative costs, incentive to diversify risk, considerably reduced overall social security costs, ability to adjust fiscal policy to economic and demographic changes. On the other hand, the disadvantages of NDC include: Mixed financial statements, Stress on individual pension funds, Difficult management in individual account

plans. In conclusion, the author indicates that NDC can be an appropriate model for social security, especially for low- and middle-income countries where demographic structures and institutional knowledge are limited.

At retirement age in defined contribution schemes, the final pension is calculated by dividing the accumulated balance by a life annuity, determined through a set of technical options. In the case of NDC schemes, pensions are backed by the discounts from the active generation, with no individualized financial funds designated for this purpose. In essence, the sole asset consists of forthcoming contributions, provided that reserve funds are not in place.

It is worth noting that, according to Börsch-Supan (2007), notional defined contribution schemes do not modify the mechanics of other systems funded on a PAYG basis and, as such, have an "Achilles heel" in terms of sustainability when the size of the workforce is reduced, with the support ratio (which represents the relationship between contributors and pensioners), being a key factor in their solvency. Thus, concerning the funding of defined contribution schemes, two options are available to states: PAYG, with the virtual capitalization of individual accounts, or advance funding with the actual capitalization of the set-up savings accounts.

Lindbeck and Persson (2003) posit that in a defined contribution scheme, the rate is externally fixed, while the benefits are internally determined. Conversely, the defined benefit scheme is marked by an endogenous contribution rate. Aguilera (2005) emphasizes the hybrid nature of an NDC scheme, which exhibits the features of an FDC while simultaneously following a PAYG system.

With regard to the literature review on this pension methodology, Williamson (2003) evaluates the NDC as a new approach to providing retirement security to the middle-aged and elderly population. The article considers the advantages and disadvantages of NDC and highlights its applicability as an income distribution mechanism for retired people.

Auerbach and Lee (2006) examine the application of NDC defined contribution pension schemes in stochastic contexts, with a focus on designing and stabilizing the proposed model. Technical terms are fully explained, and the language is formal, clear, and precise. The use of passive tone with an impersonal construction maintains objectivity throughout the text. The authors avoid biased, emotional, figurative, or ornamental language, and stick to conventional academic writing principles with consistent citation, footnote style, and formatting. The text employs the standard British variant, adhering to correct grammar,

spelling and appropriate terminology. Their aim is to provide strategies that make it possible to adopt an NDC-based social security scheme, constructed to compensate for the risks inherent in social security schemes and the market economy. Initially, the authors established a theoretical model that suits the economy with intermediate risk asset premiums. The model is based on solutions derived from a system of differential equations. Subsequently, the article highlights that whilst NDC remains sTable as long as some of the plans are in line with the assumption of infinite stocks, a flexible regulatory system must also be taken into account. Finally, the authors of the study conclude that NDC is a potentially profiTable and secure retirement plan, particularly when it is meticulously planned and takes into account the relevant market conditions. The empirical evidence indicates that NDC affords two key benefits: firstly, it decreases exposure to market volatility, and secondly, it provides the potential for better long-term returns. Therefore, this approach may serve as a successful strategy for securing retirement benefits.

2.2.7.1 POSITIVE POINTS FOR ADOPTING NDC

Williamson and Williams (2003) argue that NDC schemes aid in balancing pension costs with contributions in the long run. The belief and expectation of Swedish decision-makers that a 16% contribution rate will suffice and remain over time to secure solvency of the system exemplifies this rationale. In this context, the authors contend that NDC programs display high responsiveness to demographic and economic changes, as well as the liquidity issues arising from these fluctuations, which are addressed by reserve funds aimed at offsetting revenue shortfalls. Holzmann and Palmer (2006) argue that a universally applicable NDC system is financially sustainable in principle as it can balance assets and liabilities over the long term while maintaining an unaltered contribution rate to achieve solvency.

Another advantage, recognized by Williamson & Williams (2003), is that employees can easily view their virtual savings resulting from contributions that have been reevaluated based on a fair, clear, and previously defined criterion for all citizens. This adds transparency to NDC schemes, surpassing that of NDB schemes. The authors posit that structural reform is politically motivated by the citizens' perception of transparency which reinforces the notion that individuals will receive benefits of proportional contributions. Simultaneously, it engenders a sense of ownership over higher benefits pension rights, replacing defined benefit

pensions. Börsch-Supan (2005) contends that NDC schemes offer greater transparency and credibility due to inherent characteristics compared to the convoluted formulas of NDB schemes. Per Aguilera (2005), such transparency positively affects job market participation.

Additionally, NDC schemes afford flexibility in establishing retirement age. Essentially, the concept communicated to individuals is that retiring early leads to a reduced pension, whereas retiring later leads to a higher pension. Essentially, the concept communicated to individuals is that retiring early leads to a reduced pension, whereas retiring later leads to a higher pension. This facilitates the management of retirement timing and allows for full or partial retirement.

2.2.7.2 NEGATIVE POINTS FOR THE ADOPTION OF NDC

In Williamson and Williams (2003) report, critics of notional accounts are accused of eroding social policies beyond their redistributive capacity. They induce perverse redistributive effects by using average life expectancy to calculate pensions, which does not account for the risk of longevity by social class and disproportionately benefits the wealthiest. They assume that the wealthy tend to live longer and receive more benefits from the system.

NDC schemes are not immune to political risk as decisions regarding the type of indexation, coverage during inactivity, amounts, and duration are political in nature and susceptible to changes by the government at any time. Disney (1999) notes that NDC schemes grant explicit property rights to be claimed over future taxes. However, Scherman (2011), a renowned critic of the Swedish NDC scheme, argues that the initial operating rules, agreed upon consensually and meant to last for life, have been subject to change. This approach has subjected the model to a trial-and-error experimental methodology.

One limitation reported in the European Commission (2012) evaluation report was the level of poverty experienced by elderly women following the 2008 crisis in Sweden. At a conceptual level, Williamson and Williams (2003) suggest that women with lower salaries, especially those who are single, may be at a relative disadvantage in an NDC scheme as compared to an NDB scheme. This is due to their lower remuneration levels, less regular contribution records, and fewer years of work at a lower level in full-time employment. The responsibility of caring for children and supporting the family during times of illness typically falls on women, hindering their ability to receive equal discounts as men and resulting in inadequate responses from NDC schemes towards this societal issue.

2.3. ANALYSIS OF MINAS GERAIS RPPS

Article 41 of MF Ordinance 464/18 directs the Social Security Secretarial Services to provide studies and general data related to RPPS on their website to aid the analysis of assumptions and hypotheses adopted in actuarial valuations of the systems. The methodology presented herein relies on data already collected by the RPPS and applied in their actuarial assessments.

In the proposed structure, there will be three types of categories in which individuals will be classified: active, retired and pensioners. The active members consist of current employees who contribute to the Minas Gerais State Pension Plan. Retirement can be obtained through programmed retirement, which consists of a monthly income for life for the active insured, once the necessary conditions for granting it have been met, as listed in State Complementary Law (LCE) 64/2002. Disability retirement, which consists of a lifelong monthly income for active insurers who, following a medical examination carried out by a medical board appointed by the RPPS, are deemed to be totally incapable of working and unable to return to work. The pension is paid for as long as the disability persists. Finally, the compulsory retirement consists of a monthly income for life to the insured person who reaches the age of 75, the benefit being proportional to the length of the contribution and the other conditions established by law regarding the length of public service and the term of office.

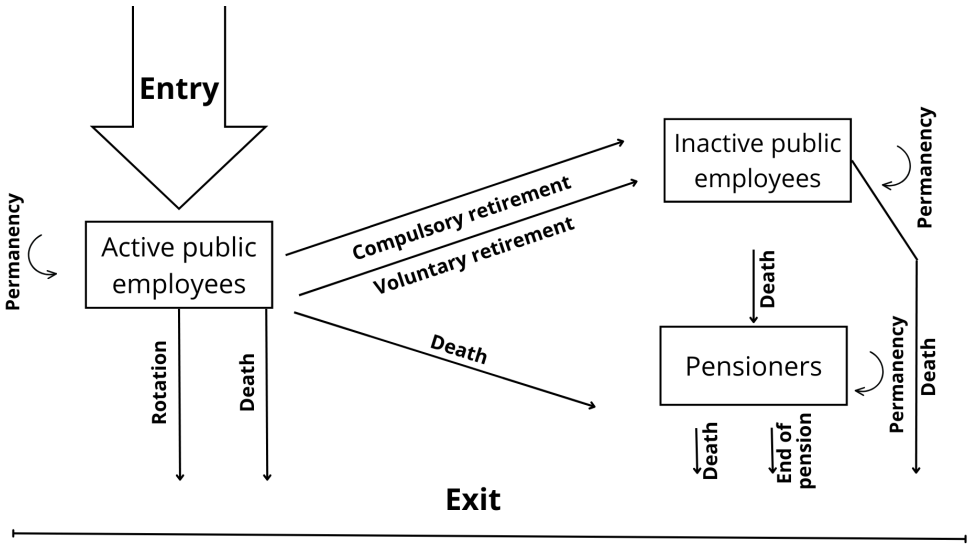
For pensioners, the death pension consists of a monthly income, for life or temporarily, depending on the situation of the insured person's beneficiary(ies) and the age of the beneficiary on the date of the event giving rise to the entitlement. The pension is paid to all beneficiaries who are eligible at the time it is granted and consists of a family portion equal to 60% of the basic benefit, plus a non-reversible portion of 10% per beneficiary in the event of disability or loss of entitlement.

2.3.1 STRUCTURE OF MINAS GERAIS RPPS

The purpose of the projections is to illustrate the yearly progression of the insured population encompassing active individuals, inactive individuals, and pensioners, thereby facilitating a fiscal and actuarial evaluation of the RPPS in the State of Minas Gerais. The

structure of the system can be exemplified by the following Figure 02, which describes the temporal dynamics of the insured.

Figure 02: Dynamics of retirees under the RPPS of Minas Gerais



Source: Own elaboration.

The insured's continuity is a residual event that occurs only in the absence of other events like turnover, retirement, death, or the end of a pension during the respective year. Continuity leads to the career advancement of active individuals, resulting in an increase in their contribution base and total compensation. To make an accurate prediction of career progress in public service, it is necessary to assume that the career configuration will adhere to the segmentation shown in the database and remain consistent throughout the projection period. With this assumption, it is feasible to estimate the expected progress of each career in terms of contribution base and compensation.

From this analysis, the following foundational information for projecting the RPPS emerges: there is initial information available on the calculation base, remuneration, and yearly growth rates for each active RPPS-insured career. As for inactive and retired individuals, assuming the immutability of the federated entity's careers, the real values of the benefits granted to them remain unchanged. The impact of continuity is restricted to ensuring insurance coverage through the RPPS.

Death, on the other hand, lacks a residual character and is represented by a randomized variable with a corresponding mortality curve. Given its significant influence on

projections, it is imperative to identify the mortality curve which best conforms to the profile of RPPS policyholders. Therefore, we consider the hypothesis that all individuals insured under RPPS, present and future, possess the same mortality curves.

Turnover refers to the departure of public employees who are currently working, whether by choice or resignation/termination. This phenomenon is typically modeled by the following hypothesis: the probability of leaving due to turnover is uniform among public employees and remains constant each year. Homogeneity implies that the likelihood of leaving is not influenced by public employees' characteristics, such as their pay or career. However, for the case analyzed, according to the simulations carried out and made available in the actuarial valuation (RELATÓRIO DE AVALIAÇÃO ATUARIAL, 2022, p.23) and given the characteristics of the insured mass, namely public employees with stability, the turnover rate of public employees will be considered zero.

Once the modeling of all the flows is completed, a temporal evolution simulation of the RPPS can be performed. To conduct a thorough analysis, we must formulate a hypothesis regarding the entry of active public employees since our previous focus was solely on their departure due to turnover, death, or retirement. This analysis employs the hypothesis of perfect substitution, which posits that the exit of a public employee from the asset base results in the entrance of another employee possessing identical characteristics. In essence, every time a public employee departs, another individual takes their place in the same career path.

Using data from the December 2022 (RELATÓRIO DE AVALIAÇÃO ATUARIAL, 2022), this study aims to evaluate the pension plan of the Minas Gerais Pension Fund (FFP-MG), a financial fund with a simple pay-as-you-go system that is part of the Minas Gerais State Public Employees Pension Plan (RPPS-MG), in light of the legal provisions of Federal Complementary Law 101/00, Law 9,717/98, MTP Regulation No. 1,467/2022, as well as State Constitutional Amendment No. 104/2020 and State Complementary Law (LCE) No. 64/02, all of which were in force at the date of the valuation. The actuarial valuation included active members, retirees and pensioners linked to the RPPS-MG, with data as of 12/31/2022. In addition, the data received is of sufficient breadth and consistency and is within the timeframe required by the legislation. Therefore, the results and conclusions presented are directly derived from these data.

2.3.1.1 DESCRIPTIVE STATISTICS

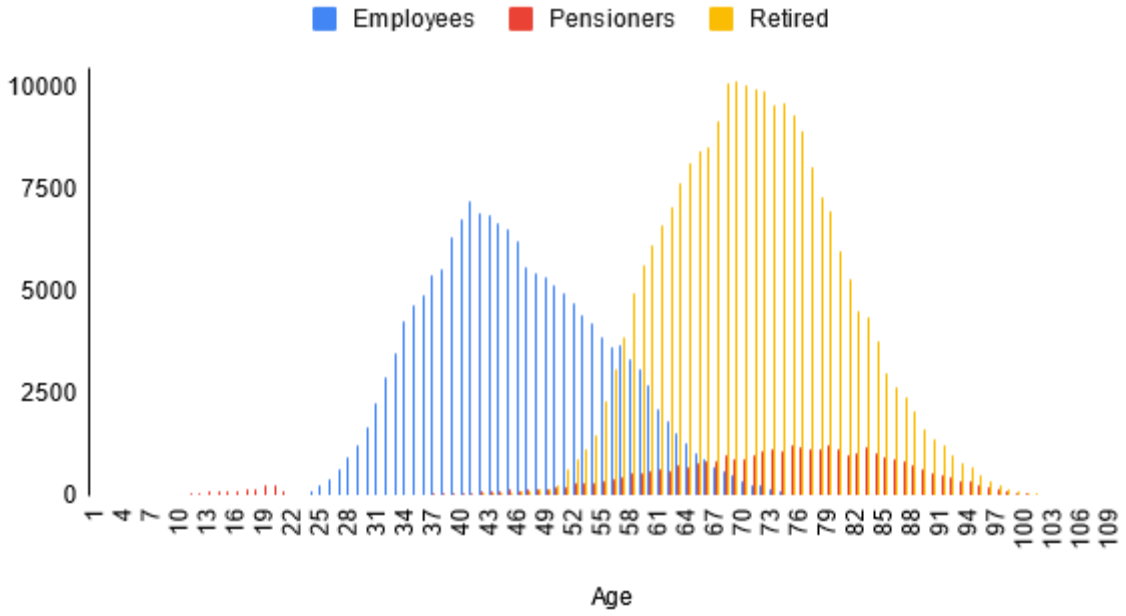
According to Table A01 in the appendix taken from the annual evaluation report for December 2022, with regard to the total number of active public employees in the Minas Gerais RPPS, we can see a decrease in the total number of public employees year on year from 2018, from 198,971 public employees to 164,950, another factor is the high percentage of women among the total, representing (61%) in 2022. The age of the youngest and oldest public employees is the same for both genders, 19 and 75 respectively, and the average age of admission to the Entity is 33.1 and 31.7 years. The lowest salary was the minimum in 2022 (R$1,212.00) and there is a large disparity between the contribution of women and men R$704.18 and 1,053.47 respectively given the average salaries of R$5,343.88 and R$7,659.42, totaling an average income of R$138,478,076.19 in December 2022 for the payment of retirees and pensioners in the current time.

Table A02 in the appendix shows the descriptive statistics for retired public employees with all the information provided by IPSEMG. It shows a total number of retirees of 240,156, 85% of whom are women. The average age of retirees is 70.8 and 72.2 for women and men respectively, as well as an average benefit received of R$4,218.79 and R$10,032.06, totaling a monthly expense for the state of R$1,219,149,048.95 in December 2022. Of the total number of pensioners, 1.98% are disabled.

Finally, Table A03 in the appendix of this paper details the data on pensioners in the Minas Gerais RPPS, with a total of 38,940 public employees, the highest Figure since 2018. Of the Figure shown, 67% are women with an average age of 71.9 years and an average income of R$6,204.73. For men, the average age is 68.1 years and they receive R$3,748.92. The average age of receipt is 53.5 and 57.8 for women and men respectively, which leads to a monthly expense for the state to make the payment of R$210,265,265.77 in December 2022.

As a way of demonstrating the information described above more clearly, the distribution between the ages of each type of individual among the groups, active, retired and pensioners in the RPPS follows the histogram shown in Figure 03 below:

Figure 03: Age distribution of the RPPS/MG



Source: Own elaboration based on Relatório de Avaliação Atuarial do IPSEMG (2022).

The ratio of active taxpayers to pensioners is 1.69. In relation to the amount collected and the amount disbursed by the state, the ratio is 10.32. For the year 2023, a financial shortfall of R$9,909,588,023.52 is estimated, generating a deficit for the state's public accounts of the same magnitude.

2.3.1.2 HOW TO STRUCTURE THE SIMULATIONS FOR FUTURE PROJECTION ANALYSIS?

The methodology described in this subsection uses the data previously described in the descriptive statistics collected from the Minas Gerais RPPS via the report made available by IPSEMG, the body responsible. With regard to the total number of active public employees, retirees and pensioners, with whom the accounting of annual income and expenditure will be carried out, the same configuration was used as for the projections found in the same report. This makes the comparison between the systems more reliable.

The demographic behavior of the population covered by the benefit plan was estimated using the following biometric Tables (RELATÓRIO DE AVALIAÇÃO ATUARIAL, 2022, p.21):

a) Mortality Table for valid employees (q_x) - working phase: AT-2000 Smoothed by 20%, segregated by sex; The change from the AT-2000 Mortality Table for Valid Employees Smoothed by 10%, adopted until the valuation of the 12/31/2021 focal date, to the AT-2000 Mortality Table for Valid Employees Smoothed by 20%, both segregated by sex, is noted as a result of the Adherence/Adjustment Test of the Hypotheses and Technical Bases of 02/07/2023.

b) Qualifying Mortality Table (q_x) - post-laboratory phase: AT-2000 Smoothed by 20%, segregated by sex; The change from the AT-2000 Qualifying Mortality Table Smoothed by 10%, adopted until the assessment of the 12/31/2021 focal date, to the AT-2000 Qualifying Mortality Table Smoothed by 20%, both segregated by sex, is noted, due to the Adherence Test/Adjustment of the Hypotheses and Technical Bases of 02/07/2023.

c) Mortality Table for invalids (q_x^i): Current mortality Table, segregated by sex, drawn up for both sexes by the Brazilian Institute of Geography and Statistics (IBGE), published on the website of the Department of Own and Supplementary Pension Schemes (SRPC), on the date of the valuation;

d) Entry into Disability Table (i_x): Álvaro Vindas. It is noted that the Álvaro Vindas Entry into Disability Table was maintained for the 12/31/2022 focal date valuation and adopted until the 12/31/2021 focal date valuation, to model the Entry into Disability event, in accordance with the Adherence/Adjustment Test of the Hypotheses and Technical Bases 07/02/2023.

As such, the values made available for the simulation estimates can be found in Table 03 below, which contains the Tables for items (a) to (d):

Table 01: Biometric Tables

		RPPS-MG Mortality Table			Minimum Required Table
Gender	Age	Smoothed AT-2000 20%	AT-2000 – Basic	AT-1983	IBGE – 2021
	At birth	87.1	84.3	83.3	80.5

Female	At 20 years	87.5	84.8	83.5	81.8
	At 62 years	87.1	86.8	85.7	85.1
	At 65 years	89.5	87.2	86.1	85.7
Male	At birth	83.1	80.1	77.9	73.4
	At 20 years	83.8	80.8	78.3	75.2
	At 62 years	84.4	84.0	82.1	81.5
	At 65 years	86.9	84.5	82.8	82.4

Source: Relatório de Avaliação Atuarial do IPSEMG (2022, p.22)

In contrast to the model used by IPSEMG, this research utilized the expected growth of the Brazilian population and the average replacement of retirees and pensioners to determine the growth of the mass of active public employees from 2023 to 2039. The model used by IPSEMG did not consider the growth of the future mass of active insured workers or the hypothesis of new entrants. As shown in Table A04 in the appendix, the number of active civil servants increased by an average of 3,838 over the years, with a range of 1,280 to 5,716 in 2026 and 2035, respectively.

Another point to consider in obtaining the results is assuming an average of 40 years dedicated to working in public office under the retirement system with a 13% salary allocation, corrected at an annual rate of 1.93%. As per the modality of acquired right for voluntary retirement by Article 144 of the ADCT of CE/89, included by State Constitutional Amendment No. 104 of September 15, 2020, an effective servant has the right to apply the retirement legislation prior to ECE No. 104/20, provided that all retirement requirements were met by September 15, 2020, the date of publication of the amendment. With 35 years of contribution for men and 30 years of contribution for women, an individual is eligible to retire as a state public employee of Minas Gerais.

2.4. RESULTS

This section will describe the results of the current scheme, as published in IPSEMG's actuarial report, as well as the results of the simulations and the policies proposed as a way of mitigating the fund's recurring negative results, which increase its actuarial liabilities year after year.

The data used for the analysis was made available by IPSEMG when it forwarded the actuarial valuation reports for the years 2015 to 2022 regarding expenses, income and public employees' data.

The GDP estimates for calculating the Notional Fund's profitability were obtained from the João Pinheiro Foundation, which provides monthly economic data for the state of Minas Gerais, while the estimates for life expectancy and population growth were obtained from the United Nations' Revision of World Population Prospects 2022 website.

2.4.1 CURRENT REGIME

In order to be able to analyze the results of each year and the impact on the accumulated net worth balance in a more coherent way, the initial value of the pension system was assumed to be zero and without real interest on the outstanding balance, i.e. the value described in the Table below as net worth in 2023 corresponds only to the difference between total income and total expenses in the same year and the results continue until 2039. The balance of net worth is cumulative with the value of each result recorded at the end of each year with the difference between the contribution of active employees and the payment of retirees and pensioners of the state of Minas Gerais. Table 02 shows all the data for IPSEMG's current PAYG system in millions of Reais:

Table 02: Actuarial result of the PAYG system of the RPPS-MG (in millions)

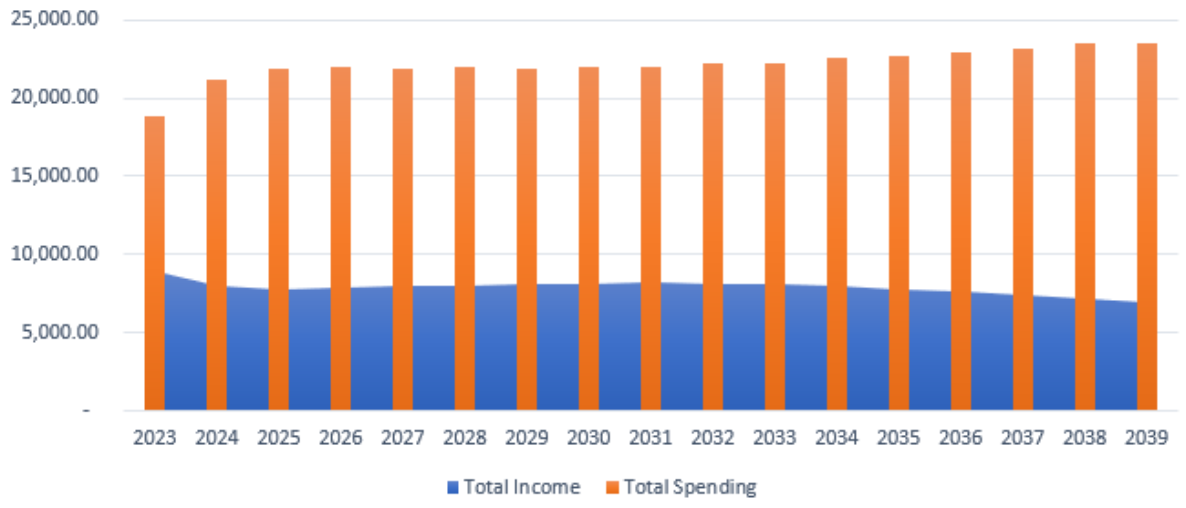
Year	Total Income	Total Spending	Result	Net Equity
2023	R$8,912.04	R$18,851.93	R$9,939.89	R$9,939.89
2024	R$7,983.65	R$21,129.36	R$13,145.71	R$23,085.60
2025	R$7,739.40	R$21,922.43	R$14,183.02	R$37,268.62
2026	R$7,842.09	R$21,958.71	R$14,116.62	R$51,385.25
2027	R$7,975.83	R$21,901.94	R$13,926.11	R$65,311.36
2028	R$8,047.31	R$21,944.17	R$13,896.86	R$79,208.22
2029	R$8,137.81	R$21,912.46	R$13,774.64	R$92,982.86
2030	R$8,149.51	R$22,009.11	R$13,859.60	R$106,842.47

2031	R$8,183.16	R$22,025.27	R$13,842.11	R$120,684.57
2032	R$8,124.97	R$22,190.08	R$14,065.11	R$134,749.68
2033	R$8,094.32	R$22,258.40	R$14,164.08	R$148,913.76
2034	R$7,939.79	R$22,532.29	R$14,592.50	R$163,506.26
2035	R$7,811.25	R$22,703.92	R$14,892.67	R$178,398.93
2036	R$7,606.58	R$22,979.76	R$15,373.18	R$193,772.11
2037	R$7,425.05	R$23,166.64	R$15,741.60	R$209,513.71
2038	R$7,163.88	R$23,463.78	R$16,299.90	R$225,813.61
2039	R$7,002.19	R$23,516.22	R$16,514.03	R$242,327.63

Source: own elaboration

Similarly, Figure 04 shows the information contained in Table 02 in relation to total income and expenditure to illustrate the magnitude of the difference between them. It can thus be established that the system is problematic and its permanence over time could be interrupted by its financial insolvency.

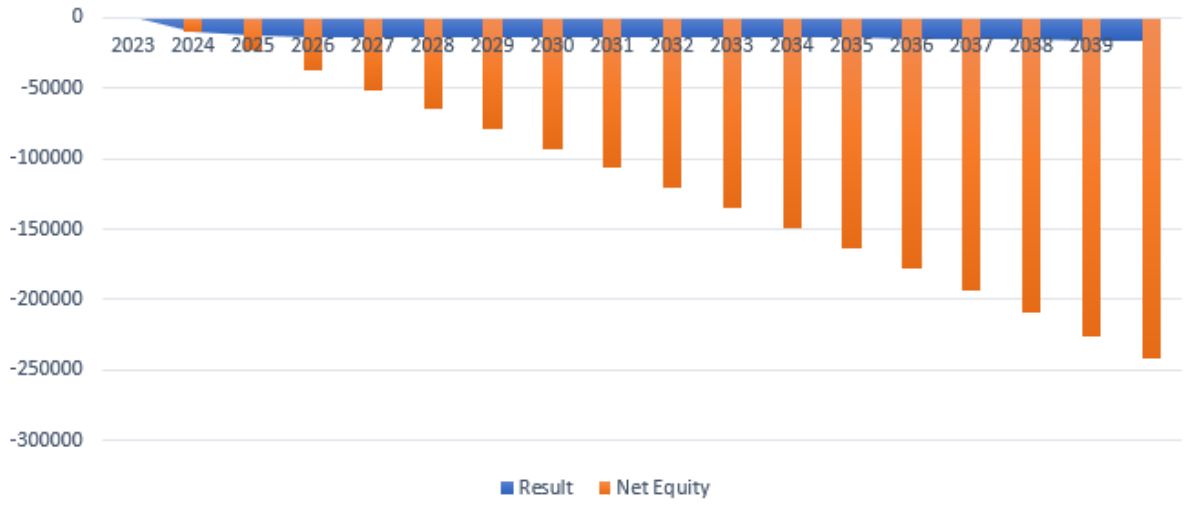
Figure 04: Difference between income and spending



Source: Own elaboration.

With regard to the deficit generated annually and the accumulated deficit from 2023 onwards, Figure 05 below shows the magnitude of the actuarial liability generated by the accumulation of the current system's debt.

Figure 05: Net Equity



Source: Own elaboration.

The result of the accumulated equity over the 17 years used for the simulation is 242,327.63. This Figure is due to the constant liabilities generated each year and, for the simulation, the cost of the debt was considered to be zero interest. This balance is just the difference between income and expenditure accumulated by the fund under the PAYG system.

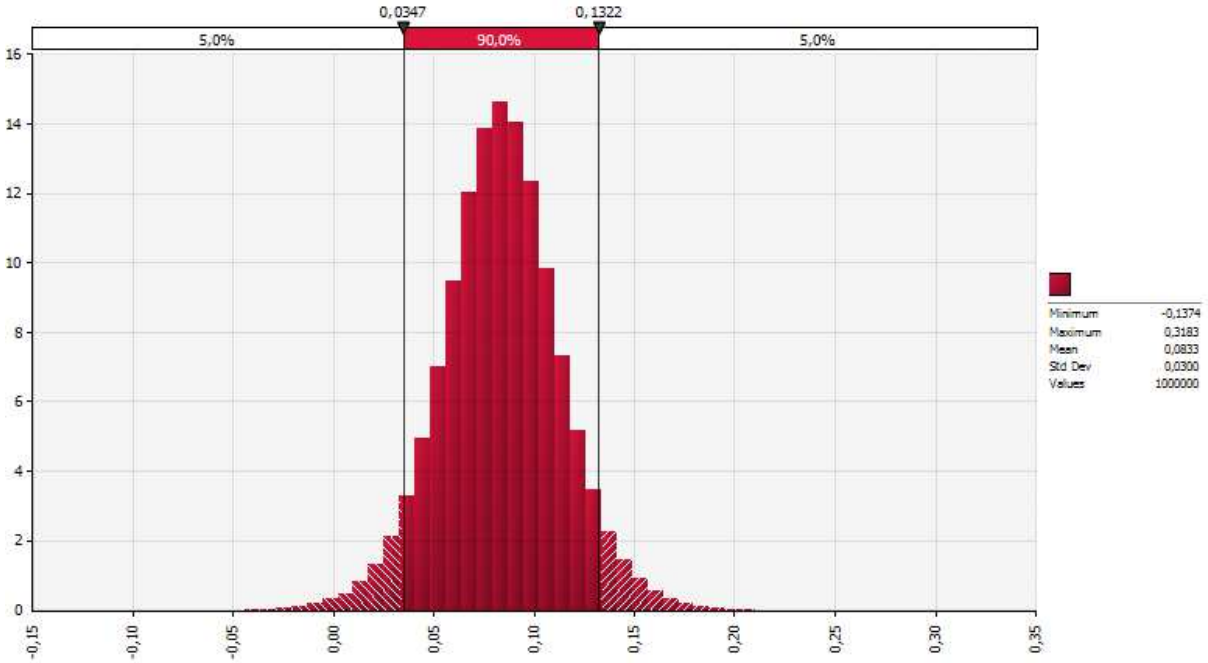
2.4.2 FUND WITH NEW ENTRANTS USING THE NOTIONAL SCHEME

Having said that, it was possible to propose a new constitution of a virtual fund for the accounting of the new pensions of these new entrants via the notional scheme, which consists of making the accounts profiTable for the subsequent payment of benefits given the estimates of population growth (n), the collection of MG (d) and the growth of the state's GDP (g). The form of profitability of these virtual accounts follows the equation described in the previous section:

$$z_t^i = [(1 + n) * (1 + g) * (1 + d)] - 1$$

The results, after 1 million iterations, were distributed as shown in Figure 06 below. It shows that the minimum generated for the Notional fund's return was -13.74%, which may be true, since the return can be negative in this methodology, but is unlikely given the variables used to establish such a return. In the maximum case, its result was 31.83% and an average annual return of 8.33%.

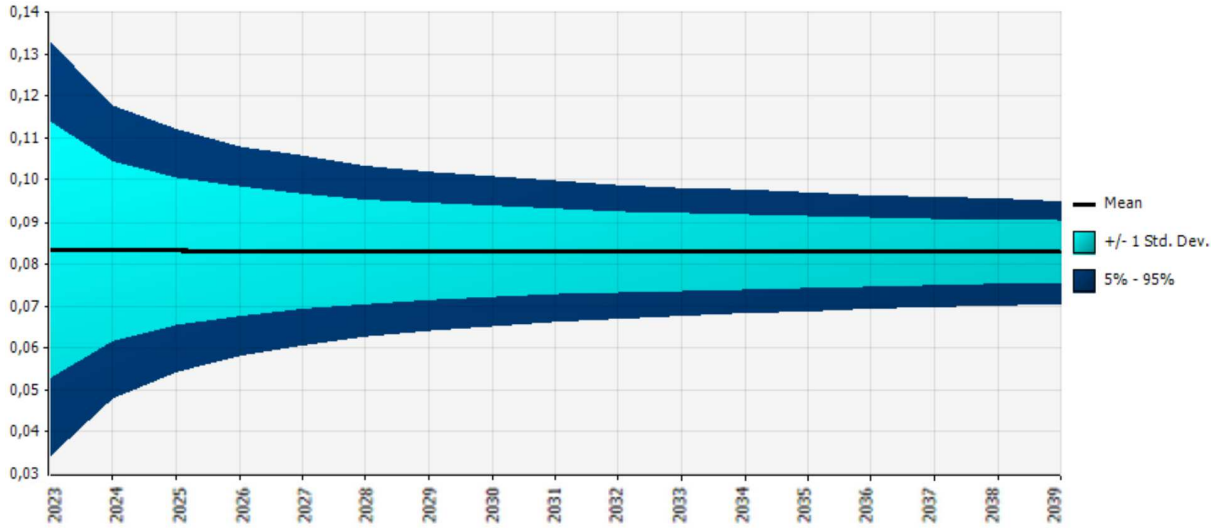
Figure 06: Distribution of the new fund's returns



Source: Own elaboration.

With a statistical significance interval of 5%, it can be seen that the rate of return on individual notional accounts tends to have an average return of 8.33%, as described in Figure 07.

Figure 07: Expected return



Source: Own elaboration.

According to IPSEMG, the fund's actuarial target is set at a return of IPCA+5.13%, according to the Monthly Investment Report for December 2023. Considering that the IPCA in 2022 was 5.79%, the new fund created does not correspond to the stipulated target, but this would no longer be necessary to maintain financial solvency, since each individual would have their individual calculation basis according to the calculated profitability.

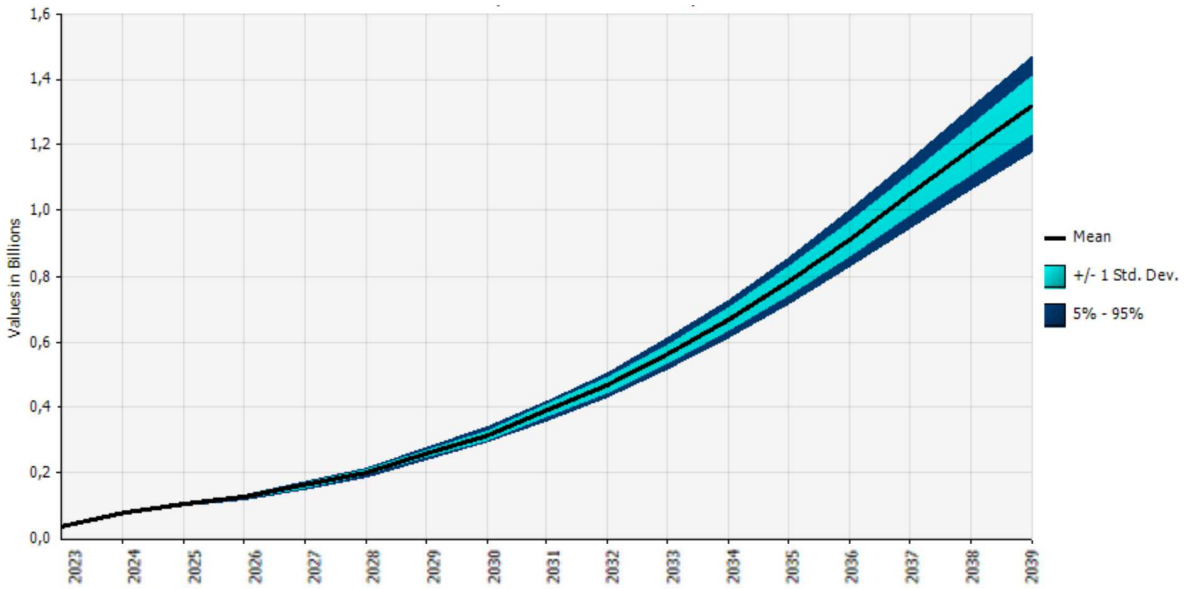
As previously stated, the notional accounts model is utilized in certain social security systems to determine retirement benefits. Under this model, each worker has an 'account' associated with their contributions to the social security system during their working life. The term 'notional' is used because this account does not necessarily reflect an actual bank account, but rather a theoretical representation of the accumulated value of the worker's contributions over time.

However, the balance is not accumulated as it is transferred to pay the full amount of pensions in force during the current period. The fund serves as a forecast for calculating the pensions of new public employees hired from 2023 onwards. These employees are fully informed of the amount they will receive upon retirement.

This fund will collect revenue from monthly contributions deducted from the payroll of the state's public employees at a fixed rate of 13%. The average remuneration for each employee, regardless of gender, is R$6,500.00. According to the simulation carried out, the

fund will yield an average of 8.3386% per year nominally, without taking into account inflation for the period. Therefore, Figure 08 displays the total accumulated balance for the analyzed period.

Figure 08: Notional fund's virtual balance



Source: Own elaboration.

Over the 17-year period, the fund is projected to reach an average of approximately 1.3 billion Reais, with increasing variation based on projected profitability. It is important to note that this fund serves as a parameter for future retirement or pension payments by the state, following capitalization with the z_t^i interest described above.

Thus, new entrants to the public service in Minas Gerais will gradually be governed by this methodology over a long period of time. This may solve future deficits caused by the annual increase in actuarial liabilities.

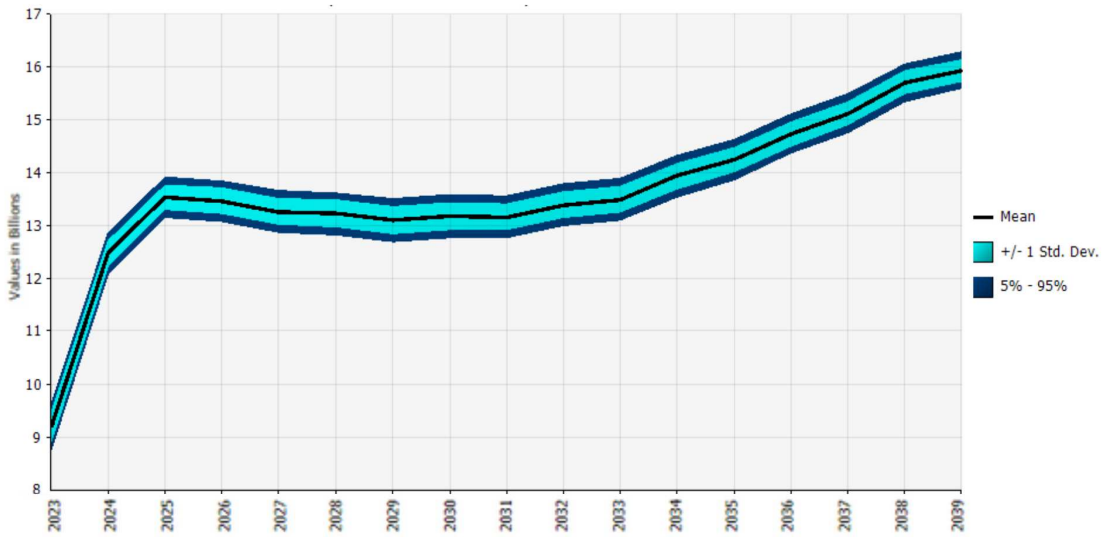
2.4.3 SETTING UP SOLVENCY INSURANCE FOR THE NOTIONAL SCHEME

This subsection applies the methodology of creating a notional account fund as described above. However, a solvency insurance for the scheme will be implemented. The insurance will consist of a 0.065% levy on the fund's income to reduce the negative net worth generated by the constant annual deficits. The insurance will be structured similarly to Credit

Default Swaps (CDS), which are derivative financial instruments that function as protection contracts against the risk of default by a debt issuer. Figure 09 below shows the charge on each individual's annual balance. The charge may vary depending on the profitability of the notional account fund balance and its profitability.

Introducing insurance for new entrants to the Minas Gerais state pension system has a slightly significant impact on public accounts, favoring a reduction in actuarial liabilities over time. Figure 09 shows the behavior of the accumulated liability curve (negative result) between 2023 and 2039. Please note that this graph indicates a reduction in actuarial liabilities.

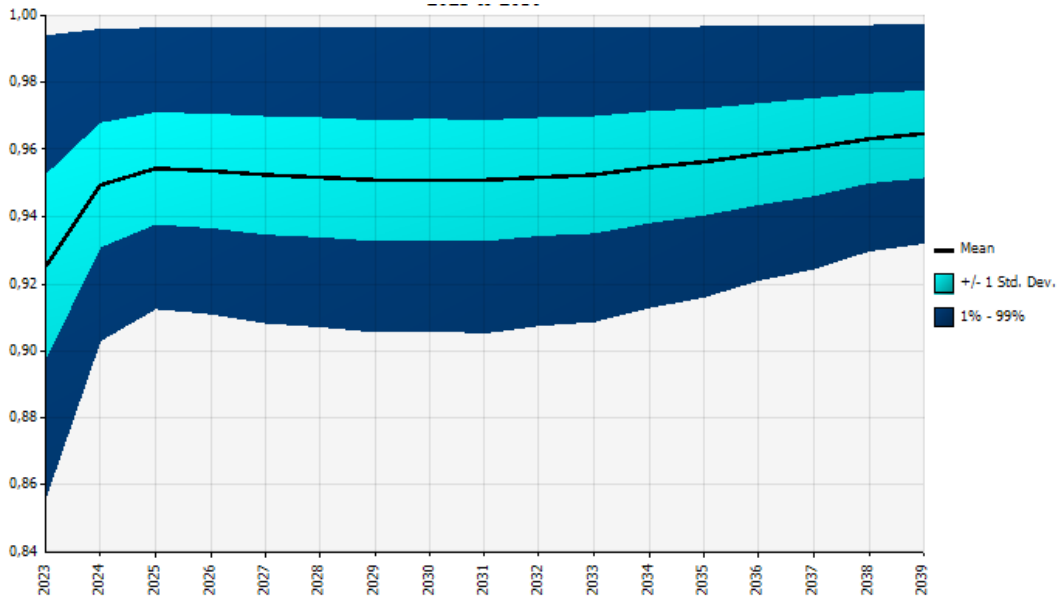
Figure 09: Accumulated actuarial liability with solvency insurance for new participants



Source: Own elaboration.

Figure 10 shows that the proposed fund is more efficient in all proposed scenarios and could generate an average reduction of 5% in the fund's actuarial liability over the years. The graph presents a ratio calculated from the value of the accumulated balance of the scheme with the notional accounts and the built-in insurance of the new entrants and the balance of the previous scheme. The result is statistically significant at 1%.

Figure 10: Ratio between actuarial results with solvency insurance for new participants



Source: Own elaboration.

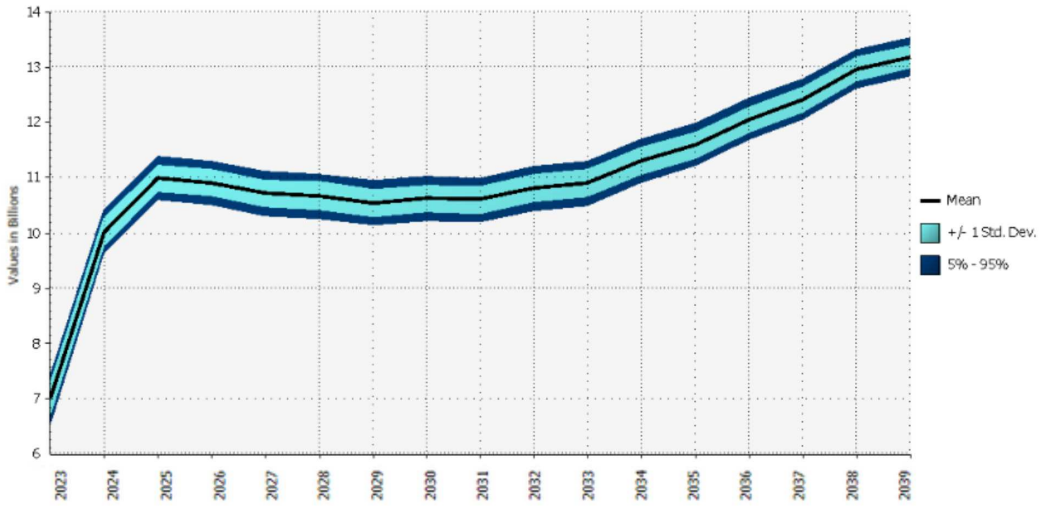
2.4.4 INTRODUCTION OF SOLVENCY INSURANCE FOR ALL PUBLIC EMPLOYEES

In addition to the solvency insurance applied to new entrants at a rate of 0.065% on the return on individual fictitious accounts, two other forms of taxation will be introduced: 0.065% on the annual salary of active public employees and a reduction of 11.64% on the average amount received by retired public employees and pensioners.

The average rebate linked to retirees and pensioners was established to equalize the amount paid by new entrants until the period tn which they would be retired. Thus, by calculating the percentage ratio between the estimated total coverage of these new public employees and the amount to be received in retirement at the end of 40 years of contributions to the system, we arrived at the rate of 11.64% that will be discounted.

Thus, taking into account the annual variation of 1.93% in the average salary of R$ 6500.00 of public employees, the annual insurance contribution is as follows $R\$6500 * 13 * 0.065\% = R\$54.92$. In this case, since the deduction is made when the salary is received, the so-called 13th is also taken into account. Thus, the value of the actuarial liability for the period analyzed, from 2023 to 2039, will be:

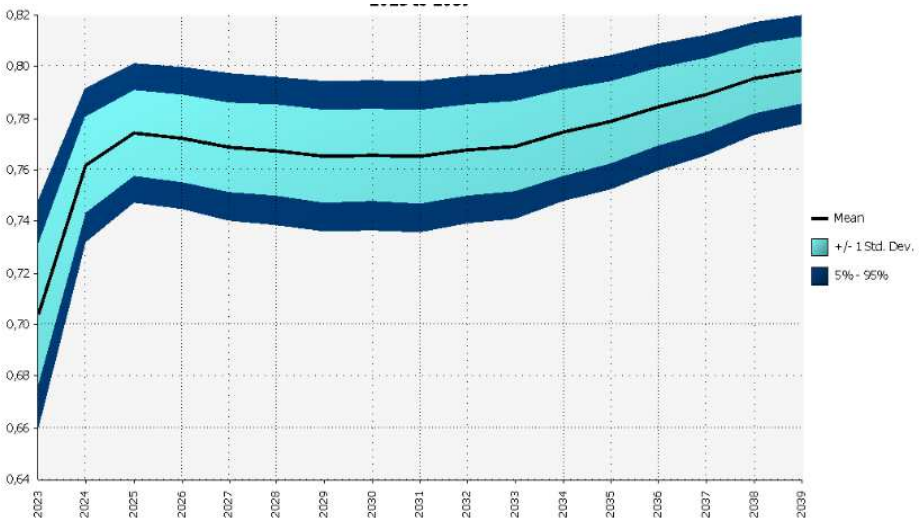
Figure 11: Accumulated actuarial liability with solvency insurance for all public employees



Source: Own elaboration.

Comparing the results of the previous sub-topic, in which public employees were not taxed under the current system, there is a big difference in the balance of liabilities generated by the social security deficit of the Minas Gerais RPPS, going from 16,514.03 million to 13,187.06 million, or a reduction of 20%. The impact of the accounts can be better seen in the following Figure. It shows the forecast of the possible results of the implementation of this policy with 1% statistical significance in the simulation carried out.

Figure 12: Ratio between actuarial results with solvency insurance for all public employee



Source: Own elaboration.

The rate of 11.64% may be considered too high for a deduction from the retirement pension of public employees in the current system, but it was adopted as an example considering any individual serving in the system, regardless of the length of service of their appointment. If we knew the distribution of the number of public employees according to each corresponding year of their entry into the system, it would be possible to tax each one proportionally according to Table 03.

Table 03: Proportionality rate for each working year

Working years	Tax	Working years	Tax	Working years	Tax	Working years	Tax
1	5.263%	11	5.493%	21	6.347%	31	8.249%
2	5.267%	12	5.545%	22	6.481%	32	8.519%
3	5.274%	13	5.603%	23	6.626%	33	8.807%
4	5.285%	14	5.668%	24	6.783%	34	9.114%
5	5.300%	15	5.740%	25	6.951%	35	9.440%
6	5.320%	16	5.820%	26	7.132%	36	9.786%
7	5.344%	17	5.908%	27	7.327%	37	10.154%
8	5.373%	18	6.004%	28	7.535%	38	10.544%
9	5.408%	19	6.109%	29	7.757%	39	10.957%
10	5.447%	20	6.223%	30	7.995%	40	11.395%

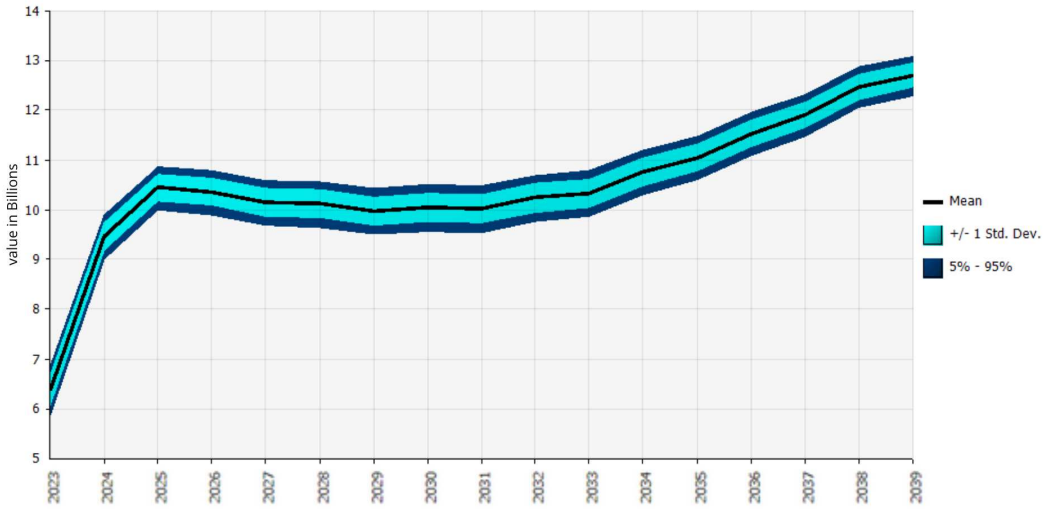
Source: Own elaboration.

In this way, it is possible to mitigate the deduction during retirement by starting to deduct salaries in proportion to the number of new entrants. Thus, according to the length of service corresponding to the individual, whether male or female, they will have their respective amount of salary deducted to cover the so-called solvency insurance of the system.

For example, if an individual has been a public employee for 10 years under the current pension scheme, a 5,447% discount will be taken from their salary until the end of their life, i.e. the discount will be taken from their current income (salary) and the amount they will receive as a retirement or pension.

For this analysis, a random linear distribution was considered in the simulation, considering the public employees in each year of their entry into the MG state government institutions that have the IPSEMG pension scheme. Since this is a more aggressive scheme from the point of view of the financial impact on family income, the result of the liabilities tends to be even lower, as shown below.

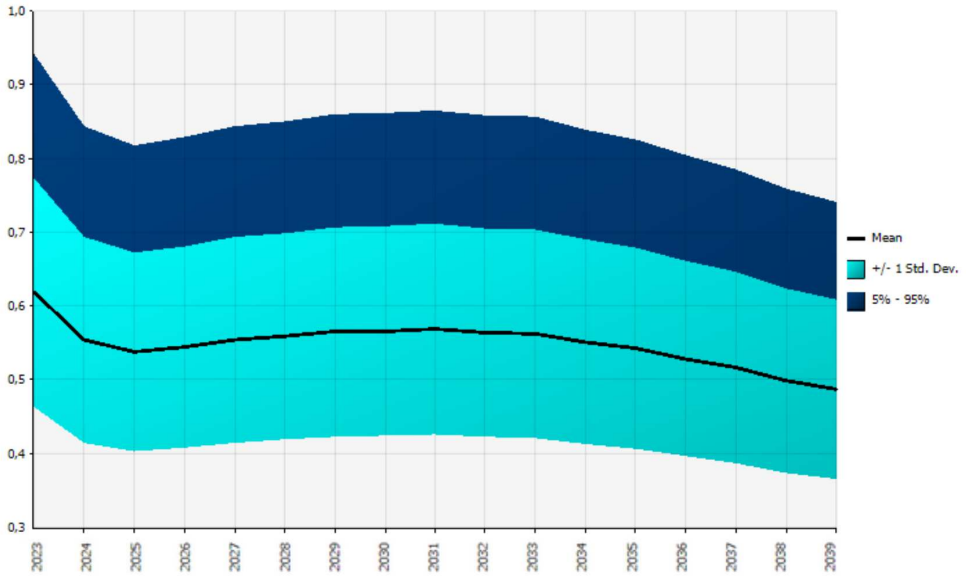
Figure 13: Accumulated actuarial liability with solvency insurance and the Proportionality rate for each working year



Source: Own elaboration.

Thus, considering the value of the liabilities of the current system and comparing it with the result of implementing the proposed insurance for all public employees more aggressively, the results with 1% statistical significance indicate that the reduction is consistent.

Figure 14: Ratio between actuarial results with solvency insurance for all public employees and different tax for each working year.



Source: Own elaboration.

2.5 CONCLUSION

An accurate study of the PAYG scheme within the Minas Gerais Social Security System (RPPS-MG) has revealed a worrying trend of deficits for the period from 2023 to 2039. The absence of real interest on the debit balance and the introduction of the pension system with an initial value of zero have exacerbated the financial insolvency of the current model, resulting in a negative accumulated net worth of 242,327.63 million reais.

Initially, the Notional account model is implemented, in which each employee's contributions create a notional account that serves as the basis for retirement benefits. However, the accumulated balance will not be retained, as it will be used to meet current pension obligations. New civil servants entering the system from 2023 will be subject to a fixed monthly contribution rate of 13%, contributing to a fund that is expected to return an average of 8.3386% per year in nominal terms.

Despite falling short of the actuarial target set by the IPSEMG, the new fund offers individualized calculation bases that promote financial sustainability. Over the 17-year projection period, the fund is expected to accumulate approximately 1.3 billion reais, helping to mitigate future deficits.

To ensure financial stability, a solvency insurance mechanism has been implemented, similar to credit default swaps. The pension system operates as a social insurance rather than a welfare or income redistribution policy, highlighting the concept of protection against system default. This insurance, levying a 0.065% rate on the Fund's income, serves to mitigate the adverse net assets impact arising from yearly deficits. Additionally, two other taxation methods will be enforced: a 0.065% rate on the annual salary of active civil servants and an 11.64% reduction on the average amount received by retired civil servants and pensioners.

Although the 11.64% rate may seem high, it is structured to equalize payments for new participants over the 40-year contribution period. The simulation shows a significant reduction in actuarial liabilities from 16,514.03 million to 13,187.06 million, a reduction of 20%.

Further statistical analysis has shown that the adoption of a more assertive solvency insurance scheme for all public employees would result in a substantial reduction in liabilities, albeit at the cost of substantial discounts on current salaries and pension benefits.

Recognizing potential concerns about the high deduction rate, a proportional approach based on years of service is proposed. Under this approach, salaries are deducted in proportion to the number of years of service, thereby easing the burden at retirement. The overall impact is expected to be less severe due to a random linear distribution of civil servants entering the system.

In this context, the transition to a notional scheme, accompanied by the introduction of solvency insurance, emerges as a strategic and effective approach to address the financial challenges inherent in the RPPS-MG PAYG scheme. The gradual integration of new entrants and the incorporation of financial safeguards converge to pursue the long-term sustainability of the public pension system. This approach seeks not only to address the identified financial gaps, but also to strengthen the structural integrity of social security in Minas Gerais.

While the results have proven effective in mitigating persistent ongoing deficits, the data acquired for the simulations conducted in this study were not sufficient to robustly unleash the full potential for improving the implementation of the proposed technique. This limitation is due to factors such as the lack of comprehensive information on the social security contribution history of the participants.

Moreover, although the study focused on Minas Gerais as a base, the identified scenario seems to be applicable to other sub-national federal entities within the Brazilian context in the future.

3 ANALYZING THEORETICAL MODELS IN THE SEARCH FOR SUSTAINABLE SOCIAL SECURITY SYSTEMS

3.1 INTRODUCTION

Demographic changes and slow GDP growth have increased attention on social security systems in several countries, including Brazil and its subnational entities. Despite recent reforms, these systems still face major challenges in balancing their actuarial liability accounts.

In situations where conditions are unsustainable and government-led reform efforts are needed to alleviate fiscal pressures, two fundamental questions arise: how can we design sustainable social security rules, and which of these sustainable configurations best serves the interests of the insured population?

To answer these questions, a robust theoretical framework based on the overlapping generations general equilibrium models developed by Paul Samuelson and Maurice Allais is necessary. This study focuses on models of consumption and credit without uncertainty to provide a relevant link to the application of social security rules in specific payment systems, such as pay-as-you-go (PAYG), capitalization, and Notional (NDC) schemes.

Barreto and Oliveira (1995) and Barreto (1997) conducted pioneering research in Brazil on the steady states of the social security overlapping generations model. Their findings suggest that more capitalized systems are generally superior in the long term, resulting in greater welfare gains. Dognini (2020) conducted a study using a model to apply the same criteria for pension formulation proposed here to the state and municipality of Rio de Janeiro, based on an NDC scheme. However, few studies have explored the dynamics of achieving a social security surplus, particularly within a state's pension system. In the state of Minas Gerais, this approach is unique, as it employs systemic reforms rather than solely demographic ones.

The objective of this second study is to analyse the impact of transitioning from the PAYG system to the Notional system on the pension system of the state of Minas Gerais. The text is free from grammatical errors, spelling mistakes, and punctuation errors. No changes in content have been made. The utility of each individual will be calculated based on their available income for consumption and the labour time invested during the transition to both a pure capitalisation system and the Notional scheme. The text adheres to conventional academic structure, employs clear and objective language, and maintains a formal register. The sentences and paragraphs create a logical flow of information with causal connections between statements. This analysis aims to evaluate changes in individual utility in relation to retirement decisions and the government's strategic choice to improve the management of the public account balance.

3.2 MATHEMATICAL MODELS

In this section, each of the types of social security schemes presented above will be presented. In addition, the type of individual representative of the economy who will exercise decision-making power over his or her retirement will be defined. This model is based on Kotamäki (2013).

However, although it is based on the work of the aforementioned author, the model proposed in this chapter aims to describe a simultaneous game between individuals and the government. Thus, each individual has the possibility to allocate consumption, savings, leisure, and retirement age, while the government will choose the type of pension system and taxes to balance its public accounts every period of time.

Each of the models and the parameters for each of the players are described below. The utility of individuals are homogeneous within and among generations. However, the optimality conditions change depending on the pension system analyzed. Then, the optimal choices of consumption, saving, and work, the first-order conditions (FCO), and the government conditions will be detailed below.

3.2.1 INDIVIDUALS

It is crucial to note that, for all of the pension systems that will subsequently be discussed, individuals' consumption function will be logarithmic. Thus, the elasticity of temporal substitution concerning consumption and leisure will be unity.

3.2.1.1 PAYG SCHEME

In this economy, there are a large number of identical individuals born in each period t . However, there are a finite number of periods in each 3-year generation denotes by i. Also individuals live their lives in a deterministic way. The objective for an individual is to select a series of consumptions, leisure activities, and asset possessions based on the factor prices, exogenous variable sequences for demographic trends and pension allocations, and an ultimate goal to maximize the discounted value of lifetime satisfaction under certain restrictions. The foresight is assumed to be perfect. The problem for an individual age-i who was born at time t is formulated as follows:

$$U_t^{i,\theta} = max_{c_t^{i,\theta}, l_t^{i,\theta}} \sum_{t=1}^{16} \beta^{t-1}\left[log\left(c_t^{i,\theta}\right) + \gamma log\left(1 - l_t^{i,\theta}\right)\right]$$

s.t.

$$c_t^{i,\theta} + s_t^{i,\theta} = w_t l_t^{i,\theta} + (1 + r)s_{t-1}^{i,\theta} - x_t^{i,\theta} - \tau_t^i \; \forall t = 1, \cdots, T^{i,\theta},$$

$$c_t^{i,\theta} + s_t^{i,\theta} = w_t l_t^{i,\theta} + (1 + r)s_{t-1}^{i-1,\theta} + p_t^{i,\theta}\left(T^{i,\theta}\right) - \tau_t^i \; \forall t = T^{i,\theta} + 1, \cdots, 16, c_t^{i,\theta} \geq 0 \; \forall \, t,$$

$$l_t^{i,\theta} \in [0, 1] \, , \forall \, t = 1, \cdots, T^{i,\theta}, \; and \; l_t^{i,\theta} = 0 \; \forall \, t = T^{i,\theta} + 1, \cdots, 16, \; \forall t,$$

$$T^{i,\theta} = 10, \cdots, 14 \; for \; \theta = f, \; and \; T^{i,\theta} = 11, \cdots, 14 \; for \; \theta = m.$$

For these equations, $c_t^{i,\theta}$ denotes consumption, $s_t^{i,\theta}$ saving, $l_t^{i,\theta}$ labour supply, so $\left(1 - l_t^{i,\theta}\right)$ represents leisure, $p_t^{i,\theta}$ pension income, $x_t^{i,\theta}$ the pension contribution. Also w_t and r represent wages and real interest rates respectively. Let us assume that the interest rate is time invariant. And τ_t^i represents a lump-sum tax. This type of tax may be different for each type of individual or group, but it is not indexed to any other variable described in the model. The term β^i is a positive parameter that denotes the yearly discount factor with $0 \leq \beta^i \leq 1$. γ

is a positive parameter that denotes the relative weight given to utility from leisure with $0 < \gamma < 1$. And θ is a parameter that represents the individual's gender. For all $c_t^{i,\theta}$ and $\left(1 - l_t^{i,\theta}\right)$, the usual Inada conditions hold with the assumed form of utility function.

Although all individual are born with zero assets and there is no saving in the last period of life because individual know that they die after retirement period, so $s_0^{i,\theta} = 0$ and $s_{16}^{i,\theta} = 0$.

According to Complementary Law No. 156, of September 22, 2020 - ALMG, new social security contribution rates for retirees and pensioners were implemented on December 22, 2020. The deductions increase progressively and are imposed on pensions that exceed three minimum wages. To simplify contribution calculations, 13% will be assumed for each individual throughout their working life.

A population of identical agents is born during each time period i, spanning a total of generations, each lasting 3 years and covering all individuals. This structural framework offers advantages, as it effectively takes into account all life cycle elements inherent in the problem at hand and supersedes the more commonly used model of two overlapping generations.

The first generation starts with individuals at the age of 32 and the last at the age of 80. The starting age was chosen based on the factor used to calculate the retirement of Brazilian public employees, which is standardized throughout the country. This is related to the contribution time, which corresponds to 35 years of contribution for men and 30 years of contribution for women. Or the individual age mechanism, in which women can retire at 63, while men can retire at 65. To make calculations easier, women were allowed to retire at the end of period 10, which corresponds to the age of 62, one year earlier than established by law for men, the age will be the same as established by law, at 65. By assumption it will be defined that women will retire in generation 10th and men in 11th. So women will work during the generation $t = 1, 2, \cdots, 10$ and men during the $t = 1, 2, \cdots, 11$.

All individuals have the option of remaining in the labour force until they reach the mandatory retirement age of 74[2], in the period 14th for women and men. Consequently, there are marginal gains in the future retirement benefits for those who choose to do so.

Death is certain, so the individual of generation t no longer exists at $t + 16$. In this case, the maximum age at which people live is the Brazilian life expectancy, which is about 80 years[3].

Formally, the demographic structure previously mentioned is presented below:

$$POP_t = \underbrace{N_1 + N_2}_{\text{Labour force}} + \cdots + \overbrace{N_{10} + N_{11}}^{\text{Retirement age}} + \cdots + \underbrace{N_{14}}_{\text{Compulsory retirement}} + N_{15} + \overbrace{N_{16}}^{\text{Death}} \quad (1)$$

$$POP_{t+1} = (1 + n)POP_t \quad (2)$$

N_t^i is the size of each generation t; POP_t is the total people adjusted by technology; n is the constant population growth rate.

As previously mentioned, x_t^i is the value of contributions up to the date of retirement. In this scenario, the calculations for men and women differ only with regard to the last payment period and, in this first case, the decision is made to retire without compulsory conditions. It is then determined as follows:

$\theta = f$ (women) $\qquad\qquad$ $\theta = m$ (men)

$$x_t^{i,f} \begin{cases} \psi w_t l_t^{i,f}, i = 1, 2, \ldots, 10 \\ 0 \; for \; i = 11, 12, \ldots, 16 \end{cases} \qquad x_t^{i,m} \begin{cases} \psi w_t l_t^{i,m}, i = 1, 2, \ldots, 11 \\ 0 \; for \; i = 12, 13, \ldots, 16 \end{cases}$$

where ψ is the pension contribution rate (13% as previously assumed) . And the pension benefits are given by:

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[2] For the State of Minas Gerais 'Public Employees' Retirement System (RPPS), the compulsory retirement age is set at 75 years However, the age of 74 was employed for computational convenience through the calculation method with 3 years in each generation

[3] According to UN estimates, the age of Brazilian life expectancy in 2023 was 76.17. That's why the age of 80 was adopted as a parameter.

$\theta = f$ (women) $\qquad\qquad$ $\theta = m$ (men)

$$p_t^{i,f}\begin{cases} 0\ for\ i = 1, 2, \dots, 10 \\ \mu_t^{rr}\dfrac{\sum_{t=1}^{11} w_t l_t^{i,f}}{10}, i = 11, 12, \dots, 16 \end{cases} \qquad p_t^{i,m}\begin{cases} 0\ for\ i = 1, 2, \dots, 11 \\ \mu_t^{rr}\dfrac{\sum_{t=1}^{12} w_t l_t^{i,m}}{11}, i = 12, 13, \dots, 16 \end{cases}$$

where μ_t^{rr} is the social security factor, that is, a mathematical formula that determines the value of IPSEMG pensions.

The pension rules of the IPSEMG provide that, if an individual exceeds the minimum period required to qualify for retirement, the payout from the security system will be increased by 2% for each year of contribution, subject to the RGPS ceiling, and will be subject to similar periodic adjustments in accordance with the RGPS standards. As each generation i represents 3 years, it will be assumed that the rate will be 6%. However, if the individual chooses to continue contributing until the mandatory retirement age, no changes are made to the method of calculating the contribution amount as described in the general case.

The aim is to encourage taxpayers to work longer, which will result in a reduction in benefits for individuals who retire before the age of 62 and 30 years of contributions for women, as well as 65 years of age and 35 years of contributions for men. The reduction in benefits will be greater for those who retire at a younger age. The social security factor was instituted by Law 9.876/99, following the 1998 Social Security Reform, to contain social security spending.

Assuming that individuals chooses to retire one generations after the minimum, the pension benefits change to:

$$p_{t+k}^{i,f}(10 + q) = \mu_t^{rr}\frac{\sum_{k=1}^{10+q} w_{t+k} l^k_{t+k}}{10+q}(1 + 0.06)^q,$$

$$p_{t+k}^{i,f}(10 + q) = \frac{\sum_{k=1}^{10+q} w_{t+k} l^{k,f}_{t+k}}{10+q}\left(\mu_t^{rr} + 0.06q\right),$$

$$p_{t+k}^{i,m}(11 + q) = \frac{\sum_{k=1}^{11+q} w_{t+k} l^{k,m}_{t+k}}{11+q}\left(\mu_t^{rr} + 0.06q\right),$$

here, q represents the additional years an individual can contribute until reaching a maximum limit of 14 ($max_t = 14$). It represents a value that is zero when the generation is below the established minimum ($min_t = m - 10$ for women or 11 for men) and the value ($t - min_i$) when $t \geq min_t$. The situation is similar for men and women. However, the transition is from the last generation to the 10th generation rather than the 11th.

Analyzing the results of each period for the restriction of individuals, we have if $\theta = f, T^{i,\theta} = 10$:

$$c_1^{i,f} + s_1^{i,f} = w_1 l_1^{i,f} - x_1^{i,f} - \tau_1^i,$$

$$c_2^{i,f} + s_2^{i,f} = w_2 l_2^{i,f} + (1 + r)s_1^{i,f} + p_2^{i,f} - \tau_2^i,$$

$$\vdots$$

$$c_{10}^{i,f} + s_{10}^{i,f} = w_{10} l_{10}^{i,f} + (1 + r)s_{t+8}^{9,f} + p_{t+9}^{10,f} - \tau_{t+9}^{10}.$$

$$c_{11}^{i,f} + s_{11}^{i,f} = w_{11} l_{11}^{i,f} + (1 + r)s_{10}^{i,f} - x_{11}^{i,f}(10) - \tau_{11}^i.$$

$$\vdots$$

$$c_{16}^{i,f} = w_{16} l_{16}^{i,f} + (1 + r)s_{15}^{i,f} - x_{16}^{i,f}(10) - \tau_{16}^i.$$

Since savings is a term that appears depending on past periods and assuming $(1 + r) = h$, we can represent a new budget constraint for the individual as follows:

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-t} + \sum_{t=1}^{10} x_t^{i,f} h^{16-t} = \sum_{k=11}^{16} p_t^{i,f} h^{16-k} + \sum_{t=1}^{10} w_t l_t^{i,f} h^{16-t}$$

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-t} + 0.13\sum_{t=1}^{10} w_t l_t^{i,f} h^{16-t} = \sum_{k=11}^{16} p_t^{i,f} h^{16-k} + \sum_{t=1}^{10} w_t l_t^{i,f} h^{16-t}$$

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-t} = \sum_{k=11}^{16} \frac{\sum_{t=1}^{10} w_t l_t^{i,f}}{10}\left(\mu_t^{rr}\right)h^{16-k} + 0.87\sum_{t=1}^{10} w_t l_t^{i,f} h^{16-t}$$

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-t} = \frac{\sum_{t=1}^{10} w_t l_t^{i,f}}{10}\left(\mu_t^{rr}\right)\left(\sum_{k=11}^{16} h^{16-k}\right) + 0.87\sum_{t=1}^{10} w_t l_t^{i,f} h^{16-t}$$

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-t} = \sum_{t=1}^{10}\left[\left(w_t l_t^{i,f}\right)\left(0.87h^{16-t} + \frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right].$$

For $\theta = f, T^{i,f} = 11$, the budget constraint can be written as:

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-i} = \sum_{t=1}^{11}\left[\left(w_t\, l_t^{i,f}\right)\left(0.87h^{16-t} + \frac{\left(\mu_t^{rr}+0.06\right)}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right].$$

$$\vdots$$

For $\theta = f, T^{i,f} = 14$, the budget constraint can be written as:

$$\sum_{t=1}^{16}\left(c_t^{i,f} + \tau_t^i\right)h^{16-t} = \sum_{t=1}^{14}\left[\left(w_t\, l_t^{i,f}\right)\left(0.87h^{16-t} + \frac{\left(\mu_t^{rr}+0.24\right)}{14}\left(\sum_{k=15}^{16} h^{16-k}\right)\right)\right].$$

For $\theta = m, T^{i,f} = 11$, the budget constraint can be written as:

$$\sum_{t=1}^{16}\left(c_t^{i,m} + \tau_t^i\right)h^{16-t} = \sum_{t=1}^{11}\left[\left(w_t\, l_t^{i,m}\right)\left(0.87h^{16-t} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right].$$

$$\vdots$$

For $\theta = m, T^{i,f} = 14$, the budget constraint can be written as:

$$\sum_{t=1}^{16}\left(c_t^{i,m} + \tau_t^i\right)h^{16-t} = \sum_{t=1}^{14}\left[\left(w_t\, l_t^{i,f}\right)\left(0.87h^{16-t} + \frac{\left(\mu_t^{rr}+0.18\right)}{14}\left(\sum_{k=15}^{16} h^{16-k}\right)\right)\right].$$

The FCO from the maximization of utility subject to the budget constraint of individuals retired by PAYG are:

For $\theta = f, T^{i,f} = 10$

$$\frac{\beta^{t-1}\gamma}{\left(1-l_t^{i,f}\right)w_t\left(0.87h^{16-t}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)} = \lambda\ \forall\ t = 1,\cdots,10, \tag{3}$$

$$\frac{\beta^{t-1}}{h^{16-t}c_t^{i,\theta}} = \lambda\ \forall\ t = 1,\cdots,16, \tag{4}$$

Then, we have that:

$$c_t^{i,f} = (\beta h)^{t-1}c_1^{i,f}\ \forall\ t = 1,\cdots,16, \tag{5}$$

$$l_t^{i,f} = 1 - \left(\frac{w_t}{w_{t-1}}\right)\frac{\beta^{t-1}\left(1-l_1^{i,f}\right)\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}{\left(0.87h^{15}+\frac{\left(\mu^{rr}+0.18\right)}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}\ \forall\ t = 1,\cdots,10,\ and \tag{6}$$

$$c_1^{i,f} = \frac{\left(1-l_1^{i,f}\right)w_t\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}{\gamma h^{15}}. \tag{7}$$

Substituting in the budget constraint, we have that:

$$\sum_{t=1}^{16}\left((\beta h)^{t-1}\frac{\left(1-l_t^{i,f}\right)w_t\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}{\gamma h^{15}}+\tau_t^i\right)h^{16-t}=$$

$$=\sum_{t=1}^{10}\left(w_t\left(0.87h^{16-t}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right)$$

$$-\sum_{t=1}^{10}\left(w_t\beta^{t-1}\left(1-l_t^{i,f}\right)\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right), \tag{8}$$

then, we have that:

$$l_1^{i,f} = 1 - \frac{\sum_{t=1}^{10}\left(w_t\left(0.87h^{16-t}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right)}{\left(\sum_{t=1}^{16}\left((\beta h)^{t-1}\frac{w_t\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}{\gamma h^{15}}+\tau_t^i\right)h^{16-t}+\sum_{t=1}^{10}\left(w_t\beta^{t-1}\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right)\right)}$$

$$c_1^{i,f} = \frac{w_t\left(\sum_{t=1}^{10}\left(w_t\left(0.87h^{16-t}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right)\right)\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}{\left(\sum_{t=1}^{16}\left((\beta h)^{t-1}\frac{w_t\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)}{\gamma h^{15}}+\tau_{t+i-1}^i\right)h^{16-t}+\sum_{t=1}^{10}\left(w_t\beta^{t-1}\left(0.87h^{15}+\frac{\mu_t^{rr}}{10}\left(\sum_{k=11}^{16} h^{16-k}\right)\right)\right)\right)\gamma h^{15}},$$

to obtain $c_1^{i,f}$ and $l_1^{i,f}$ from the FOC conditions described above.

For $\theta = f, T^{i,f} = 11, \cdots, 14$, we have that:

$$l_1^{i,f} = 1 - \frac{V_t^i}{\left(Q_t^{T^{i,f}}+H_t^i\right)} \tag{9}$$

$$c_1^{i,f} = \frac{w_t V_t^i F_t^i}{\left(Q_t^{T^{i,f}}+H_t^i\right)\gamma h^{15}}, \tag{10}$$

with

$$V_t^i = \sum_{t=1}^{T^{i,f}}\left(w_t\left(0.87h^{16-t}+\frac{\left(\mu^{rr}+0.06\left(T^{i,f}-10\right)\right)}{10+T^{i,f}}\left(\sum_{k=T^{i,f}+1}^{16} h^{16-k}\right)\right)\right),$$

$$Q_t^{T^{i,f}} = \sum_{t=1}^{16}\left((\beta h)^t \frac{w_t\left(0.87h^{15} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,f}-10\right)\right)}{10+T^{i,f}}\left(\sum_{k=T^{i,f}+1}^{16} h^{16-k}\right)\right)}{\gamma h^{15}} + \tau_t^i \right) h^{16-t}, \text{ for}$$

$\theta = f,\ T^{i,f}\ or\ \theta = m,\ T^{i,m}$

$$H_t^i = \sum_{t=1}^{T^{i,f}}\left(w_t \beta^t\left(0.87h^{16-t} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,f}-10\right)\right)}{10+T^{i,f}}\left(\sum_{k=T^{i,f}+1}^{16} h^{16-k}\right)\right)\right),$$

$$F_t^i = \left(0.87h^{15} + \frac{\left(\mu_t^{rr} + 0.06\left(T^{i,f}-10\right)\right)}{10+T^{i,f}}\left(\sum_{k=T^{i,f}+1}^{16} h^{16-k}\right)\right).$$

consumption and leisure in other periods are given by the FOC of the problem, that is:

$$c_t^{i,f} = (\beta h)^{t-1} c_1^{i,f}\ \forall\ t = 1, \cdots, 16, \tag{11}$$

$$l_t^{i,f} = 1 - \left(\frac{w_t}{w_{t-1}}\right)\frac{\beta^t\left(1-l_1^{i,f}\right)\left(0.87h^{15} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,f}-10\right)\right)}{10+T^{i,f}}\left(\sum_{k=T^{i,f}+1}^{16} h^{16-k}\right)\right)}{\left(0.87h^{16-t} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,f}-10\right)\right)}{10+T^{i,f}}\left(\sum_{k=T^{i,f}+1}^{16} h^{16-k}\right)\right)}\ \forall\ t = 1, \cdots, T^{i,f} \tag{12}$$

For $\theta = m, T^{i,m} = 11$

$$l_1^{i,m} = 1 - \frac{\sum_{t=1}^{11}\left(w_t\left(0.87h^{16-t} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right)}{\left(\sum_{t=1}^{16}\left((\beta h)^{t-1}\frac{w_t\left(0.87h^{15} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)}{\gamma h^{15}} + \tau_t^i\right)h^{16-t} + \sum_{t=1}^{11}\left(w_t\beta^t\left(0.87h^{15} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right)\right)} \tag{13}$$

$$c_1^{i,m} = \frac{w_t\left(\sum_{t=1}^{11}\left(w_t\left(0.87h^{16-t} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right)\right)\left(0.87h^{15} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)}{\left(\sum_{t=1}^{16}\left((\beta h)^{t-1}\frac{w_t\left(0.87h^{15} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)}{\gamma h^{15}} + \tau_t^i\right)h^{16-i} + \sum_{t=1}^{11}\left(w_t\beta^{t-1}\left(0.87h^{16-1} + \frac{\mu_t^{rr}}{11}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right)\right)\gamma h^{15}} \tag{14}$$

Finally, for $\theta = m, T^{i,m} = 12, \cdots, 14$, we have that:

$$l_1^{i,m} = 1 - \frac{A_t^i}{\left(Q_t^{T^{i,f}} + B_t^i\right)}, \tag{15}$$

$$c_1^{i,m} = \frac{w_t D_t^i E_t^i}{\left(Q_t^{T^{i,f}} + M_t^i\right)\gamma h^{15}}, \tag{16}$$

with

$$A_t^i = \left(0.87h^{16-t} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,m} - 11\right)\right)}{11 + T^{i,m}}\left(\sum_{k=T^{i,m}+1}^{16} h^{16-k}\right)\right),$$

$$B_t^i = \sum_{t=1}^{T^{i,m}}\left(w_t \beta^{t-1}\left(0.87h^{15} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,m} - 11\right)\right)}{11 + T^{i,m}}\left(\sum_{k=T^{i,m}+1}^{16} h^{16-k}\right)\right)\right),$$

$$D_t^i = \left(\sum_{t=1}^{T^{i,m}}\left(w_t\left(0.87h^{16-t} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,m} - 11\right)\right)}{11 + T^{i,m}}\left(\sum_{k=12}^{16} h^{16-k}\right)\right)\right)\right),$$

$$E_t^i = \left(0.87h^{15} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,m} - 11\right)\right)}{11 + T^{i,m}}\left(\sum_{k=12}^{16} h^{16-k}\right)\right),$$

$$M_t^i = \sum_{i=1}^{T^{i,m}}\left(w_t \beta^{t-1}\left(0.87h^{15} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,m} - 11\right)\right)}{11 + T^{i,m}}\left(\sum_{k=0.06\left(T^{i,m} - 11\right)+1}^{16} h^{16-k}\right)\right)\right).$$

consumption and leisure in other periods are given by:

$$c_t^{i,m} = (\beta h)^{t-1} c_1^{i,m} \; \forall \; t = 1, \cdots, 16, \tag{17}$$

$$l_t^{i,m} = 1 - \left(\frac{w_t}{w_{t-1}}\right)\frac{\beta^{t-1}\left(1 - l_1^{i,m}\right)\left(0.87h^{15} + \frac{\mu_t^{rr}}{11 + T^{i,m}}\left(\sum_{k=T^{i,m}+1}^{16} h^{16-k}\right)\right)}{\left(0.87h^{16-t} + \frac{\left(\mu^{rr} + 0.06\left(T^{i,m} - 11\right)\right)}{11 + T^{i,m}}\left(\sum_{k=T^{i,m}+1}^{16} h^{16-k}\right)\right)} \; \forall \; t = 1, \cdots, T^{i,m}. \tag{18}$$

EXAMPLE 1

Let us look at the difference in an individual's retirement choices in terms of their receipt. In this case, a 32 years old, i.e., in period $t = 1$, male individual has the option of retiring at age 65 or waiting until mandatory retirement at age 74. Their wage is set at 3[4] times the Brazilian minimum monthly wage or R$18.00 per hour[5]. The rate of return for savings[6]

[4] In 2023, the minimum wage in Brazil is R$1,320 per month

[5] considering a 40-hour work week, i.e. 8 hours a day.

[6] The yield for the year and the last 12 months is 8.03%.

will be in line with the real return on this rate in Brazil, without the inflation[7], so $r = 3.35\%$. For simplicity, we assumed for each period t, $r = 10\%$. The payout from the security system will be increased by 2% for each year of contribution or 6% for each i increased. The social security contribution is 13% of the salary received. Finally, the intertemporal discount rate will be assumed to be 0.97 and the relative weight given to utility from leisure will be 0.85.

So, we need to calculate and compare the situation with this functions:

For $t = 11$

$$l_{11}^{i,m} = 1 - \frac{A_{11}^{i}}{\left(Q_{11}^{T^{i,f}} + B_{11}^{i}\right)} = 0.786$$

$$c_{11}^{i,m} = \frac{w_{11} D_{11}^{i} E_{11}^{i}}{\left(Q_{11}^{T^{i,f}} + M_{11}^{i}\right)\gamma h^{15}} = 2,683.38$$

$$U_{11}^{i,m} = 3.2399$$

For $t = 12$

$$l_{12}^{i,m} = 11 - \frac{A_{12}^{i}}{\left(Q_{12}^{T^{i,f}} + B_{12}^{i}\right)} = 0.753$$

$$c_{12}^{i,m} = \frac{w_{12} D_{12}^{i} E_{12}^{i}}{\left(Q_{12}^{T^{i,f}} + M_{12}^{i}\right)\gamma h^{15}} = 2,769.32$$

$$U_{12}^{i,m} = 3.237$$

For $t = 13$

$$l_{13}^{i,m} = 1 - \frac{A_{13}^{i}}{\left(Q_{13}^{T^{i,f}} + B_{13}^{i}\right)} = 0.718$$

$$c_{13}^{i,m} = \frac{w_{13} D_{13}^{i} E_{13}^{i}}{\left(Q_{13}^{T^{i,f}} + M_{13}^{i}\right)\gamma h^{15}} = 2,828.77$$

$$U_{13}^{i,m} = 3.229$$

For $t = 14$

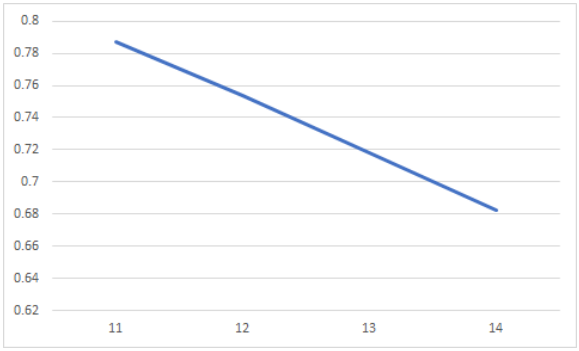
$$l_{14}^{i,m} = 1 - \frac{A_{14}^{i}}{\left(Q_{14}^{T^{i,f}} + B_{14}^{i}\right)} = 0.682$$

$$c_{14}^{i,m} = \frac{w_{14} D_{14}^{i} E_{14}^{i}}{\left(Q_{14}^{T^{i,f}} + M_{14}^{i}\right)\gamma h^{15}} = 2,862.28$$

$$U_{14}^{i,m} = 3.216$$

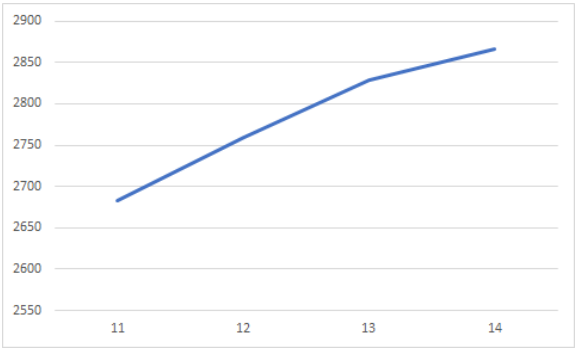
[7] The accumulated IPCA over the last 12 months is 4.68%.

Figure 15: Labour of period t in the PAYG scheme for male



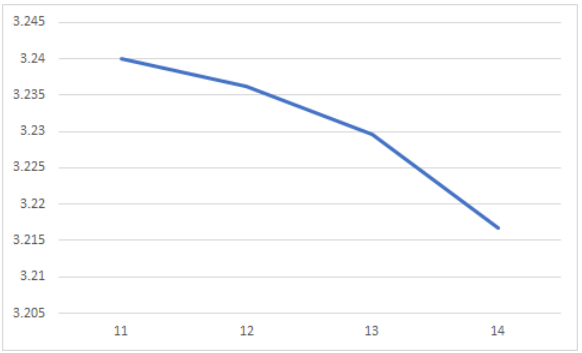
Source: Own elaboration.

Figure 16: Consumption of period t in the PAYG scheme for male



Source: Own elaboration.

Figure 17: Utility of period t in the PAYG scheme for male



Source: Own elaboration.

At the end of solving example 1, we can see that even though the worker spends less time working and has more income for consumption, in the PAYG system there is a loss of utility for the worker who decides to retire compulsorily at the end of period $t = 14$ compared to the option of retiring in the first possible period, which in this case is $t = 11$.

Examining the broader impact on the labor supply in the economy reveals a trend toward contraction. As individuals find it more advantageous to retire at the age of 65, there is a subsequent decrease in the overall supply of labor. This shift not only influences a reduction in household savings but also contributes to a decline in the capital held by firms.

EXAMPLE 2

Let us look at the difference in an individual's retirement choices in terms of their receipt. In this case, a 32 years old, i.e., in period $t = 1$, female individual has the option of retiring at age 63 or waiting until mandatory retirement at age 74. The information is the same as in the exemple 1.

For $t = 10$

$$l_{10}^{i,f} = 1 - \frac{V_{10}^{i}}{\left(Q_{10}^{T^{i,f}} + H_{10}^{i}\right)} = 0.817$$

$$c_{10}^{i,f} = \frac{w_{10} V_{10}^{i} F_{10}^{i}}{\left(Q_{10}^{T^{i,f}} + H_{10}^{i}\right)\gamma h^{15}} = 2,551.98$$

$$U_{10}^{i,f} = 3.232$$

For $t = 11$

$$l_{11}^{i,f} = 1 - \frac{V_{11}^{i}}{\left(Q_{11}^{T^{i,f}} + H_{11}^{i}\right)} = 0.786$$

$$c_{11}^{i,f} = \frac{w_{11} V_{11}^{i} F_{11}^{i}}{\left(Q_{11}^{T^{i,f}} + H_{11}^{i}\right)\gamma h^{15}} = 2,735.99$$

$$U_{11}^{i,f} = 3.248$$

For $t = 12$

$$l_{12}^{i,f} = 1 - \frac{V_{12}^{i}}{\left(Q_{12}^{T^{i,f}} + H_{12}^{i}\right)} = 0.752$$

$$c_{12}^{i,f} = \frac{w_{12} V_{12}^{i} F_{12}^{i}}{\left(Q_{12}^{T^{i,f}} + H_{12}^{i}\right)\gamma h^{15}} = 2,823.50$$

$$U_{12}^{i,f} = 3.245$$

For $t = 13$

$$l_{13}^{i,f} = 1 - \frac{V_{13}^{i}}{\left(Q_{13}^{T^{i,f}} + H_{13}^{i}\right)} = 0.717$$

$$c_{13}^{i,f} = \frac{w_{13} V_{13}^{i} F_{13}^{i}}{\left(Q_{13}^{T^{i,f}} + H_{13}^{i}\right)\gamma h^{15}} = 2,877.60$$

$$U_{13}^{i,f} = 3.236$$

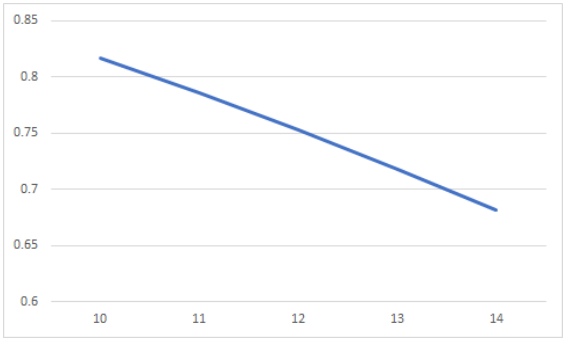
For $t = 14$

$$l_{14}^{I,f} = 1 - \frac{V_{14}^{i}}{\left(Q_{14}^{T^{i,f}} + H_{14}^{i}\right)} = 0.682$$

$$c_{14}^{i,f} = \frac{w_{14} V_{14}^{i} F_{14}^{i}}{\left(Q_{14}^{T^{i,f}} + H_{14}^{i}\right)\gamma h^{15}} = 2,899.65$$

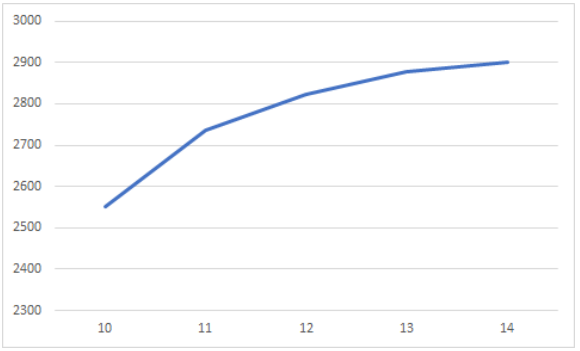
$$U_{14}^{i,f} = 3.221$$

Figure 18: Labour of period t in the PAYG scheme for female



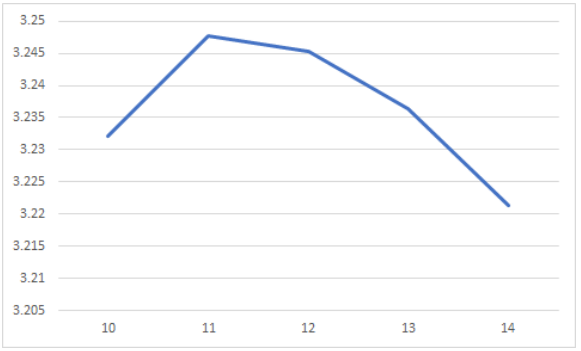
Source: Own elaboration.

Figure 19: Consumption of period t in the PAYG scheme for female



Source: Own elaboration.

Figure 20: Utility of period t in the PAYG scheme for female



Source: Own elaboration.

Looking at example 2, we can see that although the worker spends less time working and has more income for consumption, in the PAYG system there is a loss of utility for the female worker who decides to compulsorily retire at the end of period $t = 14$, but retiring in the first possible period, i.e. $t = 10$, doesn't show the best results either. That said, the option to retire in the period that results in the best utility in this case is $t = 11$. Figure 20 shows exactly the behavior depicted, indicating the highest point in $t = 11$.

With regard to the supply of labor in the economy, the trend is towards a reduction, since at the age of 63 for females it becomes more advantageous to retire. This will also lead to a reduction in household savings and, consequently, a reduction in firms' capital.

3.2.1.2 CAPITALIZATION SCHEME

The Life Cycle Hypothesis is the central theorem of economic analysis to explain how rational individuals distribute their lifetime income between consumption and savings in relation to funded pensions. Individuals are generally unconcerned with the method they use to save for retirement, whether it be through individual saving on a voluntary basis (e.g. direct investment in the stock market or voluntary savings through life insurance companies) or compulsory saving via employer-sponsored pension funds (such as defined benefit - DB or defined contribution - DC schemes). Rational individuals will adjust their individual savings downwards to account for compulsory savings made through pension fund schemes. The life-cycle hypothesis suggests that pension fund savings substitute individual savings without affecting overall savings (Feldstein, 1978). As a result, the funding of defined benefit plans does not have an impact on consumption and saving as a whole.

Moreover, in a defined benefit (DB) plan, an individual's pension benefit is calculated using a formula that takes into account numerous factors, such as years of service, salary, and age at retirement. Conversely, in a defined contribution (DC) plan, contributions are made to an individual account for each participant and invested in their chosen funds. The employer or employee is responsible for investment and longevity risks in a DB or DC plan, respectively. DB plans remain relevant in the private sector, as one in five private sector workers in the United States rely on them for retirement income, which amounts to approximately 40 million workers[8]. However, DC plans have become more prevalent, with 64% of private sector employees solely relying on them for retirement income (MUNNELL; SOTO, 2007)

For this type of social security, there is not an obligation to save through a pension fund and how much of their wage. Also each individual decides the best time to retire, respecting the minimum requirement of 62 years for women (generation $t = 10$) and 65

[8] Source: http://www.bls.gov/ncs/

years for men ($t = 11$) and a maximum of 74 years ($t = 14$), so $p_t^{i,\theta} = 0$ and $x_t^{i,\theta} = \psi w_t l_t^{i,\theta}$ if $l_t^{i,\theta} > 0$, and $p^{i,\theta}{}_t > 0$ and $x_t^{i,\theta} = 0$ if $l_t^{i,\theta} = 0, \forall t$.

The individual decides when to retire, taking into account how many generations they want to receive his own pension payments. Thus, there is no recourse to income from work after the decision to retire and there is no possibility of accumulating savings after the moment of death. Thus, the individual saves for his retirement from generation 1 to $t = T^{i,\theta}$ and receives retirement from $T^{i,\theta} + 1$ to 16, with $T^{i,\theta}$ being the moment of the decision of his choice to retire.

From moment k onwards, the income becomes just the accumulated savings and the returns obtained from the investment fund, whose average return is $e = 7.28\%$ per year[9] in Brazil. Here, in order to make the interest rate real, an average value for the inflation rate of $\pi = 5\%$ per year was established, thus, $r = e - \pi = 2.28\%$ or $r^t = 6,99\%$ for each generation. It is also important to note that there is no distinction between men and women in this scheme and the retirement income or, in that way, pension benefits. Moreover, there is a maintenance fee for pension fund accounts defined as a. However, for the result to be Pareto efficient, it will be assumed that the value of a is close to zero o in such a way that it would make the investment in savings or in the pension fund have the same return.

In addition, as pension payments will be calculated by bringing the total amount invested to the present discounting the interest rate, the individual will know the total accumulated until the last payment of their life, so the amount will be the same for everyone calculated from a simple division. Therefore:

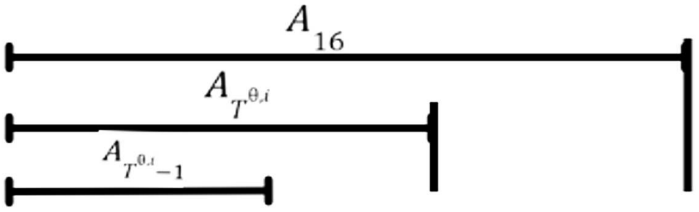
$$p_{16}^{i,\theta}\left(T^{\theta,i}\right) = p_{15}^{i,\theta}\left(T^{\theta,i}\right) = \cdots = p_{T^{\theta,i}}^{i,\theta}\left(T^{\theta,i}\right) \qquad (19)$$

In the case of funded pensions, it is important to mention that the withdrawal of each payment reduces the total amount of savings that the individual has set aside for their retirement through a pension fund, but the total amount is still subject to interest.

The individual, knowing this, decides how much will be withdrawn, proportional to the time his surplus amount will remain in the fund until generation 16, which has been

[9] Source: https://www.infomoney.com.br/onde-investir/retorno-de-fundos-de-previdencia-de-renda-fixa-e-multimercados-empata-em-10-anos-aponta-analise

defined as the time of his death, the remaining balance will be zero. The renting rate is expressed by $j = (1 + r - a)$. Thus, the following relationship can be described mathematically:



(20)

$$\sum_{t=1}^{T^{\theta,i}} \left\{ \left[\left(x_{t-1}^{i,\theta} (j)^{t} - p_{t-1}^{i,\theta}\left(T^{i,\theta}\right) \right)(j) \; - p_{t}^{i,\theta}\left(T^{i,\theta}\right) \right](j) \; - p_{t+1}^{i,\theta}\left(T^{i,\theta}\right) \right\} \ldots$$

$$\left\{ \left[\left(\sum_{t=1}^{T^{i,\theta}} x_{t-1}^{i,\theta} (j)^{t} - p_{t-1}^{i,\theta}\left(T^{i,\theta}\right) \right)(j) - p_{t}^{i,\theta}\left(T^{i,\theta}\right) \right](j) - p_{t+1}^{i,\theta}\left(T^{i,\theta}\right) \right\} \cdots - p_{16}^{i,\theta}\left(T^{i,\theta}\right) = 0 \qquad (21)$$

This case is repeated $\left(16 - T^{i,\theta}\right)$ times until $A_{16} = 0$, where A is the statement that represents the balance at the end of generation i after the payment of a retirement parcel. p_t^i is the amount to be received from each individual in each generation and the amount corresponding to each receipt is described below:

$$p_{16}^{i,\theta}\left(T^{i,\theta}\right) = \left[A_{15}(j) \right] \qquad A_{15} = \frac{p_{16}^{i,\theta}\left(T^{i,\theta}\right)}{(j)}$$

$A_{T^{\theta,i}-1}$

$$p_{15}^{i,\theta}\left(T^{i,\theta}\right) = \left\{ \left[A_{14}(j) \; - p_{14}^{i,\theta}\left(T^{i,\theta}\right) \right](j) \right\}$$

$$\frac{p_{15}^{i,\theta}\left(T^{i,\theta}\right)}{(1+r-a)} = A_{14}(j) \; - p_{14}^{i,\theta}\left(T^{i,\theta}\right) \qquad A_{14} = \frac{p_{15}^{i,\theta}\left(T^{i,\theta}\right)\left(1+\frac{1}{j}\right)}{j}$$

$$A_{13} = \frac{p_{14}^{i,\theta}\left(T^{i,\theta}\right)\left(1+\frac{1}{j}+\frac{1}{(j)^2}\right)}{j}$$

$$A_{T^{i,\theta}} = p_{T^{i,\theta}}^{i,\theta}\left(T^{i,\theta}\right)\left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(j)^{k}} \right)\frac{1}{j} = \sum_{k=1}^{16-T^{i,\theta}} \frac{p_{T^{i,\theta}}^{i,\theta}\left(T^{i,\theta}\right)}{(j)^{k}}$$

$$\sum_{t=1}^{T^{i,\theta}} x_t^{i,\theta}(j)^{T^{i,\theta}-t} = \sum_{k=1}^{16-T^{i,\theta}} \frac{p_{T^{i,\theta}}^{i,\theta}\left(T^{i,\theta}\right)}{(j)^k} = \left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(j)^k}\right) p_{T^{i,\theta}}^{i,\theta}\left(T^{i,\theta}\right)$$

$$p_{16}^{i,\theta}\left(T^{i,\theta}\right) = p_{15}^{i,\theta}\left(T^{i,\theta}\right) = \cdots = p_{T^{i,\theta}}^{i,\theta}\left(T^{i,\theta}\right) = \frac{\left(\sum_{t=1}^{T^{i,\theta}} x_t^{i,\theta}(j)^{T^{i,\theta}-t}\right)}{\left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(j)^k}\right)} \quad (22)$$

From an individual plan member's perspective, a DB pension represents a significant allocation of personal wealth into a bond-like asset. The compulsory participation in the plan may disrupt the preferred exposure to the return-risk trade-off provided by the market. If applicable, participants in the plan will adjust their entire personal portfolio of assets and liabilities to align with their preferred distribution of personal wealth (PONDS, 2003).

Thus, the individual's objective is the same and consists of selecting consumption, leisure activities and asset holdings based on factor prices, in sequences of exogenous variables that maximize their utility for demographic trends and the allocation of pensions, throughout life under certain constraints. The set of problems for an individual under the capitalization regime is:

$$U_t^{i,\theta} = max_{c_t^{i,\theta}, l_t^{i,\theta}} \sum_{t=1}^{16} \beta^{t-1}\left[log\left(c_t^{i,\theta}\right) + \gamma log\left(1 - l_t^{i,\theta}\right)\right]$$

s.t. $$c_t^{i,\theta} + s_t^{i,\theta} = w_t l_t^{i,\theta} + (1 + r)s_{t-1}^{i,\theta} + p_t^{i,\theta}\left(T^{i,\theta}\right) - x_t^{i,\theta},$$

$$c_t^i \geq 0 \; \forall \, t, i$$

$$l_t^{i,\theta} \geq 0\,, \forall\, t = 1, \cdots, T^{i,\theta},\ and\ l_t^{i,f} = 0 \; \forall\, t = T^{i,f} + 1, \cdots 4\ or\ l_t^{i,f} = 0 \; \forall\, t = T^{i,m} + 1, \cdots 3,$$

$$T^{i,f} = 10, \cdots, 14,\ and\ T^{i,m} = 11, \cdots, 14.$$

Analyzing the results of each period for the restriction of individuals, we have:

$$c_1^{i,\theta} + s_1^{i,\theta} = p_1^{i,\theta}\left(T^{i,\theta}\right) - x_1^{i,\theta}$$

$$c_2^{i,\theta} + s_2^{i,\theta} = (1 + r)s_1^{i,\theta} + p_2^{1,\theta}\left(T^{i,\theta}\right) - x_2^{i,\theta}$$

$$\vdots$$

$$c_{16}^{i,\theta} = (1 + r)s_{15}^{i,\theta} + p_{16}^{i,\theta}\left(T^{i,\theta}\right) - x_{16}^{i,\theta}$$

Since savings is a term that appears depending on past periods and assuming $(1 + r) = h$, we can represent a new budget constraint for the individual as follows:

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} = \frac{\left(\sum_{t=1}^{T^{i,\theta}}\psi w_t l_t^{i,\theta}(j)^{T^{i,\theta}-t}\right)}{\left(\sum_{k=1}^{16-T^{i,\theta}}\frac{1}{(j)^k}\right)}\left(\sum_{t=T^{i,\theta}+1}^{16}h^{16-t}\right) + \sum_{t=1}^{T^{i,\theta}}(1-\psi)w_t l_t^{i,\theta}h^{16-t}, \tag{23}$$

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} = \frac{\left(\sum_{t=1}^{T^{i,\theta}}\psi w_t l_t^{i,\theta}(j)^{T^{i,\theta}-t}\right)}{\left(\sum_{k=1}^{16-T^{i,\theta}}\frac{1}{(j)^k}\right)}\left(\sum_{t=T^{i,\theta}+1}^{16}h^{16-t}\right) + \sum_{t=1}^{T^{i,\theta}}(1-\psi)w_t l_t^{i,\theta}h^{16-t}, \tag{24}$$

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} = \sum_{t=1}^{T^{i,\theta}}\left(w_t l_t^{i,\theta}\right)\left(\frac{\left(\sum_{k=T^{i,\theta}+1}^{16}h^{16-k}\right)(j)^{T^{i,\theta}-t}}{\left(\sum_{k=1}^{16-T^{i,\theta}}\frac{1}{(j)^k}\right)} + (1-\psi)h^{16-t}\right), \tag{25}$$

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} = \sum_{t=1}^{T^{i,\theta}}\left(w_t l_t^{i,\theta}\right)B_t, \tag{26}$$

$$B_t = \left(\frac{\left(\sum_{k=T^{i,\theta}+1}^{16}h^{16-k}\right)(j)^{T^{i,\theta}-t}}{\left(\sum_{k=1}^{16-T^{i,\theta}}\frac{1}{(j)^k}\right)} + (1-\psi)h^{16-t}\right) \text{ with } t = 1,\cdots,T^{i,\theta}, \tag{27}$$

for any $T^{i,f} = 10,\cdots,14$ and $T^{i,m} = 11,\cdots,14$., after substituting the pension capitalization regime, we have that:

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} = \frac{\left(\sum_{i=1}^{T^{i,\theta}}x_t^{i,\theta}(j)^{T^{i,\theta}-t}\right)}{\left(\sum_{k=1}^{16-T^{i,\theta}}\frac{1}{(j)^k}\right)}\left(\sum_{t=T^{i,\theta}+1}^{16}h^{16-t}\right) + \sum_{t=1}^{T^{i,\theta}}(1-\psi)w_t l_t^{i,\theta}h^{16-t} \tag{28}$$

for any $T^{i,f} = 10,\cdots,14$ and $T^{i,m} = 11,\cdots,14$.

The FCO from the maximization of utility subject to the budget constraint of individuals retired by capitalization employees and retirees are:

$$\frac{\beta^t \gamma}{\left(1-l_t^{i,\theta}\right) w_t B_t} = \lambda, \ \forall t = 1, \cdots, T^{i,\theta}, \tag{29}$$

$$\frac{\beta^t}{h^{16-t} c_t^{i,\theta}} = \lambda, \ \forall t = 1, \cdots, 16, \tag{30}$$

Then, we have that:

$$c_t^{i,\theta} = (\beta h)^{t-1} c_1^{i,\theta}, \ \forall t = 1, \cdots, 16, \tag{31}$$

$$l_t^{i,\theta} = 1 - \left(\frac{w_t}{w_{t-1}}\right)\left(\frac{B_1}{B_t}\right)\beta^{t-1}\left(1 - l_1^{i,\theta}\right), \forall\, t = 1, \cdots, T^{i,\theta}, \ and \tag{32}$$

$$c_1^{i,\theta} = \frac{\left(1-l_1^{i,\theta}\right) w_t B_1}{\gamma h^{15}}. \tag{33}$$

Substituting in the budget constraint, we have that:

$$\sum_{t=1}^{16}\left((\beta h)^{t-1} \frac{\left(1-l_1^{i,\theta}\right) w_t B_1}{\gamma h^{15}}\right) h^{16-t} = \sum_{t=1}^{T^{i,\theta}}\left(\left(w_t B_t - w_t B_1 \beta^{t-1}\left(1 - l_1^{i,\theta}\right)\right)\right)$$

$$\left(1 - l_t^{1,\theta}\right)\left(\frac{w_t B_1}{\gamma h^{15}}\right)\sum_{t=1}^{16}(\beta h)^{t-1} h^{16-t} = \sum_{t=1}^{T^{i,\theta}} w_t B_t - \left(1 - l_1^{i,\theta}\right) w_t B_1 \sum_{t=0}^{T^{i,\theta}-1} \beta^t \tag{34}$$

$$\left(\frac{w_t B_1}{\gamma h^{15}}\right)\sum_{t=1}^{16}(\beta h)^{t-1} h^{16-t} - \sum_{t=1}^{T^{i,\theta}} w_t B_t + w_t B_1 \sum_{t=0}^{T^{i,\theta}-1} \beta^t = \tag{35}$$

$$= l_1^{i,\theta}\left(\left(\frac{w_t B_1}{\gamma h^{15}}\right)\sum_{t=1}^{16}(\beta h)^{t-1} h^{16-t} + w_t B_1 \sum_{t=0}^{T^{i,\theta}-1} \beta^t\right) \tag{36}$$

$$l_1^{i,\theta} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1} h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{T^{i,\theta}}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{T^{i,\theta}-1}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1} h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{T^{i,\theta}-1}\beta^t}$$

then, we have that:

$$l_1^{i,\theta} = \frac{\sum_{t=1}^{16}(\beta h)^{t-1} w_t h^{16-t} - \gamma h^{15}\sum_{t=1}^{T^{i,\theta}}\left(w_t B_t - w_1 B_1 \beta^t\right)}{\sum_{t=1}^{T^{i,\theta}} w_t \gamma h^{15} + \sum_{t=1}^{16}(\beta h)^{t-1} w_t h^{16-t}} \tag{37}$$

(38)

$$l_1^{i,\theta} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{T^{i,\theta}}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{T^{i,\theta}-1}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{T^{i,\theta}-1}\beta^t} \tag{39}$$

$$c_1^{i,\theta} = \frac{w_t}{\gamma h^{15}} - w_t\left(\frac{\sum_{t=1}^{16}(\beta h)^{t-1}w_t h^{16-t}}{\gamma h^{15}\left(\sum_{t=1}^{T^{i,\theta}} w_t \gamma h^{15} + \sum_{t=1}^{16}(\beta h)^{t-1}w_t h^{16-t}\right)} - \frac{\sum_{t=1}^{T^{i,\theta}}\left(w_t + w_t\beta^t\right)}{\sum_{t=1}^{T^{i,\theta}} w_t \gamma h^{15} + \sum_{t=1}^{16}(\beta h)^t - 1 w_t h^{16-t}}\right), \tag{40}$$

$$c_1^{i,\theta} = \frac{w_t B_1}{\gamma h^{15}} - \left(\frac{w_t B_1}{\gamma h^{15}}\right)\frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{T^{i,\theta}}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{T^{i,\theta}-1}\beta^t\right)}{\left(w_t B_1\right)\sum_{i=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{T^{i,\theta}-1}\beta^t}$$

to obtain $c_1^{i,\theta}$ and $l_1^{i,\theta}$ from the FOC conditions described above.

In this case, the upper limit for the sum will be determined by the individual, regardless of gender, while still adhering to the previously defined mandatory minimum for each period, $T^{i,f} = 10$ and $T^{i,m} = 11$ and the maximum retirement age in period $T^{i,f} = T^{i,m} = 14$.

Note that in this case there is no need for lump-sum tax, as there is no difference between the value of contributions and retirement payments. As mentioned earlier, the capitalization scheme is calculated from the balance in each individual's account. In addition, the savings balance of retired individuals serves as an increase in monthly income until the $t = 16$ period ts at zero without the possibility of inheritance or transfer between individuals.

In the same way that the PAYG scheme, when an individual retires, there are no values for $l_t^{i,\theta}$, since no work will be taken on during retirement, consequently there is no salary received after the decision for retirement.

EXAMPLE 3

Using the same information as in example 1, where a male individual has the option of retiring at 65 or waiting for compulsory retirement at 74, i.e. in period 11 or 14, let's see what

happens to his utility from the FOC calculated earlier. In this case, the contribution rate will remain at 13% of salary.

In the case of retirement in period 11, at the age of 65:

$$l_{11}^{i,m} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{11}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{10}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{10}\beta^t} = 0.711$$

$$c_{11}^{i,m} = \frac{w_t B_1}{\gamma h^{15}} - \left(\frac{w_t B_1}{\gamma h^{15}}\right)\frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{11}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{10}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=}^{10}\beta^t} = 1,372.55$$

$$U_{11}^{i,m} = max_{c_{11}^{i,m}, l_{11}^{i,m}} \sum_{t=1}^{16} \beta^{t-1}\left[log\left(c_{11}^{i,m}\right) + \gamma^i log\left(1 - l_{11}^{i,m}\right)\right] = 2.921$$

For retirement in period 12:

$$l_{12}^{i,m} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{12}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{11}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{11}\beta^t} = 0.676$$

$$c_{12}^{i,m} = \frac{w_t B_1}{\gamma h^{15}} - \left(\frac{w_t B_1}{\gamma h^{15}}\right)\frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{12}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{11}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{11}\beta^t} = 1,538.55$$

$$U_{12}^{i,m} = max_{c_{12}^{i,m}, l_{12}^{i,m}} \sum_{t=1}^{16} \beta^{t-1}\left[log\left(c_{12}^{i,m}\right) + \gamma^i log\left(1 - l_{12}^{i,m}\right)\right] = 2.952$$

For retirement in period 13:

$$l_{13}^{i,m} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{13}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{12}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{12}\beta^t} = 0.640$$

$$c_{13}^{i,m} = \frac{w_t B_1}{\gamma h^{15}} - \left(\frac{w_t B_1}{\gamma h^{15}}\right)\frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{13}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{12}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{12}\beta^t} = 1,711.26$$

$$U_{13}^{i,m} = max_{c_{13}^{i,m}, l_{13}^{i,m}} \sum_{t=1}^{16} \beta^{t-1}\left[log\left(c_{13}^{i,m}\right) + \gamma^i log\left(1 - l_{13}^{i,m}\right)\right] = 2.976$$

For compulsory retirement in period 14:

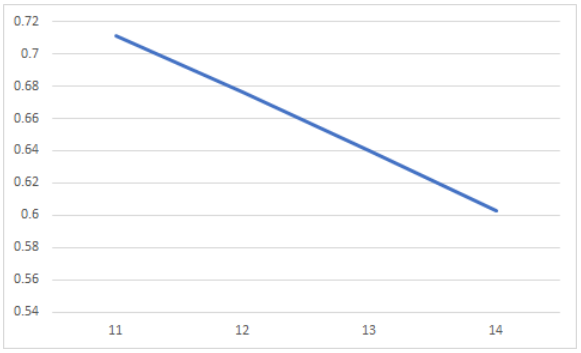
$$l_{14}^{i,m} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{14}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{13}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{10}\beta^t} = 0.602$$

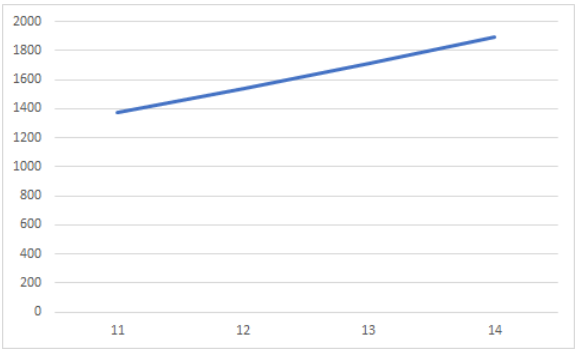
$$c_{14}^{i,m} = \frac{w_t B_1}{\gamma h^{15}} - \left(\frac{w_t B_1}{\gamma h^{15}}\right)\frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{14}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{13}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{13}\beta^t} = 1,890.08$$

$$U_{13}^{i,m} = max_{c_{14}^{i,m}, l_{14}^{i,m}} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{14}^{i,m}\right) + \gamma^i log\left(1 - l_{14}^{i,m}\right)\right] = 2.997$$

Figure 21: Labour of period t in the Capitalization scheme for male

Figure 22: Consumption of period t in the Capitalization scheme for male

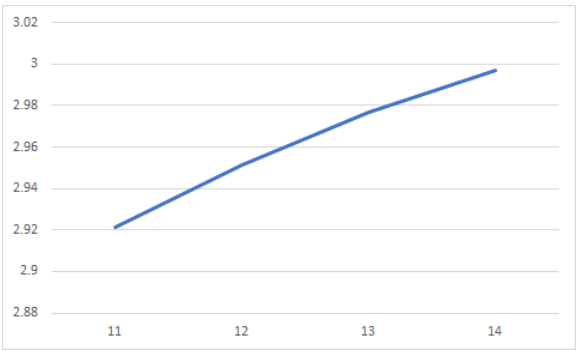




Source: Own elaboration.

Source: Own elaboration.

Figure 23: Utility of period t in the Capitalization scheme for male



Source: Own elaboration.

In this case with a capitalization scheme, unlike the PAYG scenario, delaying retirement as much as possible leads to an increase in an individual's utility. The optimal

situation is to wait until the last possible period for compulsory retirement. This results in gains in utility, as well as a reduction in workload and an increase in spending on consumption in all periods.

The implications for the economy of a higher retirement age, as the results of the model with the capitalization scheme point out, affect an increase in the workforce and an increase in household savings, as well as the capital of firms, which will be introduced in the following subtopics.

As a result, the labor market will have to adapt to the new labor supply structure. This is because, with the increase in the retirement age, even if this is by the individual's own choice, there will be a greater supply of labor.

EXAMPLE 4

Let us look at the difference in an individual's retirement choices in terms of their receipt. In this case, a 32 years old, i.e., in period $t = 1$, female individual has the option of retiring at age 63 or waiting until mandatory retirement at age 74. The information is the same as in the exemple 3.

For retirement in period 10:

$$l_{10}^{i,f} = \frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{10}\left(w_t B_t\right) - \left(w_t B_1\right)\sum_{t=0}^{9}\beta^t\right)}{\left(w_t B_1\right)\sum_{i=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{10}\beta^t} = 0.745$$

$$c_{10}^{i,f} = \frac{w_t B_1}{\gamma h^{15}} - \left(\frac{w_t B_1}{\gamma h^{15}}\right)\frac{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{10}\left(w_t B_i\right) - \left(w_t B_1\right)\sum_{i=0}^{9}\beta^t\right)}{\left(w_t B_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t B_1 \sum_{t=0}^{9}\beta^t} = 1,213.94$$

$$U_{10}^{i,f} = max_{c_{10}^{i,f}, l_{10}^{i,f}} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{10}^{i,f}\right) + \gamma^i log\left(1 - l_{10}^{i,f}\right)\right] = 2.886$$

Table 04: Results of Example 3

Periods t					
	10	11	12	13	14

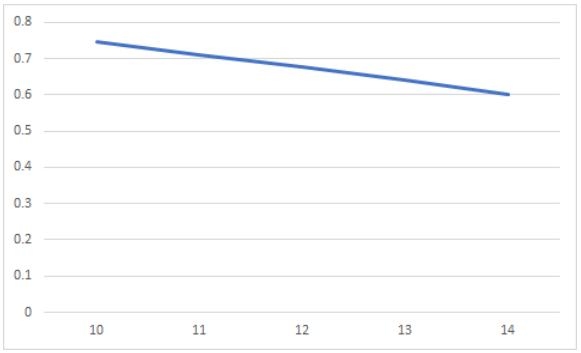
$c_t^{i,f}$	1,213.94	1,372.55	1,538.55	1,711.26	1,890.08
$l_t^{i,f}$	0.745	0.711	0.676	0.640	0.603
$U_t^{i,f}$	2.886	2.921	2.952	2.977	2.997

Source: Own elaboration.

For the results calculated in periods 11 to 14, the results for men and women do not change. This is because the capitalization model does not show gender differences, so only the result for $t = 10$ was included. Even with the addition of this new period for comparison, there was no change in the results. Therefore, it is still better to postpone retirement as much as possible until the last possible period and opt for compulsory retirement. The Figures 24 to 26 below show the results and make them easier to visualize.

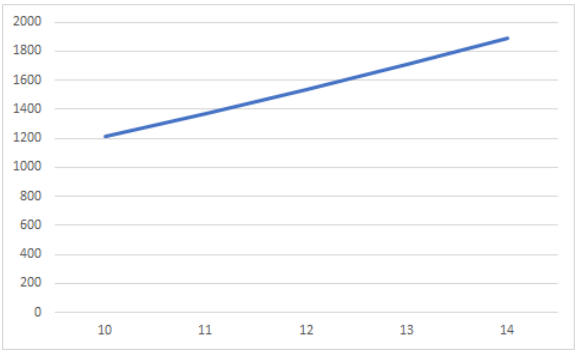
With regard to the results of the model with the capitalization scheme, as in the case of men, there will be an increase in the workforce and an increase in household savings, as well as an increase in the capital of firms.

Figure 24: Labour of period t in the Capitalization scheme for female



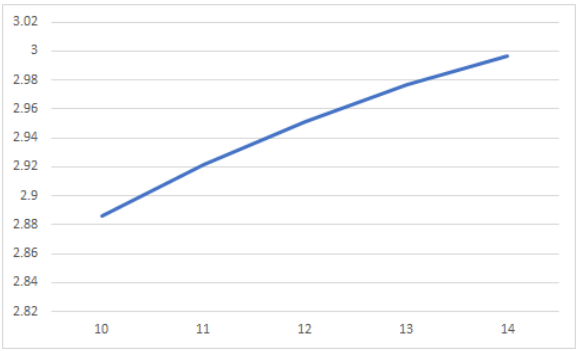
Source: Own elaboration.

Figure 25: Consumption of period t in the Capitalization scheme for female



Source: Own elaboration.

Figure 26: Utility of period t in theCapitalization scheme for female



Source: Own elaboration.

3.2.1.3 NOTIONAL SCHEME

The notional model combines PAYG funding with a pension calculation formula that depends on the individual amounts contributed, their returns (based on an index reflecting the financial health of the system) and the life expectancy of the worker at the time of retirement. A defined contribution notional accounts plan is apparently nothing more than an alternative way of calculating the amount of retirement pensions. The effect of accrued interest in an NDC system, where the internal rate is composed of population growth (n), the growth rate of the economy (g) and the growth of government tax revenues (d):

$$z_t^i = [(1 + n) * (1 + g) * (1 + d)] - 1$$

The indexers of these variables that make up profitability z_t^i can be defined in a practical way as the rate of change in the population calculated using the IBGE Census, the growth of the economy according to the rate of change in the annual Minas Gerais GDP and the change in tax revenue from this state. The largest portion of the funds collected comes through taxes, such as duties, fees and contributions, which are paid by individuals and companies.

This rate z_t^i is calculated in a different way to the capitalization scheme, because there is no established variation for the profitability of pensions, so there can be a real loss when the inflation factor is analyzed. In addition, if the population decreases or there is a downturn in

the economy or decline in government revenue ($n < 0$ or $g < 0$ or $d < 0$), the balance may have a negative return, then z_t^i can be < 0.

The balance will remain in the account, accumulating and earning interest defined according to the growth of the population and the growth of the economy, even though in some cases it may show a negative real return given inflation over the period. We can see that there are still savings accounts ($s_t^{i,\theta}$) where individuals can save part of their salary and make withdrawals when necessary, but there is no relationship with the Notional account balance.

Thus, like the other schemes described above, the individual's objective is to select consumption and leisure activities. This is done in sequences of exogenous variables that maximize their utility for demographic trends and the allocation of pensions throughout life, under certain constraints. Therefore, the utility maximization equation is:

$$U_t^{i,\theta} = max_{c_t^{i,\theta}, l_t^{i,\theta}} \sum_{t=1}^{16} \beta^{t-1}\left[log\left(c_t^{i,\theta}\right) + \gamma log\left(1 - l_t^{i,\theta}\right)\right]$$

s.t. $$c_t^{i,\theta} + s_t^{i,\theta} = w_t l_t^{i,\theta} + (1 + r)s_{t-1}^{i,\theta} + p_t^{i,\theta}\left(T^{i,\theta}\right) - x_t^{i,\theta},$$

$$c_t^i \geq 0 \; \forall \, t, i$$

$$l_t^{i,\theta} \geq 0 \, , \forall \, t = 1, \cdots, T^{i,\theta}, \; and \; l_t^{i,f} = 0 \; \forall \, t = T^{i,f} + 1, \cdots 4 \; or \; l_t^{i,f} = 0 \; \forall \, t = T^{i,m} + 1, \cdots 3,$$

$$T^{i,f} = 10, \cdots, 14, \; and \; T^{i,m} = 11, \cdots, 14.$$

Analyzing the results of each period for the restriction of individuals, we have:

$$c_1^{i,\theta} + s_1^{i,\theta} = p_1^{i,\theta}\left(T^{i,\theta}\right) - x_1^{i,\theta}$$

$$c_2^{i,\theta} + s_2^{i,\theta} = (1 + r)s_1^{i,\theta} + p_2^{1,\theta}\left(T^{i,\theta}\right) - x_2^{i,\theta}$$

$$\vdots$$

$$c_{16}^{i,\theta} = (1 + r)s_{15}^{i,\theta} + p_{16}^{i,\theta}\left(T^{i,\theta}\right) - x_{16}^{i,\theta}$$

Since savings is a term that appears depending on past periods and assuming $(1 + r) = h$, we can represent a new budget constraint for the individual as follows:

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} + \sum_{t=1}^{T^{i,\theta}} x_t^{i,\theta}h^{16-t} = \sum_{t=T^{i,\theta}+1}^{16} p_t^{i,\theta}\left(T^{i,\theta}\right)h^{16-t} + \sum_{t=1}^{T^{i,\theta}} w_t l_t^{i,\theta}h^{16-t}$$

The amount of the contribution to be paid by each individual will be similar to that defined in the capitalization scheme. In the similar case, women work at least during the periods $t = 1, 2, ..., 10$ and $t = 1, 2, ..., 11$ for men. While working, each individual must contribute to the scheme in the form of a payment to the IPSEMG and a contribution from their individual notional account. So the value of the total contribution will be given by:

$\theta = f$ (women) $\qquad\qquad$ $\theta = m$ (men)

$$x_t^{i,f} \begin{cases} \psi w_t l_t^{i,f}, i = 1,2, \dots, 10 \\ 0, i = 10,11, \dots, 16 \end{cases} \qquad x_t^{i,f} \begin{cases} \psi w_t l_t^{i,f}, i = 1,2, \dots, 11 \\ 0, i = 11,12, \dots, 16 \end{cases}$$

In this case, the contribution rate that represents the proportion of salary will be the same as that defined in the PAYG model according to IPSEMG, $\psi = 13\%$. Furthermore, the pension payment will be calculated using the identical methodology as described in the capitalization model above. This involves taking the total amount invested to date and discounting the interest rate. As a result, each individual will know the total amount accumulated until the last payment of their life, and the value will be the same for everyone, calculated through a simple division. Therefore:

$$p_{16}^{i,\theta}\left(T^{i,\theta}\right) = p_{15}^{i,\theta}\left(T^{i,\theta}\right) = \cdots = p_{T^{i,\theta}}^{i,\theta}\left(T^{i,\theta}\right) \tag{41}$$

and the total of the payment that each individual will receive is:

$$\left\{\left[\left(\sum_{t=1}^{T^{i,\theta}} x_t^{i,\theta}(1+z)^{T^{i,\theta}-t} - p_{T^{i,\theta}-1}^{i,\theta}\left(T^{i,\theta}\right)\right)(1+z)^{T^{i,\theta}-t-1} - p_{T^{i,\theta}-2}^{i,\theta}\left(T^{i,\theta}\right)\right](1+z)^{T^{i,\theta}-t-2} - p_{T^{i,\theta}-t}^{i,\theta}\left(T^{i,\theta}\right)\right\} \cdots$$

$$\sum_{t=1}^{T^{i,\theta}} x_t^{i,\theta}(1+z)^{T^{i,\theta}-t} = \sum_{k=1}^{16-T^{i,\theta}} \frac{p_{T^{i,\theta}-t}^{i,\theta}\left(T^{i,\theta}\right)}{(1+z)^k} = \left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(1+z)^k}\right) p_{T^{i,\theta}-t}^{i,\theta}\left(T^{i,\theta}\right) \tag{42}$$

Here, it is also implemented a minimum retirement age for women of 62, for men of 65 and a compulsory retirement age of 74. For this, the term represents the gender in each term of the equations described in this section.

For any $T^{i,m} = 10, \cdots, 14$ and $T^{i,m} = 11, \cdots, 14$, after substituting the pension capitalization regime, we have that:

$$\sum_{t=1}^{16}\left(c_t^{i,\theta}\right)h^{16-t} + \sum_{t=1}^{T^{i,\theta}} \psi\ w_t l_t^{i,\theta} h^{16-t} =$$

$$= \sum_{k=T^{i,\theta}+1}^{16} \left(\frac{\sum_{t=1}^{T^{i,\theta}} \psi \; w_t l_t^{i,\theta} (1+z)^{T^{i,\theta}-t}}{\left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(1+z)^k} \right)} \right) h^{16-k} + \sum_{t=1}^{T^{i,\theta}} w_t l_t^{i,\theta} h^{16-t}$$

$$\sum_{t=1}^{16} \left(c_t^{i,\theta} \right) h^{16-t} = \sum_{t=1}^{T^{i,\theta}} \left(w_t l_t^{i,\theta} \right) \left(\frac{\left(\sum_{k=T^{i,\theta}+1}^{16} h^{16-k} \right) (1+z)^{T^{i,\theta}-t}}{\left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(1+z)^k} \right)} \psi + (1 - \psi) h^{16-t} \right) \quad (43)$$

Then, we can write this constraint as

$$\sum_{t=1}^{16} \left(c_t^{i,\theta} \right) h^{16-t} = \sum_{t=1}^{T^{i,\theta}} \left(w_t l_t^{i,\theta} \right) P_t \quad (44)$$

where

$$P_t = \left(\frac{\left(\sum_{k=T^{i,\theta}+1}^{16} h^{16-k} \right) (1+z)^{T^{i,\theta}-t}}{\left(\sum_{k=1}^{16-T^{i,\theta}} \frac{1}{(1+z)^k} \right)} \psi + (1 - \psi) h^{16-t} \right) \text{ with } t = 1, \cdots, T^{i,\theta} \quad (45)$$

for any $T^{i,f} = 10, \cdots, 14$ and $T^{i,m} = 11, \cdots, 14$. Note that the solution of the problem of the agent is the same as in the capitalization model switching from B_t's to P_t's. The largest difference between the two models is how the interest rate of the regime is determined as it was discussed before.

Although the result is the same as in the previous capitalization model, there is a difference between the regimes. This is because the value of the contribution is used for the current payment of the assets, but this fact does not change the maximization for the consumer side. Another factor that differs from the capitalization model is the profitability of virtual accounts. And again, there is no need for lump-sum tax, as there is no difference between the value of contributions and retirement payments.

EXAMPLE 5

Using the same information as in example 1, where a male individual has the option of retiring at 65 or waiting for compulsory retirement at 74, i.e. in period 11 or 14, let's see what happens to his utility from the FOC calculated earlier. In this case, the contribution rate will remain at 13% of salary.

As the profitability of the notional model is endogenous, the simulated data will be used for the analysis in the next chapter of this dissertation. Thus, the values for population growth (n), the growth rate of the economy (g) and the growth of government tax revenues (d) were calculated from past data obtained from different databases which will be described later. Thus, the average nominal annual rate of return was 8.33%. Assuming the same inflation rate as in the previous examples, 4.68% will be assumed, so the total real return was 3.65% or 10.95% in each period.

In the case of retirement in period 11, at the age of 65:

$$l_{11}^{i,m} = \frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{11}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{10}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{10}\beta^t} = 0.698$$

$$c_{11}^{i,m} = \frac{w_t P_1}{\gamma h^{15}} - \left(\frac{w_t P_1}{\gamma h^{15}}\right)\frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{11}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{10}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{10}\beta^t} = 1,439.12$$

$$U_{11}^{i,m} = max_{c_{11}^{i,m}, l_{11}^{i,m}} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{11}^{i,m}\right) + \gamma^i log\left(1 - l_{11}^{i,m}\right)\right] = 2.935$$

For retirement in period 12:

$$l_{12}^{i,m} = \frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{12}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{11}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{11}\beta^t} = 0.662$$

$$c_{12}^{i,m} = \frac{w_t P_1}{\gamma h^{15}} - \left(\frac{w_t P_1}{\gamma h^{15}}\right)\frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{12}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{11}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{11}\beta^t} = 1,613.06$$

$$V_{12}^{i,m} = max_{c_{12}^{i,m}, l_{12}^{i,m}} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{13}^{i,m}\right) + \gamma^i log\left(1 - l_{12}^{i,m}\right)\right] = 2.964$$

For retirement in period 13:

$$l_{13}^{i,m} = \frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{13}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{12}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{12}\beta^t} = 0.624$$

$$c_{13}^{i,m} = \frac{w_t P_1}{\gamma h^{15}} - \left(\frac{w_t P_1}{\gamma h^{15}}\right)\frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{13}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{12}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{12}\beta^t} = 1,793.64$$

$$U_{13}^{i,m} = max_{c_{13}^{i,m} l_{13}^{i,m}} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{13}^{i,m}\right) + \gamma^i log\left(1 - l_{13}^{i,m}\right)\right] = 2.987$$

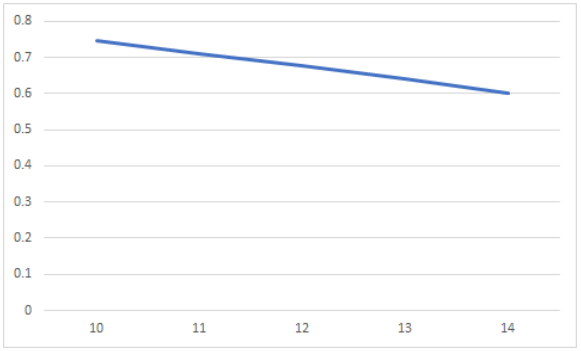
For compulsory retirement in period 14:

$$l_{14}^{i,m} = \frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{14}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{13}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{13}\beta^t} = 0.586$$

$$c_{14}^{i,m} = \frac{w_t P_1}{\gamma h^{15}} - \left(\frac{w_t P_1}{\gamma h^{15}}\right)\frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{14}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{13}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{13}\beta^t} = 1,980.20$$

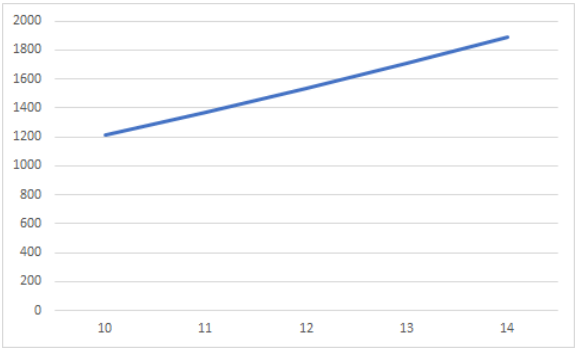
$$U_{14}^{i,m} = max_{c_{14}^{i,m} l_{14}^{i,m}} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{14}^{i,m}\right) + \gamma^i log\left(1 - l_{14}^{i,m}\right)\right] = 3.006$$

Figure 27: Labour of period t in the Notional scheme for male



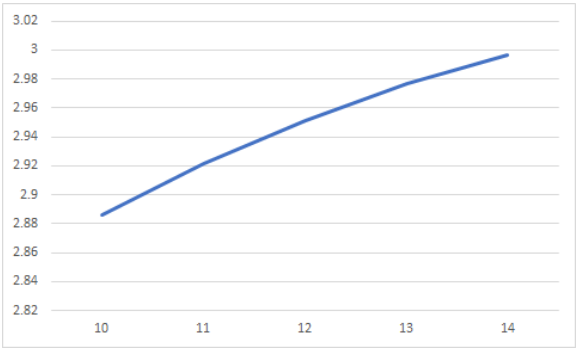
Source: Own elaboration.

Figure 28: Consumption of period t in the Notional scheme for male



Source: Own elaboration.

Figure 29: Utility of period t in the Notional scheme for male



Source: Own elaboration.

As with the capitalization scheme, the optimal situation is to wait until the last possible period for compulsory retirement. This results in utility gains as well as a reduction in workload and an increase in consumption expenditure in all periods. This is because the model follows the same calculation pattern as the capitalization system, except for the form of profitability of each individual's virtual accounts, which is calculated endogenously. In this case, the values used followed the simulation modeling described in the following chapter.

Based on the outcomes derived from the model incorporating capitalization adjustments, the economy is impacted by the decision to raise the retirement age. This change results in effects on the workforce, household savings, and corporate capital, which will be explored in subsequent sections. However, the result is different from that analyzed in the previous case with the pure capitalization system. As the labor force increases and workers endogenously work longer, then they save for longer and only spend their pensions at an older age, increasing the savings balance and, consequently, the economy's capital.

The alterations in the retirement age necessitate an adaptation in the structure of the labor market. The augmented retirement age contributes to an expanded labor supply, even if it is driven by individual preferences.

EXAMPLE 6

Using the same information as in example 5, but with a female individual, let us calculate the differences in the utility of the periods 10 to 14.

For retirement in period 10:

$$l_{10}^{i,f} = \frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{10}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{9}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{9}\beta^t} = 0.734$$

$$c_{10}^{i,f} = \frac{w_t P_1}{\gamma h^{15}} - \left(\frac{w_t P_1}{\gamma h^{15}}\right)\frac{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} - \gamma h^{15}\left(\sum_{t=1}^{10}\left(w_t P_t\right) - \left(w_t P_1\right)\sum_{t=0}^{9}\beta^t\right)}{\left(w_t P_1\right)\sum_{t=1}^{16}(\beta h)^{t-1}h^{16-t} + \gamma h^{15} w_t P_1 \sum_{t=0}^{9}\beta^t} = 1,272.53$$

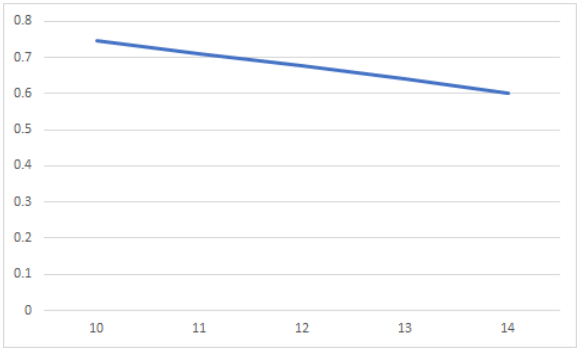
$$U_{10}^{i,f} = max_{c_{10}^{i,f}, l_{10}^{i,f},} \sum_{t=1}^{16}\beta^{t-1}\left[log\left(c_{10}^{i,f}\right) + \gamma^i log\left(1 - c_{10}^{i,f}\right)\right] = 2.901$$

Table 05: Results of Example 5

Periods i					
	10	11	12	13	14
$c_t^{i,f}$	1272.53	1439.116	1613.062	1793.642	1980.2
$l_t^{i,f}$	0.734	0.699	0.663	0.625	0.586
$U_t^{i,f}$	2.901	2.935	2.964	2.988	3.006

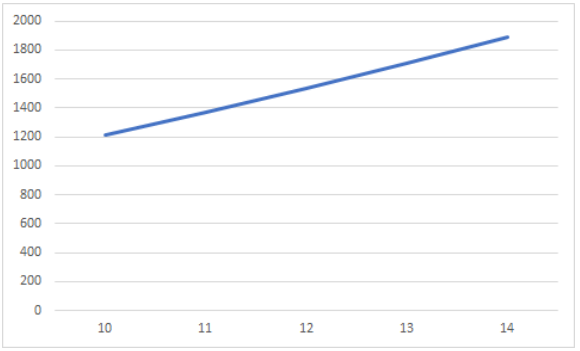
Source: Own elaboration.

Figure 30: Labour of period t in the Notional scheme for female



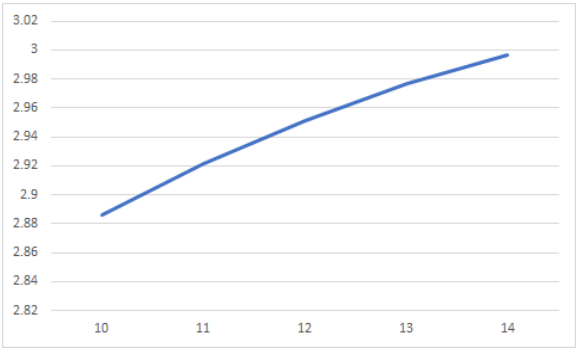
Source: Own elaboration.

Figure 31: Consumption of period t in the Notional scheme for female



Source: Own elaboration.

Figure 32: Utility of period t in the Notional scheme for female



Source: Own elaboration.

The results for females calculated in periods 11 to 14 are similar to those for men. This is because the capitalization model does not show gender differences. Therefore, only the result for $t = 10$ was included. Even with the inclusion of this new period for comparison, there was no change in the results. It is still better to postpone retirement as much as possible until the last possible period and opt for compulsory retirement.

Regarding the results of the Notional model, there will be an increase in the active population available for the labor market, because, with the increase in the workforce and, as workers endogenously work longer, they save for longer and only spend their pensions at an older age.

3.2.2 PRODUCTION SECTOR

After characterizing the consumers, the subsequent step involves describing the firms. Technical abbreviations will be defined when first used. It is assumed that a substantial number of identical firms possess a production technology, which is determined by the earlier production function. The function consists of Y, the product, K, the capital stock - depreciation at a rate δ, L, the quantity of labor and ϕ, a constant that impacts the production level.

The production sector produces a single good that is fully consumed by individuals throughout the economy. Firms operate in a competitive market and maximize profits with respect to capital and labor. In each period, firms use capital until the marginal product equals

the rental rate and employ labor until the marginal product of labor equals the wage rate. Constant returns to scale are assumed. The Cobb-Douglas[10] production function is given by:

$$Y_t = \phi(K_t)^{1-\alpha}(L_t)^{\alpha} \tag{46}$$

K_t represents the total aggregate capital present in the economy's firms. As firms have constant returns to scale, it is assumed that profit is zero for the moment of maximization and definition of the problem. The factor remunerations are characterized by the FCO of the firm's profit maximization problem, therefore the rate of return on capital, r, which is the same rate of return obtained on savings and described previously, and the remuneration of labor, w, are determined by their respective marginal productivities as expressed below.

$$r_t^i = (1 - \alpha)\phi\left(\frac{K_t}{L_t}\right)^{-\alpha} - \delta \tag{47}$$

$$w_t = \alpha\phi\left(\frac{K_t}{L_t}\right)^{(1-\alpha)} \tag{48}$$

The resource constraint of the economy holds at all times, thus the next period ts given by:

$$K_{t+1} = Y_t - C_t + (1 - \delta)K_t \tag{49}$$

δ denotes the depreciation rate of capital K and $C_t = \sum_{\theta=m,f}\sum_{t=1}^{T^{i,\theta}} c_t^{i,\theta}$.

The total amount of work that will be made available in the economy corresponds to the total number of people who make up the age group and the total hours that are maximized in the initial utility function that defined each of these individuals homogeneously.

Thus,

$$L_t = \sum_{\theta=m,f}\sum_{t=1}^{T^{i,\theta}} l_t^{i,\theta} \tag{50}$$

[10]Ellery et al (2002) suggest that the use of a Cobb-Douglas function for Brazil can be justified due to the stability of the shares of capital and labor in income, the main property of this function.

Remuneration will remain the same with wages set by w_t. Then, firms maximize profits with:

$$max_{K,L}\{F(K,L) - wL - rK\}$$

This equation represents revenue minus the cost of production.

3.2.3 GOVERNMENT

This section will analyze the dynamics of government choice given the three models described and analyzed for the individual case in the previous section.

3.2.3.1 PAYG SCHEME

Analyzing the government's restriction in the context of individual utility for participants of the social security system, it is assumed that issuing bonds will be used to address the indebtedness resulting from frequent deficits under the PAYG regime, similar to the Brazilian Treasury's approach. Objective evaluation will guide this decision. In this scenario, individuals opt for either Treasury bonds or conventional savings as their form of savings $s_t^{i,\theta}$, and there is no variation in profitability, meaning that both have the same interest rate r. As a result, the government can incur debt up to a limit determined by the amount saved by individuals.

Defining government debt as D_t, we have that:

$$max\, D_{t+k} \leq \sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} s_{t+k}^{i,\theta}$$

Debt in period t is a way of balancing the government's accounts in order to pay the pensions of individuals who are old enough to receive them. Thus, the government goes into debt by issuing bonds in t in order to pay the total amount of pensions in $(t+1)$.

$$D_t = \sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} \left\{ \left[\left(x_{t+k}^{i,\theta} - p_{t+k}^{i,\theta} + \tau_{t+k} \right)(1+r)^{T^{i,\theta}-k} + (x_{t+k}^{i,\theta} - p_{t+k}^{i,\theta} + \tau_{t+k}) \right](1+r)^{T^{i,\theta}-k-1} \right\} \cdots$$

$$\tau_{t+k} = \left(p_{t+k}^{i,\theta} - x_{t+k}^{i,\theta} \right) \quad (51)$$

Then:

$$D_t = 0$$

In our model, given any of the pension schemes PAYG, capitalization and notional, the government cannot decide on the contribution and benefit of the analyzed pension system. The government only allocates the budget constraint such that the government deficit must be compensated by issuing new debt.

In addition, constant government spending will be assumed with regard to public accounts other than those made available for social security. Also, the government taxes each young individual lump-sum amount τ_t. In the equilibrium, which will be carried out later, a lump-sum tax will be adopted so that it will be equal to the aggregate pension deficit. In this way, the government has zero debt in the future. So, the planner chooses these lump-sum taxes in such a way that revenue is equal to the public pension deficit, varying according to the government's need to maintain the balance at zero.

While agents are restricted from short selling on the asset market, the government can, since it creates its own assets to cover recurring social security deficits through PAYG. As a result, there could be a Ponzi game, in which the government can pay off current debt by issuing future debt infinitely. In addition, in order to guarantee that there will be enough savings to cover the government's debt, a high interest rate will be necessary for agents to increase the level of savings. In this way, it would imply that the debt is always bought by workers in an optimal way given the market interest rate (so that it would always be increasing).

Thus, any regime that is not financially sustainable generates a situation in which there is no equilibrium due to the indiscriminate increase in public debt and the fall in investments in other productive sectors to finance the growing public debt. To solve this type of problem, a lump-sum tax will be adopted, which will tax individuals at the same rate as the pension deficit for the period. So, given the assumptions above and according to Hosseini and Shourideh (2016), the equation that defines government revenue is:

$$G_t = \sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} x_{t+k}^{i,\theta} + \sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} \tau_{t+k}^{i,\theta} \tag{52}$$

Finally, goods and asset market clearing implies:

$$\sum_{\theta=m,f}\sum_{k=1}^{T^{i,\theta}} c_{t+k}^{i,\theta} + K_{t+k} = F(K,L) + (1-\delta)K_{t+k-1} \tag{53}$$

$$\sum_{\theta=m,f}\sum_{k=1}^{T^{i,\theta}} s_{t+k}^{i,\theta} = (K_{t+k} + D_{t+k}) \tag{54}$$

When the market clearing is calculated, it can be seen that there is a distribution of the profit of the firms present in the economy among individuals, but, as mentioned above, since the returns to scale are constant, the profit generated by the firms is zero, which in turn has no effect on the increase in income of these individuals.

3.2.3.2 CAPITALIZATION SCHEME

After analyzing the PAYG scheme in the previous subsection, it is evident that there is a relationship between the government's debts and its spending to keep the regime afloat.
Regarding the capitalization scheme, there is no public deficit to be covered with taxes. Therefore, the government will not impose lump-sum taxes since their only purpose was to counterbalance the deficit.

The government's decision is solely about the choice of regime and is not related to the reallocation of taxes or debts in the future. The government has no revenue in this case, because all the money collected from the system via the payment of retirement contributions will be returned with the interest on the investment made over time until the individual decides when to retire.
Therefore, based on the firms and the equilibrium conditions of the economy under these circumstances, we have:

$$\sum_{\theta=m,f}\sum_{k=1}^{T^{i,\theta}} c_{t+k}^{i,\theta} + K_{t+k} = F(K,L) + (1-\delta)K_{t+k-1} \tag{55}$$

$$\sum_{\theta=m,f}\sum_{k=1}^{T^{i,\theta}} s_{t+k}^{i,\theta} = K_{t+k} \tag{56}$$

Similarly, when calculating market clearing, the profit of companies in the economy is distributed among individuals. However, it is important to note that due to constant returns to scale, the companies generate zero profit, which does not increase the income of individuals.

3.2.3.3 NOTIONAL SCHEME

For the notional case, although it is very similar in terms of the distribution of values received through government contributions and the payment to current retirees, the government's revenue remains to fulfill these obligations. However the government's decision is solely about the choice of regime and is not related to the reallocation of taxes or debts in the future. The equation that defines government revenue is:

$$G_t = \sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} x_{t+k}^{i,\theta} \tag{57}$$

Therefore, based on the firms and the equilibrium conditions of the economy under these circumstances, we have the same function that on the capitalization scheme:

$$\sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} c_{t+k}^{i,\theta} + K_{t+k} = F(K, L) + (1 - \delta)K_{t+k-1} \tag{58}$$

$$\sum_{\theta=m,f} \sum_{k=1}^{T^{i,\theta}} s_{t+k}^{i,\theta} = K_{t+k} \tag{59}$$

In the same way, when calculating market clearing, the profit of companies in the economy is distributed among individuals and it does not increase the income of individuals.

3.3 CONCLUSION

Upon examining the models presented for individuals and for the government, clear patterns emerge, revealing divergent outcomes. In the PAYG system, individuals who choose to remain in the workforce for an extended period experience a linear progression of benefits in exchange for increased future returns. In contrast, both the capitalization and Notional schemes exhibit exponential gains. The data shows that individuals tend to work until the mandated retirement age, as specified in the model at $t = 14$.

Upon further examination, there is an increase in labor supply, household savings, and firm capital within the capitalization and Notional frameworks. This is in contrast to the PAYG scheme, where these parameters decrease. Extending work tenure can enhance productivity and contribute to skill retention and knowledge transfer within the workforce.

Additionally, increased household savings within the capitalization paradigm can fortify personal financial security and inject liquidity into the broader economy, fostering investment opportunities and overall economic growth.

In the notional case, the accounts are virtual. Therefore, there is no significant increase in savings compared to the PAYG case. However, savings should be lower in the notional case than in the pure capitalization case. This is because the savings caused by retirement are totally fictitious and do not increase the amount of capital in the economy. Additionally, the increase in the workforce means that workers endogenously work longer, save for longer, and only spend their retirement at an older age. As a result, the level of savings will be higher.

Extended workforce participation is positively correlated with increased capital for firms. As individuals contribute more to their retirement funds, firms have more available capital for investment. This infusion of capital enables firms to undertake more substantial projects, engage in research and development initiatives, and contribute significantly to economic advancement.

In contrast, the PAYG system relies on a more linear approach. Although individuals who choose to work longer receive higher future benefits, this mechanism does not have the exponential growth characteristic of capitalization. As a result, this linear relationship provides fewer incentives for individuals to extend their working years. Furthermore, the PAYG scheme can strain government finances, necessitating higher compensation to encourage extended workforce participation. This can lead to an increased tax burden that has the potential to impede economic growth and discourage active labor market participation.

In summary, the decision between capitalization and PAYG scheme not only affects individual retirement benefits but also has a significant impact on broader economic trends, including labor supply, household savings, and firms' capital. Capitalization and Notional models have a compounding effect that tends to create positive externalities for both individuals and the broader economy, promoting a more sustainable and robust economic environment. To clarify, this text emphasizes the motivation for individuals to continue working through the use of capitalization and notional schemes, which compound accrued

benefits. In contrast, the PAYG model requires higher compensation to encourage extended workforce participation, resulting in a higher tax burden to offset the ensuing deficit over time. In essence, the models of capitalization and Notional schemes not only incentivize longer work tenure but also have a positive impact on broader economic metrics. Meanwhile, the PAYG system presents fiscal challenges.

Despite the increase in labor supply in the economy, nothing can be said about unemployment in the economy, since the proposed model assumes full employment in all cases. Therefore, the suggestion for future research is to include this variable in order to better understand the entire movement of the economy based on the choice of individuals between the retirement schemes set out in this dissertation. In addition, according to this parameter to be included in the future, there may be a change in the choice made by the government, extending the complexity of the problem.

The study focused on examining workers' decisions regarding various pension schemes and did not analyze the Nash equilibrium of the 9x9 game, which encompasses interactions among workers, the central planner, and the production system. The results of the study are presented objectively without subjective evaluations. The study focused on examining workers' decisions regarding various pension schemes and did not analyze the Nash equilibrium of the 9x9 game, which encompasses interactions among workers, the central planner, and the production system. It is crucial to explore this aspect in future research to gain a comprehensive understanding of the overall economic behavior influenced by the choices made by these agents.

4 FINAL CONSIDERATIONS

An analysis of the simple PAYG system of the Minas Gerais Social Security System (RPPS-MG) revealed concerning deficits for the period from 2023 to 2039. These deficits are further exacerbated by the lack of real interest on the outstanding balance and the introduction of a pension system with zero initial value.

In order to address these challenges, the National Account model was implemented. This model is based on the premise that employee contributions create a virtual account that serves as the basis for retirement benefits. While the accumulated balance is utilized to fulfill current obligations, the novel fund provides individualized calculation bases that facilitate financial sustainability.

Moreover, a solvency insurance mechanism has been introduced, analogous to credit default swaps. This insurance mitigates the impact of annual deficits and other taxation methods on the salaries of active civil servants and the benefits received by pensioners.

The transition to the Notional scheme, accompanied by the introduction of solvency insurance, represents a strategic and effective approach to addressing the financial challenges of the RPPS-MG PAYG scheme. The gradual integration of new entrants and the incorporation of financial safeguards are designed to ensure the long-term sustainability of the public pension system.

In light of concerns about the high deduction rate, a proportional approach based on years of service is proposed. When viewed in a broader context, these measures emerge as viable solutions to the pension system's sustainability challenges, promoting a more robust and sustainable economic environment. The conclusions suggest that these strategies could be applicable to other subnational entities in Brazil, indicating a potential path towards a broader and more effective pension reform.

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APPENDIX

Table A01: Employees

Dez/22

Description	Female	Male	TOTAL	Dec/21	Dec/20	Dec/19	Dec/18
Number of participants	101039	63911	164950	165095	171900	184284	198971
Minor Age (years)	19	19	19	21	19	21	21
Average age (years)	46.1	45.9	45.8	45.4	44.9	45.6	44.1
Older Age (years)	75	75	75	74.9	74	74	73
Lowest Contribution Salary	R$ 1212.00	R$ 1212.00	R$ 1212.00	R$ 1100.03	R$ 1045.05	R$ 998.00	R$ 954.00
Average Contribution Salary. without Ceiling	R$ 5343.88	R$ 7659.42	R$ 6241.05	R$ 5804.60	R$ 5520.17	R$ 5049.50	R$ 4608.22
Highest Contribution Salary. without Ceiling	R$ 66994.74	R$ 70924.44	R$ 70924.44	R$ 58246.00	R$ 53975.67	R$ 55861.15	R$ 52286.93

Monthly Payroll. without Ceiling	R$ 539939986.28	R$ 489521398.25	R$ 1029461384.53	R$ 958310616.71	R$ 948916433.45	R$ 930541390.47	R$ 916901760.73
Average length of RPPS membership	11.9	11.8	11.9	12.1	12	12.1	9.3
Average time until retirement	16.0	16.3	16.1	16.3	15.6	14.4	15.7
Estimated average retirement age	62.1	61.7	61.9	61.7	60.5	60	59.8
Average Age at Entry into the Entity	33.1	31.7	32.6	32.4	32.9	33.5	34.8
Average Contribution	R$ 704.18	R$ 1053.47	R$ 839.52				
Contribution Sheet	R$ 71149507.21	R$ 67328568.98	R$ 138478076.19				
Source: Actuarial Valuation database							

Table A02: Retired

Dez/22

Description	Female	Male	TOTAL	Dec/21	Dec/20	Dec/19	Dec/18
Number of pensioners	204723	35433	240156	240799	245656	251004	244825
Average age (years)	70.8	72.2	71	70.5	69.8	69.1	68.7
Average Benefit	R$ 4218.79	R$ 10032.06	R$ 5076.49	R$ 4598.46	R$ 4468.73	R$ 4324.94	R$ 4212.06
Monthly Benefits Sheet	R$ 863682954.37	R$ 355466094.58	R$ 1219149048.95	R$ 1107198302.77	R$ 1097770653.40	R$ 1085581389.80	R$ 1031217094.72
Monthly Contribution Sheet	R$ 41678091.74	R$ 38367322.63	R$ 80045414.37				
Average Time Already on Benefit	17.4	15.6	17.2	16.2	17.1	15.3	14.5
Average retirement age	53.4	56.4	53.8	54.2	54.2	54.2	54.2
% of Disability Sheet over Total			1.98%**	2.91%**	4.16%	3.74%	3.69%

Note: ** Invalidity flags adjusted for powers/bodies IN SPREV 01
Source: Actuarial Valuation database

Table A03: Pensioners

Dez/22

Description	Female	Male	TOTAL	Dec/21	Dec/20	Dec/19	Dec/18
Number of pensioners	26173	12767	38940	38091	38171	38476	38391
Average age (years)	71.9	68.1	70.7	70.2	69.9	69.2	71.5
Lowest Benefit Quota Informed in the data (excluding zeroes)	R$984.58	R$1001.12	R$984.58	R$843.74	R$800.13		
Average Benefit Quota Informed in the data (excluding zeroes)	R$6204.73	R$3748.92	R$5399.56	R$4807.68	R$4496.63		
Highest Benefit Quota reported in the data (1) (excluding zeroes)	R$36897.86	R$35462.22	R$36897.86	R$37536.60	R$36698.43		
Monthly Benefit Quota Sheet (including zeroed ones)	R$ 162396522.41	R$ 47862473.52	R$ 210258995.93	R$ 183129451.96	R$ 171640800.88		
Standardized Monthly Benefit Quota Sheet	R$ 166441707.31	R$ 43823558.46	R$ 210265265.77	R$ 183134034.58	R$ 171642443.33	R$ 158024374.63	R$ 157903768.78
Average time on benefit	18.4	10.2	15.7	16.5	16.1	16.4	15.7
Average age of entitlement	53.5	57.8	54.9	53.8	54.1	55.1	52.8
Note:							

(1) includes payment of court decisions, payment of arrears resulting from readjustments and stock relating to the period between application and approval and stock relating to suspension and rehabilitation, Source: Data from the Actuarial Valuation database	

Table A04: Simulation results

Year		**2023**	**2024**	**2025**	**2026**	**2027**	**2028**
Recipe for the new employees		R$34,921,659.64	R$38,215,365.59	R$20,227,908.37	R$14,052,270.10	R$25,569,740.31	R$26,456,801.96
Notional Fund		R$34,921,659.64	R$76,026,303.67	R$102,544,324.30	R$125,080,701.61	R$160,999,118.85	R$200,776,343.90
Insurance	**New employees**	R$22,699.08	R$49,417.10	R$66,653.81	R$81,302.46	R$104,649.43	R$130,504.62
	Actual public employees	R$5,792,827.37	R$5,189,374.07	R$5,030,611.42	R$5,097,355.35	R$5,184,290.97	R$5,230,750.66
	Total	R$5,815,526.45	R$5,238,791.17	R$5,097,265.23	R$5,178,657.80	R$5,288,940.40	R$5,361,255.28
Total revenue		R$8,912,042,113.49	R$7,983,652,415.97	R$7,739,402,180.07	R$7,842,085,146.88	R$7,975,832,260.71	R$8,047,308,704.56
Total Expenditure		R$ 18,851,930,137.02	R$ 21,129,363,704.12	R$ 21,922,425,360.97	R$ 21,958,708,566.95	R$ 21,901,943,942.49	R$ 21,944,170,431.75
Net Equity		-R$ 6,996,255,870.58	-R$ 10,014,939,564.88	-R$ 10,980,434,514.24	-R$ 10,901,167,072.50	-R$ 10,706,000,757.77	-R$ 10,665,802,490.42
Liability	**Actual**	R$9,939.89	R$13,145.71	R$14,183.02	R$14,116.62	R$13,926.11	R$13,896.86

	With insurance	R$6,996.26	R$10,014.94	R$10,980.43	R$10,901.17	R$10,706.00	R$10,665.80
Proportion of Liabilities in relation to changes in regimes		70.39%	76.18%	77.42%	77.22%	76.88%	76.75%
Year		**2029**	**2030**	**2031**	**2032**	**2033**	**2034**
recipe for the new employees		R$40,448,020.70	R$35,542,107.32	R$47,604,428.61	R$44,550,535.60	R$59,052,210.46	R$58,905,481.87
Notional Fund		R$257,835,796.19	R$314,710,205.94	R$388,352,498.16	R$465,033,766.05	R$562,561,010.18	R$668,010,539.78
Insurance	**New employees**	R$167,593.27	R$204,561.63	R$252,429.12	R$302,271.95	R$365,664.66	R$434,206.85
	Actual public employees	R$5,289,578.16	R$5,297,180.09	R$5,319,054.80	R$5,281,232.03	R$5,261,306.04	R$5,160,866.07
	Total	R$5,457,171.43	R$5,501,741.73	R$5,571,483.92	R$5,583,503.98	R$5,626,970.70	R$5,595,072.92
Total revenue		R$8,137,812,557.62	R$8,149,507,837.28	R$8,183,161,228.34	R$8,124,972,350.99	R$8,094,316,983.36	R$7,939,793,958.89
Total Expenditure		R$ 21,912,455,844.08	R$ 22,009,111,916.54	R$ 22,025,268,954.74	R$ 22,190,080,055.11	R$ 22,258,396,585.62	R$ 22,532,294,942.20
Net Equity		-R$ 10,539,633,482.83	-R$ 10,612,322,114.12	-R$ 10,590,068,850.40	-R$ 10,798,722,202.45	-R$ 10,892,253,686.06	-R$ 11,301,705,714.04
Liability	**Actual**	R$13,774.64	R$13,859.60	R$13,842.11	R$14,065.11	R$14,164.08	R$14,592.50
	With insurance	R$10,539.63	R$10,612.32	R$10,590.07	R$10,798.72	R$10,892.25	R$11,301.71
Proportion of Liabilities in		76.51%	76.57%	76.51%	76.78%	76.90%	77.45%

relation to changes in regimes							
Year		**2035**	**2036**	**2037**	**2038**	**2039**	
recipe for the new employees		R$62,790,128.92	R$60,517,504.10	R$62,273,657.29	R$49,271,429.78	R$36,269,202.28	
Notional Fund		R$786,069,188.72	R$911,622,913.22	R$1,049,320,603.85	R$1,185,408,623.12	R$1,319,753,793.24	
Insurance	**New employees**	R$510,944.97	R$592,554.89	R$682,058.39	R$770,515.61	R$857,839.97	
	Actual public employees	R$5,077,314.75	R$4,944,274.43	R$4,826,279.67	R$4,656,521.79	R$4,551,425.68	
	Total	R$5,588,259.72	R$5,536,829.32	R$5,508,338.06	R$5,427,037.40	R$5,409,265.64	
Total revenue		R$7,811,253,460.15	R$7,606,576,041.00	R$7,425,045,644.93	R$7,163,879,681.87	R$7,002,193,351.09	
Total Expenditure		R$22,703,918,694.06	R$22,979,757,594.12	R$23,166,644,391.61	R$23,463,777,113.66	R$23,516,221,618.74	
Net Equity		-R$ 11,592,615,407.00	-R$ 12,058,138,168.71	-R$ 12,419,964,226.81	-R$ 12,965,530,624.64	-R$ 13,187,056,199.13	
Liability	**Actual**	R$14,892.67	R$15,373.18	R$15,741.60	R$16,299.90	R$16,514.03	
	With insurance	R$11,592.62	R$12,058.14	R$12,419.96	R$12,965.53	R$13,187.06	
Proportion of Liabilities in relation to changes in regimes		77.84%	78.44%	78.90%	79.54%	79.85%	