UNIVERSIDADE FEDERAL DE MINAS GERAIS
Escola de Engenharia
Programa de Pós-Graduação em Engenharia Elétrica

Paulo Sérgio Pereira Pessim

APPROACHES TO DISTRIBUTED CONTROL DESIGN: addressing multiple challenges in interconnected and multi-agent systems

Belo Horizonte
2025

Paulo Sérgio Pereira Pessim

APPROACHES TO DISTRIBUTED CONTROL DESIGN: addressing multiple challenges in interconnected and multi-agent systems

A thesis presented to the Graduate Program in Electrical Engineering (PPGEE) of the Federal Univesity of Minas Gerais (UFMG) in partial fulfillment of the requirements to obtain the degree of Doctor in Electrical Engineering.

Supervisor: Reinaldo Martínez Palhares

Co-supervisor: Márcio Júnior Lacerda

Co-supervisor: Pedro Henrique Silva Coutinho

Belo Horizonte
2025

Pessim, Paulo Sérgio Pereira.
P475a Approaches to distributed control design [recurso eletrônico] : addressing multiple challenges in interconnected and multi-agent systems / Paulo Sérgio Pereira Pessim. – 2025.
1 recurso online (167 f. : il., color.) : pdf.

Orientador: Reinaldo Martinez Palhares.
Coorientadores: Márcio Júnior Lacerda, Pedro Henrique Silva Coutinho.

Tese (doutorado) – Universidade Federal de Minas Gerais, Escola de Engenharia.

Inclui bibliografia.

1. Engenharia elétrica – Teses. 2. Sistemas multiagentes – Teses. 3. Programação linear – Teses. 4. Fuzzy systems – Teses. I. Palhares, Reinaldo Martinez. II. Lacerda, Márcio Júnior. III. Coutinho, Pedro Henrique Silva. IV. Universidade Federal de Minas Gerais. Escola de Engenharia. V. Título.

CDU: 621.3(043)

Ficha catalográfica elaborada pelo bibliotecário Marcio A. A. Gomes CRB/6 2812
Biblioteca Prof. Mário Werneck, Escola de Engenharia da UFMG

UNIVERSIDADE FEDERAL DE MINAS GERAIS

Escola de Engenharia

COLEGIADO DO CURSO DE GRADUAÇÃO / PÓS-GRADUAÇÃO EM Engenharia Elétrica

FOLHA DE APROVAÇÃO

"Approaches to Distributed Control Design: Addressing Multiple Challenges in Interconnected and Multi-agent Systems"

Paulo Sérgio Pereira Pessim

Tese de Doutorado submetida à Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação em Engenharia Elétrica da Escola de Engenharia da Universidade Federal de Minas Gerais, como requisito para obtenção do grau de Doutor em Engenharia Elétrica.

Aprovada em 09 de julho de 2025.

Por:

Prof. Dr. Reinaldo Martínez Palhares
Universidade Federal de Minas Gerais - Orientador

Prof. Dr. Márcio Júnior Lacerda
London Metropolitan University - Coorientador

Prof. Dr. Pedro Henrique Silva Coutinho
Universidade do Estado do Rio de Janeiro (UERJ) - Coorientador

Prof. Dr. Eduardo Stockler Tognetti
Universidade de Brasília (UNB)

Prof. Dr. João Yoshiyuki Ishihara
Universidade de Brasília (UNB)

Prof. Dr. Roberto Kawakami Harrop Galvão

Instituto Tecnológico de Aeronáutica (ITA)

Prof. Dr. Rodrigo Farias Araújo
Universidade do Estado do Amazonas (UEA)

Documento assinado eletronicamente por **Reinaldo Martinez Palhares**, **Professor do Magistério Superior**, em 09/07/2025, às 14:30, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

Documento assinado eletronicamente por **Márcio Júnior Lacerda**, **Usuário Externo**, em 10/07/2025, às 10:09, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

Documento assinado eletronicamente por **João Yoshiyuki Ishihara**, **Usuário Externo**, em 11/07/2025, às 08:54, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

Documento assinado eletronicamente por **Roberto Kawakami Harrop Galvão**, **Usuário Externo**, em 22/07/2025, às 08:43, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

Documento assinado eletronicamente por **Pedro Henrique Silva Coutinho**, **Usuário Externo**, em 22/07/2025, às 08:49, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

Documento assinado eletronicamente por **Rodrigo Farias Araujo**, **Usuário Externo**, em 22/07/2025, às 10:38, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

Documento assinado eletronicamente por **Eduardo Stockler Tognetti**, **Usuário Externo**, em 22/07/2025, às 10:48, conforme horário oficial de Brasília, com fundamento no art. 5º do Decreto nº 10.543, de 13 de novembro de 2020.

A autenticidade deste documento pode ser conferida no site https://sei.ufmg.br/sei/controlador_externo.php?acao=documento_conferir&id_orgao_acesso_externo=0, informando o código verificador **4356609** e o código CRC **AF2FC406**.

Referência: Processo nº 23072.241570/2025-76 SEI nº 4356609

AGRADECIMENTOS

Primeiramente, gostaria de agradecer ao meu orientador Prof. Reinaldo Palhares, por todo incentivo e confiança, desde o primeiro dia do doutorado. Sou extremamente grato por todos os ensinamentos e conselhos compartilhados. Espero fazer bom uso de toda a experiência adquirida com o senhor ao longo do restante de minha carreira como pesquisador. Gostaria de agradecer também, aos meus coorientadores, Prof. Márcio Júnior Lacerda e Prof. Pedro Henrique Coutinho. O suporte oferecido por ambos foi essencial para o desenvolvimento deste trabalho. Desejo estender a parceria, as colaborações e amizade com todos. Além de orientadores, são pessoas que respeito muito e que certamente contribuíram para o meu crescimento pessoal. Muito obrigado por tudo!

Também sou grato ao Prof. Vincenç Puig que me recebeu na Universitat Politècnica de Catalunya durante parte do doutorado. Foi um período extremamente desafiador, que se tornou leve e produtivo devido a sua gentileza, suporte e atenção. Estendo o agradecimento a todos os amigos que conheci na UPC e em Barcelona. A companhia e suporte de vocês tornaram essa experiência ainda mais proveitosa e divertida.

Gostaria de agradecer também aos meus amigos do Laboratório D!FCOM: Ariany, Davi, Emanuel, Emerson, Igor, Iury, Jonatan, Nathan, Márcia, Pedro Moreira, Pedro Pires, e Vitor por todo o suporte e companheirismo. Em especial, gostaria de agradecer à Márcia e ao Emanuel por confiarem no meu trabalho e me convidarem para colaborar em muitos de seus artigos.

Agradeço também aos amigos que conheci em Belo Horizonte ao longo desse período: Amanda, Diego, Rebeca, Samla, e toda a turma de Sete Lagoas que conheci pela convivência com vocês.

Agradeço a toda a minha família, em especial à minha mãe, um grande exemplo de força e perseverança e que sempre me apoiou em todas as minhas decisões.

Por fim, gostaria de agradecer aos membros do comitê de tese por terem aceitado julgar este trabalho, à toda comunidade da UFMG, e ao CNPq pelo apoio financeiro.

RESUMO

Avanços na tecnologia de automação aumentam a segurança e a eficiência dos processos industriais. Como resultado, atualmente operamos em uma era interconectada, na qual dispositivos se comunicam e colaboram para resolver problemas complexos. Contudo, essa interconectividade traz consigo novos obstáculos, demandando estratégias robustas para assegurar um controle confiável e eficaz em aplicações do mundo real. Esta tese aborda alguns desses desafios via o projeto de controladores distribuídos, aplicáveis tanto a sistemas interconectados quanto a sistemas multiagentes. No contexto de sistemas interconectados, esta tese concentra-se em dois cenários distintos. O primeiro cenário trata do projeto de controladores distribuídos para sistemas interconectados não lineares sujeitos a retardos variantes no tempo na dinâmica local e nas interconexões entre os subsistemas. Já o segundo cenário aborda o projeto simultâneo de controladores distribuídos e mecanismos locais de acionamento baseados em eventos (ETMs, do inglês, *Event-triggered mechanims*) para reduzir o consumo de recursos de comunicação. Em ambos os casos, o sistema não linear é representado por modelos fuzzy Takagi-Sugeno (TS) com consequentes não lineares (N-TS), que simplificam a representação em contextos interconectados em comparação a abordagem clássica para modelagem Takagi-Sugeno. Já no contexto de sistemas multiagentes, esta tese aborda o consenso sem líder e o consenso de formação com seguimento de líder para agentes modelados como sistemas lineares com parâmetros variantes (LPV, do inglês, *Linear Parameter-Varying*). Diferentemente da maioria das abordagens existentes, os métodos propostos consideram perturbações internas causadas por discrepâncias entre parâmetros de escalonamento dos agentes. Em ambas as estruturas de consenso, são propostos protocolos distribuídos baseados em observadores para lidar com estados não medidos. Todos os métodos de projeto propostos nesta tese são formulados via condições suficientes expressas como desigualdades matriciais lineares (LMIs, do inglês, *Linear Matrix Inequalities*), obtidas das teorias de Lyapunov e Lyapunov-Krasovskii. Simulações numéricas ilustram a eficácia das abordagens de controle distribuídas propostas para lidar com esses desafios.

Palavras-chave: controle distribuído; sistemas interconectados. sistemas multi-agentes; consenso sem líder; consenso líder-seguidor; desigualdades matriciais lineares; sistemas lineares com parâmetros variantes; sistemas fuzzy Takagi-Sugeno.

ABSTRACT

Advancements in automation technology improve the safety and efficiency of industrial processes. As a result, we now operate in an interconnected era where devices communicate and collaborate to solve complex problems. However, this interconnectivity introduces new challenges and requires robust strategies to ensure reliable and effective control in modern real-world applications. This thesis addresses some of these challenges by designing distributed control approaches for interconnected as well multi-agent systems. In the context of interconnected systems, this thesis focuses on two different scenarios. In particular, the first scenario addresses the design of distributed controllers for delayed interconnected nonlinear systems with time-varying delays in both the local subsystems' dynamics and the physical interconnections among the subsystems. Moreover, the second scenario addresses the codesign of distributed controllers and local event-triggered mechanisms (ETMs) to reduce the consumption of communication resources. In both cases, the nonlinear system is represented by Takagi-Sugeno (TS) fuzzy models with nonlinear consequents (N-TS fuzzy), which simplify representation in interconnected contexts compared to standard TS fuzzy models. Furthermore, for multi-agent systems, this thesis tackles leaderless and leader-following formation consensus for agents modeled as Linear Parameter-Varying (LPV) systems. Unlike most existing approaches, the proposed methods account for internal perturbations caused by scheduling parameter mismatches among agents. In both consensus frameworks, distributed observer-based protocols are proposed to handle unmeasured states. All the design methods proposed in this thesis are performed with sufficient conditions formulated via linear matrix inequalities (LMIs) derived from the Lyapunov and Lyapunov-Krasovskii theory. Numerical simulations illustrate the effectiveness of the proposed distributed control approaches in addressing these challenges.

Keywords: distributed control; interconnected systems; multi-agent systems; leaderless consensus; leader-following consensus; linear matrix inequalities; linear parameter varying systems; Takagi-Sugeno fuzzy systems.

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ETC	Event-triggered control
ETM	Event-triggering mechanism
LMI	Linear matrix inequality
LPV	Linear parameter-varying
LTI	Linear time-invariant
MIET	Minimum inter-event time
IS	Interconnected system
MAS	Multi-Agent Systems
TS	Takagi-Sugeno
N-TS	Takagi-Sugeno fuzzy model with nonlinear consequent

LIST OF SYMBOLS

General Notation

$\mathbb{B}$	The Boolean domain $\{0, 1\}$
$\mathbb{B}^p$	The p-ary Cartesian power of the Boolean set $\mathbb{B}$
$\mathscr{P}(\mathbf{i})$	The set of permutations of a multi-index $\mathbf{i} = (i_1, \ldots, i_p) \in \mathbb{B}^p$
$\mathbb{B}^{p+}$	The set $\{\mathbf{i} \in \mathbb{B}^p : i_j \leq i_{j+1}, j \in \mathbb{N}_{\leq p-1}\}$
$\mathbb{R}$	The set of real numbers
$\mathbb{R}_{\geq 0\ (>0)}$	The set of non-negative (positive) real numbers
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{m \times n}$	The set of real matrices of order m by n
$\mathbb{N}$	The set of natural numbers, $\mathbb{N} = \{1, 2, \ldots\}$
$\mathbb{N}_{\leq k}$	The set $\{1, 2, \ldots, k\}$ for a given $k \in \mathbb{N}$
$\mathbb{N}_0$	The set $\{0\} \cup \mathbb{N}$
$\underline{\lambda}(A)$	The minimum eigenvalue of a matrix A
$\overline{\lambda}(A)$	The maximum eigenvalue of a matrix A
$x^\top$ or $A^\top$	Transpose of a vector x or a matrix A
$\|x\|$	The Euclidean norm of a vector x
$\lvert x \rvert$	The absolute value of a real number x
$\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$ or $\begin{bmatrix} A & \star \\ B^\top & C \end{bmatrix}$	Shorthand notation for $\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$
$\mathrm{He}(A)$	Shorthand notation for $A + A^\top$, where A is any matrix expression
$P > 0\ (\geq 0)$	Indicates that P is a symmetric positive (semi-)definite matrix
$\mathrm{diag}(X_1, \ldots, X_n)$	The diagonal matrix whose entries are $X_1, \ldots, X_n$, $n \in \mathbb{N}$
$\mathrm{tr}(X)$	The trace of a square matrix X
$\otimes$	The Kronecker product

I_n	Identity matrix of order n. If the order is clear in the context, it is omitted
$0_{n,m}$	Null matrix of order n by m. If the order is clear in the context, it is omitted
$\mathcal{G}(\mathcal{V},\mathcal{E})$	The graph considered to represent the interconnections or the communication networks
$\mathcal{V}$	The vertex set associated with the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$
$\mathcal{E}$	The edges set associated with the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$
$\mathcal{N}_i$	The neighboring set of an agent or subsystem i associated with the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$
$\mathcal{D}$	The degree matrix associated with the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$
$\mathcal{A}$	The adjacency matrix associated with the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$
$\mathcal{L}$	The Laplacian matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$ associated with the graph $\mathcal{G}(\mathcal{V},\mathcal{E})$
η	A Diagonal matrix $\eta = \mathrm{diag}(\eta_i, \ldots, \eta_N)$ composed by the pinning parameters that indicates if the communication of the leader with the i-th agent exists.
$\bar{\mathcal{L}}$	The matrix associated with the overall communication $\bar{\mathcal{L}} = \mathcal{L} + \eta$ in the leader-following scenario.
$\mathscr{D}$	The validity domain of the N-TS fuzzy models.
h_l	The limits (hyperplanes) of the validity domain $\mathscr{D}$ of the N-TS fuzzy models.
ϵ	A Scalar parameter considered in the definition of the LMIs.
$(x_1, \ldots, x_N)$	For given vectors $x_i \in \mathbb{R}^{n_i}$, define $(x_1, \ldots, x_N) = [x_1^\top, \ldots, x_N^\top]^\top \in \mathbb{R}^n$, where $n = \sum_{i=1}^N n_i$
$\mathrm{co}\{x, y\}$	The convex hull of two points x and y
$A^\dagger$	The Moore-Penrose pseudo-inverse a matrix A

Chapter-Specific Notation: Chapter 3

$h_i(t)$	The time-varying delay of the local dynamics of the i-th subsystem.

h_0	The lower bound for the time-varying delays $h_i(t)$.
τ_0	The upper bound for the time-varying delays $h_i(t)$.
$\tau_{ij}(t)$	The time-varying delay of the interconnection between the i-th and j-th subsystems.
τ_0	The lower bound for the time-varying delays $\tau_{ij}(t)$.
τ_1	The upper bound for the time-varying delays $\tau_{ij}(t)$.
d	The maximum upper bound of the system delays $d = \max\{h_1, \tau_1\}$.
φ_i	The set of initial conditions over the interval $[-d, 0]$.
$\mathscr{R}_0$	The safe operation region.
δ	A scalar parameter computed considering the eigenvalues of the matrices of the Lyapunov-Krasovskii functional and the bounds of the delays.
γ	A scalar parameter computed considering the eigenvalues of the matrices of the Lyapunov-Krasovskii functional and the bounds of the delays.
$\mathscr{X}_\delta$	The set of admissible initial conditions.
ρ	A scalar parameter computed considering the eigenvalues of the matrices of the Lyapunov-Krasovskii functional and the bounds of the delays.
ω_i	Scalar parameters computed considering the bounds of the delays.
$\bar{\eta}$	A Scalar weight of the optimization problem.
$\mathscr{C}([-d, 0], \mathbb{R}^n)$	Is the space of continuous functions with finite norm $\|\varphi\|_c = \sup_{-d \leq t \leq 0} \|\varphi(t)\|$
$\mathscr{C}^1([-d, 0], \mathbb{R}^n)$	Is the space of continuously differentiable functions with finite norm $\|\varphi\|_{c^1} = \|\varphi(t)\|_c + \|\dot{\varphi}(t)\|_c$
x_t	Is the the segment $x_t(s) = x(t+s)$, $\forall s \in [-d, 0]$

Chapter-Specific Notation: Chapter 4

t_k^i	The k-th triggering instant of the i-th event-triggered mechanism.
$\hat{x}_i(t)$	The last state information of the i-th sent by the i-th event-triggered mechanism.

$e_i(t)$	The transmission error that measures the deviation between the current state and the latest transmission.
J_i	The Upper bounds of the partial derivatives of the nonlinear interconnections $\phi_i(x)$.
$\Gamma_i(\zeta_i, \zeta_j)$	The i-th local triggered function
$\mu_i(\zeta_i, \zeta_j)$	The i-th local function introduced to cope with the asynchronous phenomenon.
$\mathscr{R}$	The estimate of the domain of attraction of the origin.
τ_i^*	The i-th local minimum inter-event time.
$\bar{\tau}_i$	The i-th local maximum inter-event time.
τ_m	The i-th median of the average inter-event time.
ε	The scalar bound of the ε-constraint optimization.

Chapter-Specific Notation: Chapter 6

$\rho_i(t)$	The vector of time-varying scheduling parameters of the i-th agent.
$\hat{x}_i(t)$	The estimated state-vector of the i-th agent.
$z_i(t)$	The local estimation error of the i-th agent.
$\delta_i(t)$	The local state-consensus error of the i-th agent.
$e(t)$	The combined augmented error system.
$\omega(t)$	The internal perturbation vector
ε^*	The upper bound of the Ellipsoid of the Practical consensus definition.
γ	The supremum value of the infinity norm of the internal perturbation vector $\gamma \triangleq \sup_{t\geq 0}(\|\omega(t)\|_\infty)$.
b	A Positive scalar parameter associated with the attractive region.
a	A Positive scalar parameter associated with the decay rate of the convergence to the attractive region.
$\mathscr{X}_\infty$	The attractive region of convergence of the error system.
λ_m	The Eigenvalues of the Laplacian matrix $\mathcal{L}$.

T	A orthonormal matrix composed by the eigenvectors of $\mathcal{L}$.

Chapter-Specific Notation: Chapter 7

$\rho_i(k)$	The vector of time-varying scheduling parameters of the i-th agent.
$\hat{x}_i(k)$	The estimated state-vector of the i-th agent.
$z_i(k)$	The local estimation error of the i-th agent.
$\delta_i(k)$	The local consensus tracking error of the i-th agent.
$r_i(k)$	The local formation compensation of the i-th agent.
$\nu_i(k)$	The local parameter-mismatch compensation of the i-th agent.
$e(k)$	The combined augmented error system.
$\omega(k)$	The internal perturbation vector
$s(k)$	The desired trajectory of the leader
f_i	The desired constant formation vector of the i-th agent with respect to the leader trajectory.
γ	The ℓ_∞ performance level.
$\varphi(e(0), \|w\|_{\ell_\infty})$	The upper bound of the norm of the augmented error system.
σ	A Scalar parameter $\sigma \in (0, 1)$ associated with the decay rate of the error system.
$\bar{\lambda}_m$	The Eigenvalues of the Laplacian matrix $\bar{\mathcal{L}}$.
$\bar{T}$	An orthonormal matrix composed by the eigenvectors of $\bar{\mathcal{L}}$.

CONTENTS

Part I

General Introduction and Addressed Challenges

1 INTRODUCTION

1.1 Interconnected and multi-agent systems

The human capacity for critical thinking allows the design of tools and techniques to modify the environment. History shows that since the Paleolithic age, when the first tools were created to ensure survival, the complexity of humanity's existence has constantly grown, and consequently, the need for technological innovations has increased at an exponential rate. The improvement of society's quality of life can be seen as the main motivation for the development of new techniques, once the increase in population drew attention to the necessity of simplifying relationships and automating processes. This technological evolution is a slow and gradual process that requires the cooperation of several fields of science. As highlighted by Bernstein & Bushnell [1]:

> "An interesting aspect of the history of technology is the way in which an innovation was developed. In many cases, inventions were the result of numerous people making small advances until a critical point was reached."

It is beyond question that the field of control engineering has directly contributed to the inception of several technological breakthroughs. These previous efforts have led to an interconnected era, where we can interact and communicate with each other even in distant parts of the world. One can observe that many modern devices are composed of interconnected elements that might engage in collaboration with one another through information exchange. Therefore, it is crucial to design new control strategies that can effectively address the main challenges presented by these complex systems.

In this context, the literature has introduced several definitions to characterize such scenarios. Among them, the so-called Interconnected Systems (ISs), and Multi-Agent Systems (MASs) are of particular interest in this thesis. The first reference to ISs was found in the notes of Elden [2], in 1921, where the operation of the interconnection between the power systems of the Eastern Massachusetts Electric Company and the New England Power Company is discussed. Since then, processes such as multi-robot control, distributed intelligence, oscillator synchronization, social interaction modeling, transportation networks, power systems, chemical reactors, and aerospace systems, among many others, have been defined as ISs [3]. Similarly, as discussed by Chen & Ren [4], the study of MASs stretches back through history, having emerged as a distinct field at the beginning of the 21^{st} century. The potential applications of MASs also include processes such as satellite formation, robotics, electric power systems, and intelligent transportation systems [5].

Due to their similarities, overlaps in the terminology of ISs and MASs are common in the literature. For instance, the approaches of [6, 7] define the systems they address as interconnected multi-agent systems, while [8] refers to physically interconnected multi-agent systems. Other works, such as [9, 10], also include the notion of large-scale into interconnected and multi-agent systems, respectively. However, as discussed in [11], the concept of large-scale can be highly subjective. An alternative way to distinguish these systems is to consider whether decomposition techniques are necessary to manage their complexity [10, 11]. To properly define the class of systems addressed in this thesis, the same arguments presented in [12] are considered, where ISs and MASs are distinguished by evaluating if the interactions among the components of the overall system are raised by physical connections or introduced into the control setup to deal with cooperative goals. More specifically, the following definitions are considered:

Definition 1.1: Interconnected Systems (Adapted from [12])

Interconnected systems (ISs) are referred to the class of systems composed of a set of physically coupled interacting subsystems with individual control objectives.

Definition 1.2: Multi-agent Systems (Adapted from [12])

Multi-agent systems (MASs) are referred to the class of systems that are composed of a set of autonomous subsystems (usually known as agents), that interact trough the exchange of information and work together to achieve cooperative control goals.

From Definitions 1.1 and 1.2, one can perceive that only on ISs certain variables associated with a subsystem may directly affect the dynamics of the other subsystems. In practice, the structure of a global closed-loop interconnected system depends on the number of subsystems, how the subsystems are physically interconnected, and how the communications (between the subsystems to the local controllers and among the controllers) are performed. Similarly, the structure of a global closed-loop MAS depends on the number of agents and on how the communications (between the agents to the local controllers and among the controllers) are defined. Thus, a general structure of an overall closed-loop system for both ISs and MASs is not easy to depict.

Broadly speaking, these general structures can be divided into three main components: the physical system, the communication framework, and the control setup. For illustration purposes, Figures 1.1 and 1.2 present conceptual simplifications of these structures. The structure of a IS, emphasizing the physical interconnections among the subsystems, is presented in Figure 1.1. Meanwhile, Figure 1.2 illustrates the MASs structure, where the interactions are restricted to the exchange of information in the control setup, in order to achieve cooperative control.

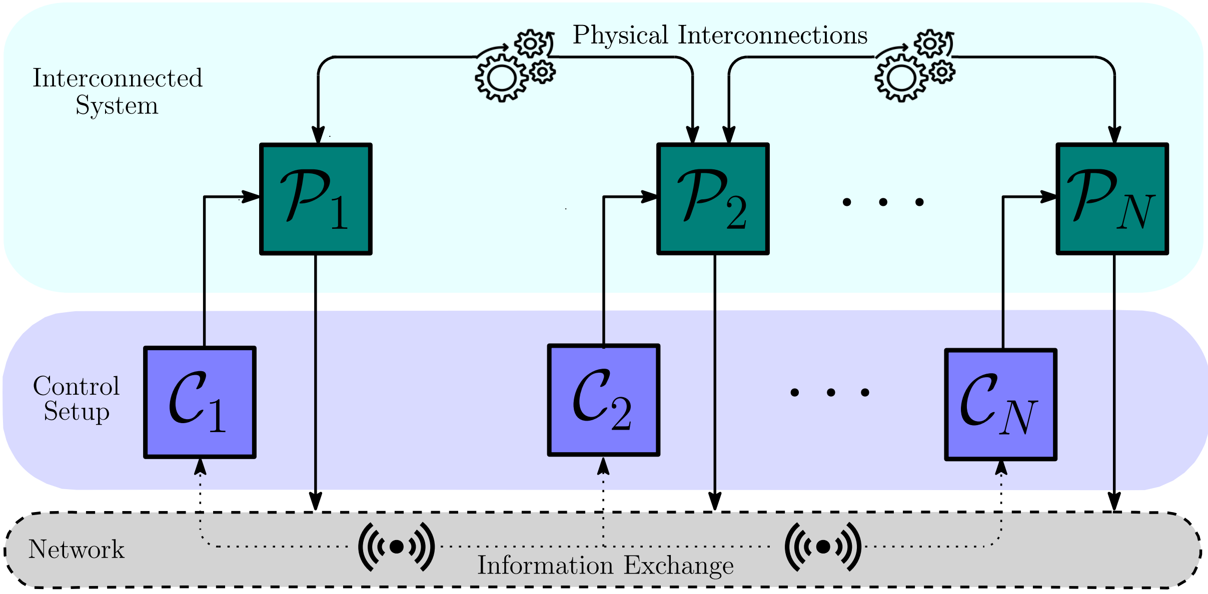

Figure 1.1 – Conceptual simplification of the general structure of a closed-loop Interconnected Systems.

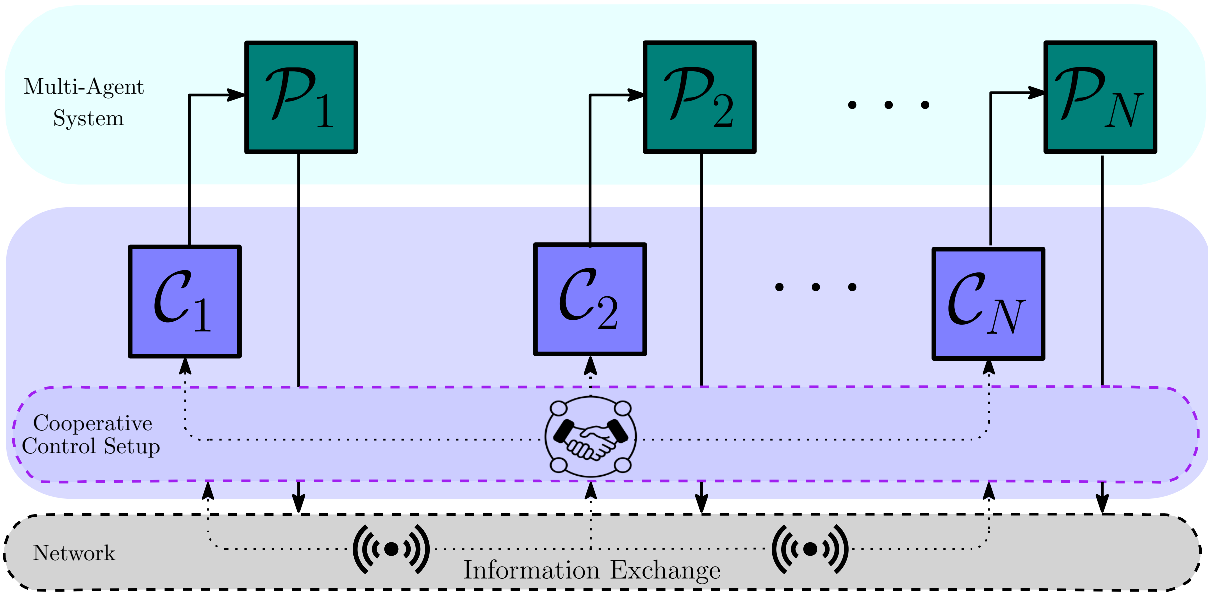

Figure 1.2 – Conceptual simplification of the general structure for a closed-loop Multi-Agent Systems.

Based on the aforementioned discussion, the importance of the information exchange among subsystems or agents is evident. From the availability of information to the design and implementation of the control law, the main structures of control setup are categorized as *centralized control*, *decentralized control*, and *distributed control*. Due to the characteristics of the systems, it is reasonable to expect an increase in the design complexity and the required resources with the growth of the number of subsystems or agents. Therefore, a trade-off between the performance of the closed-loop system and these required resources is desirable.

In addition to hardware, the main concerns in these cases pertain to the communication and computational burden of each strategy.

The next section provides a discussion on the advantages and disadvantages of the main structures for control setup.

1.2 Main control structures

1.2.1 Centralized control

In the centralized approach, a single controller is responsible for ensuring the stability of the entire system. The control unit requires measurements of all subsystems or agents and defines all control actions. A general illustration of the centralized structure is depicted in Figure 1.3.

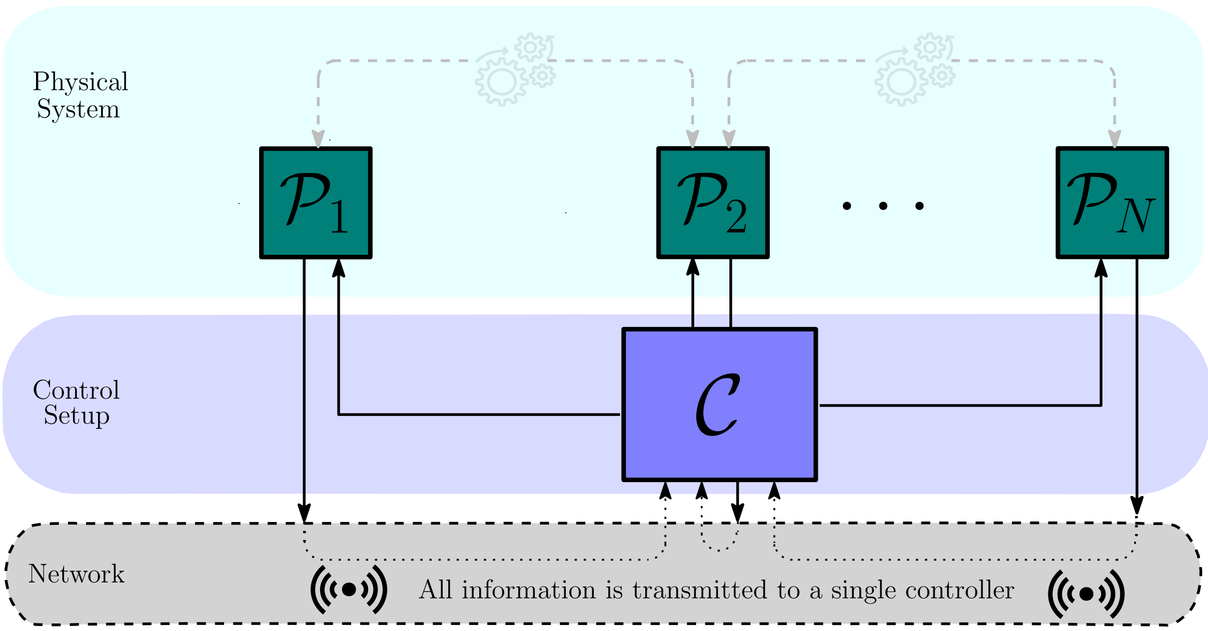

Figure 1.3 – Centralized control.

Due to the knowledge of global information, the main advantage of this approach is the possibility of obtaining appropriate levels for closed-loop performance. However, the high communication resources necessary to collect and transmit all data to the central controller and the high computational resources required to simultaneously process all data for the design of the control inputs can be considered as the major drawbacks of this approach. Moreover, note that if the measurements of one single physical system are compromised, there is an impact on the performance of the global system.

1.2.2 Decentralized control

In the decentralized approach, each subsystem has its own local controller. The controllers have access only to the measurements of their respective subsystems, that is, there is no exchange of information among the controllers. A general illustration of the decentralized structure is depicted in Figure 1.4.

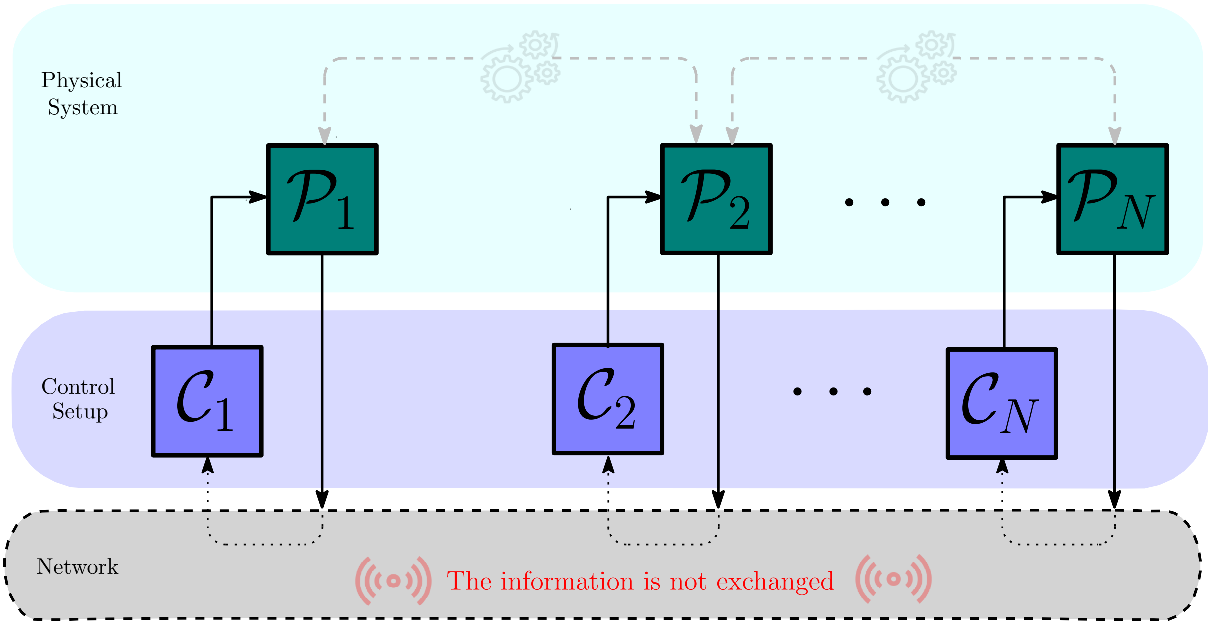

Figure 1.4 – Decentralized control.

Compared with the centralized approach, the decentralized structure provides a reduction in the required communication and computational resources. However, due to the lack of information, global closed-loop performance may be affected. Furthermore, note that when the decentralized structure is considered in ISs, the interconnections among the subsystems are ignored in the local control laws. Overlooking the interconnection effects in the structure of the controllers introduces additional difficulties and may lead the design process to infeasible solutions, as there is no guarantee that a set of independent local controllers can always be designed. Unlike the centralized approach, if the measurements of one single unit are compromised, only the corresponding control input is damaged at first. However, due to the interconnections, a cascade effect might occur, affecting the other subsystems.

1.2.3 Distributed control

In the distributed approach, stabilization of the global closed-loop system is performed with a set of local controllers that are capable of exchanging information with the controllers of neighboring subsystems or agents. For ISs, a suitable choice for the communication network might be to guarantee that the controllers are interconnected in the same way as the physical interconnections. A general illustration of the distributed structure is depicted in Figure 1.5.

Distributed control establishes a compromise between closed-loop performance and the required resources. On the one hand, this strategy provides greater modularity and economy of resources, in comparison with centralized control. On the other hand, due to the exchange of information in the neighborhood, the distributed control is able to improve the closed-loop performance compared to the decentralized approach. Thus, it is possible to conclude that the distributed control structure pursues an equilibrium between the main advantages of the previous approaches. Due to this, the focus of this thesis is on exploring the benefits of the distributed control structure for different challenges in the context of ISs and MASs.

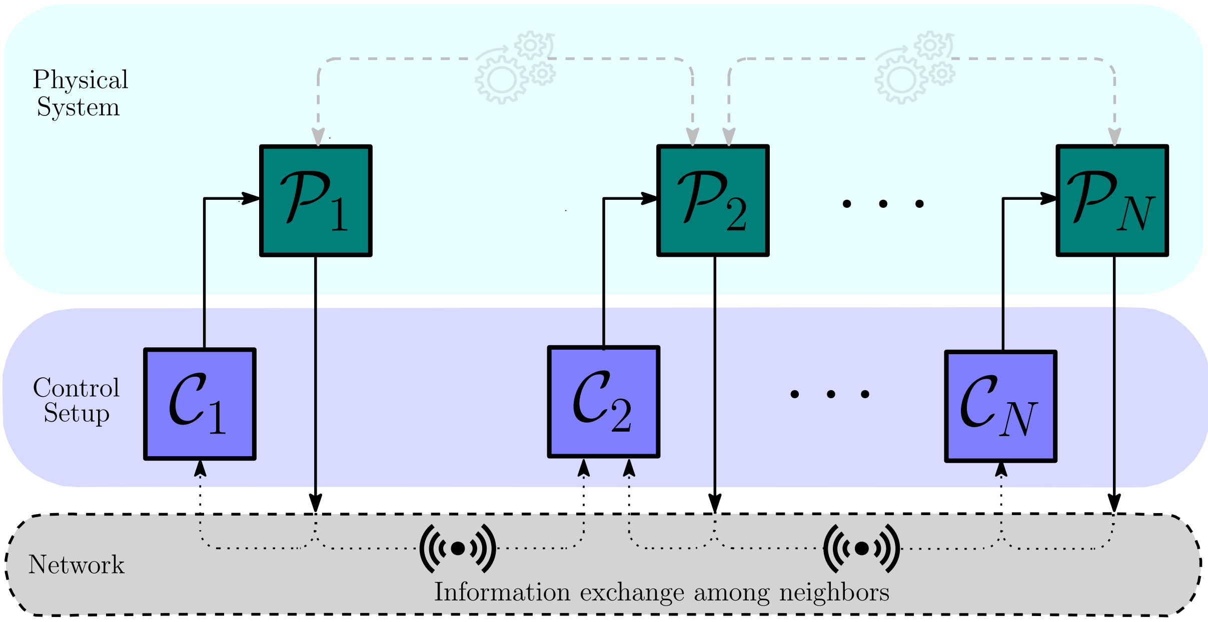

Figure 1.5 – Distributed control.

In a more general setting, where the overall IS or MAS consists of N subsystems or agents, the interactions among them must be properly defined, and in such cases graph theory is employed. An example of an undirected graph is depicted in Figure 1.6. In the case of ISs, the arrows (edges) indicate that the dynamics of the i-th subsystem directly depends on the states of the j-th subsystems that are interconnected to it. The edges are undirected, that is, they point in both directions, to indicate that the interconnections are mutual. Moreover, in the case of MAS, the graph represents the communication topology. Therefore, the local controllers will send and receive information from the other controllers according to the edges of the undirected graph. More information on the concepts of graph theory considered in this thesis is presented in Appendix A.

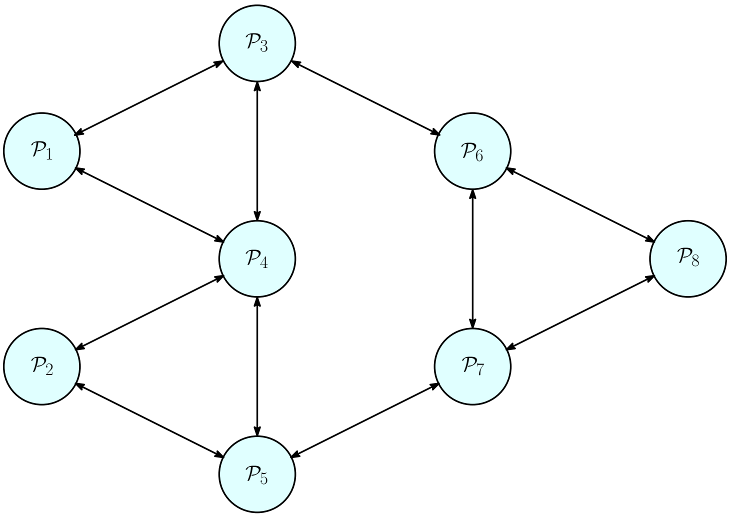

Figure 1.6 – Example of an undirected graph.

Similarly to the control setup, several modeling approaches can also be used to describe the physical dynamics of ISs and MASs. Some of these approaches are further discussed in the following section, which focuses on modeling at the physical level.

1.3 Modeling of interconnected and multi-agent systems

The modeling of a system's dynamic behavior is crucial in the field of automatic control. Among various modeling approaches, nonlinear dynamics is known to offer a richer representation, covering a wide range of real-world applications [13, 14]. As discussed in [15], it is often possible to approximate nonlinear systems using models such as Linear Time-Invariant (LTI) systems, uncertain LTI systems, Takagi-Sugeno (TS) fuzzy models, Linear Parameter-Varying (LPV) systems, and quasi-LPV systems. Consequently, it is not surprising that the literature in the fields of ISs and MAS covers works that address both nonlinear dynamics and their approximations. Thus, the goal of this section is to specify the modeling approach adopted in this thesis.

First, for ISs, the aim is to investigate continuous-time nonlinear models that consists of N interconnected nonlinear subsystems described as follows:

$$\mathcal{P}_i : \dot{x}_i(t) = f_i(x_i(t), u_i(t)) + \sum_{j \in \mathcal{N}_i} g_{ij}(x_i(t), x_j(t)),$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector of the i-th subsystem, $u_i(t) \in \mathbb{R}^{m_i}$ is the i-th control input, $\mathcal{N}_i$ is the neighborhood, and f_i and g_{ij} are nonlinear functions that represents the local nonlinear dynamics, and the nonlinear interconnections among the subsystems respectively.

Nonlinear dynamical systems are known to be more complex and challenging to control than linear systems. Moreover, the nonlinear control techniques that exist have particularities that can make the design process onerous for complex applications. Therefore, the aforementioned approximations for the nonlinear dynamics are often considered to simplify the control design process. Among them, this thesis places particular emphasis on the TS fuzzy approach. In this modeling, the nonlinear behavior is represented with a polytopic embedding constructed by a convex summation of linear models. The main advantage of this approach is the possibility to formulate the control design by exploring the Lyapunov theory and the model's convexity, obtaining sufficient conditions in terms of Linear Matrix Inequalities (LMIs).

Note that in the context of ISs, the nonlinearities of the model may occur in the local subsystems dynamics, and in the interconnections among the subsystems, which makes the control design even more challenging, once representing all these nonlinearities by a set of fuzzy rules might lead to the curse of dimensionality problem. To overcome this issue, the main strategies consist of considering only the local nonlinearities in fuzzy modeling, and assuming that the nonlinear interconnections satisfy some bounding properties such as norm bounding [16], quadratic bounding [17], or sector bounding [18]. Therefore, the approach proposed in this thesis assumes that the nonlinear interconnections are sector-bounded. In this case, a Takagi-Sugeno fuzzy model with nonlinear consequents (N-TS fuzzy) [19, 20] is obtained for each subsystem of IS.

A variety of possible dynamical models have also been investigated in the literature for MASs. Among them, it is possible to highlight the single and double integrator models (that can be seen as special cases of a general linear dynamics), and nonlinear models such as the Lagrangian and unicycle systems [4]. Furthermore, MASs can be classified as homogeneous (when the dynamics of all agents are equal), or heterogeneous (when each agent is described by an individual representation) [21]. In the context of MASs, this thesis centers its attention on the particular cases where the dynamics of the agents can be represented by LPV models.

Similarly to TS fuzzy models, the LPV approach also represents the system dynamics in a polytopic embedding. However, in the LPV representation, the convex summation of linear models can be performed in terms of time-varying scheduling parameters, which are functions of measured exogenous signals. For more details about the TS fuzzy, N-TS fuzzy, and LPV modeling approaches employed in this thesis, the reader is referred to the seminal works, survey papers, and previous thesis available in [11, 15, 22–29].

1.4 Addressed challenges

The general scope of this thesis has been outlined in previous discussions. Building on this context, this section now focuses on the specific challenges (problems) investigated in this work.

1.4.1 Handle the presence of time-delays in interconnected systems

Time delay is an intrinsic phenomenon of several real-world problems in the fields of biology, chemistry, economics, mechanics, physics, physiology, population dynamics, and engineering sciences. Examples of the modeling of such processes as Delay Differential Equations (DDE) are presented in [30], and the motivations for the study of time-delay systems are discussed in [31].

As highlighted in a historical note in [32], the first equations with delay were studied in the 18th century, and ever since, this topic has been attracting the attention of many researchers, especially in the control community, after the early results of robust control for systems with uncertain delays. In control systems, the effects of the time delays are not restricted to the system dynamics. They can appear in the state, in the control input, or in the measurements, being classified as transport, communication, and measurement delays, respectively, [32].

It is well known that the presence of time delays can cause harmful effects on the system's operation, and if not properly managed, it can even lead the system to instability. Therefore, the development of control approaches capable of dealing with the presence of delays is a very important and challenging issue. Several results have been proposed in the literature, mostly based on the methods of Lyapunov-Krasovskii and Lyapunov-Razumikhin, which are able to provide constructive finite-dimensional conditions. More information on analysis and control approaches for time-delay systems can be found in the works [31–35] (and the references therein).

In continuous-time nonlinear ISs, the effects of time-varying delays in the physical system might appear in both the local subsystem's dynamics and the physical interconnections among the subsystems. The dynamics of each subsystem, in this case, can be described as:

$$\mathcal{P}_i : \dot{x}_i(t) = f_i(x_i(t), x_i(t - h_i(t)), u_i(t)) + \sum_{j \in \mathcal{N}_i} g_{ij}(x_i(t), x_j(t - \tau_{ij}(t))),$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector of the i-th subsystem, $u_i \in \mathbb{R}^{m_i}$ is the i-th subsystem control input, $h_i(t)$ is the internal time-varying delay of the subsystem's dynamics, and $\tau_{ij}(t)$ is the time-varying delay induced by the connection between the subsystems $\mathcal{P}_i$ and $\mathcal{P}_j$.

Based on the aforementioned discussion, the first challenge addressed by this thesis is as follows:

> To propose a novel distributed control approach, in terms of LMI-based conditions, for the stabilization of continuous time-delay interconnected nonlinear systems represented by polytopic N-TS fuzzy models.

1.4.2 Preserve communication resources in the control of interconnected systems

It is evident that a reliable communication network is essential for a successful implementation of control approaches for ISs. Unfortunately, in real-world applications, the ideal operation of such networks is almost impossible, and to address more realistic scenarios, it is necessary to consider network-induced phenomena that might prevent the correct operation of the closed-loop system.

For instance, consider that the communication network has its own bandwidth limitations. In this case, the system measurements traffic, or the control input traffic, may cause congestion in the Sensor-Controller (S-C) or Controller-Actuator (C-A) communication channels. This congestion is capable of causing problems such as network-induced delays, data packet dropouts, and data packet disorders. A complete discussion of the mentioned issues and other network constraints, such as quantization errors, network channel fading, time-varying networks, and time-varying sample intervals, can be consulted in the survey papers [36–40], and in the references therein. These aspects highlight several challenges of the control design for ISs. The possible consequences make clear the importance of designing control approaches to cope with these network-induced effects. Moreover, it is also clear that developing resource-aware control approaches that reduce the usage of communication resources is equally important.

The way information is transmitted in the control loop directly influences the burden of communication resources. The traditional time-triggered approach considers that data sampling and transmissions are performed in instants of time determined by clocks. In this case, the sampling mainly occurs according to three different strategies: i) periodically with a constant sampling period, ii) periodically with a time-varying sampling period, or iii) with stochastic sampling. Since these strategies are only driven by time, they cannot verify if transmissions are not required. This inefficient transmission results in a waste of communication resources and may cause an overload in channels [37].

An effective way to reduce the number of unnecessary transmissions is the Event-Triggered Control (ETC) approach. In this strategy, an Event-Triggered Mechanism (ETM) that takes into account the evolution of the system states (or outputs) and their deviation from the last transmission to define when the transmissions occur. More precisely, new transmissions are triggered by ETM if the triggering function exceeds a threshold.

Due to its efficiency, the ETC theory has been widely explored by the control community [41–47]. Several results have been reported in the literature considering optimization-based and Lyapunov-based formulations that employ modeling tools such as hybrid system models, perturbed models, and time-delay models. Moreover, it is also possible to classify the ETC methods according to the technique considered to monitor the measurements of the system (continuous, continuous with regularization, and periodic), according to the triggering function (static, adaptive, and dynamic), and according to the design process (emulation and co-design). The works [48, 49] present a complete overview of the definitions, differences, and advantages of these strategies.

Therefore, the goal is to consider the ETC theory to reduce the number of transmissions in distributed control of ISs. More specifically, the second challenge addressed by the thesis is:

To propose a distributed event-triggered control approach, in terms of LMI-based conditions, to preserve communication resources in the stabilization of continuous-time interconnected nonlinear systems represented by polytopic N-TS fuzzy models.

1.4.3 Manage different scheduling parameters and unmeasurable states in LPV multi-agent system

Influenced by the cooperative behaviors of the natural world, the study of MASs enabled significant advancements in practical applications in several fields. An interesting feature of MASs is their cooperative actions, which allow them to solve complex problems by modifying the agents' behavior to achieve a common goal. From the control design perspective, consensus, formation, and flocking control have been the three major tasks extensively investigated in recent years [50, 51].

Broadly speaking, the consensus problem seeks to achieve an agreement among all agents, that is, all agents agree upon and converge to a single value. In this case, the control input is defined as the consensus protocol, which is the way in which the agents will exploit their collective behavior to reach a consensus [4]. As explained in [4, 50], the consensus problem can be divided into leaderless and leader-following problems, as described as follows:

Definition 1.3: Leaderless consensus (Adapted from [4, 50])

A general MAS with N agents achieves a leaderless consensus if

$$\lim_{t\to\infty} ||z_i(t) - z_j(t)|| = 0, \quad \forall j \neq i, \;\; i, j \in \mathbb{N}_{\leq N},$$

where $z_i(t)$ and $z_j(t)$ represent the state or output of the i-th and j-th agent.

Definition 1.4: Leader-following consensus (Adapted from [4, 50])

A general MAS with N agents and a leader achieves a leader-following consensus if

$$\lim_{t\to\infty} ||z_i(t) - z_0(t)|| = 0, \quad \forall i \in \mathbb{N}_{\leq N}$$

where $z_i(t)$ represents the state or output of the i-th agent, and $z_0(t)$ is a common desired trajectory of the leader for all agents to track.

Moreover, in the formation problem, the main goal is to guarantee that the group of agents forms and maintain a geometric formation to achieve a coordinated goal:

Definition 1.5: Leaderless formation consensus (Adapted from [4, 50])

A general leaderless formation consensus problem is to design a consensus protocol for a MAS to achieve

$$\lim_{t\to\infty} ||z_i(t) - z_j(t) - (f_i - f_j)|| = 0, \quad \forall j \neq i, \ \ i, j \in \mathbb{N}_{\leq N},$$

where $z_i(t)$ and $z_j(t)$ represent the state or output of the i-th and j-th agent, and $(f_i - f_j)$ is the formation deviation between the i-th and j-th agents.

Definition 1.6: Leader-following formation consensus (Adapted from [4, 50])

A general leader-follower formation consensus problem is to design a consensus protocol for an MAS to achieve

$$\lim_{t\to\infty} ||z_i(t) - z_0(t) - f_{i0}|| = 0, \quad \forall i \in \mathbb{N}_{\leq N},$$

where $z_i(t)$ represents the state or output of the i-th agent, $z_0(t)$ the common desired trajectory for all agents to track and f_{i0} is the desired formation deviation from $z_0(t)$.

These problems and their variations have been extensively addressed in multiple scenarios, as can be seen in several existing survey papers [4, 50–59] (and the references therein). However, as discussed in Section 1.3, the focus of this thesis is on the case of LPV MASs. In general, an LPV system can be described by:

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) = C(\rho(t))x(t) \end{cases} \quad \text{(continuous-time)}$$

$$\begin{cases} x(k+1) = A(\rho(k))x(k) + B(\rho(k))u(k) \\ y(k) = C(\rho(k))x(k) \end{cases} \quad \text{(discrete-time)}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the input, $y \in \mathbb{R}^{n_y}$ is the output, and $\rho = [\rho_1, \rho_2, \ldots, \rho_p]^\top$ is a vector of time-varying parameters, which are functions of exogenous signals. This system can be represented by a polytopic model as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N_v} \alpha_i(\rho(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{N_v} \alpha_i(\rho(t)) C_i x(t) \end{cases} \quad \text{(continuous-time)}$$

$$\begin{cases} x(k+1) = \sum_{i=1}^{N_v} \alpha_i(\rho(k))(A_i x(k) + B_i u(k)) \\ y(k) = \sum_{i=1}^{N_v} \alpha_i(\rho(k)) C_i x(k) \end{cases} \quad \text{(discrete-time)}$$

being N_v the number of vertices of the polytopic domain, A_i, B_i and C_i, are constant matrices representing the vertices, and $\alpha_i(\rho(\cdot))$ are the time-varying parameters satisfy the convex sum property:

$$\sum_{h=1}^{N_v} \alpha_h(\rho(\cdot)) = 1, \ \ \text{and} \ \ \alpha_h(\rho(\cdot)) \geq 0, \ \ \forall h \in \mathbb{N}_{\leq N_v}.$$

Since the time-varying parameters $\rho(\cdot)$ are functions of exogenous signals, one interesting feature that may occur in the context of MASs is: *A homogeneous LPV MASs where all agents belong to the same polytopic domain, may present heterogeneity in its dynamics, if each agent has different time-varying parameters $\rho_i(\cdot)$.* In this case, the convex sum of the vertices can result in a different matrix of the polytopic domain, even if the agents have identical structures. As discussed in [52], consensus protocols designed for completely homogeneous MASs are not suitable if all agents do not have exactly the same dynamics, and challenges might arise due to various aspects.

In the cases where it is not possible to overcome these challenges posed by physical characteristics or communication constraints, exact consensus might not be achieved. An alternative solution is to guarantee that the consensus error can be confined in a bounded region around the origin, ensuring a *practical consensus* [60]. Following [60–62], the notion of practical consensus can be described as follows:

Definition 1.7: Practical consensus (Adapted from [60–62])

Defining the ellipsoid

$$\mathcal{E}(P,\varepsilon) \triangleq \{e \in \mathbb{R}^{Nn} : e^\top P\, e \leq \varepsilon\},$$

where $P > 0$ is a positive definite matrix, $\varepsilon > 0$ is a given positive scalar and e is the consensus error. A MAS is said to achieve a practical consensus if

$$\lim_{t\to\infty} d(e(t,e_0), \mathcal{E}(P,\varepsilon)) = 0,$$

where $e(t,e_0)$ represents the trajectory of the error system with the initial condition e_0, and $d(e(t,e_0), \mathcal{E}(P,\epsilon))$ represents the distance between $e(t,e_0)$ and $\mathcal{E}(P,\varepsilon)$.

As discussed previously, the information available for the design of the consensus law is a key feature in achieving the defined consensus. Besides the information obtained from other agents, the accessibility to local measurements is equally important. However, in several practical applications, some of the systems states might not be available for measurement, or there is a high cost for obtaining all the sensors required [15, 63]. In such cases, the design of observer-based consensus protocols can provide a viable solution [64]. Thus, the challenges addressed by this thesis in the context of LPV MASs are:

To propose a distributed observer-based consensus approach, in terms of LMI-based conditions, capable of guaranteeing the leaderless practical state-consensus of continuous-time LPV MASs with different scheduling parameters and unmeasurable states.

To propose a distributed observer-based consensus approach, in terms of LMI-based conditions, capable of guaranteeing the leader-follower formation consensus of discrete-time LPV MASs with different scheduling parameters and unmeasurable states.

1.5 Thesis outline and contributions – The Big Picture

The big picture of the thesis organization is summarized in Figure 1.7, where it is possible to see that it is divided into four main parts.

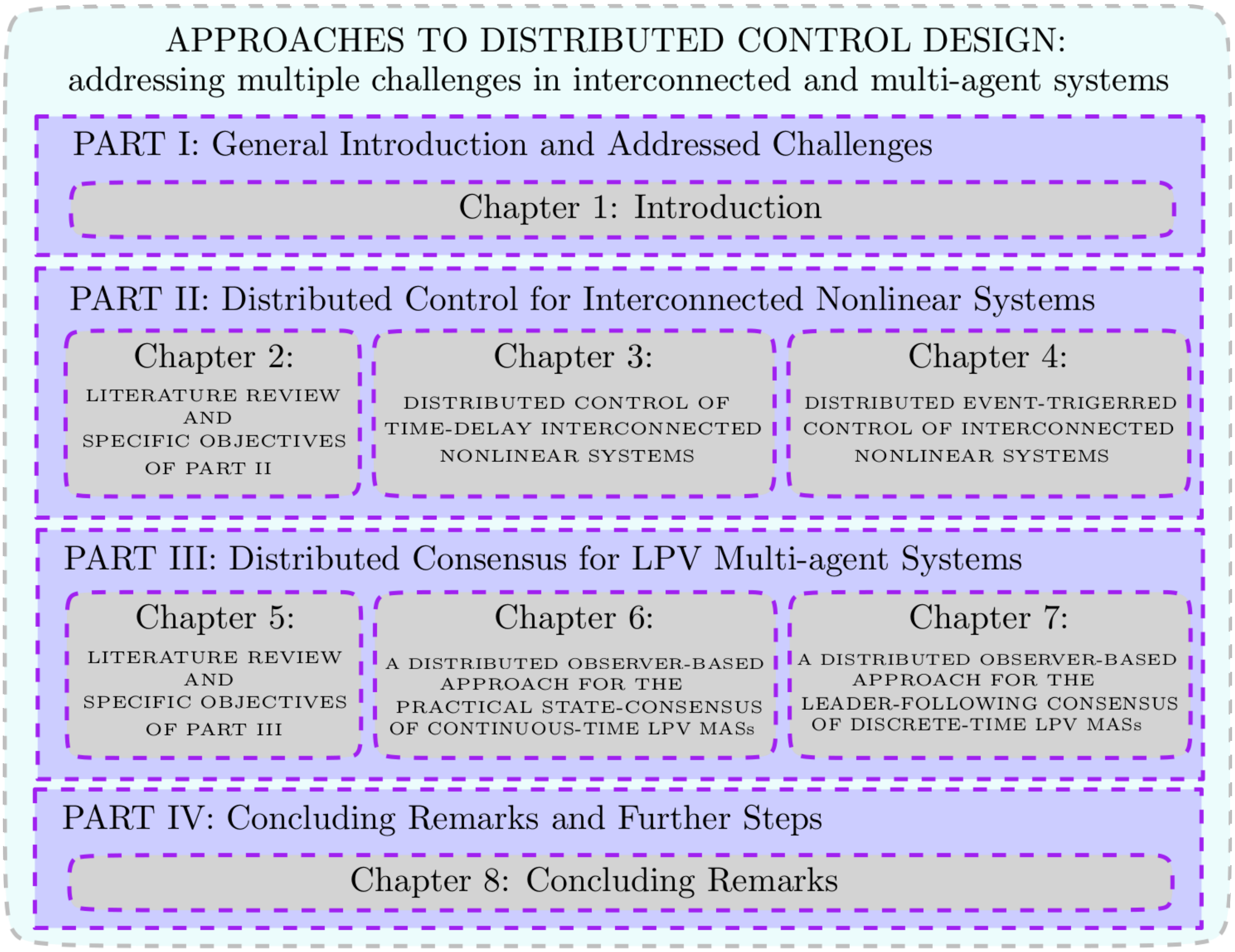

Figure 1.7 – Thesis organization – The big picture.

The Part I encompasses this Chapter 1, where the general introduction and the addressed challenges are discussed. In Parts II and III, the proposed distributed control approaches for interconnected nonlinear systems and distributed consensus protocols for LPV MASs are presented, respectively. Moreover, in Part IV, concluding remarks and suggestions for future work are discussed. The related contributions of each chapter are described as follows:

- **PART II - Distributed Control for Interconnected Nonlinear Systems**

  - Chapter 2: Provides a literature review on the existing approaches for interconnected systems in the contexts of time-delay and event-based control approaches. Moreover, the specific objectives of Chapters 3 and 4 are stated.

  - Chapter 3: Concerns the challenge of designing a distributed control strategy for continuous-time interconnected nonlinear systems subject to time-varying delays in both local subsystem dynamics and physical interconnections among subsystems. Based on Lyapunov-Krasovskii stability theory, the proposed method develops synthesis conditions for constructing a distributed control law that ensures local asymptotic stability of the closed-loop system at the origin. A guaranteed set of admissible initial conditions is formulated to ensure the validity of the closed-loop N-TS fuzzy model considered to represent the system. Furthermore, the chapter introduces a quasi-convex optimization procedure to enlarge the set of initial conditions. The results of this Chapter are published in [65].

  - Chapter 4: Concerns the challenge of co-designing an asynchronous distributed event-triggered control strategy for continuous-time nonlinear interconnected systems. Since state information is transmitted to local controllers only at triggering instants, a key complexity arises from the mismatch between the premise variables of the subsystems and the ETC setup. To address this, the chapter proposes a novel distributed event-triggered mechanism (ETM) that actively compensates for these mismatches. The chapter also introduces a multi-objective optimization procedure to simultaneously enlarge the estimated domain of attraction and minimize the consumption of communication resources. Furthermore, the proof of the existence of a strictly positive minimum inter-event time to exclude Zeno behavior is also presented. The results of this Chapter are published in [66].

- **PART III - Distributed Consensus for LPV Multi-agent Systems**

  - Chapter 5: Provides a literature review on the existent approaches for the consensus of LPV MASs.

  - Chapter 6: Concerns the challenge of designing a distributed gain-scheduled observer-based framework to achieve practical state consensus in continuous-time LPV MASs, where each agent operates under distinct time-varying scheduling parameters. A central problem arises from the heterogeneity of scheduling parameters across agents, which introduces non-synchronization effects modeled as internal disturbances. The chapter presents sufficient synthesis conditions for designing an observer-based consensus protocol that guarantees the exponential convergence of the trajectories of the consensus error to a bounded region around the origin. The results of this Chapter are published in [67].

- Chapter 7: Concerns the challenge of designing a distributed gain-scheduled observer-based framework to achieve leader-following formation in discrete-time LPV MASs, where each agent operates under distinct time-varying scheduling parameters. Similar to Chapter 6, the framework explicitly addresses parameter mismatches and formation requirements as internal perturbations. However, in this leader-following formation scenario, the mitigation of these perturbations via compensation signals is investigated. The chapter further establishes sufficient design conditions to guarantee that the combined dynamics of estimation and tracking errors remain bounded in the cases where the proposed compensation signals cannot be designed. The results of this Chapter are published in [68].

- **PART IV - Concluding Remarks and Further Steps**
  - Chapter 8: Summarizes the main contributions of this thesis and presents the concluding remarks and suggestions for future work.

Part II

Distributed Control for Interconnected Nonlinear Systems

2 LITERATURE REVIEW AND SPECIFIC OBJECTIVES OF PART II

This chapter presents a literature review of existing approaches for interconnected systems in the scenarios of time-delay §2.1, and event-based control §2.2. Moreover, based on the pre-organized discussion, the specific objectives addressed in the remaining Chapters of Part II are defined in §2.3.

2.1 Literature review of time-delay interconnected systems

Within the context of ISs, the system's intricate structure permits the occurrence of time-delays not only in the local dynamics of the subsystems but also in the interconnections among them. Various approaches have been proposed in the literature to address each specific case. To provide an overview of the control methods for time-delay interconnected systems, the main results were summarized in the literature review presented in Table 2.1. The approaches were classified based on three criteria: the location of the time-delays (local dynamics of the subsystems, interconnections among the subsystems, or in both), the control structure (Decentralized or Distributed control), and the class of model representing the subsystems (Nonlinear, Linear, TS fuzzy or N-TS fuzzy).

Table 2.1 – Literature Review: Control for Time-delay Interconnected Systems

Delay location		Nonlinear	Linear	TS fuzzy	N-TS fuzzy
Local Dynamics	Decentralized	[69–79]	[80]	[81–89]	×
	Distributed	×	[90–92]	×	×
Interconnections	Decentralized	[93–122]	[123–131]	[132–137]	×
	Distributed	×	[138]	[139]	×
Both	Decentralized	[140–147]	[148–153]	[154]	×
	Distributed	[155]	[156, 157]	×	×

Control Approaches for Time-Delay Interconnected Nonlinear Systems

The works [69–79] focus on the design of decentralized controllers for systems with time-delay only in the nonlinear dynamics of the subsystems. To cope with unknown functions, delays, and unmeasured states, the approaches of [69–73] perform the design of adaptive controllers based on universal approximators and recursive techniques. More specifically, the design of a decentralized adaptive neural output-feedback control scheme is proposed in [71], adaptive neural network memory output-feedback controllers are designed in [72], and a decentralized adaptive implicit inverse control scheme is presented in [73]. Moreover, in the approaches of [74–76], the design is based on the combination of dynamic gain techniques with the backstepping approach, and Lyapunov-Krasovskii stability arguments. Finally, the works [77–79] propose delay-dependent LMI conditions for the design of state-feedback [77, 79], and memory static output-feedback controllers [78].

The design of decentralized controllers for ISs with nonlinear subsystems and time-delay only in the interconnections among the subsystems is addressed in [93–122]. Notice that this setup has been the most explored in the literature. Similarly to the first group of references, the majority of these approaches [93, 94, 97, 99–109, 111–114, 117–122] focus on the development of adaptive controllers based on universal approximators and recursive techniques. The design of observer-based output-feedback controllers is performed in [95, 96] based on the dynamic-gain approach. A delay-dependent sufficient condition is proposed in [101] for the design of decentralized $\mathcal{H}_\infty$ state-feedback controllers. The $\mathcal{H}_\infty$ control problem is also addressed in [115, 116], using neural networks to obtain a linear differential inclusion state-space representation, that allows the synthesis of fuzzy controllers through LMIs. Moreover, in [110] the design of dynamic output-feedback controllers is carried out by combining the backstepping technique with Nussbaum functions and Lyapunov-Krasovskii stability arguments.

The approaches of [140–147], deal with the decentralized control problem for ISs with nonlinear subsystems and time-delay in both the local subsystems' dynamics and the physical interconnections. The design of model reference adaptive controllers is investigated in [140–142]. Two distinct cases are considered in [147]. If the bounds of the uncertain interconnection are available, the design of the output-feedback decentralized controllers is performed with the backstepping method. Otherwise, if the bounds are unknown, neural network approximations are considered to obtain an adaptive control law. In [144] a delay-dependent static output-feedback variable structure control is obtained considering the Lyapunov-Razumikhin approach. Furthermore, in [145,146,151] the design of robust decentralized state-feedback controllers [146], decentralized delay-dependent state-feedback controllers [145], and decentralized dynamic output-feedback controllers for discrete-time systems [151], are performed through LMIs conditions.

It is important to emphasize that the performed literature review shown that little attention has been paid to the design of distributed control approaches for time-delay interconnected nonlinear systems, regardless of the location of the time-delays. The only exception is the approach of [155], where the reference tracking problem for interconnected nonlinear system with delays in both local dynamics and interconnections is investigated. The proposed method consist on the design of distributed preview tracking controllers, obtained trough sufficient conditions based on Lyapunov-Krasovskii stability arguments with the focus on $\mathcal{H}_\infty$ disturbance attenuation.

Control Approaches for Time-Delay Interconnected Linear Systems

Shifting our attention to approaches that employ linear models to represent the subsystems, it is also challenging to find results for distributed control. In [90–92], the authors present sufficient conditions in the form of LMIs for the design of distributed filters and controllers [90], distributed preview controllers for discrete-time systems with time-varying delays and polytopic uncertainties [91], and robust distributed controllers for the primary frequency control of time-delay power systems [92]. It is important to note that these conditions consider the presence of delays only in the local dynamics of the subsystems.

The partial state consensus problem is investigated in [138] with the design of distributed controllers for a chain of interconnected delayed systems with interconnection delays. In [157] the design of distributed controllers and distributed anti-windup compensators are proposed for the robust stabilization of spatially interconnected delayed systems with input saturation. The control of wide-area power systems with state and interconnection delays is performed in [156] with distributed sliding mode controllers.

Furthermore, notice that the remaining approaches focus on the design of decentralized controllers for linear ISs with time-delays only in the local dynamics [80], only in the interconnections [123–131], or both in the dynamics of local subsystems and physical interconnections [148–153].

Control Approaches for Time-Delay Interconnected TS Fuzzy Systems

Specifically in the context of TS fuzzy delayed ISs, one may cite the approaches [81–88, 132–137, 139, 154]. In [81, 82] decentralized control schemes are proposed for nonlinear interconnected systems with multiple time-delays in their local dynamics. The modeling error is considered as a perturbation, and a robust design is proposed. Decentralized $\mathcal{H}_\infty$ controllers are also designed by the approaches of [83, 86, 87]. In [83, 87] the system is subject to delays in the dynamics and outputs of the subsystems, and in [86] the delays affect only the local dynamics. The decentralized approach proposed in [86] is made up of sampled-data local controllers with memory parameters. Combining stability arguments of the Lyapunov-Krasovskii theory with the scaled small gain theorem, the method proposed in [87] can design decentralized dynamic output-feedback controllers.

An Input-Output stability approach is addressed in [134]. Similarly to [87], a combination of the Lyapunov-Krasovskii theory with the scaled small gain theorem is considered to provide a reachable set estimation and design decentralized controllers. In [136], the finite-time control problem is investigated with the design of decentralized dynamic output-feedback controllers for time-delayed systems subject to intermittent actuator and sensor faults. Moreover, the finite-time stabilization for interconnected systems with nonlinear discontinuous interconnections is addressed in [139]. The system is modeled with the interval type-2 (IT2) TS fuzzy approach, and the proposed controllers are delay-dependent once the discontinuous and delayed terms are included in the control scheme. Finally, notice that only the approach of [154] considers the presence of time delays in both local dynamics of the interconnections, and only [139] proposes a distributed structure in the context of delayed TS fuzzy ISs.

From the literature review performed, it becomes evident that limited attention has been paid to the design of distributed controllers for time-delay interconnected nonlinear systems. Furthermore, it is also possible to notice that the approaches employing TS fuzzy models are mainly focused on the design of decentralized controllers. Although the distributed control problem for local stabilization of *non-delayed* interconnected N-TS fuzzy systems has been addressed by [18], the literature review was unable to find any approach that investigates the local control design for interconnected N-TS systems in the presence of time-delays, which is more involved than in the non-delayed case.

In the next section, the literature review is extended to include existing approaches of event-based control for interconnected systems.

2.2 Literature review on event-based control for interconnected systems

To provide an overview of the ETC methods for ISs, the main results were summarized in the literature review presented in Table 2.2. The approaches were classified based on two criteria: the control structure (Decentralized or Distributed control), and the class of models representing the subsystems (Nonlinear, Linear, TS fuzzy , or N-TS fuzzy).

Table 2.2 – Literature Review

	Nonlinear	Linear	T-S Fuzzy	N-TS Fuzzy
Decentralized	[158–197]	[198–202]	[203–208]	×
Distributed	[209–218]	[219–229]	[230–232]	×

Decentralized Event-triggered Control Approaches for Interconnected Nonlinear Systems

It is evident that the development of event-triggered decentralized approaches for the control of interconnected nonlinear systems is currently attracting significant attention [158–197]. Among these approaches, the main focus is the development of event-triggered adaptive techniques [158–169, 171–187, 195]. Similarly to the case of time-delay interconnected nonlinear system, these works are mainly based on the combination of universal approximators with classical methods for the control of nonlinear systems. It is possible to highlight approaches derived considering dynamic surface control [158–160], adaptive dynamic programming algorithms [161–164], adaptive critic learning [165–169], reinforcement learning [170, 171], fuzzy logic systems [172–174], cyclic small gain [175], and neural networks [176–181]. Approaches that combine game theory with reinforcement learning have also been proposed in [191–193]. Furthermore, the results of [188–190] are established considering Input-to-State stability arguments, and the adaptive tracking problem is addressed in [182–187].

Notice that none of the previously mentioned event-based decentralized approaches for interconnected nonlinear systems have provided constructive conditions in the form of LMIs. The literature review performed identifies [196, 197] as the only works focusing on control design using LMIs. In [196], the design of robust decentralized event-triggered model predictive controllers for networked Lipchitz systems is performed considering infinite-horizon cost functions. Furthermore, co-design conditions based on Lyapunov-Krasovskii stability arguments are proposed in [197], to simultaneously design the decentralized event-triggered controllers and the matrices of the event-triggered mechanisms.

Distributed Event-triggered Control Approaches for Interconnected Nonlinear Systems

The distributed event-triggered control for interconnected nonlinear systems has been addressed in the works [209–218]. In particular, [210] proposes a distributed event-based control scheme for uncertain input-affine interconnected nonlinear systems based on a hybrid learning approach that integrates approximate dynamic programming with online exploration. In [211], the adaptive dynamic programming approach is also explored for the development of a distributed control scheme for nonlinear interconnected systems with strong interconnections. Furthermore, in [214], the same approach is employed for the design of an event-triggered distributed $\mathcal{H}_\infty$ constrained control algorithm. The $\mathcal{H}_\infty$ performance is also considered in [215] for the design of an observer-based distributed approach with dynamic dual side ETMs, and in [217] for an anti-disturbance control approach based on the design of distributed extended state observer and output-feedback.

Additionally, in [213], the adaptive dynamic programming approach is used to address the robust tracking control problem for nonlinear interconnected systems with uncertain interconnections. In [212], the focus is on a dynamic event-triggered distributed fault-tolerant control method for uncertain nonlinear interconnected systems, which also employs the adaptive dynamic programming technique. Moreover, a fault-compensation control scheme is proposed in [216] considering distributed adaptive sliding mode controllers. Finally, event-triggered distributed model predictive control algorithms are proposed in [209, 218] for interconnected systems with nonlinear coupling terms and bounded disturbances.

Event-triggered Control Approaches for Interconnected Linear Systems

For ISs with linear models representing the subsystems, the development of event-based decentralized approaches has been explored in [198–202]. Furthermore, event-based distributed control methods have been proposed in [219–229]. Unlike the approaches for nonlinear subsystems, the results of several works in this context are based on LMIs conditions

[198–202, 219–222]. Among then, co-design conditions with $\mathcal{H}_\infty$ performance were proposed in [199, 200] for the design of event-triggered dynamic output-feedback controllers, and in [202], for the design of event-triggered state-feedback controllers. Meanwhile, event-based co-design conditions are also proposed in [201, 221] for the design of static output-feedback controllers, in [220] for the design of state-feedback controllers for ISs with sparse connections, and in [219] for an observer-based approach to solve the secure load frequency control problem. Finally, a data-driven approach is proposed in [222] for unknown discrete-time interconnected systems.

Event-triggered Control Approaches for Interconnected TS fuzzy Systems

In the context of TS fuzzy modeling for the subsystems, the development of event-based decentralized approaches has been carried out in [203–208]. Additionally, event-based distributed control methods were proposed in [230–232]. In [205], a decentralized event-triggered state-feedback fuzzy approach was introduced that considers the presence of network-induced delays and time-driven samplers. The problem is formulated considering the input-delay and perturbed system approaches, and the design conditions are obtained using Lyapunov-Krasovskii stability arguments. In the works [206, 207], a two-channel triggering strategy has been proposed for the development of event-based decentralized observer-based output-feedback [206], and event-based decentralized dynamic output-feedback control approaches. In both cases, the presence of ETM is assumed in the S-C and C-A channels. As in [205], the proposed methods are also formulated considering the input-delay approach and Lyapunov-Krasovskii stability arguments. However, the presence of network-induced delays is not considered in [206, 207].

Furthermore, a decentralized $\mathcal{H}_\infty$ control approach is proposed in [208] for nonlinear large-scale systems modeled by affine fuzzy systems with unknown interconnections and network-induced delays. Meanwhile, the approaches of [194, 203] propose the use of interval type-2 TS fuzzy models to represent the subsystems. In [203], an observer-based decentralized output-feedback control method is proposed for systems with actuator saturation. Moreover, [194] presents an exponential gain approach to achieve fixed-time stabilization for systems with unknown interconnections and bounded perturbations.

In distributed methods [230–232], the type-2 TS fuzzy modeling is also employed [230, 231]. In [230] a distributed output-feedback load frequency control approach is developed for nonlinear interconnected power systems with switching topology. Similarly, in [231] a distributed output-feedback approach is proposed considering multi-rate samplers and time-driven zero-order holds. Furthermore, in [232], a distributed event-based output-feedback approach is proposed for ISs where the subsystems are represented by discrete-time TS fuzzy models.

Unlike the previously discussed classes of systems, when the TS fuzzy approach is considered to model the subsystems, event-based control designs must account for additional challenges arising from the aperiodic update of information. For instance, in the design fuzzy controllers relying on state-dependent premise variables, the aperiodic update of the measurements induces a difference between the premise variables available to the fuzzy controller, and the real-time premise variables of the subsystems. If this asynchronous phenomenon is not addressed, fuzzy controllers are reduced to traditional linear control laws. This issue increases the conservativeness of the design condition, potentially affecting the feasibility of the solution and the efficiency of the proposed method.

Unfortunately, the phenomenon of asynchronism has been neglected in the results of [203, 207]. In [205, 206, 231], this problem was handled considering prior knowledge of the deviation bounds between asynchronous membership functions. In [230] a three-step procedure is considered to obtain a relation between the current state and the last transmitted output, from which the deviation bounds of the product of the asynchronous membership functions are obtained. Moreover, in [208, 232] partitions in the state-space are considered to deal with this issue. However, this bounding strategy requires that the deviation bounds be satisfied during the operation, thus limiting the operation range of the closed-loop system.

Similarly to the case of time-delay interconnected nonlinear systems, the literature review reveals a lack of approaches that address the local event-based control problem for interconnected N-TS fuzzy systems. With the main existing methods now established, the specific objectives of Part II of this thesis are outlined in the next section.

2.3 Specific objectives of part II

Although different challenges are addressed in the context of delayed and event-based control for interconnected nonlinear systems, two common features can be highlighted: the design of distributed controllers and the N-TS fuzzy modeling of the subsystems. Furthermore, it is known that in general, the polytopic embedding resultant from the TS fuzzy and N-TS fuzzy modeling represents the nonlinear dynamics in a compact set (validity domain) of the state-space. If the system trajectories evolve outside of this domain, the stability guarantees may be lost, leading to implementation issues on the original nonlinear system. Therefore, a common issue that must be addressed in both cases is to ensure that the trajectories of the closed-loop system do not evolve outside of the validity domain. Thus, it is important to perform a local analysis to ensure the correct operation of the closed-loop system. To address this issue, along with the gaps in the literature discussed in Sections 2.1 and 2.2, the specific objectives of this thesis in the context of ISs are:

- To propose sufficient LMI-based synthesis conditions, using Lyapunov-Krasovskii stability arguments, for the design of distributed control laws to the stabilization of time-delay interconnected nonlinear systems subject to time-varying delays in both the subsystems' dynamics and nonlinear interconnections.

- To propose sufficient LMI-based synthesis conditions, using stability arguments from Lyapunov theory, for the simultaneous co-design of a set of local ETMs and event-based distributed control laws to stabilize interconnected nonlinear systems while preserving communication resources.
- To provide an estimate of the domain of attraction of the equilibrium of the global closed-loop system (in the scenario of non-delayed nonlinear interconnected systems), and an estimate of the set of admissible initial conditions (in the scenario of time-delay nonlinear interconnected systems) to ensure the safe operation of the proposed distributed control approaches.
- To propose an optimization procedure capable of enlarging the estimate of the set of admissible initial conditions in the scenario of time-delay nonlinear interconnected systems.
- To propose a multi-objective optimization procedure to enlarge the estimate of the domain of attraction, and minimize the number of state transmissions in the proposed event-based distributed control setup.
- Effortlessly deal with the influence of the asynchronous state-dependent membership functions in the proposed event-based distributed control setup.
- To prove the existence of a minimum inter-event time (MIET) ensuring that in the proposed event-based distributed control setup the Zeno behavior does not occur.

3 DISTRIBUTED CONTROL OF TIME-DELAY INTERCONNECTED NONLINEAR SYSTEMS

This chapter investigates the distributed control of interconnected nonlinear systems subject to time-varying delays in both local subsystem dynamics and physical interconnections. To address the complexity of such systems, the N-TS fuzzy model is employed, a framework offering reduced structural complexity compared to conventional TS fuzzy models. Building on Lyapunov-Krasovskii stability theory, we propose a synthesis condition to design a distributed control law that ensures local asymptotic stability of the closed-loop system's origin. Crucially, this approach guarantees a well-defined set of admissible initial conditions, within which the validity of the N-TS fuzzy model is rigorously maintained. Furthermore, a quasi-convex optimization framework is formulated to systematically enlarge the admissible initial condition set, thereby enhancing the practical applicability of the proposed methodology.

The remainder of this chapter is organized as follows. The problem formulation is presented in §3.1. The proposed design condition and the optimization issues are defined in §3.2. Two numerical examples are presented in §3.3 to illustrate the application of the proposed methodology. Conclusions are drawn in §3.4.

3.1 Problem Formulation

Consider the i-th nonlinear time-delay subsystem given by the following N-TS fuzzy model:

$$\begin{aligned}\mathcal{P}_i\colon \dot{x}_i(t)=&A_i(z_i(x_i(t)))x_i(t)+A_{di}(z_i(x_i(t)))x_i(t-h_i(t))\\&+B_i(z_i(x_i(t)))u_i(t)+\sum_{j\in\mathcal{N}_i} g_{ij}(x_i(t),x_j(t-\tau_{ij}(t)),\end{aligned} \tag{3.1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}^{m_i}$ is the input, $z_i = (z_{i1}, z_{i2}, \ldots, z_{ip_i}) \in \mathbb{R}^{p_i}$ is the vector of premise variables $z_{ik} : \mathbb{R}^{n_i} \to \mathbb{R}$, $k \in \mathbb{N}_{\leq p_i}$,

$$g_{ij}(x_i, x_j) = A_{ij}x_j + G_{ij}(z_i)\phi_{ij}(x_i, x_j),$$

is the function that models the interconnection between the subsystems $\mathcal{P}_j$ and $\mathcal{P}_i$, $\phi_{ij} : \mathbb{R}^{n_i} \times \mathbb{R}^{n_j} \to \mathbb{R}^{n_{\phi,i}}$ are the nonlinear interconnections, $h_i(t)$ are the internal time-varying delays of the subsystem's dynamics, $\tau_{ij}(t)$ is the time-varying delay induced from the connection between $\mathcal{P}_i$ and $\mathcal{P}_j$, and $\mathcal{N}_i$ is the neighboring set of the i-th subsystem (see Appendix A).

Let $w_0^{ik}(z_{ik}) = (z_{ik}^1 - z_{ik}) \,/\, (z_{ik}^1 - z_{ik}^0)$, and $w_1^{ik}(z_{ik}) = 1 - w_0^{ik}(z_{ik})$, be the membership functions defined such that each premise variable z_{ik} is written as $z_{ik} = w_0^{ik}(z_{ik})z_{ik}^0 + w_1^{ik}(z_{ik})z_{ik}^1$, with $z_{ik}^0 = \inf_{x_i\in\mathscr{D}_i} z_{ik}$ and $z_{ik}^1 = \sup_{x_i\in\mathscr{D}_i} z_{ik}$. By defining the normalized membership functions

$$\alpha_{\mathbf{i}_i}(z_i) = \prod_{k=1}^{p_i} w_{i_k}^{ik}(z_{ik}),$$

it is possible to verify that

$$\sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) = 1, \text{ and } \alpha_{\mathbf{i}_i}(z_i) \geq 0, \ \forall \mathbf{i}_i \in \mathbb{B}^{p_i}.$$

Therefore, the i-th subsystem $\mathcal{P}_i$ described as (3.1) can be equivalently rewritten as the following N-TS fuzzy model:

$$\begin{aligned} \mathcal{P}_i \colon \dot{x}_i(t) = & \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i)\Big(A^i_{\mathbf{i}_i} x_i(t) + B^i_{\mathbf{i}_i} u_i(t) + A^i_{d\mathbf{i}_i} x_i(t-h_i(t))\Big) \\ & + \sum_{j \in \mathcal{N}_i} A_{ij} x_j(t-\tau_{ij}(t)) + \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) G^{ij}_{\mathbf{i}_i} \phi_{ij}(x_i(t), x_j(t-\tau_{ij}(t))), \end{aligned} \tag{3.2}$$

where $\mathbf{i}_i$ is a p_i-dimensional multi-index, and $A^i_{\mathbf{i}_i}$, $B^i_{\mathbf{i}_i}$, $A^i_{d\mathbf{i}_i}$, and $G^{ij}_{\mathbf{i}_i}$ are the vertices.

Furthermore, the time-varying delays satisfy the following assumption.

Assumption 3.1: Bounded delays

The time-varying delays of the interconnections and the time-varying delays of each subsystem are subject to the same upper and lower bounds.

In this case, it is possible to define single time-varying delays $\tau(t)$, and $h(t)$, satisfying

$$h_0 \leq h(t) \leq h_1 \quad \text{and} \quad \tau_0 \leq \tau(t) \leq \tau_1,$$

for positive scalars h_k, τ_k, $\forall\, k \in \mathbb{B}$. This Assumption that the time-varying delays are bounded is commonly considered in the literature of time-delay systems (see [33, 35, 233]). Note that knowledge of the variation in the time-varying delays is not required.

Therefore, each subsystem and interconnection can be subject to different time-varying delays, as long as they respect the general bounds. Based on these bounds, the initial condition for each subsystem $\mathcal{P}_i$ is

$$x_i(s) = \varphi_i(s), \quad \forall s \in [-d, 0],$$

where $\varphi_i \in \mathscr{C}([-d, 0], \mathbb{R}^{n_i})$ corresponds to the set of initial conditions over the interval $[-d, 0]$, with $d \triangleq \max\{h_1, \tau_1\}$.

The i-th local fuzzy controller is given by

$$\mathcal{C}_i \colon u_i(t) = K_i(z_i(x_i(t)))x_i(t) + \sum_{j \in \mathcal{N}_i} \mathcal{K}_{ij}(x_i(t), x_j(t)), \tag{3.3}$$

where

$$K_i(z_i(x_i(t))) = \sum_{\mathbf{j}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{j}_i}(z_i) K^i_{\mathbf{j}_i}$$

is the decentralized term, and

$$\mathcal{K}_{ij}(x_i, x_j) = L_{ij} x_j + M_{ij}(z_i(x_i(t)))\phi_{ij}(x_i, x_j), \tag{3.4}$$

is the distributed term, with $L_{ij} \in \mathbb{R}^{m_i \times n_j}$ and

$$M_{ij}(z_i(x_i(t))) = \sum_{\mathbf{j}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{j}_i}(z_i) M^{ij}_{\mathbf{j}_i},$$

for all $j \in \mathcal{N}_i$.

Notice that (3.4) does not depend on the delayed state information of the adjacent subsystems because the delay $\tau(t)$ are only induced in the (physical) interconnections among the subsystems. Moreover, the local controllers $\mathcal{C}_i$ are assumed to be interconnected with the same topology as the subsystems $\mathcal{P}_i$, then the graph representing the local controllers' communication is the same undirected graph $\mathcal{G}$ (see Appendix A).

Remark 3.1

The proposed distributed control law (3.3) *can be reduced to a decentralized control law when considering $L_{ij} = 0$ and $M_i(z_i(x_i(t)) = 0$ for all $i \in \mathcal{V}$ and $j \in \mathcal{N}_i$. Moreover, when considering $L_{ij} \neq 0$ and $M_i(z_i(x_i(t)) = 0$ the proposed control law can be reduced to a distributed control law that is linear with respect to state variables from neighboring subsystems. Notice that* (3.3) *is distributed and nonlinear with respect to state variables from neighboring subsystems due to dependence on $\phi_{ij}(x_i, x_j)$.*

The matrix-value functions $A_i(z_i(x_i(t)))$, $B_i(z_i(x_i(t)))$, $K_i(z_i(x_i(t)))$, $A_{di}(z_i(x_i(t)))$, $G_{ij}(z_i(x_i(t)))$, and $M_{ij}(z_i(x_i(t)))$ are obtained with the N-TS fuzzy modeling of the delayed nonlinear dynamics of the subsystems, and defined in a validity domain represented by a convex polytope (the reader is referred to the Appendix B for additional details and examples on TS fuzzy and N-TS fuzzy modeling). To alleviate the notation the dependence of this matrices on $z_i(x_i(t))$ is omitted. Thus, they are simply denoted by $A_i(x_i)$, $B_i(x_i)$, $K_i(x_i)$, $A_{di}(x_i)$, $G_{ij}(x_i)$, and $M_{ij}(x_i)$ hereafter.

It follows from $\mathcal{P}_i$ in (3.1), $\mathcal{C}_i$ in (3.3), and Assumption 3.1 that the closed-loop dynamics of each subsystem is

$$\begin{aligned}\dot{x}_i(t) &= (A_i(x_i) + B_i(x_i)K_i(x_i))\,x_i(t) + A_{di}(x_i)x_i(t-h(t)) \\ &\quad + \sum_{j\in\mathcal{N}_i} B_i(x_i)L_{ij}x_j(t) + B_i(x_i)M_{ij}(x_i)\phi_{ij}(x_i(t), x_j(t)) \\ &\quad + \sum_{j\in\mathcal{N}_i} A_{ij}x_j(t-\tau(t)) + G_{ij}(x_i)\phi_{ij}(x_i(t), x_j(t-\tau(t)).\end{aligned} \tag{3.5}$$

Furthermore, the global interconnected closed-loop system $(\mathcal{P}, \mathcal{C})$ can be compactly written as follows:

$$\begin{aligned}\dot{x}(t) = &A_{cl}(x)x(t) + A_d(x)x(t-h(t)) + B(x)M(x)\phi(\chi(t)) \\ &+ Hx(t-\tau(t)) + G(x)\phi(\chi(t-\tau(t))),\end{aligned} \tag{3.6}$$

where $A_{cl}(x) \triangleq A(x) + B(x)(K(x) + Z)$, $x = (x_1, \dots, x_N) \in \mathbb{R}^n$, $u = (u_1, \dots, u_N) \in \mathbb{R}^m$, $\phi(\chi) = (\phi_1(\chi_1), \dots, \phi_N(\chi_N)) \in \mathbb{R}^{n_\phi}$, $\chi = (\chi_1, \dots, \chi_N) \in \mathbb{R}^{Nn}$, with

$$\chi_i(t - \tau(t)) = (\underline{x}_i(t - \tau(t)), x_i(t), \bar{x}_i(t - \tau(t))) \in \mathbb{R}^n,$$
$$\phi_i\left(\chi_i(t - \tau(t)\right) = \left(\phi_{ik_1}(x_i(t), x_{k_1}(t - \tau(t))), \dots, \phi_{ik_{d_i}}(x_i(t), x_{k_{d_i}}(t - \tau(t)))\right) \in \mathbb{R}^{d_i},$$

$\underline{x}_i = (x_1, \dots, x_{i-1}) \in \mathbb{R}^{\underline{n}_i}$, $\bar{x}_i = (x_{i+1}, \dots, x_N) \in \mathbb{R}^{\bar{n}_i}$, $\underline{n}_i \triangleq n - \sum_{k=i}^{N} n_k$, $\bar{n}_i \triangleq n - \sum_{k=1}^{i} n_k$, $\varphi \in \mathscr{C}([-d, 0], \mathbb{R}^n)$ is the initial condition, and

$$\begin{aligned} A(x) &= \mathrm{diag}(A_1(x_1), \dots, A_N(x_i)), & A_d(x) &= \mathrm{diag}(A_{d1}(x_1), \dots, A_{dN}(x_N)), \\ G(x) &= \mathrm{diag}(G_1(x_1), \dots, G_N(x_N)), & B(x) &= \mathrm{diag}(B_1(x_1), \dots, B_N(x_N)), \\ K(x) &= \mathrm{diag}(K_1(x_1), \dots, K_N(x_N)), & M(x) &= \mathrm{diag}(M_1(x_1), \dots, M_N(x_N)), \end{aligned}$$

with

$$G_i(x_i) = \begin{bmatrix} G_{ik_1}(x_i) & \cdots & G_{ik_{d_i}}(x_i) \end{bmatrix}, \quad M_i(x_i) = \begin{bmatrix} M_{ik_1}(x_i) & \cdots & M_{ik_{d_i}}(x_i) \end{bmatrix},$$

for all $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\leq d_i}$. The elements of $H = [H_{ij}]$ and $Z = [Z_{ij}]$, for all $i, j \in \mathcal{V}$, are defined in terms of the adjacency matrix $\mathcal{A}$ (see Appendix A) as

$$H_{ij} = \begin{cases} 0 & \text{if } a_{ij} = 0 \\ A_{ij} & \text{if } a_{ij} = 1, \end{cases} \qquad Z_{ij} = \begin{cases} 0 & \text{if } a_{ij} = 0 \\ L_{ij} & \text{if } a_{ij} = 1. \end{cases}$$

Remark 3.2

Since the terms of the degree matrix $\mathcal{D}$ are considered to construct $G_i(x_i)$, and $M_i(x_i)$, and the terms of the adjacency matrix $\mathcal{A}$ are considered to construct H, and Z, the topology of the global systems, represented by the undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is directly reflected in the closed-loop system.

Remark 3.3

The vector $\chi_i(\cdot) = (\underline{x}_i(\cdot), x_i(t), \bar{x}_i(\cdot))$ is introduced to represent the situation in which the nonlinear interconnections of the i-th subsystem depends on the time-delayed information of the neighbor subsystems. Notice that $\chi_i(t) = (\underline{x}_i(t), x_i(t), \bar{x}_i(t)) = x(t)$.

Notice that the matrix-valued functions of the subsystems and local controllers are written in a validity domain represented by a convex polytope (see Appendix B). Consequently, the matrices of the global closed-loop system (3.6) can be written as

$$\begin{aligned} A(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(z) A_{\mathbf{i}}, & A_d(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(z) A_{d\mathbf{i}}, & K(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(z) K_{\mathbf{i}}, \\ B(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(z) B_{\mathbf{i}}, & G(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(z) G_{\mathbf{i}}, & M(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(z) M_{\mathbf{i}}, \end{aligned} \tag{3.7}$$

for all $x \in \mathscr{D}$, where $\mathbf{i}$=$(\mathbf{i}_1, \mathbf{i}_2, \dots, \mathbf{i}_N)$ is a multi-dimensional index on $\mathbb{B}^p = \mathbb{B}^{p_1} \times \cdots \times \mathbb{B}^{p_N}$, $\alpha_{\mathbf{i}}(x) = \alpha_{\mathbf{i}_1}(z_1) \cdots \alpha_{\mathbf{i}_N}(z_N)$, and

$$\mathscr{D} = \{x \in \mathbb{R}^n : h_l^\top x \leq 1,\, l \in \mathbb{N}_{\leq n_f}\}, \tag{3.8}$$

is the validity domain of the global time-delay interconnected system, with $h_l = [0_{1\times \underline{n}_i}\ h_{i\ell_i}^\top\ 0_{1\times \overline{n}_i}]^\top$, and $l = 2(i-1) + \ell_i$, $\ell_i \in \mathbb{N}_{\leq n_{fi}}$, $i \in \mathcal{V}$, and n_f the the number of half-spaces defining the polytope. The domain $\mathscr{D}$ is a compact set that contains the origin $x = 0$, and the model (3.6) is valid only if the trajectories remain within $\mathscr{D}$, since convexity is lost otherwise.

Notice that in the proposed modeling, the nonlinear interconnections ϕ_i are introduced in the consequent parts, and only the local nonlinearities are considered to build the polytopic embedding via the sector nonlinearity approach, which reduces the number of fuzzy rules. The advantages of reducing the number of fuzzy rules are put in evidence in [18] for the non-delayed case.

Furthermore, nonlinear interconnections $\phi_i(\cdot)$ are considered to satisfy the sector-bounded property, as established in the following assumption.

Assumption 3.2: Sector bounded nonlinear interconnections

Each nonlinearity $\phi_i : \mathbb{R}^n \to \mathbb{R}^{d_i}$, $i \in \mathcal{V}$, belongs to the sector $[0, \Omega_i]$ in a compact domain $\mathscr{D} \subset \mathbb{R}^n$.

From Assumption 3.2, since ϕ_i is constructed clustering $\phi_{ik_{i\ell}}$ for $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\leq d_i}$, then all $\phi_{ik_{i\ell}}$ also belong to the sectors $[0, \Omega_{ij}]$. Thus, Ω_i can be defined as

$$\Omega_i = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \cdots & \Omega_{iN} \end{bmatrix} \in \mathbb{R}^{d_i \times n},$$

with $\Omega_{ij} \in \mathbb{R}^{d_i \times n_j}$ and $\Omega_{ij} = 0\ \forall j \notin \mathcal{N}_i$. If Assumption 3.2 is satisfied, the conditions of Lemma C.2 also hold. This Lemma presents an inequality that is considered to obtain the proposed conditions to design the distributed control law.

Based on the aforementioned definitions, the problem addressed in this chapter is stated as follows.

Problem 3.1

Consider the closed-loop distributed control system $(\mathcal{P}, \mathcal{C})$ in (3.6), where each subsystem $\mathcal{P}_i$ is given as in (3.1), and the local controllers $\mathcal{C}_i$ are as in (3.3). Assuming that Assumptions 3.1 and 3.2 are satisfied, for a given equivalent polytopic representation of the closed-loop system (3.7), valid in the compact region $\mathscr{D}$ in (3.8), design the gains of the distributed control law such that the origin $x = 0$ of the closed-loop system (3.6) is asymptotically stable. Moreover, find a set of admissible initial conditions $\mathscr{X}_\delta$, such that any closed-loop trajectory $x(t)$ with initial condition $\varphi \in \mathscr{X}_\delta$ remains confined in $\mathscr{D}$.

3.2 Main Results

3.2.1 Distributed control design conditions

As discussed in the previous sections, the N-TS fuzzy model (3.6) is only valid on the domain $\mathscr{D}$. To ensure the correct operation of the closed-loop system, a set of admissible initial conditions $\mathscr{X}_\delta$ is defined in the following proposition.

Proposition 3.1: Stability analysis

Let $V_0 : \mathbb{R}^n \to \mathbb{R}$ be a positive definite and radially unbounded function and $V : \mathscr{C}^1([-d,0],\mathbb{R}^n) \times \mathscr{C}([-d,0],\mathbb{R}^n) \to \mathbb{R}$ be a functional such that

$$V_0(x(t)) \leq V(x_t, \dot{x}_t), \quad \forall t > 0, \tag{3.9}$$

$$\kappa\|\varphi(0)\|^2 \leq V(\varphi, \dot{\varphi}) \leq \gamma\|\varphi\|_{c^1}^2 + \rho\|\dot{\varphi}\|_c^2, \tag{3.10}$$

where κ, γ, and ρ are positive scalars and $x_t : [-d,0] \to \mathbb{R}^n$ the segment $x_t(s) = x(t+s)$, $\forall s \in [-d,0]$. Let the sets

$$\mathscr{R}_0 = \{x \in \mathbb{R}^n : V_0(x) \leq 1\}, \tag{3.11}$$

$$\mathscr{X}_\delta = \left\{\varphi \in \mathscr{C}^1([-d,0],\mathbb{R}^n), \dot{\varphi} \in \mathscr{C}([-d,0],\mathbb{R}^n) : \|\varphi\|_{c^1} \leq \delta,\ \|\dot{\varphi}\|_c \leq \delta\right\}, \tag{3.12}$$

with

$$\delta = 1/\sqrt{\gamma + \rho}. \tag{3.13}$$

If the following conditions are satisfied

$$\dot{V}(x_t, \dot{x}_t) < 0, \quad \forall x_t \in \mathscr{D}_a, \tag{3.14}$$

$$\mathscr{R}_0 \subset \mathscr{D}, \tag{3.15}$$

where

$$\mathscr{D}_a = \{\varphi \in \mathscr{C}^1([-d,0],\mathbb{R}^n) : \varphi(s) \in \mathscr{D}, \forall s \in [-d,0]\},$$

then, for any initial condition $\varphi \in \mathscr{X}_\delta$, the trajectories of (3.6) converge asymptotically to the origin remaining confined in $\mathscr{R}_0$ and, consequently, in $\mathscr{D}$.

Proof. Assume that conditions (3.14)–(3.15) are satisfied. Condition (3.14) ensures that the origin of the closed-loop system (3.6) is asymptotically stable, provided that $x(t)$ is confined to a region

$$\mathscr{R}_a = \{x_t \in \mathscr{C}([-d,0],\mathbb{R}^n) : V(x_t, \dot{x}_t) \leq 1\} \subset \mathscr{D}_a.$$

Integrating (3.14) from 0 to t, yields $V(x_t, \dot{x}_t) \leq V(\varphi, \dot{\varphi})$, which from (3.10) ensures that

$$V_0(x(t)) \leq V(x_t, \dot{x}_t) \leq V(\varphi, \dot{\varphi}) \leq \gamma\|\varphi\|_{c^1}^2 + \rho\|\dot{\varphi}\|_c^2.$$

Provided that $\varphi \in \mathscr{X}_\delta$, it is obtained

$$V_0(x(t)) \leq V(x_t, \dot{x}_t) \leq V(\varphi, \dot{\varphi}) \leq \delta^2(\gamma + \rho),$$

and the selection of δ as in (3.13) implies

$$V_0(x(t)) \leq V(x_t, \dot{x}_t) \leq V(\varphi, \dot{\varphi}) \leq 1.$$

As a result, it is possible to conclude that for any $\varphi \in \mathscr{X}_\delta$, then $x(t) \in \mathscr{R}_0$, $\forall t > 0$. Since (3.15) ensures that $\mathscr{R}_0 \subset \mathscr{D}$, consequently $x(t) \in \mathscr{D}$, $\forall t > 0$. This concludes the proof. □

Proposition 3.1 introduces the conditions required to guarantee the local asymptotic stability of the origin of (3.6), providing estimates of the set of admissible initial conditions $\mathscr{X}_\delta$. However, this result does not provide a constructive procedure for designing the distributed control law. In the sequel, the idea is to obtain constructive conditions such that Proposition 3.1 are satisfied based on the following Lyapunov-Krasovskii functional candidate

$$V(x_t, \dot{x}_t) = \sum_{k=1}^{4} V_k(x_t) + V_5(\dot{x}_t) + V_6(\dot{x}_t), \tag{3.16}$$

where $V_k(x_t) = \sum_{i=1}^{N} V_{ki}(x_{it})$, for $k \in \mathbb{N}_{\leq 4}$, and $V_m(\dot{x}_t) = \sum_{i=1}^{N} V_{mi}(\dot{x}_{it})$, for $m \in \{5, 6\}$, with

$$\begin{aligned}
V_{1i}(x_{it}) =& \eta_{hi}^\top(t) P_{1i} \eta_{hi}(t),\\
V_{2i}(x_{it}) =& \eta_{\tau i}^\top(t) P_{2i} \eta_{\tau i}(t),\\
V_{3i}(x_{it}) =& \int_{t-h_0}^{t} x_i^\top(s) Q_{1i} x_i(s) ds + \int_{t-h_1}^{t-h_0} x_i^\top(s) Q_{2i} x_i(s) ds,\\
V_{4i}(x_{it}) =& \int_{t-\tau_0}^{t} x_i^\top(s) Q_{3i} x_i(s) ds + \int_{t-\tau_1}^{t-\tau_0} x_i^\top(s) Q_{4i} x_i(s) ds,\\
V_{5i}(\dot{x}_{it}) =& h_0 \int_{-h_0}^{0} \int_{t+\theta}^{t} \dot{x}_i^\top(s) R_{1i} \dot{x}_i(s) ds d\theta + h_{10} \int_{-h_1}^{h_0} \int_{t+\theta}^{t} \dot{x}_i^\top(s) R_{2i} \dot{x}_i(s) ds d\theta,\\
V_{6i}(\dot{x}_{it}) =& \tau_0 \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \dot{x}_i^\top(s) R_{3i} \dot{x}_i(s) ds d\theta + \tau_{10} \int_{-\tau_1}^{\tau_0} \int_{t+\theta}^{t} \dot{x}_i^\top(s) R_{4i} \dot{x}_i(s) ds d\theta,
\end{aligned}$$

being $h_{10} = h_1 - h_0$, $\tau_{10} = \tau_1 - \tau_0$,

$$\begin{aligned}
\eta_{hi}(t) &= (x_i(t), h_0 \psi_i(h_0, 0), h_{10} \psi_i(h_1, h_0)),\\
\eta_{\tau i}(t) &= (x_i(t), \tau_0 \psi_i(\tau_0, 0), \tau_{10} \psi_i(\tau_1, \tau_0)),
\end{aligned}$$

and $\psi_i(a, b) \triangleq \frac{1}{a-b} \int_{t-a}^{t-b} x_i(s) ds$. The matrices P_{ji}, $j \in \mathbb{N}_{\leq 2}$, Q_{ki}, R_{ki}, $k \in \mathbb{N}_{\leq 4}$, for $i \in \mathbb{N}_{\leq N}$, are all symmetric and positive definite matrices.

With the definition of the Lyapunov-Krasovskii functional (3.16), the following proposition introduces a systematic procedure to compute the positive scalars κ, γ, and ρ required to estimate the set of admissible initial conditions $\mathscr{X}_\delta$ that meet the requirements established in Proposition 3.1.

Proposition 3.2: Estimate the set of admissible initial conditions

For the Lyapunov-Krasovskii functional candidate $V(x_t, \dot{x}_t)$ defined in (3.16) and the function

$$V_0(x) = x^\top P_0 x, \tag{3.17}$$

where

$$P_0 = \mathrm{diag}(U_1^\top (P_{11} + P_{21})U_1, \ldots, U_N^\top (P_{1N} + P_{2N})U_N)$$

and $U_i = [I_{n_i}\, 0_{n_i \times n_i}\, 0_{n_i \times n_i}]^\top$, the conditions (3.9)-(3.10) are satisfied with

$$\kappa = \underline{\lambda}(P_0), \tag{3.18}$$

$$\gamma = \sum_{i=1}^{N} \left(\sum_{k=1}^{2} \omega_k \bar{\lambda}(P_{ki}) + \sum_{j=1}^{4} \omega_{j+2} \bar{\lambda}(Q_{ji}) \right), \tag{3.19}$$

$$\rho = \sum_{i=1}^{N} \sum_{l=1}^{4} \omega_{l+6} \bar{\lambda}(R_{li}), \tag{3.20}$$

being $\omega_1 = 1 + h_0^2 + h_{10}^2$, $\omega_2 = 1 + \tau_0^2 + \tau_{10}^2$, $\omega_3 = h_0$, $\omega_4 = h_{10}$, $\omega_5 = \tau_0$, $\omega_6 = \tau_{10}$, $\omega_7 = h_0^3/3$, $\omega_8 = h_{10}(h_1^2 - h_0^2)/2$, $\omega_9 = \tau_0^3/3$, and $\omega_{10} = \tau_{10}(\tau_1^2 - \tau_0^2)/2$.

Proof. Note that $V_0(x) \leq V_1(x_t) + V_2(x_t)$, then it is possible to conclude that $V_0(x) \leq V(x_t, \dot{x}_t)$, since all terms of the functional are positive definite, which shows that (3.9) is satisfied. Moreover, for $t \in [-d, 0]$, $x_t = \varphi$, which allows to write

$$V(\varphi, \dot{\varphi}) \leq \sum_{i=1}^{N} \left(\sum_{k=1}^{2} \omega_k \bar{\lambda}(P_{ki}) + \sum_{j=1}^{4} \omega_{j+2} \bar{\lambda}(Q_{ji}) \right) \|\varphi_i\|_{c^1}^2 + \sum_{i=1}^{N} \sum_{l=1}^{4} \omega_{l+6} \bar{\lambda}(R_{li}) \|\dot{\varphi}_i\|_c^2.$$

As $\|\varphi_i\|_{c^1}^2 \leq \|\varphi\|_{c^1}^2$ and $\|\dot{\varphi}_i\|_c^2 \leq \|\dot{\varphi}\|_c^2$, it follows that $V(\varphi, \dot{\varphi}) \leq \gamma \|\varphi\|_{c^1}^2 + \rho \|\dot{\varphi}\|_c^2$, with γ and ρ defined in (3.19) and (3.20), respectively. Finally, since $V_0(x) \leq V(x_t, \dot{x}_t)$ and $\underline{\lambda}(P_0)\|x\|^2 \leq V_0(x)$, the lower bound to $V(\varphi, \dot{\varphi})$ can be obtained directly by taking κ as in (3.18), which proves that (3.10) is satisfied. This concludes the proof. □

Remark 3.4

Three different regions are considered in Propositions 3.1 and 3.2: the validity domain previously discussed $\mathscr{D}$*, the safe operating region* $\mathscr{R}_0$*, and the set of admissible initial conditions* $\mathscr{X}_\delta$*. It is known that finding the exact region of attraction, i.e., the set of all initial conditions whose trajectories of the closed-loop system asymptotically converge to the origin, is a challenging task. In the non-delayed case, as in [18], it is possible to estimate this region considering the largest level set of quadratic Lyapunov functions* $V_0 : \mathbb{R}^n \to \mathbb{R}_{>0}$ *in the Euclidean space, obtaining regions* $\mathscr{R}_0$ *that guarantee the safe operation of the closed-loop system. However, in the delayed case as addressed here, the trajectories initiating at this safe operation region* $\mathscr{R}_0$ *do not necessarily converge to the origin, once conditions are obtained considering a Lyapunov-Krasovskii functional* $V : \mathscr{C}^1([-d,0],\mathbb{R}^n) \times \mathscr{C}([-d,0],\mathbb{R}^n) \to \mathbb{R}$ *that is not defined only in the Euclidean space, but also in the space of continuously differentiable functions.*

Remark 3.5

By imposing restrictions on initial conditions, as made in Propositions 3.1 and 3.2, it is possible to estimate the set of admissible initial conditions $\mathscr{X}_\delta$ *that represents a relation between the initial conditions in the space of continuously differentiable functions with the initial conditions in the Euclidean space. Thus, due to this relation, for any initial condition taken in* $\mathscr{X}_\delta$*, the trajectories will converge asymptotically to the origin remaining confined in* $\mathscr{R}_0$*. Finally, since* $\mathscr{R}_0 \subset \mathscr{D}$*, it is possible to conclude that the initial conditions* $\varphi \in \mathscr{X}_\delta$ *are sufficient to ensure that the validity domain* $\mathscr{D}$ *will not be violated. This guarantee is not provided in the existent work in the literature of interconnected T-S fuzzy systems with time-delays.*

In the sequel, a *delay-dependent* condition is introduced to design the distributed control laws (3.3) such that (3.14) in Proposition 3.1 is satisfied with the functional (3.16), thus guaranteeing the asymptotic stability of the origin of the closed-loop system (3.6).

Theorem 3.1

Consider the system (3.6) and assume that Assumptions 3.1 and 3.2 are satisfied for given τ_k and h_k, $k \in \mathbb{B}$, and $\Omega_i \in \mathbb{R}^{d_i \times n}$. Let the positive scalar ϵ and $h_{i\ell_i}$ be given. If there exist symmetric positive definite matrices $\widetilde{P}_{1i}, \widetilde{P}_{2i} \in \mathbb{R}^{3n_i \times 3n_i}$, $\widetilde{Q}_{1i}, \widetilde{Q}_{2i}, \widetilde{Q}_{3i}, \widetilde{Q}_{4i}, \widetilde{R}_{1i}$, $\widetilde{R}_{2i}, \widetilde{R}_{3i}$, $\widetilde{R}_{4i} \in \mathbb{R}^{n_i \times n_i}$, and matrices $X_i \in \mathbb{R}^{n_i \times n_i}$, $\Lambda_i \in \mathbb{R}^{d_i \times d_i}$, $\widetilde{L}_{ij} \in \mathbb{R}^{m_i \times n_j}$, $\widetilde{M}_i(x_i) \in \mathbb{R}^{m_i \times d_i}$, $\widetilde{Y}_{1i}, \widetilde{Y}_{2i}, \widetilde{Y}_{3i}, \widetilde{Y}_{4i} \in \mathbb{R}^{2n_i \times 2n_i}$, and $\widetilde{K}_i(x_i) \in \mathbb{R}^{m_i \times n_i}$, $\forall i \in \mathbb{N}_{\leq N}, \forall j \in \mathcal{N}_i$, such that

$$\sum_{(\mathbf{i},\mathbf{j})\in\mathscr{P}(\mathbf{m},\mathbf{n})} \Upsilon_{\mathbf{ij}}(h_w,\tau_k,\epsilon) < 0,\ \forall \mathbf{m},\mathbf{n}\in\mathbb{B}^{p+},\ w,k\in\mathbb{B}, \tag{3.21}$$

$$\begin{bmatrix} U_i^\top \left(\widetilde{P}_{1i}+\widetilde{P}_{2i}\right) U_i & \star \\ h_{i\ell_i}X_i & 1 \end{bmatrix} \geq 0,\ \forall \ell_i\in\mathbb{N}_{\leq n_{f_i}},\ i\in\mathcal{V}, \tag{3.22}$$

where

$$\Upsilon_{\mathbf{ij}}(h,\tau,\epsilon) \triangleq \begin{bmatrix} \widetilde{\Theta}_{\mathbf{ij}}(h,\tau,\epsilon) & \mathcal{W}_h^\top \widetilde{\mathcal{Y}}_{w+1} & \mathcal{W}_\tau^\top \widetilde{\mathcal{Y}}_{k+3} \\ \star & -\widetilde{\mathcal{R}}_2 & 0 \\ \star & \star & -\widetilde{\mathcal{R}}_4 \end{bmatrix}, \tag{3.23}$$

$$\begin{aligned}
&\widetilde{\Theta}_{\mathbf{ij}}(h,\tau,\epsilon) = \widetilde{\Phi}(h,\tau) + \widetilde{\Pi}_0 + \widetilde{\Pi}_\tau + \mathrm{He}(\widetilde{\mathcal{X}}(\epsilon)\widetilde{\mathcal{B}}_{\mathbf{ij}}),\\
&\widetilde{\Phi}(h,\tau) = \sum_{i\in\mathcal{V}} \left(\widetilde{\Phi}_{hi}(h) + \widetilde{\Phi}_{\tau i}(\tau)\right),\\
&\widetilde{\Phi}_{hi}(h) = \mathrm{He}(G_{hi}^\top(h)\widetilde{P}_{1i}J_{hi}) + \widetilde{\Sigma}_{hi} - Z_{hi}^\top \widetilde{\mathcal{R}}_{1i} Z_{hi} + F_i^\top \left(h_0^2\widetilde{R}_{1i} + h_{10}^2\widetilde{R}_{2i}\right) F_i\\
&\qquad - W_{hi}^\top \widetilde{\Psi}_{hi}(h) W_{hi},\\
&\widetilde{\Phi}_{\tau i}(\tau) = \mathrm{He}(G_{\tau i}^\top(\tau)\widetilde{P}_{2i}J_{\tau i})) + \widetilde{\Sigma}_{\tau i} - Z_{\tau i}^\top \widetilde{\mathcal{R}}_{3i} Z_{\tau i} + F_i^\top \left(\tau_0^2\widetilde{R}_{3i} + \tau_{10}^2\widetilde{R}_{4i}\right) F_i\\
&\qquad - W_{\tau i}^\top \widetilde{\Psi}_{\tau i}(\tau) W_{\tau i},\\
&\widetilde{\Pi}_0 = -2E_0^\top \Lambda E_0 + \mathrm{He}(E_0^\top \Omega(I_N\otimes X)(\mathbf{1}_N^\top \otimes N_0)),\\
&\widetilde{\Pi}_\tau = -2E_\tau^\top \Lambda E_\tau + \mathrm{He}(E_\tau^\top \Omega(I_N\otimes X)T),\\
&\widetilde{\Sigma}_{hi} = v_{i2}^\top \widetilde{Q}_{1i} v_{i2} + v_{i3}^\top(\widetilde{Q}_{2i} - \widetilde{Q}_{1i})v_{i3} - v_{i5}^\top \widetilde{Q}_{2i} v_{i5},\\
&\widetilde{\Sigma}_{\tau i} = v_{i2}^\top \widetilde{Q}_{3i} v_{i2} + v_{i6}^\top(\widetilde{Q}_{4i} - \widetilde{Q}_{3i})v_{i6} - v_{i8}^\top \widetilde{Q}_{4i} v_{i8},\\
&\widetilde{\mathcal{X}}(\epsilon) = \epsilon F^\top + N_0^\top + N_h^\top + N_\tau^\top,\ \widetilde{\mathcal{A}}_{\mathbf{ij}} = A_{\mathbf{i}}X + B_{\mathbf{i}}\widetilde{K}_{\mathbf{j}} + B_{\mathbf{i}}\widetilde{Z},\\
&\widetilde{\mathcal{B}}(z) = -XF + \widetilde{\mathcal{A}}_{\mathbf{ij}}N_0 + A_{d\mathbf{i}}XN_h + HXN_\tau + B_{\mathbf{i}}\widetilde{M}_{\mathbf{j}}E_0 + G_{\mathbf{i}}\Lambda E_\tau,\\
&\widetilde{\mathcal{R}}_{1i} = \mathrm{diag}(\widetilde{R}_{1i}, 3\widetilde{R}_{1i}),\ \widetilde{\mathcal{R}}_{2i} = \mathrm{diag}(\widetilde{R}_{2i}, 3\widetilde{R}_{2i}),\ \Omega = \mathrm{diag}(\Omega_1,\ldots,\Omega_N),\\
&\widetilde{\mathcal{R}}_{3i} = \mathrm{diag}(\widetilde{R}_{3i}, 3\widetilde{R}_{3i}),\ \widetilde{\mathcal{R}}_{4i} = \mathrm{diag}(\widetilde{R}_{4i}, 3\widetilde{R}_{4i}),\ X = \mathrm{diag}(X_1,\ldots,X_N),\\
&\Lambda = \mathrm{diag}(\Lambda_1,\ldots,\Lambda_N),\ \widetilde{\mathcal{R}}_k = \mathrm{diag}(\widetilde{\mathcal{R}}_{k1},\ldots,\widetilde{\mathcal{R}}_{kN}),\ k\in\{2,4\},\ F_i = v_{i1},\\
&\widetilde{\Psi}_{hi}(h) = \begin{bmatrix} \widetilde{\mathcal{R}}_{2i} & 0 \\ 0 & \widetilde{\mathcal{R}}_{2i} \end{bmatrix} + \tfrac{h_1-h}{h_{10}} \begin{bmatrix} \widetilde{\mathcal{R}}_{2i} & \widetilde{Y}_{2i} \\ \widetilde{Y}_{2i}^T & 0 \end{bmatrix} + \tfrac{h-h_0}{h_{10}} \begin{bmatrix} 0 & \widetilde{Y}_{1i} \\ \widetilde{Y}_{1i}^T & \widetilde{\mathcal{R}}_{2i} \end{bmatrix},\\
&\widetilde{\Psi}_{\tau i}(\tau) = \begin{bmatrix} \widetilde{\mathcal{R}}_{4i} & 0 \\ 0 & \widetilde{\mathcal{R}}_{4i} \end{bmatrix} + \tfrac{\tau_1-\tau}{\tau_{10}} \begin{bmatrix} \widetilde{\mathcal{R}}_{4i} & \widetilde{Y}_{4i} \\ \widetilde{Y}_{4i}^\top & 0 \end{bmatrix} + \tfrac{\tau-\tau_0}{\tau_{10}} \begin{bmatrix} 0 & \widetilde{Y}_{3i} \\ \widetilde{Y}_{3i}^\top & \widetilde{\mathcal{R}}_{4i} \end{bmatrix},\\
&\widetilde{\mathcal{Y}}_k = \mathrm{diag}(\widetilde{\mathcal{Y}}_{k1},\ldots,\widetilde{\mathcal{Y}}_{kN}),\ \widetilde{\mathcal{Y}}_{ki} = \begin{cases} \left[\widetilde{Y}_{ki}^\top\ 0\right]^\top & k\in\{1,3\},\\ \left[0\ \widetilde{Y}_{ki}\right]^\top & k\in\{2,4\}, \end{cases}\\
&v_{ij} = \left[0_{n_i\times 14\underline{n}_i+(j-1)n_i} \quad I_{n_i} \quad 0_{n_i\times(14-j)n_i+14\overline{n}_i+2n_\phi}\right],\\
&E_0 = \left[0_{n_\phi\times 14n} \quad I_{n_\phi} \quad 0_{n_\phi\times n_\phi}\right],\ E_\tau = \left[0_{n_\phi\times 14n} \quad 0_{n_\phi\times n_\phi} \quad I_{n_\phi}\right],\\
&\mathcal{W}_h = \left[W_{h1}^\top \quad \cdots \quad W_{hN}^\top\right]^\top,\ \mathcal{W}_\tau = \left[W_{\tau 1}^\top \quad \cdots \quad W_{\tau N}^\top\right]^\top.
\end{aligned}$$

$$J_{hi} = \begin{bmatrix} v_{i1} \\ v_{i2} - v_{i3} \\ v_{i3} - v_{i5} \end{bmatrix}, \; G_{hi}(h) = \begin{bmatrix} v_{i2} \\ h_0 v_{i9} \\ (h - h_0)v_{i10} + (h_1 - h)v_{i11} \end{bmatrix},$$

$$J_{\tau i} = \begin{bmatrix} v_{i1} \\ v_{i2} - v_{i6} \\ v_{i6} - v_{i8} \end{bmatrix}, \; G_{\tau i}(\tau) = \begin{bmatrix} v_{i2} \\ \tau_0 v_{i12} \\ (\tau - \tau_0)v_{i13} + (\tau_1 - \tau)v_{i14} \end{bmatrix},$$

$$W_{hi} = \begin{bmatrix} v_{i3} - v_{i4} \\ v_{i3} + v_{i4} - 2v_{i10} \\ v_{i4} - v_{i5} \\ v_{i4} + v_{i5} - 2v_{i11} \end{bmatrix}, \; W_{\tau i} = \begin{bmatrix} v_{i6} - v_{i7} \\ v_{i6} + v_{i7} - 2v_{i13} \\ v_{i7} - v_{i8} \\ v_{i7} + v_{i8} - 2v_{i14} \end{bmatrix}, \; T = \begin{bmatrix} T_1 \\ \vdots \\ T_N \end{bmatrix}, \; T_i = \begin{bmatrix} \underline{T}_i \\ v_{i2} \\ \overline{T}_i \end{bmatrix},$$

$$\underline{T}_i = \begin{bmatrix} v_{17} \\ \vdots \\ v_{(i-1)7} \end{bmatrix}, \; \overline{T}_i = \begin{bmatrix} v_{(i+1)7} \\ \vdots \\ v_{N7} \end{bmatrix}, \; N_0 = \begin{bmatrix} v_{12} \\ \vdots \\ v_{N2} \end{bmatrix}, \; Z_{hi} = \begin{bmatrix} v_{i2} - v_{i3} \\ v_{i2} + v_{i3} - 2v_{i9} \end{bmatrix},$$

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix}, \; N_h = \begin{bmatrix} v_{14} \\ \vdots \\ v_{N4} \end{bmatrix}, \; N_\tau = \begin{bmatrix} v_{17} \\ \vdots \\ v_{N7} \end{bmatrix}, \; Z_{\tau i} = \begin{bmatrix} v_{i2} - v_{i6} \\ v_{i2} + v_{i6} - 2v_{i12} \end{bmatrix},$$

then, invoking Proposition 3.1, we have that for any initial condition $\varphi \in \mathscr{X}_\delta$, for δ as in (3.13) with γ and ρ as in (3.19) and (3.20), respectively, the trajectories of the closed-loop system (3.6) converge asymptotically to the origin remaining confined in $\mathscr{R}_0 \subset \mathscr{D}$, being $\mathscr{R}_0$ defined in (3.15) with $V_0(x)$ given in (3.17). The gains of the distributed controller are $K_i(x_i) = \widetilde{K}_i(x_i)X_i^{-1}$, $M_i(x_i) = \widetilde{M}_i(x_i)\Lambda^{-1}$, and $L_{ij} = \widetilde{L}_{ij}X_j^{-1}$.

Proof. The proof is carried out in two steps. In the first step, conditions (3.21) ensure the negative definiteness of the time-derivative of the Lyapunov-Krasovskii candidate (3.16) and, consequently, (3.14) hold. In the second step, (3.22) ensures that $\mathscr{R}_0 \subset \mathscr{D}$, and, consequently, (3.15) holds. Having (3.14) and (3.15) satisfied, it is possible to prove the local asymptotic stability of the origin of (3.6) by invoking Proposition 3.1.

1) Negativeness of $\dot{V}(x_t, \dot{x}_t)$:

Consider the augmented vector

$$\xi(t) = (\zeta(t), \phi(\chi(t)), \phi(\chi(t - \tau(t)))) \in \mathbb{R}^{n_\xi}, \quad n_\xi = 14n + 2n_\phi$$

where $\zeta = (\zeta_1, \ldots, \zeta_N)$, $\zeta_i(t) = (\dot{x}_i(t), x_i(t), \sigma_{hi}(t), \sigma_{\tau i}(t), \varsigma_{hi}(t), \varsigma_{\tau i}(t))$, and

$$\sigma_{hi}(t) = (x_i(t - h_0), x_i(t - h(t)), x_i(t - h_1)), \; \varsigma_{hi}(t) = (\psi_i(h_0, 0), \psi_i(h_0, h(t)), \psi_i(h(t), h_1)),$$
$$\sigma_{\tau i}(t) = (x_i(t - \tau_0), x_i(t - \tau(t)), x_i(t - \tau_1)), \; \varsigma_{\tau i}(t) = (\psi_i(\tau_0, 0), \psi_i(\tau_0, \tau(t)), \psi_i(\tau(t), \tau_1)).$$

The time-derivative of (3.16) can be expressed as

$$\begin{aligned}\dot{V}(x_t,\dot{x}_t) =& \sum_{i=1}^{N}\Big[\xi(t)^\top\Big(\mathrm{He}\left(G_{hi}^\top(h(t))P_{1i}J_{hi}\right)+\Sigma_{hi}+\mathrm{He}\left(G_{\tau i}^\top(\tau(t))P_{2i}J_{\tau i}\right)\\ &+\Sigma_{\tau i}+F_i^\top\left(h_0^2R_{1i}+h_{10}^2R_{2i}\right)F_i+F_i^\top\left(\tau_0^2R_{3i}+\tau_{10}^2R_{4i}\right)F_i\Big)\xi(t)\\ &-h_0\int_{t-h_0}^{t}\dot{x}_i^\top(s)R_{1i}\dot{x}_i(s)ds-h_{10}\int_{t-h_1}^{t-h_0}\dot{x}_i^\top(s)R_{2i}\dot{x}_i(s)ds\\ &-\tau_0\int_{t-\tau_0}^{t}\dot{x}_i^\top(s)R_{3i}\dot{x}_i(s)ds-\tau_{10}\int_{t-\tau_1}^{t-\tau_0}\dot{x}_i^\top(s)R_{4i}\dot{x}_i(s)ds\Big].\end{aligned}$$

being $\Sigma_{hi}=v_{i2}^\top Q_{1i}v_{i2}+v_{i3}^\top(Q_{2i}-Q_{1i})v_{i3}-v_{i5}^\top Q_{2i}v_{i5}$, and $\Sigma_{\tau i}=v_{i2}^\top Q_{3i}v_{i2}+v_{i6}^\top(Q_{4i}-Q_{3i})v_{i6}-v_{i8}^\top Q_{4i}v_{i8}$. From the Wirtinger-based integral inequality (Lemma C.5), and the delay-dependent reciprocally convex lemma (Lemma C.4), we have that it is possible to obtain the following upper-bound for the global functional

$$\dot{V}(x_t,\dot{x}_t)\le\xi^\top(t)\left[\sum_{i\in\mathcal{V}}\left(\Phi_{hi}(h)+\Phi_{\tau i}(\tau)+W_{hi}^\top\Xi_{hi}(h(t))W_{hi}+W_{\tau i}^\top\Xi_{\tau i}(\tau(t))W_{\tau i}\right)\right]\xi(t),$$

with $\mathcal{R}_{1i}=\mathrm{diag}(R_{1i},3R_{1i})$, $\mathcal{R}_{2i}=\mathrm{diag}(R_{2i},3R_{2i})$, $\mathcal{R}_{3i}=\mathrm{diag}(R_{3i},3R_{3i})$, $\mathcal{R}_{4i}=\mathrm{diag}(R_{4i},3R_{4i})$,

$$\Psi_{hi}(h)=\begin{bmatrix}\mathcal{R}_{2i}&0\\0&\mathcal{R}_{2i}\end{bmatrix}+\tfrac{h_1-h}{h_{10}}\begin{bmatrix}\mathcal{R}_{2i}&Y_{2i}\\{Y_{2i}}^T&0\end{bmatrix}+\tfrac{h-h_0}{h_{10}}\begin{bmatrix}0&Y_{1i}\\{Y_{1i}}^T&\mathcal{R}_{2i}\end{bmatrix},$$

$$\Xi_{hi}(h)=\begin{bmatrix}\frac{h_1-h}{h_{10}}Y_{1i}\mathcal{R}_{2i}^{-1}Y_{1i}^\top&0\\0&\frac{h-h_0}{h_{10}}Y_{2i}^\top\mathcal{R}_{2i}^{-1}Y_{2i}\end{bmatrix},$$

$$\Psi_{\tau i}(\tau)=\begin{bmatrix}\mathcal{R}_{4i}&0\\0&\mathcal{R}_{4i}\end{bmatrix}+\tfrac{\tau_1-\tau}{\tau_{10}}\begin{bmatrix}\mathcal{R}_{4i}&Y_{4i}\\Y_{4i}^\top&0\end{bmatrix}+\tfrac{\tau-\tau_0}{\tau_{10}}\begin{bmatrix}0&Y_{3i}\\Y_{3i}^\top&\mathcal{R}_{4i}\end{bmatrix},$$

$$\Xi_{\tau i}(\tau)=\begin{bmatrix}\frac{\tau_1-\tau}{\tau_{10}}Y_{3i}\mathcal{R}_{4i}^{-1}Y_{3i}^\top&0\\0&\frac{\tau-\tau_0}{\tau_{10}}Y_{4i}^\top\mathcal{R}_{4i}^{-1}Y_{4i}\end{bmatrix},$$

$$\Phi_{hi}(h)=\mathrm{He}(G_{hi}^\top(h)P_{1i}J_{hi})+F_i^\top\left(h_0^2R_{1i}+h_{10}^2R_{2i}\right)F_i+\Sigma_{hi}-Z_{hi}^\top\mathcal{R}_{1i}Z_{hi}-W_{hi}^\top\Psi_{hi}(h)W_{hi},$$

$$\Phi_{\tau i}(\tau)=\mathrm{He}(G_{\tau i}^\top(\tau)P_{2i}J_{\tau i}))+F_i^\top\left(\tau_0^2R_{3i}+\tau_{10}^2R_{4i}\right)F_i+\Sigma_{\tau i}-Z_{\tau i}^\top\mathcal{R}_{3i}Z_{\tau i}-W_{\tau i}^\top\Psi_{\tau i}(\tau)W_{\tau i}.$$

Considering the property of the sector-bounded nonlinearities (Lemma C.2), it follows from S-procedure arguments that

$$\dot{V}(x_t,\dot{x}_t)-2\sum_{i\in\mathcal{V}}\mathcal{S}_i(\chi_i(t),\Lambda_i)-2\sum_{i\in\mathcal{V}}\mathcal{S}_i(\chi_i(t-\tau(t)),\Lambda_i)<0. \tag{3.24}$$

With the previous definitions, it is possible to conclude that condition (3.24) is satisfied if $\xi^\top(t)\mathcal{Q}(h(t),\tau(t))\xi(t)<0$, where

$$\begin{aligned}\mathcal{Q}(h,\tau)&=\Phi(h,\tau)+\Pi_0+\Pi_\tau+\sum_{i\in\mathcal{V}}\left(W_{hi}^\top\Xi_{hi}(h)W_{hi}+W_{\tau i}^\top\Xi_{\tau i}(\tau)W_{\tau i}\right),\\ \Phi(h,\tau)&=\sum_{i\in\mathcal{V}}\left(\Phi_{hi}(h)+\Phi_{\tau i}(\tau)\right),\\ \Pi_0&=-2E_0^\top\Lambda^{-1}E_0+\mathrm{He}(E_0^\top\Lambda^{-1}\Omega(\mathbf{1}_N^\top\otimes N_0)),\\ \Pi_\tau&=-2E_\tau^\top\Lambda^{-1}E_\tau+\mathrm{He}(E_\tau^\top\Lambda^{-1}\Omega T).\end{aligned}$$

Moreover, it follows from the closed-loop dynamics in (3.6) that $\mathcal{B}(x)\xi(t) = 0$, being $\mathcal{B}(x) = -F + A_{cl}(x)N_0 + A_d(x)N_h + HN_\tau + B(x)M(x)E_0 + G(x)E_\tau$. Then, considering Lemma C.1 we have that

$$\mathcal{Q}(h(t), \tau(t)) + \mathcal{X}(\epsilon)\mathcal{B}(x) + \mathcal{B}^\top(x)\mathcal{X}(\epsilon)^\top < 0, \tag{3.25}$$

ensures the asymptotic stability of the closed-loop equilibrium with $\mathcal{X}(\epsilon) = \epsilon F^\top X^{-\top} + N_0^\top X^{-\top} + N_h^\top X^{-\top} + N_\tau^\top X^{-\top}$. As the condition in (3.25) is affine in h and τ, it is ensured if the following conditions obtained with the application of the Schur complement N times hold:

$$\begin{bmatrix} \Theta(x, h_w, \tau_k, \epsilon) & \mathcal{W}_h^\top \mathcal{Y}_{w+1} & \mathcal{W}_\tau^\top \mathcal{Y}_{k+3} \\ \star & -\mathcal{R}_2 & 0 \\ \star & \star & -\mathcal{R}_4 \end{bmatrix} < 0, \tag{3.26}$$

where $\Theta(x, h, \tau, \epsilon) = \Phi(h, \tau) + \Pi_0 + \Pi_\tau + \mathrm{He}(\mathcal{X}(\epsilon)\mathcal{B}(x))$, $w, k \in \mathbb{B}$, $\mathcal{R}_k = \mathrm{diag}(\mathcal{R}_{k1}, \ldots, \mathcal{R}_{kN})$, $k \in \{2, 4\}$, and the others matrices are as defined in Theorem 3.1. Let $\mathcal{U} = \mathrm{diag}(\mathcal{U}_1, I_2 \otimes \Lambda, \mathcal{U}_2)$, $\mathcal{U}_1 = \mathrm{diag}((I_{14} \otimes X_1), \ldots, (I_{14} \otimes X_N))$, $\mathcal{U}_2 = \mathrm{diag}(I_{4N} \otimes X_1), \ldots, (I_{4N} \otimes X_N))$. By multiplying (3.26) with $\mathcal{U}^\top$ on the left and $\mathcal{U}$ on the right, it results in

$$\Upsilon(x, h_w, \tau_k, \epsilon) \triangleq \begin{bmatrix} \widetilde{\Theta}(x, h_w, \tau_k, \epsilon) & \mathcal{W}_h^\top \widetilde{\mathcal{Y}}_{w+1} & \mathcal{W}_\tau^\top \widetilde{\mathcal{Y}}_{k+3} \\ \star & -\widetilde{\mathcal{R}}_2 & 0 \\ \star & \star & -\widetilde{\mathcal{R}}_4 \end{bmatrix} < 0, \tag{3.27}$$

for $w, k \in \mathbb{B}$. Note that the conditions in (3.27) are nonlinear since they are written in terms of the normalized membership functions. To recast a finite set of solvable LMI conditions, the relaxations developed in [234] are used, resulting in (3.23). Thus, if (3.21) hold, then (3.25) and (3.14) are satisfied as well, and the origin of the closed-loop system (3.6) is asymptotically stable.

2) $\mathscr{R}_0 \subset \mathscr{D}$:

By multiplying (3.22) by $\mathrm{diag}(X_i^{-\top}, 1)$ on the left and its transpose on the right yields

$$\begin{bmatrix} U_i^\top (P_{1i} + P_{2i}) U_i & \star \\ h_{i\ell_i} & 1 \end{bmatrix} \geq 0, \; \ell_i \in \mathbb{N}_{\leq n_{f_i}}, \, \forall i \in \mathcal{V}. \tag{3.28}$$

Then pre and post-multiplying (3.28) by $\begin{bmatrix} -x_i^\top & 1 \end{bmatrix}$ results in

$$x_i^\top U_i^\top (P_{1i} + P_{2i}) U_i x_i - x_i^\top h_{i\ell_i} - h_{i\ell_i} x_i + 1 \geq 0,$$

for all $\ell_i \in \mathbb{N}_{\leq n_e}$, $i \in \mathcal{V}$. These inequalities imply

$$x^\top P_0 x - x^\top h_l - h_l x + 1 \geq 0, \quad l \in \mathbb{N}_{\leq n_f}.$$

Since for all $x \in \mathscr{R}_0$ we have $x^\top P_0 x \leq 1$, which implies that $h_l x \leq 1$, and guarantees the inclusion $\mathscr{R}_0 \subset \mathscr{D}$. Thus, we have that (3.15) holds. This concludes the proof. □

Remark 3.6

Note that it is possible to construct particular cases considering the time-varying delays either in the interconnections or the subsystems' dynamics individually, as discussed as follows:

(i) Delay only in the interconnections:

Disregarding the time-varying delay $h(t)$, the closed-loop system is

$$\dot{x}(t) = A_{cl}(x)x(t) + Hx(t-\tau(t)) + G(x)\phi(\chi(t-\tau(t))) + B(x)M(x)\phi(\chi(t)),$$

where $A_{cl}(x)$ and the set of initial conditions are defined as in (3.6) *with $d = \tau_1$. Following the same reasoning presented in the proof of Theorem 3.1, the design condition can be obtained with a Lyapunov-Krasovskii functional candidate composed of the terms $V_2(x_t), V_4(x_t)$, and $V_6(x_t)$ of* (3.16).

(ii) Delay only in the subsystems' dynamics:

In this case, the closed-loop system can be expressed as

$$\dot{x}(t) = A_{cl}(x)x(t) + A_d x(t-h(t)) + G_{cl}(x)\phi(\chi(t)),$$

where $A_{cl}(x) \triangleq A(x) + B(x)(K(x) + Z) + H$, $G_{cl}(x) \triangleq G(x) + B(x)M(x)$, and the initial conditions defined with $d = h_1$. In this case, the corresponding set of LMIs can be obtained with a Lyapunov-Krasovskii functional candidate composed of the terms $V_1(x_t), V_3(x_t)$, and $V_5(x_t)$ of (3.16).

3.2.2 Enlargement of the Set of Admissible Initial Conditions

The previous result already guarantees that the system's closed-loop trajectories will remain enclosed in the estimated safe operation region. However, our goal in this section is to enlarge the estimated set of admissible initial conditions with the following optimization problem:

$$\min \ \mathcal{J} \triangleq \sum_{i=1}^{N} \sum_{j=1}^{10} \omega_j \ \text{trace}\left(E_{ij}\right) + \bar{\eta}\beta_i \tag{3.29}$$

$$\text{s.t.}: \begin{cases} \text{LMIs in (3.21), (3.22)}, \\ O_{ij} - E_{ij} \leq 0, \ O_{ij} \in \mathcal{O}_i, \ i \in \mathcal{V}, \ j \in \mathbb{N}_{\leq 10}, \\ \begin{bmatrix} X_i + X_i^\top - I_{n_i} & I_{n_i} \\ I_{n_i} & \beta_i I_{n_i} \end{bmatrix} \geq 0, \ i \in \mathcal{V}, \end{cases}$$

where E_{ij} are decision variables with appropriate dimensions, β_i are scalar variables, $\bar{\eta}$ is a scalar weight, ω_j are the scalar terms defined in Proposition 3.2, and $O_{ij} \in \mathcal{O}_i$ is the j-th matrix in the set

$$\mathcal{O}_i = \left\{ \tilde{P}_{1i}, \tilde{P}_{2i}, \tilde{Q}_{1i}, \tilde{Q}_{2i}, \tilde{Q}_{3i}, \tilde{Q}_{4i}, \tilde{R}_{1i}, \tilde{R}_{2i}, \tilde{R}_{3i}, \tilde{R}_{4i} \right\}.$$

The minimization of $\mathcal{J}$ tends to enlarge the radius of $\mathscr{X}_\delta$, since the minimization of the sum of $\mathrm{trace}(E_{ij})$ together with the constraints $O_j - E_{ij} \leq 0$ imply in the decreasing of the maximum eigenvalues of the matrices of the functional (3.16). The last constraints imply that $(X_i + X_i^\top - I_{n_i})^{-1} \leq \beta_i I_n$. Since $(X_i - I_{n_i})^\top (X_i - I_{n_i}) \geq 0$, then $(X_i^\top + X_i - I_{n_i})^{-1} \geq X^{-\top} X^{-1}$, from which it follows that $\beta_i I_{n_i} \geq X^{-\top} X^{-1}$. Inclusion of $\bar{\eta}\beta_i$ in the objective function $\mathcal{J}$ tends to reduce the eigenvalues of $X_i^{-\top} X^{-1}$. This is useful to further reduce the eigenvalues of the matrices of the functional since they are reconstructed performing a congruence transformation with $X_i^{-\top}$. Therefore, the optimization problem tends to reduce γ and ρ given in (3.19) and (3.20), respectively, which leads to the enlargement of (3.12) with δ given in (3.13). Using similar arguments, it is possible to conclude that the volume of the region $\mathscr{R}_0 \subset \mathscr{D}$ in (3.11) is also enlarged with $V_0(x)$ given in (3.17).

3.3 Numerical Examples

This section presents two numerical examples to illustrate the effectiveness of the proposed approach in designing distributed controllers for time-delay interconnected nonlinear systems

Example 1: To illustrate the importance of estimating the set of admissible initial conditions

Consider the interconnected system with two subsystems

$$\begin{aligned}\dot{x}_1(t) =& x_1(t) + 2x_1^3(t)\cos(2x_1(t)) + 3x_1(t-h(t)) - 3x_2(t-\tau(t)) \\ & + 0.5\arctan^5(x_1(t) - x_2(t-\tau(t))) + u_1(t), \\ \dot{x}_2(t) =& x_2(t) + 2x_2^3(t) - 3x_1(t-\tau(t)) - 3x_2(t-h(t)) \\ & + 0.5\arctan^5(x_2(t) - x_1(t-\tau(t))) + x_2(t)^2 u_2(t).\end{aligned}$$

Notice that the N-TS fuzzy modeling of this system is discussed in Appendix B. For the minimum and maximum allowable delays $\tau_0 = 0.05$, $\tau_1 = 0.20$, $h_0 = 0.05$, and $h_1 = 0.10$, the optimization problem (3.29) is solved by employing a grid search over the parameters ϵ and $\bar{\eta}$. The solution with $(\epsilon^*, \bar{\eta}^*) = (0.7, 3 \times 10^5)$ leads to the enlarged set of admissible initial conditions $\mathscr{X}_\delta$ with $\delta = 2.8621$. To illustrate the importance of estimating this set, the designed distributed control law was used in several simulations with different initial conditions. The closed-loop trajectories for each initial condition are depicted in Figure 3.1. The initial conditions were defined in terms of the three different regions discussed in Remark 3.4. These regions are represented as follows:

- The validity domain $\mathscr{D}$: Square box in black (——);
- Safe operating region $\mathscr{R}_0$: Ellipse in cyan (——);
- Set of admissible initial conditions $\mathscr{X}_\delta$: Circle in red (——).

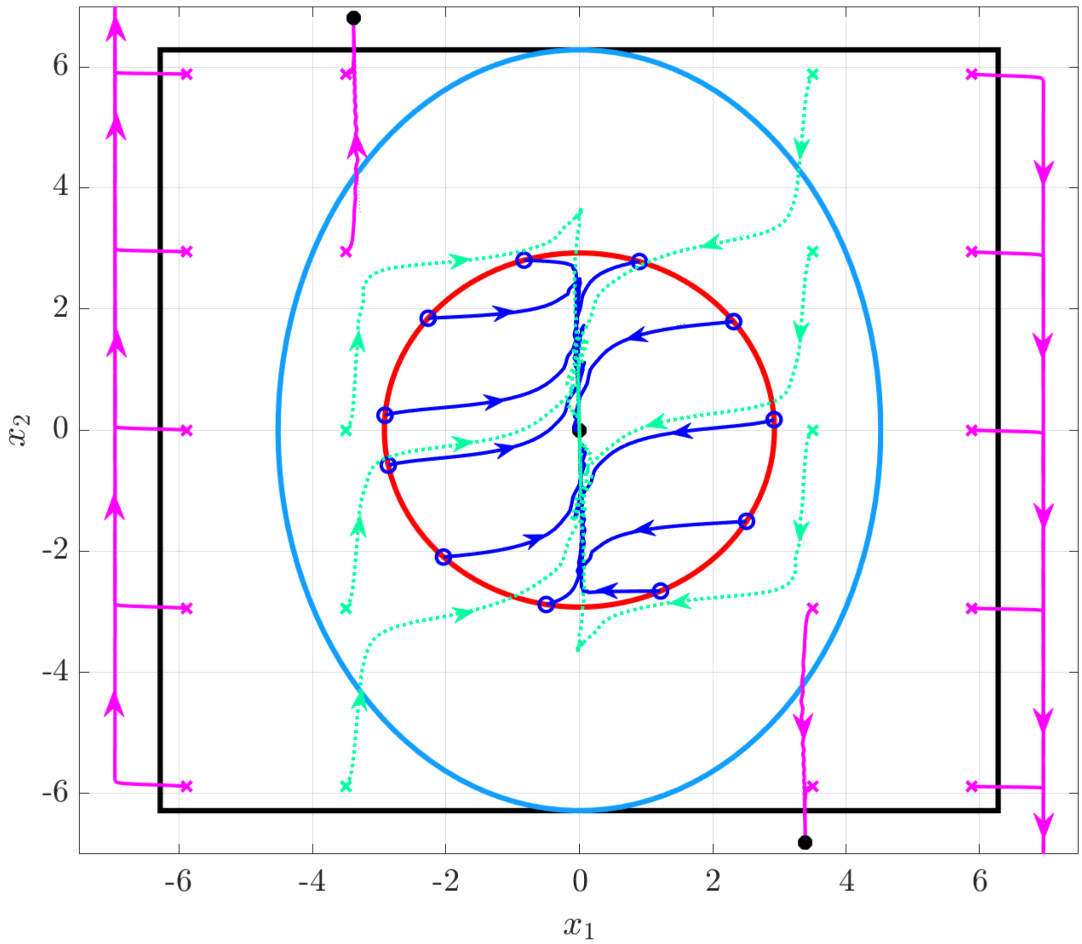

Figure 3.1 – The closed-loop trajectories for different initial conditions. The region of validity $\mathscr{D}$ is the square in black (——), the safe operating region $\mathscr{R}_0$ is the ellipse in cyan (——), and the set of admissible initial conditions $\mathscr{X}_\delta$ is the circle in red (——) - Example 1.

On the one hand, as expected, all the closed-loop trajectories depicted in blue (——), are initiating in $\mathscr{X}_\delta$ and asymptotically converging to the origin without leaving the validity domain $\mathscr{D}$. On the other hand, it is clearly illustrated that there is no guarantee that trajectories initiating outside of $\mathscr{X}_\delta$, will be constrained within $\mathscr{D}$ or converge to the origin. Notice that only the trajectories in green (······) converge to the origin, but the ones in magenta (——) leave the region of validity $\mathscr{D}$ and converge to other equilibrium points. Some of the trajectories in magenta (——) have initial conditions inside of $\mathscr{R}_0$, but since the convergence to the origin is not ensured in this set, the trajectories ended up leaving the validity domain $\mathscr{D}$.

Recall that the set of admissible initial conditions $\mathscr{X}_\delta$ is obtained by imposing the restrictions presented in Propositions 3.1 and 3.2. From the results presented, it is possible to conclude that this set is essential to ensure the correct operation of the closed-loop system, since otherwise there is no guarantee that the validity domain $\mathscr{D}$ is respected. Moreover, to the best of the authors' knowledge, there is no approach in the literature of time-delay interconnected nonlinear systems that also provides such a guarantee. Thus, a direct comparison of the results obtained is not possible.

Although the simulations were performed with the time-varying delays

$$\tau(t)=\frac{\tau_1+\tau_0}{2}+\frac{(\tau_1-\tau_0)\sin(40\pi t+\pi/2)}{2},\ h(t)=\frac{h_1+h_0}{2}+\frac{(h_1-h_0)\cos(40\pi t+\pi/2)}{2},$$

any time-varying function that respects the bounds of the minimum an maximum allowable delay can be considered. The initial condition for all cases is $\varphi(s)=x_0, \forall s\in[-d,0]$. Moreover, to illustrate the importance of the proposed optimization problem, notice that disregarding the procedure of section 3.2.2 the obtained solution leads to a set $\mathscr{X}_\delta$ with $\delta=0.0033$. Thus, it is possible to conclude that, with the optimization procedure, the radius of the admissible initial conditions set is enlarged 867.30 times.

Example 2: Interconnected Power Network

The aim of this example is to synchronize a power network composed of 7 generators. The interconnections among the generators are described according to the undirected graph in Figure 3.2.

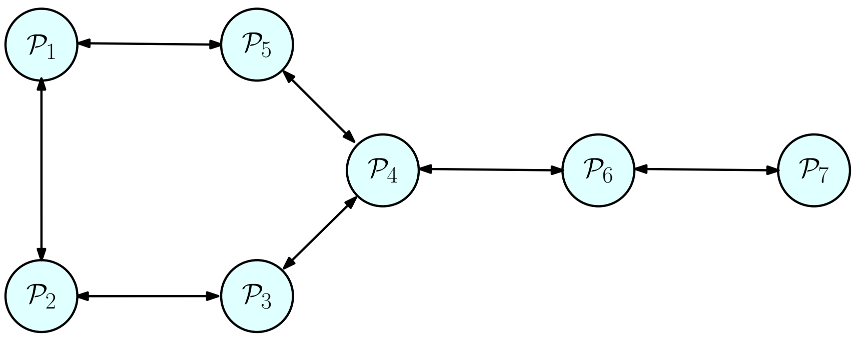

Figure 3.2 – Graph representing the interconnections among generators – Example 2.

The nonlinear dynamics of each generator is [235]:

$$\begin{aligned}
\dot{x}_{i1}(t) &= x_{i2}(t),\\
\dot{x}_{i2}(t) &= -\tfrac{D_i}{M_i}x_{i2}(t)-\tfrac{1}{M_i}x_{i3}(t)-\tfrac{1}{M_i}\sum_{j\in\mathcal{N}_i}\bar{\phi}_{ij}(x_i(t),x_j(t-\tau(t))),\\
\dot{x}_{i3}(t) &= -\tfrac{1}{T_i}x_{13}(t)+\tfrac{1}{T_i}x_{14}(t),\\
\dot{x}_{i4}(t) &= \tfrac{1}{K_i}x_{12}(t)-\tfrac{R_i}{K_i}x_{14}(t)+\tfrac{1}{K_i}u_i(t),
\end{aligned}$$

where the state variables $(x_{i1}, x_{i2}, x_{i3}, x_{i4})$ describe the phase angle difference, angular velocity difference, mechanical input difference, and valve position difference respectively. In this case, time-varying delays affect only the interconnections among the generators. The parameters M_i, D_i, T_i, K_i, and R_i are, respectively, the inertia constant, damping coefficient, turbine time constant, governor time constant, and droop characteristic. Their values are depicted in Table 3.1.

Table 3.1 – Parameters of the interconnected power network – Example 2

Subsystem	M_i	D_i	T_i	K_i	R_i
$\mathcal{P}_1$	0.5	1.2	0.1	1	0.1
$\mathcal{P}_2$	0.3	1.0	0.1	1	0.1
$\mathcal{P}_3$	1.0	0.3	0.1	1	0.1
$\mathcal{P}_4$	0.2	1.5	0.1	1	0.1
$\mathcal{P}_5$	0.9	0.9	0.1	1	0.1
$\mathcal{P}_6$	0.1	0.4	0.1	1	0.1
$\mathcal{P}_7$	0.4	0.9	0.1	1	0.1

Moreover, the sector-bound nonlinear interconnections are

$$\bar{\phi}_{ij}(x_i, x_j) = Y_{ij} \sin(x_{i1} - x_{j1}) \in \mathrm{co}\{\Omega_{Li}x, \Omega_{Ui}x\}, \tag{3.30}$$

where Y_{ij} is the admittance between the i-th and the j-th generator, $\Omega_{Li} = (2/\pi)\Omega_{Ui}$, and

$$\Omega_{Ui} = \begin{bmatrix} \Omega_{Ui1} & \Omega_{Ui2} & \dots & \Omega_{UiN} \end{bmatrix},$$

with $\Omega_{Uij} = \begin{bmatrix} \Omega_{Uijk_{i1}}^\top & \dots & \Omega_{Uijk_{i\ell}}^\top \end{bmatrix}^\top \in \mathbb{R}^{d_i \times n_j}$, being

$$\Omega_{Uijk_\ell} = \begin{cases} [Y_{ik_{i\ell}} \quad 0 \quad 0 \quad 0], & \text{if } j = i, \\ [-Y_{ik_{i\ell}} \quad 0 \quad 0 \quad 0], & \text{if } j = k_{i\ell}, \\ [0 \quad 0 \quad 0 \quad 0], & \text{otherwise} \end{cases}$$

for all $j \in \mathcal{V}$, $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\leq d_i}$. Notice that the sector in (3.30) is valid for all $\mathscr{D}_i = \{x_i \in \mathbb{R}^4 : |x_{i1}| \leq \pi/4\}$, $\forall i \in \mathcal{V}$. Although the sector-bound nonlinearities (3.30) do not satisfy Assumption 3.2, one can perform a simple loop transformation to obtain

$$\phi_{ij}(x_i, x_j) = \bar{\phi}_{ij}(x_i, x_j) - \Omega_{Li}x \in \mathrm{co}\{0, \Omega_i x\},$$

where $\Omega_i = (1 - 2/\pi)\Omega_{Ui}$. The admittances are $Y_{12} = 1.2819$, $Y_{15} = 0.7668$, $Y_{23} = 1.0752$, $Y_{34} = 0.8634$, $Y_{45} = 0.2443$, $Y_{46} = 0.5580$, and $Y_{67} = 0.3343$.

Consider that the minimum and maximum allowable delays are $\tau_0 = 0.05$ and $\tau_1 = 0.30$, respectively. Disregarding the optimization problem a set of admissible initial conditions

$\mathscr{X}_\delta$ with $\delta = 0.0050$ is obtained. Meanwhile, performing a grid search over the parameters $(\epsilon, \bar{\eta})$, a solution with $(\epsilon^*, \bar{\eta}^*) = (0.44531, 5.7243)$ leading to $\delta = 0.1134$, which results in an enlargement of 22.68 times of the set of admissible initial condition estimation. The time series of the angular positions $x_{i1}(t)$ and the control input signals of the local controllers $u_i(t)$, $\forall i \in \mathcal{V}$, are presented in Figure 3.3. The simulation is performed considering the initial condition $\varphi(s) = x_0$, $\forall s \in [-\tau_1, 0]$, with x_0 taken inside of $\mathscr{X}_\delta$. As expected, the trajectories converge asymptotically to the origin remaining inside of the validity domain.

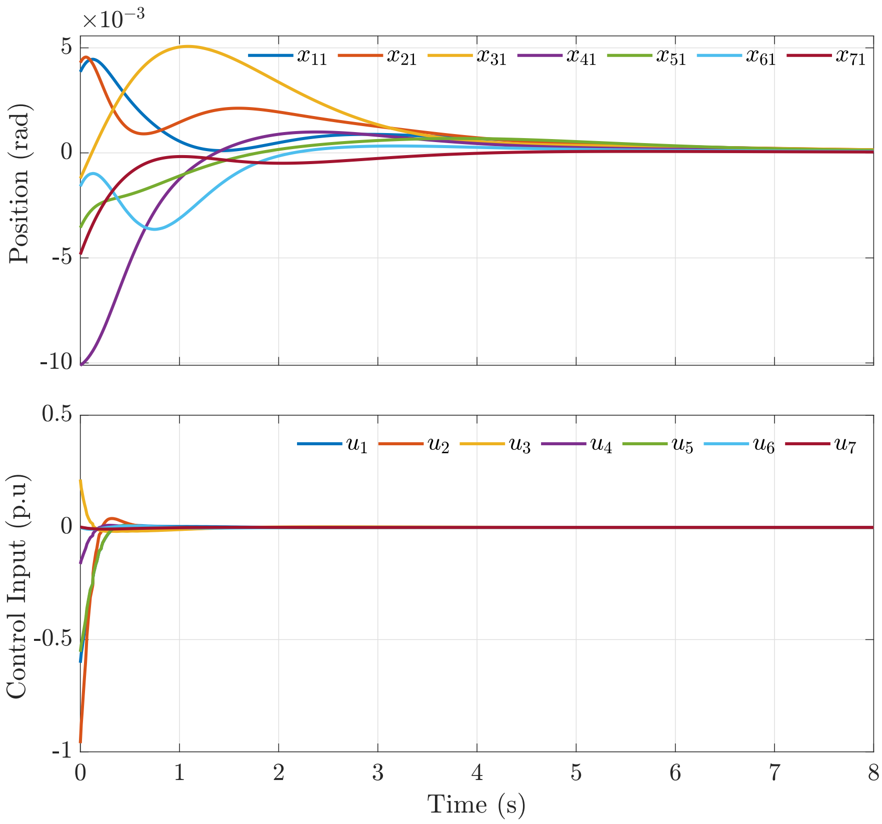

Figure 3.3 – Trajectories of the closed-loop phase angle difference of each generator for initial condition chosen inside of the estimated set $\mathscr{X}_\delta$ – Example 2.

The case when different time-varying delays are considered

To illustrate the capability of the proposed approach to deal with different time-varying delays, consider the results presented in Figure 3.4. In this experiment, each time-varying delay $\tau_{ij}(t)$ varies according to a specific trigonometric function that satisfies the bounds defined for $\tau(t)$, that is, $0.05 \leq \tau_{ij}(t) \leq 0.3$. Note that the designed distributed control law is capable of ensuring the synchronism of the power network. Thus, if the interval obtained for the general time-varying delay considered in the synthesis condition is respected, each time-varying delay can be modeled separately.

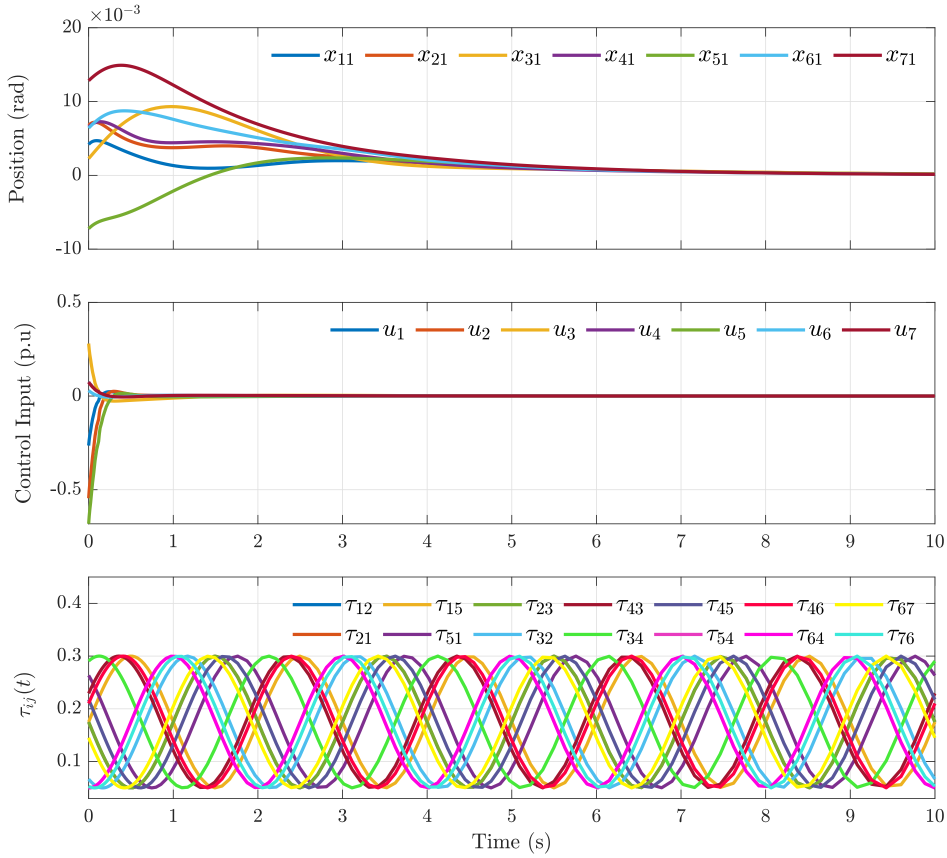

Figure 3.4 – Trajectories of the closed-loop phase angle difference of each generator for initial condition chosen inside of the estimated set $\mathscr{X}_\delta$ considering different delays τ_{ij} – Example 2.

3.4 Conclusion

This chapter addressed the distributed control of interconnected time-delay nonlinear systems with time-delayed interconnections. Considering an N-TS fuzzy representation of the interconnected system, a design condition has been proposed to design the distributed control law such that the origin of the closed-loop time-delay interconnected system is locally asymptotically stable. To ensure that closed-loop trajectories do not leave the validity domain of the N-TS fuzzy model, a characterization of the set of admissible initial conditions has been proposed. Numerical examples have been provided to illustrate the effectiveness of the proposed approach.

4 DISTRIBUTED EVENT-TRIGGERED CONTROL OF INTERCONNECTED NONLINEAR SYSTEMS

This chapter addresses the asynchronous distributed event-triggered control of continuous-time nonlinear interconnected systems. The nonlinear dynamics of the subsystems are represented by N-TS fuzzy models via the sector nonlinearity approach. Moreover, the nonlinear interconnections among the subsystems are assumed to be known and sector-bounded. In the ETC setup, where the state information is asynchronously available to the local controllers only in specific time instants, it is necessary to deal with the asynchronous premise variables once they introduce extra difficulties to derive suitable co-design conditions. To deal with this issue, it is proposed a new triggering strategy such that local ETMs are properly designed to counteract the effects of asynchronous premise variables. With this new distributed ETM, a co-design condition is proposed and the existence of a strictly positive minimum inter-event time is proved to exclude Zeno behavior. Moreover, a multi-objective optimization procedure is introduced to enlarge the estimate of the domain of attraction of the closed-loop equilibrium and minimize the number of transmissions provided by the ETMs. Finally, the approach is validated through a numerical example of the synchronization of interconnected oscillators.

The remainder of this chapter is organized as follows. The problem formulation is presented in §4.1. The proposed emulation condition and the optimization issues are defined in §4.2. A numerical example is presented in §4.3 to illustrate the application of the proposed methodology. Conclusions are drawn in §4.4.

4.1 Problem formulation

The Distributed Event-Triggered Control Setup

Consider a nonlinear interconnected system composed of a set of subsystems $\mathcal{P} = \{\mathcal{P}_1, \ldots, \mathcal{P}_N\}$. The goal is to design a distributed event-triggered controller composed of a set of local controllers $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_N\}$ and local event-triggered mechanisms (ETMs), such that the origin of the closed-loop system is asymptotically stable. The ETC setup is shown in Figure 4.1. In the considered setup, the state measurements of each subsystem, $x_i(t)$, are transmitted through the network in appropriate instants of time t_k^i, determined by local ETMs. The discrete signal $x_i(t_k^i)$ is sent to a zero-order hold (ZOH) mechanism resulting in a piecewise continuous signal $\hat{x}_i(t) = x_i(t_k^i)$, $\forall t \in [t_k^i, t_{k+1}^i)$, that is available to the local controllers.

Consider that each subsystem of the interconnected system $\mathcal{P}$ is described as

$$\mathcal{P}_i: \ \dot{x}_i = A_i(x_i)x_i + B_i(x_i)u_i + \sum_{j \in \mathcal{N}_i} g_{ij}(x_i, x_j), \tag{4.1}$$

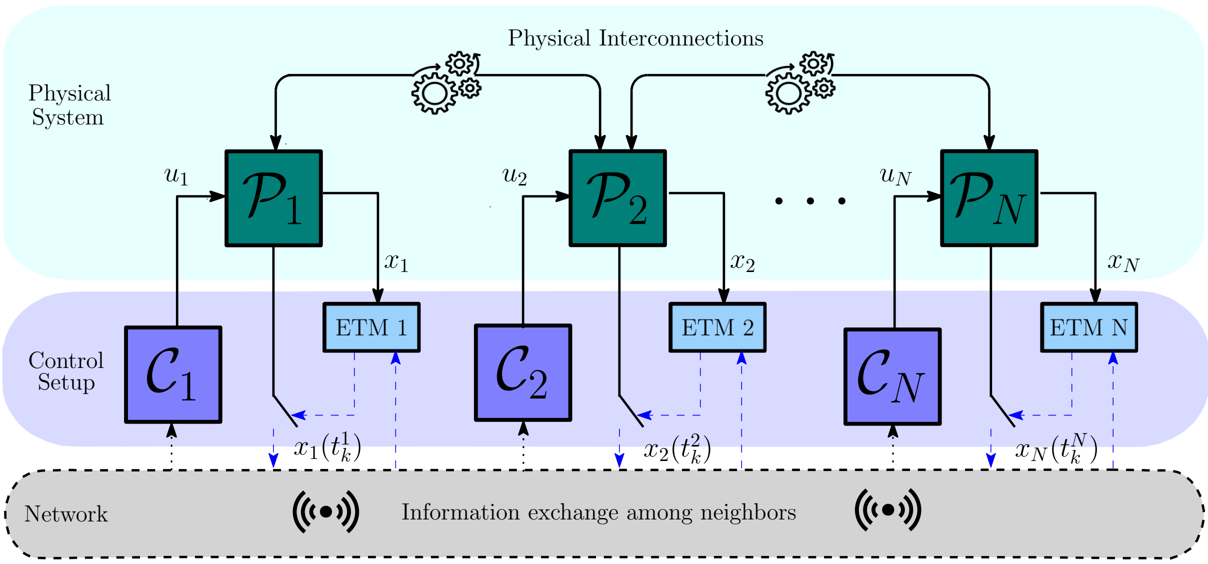

Figure 4.1 – Distributed ETC setup, where $\mathcal{P}_i$ are the subsystems, $\mathcal{C}_i$ are the local controllers, x_i are the continuous state measurements, $x_i(t_k^i)$ are the state measurements available to the controllers, and $u_i(t)$ are the distributed control inputs.

where $x_i \in \mathbb{R}^{n_i}$ is the state of the i-th subsystem, $u_i \in \mathbb{R}^{m_i}$ is the i-th control input, and

$$g_{ij}(x_i, x_j) = A_{ij}x_j + G_{ij}(x_i)\phi_{ij}(x_i, x_j), \tag{4.2}$$

is the function that represent the interconnection between $\mathcal{P}_i$ and the subsystem's $\mathcal{P}_j$ in the neighborhood $\mathcal{N}_i$ of $\mathcal{P}_i$. The interconnections among the subsystems are composed of sector-bounded functions ϕ_{ij} representing the nonlinear interconnections and matrices A_{ij} representing the linear interconnections.

Moreover, the proposed i-th event-based control law is

$$\mathcal{C}_i : u_i = K_i(\hat{x}_i)\hat{x}_i + \sum_{j \in \mathcal{N}_i} \mathcal{K}_{ij}(\hat{x}_i, \hat{x}_j), \tag{4.3}$$

where

$$K_i(\hat{x}_i) = \sum_{\mathbf{j}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{j}_i}(\hat{z}_i) K^i_{\mathbf{j}_i}$$

is the decentralized term of the local fuzzy controller, and the distributed term is

$$\mathcal{K}_{ij}(\hat{x}_i, \hat{x}_j) = L_{ij}\hat{x}_j + M_{ij}(\hat{x}_i)\phi_{ij}(\hat{x}_i, \hat{x}_j),$$

with $L_{ij} \in \mathbb{R}^{m_i \times n_j}$ and

$$M_{ij}(\hat{x}_i) = \sum_{\mathbf{j}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{j}_i}(\hat{z}_i) M^{ij}_{\mathbf{j}_i},$$

for all $j \in \mathcal{N}_i$.

Remark 4.1

Similar to Chapter 3, the dependency of matrix-value functions on the premise variables $z_i(x_i(t))$ *and* $z_i(\hat{x}_i(t))$ *is omitted. Thus, they are simply denoted by* $A_i(x_i)$, $B_i(x_i)$, $G_{ij}(x_i)$, $K_i(\hat{x}_i)$, *and* $M_{ij}(\hat{x}_i)$. *Moreover, the discussion performed in Remark 3.1 can be extended to the proposed event-based control law* (4.3).

It follows from $\mathcal{P}_i$ in (4.1), and $\mathcal{C}_i$ in (4.3), that the closed-loop dynamics of each subsystem is

$$\begin{aligned}\dot{x}_i = A_i(x_i)x_i + B_i(x_i)K_i(\hat{x}_i)(x_i + e_i) + \sum_{j\in\mathcal{N}_i} \left[A_{ij}x_j + B_i(x_i)L_{ij}(x_j + e_j)\right] \\ + \sum_{j\in\mathcal{N}_i} \left[G_{ij}(x_i)\phi_{ij}(x_i, x_j) + B_i(x_i)M_{ij}(\hat{x}_i)\phi_{ij}(\hat{x}_i, \hat{x}_j)\right],\end{aligned} \tag{4.4}$$

where

$$e_i(t) = \hat{x}_i - x_i, \quad \forall t \in [t^i_k, t^i_{k+1}), \tag{4.5}$$

is the transmission error that measures the deviation between the current state x_i and its value at the latest transmission $\hat{x}_i$. Notice that the closed-loop dynamics (4.4) reads

$$\begin{aligned}\dot{x}_i = A_i(x_i)x_i + B_i(x_i)K_i(x_i)(x_i + e_i) + B_i(x_i)(\delta_i + \xi_i) - G_i(x_i)\rho_i(e, x) \\ + \left[G_i(x_i) + B_i(x_i)M_i(x_i)\right]\phi_i(\hat{x}) + \sum_{j\in\mathcal{N}_i} \left[A_{ij}x_j + B_i(x_i)L_{ij}(x_j + e_j)\right],\end{aligned} \tag{4.6}$$

where $e = (e_1, \dots, e_N) \in \mathbb{R}^n$, $x = (x_1, \dots, x_N) \in \mathbb{R}^n$,

$$\begin{aligned}
&\phi_i(x) = (\phi_{ik_{i1}}(x_i, x_{k_1}), \dots, \phi_{ik_{id_i}}(x_i, x_{k_{d_i}})) \in \mathbb{R}^{d_i},\\
&\rho_i(e, x) = \phi_i(\hat{x}) - \phi_i(x), \quad \zeta_i = (x_i, e_i),\\
&\xi_i(\zeta_i, \zeta_j) = \left[M_i(\hat{x}_i) - M_i(x_i)\right]\phi_i(\hat{x}),\\
&\delta_i(\zeta_i) = \left[K_i(\hat{x}_i) - K_i(x_i)\right](x_i + e_i),\\
&M_i(x_i) = \begin{bmatrix} M_{ik_1}(x_i) & \cdots & M_{ik_{d_i}}(x_i)\end{bmatrix},\\
&G_i(x_i) = \begin{bmatrix} G_{ik_1}(x_i) & \cdots & G_{ik_{d_i}}(x_i)\end{bmatrix},
\end{aligned}$$

for all $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\leq d_i}$.

Notice that the collection of nonlinearities $\phi_i(x)$ is obtained considering all the nonlinear interconnections of the i-th subsystem. It is considered that the functions $\phi_i(x)$ in (4.6) and its partial derivatives bounded as established in the following assumption.

Assumption 4.1

The nonlinearities $\phi_i(x) : \mathbb{R}^n \to \mathbb{R}^{d_i}$, $i \in \mathcal{V}$, belongs to the sector $[0, \Omega_i]$, and its partial derivatives are bounded as

$$0 \leq \frac{\partial \phi_i(x)}{\partial x} \leq J_i,$$

inside a compact domain $\mathscr{D} \subset \mathbb{R}^n$.

Remark 4.2

The purpose of Assumption 4.1 is to exploit the sector properties presented in Lemmas C.2 and C.3. Notice that these conditions are not restrictive for practical uses once it is possible to perform loop transformations in the original nonlinearities. For instance, consider that the system (4.6) *is defined with* $\tilde{\phi}_i(x) \in [\Omega_{Li}, \Omega_{Ui}]$. *Applying a loop transformation, it is possible to obtain* $\bar{\phi}_i(x) = \tilde{\phi}_i(x) - \Omega_{Li}x \in [0, \Omega_i]$, *with* $\Omega_i = \Omega_{Ui} - \Omega_{Li}$. *Similarly, if* $J_{Li} \leq \frac{\partial \bar{\phi}_i(x)}{\partial x} \leq J_{Ui}$, *another loop transformation can be performed to obtain* $\phi_i(x) = \bar{\phi}_i(x) - J_{Li}x$, *with* $J_i = J_{Ui} - J_{Li}$.

Interconnected Closed-Loop System

The global interconnected system $(\mathcal{P}, \mathcal{C})$ can be written as

$$\dot{x} = A_{\text{cl}}(x)x + G_{\text{cl}}(x)\phi(\hat{x}) + B(x)(K(x) + \bar{L})e + B(x)(\delta + \xi) - G(x)\rho(x, e), \tag{4.7}$$

where $u = (u_1, \ldots, u_N) \in \mathbb{R}^m$, $\zeta = (\zeta_1, \ldots, \zeta_N) \in \mathbb{R}^{2n}$, $z = (z_1, \ldots, z_N) \in \mathbb{R}^p$, $\phi(x) = (\phi_1(x), \ldots, \phi_N(x)) \in \mathbb{R}^{n_\phi}$, $\rho(e, x) = (\rho_1(e, x), \ldots, \rho_N(e, x)) \in \mathbb{R}^{n_\phi}$,

$$\begin{aligned}
&A_{\text{cl}}(x) = A(x) + B(x)K(x) + \bar{A} + B(x)\bar{L}, \quad G_{\text{cl}}(x) = G(x) + B(x)M(x),\\
&M(x) = \text{diag}(M_1(x_1), \ldots, M_N(x_N)), \quad A(x) = \text{diag}(A_1(x_1), \ldots, A_N(x_N)),\\
&B(x) = \text{diag}(B_1(x_1), \ldots, B_N(x_N)), \quad K(x) = \text{diag}(K_1(x_1), \ldots, K_N(x_N))\\
&G(x) = \text{diag}(G_1(x_1), \ldots, G_N(x_N)), \quad \delta(\zeta) = [K(x+e) - K(x)](x+e),\\
&\xi(\zeta) = [M(x+e) - M(x)]\phi(x+e),
\end{aligned}$$

The elements of $\bar{A} = [\bar{A}_{ij}]$ and $\bar{L} = [\bar{L}_{ij}]$, for all $i, j \in \mathcal{V}$, are defined in terms of the adjacency matrix $\mathcal{A}$ (see Appendix A) as follows:

$$\bar{A}_{ij} = \begin{cases} 0 & \text{if } a_{ij} = 0, \\ A_{ij} & \text{if } a_{ij} = 1, \end{cases} \quad \bar{L}_{ij} = \begin{cases} 0 & \text{if } a_{ij} = 0, \\ L_{ij} & \text{if } a_{ij} = 1. \end{cases}$$

Based on the N-TS fuzzy modeling of the subsystems, it is possible to write the matrix-valued functions of the interconnected closed-loop system (4.7) as:

$$\begin{aligned}
A(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(x)A_{\mathbf{i}}, \quad B(x) = \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(x)B_{\mathbf{i}},\\
G(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(x)G_{\mathbf{i}}, \quad K(x) = \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(x)K_{\mathbf{i}},\\
M(x) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \alpha_{\mathbf{i}}(x)M_{\mathbf{i}},
\end{aligned} \tag{4.8}$$

for all $x \in \mathscr{D}$, where $\mathbf{i} = (\mathbf{i}_1, \mathbf{i}_2, \ldots, \mathbf{i}_N)$ is a multi-dimensional index on $\mathbb{B}^p$, being $\mathbb{B}^p = \mathbb{B}^{p_1} \times \cdots \times \mathbb{B}^{p_N}$, $\alpha_{\mathbf{i}}(x) = \alpha_{\mathbf{i}_1}(x_1) \cdots \alpha_{\mathbf{i}_N}(x_N)$ are the membership functions, and

$$\mathscr{D} = \{x \in \mathbb{R}^n : h_l^\top x \leq 1, \, l \in \mathbb{N}_{\leq n_f}\}, \tag{4.9}$$

with $h_l = [0_{1\times \underline{n}_i} \, h_{i\ell_i}^\top \, 0_{1\times \overline{n}_i}]^\top$, and $l = 2(i-1) + \ell_i$, $\ell_i \in \mathbb{N}_{\leq n_{fi}}$, $i \in \mathcal{V}$. Recall that the model (4.7) is valid only for $x \in \mathscr{D}$ (the compact set containing the origin $x = 0$).

The Local Event-Triggered Control Scheme

In this paper, event-based communication is considered to save communication resources. It is assumed that the transmissions are performed asynchronously, that is, the state information of each subsystem is updated independently. The sequence of transmissions of each subsystem is scheduled according to the following event-triggering mechanism:

$$t_0^i = 0, t_{k+1}^i = \inf\{t > t_k^i : \Gamma_i(\zeta_i(t), \zeta_j(t)) < 0\},\ i \in \mathcal{V}, \tag{4.10}$$

and the trigger function is given by

$$\Gamma_i(\zeta_i, \zeta_j) = \zeta_i^\top \Psi_i \zeta_i - \mu_i(\zeta_i, \zeta_j), \tag{4.11}$$

where

$$\Psi_i = \begin{bmatrix} \Psi_x^i & \Psi_{xe}^i \\ \star & -\Psi_e^i \end{bmatrix}, \tag{4.12}$$

is a symmetric matrix to be designed, with $\Psi_x^i \in \mathbb{R}^{n_i \times n_i}$ and $\Psi_e^i \in \mathbb{R}^{n_i \times n_i}$ being symmetric and positive definite matrices. Moreover, the function introduced to cope with the internal perturbation caused by the asynchronous phenomenon is

$$\mu_i(\zeta_i, \zeta_j) = 2x_i^\top P_i^{-1} B_i(x_i)\Big(\delta_i(\zeta_i) + \xi_i(\zeta_i, \zeta_j)\Big), \tag{4.13}$$

with $P_i \in \mathbb{R}^{n_i \times n_i}$ being symmetric and positive definite matrices, and $\delta_i(\zeta_i)$, $\xi_i(\zeta_i, \zeta_j)$ as defined in (4.6).

Notice that between event times, the states $\hat{x}_i$ and x_i are mismatched. Due to this difference, the state-dependent membership functions $\alpha_{\mathbf{i}_i}(x_i)$ and $\alpha_{\mathbf{i}_i}(\hat{x}_i)$ are asynchronous. Consequently, the usual Linear Matrix Inequalities (LMIs) relaxations cannot be employed for the linearization of terms such as $B(x_i)K(\hat{x}_i)$ and $B(x_i)M(\hat{x}_i)$ once the normalized membership functions of the matrix-valued functions are not the same. This problem is generally addressed by limiting the deviation bounds of the membership functions [205, 230], or assuming partitions in the state space [208, 232]. However, these assumptions may introduce conservativeness into the co-design conditions. To deal with this phenomenon in the distributed control co-design for nonlinear interconnected systems, this paper considers a cancellation-based strategy [236], in which the inclusion of the term μ_i in the triggering mechanism allows the development of conditions in the form of LMIs that do not impose any restriction on the membership functions. Moreover, since the proposed control input (4.3) is distributed, there is another asynchronism between the nonlinear interconnections $\phi_{ij}(x_i, x_j)$ and $\phi_{ij}(\hat{x}_i, \hat{x}_j)$. The proposed approach is also capable of dealing with the influence of this additional source of asynchronism.

Remark 4.3

Notice that although the first part of $\Gamma_i(\zeta_i, \zeta_j)$ in (4.11) given by $\zeta_i^\top \Psi_i \zeta_i$ is standard in the general literature of ETC, the proposed ETM in (4.10) provides characteristics that have not been explored so far in the literature of distributed ETC of continuous-time interconnected nonlinear systems. The main novelty is the inclusion in $\Gamma_i(\zeta_i, \zeta_j)$ of the new term $\mu_i(\zeta_i, \zeta_j)$ defined in (4.13) to cope with the internal perturbation caused by the asynchronous phenomenon.

Based on the previous definitions, the problem addressed in this chapter is stated as follows.

Problem 4.1

Consider a nonlinear interconnected system with subsystems described as (4.1) and an event-triggered distributed control policy with local control laws as in (4.3) whose sequences of transmissions is generated by the local ETMs (4.10)-(4.13). Design the gains $K_i(\hat{x}_i)$, L_{ij}, and $M_{ij}(\hat{x}_i)$ of (4.3), and the trigger parameters Ψ_i of (4.10), $\forall\, i \in \mathcal{V}$, $j \in \mathcal{N}_i$, such that the closed-loop system (4.7) is locally asymptotically stable. Moreover, provide an estimate of the domain of attraction $\mathscr{R} = \{x \in \mathbb{R}^n : x^\top P^{-1} x \leq 1\}$, $P = P^\top > 0$, to ensure that closed-loop trajectories starting in $\mathscr{R} \subset \mathscr{D}$ do not evolve outside the domain $\mathscr{D}$, and converge asymptotically to the origin.

4.2 Main Results

4.2.1 Excluding Zeno behavior

This analysis aims to prove that the triggering strategy proposed in (4.10) does not lead to Zeno behavior. The existence of a strictly positive minimum inter-event time (MIET) is ensured by the following Lemma.

Lemma 4.1

Consider the global closed-loop interconnected system (4.7). Given the local ETMs as in (4.10)-(4.13), there exists a MIET $\tau_i^* \in \mathbb{R}_{>0}$ such that $t_{k+1}^i - t_k^i \geq \tau_i^*$.

Proof. From (4.10), transmissions are triggered when

$$\mathcal{G}_i(x_i, e_i) > 1 - \mathcal{V}_i(x_i, e_i),$$

with

$$\mathcal{G}_i(x_i, e_i) = \frac{e_i^\top \Psi_e^i e_i}{x_i^\top \Psi_x^i x_i}, \quad \mathcal{V}_i(x_i, e_i) = \frac{\mu_i(\zeta_i, \zeta_j) - 2x_i^\top \Psi_{xe}^i e_i}{x_i^\top \Psi_x^i x_i}.$$

Notice that at $t = t_k^i$, one has $e_i = \mu_i(\zeta_i, \zeta_j) = 0$. Consequently, $\mathcal{G}_i(x_i, e_i) = \mathcal{V}_i(x_i, e_i) = 0$. Meanwhile, for $t \in [t_k^i, t_{k+1}^i)$, that is, before a new event is triggered, $\mathcal{G}_i(x_i, e_i) \geq 0$ must evolve from 0 to $1-\mathcal{V}_i(x_i, e_i)$. Therefore, it is possible to conclude that $\mathcal{V}_i(x_i, e_i) < 1, \; \forall t \in [t_k^i, t_{k+1}^i)$.

Defining $\mathcal{Z}_i = \frac{\bar{\lambda}(\Psi_e^i)}{\underline{\lambda}(\Psi_x^i)}$, one has $\mathcal{G}_i(x_i, e_i) \leq \mathcal{Z}_i \frac{||e_i(t)||^2}{||x_i(t)||^2}$. Thus, in the worst case, transmissions are not triggered while

$$\frac{||e_i(t)||}{||x_i(t)||} \leq \frac{1}{\sqrt{\mathcal{Z}_i}} \sqrt{1 - \mathcal{V}_i(x_i, e_i)}. \tag{4.14}$$

By following the same steps presented in [237], the dynamics of $\frac{||e_i(t)||}{||x_i(t)||}$ is bounded as follows

$$\frac{d}{dt}\left(\frac{||e_i(t)||}{||x_i(t)||}\right) \leq \frac{||\dot{e}_i(t)||}{||x_i(t)||} + \frac{||e_i(t)|| \; ||\dot{x}_i(t)||}{||x_i(t)||^2}. \tag{4.15}$$

Considering the closed-loop dynamics in (3.5) and provided that the state-dependent matrices are bounded for all $x_i, \hat{x}_i \in \mathscr{D}_i$, one has that

$$||\dot{e}_i(t)|| = ||\dot{x}_i(t)|| \leq C_1||x_i(t)|| + C_2||e_i(t)|| + \mathcal{W}_i(x, e),$$

with $C_1 = ||A_i(x_i) + B_i(x_i)K_i(\hat{x}_i)||, \; C_2 = ||B_i(x_i)K_i(\hat{x}_i)||$,

$$\begin{aligned} \mathcal{W}_i(x, e) = \Bigg\| \sum_{j \in \mathcal{N}_i} & A_{ij}x_j + B_i(x_i)L_{ij}(x_j + e_j) \\ & + G_{ij}(x_i)\phi_{ij}(x_i, x_j) + B_i(x_i)M_{ij}(\hat{x}_i)\phi_{ij}(\hat{x}_i, \hat{x}_j)\Big\| \end{aligned}$$

Then, substituting the last results in (4.15) leads to

$$\frac{d}{dt}\left(\frac{||e_i(t)||}{||x_i(t)||}\right) \leq \frac{C_1||x_i(t)|| + C_2||e_i(t)|| + \mathcal{W}_i(x, e)}{||x_i(t)||} \tag{4.16}$$

$$+ \frac{||e_i(t)|| \, (C_1||x_i(t)|| + C_2||e_i(t)|| + \mathcal{W}_i(x, e))}{||x_i(t)||^2}. \tag{4.17}$$

Defining $\widetilde{\mathcal{W}_i}(x, e) = \frac{\mathcal{W}_i(x,e)}{||x_i(t)||}$, $s_i = \sup_{x \in \mathcal{D}, \, e \in \mathcal{D}_e} \widetilde{\mathcal{W}_i}(x, e) \in \mathbb{R}_{\geq 0}$, and $\varphi_i(t) = \frac{||e_i(t)||}{||x_i(t)||}$, it is possible to write the estimate

$$\dot{\varphi}_i(t) \leq \nu_0^i + \nu_1^i \varphi_i(t) + \nu_2^i \varphi_i^2(t),$$

with $\nu_0^i = C_1 + s_i$, $\nu_1^i = C_1 + C_2 + s_i$, and $\nu_2^i = C_2$, from which $\varphi_i(t) \leq \psi_i(t, \psi_0^i)$, where $\psi_i(t, \psi_0^i)$ is the solution of the initial value problem $\dot{\psi}_i(t) = \nu_0^i + \nu_1^i \psi_i(t) + \nu_2^i \psi_i^2(t)$, with $\psi_i(0, \psi_0^i) = \psi_0^i$, being ν_0^i, ν_1^i, and ν_2^i non-negative constants. By following the same arguments of [236], we have that

1. $0 \leq \mathcal{V}_i(x_i, e_i) < 1$: Since $\mathcal{G}_i(x_i, e_i) < 1$, we have that $\mathcal{G}_i(x_i, e_i)$ takes more time to evolve from 0 to $1 - \mathcal{V}_i(x_i, e_i)$ than $\psi_i(t, 0)$ to reach $\frac{1}{\sqrt{\mathcal{Z}_i}}\sqrt{1 - \mathcal{V}_i(x_i, e_i)}$ for the first time.

2. $\mathcal{V}_i(x_i, e_i) < 0$: In this case, as $1 - V_i(x_i, e_i) > 1$, $\psi_i(t, 0)$ takes less time to evolve from 0 to $\frac{1}{\sqrt{\mathcal{Z}_i}}$ than to $\frac{1}{\sqrt{\mathcal{Z}_i}}\sqrt{1 - \mathcal{V}_i(x_i, e_i)}$. Therefore, $\mathcal{G}_i(x_i, e_i)$ requires less time to evolve from 0 to 1 than $\psi_i(t, 0)$ to reach $\frac{1}{\sqrt{\mathcal{Z}_i}}$.

Thus, it is possible to conclude that the inter-event times are bounded by the time that ψ_i takes to evolve from 0 to $\frac{\bar{\mathcal{Z}}_i}{\sqrt{\mathcal{Z}_i}}$, with $\bar{\mathcal{Z}}_i = \min(1, \sqrt{1 - \mathcal{V}_i(x_i, e_i)})$. The solution of

$$\int \frac{\mathrm{d}\psi_i}{\nu_0^i + \nu_1^i \psi_i(t) + \nu_2^i \psi_i(t)^2} = \int \mathrm{d}t,$$

is computed according to the following cases

- Case 1: $(\nu_1^i)^2 - 4\nu_0^i\nu_2^i = 0$,
- Case 2: $(\nu_1^i)^2 - 4\nu_0^i\nu_2^i < 0$,
- Case 3: $(\nu_1^i)^2 - 4\nu_0^i\nu_2^i > 0$.

The inter-event times are then bounded by the solution $\tau_i^* \in \mathbb{R}_{>0}$ of $\psi_i(\tau_i^*, 0) = \frac{\bar{\mathcal{Z}}_i}{\sqrt{\mathcal{Z}_i}}$ obtained for each case:

- Case 1:

$$\tau_i^* = \frac{4\bar{\mathcal{Z}}_i \nu_2^i}{2\nu_2^i \nu_1^i \bar{\mathcal{Z}}_i + (\nu_1^i)^2 \sqrt{\mathcal{Z}_i}},$$

- Case 2:

$$\tau_i^* = \frac{2\arctan\left(\frac{\bar{\mathcal{Z}}_i(4\nu_0^i\nu_2^i - (\nu_1^i)^2)}{(\nu_1^i\bar{\mathcal{Z}}_i + 2\nu_0^i\sqrt{\mathcal{Z}_i})\sqrt{(4\nu_0^i\nu_2^i - (\nu_1^i)^2)}}\right)}{\sqrt{(4\nu_0^i\nu_2^i - (\nu_1^i)^2)}},$$

- Case 3:

$$\tau_i^* = \frac{\ln\left(\frac{\bar{\mathcal{Z}}_i\nu_1^i + \bar{\mathcal{Z}}_i\sqrt{{\nu_1^i}^2 - 4\nu_2^i\nu_0^i} + 2\bar{\mathcal{Z}}_i\nu_0^i}{\bar{\mathcal{Z}}_i\nu_1^i - \bar{\mathcal{Z}}_i\sqrt{{\nu_1^i}^2 - 4\nu_2^i\nu_0^i} + 2\bar{\mathcal{Z}}_i\nu_0^i}\right)}{\sqrt{{\nu_1^i}^2 - 4\nu_2^i\nu_0^i}},$$

which is not null for all cases, thus ensuring $t_{k+1}^i - t_k^i \geq \tau_i^* > 0$, and excluding the existence of Zeno behavior. This concludes the proof. □

4.2.2 Co-design Condition

A sufficient condition to design the control laws (4.3) and the ETMs (4.10)-(4.13) is stated as follows.

Theorem 4.1

Consider the system (4.7) and assume that Assumption 4.1 is satisfied for given Ω_i, $J_i \in \mathbb{R}^{d_i \times n}$. If there exists symmetric positive definite matrices P_i, $\widetilde{\Psi}_x^i$, $\widetilde{\Psi}_e^i \in \mathbb{R}^{n_i \times n_i}$, matrices $\widetilde{K}_{\mathbf{j}} \in \mathbb{R}^{m \times n}$, $\widetilde{M}_{\mathbf{j}} \in \mathbb{R}^{m \times n_\phi}$, $\mathbf{j} \in \mathbb{B}^p$, $\widetilde{\Psi}_{xe}^i \in \mathbb{R}^{n_i \times n_i}$, and $\widetilde{L} \in \mathbb{R}^{m \times n}$, and diagonal matrices

$\Lambda_{s_i}, \Lambda_{h_i} \in \mathbb{R}^{d_i \times d_i}$ such that

$$\sum_{(\mathbf{i},\mathbf{j})\in\mathscr{P}(\mathbf{m},\mathbf{n})} \Upsilon_{\mathbf{ij}} < 0,\ \forall \mathbf{m},\mathbf{n} \in \mathbb{B}^{p+}, \tag{4.18}$$

$$\begin{bmatrix} P & P^\top h_\ell \\ \star & 1 \end{bmatrix} \geq 0, \quad \forall \ell \in \mathbb{N}_{\leq n_f}, \tag{4.19}$$

with

$$\Upsilon_{\mathbf{ij}} = \begin{bmatrix} \widetilde{\Theta}_{\mathbf{ij}} & \star \\ P\chi_1 & -\widetilde{\Psi}_x \end{bmatrix},$$

$$\begin{aligned}
\widetilde{\Theta}_{\mathbf{ij}} = {} & \chi_1{}^\top \left(\mathrm{He}\left(A_{\mathbf{i}}P + B_{\mathbf{i}}\widetilde{K}_{\mathbf{j}} + \bar{A}P + B_{\mathbf{i}}\widetilde{L}\right)\right)\chi_1 + \mathrm{He}\left(\chi_1^\top\left(G_{\mathbf{i}}\Lambda_s + B_{\mathbf{i}}\widetilde{M}_{\mathbf{j}}\right)\chi_3\right) \\
& + \mathrm{He}\left(\chi_1^\top\left(B_{\mathbf{i}}(\widetilde{L} + \widetilde{K}_{\mathbf{j}}) + \widetilde{\Psi}_{xe}\right)\chi_2\right) - \chi_2{}^\top\widetilde{\Psi}_e\,\chi_2 - 2\chi_3{}^\top\Lambda_s\,\chi_3 - 2\chi_4{}^\top\Lambda_h\,\chi_4 \\
& - \mathrm{He}\left(\chi_1^\top\left(G_{\mathbf{i}}\Lambda_h\right)\chi_4\right) + \mathrm{He}\left(\chi_3^\top\Omega(I_N \otimes P)(1_N \otimes \chi_1)\right) \\
& + \mathrm{He}\left(\chi_3^\top\Omega(I_N \otimes P)(1_N \otimes \chi_2)\right) + \mathrm{He}\left(\chi_4^\top J(I_N \otimes P)(1_N \otimes \chi_2)\right), \\
\chi_1 = {} & \begin{bmatrix} I_{n\times n} & 0_{n\times n} & 0_{n\times n_\phi} & 0_{n\times n_\phi} \end{bmatrix}, \\
\chi_2 = {} & \begin{bmatrix} 0_{n\times n} & I_{n\times n} & 0_{n\times n_\phi} & 0_{n\times n_\phi} \end{bmatrix}, \\
\chi_3 = {} & \begin{bmatrix} 0_{n_\phi\times n} & 0_{n_\phi\times n} & I_{n_\phi\times n_\phi} & 0_{n_\phi\times n_\phi} \end{bmatrix}, \\
\chi_4 = {} & \begin{bmatrix} 0_{n_\phi\times n} & 0_{n_\phi\times n} & 0_{n_\phi\times n_\phi} & I_{n_\phi\times n_\phi} \end{bmatrix}, \\
\widetilde{\Psi}_{xe} = {} & \mathrm{diag}(\widetilde{\Psi}^1_{xe}, \ldots, \widetilde{\Psi}^N_{xe}),\ \ \widetilde{\Psi}_x = \mathrm{diag}(\widetilde{\Psi}^1_x, \ldots, \widetilde{\Psi}^N_x),\ \ \widetilde{\Psi}_e = \mathrm{diag}(\widetilde{\Psi}^1_e, \ldots, \widetilde{\Psi}^N_e), \\
P = {} & \mathrm{diag}(P_1, \ldots, P_N),\ \ \Lambda_s = \mathrm{diag}(\Lambda_{s_1}, \ldots, \Lambda_{s_N}),\ \ \Lambda_h = \mathrm{diag}(\Lambda_{h_1}, \ldots, \Lambda_{h_N}), \\
\Omega = {} & \mathrm{diag}(\Omega_1, \ldots, \Omega_N),\ J = \mathrm{diag}(J_1, \ldots, J_N),
\end{aligned}$$

then the distributed controllers (4.3) with $K_{\mathbf{j}} = \widetilde{K}_{\mathbf{j}}P^{-1}$, $M_{\mathbf{j}} = \widetilde{M}_{\mathbf{j}}\Lambda_s{}^{-1}$, $\bar{L} = \widetilde{L}P^{-1}$, and the ETMs (4.10)-(4.13) with $\Psi^i_e = P_i^{-1}\widetilde{\Psi}^i_e P_i^{-1}$, $\Psi^i_{xe} = P_i^{-1}\widetilde{\Psi}^i_{xe}P_i^{-1}$, and $\Psi^i_x = \widetilde{\Psi}^{i-1}_x$, guarantee that the state trajectories of the closed-loop system (4.7) with initial conditions taken inside of

$$\mathscr{R} = \{x \in \mathbb{R}^n : V(x) \leq 1\}, \tag{4.20}$$

with Lyapunov function given by,

$$V(x) = x^\top P^{-1} x, \tag{4.21}$$

do not leave $\mathscr{R} \subset \mathscr{D}$, and converge asymptotically to the zero equilibrium.

Proof. First, note that all P_i, $i \in \mathbb{N}_N$ are positive definite matrices. Thus, it is possible to conclude that $V(x)$ as in (4.21) is positive definite.

Furthermore, assume that the conditions in (4.18) are feasible. From the convex property of the membership functions, the LMIs in (4.18) imply

$$\sum_{\mathbf{m}\in\mathbb{B}^{p+}}\sum_{\mathbf{n}\in\mathbb{B}^{p+}}\alpha_{\mathbf{m}}(x)\alpha_{\mathbf{n}}(x)\Bigg(\sum_{(\mathbf{i},\mathbf{j})\in\mathscr{P}(\mathbf{m},\mathbf{n})}\Upsilon_{\mathbf{ij}}\Bigg)<0. \tag{4.22}$$

Multiplying (4.22) by $\mathrm{diag}(P^{-1},P^{-1},\Lambda_s^{-1},\Lambda_h^{-1},I)$ on the left and on the right, and performing the change of variables $K_{\mathbf{j}}=\widetilde{K}_{\mathbf{j}}P^{-1}$, $M_{\mathbf{j}}=\widetilde{M}_{\mathbf{j}}\Lambda_s{}^{-1}$, $\bar{L}=\widetilde{L}P^{-1}$, $\Psi_{xe}=P^{-1}\widetilde{\Psi}_{xe}P^{-1}$, and $\Psi_e=P^{-1}\widetilde{\Psi}_eP^{-1}$, it follows that

$$\sum_{\mathbf{i}\in\mathbb{B}^p}\sum_{\mathbf{j}\in\mathbb{B}^p}\alpha_{\mathbf{i}}(x)\alpha_{\mathbf{j}}(x)\begin{bmatrix}\Theta_{\mathbf{ij}} & \star\\ \chi_1 & -\widetilde{\Psi}_x\end{bmatrix}<0, \tag{4.23}$$

with

$$\begin{aligned}\Theta_{\mathbf{ij}}&=\chi_1^\top\mathrm{He}\left(P^{-1}\left(A_{\mathbf{i}}+B_{\mathbf{i}}K_{\mathbf{j}}+\bar{A}+B_{\mathbf{i}}\bar{L}\right)\right)\chi_1+\mathrm{He}\left(\chi_1^\top\left(P^{-1}\left(G_{\mathbf{i}}+B_{\mathbf{i}}M_{\mathbf{j}}\right)\right)\chi_3\right)\\&+\mathrm{He}\left(\chi_1^\top\left(P^{-1}B_{\mathbf{i}}(\bar{L}+K_{\mathbf{j}})+\Psi_{xe}\right)\chi_2\right)-\chi_2^\top\Psi_e\,\chi_2+-2\chi_3{}^\top\Lambda_s^{-1}\chi_3-2\chi_4{}^\top\Lambda_h^{-1}\chi_4\\&-\mathrm{He}\left(\chi_1^\top\left(P^{-1}G_{\mathbf{i}}\right)\chi_4^\top\right)+\mathrm{He}\left(\chi_3^\top\Lambda_s^{-1}\Omega(1_N\otimes\chi_1)\right)+\mathrm{He}\left(\chi_3^\top\Lambda_s^{-1}\Omega(1_N\otimes\chi_2)\right)\\&+\mathrm{He}\left(\chi_4^\top\Lambda_h^{-1}J(1_N\otimes\chi_2)\right).\end{aligned}$$

From Schur complement and the change of variables $\Psi_x=\widetilde{\Psi}_x^{-1}$, the inequalities in (4.23) imply

$$\begin{aligned}&\chi_1^\top\mathrm{He}\left(P^{-1}A_{\mathrm{cl}}(x)\right)\chi_1-\chi_2^\top\Psi_e\,\chi_2-2\chi_3^\top\Lambda_s^{-1}\chi_3+\chi_1^\top\Psi_x\,\chi_1-2\chi_4^\top\Lambda_h^{-1}\chi_4\\&+\mathrm{He}\left(\chi_1^\top\left(P^{-1}B(x)(\bar{L}+K(x))+\Psi_{xe}\right)\chi_2\right)-\mathrm{He}\left(\chi_1^\top\left(P^{-1}G(x)\right)\chi_4\right)\\&+\mathrm{He}\left(\chi_1^\top\left(P^{-1}G_{\mathrm{cl}}(x)\right)\chi_3\right)+\mathrm{He}\left(\chi_3^\top\Lambda_s^{-1}\Omega(1_N\otimes\chi_1)\right)+\mathrm{He}\left(\chi_3^\top\Lambda_s^{-1}\Omega(1_N\otimes\chi_2)\right)\\&+\mathrm{He}\left(\chi_4^\top\Lambda_h^{-1}J(1_N\otimes\chi_2)\right)<0.\end{aligned} \tag{4.24}$$

Multiplying (4.24) by $\begin{bmatrix}x^\top & e^\top & \phi^\top(\hat{x}) & \rho^\top(e,x)\end{bmatrix}$ on the left and its transpose on the right, it is possible to obtain

$$\begin{aligned}&2x^\top P^{-1}\Big[A_{\mathrm{cl}}(x)x+\left(B(x)(K(x)+\bar{L})\right)e\Big]+2x^\top P^{-1}\Big[G_{\mathrm{cl}}(x)\phi(\hat{x})-G(x)\rho^\top(e,x)\Big]\\&-2\phi^\top(\hat{x})\Lambda_s^{-1}\phi(\hat{x})+2\phi^\top(\hat{x})\Lambda_s^{-1}\Omega(1_N\otimes\hat{x})-2\rho^\top(x,e)\Lambda_h^{-1}\rho(x,e)\\&+2\rho^\top(x,e)\Lambda_h^{-1}J(1_N\otimes e)+2x^\top(t)\Psi_{xe}\,e(t)-e^\top(t)\Psi_e\,e(t)+x^\top(t)\Psi_x\,x(t)<0.\end{aligned} \tag{4.25}$$

Notice that $\sum_{i=1}^N\mu_i(\zeta_i,\zeta_j)=2x^\top P^{-1}B(x)\left(\delta(\zeta)+\xi(\zeta)\right)$. By adding this term on both sides of (4.25), one has

$$\dot{x}^\top P^{-1}x+x^\top P^{-1}\dot{x}+\sum_{i=1}^N\left(\Gamma_i(\zeta_i,\zeta_j)-2\mathcal{S}_i(\hat{x},\Lambda_{s_i})-2\mathcal{H}_i(e,x,\Lambda_{h_i})\right)<0. \tag{4.26}$$

with $\dot{x}$ as in (4.7), $\Gamma_i(\zeta_i,\zeta_j)$ as in (4.11), $\mathcal{S}_i(\hat{x},\Lambda_i)$ as in (C.1), and $\mathcal{H}_i(e,x,\Lambda_{h_i})$ as in (C.2). Thus, considering the property of the sector-bounded nonlinearities, it is possible to conclude from S-procedure arguments that (4.26) ensures $\dot{V}(x)<0$, with $V(x)$ as in (4.21), guaranteeing that the origin of the global closed-loop system (4.7) is locally asymptotically stable. Moreover, the condition (4.19) ensures that $\mathscr{R}\subset\mathscr{D}$ (see [18] for details). This concludes the proof. □

Remark 4.4

Theorem 4.1 provides a co-design condition without any assumptions on the deviation bounds between the membership functions. This is possible due to the term $\mu_i(\zeta_i, \zeta_j)$ in the proposed ETM (4.10)*, which can vanish the effects of asynchronous premise variables by canceling the terms with the product $B(x_i)K(\hat{x}_i)$, and $B(x_i)M(\hat{x}_i)$. The computation of $\mu_i(\zeta_i, \zeta_j)$ requires that the states of the local and neighboring subsystems be continuously available for the local ETMs. Therefore, the proposed approach might not be suitable for applications where this information is unknown.*

4.2.3 A Multi-objective optimization problem

There are two different objectives related to the solution of Problem 4.1: the minimization of the number of transmissions and the maximization of the estimated domain of attraction. The first objective implies a reduction of the bandwidth usage, and it can be achieved by minimizing the eigenvalues of the matrices Ψ_e^i, Ψ_{xe}^i, and maximizing the eigenvalues of Ψ_x^i. Considering the decision variables of Theorem 4.1, similar to [236], this objective can be carried out by minimizing

$$\bar{J}_1 = \mathrm{tr}(\tilde{\Psi}_x + \tilde{\Psi}_e + \tilde{\Psi}_{xe}).$$

Assuming that the conditions of Theorem 4.1 hold, with the constraint $P^{-1} \leq Q$, being Q a symmetric matrix with appropriate dimension, the minimization of

$$\bar{J}_2 = \mathrm{tr}(Q),$$

results in the minimization of the eigenvalues of P^{-1}. Thus, the volume of the ellipsoidal set $\mathscr{R}$ in (4.20) is also enlarged.

A multi-objective optimization problem can be formulated to deal with both objective functions by applying the ε-constraint approach [238]. Here, the objective function $\bar{J}_2$ is minimized under $\bar{J}_1$ constrained by $\varepsilon \in \mathbb{R}_{>0}$, as presented in the following proposed optimization problem:

$$\begin{aligned} \text{minimize} \quad & \bar{J}_2 \\ \text{subject to} \quad & P > 0,\ \widetilde{\Psi}_x > 0,\ \widetilde{\Psi}_e > 0, \\ & \begin{bmatrix} Q & I_n \\ \star & P \end{bmatrix} \geq 0, \\ & (4.18),\ (4.19),\ \mathrm{tr}(\widetilde{\Psi}_x + \widetilde{\Psi}_e + \widetilde{\Psi}_{xe}) \leq \varepsilon. \end{aligned} \tag{4.27}$$

For fixed values of ε, the optimization problem is a convex one, and the Pareto front is estimated by solving it for several values of ε.

4.3 Distributed ETC of Interconnected Van Der Pol Oscillators

Consider a nonlinear interconnected system composed of four Van Der Pol Oscillators coupled according to the graph in Figure 4.2.

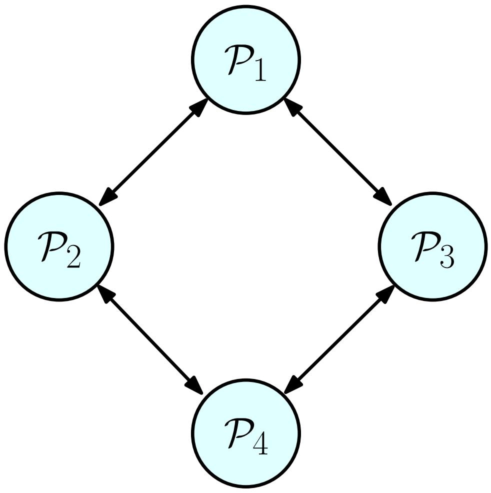

Figure 4.2 – Graph describing the interconnections among the Van Der Pol oscillators - Example 1.

Each subsystem is described as [10]:

$$\begin{aligned}\dot{x}_{i1}(t) &= x_{i2}(t),\\ \dot{x}_{i2}(t) &= -x_{i1}(t) + \mu_i \left(1 - x_{i1}^2(t)\right) x_{i2}(t)\\ &\quad + g_i(x_{i1}(t))u_i(t) + \bar{\gamma}_i \sum_{j\in\mathcal{N}_i} \sin\left(x_{j1}(t) - x_{i1}(t)\right).\end{aligned} \tag{4.28}$$

where $g_i(x_{i1}(t)) = (1/(0.4 + 0.1x_{i1}^2(t))$. The parameters μ_i and $\bar{\gamma}_i$ are chosen as $\mu_1 = 0.5$, $\mu_2 = 0.35$, $\mu_3 = 0.6$, $\mu_4 = 1$, and $\bar{\gamma}_1 = -0.5$, $\bar{\gamma}_2 = 0.2$, $\bar{\gamma}_3 = 0.4$, $\bar{\gamma}_4 = -0.1$. The functions $\bar{\phi}_i(x_i, x_j)$, defined gathering $\bar{\phi}_{ij} = \sin(x_{ji} - x_{i1})$, do not satisfy Assumption 4.1, since they belong to the sector $[\Omega_{Li}, \Omega_{Ui}]$ where $\Omega_{Li} = [\Omega_{Li1} \ \dots \ \Omega_{LiN}]$ and $\Omega_{Ui} = [\Omega_{Ui1} \ \dots \ \Omega_{UiN}]$ are constructed as $\Omega_{Lim} = [\Omega_{Limk_{i1}}^\top \ \dots \ \Omega_{Limk_{i\ell}}^\top]^\top$, $\Omega_{Uim} = [\Omega_{Uimk_{i1}}^\top \ \dots \ \Omega_{Uimk_{i\ell}}^\top]^\top \in \mathbb{R}^{d_i \times n_j}$, with

$$\Omega_{Limk_\ell} = \begin{cases} [-1 \quad 0], & \text{if } m = i,\\ [1 \quad 0], & \text{if } m = k_{i\ell},\\ [0 \quad 0], & \text{otherwise,}\end{cases} \quad \Omega_{Uimk_\ell} = \begin{cases} [0.22 \quad 0], & \text{if } m = i,\\ [-0.22 \quad 0], & \text{if } m = k_{i\ell},\\ [0 \quad 0], & \text{otherwise.}\end{cases}$$

Notice that it is possible to perform a loop transformation and define $\widetilde{\phi}_{ij} = x_{i1} - x_{j1} + \sin(x_{ji} - x_{i1})$, which belong to the sector $[0, \Omega_i]$, as required by Assumption 4.1. The matrices

$\Omega_i = [\Omega_{i1} \ \dots \ \Omega_{iN}]$ are defined as $\Omega_i = \Omega_{Ui} - \Omega_{Li}$, with $\Omega_{im} = [\Omega_{imk_{i1}}^\top \ \dots \ \Omega_{imk_{i\ell}}^\top]^\top \in \mathbb{R}^{d_i \times n_j}$,

$$\Omega_{imk_\ell} = \begin{cases} [1.22 \quad 0], & \text{if } m = i, \\ [-1.22 \quad 0], & \text{if } m = k_{i\ell}, \\ [0 \quad 0], & \text{otherwise,} \end{cases}$$

for all $j \in \mathcal{N}_i$, $m \in \mathcal{V}$, $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\le d_i}$.

Therefore, an exact NT-S fuzzy model with 4 rules can be constructed with

$$A_{00}^i = A_{01}^i = \begin{bmatrix} 0 & 1 \\ -1 - 2\bar{\gamma}_i & -8\mu_i \end{bmatrix}, \ B_{00}^i = B_{10}^i = \begin{bmatrix} 0 \\ 2.5 \end{bmatrix},$$
$$A_{10}^i = A_{11}^i = \begin{bmatrix} 0 & 1 \\ -1 - 2\bar{\gamma}_i & \mu_i \end{bmatrix}, \ B_{01}^i = B_{11}^i = \begin{bmatrix} 0 \\ 0.7692 \end{bmatrix},$$
$$A_{ij} = \begin{bmatrix} 0 & 0 \\ \bar{\gamma}_i & 0 \end{bmatrix}, \quad G_{00}^i = G_{01}^i = G_{10}^i = G_{11}^i = \begin{bmatrix} 0 & 0 \\ \bar{\gamma}_i & \bar{\gamma}_i \end{bmatrix}.$$

where $z_{i1}(t) = x_{i1}^2$, and $z_{i2}(t) = g_i(x_{i1}(t))$, $\mathscr{D}_i = \{x_i \in \mathbb{R}^2 : |x_{i1}| \le 3\}$ $\forall i \in \mathcal{V}$, $z_{i1}^0 = 0$, $z_{i1}^1 = 9$, $z_{i2}^0 = 0.7692$, and $z_{i2}^1 = 2.5$. Consequently, the weighting functions can be obtained as in (B.6). and the normalized state-dependent membership functions as $\alpha_{00}(z_i) = w_0^{i1}(z_{i1})w_0^{i2}(z_{i2})$, $\alpha_{01}(z_i) = w_0^{i1}(z_{i1})w_1^{i2}(z_{i2})$, $\alpha_{10}(z_i) = w_1^{i1}(z_{i1})w_0^{i2}(z_{i2})$, $\alpha_{11}(z_i) = w_1^{i1}(z_{i1})w_1^{i2}(z_{i2})$.

Notice that with the previously performed steps, we are able to represent system (4.28) as a T-S fuzzy model (4.1). Considering the closed-loop subsystems in (4.6), we have that the nonlinear functions composing $\rho_i(e,x)$ must satisfy Assumption 4.1 to guarantee that Lemma (C.2) holds. For $\widetilde{\phi}_{ij}=x_{i1} - x_{j1} + \sin(x_{ji} - x_{i1})$, it possible to compute that

$$J_{Li} \le \frac{\partial \widetilde{\phi}_i(x)}{\partial x} \le J_{Ui},$$

with $J_{Li}=[J_{Li1} \ \dots \ J_{LiN}]$ and $J_{Ui}=[J_{Ui1} \ \dots \ J_{UiN}]$ constructed as $J_{Lim}=[J_{Limk_{i1}}^\top \ \dots \ J_{Limk_{i\ell}}^\top]^\top$, $J_{Uim} = [J_{Uimk_{i1}}^\top \ \dots \ J_{Uimk_{i\ell}}^\top]^\top \in \mathbb{R}^{d_i \times n_j}$, where

$$J_{Limk_\ell} = \begin{cases} [-2 \quad 0], & \text{if } m = k_{i\ell}, \\ [0 \quad 0], & \text{otherwise,} \end{cases} \qquad J_{Uimk_\ell} = \begin{cases} [2 \quad 0], & \text{if } m = i, \\ [0 \quad 0], & \text{otherwise.} \end{cases}$$

Another loop transformation can be performed to obtain $\phi_i(x) = \widetilde{\phi}_i(x) - J_{Li}x$ with $\widetilde{\phi}_{ij} = x_{i1} + x_{j1} + \sin(x_{ji} - x_{i1})$, that satisfies Assumption 4.1 considering $J_i = J_{Ui} - J_{Li}$. Notice that this second loop transformation is performed only in the nonlinear terms of $\widetilde{\rho}_i(e,x)$, that is,

$$\begin{aligned} \widetilde{\rho}_i(e,x) &= \widetilde{\phi}_i(\hat{x}) - \widetilde{\phi}_i(x) \\ &= (\phi_i(\hat{x}) + J_{Li}\hat{x}) - (\phi_i(x) + J_{Li}x) \\ &= (\phi_i(\hat{x}) - \phi_i(x)) + J_{Li}(x - \hat{x}) \\ &= \rho_i(e,x) + J_{Li}e. \end{aligned}$$

Therefore, with $\rho_i(e, x) = \phi_i(x) - \phi_i(\hat{x})$ the conditions of Lemma (C.2) holds. Moreover, notice that it is required to replace $\widetilde{\rho}(e, x)$ by $\rho(e, x) + J_L e$ in the global closed-loop (4.7) to compensate the transformations.

The optimization problem in (4.27) was solved for 20 values of ε in a logarithmic grid between 10^0 and 10^4, and the cost obtained for the objective function $\bar{J}_2$, for each one of the 20 points, are depicted in Figure 4.3. From the results of Figure 4.3, it can be noticed that larger values of ε result in smaller values for $\bar{J}_2$, resulting in the increase of the volume of the estimated domain of attraction. For $\varepsilon = 10^0$, the solution of the optimization problem results in $\bar{J}_2 = 265.33$, and with $\varepsilon = 10^4$ the obtained solution is $\bar{J}_2 = 0.8901$. Since $\bar{J}_1$ is constrained by ε, the increase of ε will result in a higher number of events. Thus, the choice of ε must be made to achieve a compromise between the two objectives.

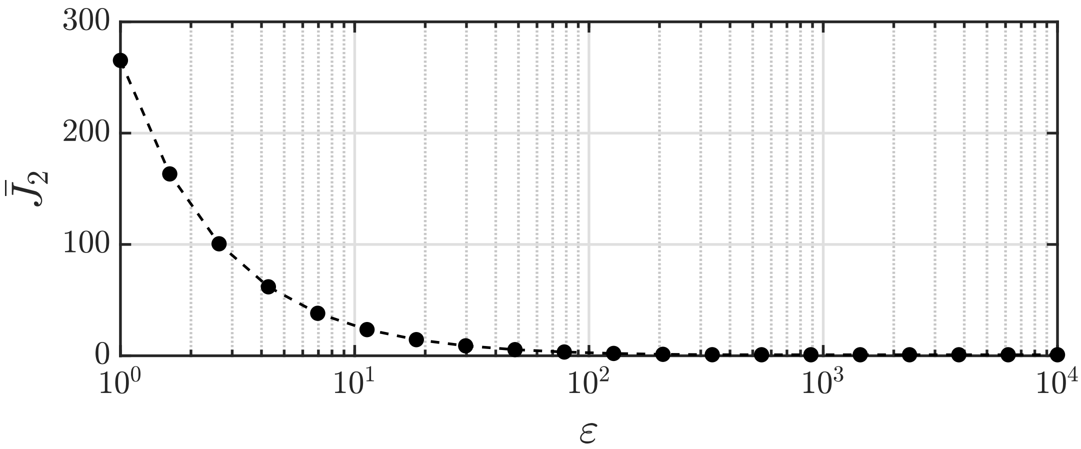

Figure 4.3 – The values of $\bar{J}_2$ obtained by solving the proposed optimization problem with respect to fixed values for the upper-bound ε of $\bar{J}_1$ - Example 1.

The trajectories of the closed-loop system in a simulation are presented in Figure 4.4. The distributed control law composed by (4.3) and the local ETMs (4.10)-(4.13) were designed by solving the optimization problem in (4.27) with $\varepsilon = 100$, resulting in $\bar{J}_2 = 2.6515$. The initial condition x_0 was chosen inside of the enlarged estimation for the DoA, that is, $x_0^\top P^{-1} x_0 < 1$.

The results in Figure 4.4 illustrate the efficacy of the proposed approach in ensuring the asymptotic stability of the origin of the closed-loop interconnected system (4.7). Notice that the control inputs $u_i(t)$ are asynchronously updated according to the event times of the corresponding ETM, being $(21, 19, 24, 22)$ the number of events of each subsystem $(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4)$ respectively. The inter-event times are depicted in Figure 4.5. From Figure 4.6, it is possible to notice that the inter-event times correspond to the times required for $\mathcal{G}_i(x_i, e_i)$ evolve from 0 to $1 - \mathcal{V}_i(x_i, e_i)$, as expected from the proof of the existence of τ_i^* in Lemma 4.1. Therefore, the obtained results emphasize the arguments provided in Lemma 4.1 to ensure the Zeno-freeness property.

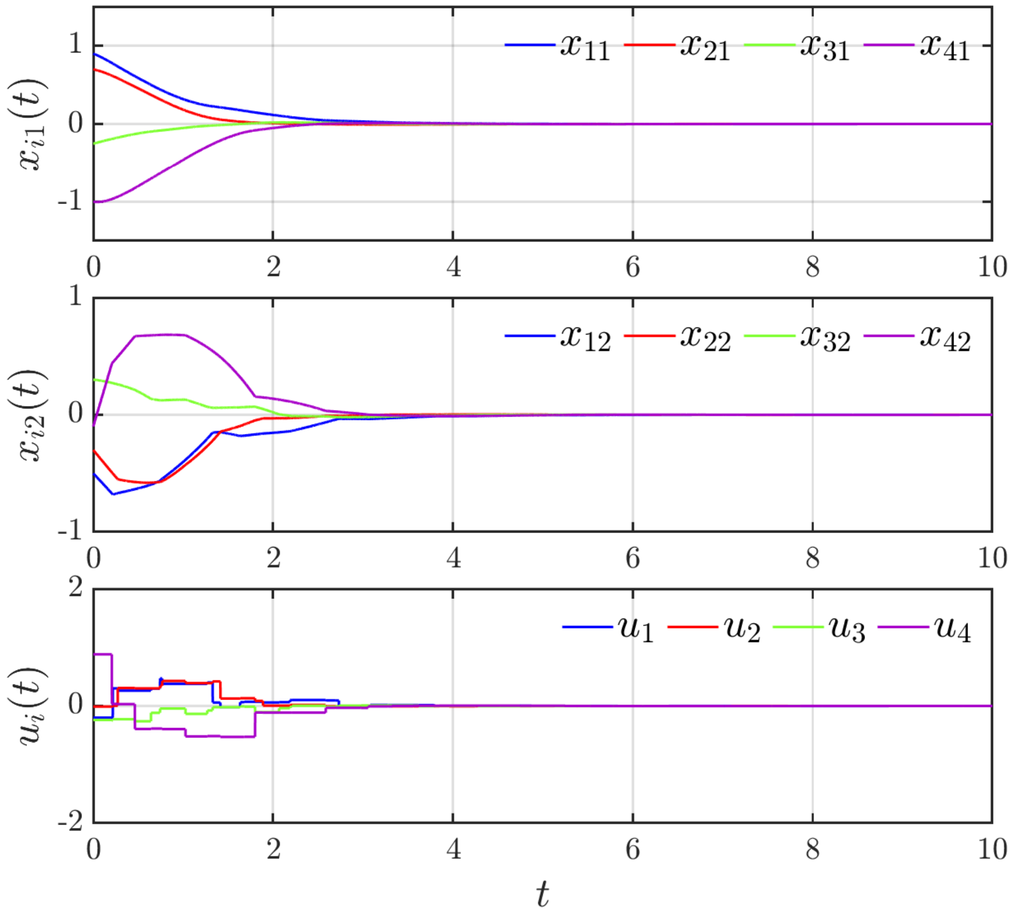

Figure 4.4 – Closed-loop trajectories of system (4.28) with the designed distributed control inputs (4.3) under the proposed ETMs (4.10)-(4.13).

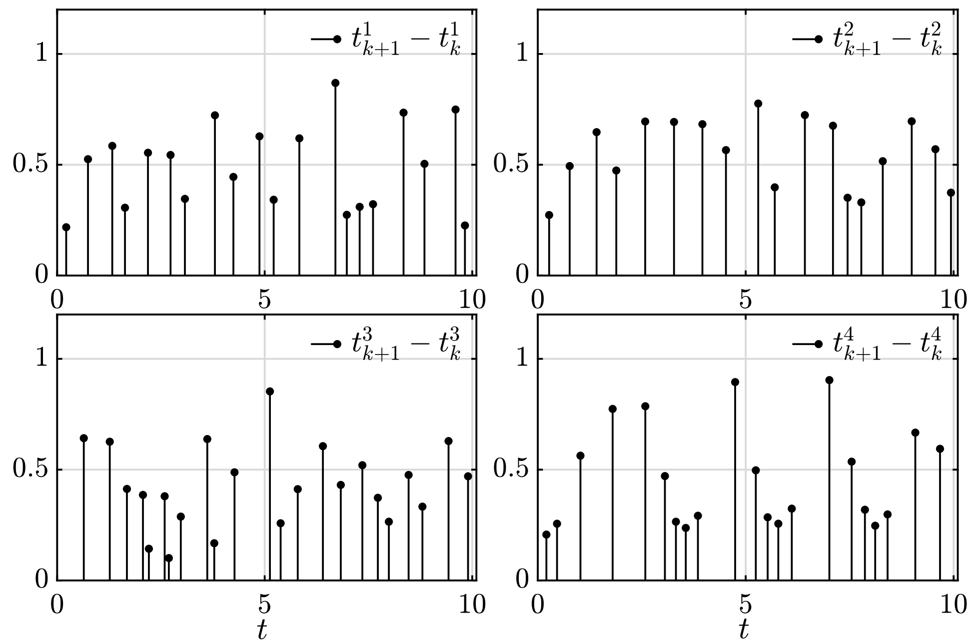

Figure 4.5 – Inter-event times of each subsystem originated with the implementation of the proposed ETMs (4.10)-(4.13) in the simulations of Figure 4.4.

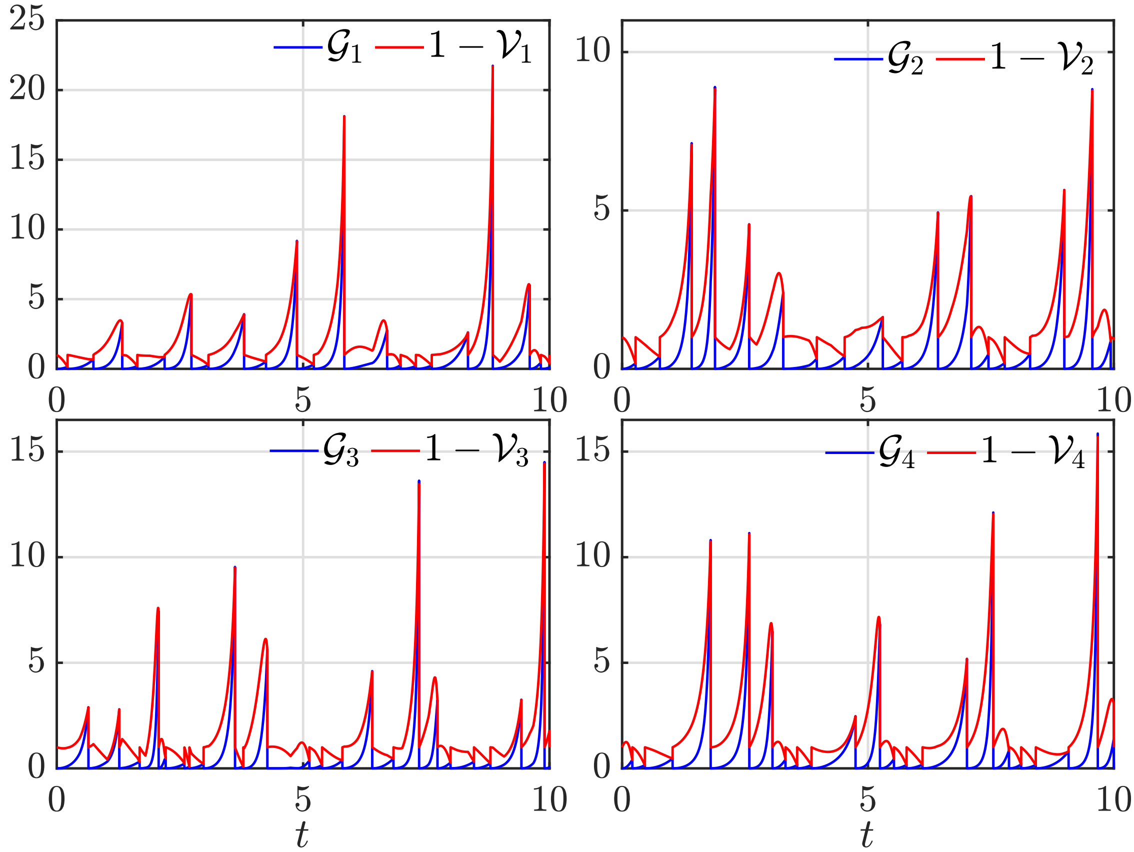

Figure 4.6 – Functions $\mathcal{G}_i(x_i, e_i)$ and $1 - \mathcal{V}_i(x_i, e_i)$ computed in the simulations of Figure 4.4.

Moreover, the number of events (N_e), minimum inter-event times (τ_i^*), maximum inter-event times $(\bar{\tau}_i)$, and the median of average inter-event times (τ_a) where computed in 100 simulations of 10s with different initial conditions chosen in the border of the enlarged estimation of the DoA obtained with $\varepsilon = 100$. The results for each subsystem are presented in Table 4.1, and are similar even considering the asynchronous transmissions. Furthermore, it is also possible to notice that all MIETs (τ_i) are strictly positive, confirming the exclusion of the Zeno behavior in the proposed distributed ETC setup.

Table 4.1 – Mean of the number of events (N_e), minimum inter-event times (τ_i^*), maximum inter-event times $(\bar{\tau}_i)$, and the median of average inter-event times (τ_m) for each subsystem in 100 simulations with different initial conditions.

Subsystem	N_e	τ_i^* (s)	$\bar{\tau}_i$ (s)	τ_m (s)
$\mathcal{P}_1$	22	0.2094	0.7763	0.4684
$\mathcal{P}_2$	22	0.2231	0.7646	0.4803
$\mathcal{P}_3$	23	0.1584	0.7334	0.4525
$\mathcal{P}_4$	22	0.2143	0.8884	0.4615

4.4 Conclusions

In this chapter an asynchronous distributed event-triggered control approach has been proposed for the stabilization of nonlinear interconnected NT-S fuzzy systems. In the proposed method, transmissions are triggered regarding a distributed ETC scheme properly defined to cancel the effect of asynchronous premise variables and nonlinear interconnections. With this cancellation scheme, a sufficient condition has been proposed to co-design the ETMs and the gains of the distributed control law. Moreover, a procedure to estimate the domain of attraction has been provided. To enlarge this estimate and reduce the number of transmissions, a convex multi-objective optimization problem has been presented. From numerical simulations, the effectiveness of the approach in guaranteeing the asymptotic stability of the origin of the global interconnected system is demonstrated. Finally, the exclusion of Zeno behavior has been formally proven, thus enabling the implementation of the distributed ETC scheme.

Part III

Distributed Consensus for LPV Multi-agent Systems

5 LITERATURE REVIEW AND SPECIFIC OBJECTIVES OF PART III

This chapter presents a review of the existent approaches for LPV MASs §5.1. Moreover, based on the pre-organized discussion, the specific objectives addressed in the remaining Chapters of Part III are defined in §5.2.

5.1 Literature review of LPV MAS systems

Due to its wide application range, which encompasses several modeling approaches for agents, and distinct consensus problems, the literature of multi-agent system is more complex and diverse than the previous case of ISs. Consequently, obtaining a complete overview of all the existent approaches to the consensus of MAS is an exhaustive task that goes beyond the scope of this thesis. Therefore, this review of the literature is restricted to the existing approaches for MAS represented by LPV models. More information on approaches for other classes of MAS can be found in the various survey papers existent (and the references within) [4, 50–59].

To provide an overview of the consensus methods within the LPV context, the main results are summarized in the literature review presented in Table 5.1. The approaches were classified based on two criteria: the type of LPV system considered to describe the agents (completely homogeneous LPV MAS, LPV MAS with the homogeneous polytopic domain but with different scheduling parameters, or heterogeneous LPV MAS), and the addressed consensus problem (Leaderless, Leader-following, Output regulation, or Practical Consensus). Moreover, notice that the systems were also distinguished as continuous-time LPV MAS or discrete-time LPV MAS.

Table 5.1 – Literature Review: Consensus for LPV MAS

		Leaderless	Leader-following	Output	Practical
Continuous	Homogeneous	[239–242]	[243, 244]	×	×
	Different parameters	×	×	×	×
	Heterogeneous	[245]	[246]	[247–249]	×
Discrete	Homogeneous	[250]	×	[251, 252]	×
	Different parameters	×	×	×	×
	Heterogeneous	×	×	×	×

The leaderless consensus problem of homogeneous continuous-time LPV MAS has been investigated in [239–242]. Among these results, the approach of [241] proposes a gain-scheduled observer-based consensus protocol, in which both the controller and the observer gains depend on the time-varying scheduling parameters. The design is performed through sufficient LMI conditions obtained considering the Polya's Theorem. A similar gain-scheduled observer-based approach is also considered in [242]. However, differently from [241], the presence of additive and multiplicative faults in the sensors and actuators is considered. To deal with this faulty scenario, virtual actuators and sensors are introduced to compose a fault-tolerant consensus protocol. Moreover, the presence of actuator faults is also considered in [240]. In the proposed method, the faults are modeled as an polytopic uncertainty and a robust observer-based reliable protocol is proposed. Furthermore, a gain-scheduled consensus protocol is proposed in [239] considering the presence of time-varying delays in communication among agents.

The use of homogeneous continuous-time LPV models for agents is also explored in [243, 244]. As in [241, 242], observer-based protocols are introduced, but with attention directed to the leader-following consensus problem. Additionally, the presence of actuator faults is considered in [243]. Furthermore, the consensus of heterogeneous continuous-time LPV MAS is investigated in [245]. Although the approach is not explicitly focused on the leader-following problem, the consensus is based on an internal dynamics method, in which the agents try to track a controlled local internal dynamics that must achieve a consensus with the homogeneous internal dynamics of the other agents. Moreover, the leader-following consensus problem has also been addressed in [246] for MASs with heterogeneous parameter-dependent linear fractional transformation dynamics. Further investigations in MASs with continuous-time heterogeneous LPV modeling are performed in [247–249]. These approaches focused on the output regulation problem considering distributed observers [247], distributed adaptive observers [248], and distributed event-triggered adaptive observers [249].

In contrast to the previously discussed scenario of continuous-time LPV MASs, approaches for discrete-time LPV MASs have received limited attention. The literature review identified only the works of [250–252] addressing this scenario. In [250], a leaderless consensus approach is proposed based on finite frequency fault estimators and adaptive event-triggered mechanisms. The fault-tolerant consensus protocol considers a compensation scheme using the estimations of the faults to mitigate its effects. It is worth mentioning that a completely homogeneous modeling with parameter-independent input and output matrices is considered. Moreover, the design of the consensus and observer gains is performed with LMI conditions obtained considering the pole assignment and $\mathcal{H}_\infty$ performance. Finally, the event-triggered ℓ_2-optimal output formation is investigated in [251, 252] for non-holonomic vehicles modeled as polytopic LPV models. Although the problem formulation accounts for different parameters, [251, 252] also assumes a homogeneous scheduling among all agents.

Furthermore, although not included in Table 5.1, it is worth mentioning the approaches

[253–257] in the context of nonlinear Lipschitz MASs. Notice that these works use the LPV framework to represent local Lipschitz nonlinearities in the agents' nonlinear models. In summary, with the main consensus approaches for LPV MASs now established, the specific objectives of Part III of this thesis are presented in the next section.

5.2 Specific objectives of part III

From the discussion performed in Section 5.1, it becomes evident that limited attention has been given to the design of distributed consensus protocols for LPV MASs. The literature review was unable to find any approach that investigates the practical consensus for the scenario with different scheduling parameters in both continuous and discrete-time. The first work to evaluate the influence of different scheduling parameters was [241], where it is shown that if a consensus protocol designed considering equal parameters is implemented in an continuous-time LPV MAS with different parameters, a non-synchronization scenario appears. Although [241] acknowledges the non-synchronization effects raised by the difference among the scheduling parameters, it does not offer a solution to deal with this problem, by assessing or setting bounds on the consensus error within this more intricate context. Furthermore, the literature review was also unable to find any approach that investigates the leader-following consensus for discrete-time LPV MASs, even for the scenario of completely homogeneous agents. Thus, to address these gaps in the literature, the specific objectives of this thesis in the context of LPV MASs are:

- To propose sufficient LMI-based synthesis conditions, using Lyapunov stability arguments, for the design of a distributed observer-based consensus protocol to achieve practical leaderless consensus of continuous-time LPV MASs with distinct time-varying scheduling parameters.
- To propose sufficient LMI-based synthesis conditions, using Lyapunov stability arguments, for the design of a distributed observer-based consensus protocol to achieve the practical leader-following formation consensus of discrete-time LPV MASs with distinct time-varying scheduling parameters.
- To guarantee that the trajectories of the consensus error, when disturbed by internal perturbations that arise due to the difference among the scheduling parameters, converge exponentially to an attractive bounding region in both scenarios of practical leaderless consensus and practical leader-following formation consensus.
- To propose a compensated distributed observer-based consensus protocol to ensure, when possible, the exact leader-following tracking formation of discrete-time LPV MASs with heterogeneous scheduling parameters.

6 A DISTRIBUTED OBSERVER-BASED APPROACH FOR THE PRACTICAL STATE CONSENSUS OF LPV MULTI-AGENT SYSTEMS

This chapter proposes a distributed gain-scheduled observer-based framework to achieve practical state consensus for linear parameter-varying (LPV) multi-agent systems (MAS). In the considered scenario, scheduling parameter mismatches among agents are explicitly modeled as bounded disturbances. Using Lyapunov theory, sufficient conditions are derived to design a gain-scheduled observer-based consensus protocol. This protocol guarantees that the consensus error trajectories converge exponentially to a bounded region, ensuring practical consensus. Furthermore, numerical simulations demonstrate the efficacy of the proposed approach, highlighting its advantages over non-synchronized outcomes typically observed in LPV MAS under heterogeneous scheduling parameters.

The remainder of this chapter is organized as follows. The problem formulation is presented in §6.1. The proposed design conditions are defined in §6.2. Two numerical examples are presented in §6.3 to illustrate the application of the proposed methodology. Conclusions are drawn in §6.4.

6.1 Problem formulation

Consider a MAS composed of N agents in the form

$$\begin{aligned}\dot{x}_i(t) &= A(\rho_i(t))x_i(t) + B(\rho_i(t))u_i(t),\\ y_i(t) &= C(\rho_i(t))x_i(t),\end{aligned} \tag{6.1}$$

where $x_i \in \mathbb{R}^{n_x}$ is the state vector, $u_i \in \mathbb{R}^{n_u}$ is the control input, $y_i \in \mathbb{R}^{n_y}$ is the output, and $\rho_i(t) \in \mathbb{R}^p$ is the vector of time-varying scheduling parameters, which are functions of measured exogenous signals.

The parameter-dependent matrices $A(\rho_i(t)) \in \mathbb{R}^{n_x \times n_x}$, $B(\rho_i(t)) \in \mathbb{R}^{n_x \times n_u}$, and $C(\rho_i(t)) \in \mathbb{R}^{n_y \times n_x}$ are functions of the scheduling parameters and belong to a polytopic domain. Then, they can be written as follows:

$$\begin{bmatrix} A(\rho_i(t)) & B(\rho_i(t)) \\ C(\rho_i(t)) & \cdot \end{bmatrix} = \sum_{h=1}^{N_v} \alpha_h(\rho_i(t)) \begin{bmatrix} A_h & B_h \\ C_h & \cdot \end{bmatrix}, \tag{6.2}$$

where A_h, B_h, and C_h are the vertices of the polytopic domain, N_v is the number of vertices and $\alpha_h(\rho_i(t))$ satisfies the convex sum property:

$$\sum_{h=1}^{N_v} \alpha_h(\rho_i(t)) = 1, \text{ and } \alpha_h(\rho_i(t)) \geq 0, \ \forall i \in \mathcal{V}.$$

The addressed LPV-MAS (6.1) comprehends N same-order agents represented by homogeneous LPV models with the same vertices. That is, all agents share the same polytopic domain.

However, it is assumed that each agent has independent time-varying scheduling parameters, which can induce heterogeneity among the agents. Moreover, also notice that differently of interconnected system, the agents do not share any common states, being the local open-loop dynamics completely decoupled.

Consider the following gain-scheduled observer-based distributed consensus protocol

$$u_i = K(\rho_i(t)) \sum_{j \in \mathcal{N}_i} (\hat{x}_i - \hat{x}_j), \quad \forall i \in \mathcal{V}, \tag{6.3}$$

where $K(\rho_i(t)) \in \mathbb{R}^{n_u \times n_x}$ is the gain-scheduled function, $\hat{x}_i \in \mathbb{R}^{n_x}$ is the estimated state of the local agent, and $\hat{x}_j \in \mathbb{R}^{n_x}$ are the estimated states of the agents that compose the neighborhood $\mathcal{N}_i$. For each agent, the local estimated states are given by

$$\dot{\hat{x}}_i(t) = A(\rho_i(t))\hat{x}_i(t) + B(\rho_i(t))u_i(t) + L(\rho_i(t))\Big(C(\rho_i(t))\hat{x}_i(t) - y_i(t)\Big), \tag{6.4}$$

where $L(\rho_i(t)) \in \mathbb{R}^{n_x \times n_y}$ is the gain-scheduled observer function.

The gain-scheduled functions associated with the proposed distributed consensus protocol and the local observers belong to the same polytopic domain and can be expressed as:

$$K(\rho_i(t)) = \sum_{l=1}^{N_v} \alpha_l(\rho_i(t))K_l, \quad L(\rho_i(t)) = \sum_{l=1}^{N_v} \alpha_l(\rho_i(t))L_l,$$

where K_l and L_l are the vertices to be designed. For notational simplicity, the time dependency of $\rho_i(t)$ is omitted, and only ρ_i is considered hereafter.

Remark 6.1

In the proposed gain-scheduled observer-based distributed consensus protocol, the difference between the state of agent i and those of its neighbors j, i.e., $x_i - x_j$, cannot be directly used, since not all states are measurable. Each agent has its own local observer $\hat{x}_i$, and the estimated states are transmitted to all agents in its neighborhood. Similar to previous distributed approaches, the communication among agents follows an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ (see Appendix A).

Defining $z_i = \hat{x}_i - x_i$, as the local estimation error, $z = (z_1, z_2, \dots, z_N)$, and $\rho = (\rho_1, \rho_2, \dots, \rho_N)$, it is possible to write

$$\dot{z} = \sum_{h=1}^{N_v} \sum_{l=1}^{N_v} \Big(\alpha_{hl}(\rho) \otimes I_{n_x}\Big)\Big(I_N \otimes (A_h + L_l C_h)\Big)z, \tag{6.5}$$

where $\alpha_{hl}(\rho) = \text{diag}\Big(\alpha_h(\rho_1)\alpha_l(\rho_1), \dots, \alpha_h(\rho_N)\alpha_l(\rho_N)\Big)$.

By substituting the consensus protocol (6.3) into (6.1), we have that

$$\dot{x}_i = A(\rho_i)x_i + B(\rho_i)K(\rho_i) \sum_{j \in \mathcal{N}_i} \Big((z_i + x_i) - (z_j + x_j)\Big).$$

Therefore, the global dynamics of all agents $x = (x_1, x_2, \ldots, x_N)$ can be written as

$$\dot{x} = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Big(\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big[\Big(I_N\otimes A_h + \mathcal{L}\otimes B_h K_l\Big)x + \Big(\mathcal{L}\otimes B_h K_l\Big)z\Big]. \tag{6.6}$$

Let the consensus error be defined as $\delta_i = x_i - \bar{x}$, being $\bar{x} = (1/N)\sum_{j=1}^{N} x_j$. Then, the relation between the global consensus error $\delta = (\delta_1, \delta_2, \ldots, \delta_N)$, and the states x is given by $\delta = \Upsilon x$, with $\Upsilon = (\bar{\Upsilon}\otimes I_{n_x})$ and $\bar{\Upsilon} = (I_N - (1/N)\mathbf{1}\mathbf{1}^\top)$. Hence, the consensus error can be defined as

$$\dot{\delta} = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Upsilon\Big\{\Big(\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big[\Big(I_N\otimes A_h + \mathcal{L}\otimes B_h K_l\Big)x + \Big(\mathcal{L}\otimes B_h K_l\Big)z\Big]\Big\}.$$

Provided that $(\mathcal{L}\otimes B_h K_l\Big)x = (\mathcal{L}\otimes B_h K_l\Big)\delta$, and $x = \delta + (\mathbf{1}\otimes\bar{x})$, it is possible to write

$$\dot{\delta} = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Upsilon\Big\{\Big(\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big[\Big(I_N\otimes A_h + \mathcal{L}\otimes B_h K_l\Big)\delta + \Big(\mathcal{L}\otimes B_h K_l\Big)z\Big] + f_{hl}(\rho,\bar{x})\Big\},$$

being $f_{hl}(\rho,\bar{x}) = (\alpha_{hl}(\rho)\mathbf{1}\otimes A_h\bar{x}\Big)$.

Finally, defining the augmented error systems as $e = (z, \delta)$ we obtain

$$\dot{e} = \bar{A}(\rho)e + B_\omega\,\omega, \tag{6.7}$$

with

$$\begin{aligned}
\bar{A}(\rho) &= \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\begin{bmatrix}\bar{A}_{hl}^{11} & 0\\ \bar{A}_{hl}^{21} & \bar{A}_{hl}^{22}\end{bmatrix}, \quad B_\omega = \begin{bmatrix}\mathbf{0}\\ \mathbf{1}\end{bmatrix},\\
\bar{A}_{hl}^{11} &= \alpha_{hl}(\rho)\otimes(A_h + L_l C_h),\\
\bar{A}_{hl}^{21} &= \Big(\bar{\Upsilon}\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big(\mathcal{L}\otimes B_h K_l\Big),\\
\bar{A}_{hl}^{22} &= \Big(\bar{\Upsilon}\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big(I_N\otimes A_h + \mathcal{L}\otimes B_h K_l\Big),\\
\omega &= \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Upsilon f_{hl}(\rho,\bar{x}).
\end{aligned}$$

Remark 6.2

Notice that the augmented error system is subject to an internal perturbation ω due to the difference among the time-varying parameters of the agents. From the definition of (6.7), it is possible to conclude that $e = 0$ if and only if $\omega = 0$, and $\hat{x}_i = x_i = \bar{x}, \forall i \in \mathcal{V}$. Similarly to [241], an exact consensus (or synchronization scenario) can be achieved by selecting $\rho_1 = \rho_2 = \ldots = \rho_N$. If all agents have the same scheduling parameters, then $\bar{\Upsilon}\alpha_{hl}(p)\mathbf{1} = \mathbf{0}\ \forall h, l = 1, \ldots, N_v$, which is a sufficient condition to ensure $\omega = 0$. For the case where the scheduling parameters are different, the trajectories of w depend on the vertices A_h of the system matrices, on the mean of the states $\bar{x}$, and on the time-varying difference among the scheduling functions $\alpha_{hl}(\rho)$. Since in general $\omega \neq 0$, the exact consensus cannot be achieved, resulting in the non-synchronization scenario discussed in [241]. For this reason, the development of a practical consensus approach is the main goal of this chapter.

Defining the ellipsoid

$$\mathcal{E}(P,\varepsilon^*) \triangleq \{e \in \mathbb{R}^{2Nn_x} : e^\top P\, e \leq \varepsilon^*\},$$

where $P > 0$ is a symmetric positive definite matrix and $\varepsilon^* > 0$ is a given positive scalar, the notion of practical consensus considered follows the same lines established in Definition 1.7, where the MAS (6.1) is said to achieve practical leaderless consensus if

$$\lim_{t\to\infty} d(e(t,e_0),\mathcal{E}(P,\varepsilon^*)) = 0,$$

being $e(t,e_0)$ the trajectory of the error system (6.7) with initial condition $e_0 \in \mathbb{R}^{2Nn_x}$, and $d(e(t,e_0),\mathcal{E}(P,\varepsilon^*))$ the distance between $e(t,e_0)$ and $\mathcal{E}(P,\varepsilon^*)$.

Based on the augmented error dynamics (6.7) the following assumption is considered.

Assumption 6.1

The internal perturbation ω is bounded by

$$||\omega(t)||_\infty \leq \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} ||\Upsilon f_{hl}(\rho,\bar{x})||_\infty, \tag{6.8}$$

from which it is possible to define $\gamma \triangleq \sup_{t\geq 0}(||\omega(t)||_\infty)$.

Remark 6.3

As highlighted in Remark 6.2 the trajectories of the internal perturbations depend on the mean of the states $\bar{x}$. Therefore, the value of γ, computed as the supremum of the infinity norm of $\omega(t)$ for all $t \geq 0$, will be finite only if $\bar{x}$ is also finite. If the dynamics of (6.6) are not unstable, this condition is directly satisfied.

The following analysis conditions are considered in the design of the proposed gain-scheduled observer-based consensus protocol.

Lemma 6.1: Adapted from [62, 258]

If there exist positive scalars $a > 0$, $b > 0$, and symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x\times n_x}$ and $P_2 \in \mathbb{R}^{n_x\times n_x}$ such that the Lyapunov function

$$V(e) = e^\top P\, e, \quad P = \mathrm{diag}\Big((I_N \otimes P_1),(I_N \otimes P_2)\Big), \tag{6.9}$$

satisfies

$$\dot{V}(e) + aV(e) - b\,\omega^\top\omega < 0, \tag{6.10}$$

the trajectories $e(t,e_0)$ of the error system will converge exponentially to the attractive region

$$\mathscr{X}_\infty \triangleq \left\{e \in \mathbb{R}^{2Nn_x} : e^\top P e \leq \frac{\gamma^2 b}{a}\right\}, \tag{6.11}$$

with a decay rate of $a/2$.

Proof. The proof follows the same reasoning performed in [62, 258]. Therefore, the proof is omitted. □

Finally, the problem addressed in this Chapter is stated as follows.

Problem 6.1

Given the LPV MAS described in (6.1), determine a condition to design both the distributed observer-based gain-scheduled consensus protocol (6.3), and the gain-scheduled state-observer (6.4) such that the trajectories $e(t, e_0)$ of the error system (6.7) converges exponentially towards the attractive region (6.11), and the practical consensus is achieved.

6.2 Main results

This section presents the proposed LMI-based design conditions of the distributed gain-scheduled observer-based consensus protocol (6.3).

Theorem 6.1

Consider closed-loop error dynamics (6.7) computed with the LPV MAS (6.1), the distributed gain-scheduled consensus protocol (6.3), and the gain-scheduled observer (6.4). Given positive scalars $a, b \in \mathbb{R}$, $\epsilon \in \mathbb{R}$, and the $\lambda_{\mathbf{m}}$ non-zero eigenvalues of the Laplacian matrix $\mathcal{L}$, $\forall \mathbf{m} = 2, \ldots, N$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{P}_2 \in \mathbb{R}^{n_x \times n_x}$, matrices $X_1 \in \mathbb{R}^{n_x \times n_x}$, $X_2 \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{K}_l \in \mathbb{R}^{n_u \times n_x}$, and $\widetilde{L}_l \in \mathbb{R}^{n_x \times n_y}$, such that the following inequalities hold

$$\Phi_{hh} < 0, \qquad \text{if } h = l, \tag{6.12}$$

$$\Phi_{hl} + \Phi_{lh} < 0, \qquad \text{if } h < l, \tag{6.13}$$

$$\Psi_{hh\mathbf{m}} < 0, \qquad \text{if } h = l, \ \forall\, \mathbf{m} = 2, \ldots, N, \tag{6.14}$$

$$\Psi_{hl\mathbf{m}} + \Psi_{lh\mathbf{m}} < 0, \qquad \text{if } h < l, \ \forall\, \mathbf{m} = 2, \ldots, N, \tag{6.15}$$

with

$$\Phi_{hl} = \begin{bmatrix} -\epsilon \mathrm{He}(X_1) & \star \\ \epsilon \widetilde{\Theta}_{hl}^{\top} + P_1 - X_1 & aP_1 + \mathrm{He}(\widetilde{\Theta}_{hl}) \end{bmatrix}, \tag{6.16}$$

$$\Psi_{hl\mathbf{m}} = \begin{bmatrix} -\epsilon \mathrm{He}(X_2) & \star & \star \\ \epsilon \widetilde{\Gamma}_{hl\mathbf{m}}^{\top} + \widetilde{P}_2 - X_2 & a\widetilde{P}_2 + \mathrm{He}(\widetilde{\Gamma}_{hl\mathbf{m}}) & \star \\ \epsilon I_{nx} & I_{nx} & -bI_{nx} \end{bmatrix}, \tag{6.17}$$

$$\widetilde{\Theta}_{hl} = X_1 A_h + \widetilde{L}_l C_h, \tag{6.18}$$

$$\widetilde{\Gamma}_{hl\mathbf{m}} = A_h X_2 + \lambda_m B_h \widetilde{K}_l, \tag{6.19}$$

for all $h, l \in \mathbb{N}_{\leq N_v}$. Then, the trajectories of the error dynamics (6.7) converge exponentially to the attractive region $\mathscr{X}_\infty$ defined in (6.11) with a decay rate of $a/2$.

Moreover, the gains of the consensus protocol (6.3), the gains of the observer (6.4), and the matrices of the Lyapunov function (6.9) are given, respectively, by $K_l = \widetilde{K}_l X_2^{-1}$, $L_l = X_1^{-1}\widetilde{L}_l$, P_1, and $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$.

Proof. Firstly by applying a separation principle argument, we have that the exponential convergence of (6.7) to the attractive region (6.11) can be analysed considering the subsystems

$$\dot{z} = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Big(\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big(I_N \otimes (A_h + L_l C_h)\Big)z, \tag{6.20}$$

$$\dot{\delta} = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Big(\bar{\Upsilon}\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big(I_N \otimes A_h + \mathcal{L}\otimes B_h K_l\Big)\delta + \omega. \tag{6.21}$$

Notice that the internal perturbation ω does not affect the state-estimation subsystem (6.20). Moreover, considering $V(z) = z^\top(I_N \otimes P_1)z$ it is possible to compute

$$\dot{V}(z) = \dot{z}^\top(I_N \otimes P_1)z + z^\top(I_N \otimes P_1)\dot{z}, \tag{6.22}$$

that can be written as

$$\dot{V}(z) = 2\sum_{h=1}^{N_v}\sum_{l=1}^{N_v} z^\top\Big(\alpha_{hl}(\rho)\otimes I_{n_x}\Big)\Big(I_N \otimes P_1\left(A_h + L_l C_h\right)\Big)z.$$

Replacing (6.18) in (6.16), and performing the exchange of variables $X_1 L_l = \widetilde{L}_l$, we have $\Theta_{hl} = X_1 A_h + X_1 L_l C_h$, and

$$\Phi_{hl} = \begin{bmatrix} -\epsilon \mathrm{He}(X_1) & \star \\ \epsilon\Theta_{hl}^\top + P_1 - X_1 & aP_1 + \mathrm{He}(\Theta_{hl}) \end{bmatrix}. \tag{6.23}$$

Multiplying (6.23) by $[A_h^\top + C_h^\top L_l^\top \;\; I_{n_x}]$ on the left, and its transpose in the right, results in

$$\Phi_{hl} = \mathrm{He}\left(P_1(A_h + L_l C_h)\right) + aP_1. \tag{6.24}$$

Considering the convex property of the time-varying parameters, it is possible to write (6.24) as

$$\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\alpha_{hl}(\rho)\Phi_{hl} = \sum_{h=1}^{N_v}\alpha_{hh}(\rho)\Phi_{hh} + \sum_{h=1}^{N_v}\sum_{l>h}^{N_v}\alpha_{hl}(\rho)\left(\Phi_{hl} + \Phi_{lh}\right),$$

from which it is possible to conclude that the LMIs (6.12)-(6.13) are sufficient to guarantee that $\dot{V}(z) + aV(z) < 0$, with $\dot{V}(z)$ as in (6.22).

Now considering the consensus-error (6.21), and choosing $V(\delta) = \delta^\top(I_N \otimes P_2)\delta$, it is possible to compute $\dot{V}(\delta)$ similarly to (6.22). Moreover, from the spectral decomposition of the Laplacian matrix $\mathcal{L}$, it is possible to define the exchange of coordinates $\widetilde{\delta} = (T^{-1}\otimes I_{n_x})\delta$. Applying the exchange of coordinates in (6.21) and in $\dot{V}(\delta)$, we obtain

$$\dot{\widetilde{\delta}} = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Big(\widetilde{\alpha}_{hl}(\rho)\otimes I_{n_x}\Big)\Big(I_N \otimes A_h + \Lambda \otimes B_h K_l\Big)\widetilde{\delta} + \widetilde{\omega}, \tag{6.25}$$

where $\tilde{\alpha}_{hl}(\rho) = T^{-1}\bar{\Upsilon}\alpha_{hl}(\rho)T$, $\tilde{\omega} = (T^{-1} \otimes I_{n_x})\omega$, and

$$\dot{V}(\tilde{\delta}) = 2\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\tilde{\delta}^\top\Big(\tilde{\alpha}_{hl}(\rho)\otimes I_{n_x}\Big)\Big(I_N \otimes P_2A_h + \Lambda \otimes P_2B_hK_l\Big)\tilde{\delta} + 2\tilde{\delta}^\top(I_N \otimes P_2)\tilde{\omega}. \quad (6.26)$$

Replacing (6.19) in (6.17), multiplying (6.17) on the left by $\mathrm{diag}(X_2^{-\top}, X_2^{-\top}, I_{n_x})$ and its transpose on the right, and performing the exchange of variables $\widetilde{K}_l = K_lX_2$, and $P_2 = X_2^{-\top}\widetilde{P}_2X_2^{-1}$, we have $\Gamma_{hlm} = X_2^{-\top}A_h + \lambda_m X_2^{-\top}B_hK_l$, and

$$\Psi_{hl\mathbf{m}} = \begin{bmatrix} -\epsilon\mathrm{He}(X_2^{-1}) & \star & \star \\ \epsilon\Gamma_{hl\mathbf{m}}^\top + P_2 - X_2^{-\top} & aP_2 + \mathrm{He}(\Gamma_{hl\mathbf{m}}) & \star \\ \epsilon X_2^{-1} & X_2^{-1} & -bI_{n_x} \end{bmatrix}. \quad (6.27)$$

Multiplying (6.27) by

$$\mathcal{B}^\top = \begin{bmatrix} A_h^\top + \lambda_m K_l^\top B_h^\top & I_{n_x} & 0 \\ I_{n_x} & 0 & I_{n_x} \end{bmatrix},$$

on the left, and $\mathcal{B}$ on the right results in

$$\Psi_{hl\mathbf{m}} = \begin{bmatrix} \mathrm{He}(P_2\,(A_h + \lambda_m B_hK_l) + aP_2 & \star \\ P_2 & -bI_{n_x} \end{bmatrix}. \quad (6.28)$$

Finally, performing a dimension adjustment and then multiplying (6.28) by $\varpi^\top = [\tilde{\delta}^\top \;\; \tilde{\omega}^\top]$ on the left, and its transpose on the right, we obtain

$$\begin{aligned}\varpi^\top\Psi_{hl\mathbf{m}}\varpi = {} & \tilde{\delta}^\top\left(\mathrm{He}\Big(I_N \otimes P_2\,(A_h + \lambda_m B_hK_l)\Big)\right)\tilde{\delta} \\ & + \tilde{\delta}^\top(I_N \otimes aP_2)\,\tilde{\delta} + \tilde{\omega}^\top\,(I_N \otimes P_2)\,\tilde{\delta} + \tilde{\delta}^\top\left(I_N \otimes P_2^\top\right)\tilde{\omega} - b\,\tilde{\omega}^\top\tilde{\omega}.\end{aligned}$$

Considering the convex property of the time-varying parameters, and exploring the fact that $\lambda_1 = 0$ and $\tilde{\delta}_1 = 0$ in the transformed subsystem, it is possible to conclude that the LMIs (6.14)-(6.15) are sufficient to guarantee that $\dot{V}(\tilde{\delta}) + aV(\tilde{\delta}) - b\,\tilde{\omega}^\top\tilde{\omega} < 0$, with $\dot{V}(\tilde{\delta})$ as in (6.26).

Moreover, notice that since P_1 and P_2 are symmetric positive definite matrices, we have $V(z) > 0$ and $V(\delta) > 0$, for all $z, \delta \neq 0$. Thus, it is possible to conclude that $V(e) = V(z) + V(\delta)$ as in (6.9) is positive definite. Furthermore, from the previously performed steps, it is possible to conclude that if the LMIs (6.12)-(6.15) hold, the derivative $\dot{V}(e) = \dot{V}(z) + \dot{V}(\delta)$ satisfies (6.10). Thus, all the conditions of Lemma 6.1 hold and the trajectories of the error system will converge exponentially to the attractive region (6.11). This concludes the proof. □

Remark 6.4

Note that differently from [241], the observer gains of Theorem 6.1 are recovered independently of the Lyapunov function (6.9). *Therefore, in the proposed approach, the design can be performed without imposing particular structures on the Lyapunov matrices in* (6.9).

6.3 Numerical examples

6.3.1 Example 1

In this example, a comparison of the proposed approach with the method developed in [241] to demonstrate its advantages. First, it is illustrated that the proposed practical consensus approach can improve the state consensus and reduce the consensus error for LPV MAS with different time-varying parameters. Moreover, it is also shown that the observer-based gain-scheduled approach in Theorem 6.1 achieves feasible results in cases where the observer-based gain-scheduled method developed in [241] fails.

Consider the LPV MAS, composed of 4 agents with the following two-vertex polytopic representation:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad A_2 = R_p^{-1} \begin{bmatrix} 0 & 1+p \\ -1 & 0 \end{bmatrix} R_p, \quad R_p = \begin{bmatrix} \cos(\bar{\beta}) & -\sin(\bar{\beta}) \\ \sin(\bar{\beta}) & \cos(\bar{\beta}) \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 \\ 1+10p \end{bmatrix}, \; C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 1+10p & 0 \end{bmatrix}, \; \bar{\beta} = \arctan(p),$$

and communication described by the Laplacian matrix

$$\mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

As explained in [241], with $p = 0$ the system is reduced to an LTI system, and for increasing values of $p > 0$, the difference between the two vertices increases. Thus, the goal is to compare the performance of the design in terms of p.

Considering $p = 0.49$ (the maximum value of p such that the conditions of [241] are feasible), $a = 0.9526$, $b = 1.6554$, and $\epsilon = 0.001$, the solution of the gain-scheduled observer based practical consensus protocol proposed in Theorem 6.1 is given by:

$$L_1^\top = \begin{bmatrix} -1.1236 & -0.6837 \end{bmatrix}, \; L_2^\top = \begin{bmatrix} -0.3373 & -0.2873 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -2.4882 & -0.7343 \end{bmatrix}, \; K_2 = \begin{bmatrix} -1.0936 & -0.2344 \end{bmatrix}.$$

The values of the scalar parameters a, b, and ϵ were defined by a grid search. Similarly to the non-synchronization scenario of [241], we perform simulations starting from the initial conditions $x_1(0) = [1 \;\; 1]^\top$, $x_2(0) = [1 \;\; 3]^\top$, $x_3(0) = [3 \;\; 1]^\top$, $x_4(0) = [3 \;\; 3]^\top$, and with different scheduling parameters, $\alpha_1(\rho_1(t)) = (1+\sin(2t))/2$; $\alpha_1(\rho_2(t)) = (1+\cos(0.1t))/2$; $\alpha_1(\rho_3(t)) = (1+\sin(0.5t))/2$; $\alpha_1(\rho_4(t)) = (1+\cos(0.05t))/2$; and $\alpha_2(\rho_i(t)) = (1-\alpha_1(\rho_i(t))), \forall i \in \mathcal{V}$. The simulation results are depicted in Figures 6.1-6.2.

In Figure 6.1, it is clear that closed-loop LPV MAS achieves practical consensus, even in the case of different scheduling parameters. Notice that the agents are maintaining the practical consensus while converging together toward the origin. Moreover, from Figure 6.2, it is possible to see the oscillations of the consensus error around the zero equilibrium, confirming once again that the practical consensus is achieved. When the agents are close to the origin, we have $\bar{x} \approx 0$. Consequently, the internal perturbation that depends on $\bar{x}$ is also $w \approx 0$ even with different scheduling parameters.

To illustrate the benefits of the proposed practical consensus, we will directly compare the norm of the augmented error vector $e(t)$, which comprises the estimation and consensus errors, with the norm of error in the non-synchronization scenario obtained with the approach proposed in [241]. From the results presented in Figure 6.3, we observe that, as expected, the convergence error obtained with the proposed approach in (——) is upper-bounded by the convergence error of the approach in [241] (- - - -). This result clearly demonstrates the superiority of the designed consensus protocol in dealing with the presence of different scheduling parameters compared to the current literature. It is important to highlight that, unlike the proposed method, the conditions of [241] were not designed to deal with this more general case, which makes clear the contribution and importance of the result in Theorem 6.1.

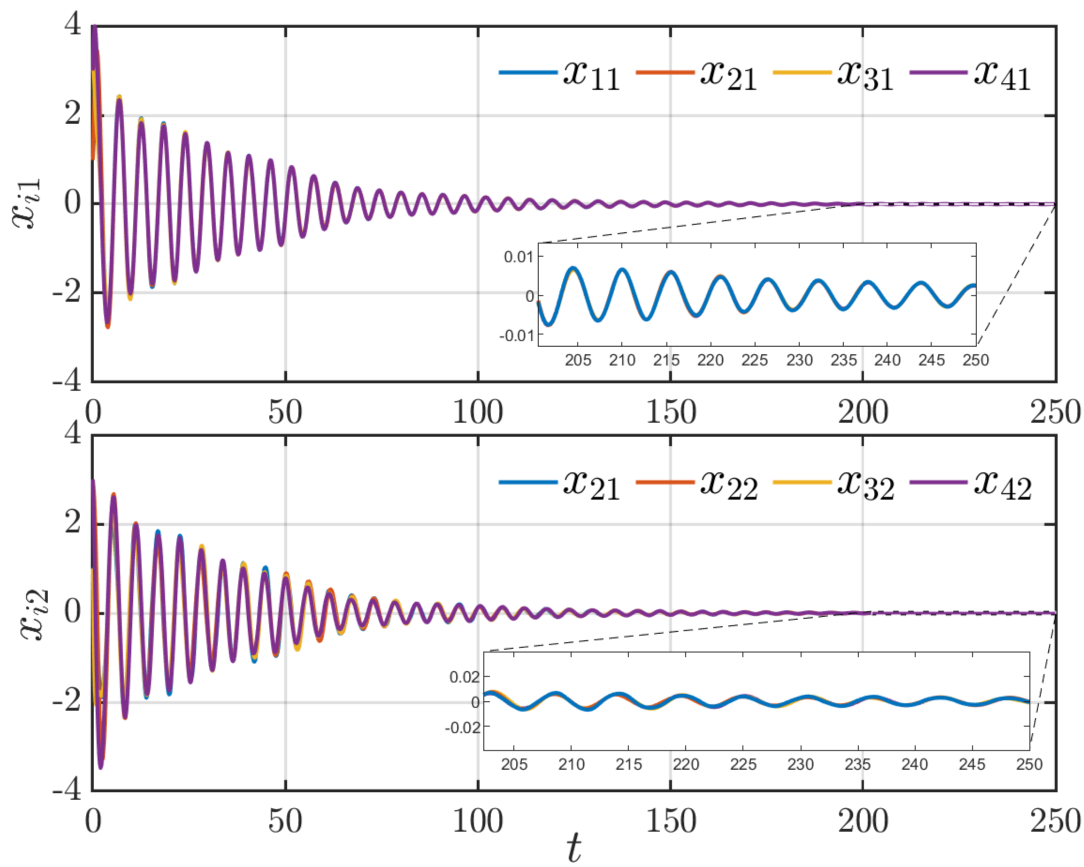

Figure 6.1 – State trajectories of the closed-loop LPV MAS with the proposed practical consensus for $p = 0.49$ – Example 6.3.1.

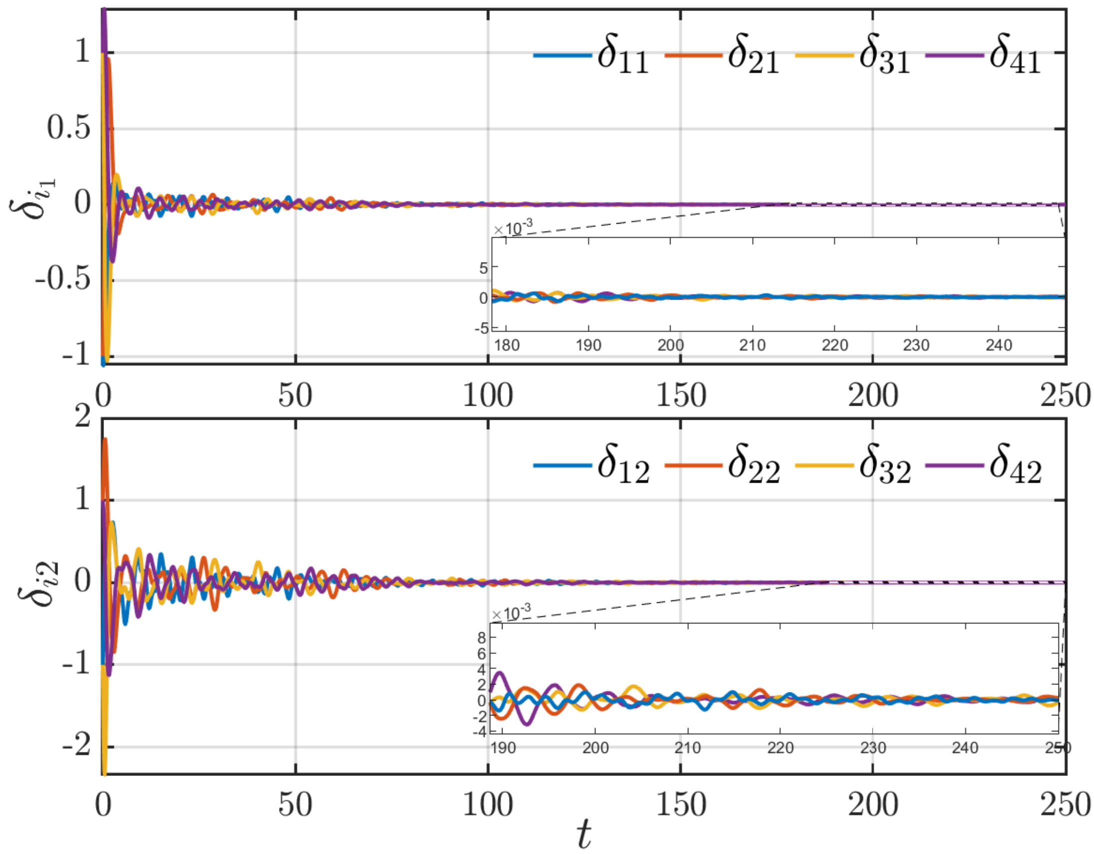

Figure 6.2 – Trajectories of the consensus error obtained with the proposed gain-scheduled observer-based practical consensus.

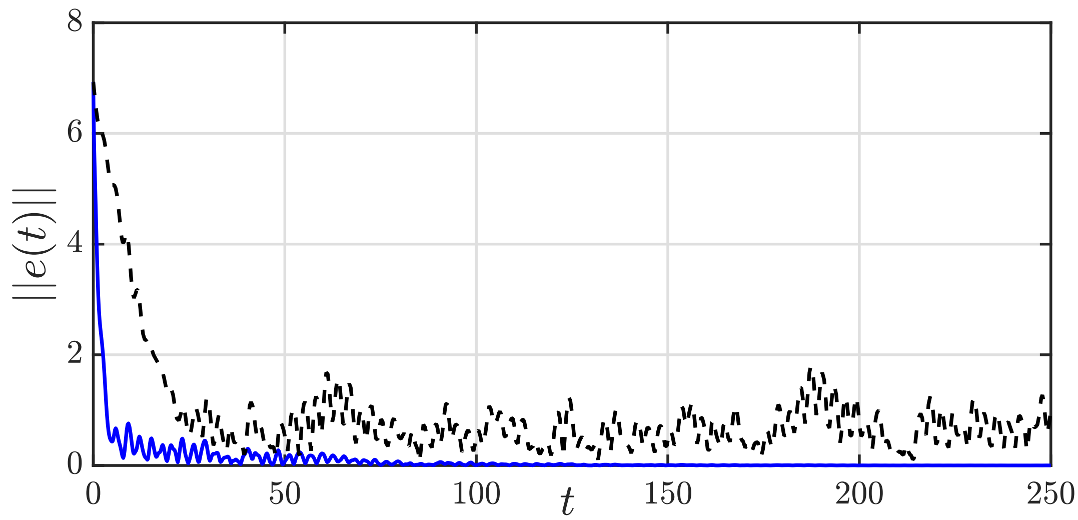

Figure 6.3 – Norm of the augmented error system considering the proposed approach (——) and the method of [241] (- - - -) – Example 6.3.1.

Now consider the LPV MAS with $p = 1$. In this case, the approach in [241] fails to provide feasible solutions. However, selecting by grid-search: $a = 1$, $b = 0.1$, and $\epsilon = 0.001$, it is possible, via Theorem 6.1, to have a solution to the gain-scheduled observer-based practical consensus protocol, namely:

$$L_1^\top = \begin{bmatrix} -1.2200 & -0.7220 \end{bmatrix}, \; L_2^\top = \begin{bmatrix} -0.2873 & -0.1977 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -5.2191 & -1.0451 \end{bmatrix}, \; K_2 = \begin{bmatrix} -1.4950 & -0.2352 \end{bmatrix}.$$

Similarly to [241], if the scheduling parameters are equal, we have that $\gamma = 0$, and the protocol proposed in Theorem 6.1 can achieve the exact consensus.Therefore, it is possible to conclude that the proposed approach also contributes to improving the completely homogeneous case, since it proposes less conservative design conditions, that obtain feasible solutions for bigger values of p compared to [241]. To illustrate this fact, Figures 6.4-6.5 depict the state trajectories obtained with a simulation considering the solution for $p = 1$, with different parameters and with $\alpha_1(\rho_i(t)) = (1 + \sin(2t))/2$, $\alpha_2(\rho_i(t)) = (1 - \alpha_1(\rho_i(t))), \forall i \in \mathcal{V}$, respectively.

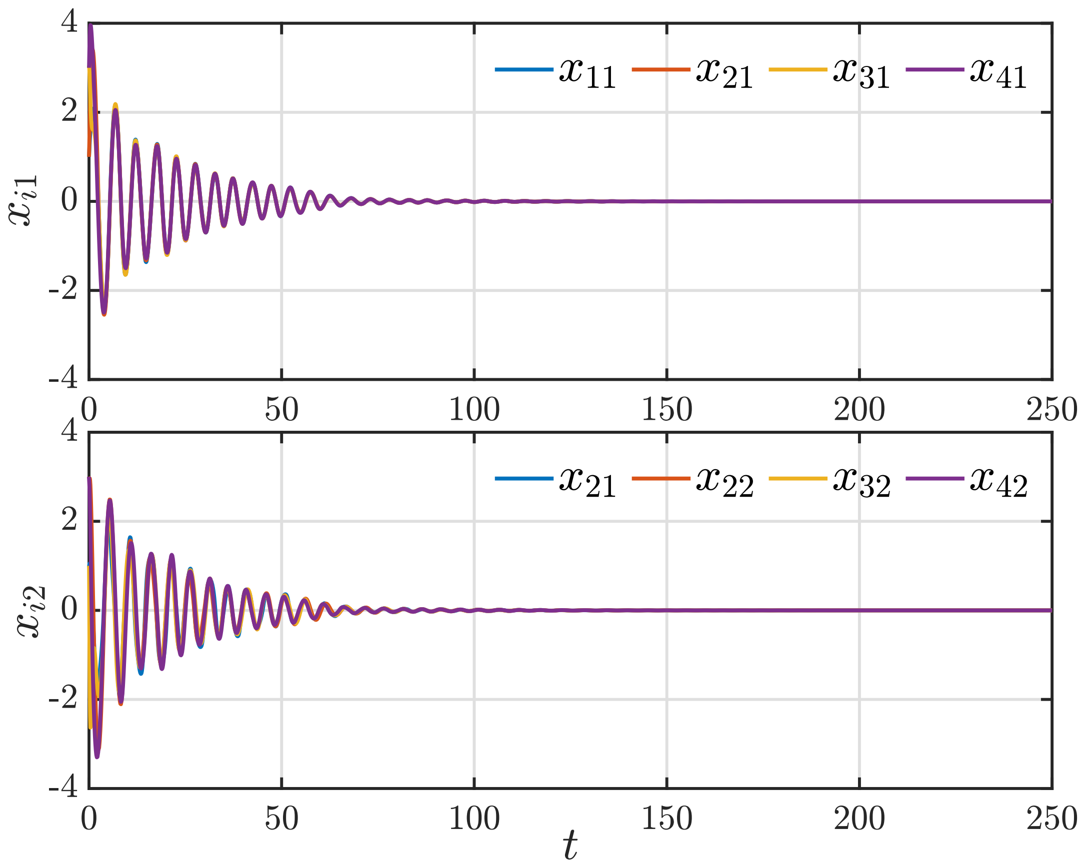

Figure 6.4 – State trajectories of the closed-loop LPV MAS for the case of $p = 1$ and different parameters – Example 6.3.1.

It can be seen that practical consensus is successfully achieved as depicted in Figure 6.4, and that exact consensus occurs as depicted in Figure 6.5. Similarly to the previous case, when different parameters are considered in the simulations, the trajectories of the agents maintain the practical consensus while converging towards the origin. This behavior is possible because the combination of the proposed protocol with different parameters results in stable heterogeneous dynamics. Meanwhile, when considering the same parameter for all agents, the closed-loop MAS is completely homogeneous and the exact consensus is achieved.

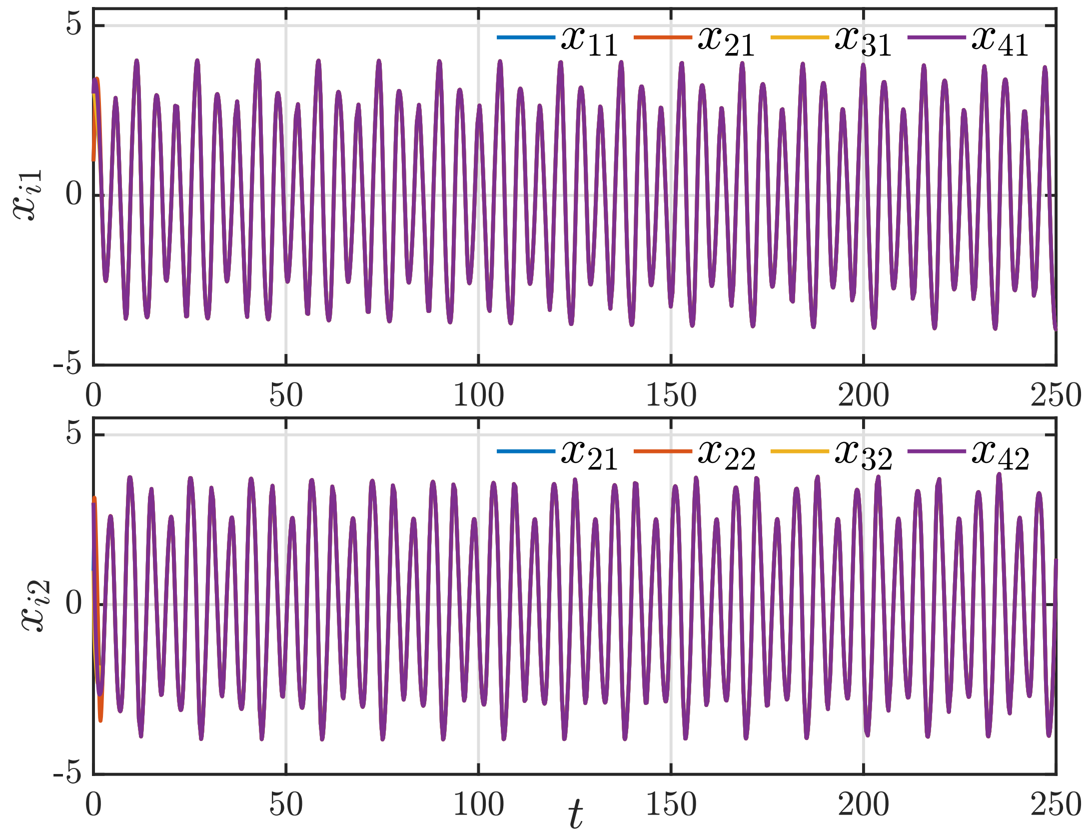

Figure 6.5 – State trajectories of the closed-loop LPV MAS for the case of $p = 1$ and equal parameters – Example 6.3.1.

6.3.2 Example 2

Consider an adaptation of the LPV MAS presented in Example 1 of [240]. The matrices $A(\bar{\eta})$ and $B(\bar{\eta})$ are assumed to remain the same, but the output matrix is replaced by $C(\bar{\eta})$. The system is composed of a four-vertex polytopic representation given by:

$$A(\bar{\eta}) = \begin{bmatrix} 0 & -0.18 \\ 1.07 + 0.3\bar{\eta}_1 + 0.4\bar{\eta}_2 & 0 \end{bmatrix}, \; B(\bar{\eta}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; C(\bar{\eta}) = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

where $\bar{\eta}_1$ and $\bar{\eta}_2$ belong to the intervals $0.1 \leq \bar{\eta}_1 \leq 0.2$, $0.3 \leq \bar{\eta}_2 \leq 0.4$.

Moreover, consider that the system has $N = 10$ agents and the Laplacian matrix encompasses the following neighborhood: $\mathcal{N}_1 = \{2\}$, $\mathcal{N}_2 = \{1, 3, 4\}$, $\mathcal{N}_3 = \{2, 5\}$, $\mathcal{N}_4 = \{2, 5\}$, $\mathcal{N}_5 = \{3, 4, 6\}$, $\mathcal{N}_6 = \{5, 7, 8\}$, $\mathcal{N}_7 = \{6, 9\}$, $\mathcal{N}_8 = \{6, 9\}$, $\mathcal{N}_9 = \{7, 8, 10\}$, $\mathcal{N}_{10} = \{9\}$.

Selecting $a = 1$, $b = 0.001$ and $\epsilon = 0.001$, the solution of Theorem 6.1 results in the observer-based practical consensus protocol given by:

$$\begin{aligned} K_1 &= \begin{bmatrix} -3.0498 & -6.5706 \end{bmatrix}, & K_2 &= \begin{bmatrix} -3.0663 & -6.5793 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -3.0625 & -6.5809 \end{bmatrix}, & K_4 &= \begin{bmatrix} -3.0829 & -6.6023 \end{bmatrix}, \\ L_1^\top &= \begin{bmatrix} -1.2118 & 2.8951 \end{bmatrix}, & L_2^\top &= \begin{bmatrix} -1.2118 & 2.8551 \end{bmatrix}, \\ L_3^\top &= \begin{bmatrix} -1.2118 & 2.8651 \end{bmatrix}, & L_4^\top &= \begin{bmatrix} -1.2118 & 2.8251 \end{bmatrix}. \end{aligned}$$

To evaluate the effectiveness of the proposed consensus protocol, we have performed temporal simulations with initial conditions within the range $x_1, x_2 \in [-5, 5]$. Similarly to Example 1, we have considered a different trigonometric variation for the parameters $\bar{\eta}_1^i(t)$ and $\bar{\eta}_2^i(t)$ within its specific bounds. The state trajectories of the closed-loop LPV MAS, equipped with the proposed distributed observer-based consensus protocol and considering different scheduling parameters are depicted in Figure 6.6.

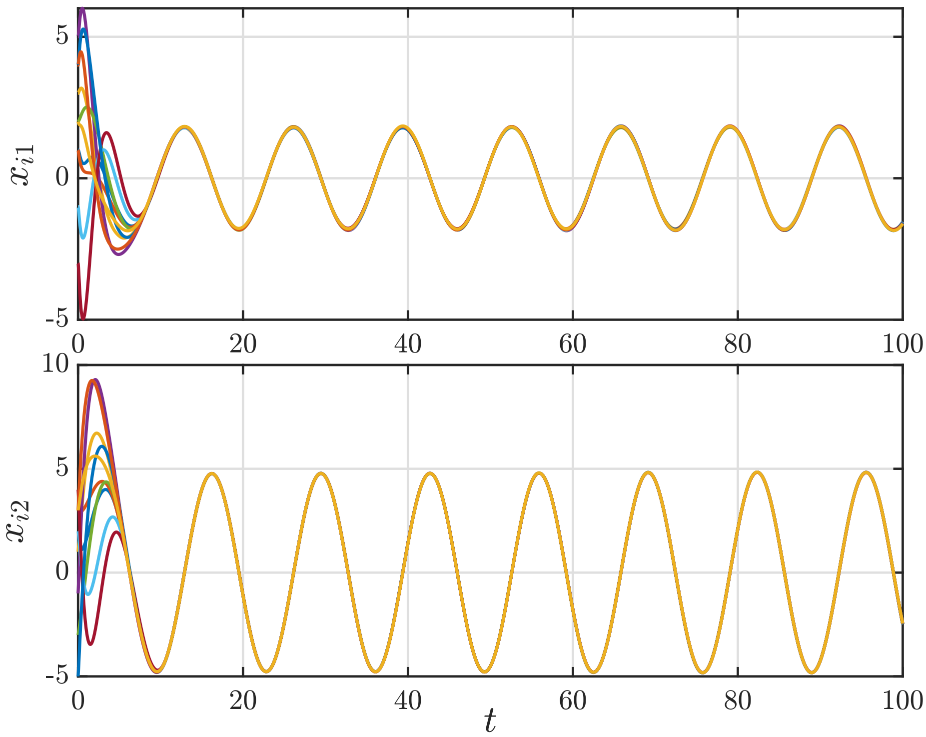

Figure 6.6 – State trajectories of the closed-loop LPV MAS of Example 6.3.2.

From the simulation results presented in Figure 6.6, it is possible to conclude that the practical consensus is successfully achieved. Furthermore, with the simulated state trajectories, we can compute the internal perturbation $\omega(t)$ and γ, for $0 \le t \le 100$. Thus, an estimate of the attractive region (6.11) can be performed with an upper bound $r = \sqrt{\frac{\gamma^2 b}{\underline{\lambda}(P)}}$ for the norm of the augmented error system. The trajectory of the norm and the upper bound obtained r are presented in Figure 6.7. Since an exact consensus has not been reached, the norm of the error system does not converge to the origin. However, notice that, as expected, the norm in steady state is constrained in the estimated attractive region.

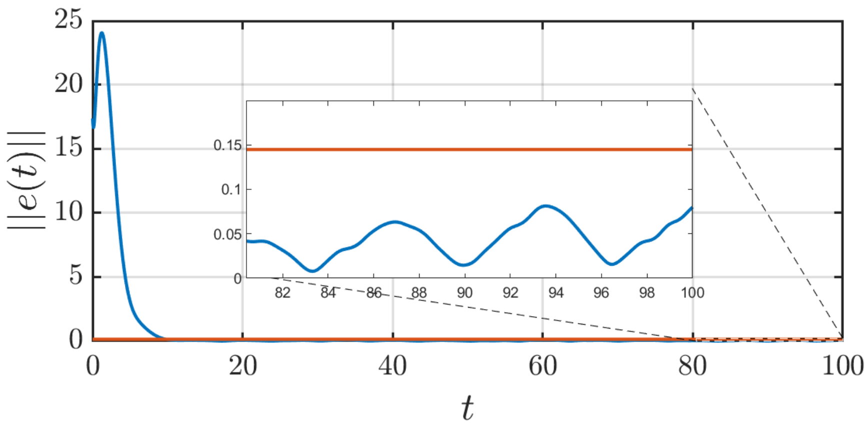

Figure 6.7 – Norm of the consensus error with the obtained bound for the attractive region - Example 6.3.2.

6.4 Conclusions

This chapter has presented a novel distributed gain-scheduled observer-based approach for the practical consensus of multi-agent systems represented by continuous-time LPV models. It has been shown that due to mismatched scheduling parameters among the agents, the exact state-consensus cannot be achieved with a classical distributed consensus protocol. To cope with the resultant internal perturbations, the proposed method ensures an alternative solution where the trajectories of the consensus error exponentially converge to a bounded region around the origin. Furthermore, the effectiveness of proposed protocol was illustrated through numerical methods compared with existent gain-scheduled observer-based approaches.

7 A DISTRIBUTED OBSERVER-BASED APPROACH FOR THE LEADER-FOLLOWING FORMATION CONSENSUS OF DISCRETE-TIME LPV MULTI-AGENT SYSTEMS

This chapter addresses the leader-following formation consensus problem for multi-agent systems (MASs) represented by discrete-time linear parameter-varying (LPV) models. The scenario in which each agent can be modeled with different time-varying scheduling parameters is investigated with the design of compensation signals. Using Lyapunov stability arguments, sufficient conditions are derived for designing a distributed gain-scheduled observer-based consensus protocol that ensures formation tracking. Furthermore, the scenario where the effects of the desired formation and mismatches on the scheduling-parameters are considered as internal perturbations is also investigated. Under this assumption, we propose sufficient design conditions to ensure that the combined estimation and tracking error dynamics is ℓ_∞-norm bounded. The effectiveness of the proposed leader-following framework is illustrated through numerical examples.

The remainder of this chapter is organized as follows. The problem formulation is presented in §7.1. The proposed design conditions are defined in §7.2. Two numerical examples are presented in §7.3 to illustrate the application of the proposed methodology. Conclusions are drawn in §7.4.

7.1 Problem formulation

Consider an MAS comprising a leader and N followers represented by discrete-time LPV systems. The dynamics of the followers are given by

$$\begin{aligned} x_i(k+1) &= A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k), \\ y_i(k) &= C(\rho_i(k))x_i(k), \end{aligned} \tag{7.1}$$

where $x_i(k) \in \mathbb{R}^{n_x}$ is the state vector, $u_i(k) \in \mathbb{R}^{n_u}$ is the control input, $y_i(k) \in \mathbb{R}^{n_y}$ is the output, and $\rho_i(k) \in \mathbb{R}^p$ is the vector of time-varying scheduling parameters, which are functions of measured exogenous signals. Meanwhile, the leader system is described as

$$s(k+1) = A(\rho_s(k))s(k), \tag{7.2}$$

where $s(k) \in \mathbb{R}^{n_x}$ is the state vector and $\rho_s(k) \in \mathbb{R}^p$ is the vector of time-varying scheduling parameters of the leader.

The parameter-dependent matrices of the followers $A(\rho_i(k)) \in \mathbb{R}^{n_x \times n_x}$, $B(\rho_i(k)) \in \mathbb{R}^{n_x \times n_u}$, and $C(\rho_i(k)) \in \mathbb{R}^{n_y \times n_x}$, the parameter-dependent matrix of the leader $A(\rho_s(k))$, and all the parameter-dependent matrices to be designed in this thesis belong to a polytopic domain and can be generically written as functions of the scheduling parameters as follows:

$$A(\rho_i(k)) = \sum_{h=1}^{N_v} \alpha_h(\rho_i(k))A_h, \quad A(\rho_s(k)) = \sum_{h=1}^{N_v} \alpha_h(\rho_i(k))A_h,$$
$$C(\rho_i(k)) = \sum_{h=1}^{N_v} \alpha_h(\rho_i(k))C_h, \quad B(\rho_i(k)) = \sum_{h=1}^{N_v} \alpha_h(\rho_i(k))B_h,$$

where N_v is the number of vertices of the polytopic domain and $\alpha_h(\rho_i(k))$ satisfy the convex sum property:

$$\sum_{h=1}^{N_v} \alpha_h(\rho_i(k)) = 1, \text{ and } \alpha_h(\rho_i(k)) \geq 0, \quad \forall i \in \mathbb{N}_{\leq N}.$$

Notice that similarly to Chapter 6, the parameter-dependent matrices of the agents are homogeneous with respect to the vertices of the polytopic domain. However, they have independent time-varying scheduling parameters, which induces heterogeneity in global dynamics. Moreover, the leader-dynamics $A(\rho_s(k))$ also share the same vertices of the followers' representation.

The main goal of this chapter is to design distributed observer-based consensus laws $u_i(k)$, $\forall i \in \mathbb{N}_{\leq N}$, such that the followers described in (7.1) track the trajectory generated by the leader-dynamics (7.2) according to a desired formation, as established in Definition 7.1.

Definition 7.1: Adapted from [259, 260]

The MAS (7.1)-(7.2) achieves leader-following formation consensus if

$$\lim_{k\to\infty} ||x_i(k) - s(k) - f_i|| = 0, \quad \forall i \in \mathbb{N}_{\leq N}, \tag{7.3}$$

for any initial condition $x_i(0)$, where f_i denotes the desired constant formation vector of agent i with respect to the leader trajectory.

To ensure that the MAS achieves the leader-following formation consensus (7.3), the proposed gain-scheduled observer-based distributed consensus law is:

$$u_i(k) = K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i}\left[\bar{x}_i(k) - \bar{x}_j(k)\right] + \eta_i\left(\bar{x}_i(k) - s(k)\right)\right) + \nu_i(k) + r_i(k), \tag{7.4}$$
$$\nu_i(k) = M(\rho_i(k))f_i \tag{7.5}$$
$$r_i(k) = \eta_i R(\rho_i(k))s(k), \tag{7.6}$$

where $K(\rho_i(k)) \in \mathbb{R}^{n_u\times n_x}$, $M(\rho_i(k)) \in \mathbb{R}^{n_u\times n_x}$, and $R(\rho_i(k)) \in \mathbb{R}^{n_u\times n_x}$, are the gain-scheduled functions to be designed, $\bar{x}_i(k) = \hat{x}_i(k) - f_i$, $\hat{x}_i \in \mathbb{R}^{n_x}$ is the estimated state of the local agent, and η_i are the pinning that indicate whether the i-th follower has access to the leader dynamics ($\eta_i = 1$) or not ($\eta_i = 0$).

For each agent, the local estimated state is given by

$$\hat{x}_i(k+1) = A(\rho_i(k))\hat{x}_i(k) + B(\rho_i(k))u_i(k) + L(\rho_i(k))\Big(C(\rho_i(k))\hat{x}_i(k) - y_i(k)\Big), \tag{7.7}$$

where $L(\rho_i(k)) \in \mathbb{R}^{n_x \times n_y}$ is the gain-scheduled observer gain to be designed.

The gain-scheduled functions associated with the proposed distributed consensus protocol and the local observers belong to the same polytopic domain and can be expressed as:

$$K(\rho_i(t)) = \sum_{l=1}^{N_v} \alpha_l(\rho_i(t))K_l, \quad L(\rho_i(t)) = \sum_{l=1}^{N_v} \alpha_l(\rho_i(t))L_l,$$
$$M(\rho_i(t)) = \sum_{l=1}^{N_v} \alpha_l(\rho_i(t))M_l, \quad R(\rho_i(t)) = \sum_{l=1}^{N_v} \alpha_l(\rho_i(t))R_l,$$

where K_l, L_l, M_l and R_l are the vertices to be designed.

Remark 7.1

The proposed observer-based gain-scheduled distributed consensus law (7.4) *can be partitioned into three components. The term* $K(\rho_i(k))\Big(\sum_{j\in\mathcal{N}_i}\big[\bar{x}_i(k) - \bar{x}_j(k)\big] + \eta_i\left(\bar{x}_i(k) - s(k)\right)\Big)$*, concerns the distributed relative information with respect to the neighboring agents and the leader. Notice that if* $f_i = 0, \forall i \in \mathbb{N}_{\leq N}$*, this term is reduced to a leader-following consensus protocol. The components* $\nu_i(k)$ *and* $r_i(k)$ *are compensation signals introduced to deal with the desired formation* f_i*, and the mismatch between the scheduling parameters of the agents and the leader, respectively. Similar compensation signals can be found in the literature on formation tracking for MASs [260–263].*

The proposed design conditions will be developed considering a global error composed of local estimation and consensus-tracking errors. From the discrete-time LPV model of the agents (7.1), and the defined local observer structure (7.7), it is possible to write the dynamics of the local estimation errors $z_i(k) = x_i(k) - \hat{x}_i(k)$ as

$$\begin{aligned} z_i(k+1) &= A(\rho_i(k))\left(x_i(k) - \hat{x}_i(k)\right) - L(\rho_i(k))\Big(C(\rho_i(k))\hat{x}_i(k) - y_i(k)\Big) \\ z_i(k+1) &= \left(A(\rho_i(k)) + L(\rho_i(k))C(\rho_i(k))\right)z_i(k). \end{aligned} \tag{7.8}$$

Let $\alpha_{hl}(\rho_k)$=diag$(\alpha_h(\rho_1(k))\alpha_l(\rho_1(k)), \ldots, \alpha_h(\rho_N(k))\alpha_l(\rho_N(k))$, $\rho(k)$=$(\rho_1(k), \ldots, \rho_N(k))$, and $z(k)$=$(z_1(k), \ldots, z_N(k))$, we have that

$$z(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}(\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes (A_h + L_l C_h)). \tag{7.9}$$

Meanwhile, by setting $\delta_i(k) = x_i(k) - s(k) - f_i$ as the local leader-following consensus tracking error, it is possible to compute

$$\delta_i(k+1) = A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k) - A(\rho_s(k))s(k) - f_i.$$

By the definitions of local estimation and consensus-tracking error, we have $\hat{x}_i(k) = x_i(k) - z_i(k)$, and $x_i(k) = \delta_i(k) + s(k) + f_i$, resulting in $\hat{x}_i(k) = \delta_i(k) + s(k) + f_i - z_i(k)$. Replacing

$\hat{x}_i(k)$ in the consensus law (7.4), we obtain

$$u_i(k)=K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i}\Big[(\delta_i(k)-z_i(k))-(\delta_j(k)-z_j(k))\Big]+\eta_i\left(\delta_i(k)-z_i(k)\right)\right)$$
$$+\nu_i(k)+r_i(k),\ \ \forall i\in\mathbb{N}_{\leq N},$$

allowing us to write the dynamics of the local consensus tracking error as

$$\begin{aligned}\delta_i(k+1) &= A(\rho_i(k))\delta_i(k)+\Big(A(\rho_i(k))-I+B(\rho_i(k))M(\rho_i(k))\Big)f_i\\ &+\Big(A(\rho_i(k))-A(\rho_s(k))+\eta_iB(\rho_i(k))R(\rho_i(k))\Big)s(k)\\ &+\eta_iB(\rho_i(k))K(\rho_i(k))\left(\delta_i(k)-z_i(k)\right)\\ &+B(\rho_i(k))K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i}\Big[(\delta_i(k)-z_i(k))-(\delta_j(k)-z_j(k))\Big]\right).\end{aligned}$$

Let $\alpha_{hlq}(\bar{\rho}_k)=\operatorname{diag}(\alpha_h(\rho_1(k))\alpha_l(\rho_1(k))\alpha_q(\rho_s(k)),\ldots,\alpha_h(\rho_N(k))\alpha_l(\rho_N)(k)\alpha_q(\rho_s(k)))$, $f=(f_1,\ldots,f_N)$, $\eta=\operatorname{diag}(\eta_1,\ldots,\eta_N)$, $\bar{s}(k)=(\mathbf{1}\otimes s(k))$, and $\delta(k)=(\delta_1(k),\ldots,\delta_N(k))$. Then, similarly to the estimation error, it is possible to write

$$\begin{aligned}\delta(k+1) = &\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}(\alpha_{hl}(\rho_k)\otimes I_{n_x})\Big\{(I_N\otimes A_h+\bar{\mathcal{L}}\otimes B_hK_l)\delta(k)\\ &-(\bar{\mathcal{L}}\otimes B_hK_l)z(k)+(I_N\otimes(A_h+B_hM_l-I_{n_x}))\,f\Big\}\\ &+\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\sum_{q=1}^{N_v}(\alpha_{hlq}(\bar{\rho}_k)\otimes I_{n_x})(I_N\otimes(A_h-A_q)-\eta\otimes B_hR_l)\bar{s}(k).\end{aligned} \tag{7.10}$$

Finally, defining the augmented error system $e(k)=(z(k),\delta(k))$, we have

$$e(k+1)=\bar{A}(\rho_k)e(k)+\bar{B}_ww(k), \tag{7.11}$$

where $w(k)=w_f(k)+w_s(k)$, and

$$\begin{aligned}\bar{A}(\rho_k)&=\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\begin{bmatrix}\alpha_{hl}(\rho_k)\otimes(A_h+L_lC_h) & 0\\ -\alpha_{hl}(\rho_k)\bar{\mathcal{L}}\otimes B_hK_l & \alpha_{hl}(\rho_k)\otimes A_h+\alpha_{hl}(\rho_k)\bar{\mathcal{L}}\otimes B_hK_l\end{bmatrix},\\ w_f(k)&=\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\left(\alpha_{hl}(\rho_k)\otimes(A_h+B_hM_l-I_{n_x})\right)f,\ \bar{B}_w=\begin{bmatrix}\mathbf{0}\\ \mathbf{1}\end{bmatrix},\\ w_s(k)&=\sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\sum_{q=1}^{N_v}\left((\alpha_{hlq}(\bar{\rho}_k)\otimes I_{n_x})(I_N\otimes(A_h-A_q)-\eta\otimes B_hR_l)\right)\bar{s}(k).\end{aligned}$$

With the error system (7.11), it is possible to conclude that the leader-following formation consensus (7.3) can be achieved only if $w(k)=0$. The term $w(k)$ can be seen as an internal perturbation that arises in the closed-loop error system due to the desired formation ($w_f(k)$), and heterogeneity among the scheduling parameters of the agents and the leader ($w_s(k)$). The compensation signals (7.5)-(7.6) introduced in the consensus law (7.4) are designed to compensate the signal $w(k)$ such that the augmented error system (7.11) is reduced to $e(k+1) = \bar{A}(\rho_k)e(k)$, which allows the design of the control and the observer gains, $K(\rho_i(k))$ and $L(\rho_i(k))$, by employing a separation principle argument.

Remark 7.2

As discussed in [260], without the compensation signal (7.5)*, the leader-following formation consensus* (7.3) *is achieved only for specific formations* f_i *that satisfy* $\Big(A(\rho_i(k)) - I\Big)f_i = 0$*. Therefore, the compensation signal* (7.5) *is essential to extend the range of feasible formations. Another outstanding contribution of this Chapter is to deal with heterogeneity among the scheduling parameters of the agents and the leader by means of the additional compensation signal* (7.6)*. Similarly, if* (7.6) *is not introduced in the consensus law, the leader-following formation consensus* (7.3) *can be achieved by forcing that all scheduling parameters are equal, that is,* $\rho_1(k) = \rho_2(k) = \ldots = \rho_N(k) = \rho_s(k)$*, once it results in* $A(\rho_i(k)) = A(\rho_s(k))$*,* $\forall\, i \in \mathbb{N}_{\leq N}$*. Thus,* (7.6) *allows for exact leader-following formation in a more general scenario.*

The design of the compensation signal (7.5) requires only local information. However, to cope with the heterogeneity among the scheduling parameters, the design of (7.6) requires access to the states and scheduling parameters of the leader, as defined in the next Assumption.

Assumption 7.1

The leader states $s(k)$, and scheduling parameters $\rho_s(k)$ are available for all agents of the MASs (6.1), that is, $\eta_i = 1$, $\forall\, i \in \mathbb{N}_{\leq N}$.

Based on the previous discussion, the first problem addressed in this Chapter can be stated as follows.

Problem 7.1

Given the LPV MAS described in (7.1), assuming that Assumption 7.1 holds, design the distributed observer-based consensus law (7.4), and the observer (7.7), such that the origin of the error system (7.11) is exponentially stable and the leader-following formation consensus (7.3) is achieved.

In this Chapter, the case where Assumption 7.1 does not hold and the compensation signals cannot be designed is also investigated. In this scenario, it is required that at least one follower agent can obtain the information from the leader. Moreover, we have that if $M(\rho_i(k)) = 0_{n_u \times n_{n_x}}$ and $R(\rho_i(k)) = 0_{n_u \times n_{n_x}}$, the internal perturbations of the augmented error system (7.11) can be rewritten as

$$w_f(k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \left(\alpha_{hl}(\rho_k) \otimes (A_h - I_{n_x})\right) f, \tag{7.12}$$

$$w_s(k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\sum_{q=1}^{N_v} \left(\alpha_{hlq}(\bar{\rho}_k) \otimes (A_h - A_q)\right) \bar{s}(k). \tag{7.13}$$

For the sequence of vectors $\{w_f(k)\}_{k\in\mathbb{Z}^+}$, and $\{w_s(k)\}_{k\in\mathbb{Z}^+}$, define $||w_f||_{\ell_\infty}=\sup_{k\geq 0}||w_f(k)||$, and $||w_s||_{\ell_\infty} = \sup_{k\geq 0}||w_s(k)||$. Consequently, considering that the leader dynamics is not unstable and the desired formations are finite, we have that

$$||w||_{\ell_\infty} \leq ||w_f||_{\ell_\infty} + ||w_s||_{\ell_\infty} < \infty$$

Due to the effects of these internal perturbations, the leader-following consensus (7.3) cannot be achieved. Therefore, the second problem addressed in this paper is to ensure that in the absence of compensation signals, the error system (7.11) is bounded, as described in the following problem.

Problem 7.2

Given the LPV MAS described in (7.1), if Assumption 7.1 does not hold, design a distributed observer-based consensus law in the form

$$u_i(k) = K(\rho_i(k)) \left(\sum_{j\in\mathcal{N}_i} \bar{x}_i(k) - \bar{x}_j(k) + \eta_i\left(\bar{x}_i(k) - s(k)\right) \right), \tag{7.14}$$

such that the augmented error system (7.11), with the internal perturbations given by (7.12) and (7.13), is bounded for any initial condition $e(0)$, and any sequence $\{w(k)\}_{k\in\mathbb{Z}^+} \in \ell_\infty$. That is, there exists an upper bound $\varphi(e(0), ||w||_{\ell_\infty})$ such that

$$||e(k)|| \leq \varphi(e(0), ||w||_{\ell_\infty}), \ \ \forall k \geq 0,$$

and

$$\lim_{k\to\infty} \sup ||e(k)|| < \gamma ||w||_{\ell_\infty}, \tag{7.15}$$

where $\gamma > 0$ corresponds to the ℓ_∞ performance level.

Remark 7.3

Note that Problem 7.1 considers the case where both compensation signals (7.5) *and* (7.6) *can be properly designed. In contrast, Problem 7.2 considers the case where neither of them can be designed. The situations where only* (7.5) *or only* (7.6) *is designed are special cases of Problem 7.2, corresponding to* $w(k) = w_s(k)$ *and* $w(k) = w_f(k)$*, respectively.*

7.2 Main Results

This section presents the main results. First, the conditions to ensure the exponential and ℓ_∞ stability of the augmented error system are defined based on the Lyapunov Theory. Then, assuming that the compensation signals are properly designed, sufficient conditions are provided for the design of the compensated consensus law able to ensure the leader-following consensus (7.3). Finally, sufficient conditions are presented for the design of the uncompensated consensus law that provides the bounding guarantees for the ℓ_∞ gain (7.15).

7.2.1 Exponential and ℓ_∞ stability analysis conditions

The proposed approach is based on the Lyapunov theory presented in the following analysis condition.

Lemma 7.1

If there exist positive scalars γ, $\sigma \in (0,1)$, and symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x \times n_x}$ and $P_2 \in \mathbb{R}^{n_x \times n_x}$, such that the Lyapunov function

$$V(e(k)) = e(k)^\top P\, e(k), \quad P = \text{diag}((I_N \otimes P_1), (I_N \otimes P_2)), \tag{7.16}$$

satisfies

$$\Delta_{Vk} + \sigma\Big(V(e(k)) - \gamma w(k)^\top w(k)\Big) < 0, \tag{7.17}$$

$$\begin{bmatrix} P & \star \\ I & \gamma I \end{bmatrix} > 0, \tag{7.18}$$

where $\Delta_{Vk} = V(e(k+1)) - V(e(k))$, then the augmented error system (7.11), with the rewritten internal perturbations given by (7.12) and (7.13) is bounded by

$$||e(k)|| < \sqrt{\gamma(1-\sigma)^k V(e(0)) + \gamma^2 ||w||^2_{\ell_\infty}}, \tag{7.19}$$

and ℓ_∞-stable with performance level γ. Moreover, if $w(k) = 0\ \forall k \geq 0$, the augmented error system (7.11) is exponentially stable with respect to the origin, and the leader-following formation consensus (7.3) is achieved.

Proof. The proof follows the same reasoning as in [264]. First, notice that condition (7.17) can be written as

$$V(e(k+1)) < (1-\sigma)V(e(k)) + \sigma\gamma w(k)^\top w(k)),$$

which recursively implies that

$$\begin{aligned} V(e(k)) &< (1-\sigma)^k V(e(0)) + \sigma\gamma \sum_{i=0}^{k-1} (1-\sigma)^i ||w(k-1-i)||^2, \quad \forall k \geq 1, \\ &< (1-\sigma)^k V(e(0)) + \gamma ||w||^2_{\ell_\infty}, \end{aligned} \tag{7.20}$$

once $\sigma \in (0,1)$. By applying the Schur complement in (7.18) we have

$$P > \gamma^{-1} I,$$

which is equivalent to

$$\gamma e(k)^\top P e(k) > e(k)^\top e(k). \tag{7.21}$$

By combining (7.20) and (7.21), it is possible to conclude that

$$||e(k)||^2 < \gamma V(e(k)) < \gamma(1-\sigma)^k V(e(0)) + \gamma^2 ||w||^2_{\ell_\infty}, \quad \forall k \geq 1,$$

resulting in the bound defined in (7.19). Moreover, notice that if $w(k) = 0$, the condition (7.20) is reduced to $V(e(k)) < (1-\sigma)^k V(e(0))$, guaranteeing that the origin of the error system is exponentially stable. This concludes the proof. □

7.2.2 Compensated-consensus design conditions

In the sequel, the design conditions proposed to solve Problem 7.1 are presented.

Theorem 7.1

On the basis that Assumption 7.1 holds, consider: the augmented error dynamics (7.11) obtained with discrete-time LPV MAS (7.1), the gain-scheduling consensus protocol (7.4), and the observer (7.7). Given positive scalars $\sigma \in (0,1)$, $\epsilon \in \mathbb{R}^+$, and the eigenvalues $\bar{\lambda}_{\mathbf{m}}$ of $\bar{\mathcal{L}}$, $\forall\, \mathbf{m} = 1, \ldots, N$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{P}_2 \in \mathbb{R}^{n_x \times n_x}$, matrices $X_1 \in \mathbb{R}^{n_x \times n_x}$, $X_2 \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{K}_l \in \mathbb{R}^{n_u \times n_x}$, and $\widetilde{L}_l \in \mathbb{R}^{n_x \times n_y}$, such that the following inequalities hold

$$\Psi_{hh} < 0, \qquad \text{if } h = l, \tag{7.22}$$

$$\Psi_{hl} + \Psi_{lh} < 0, \qquad \text{if } h < l, \tag{7.23}$$

$$\Phi_{hh\mathbf{m}} < 0, \qquad \text{if } h = l,\ \forall\, \mathbf{m} = 1, \ldots, N, \tag{7.24}$$

$$\Phi_{hl\mathbf{m}} + \Phi_{lh\mathbf{m}} < 0, \qquad \text{if } h < l,\ \forall\, \mathbf{m} = 1, \ldots, N, \tag{7.25}$$

with

$$\Psi_{hl} = \begin{bmatrix} P_1 - \epsilon\,\mathrm{He}(X_1) & \star \\ \epsilon\,\widetilde{\Theta}_{hl}^\top & (\sigma-1)P_1 \end{bmatrix}, \tag{7.26}$$

$$\Phi_{hl\mathbf{m}} = \begin{bmatrix} \widetilde{P}_2 - \epsilon\,\mathrm{He}(X_2) & \star \\ \epsilon\,\widetilde{\Gamma}_{hl\mathbf{m}}^\top & (\sigma-1)\widetilde{P}_2 \end{bmatrix}, \tag{7.27}$$

$$\widetilde{\Theta}_{hl} = X_1 A_h + \widetilde{L}_l C_h, \tag{7.28}$$

$$\widetilde{\Gamma}_{hl\mathbf{m}} = A_h X_2 + \bar{\lambda}_m B_h \widetilde{K}_l, \tag{7.29}$$

for all $h, l \in \mathbb{N}_{\leq N_v}$, and if there exist compensation gains $M(\rho_i(k))$ and $R(\rho_i(k))$, such that the following conditions

$$\Big(A(\rho_i(k)) - I + B(\rho_i(k))M(\rho_i(k))\Big) f_i = 0, \tag{7.30}$$

$$\Big(A(\rho_i(k)) - A(\rho_s(k)) + B(\rho_i(k))R(\rho_i(k))\Big) s(k) = 0, \tag{7.31}$$

hold for all $i \in \mathbb{N}_{\leq N}$. Then $\{w(k) = 0\}_{k\in\mathbb{Z}^+}$, the error dynamics (7.11) is exponentially stable and the gains of the observer (7.7), the matrices of the Lyapunov function (7.16), and the remaining gains of the consensus protocol are given, respectively, by $L_l = X_1^{-1}\widetilde{L}_l$, $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$, P_1, and $K_l = \widetilde{K}_l X_2^{-1}$.

Proof. First, notice that matrices P_1 and $\widetilde{P}_2$ are symmetric and positive definite. Therefore, the Lyapunov function (7.16) is also positive definite. Moreover, considering that the conditions (7.30)-(7.31) hold we have that $\{w(k) = 0\}_{k\in\mathbb{Z}^+}$, and by employing the separation principle argument, the exponential stability of the augmented error system (7.11) can be evaluated considering the following subsystems

$$z(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \left(\alpha_{hl}(\rho_k) \otimes (A_h + L_l C_h)\right) z(k),$$

$$\delta(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \Big((\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes A_h + \bar{\mathcal{L}} \otimes B_h K_l)\Big)\,\delta(k).$$

Defining $V(z(k)) = z(k)^\top (I_N \otimes P_1) z(k)$, the condition $V(z(k+1)) + (\sigma - 1)\,V(z(k)) < 0$, is equivalent to

$$z(k+1)^\top (I_N \otimes P_1) z(k+1) + (\sigma-1) z(k)^\top (I_N \otimes P_1) z(k) < 0.$$

Replacing (7.28) in (7.26), and performing the change of variables $\widetilde{L}_l = X_1\,L_l$, results in

$$\begin{bmatrix} P_1 - \epsilon\,\mathrm{He}(X_1) & \epsilon(X_1 A_h + X_1 L_l C_h) \\ \star & (\sigma-1)P_1 \end{bmatrix}. \tag{7.32}$$

Multiplying (7.32) by $[A_h^\top + C_h^\top L_l^\top \; I_{n_x}]$, on the left and its transpose in the right, we obtain

$$\Psi_{hl} = (A_h + L_l C_h)^\top P_1 \,(A_h + L_l C_h) + (\sigma - 1)P_1,$$

from which it is possible to conclude that due to the convex properties of time-varying parameters, the LMIs (7.22)-(7.23) are sufficient to guarantee $V(z(k\ +\ 1))\ +\ (\sigma\ -\ 1)\ V(z(k)) < 0$. Furthermore, by defining

$$V(\delta(k)) = \delta(k)^\top (I_N \otimes P_2)\delta(k), \tag{7.33}$$

considering the spectral decomposition $\bar{\mathcal{L}} = \bar{T}\bar{\Lambda}\bar{T}^{-1}$, and the exchange of coordinates $\widetilde{\delta}(k) = (\bar{T}^{-1} \otimes I_{n_x})\delta(k)$, $\widetilde{\alpha}_{hl}(\rho k) = \bar{T}^{-1}\alpha_{hl}(\rho k)\bar{T}$, it is possible to write the condition $V(\delta(k+1)) + (\sigma - 1)V(\delta(k)) < 0$ as

$$\widetilde{\delta}(k+1)^\top (I_N \otimes P_2)\widetilde{\delta}(k+1) + (\sigma-1)\widetilde{\delta}(k)^\top (I_N \otimes P_2)\widetilde{\delta}(k) < 0,$$

with

$$\widetilde{\delta}(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \Big((\widetilde{\alpha}_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes A_h + \bar{\Lambda} \otimes B_h K_l) \Big)\, \widetilde{\delta}(k).$$

Multiplying (7.27) by $\mathrm{diag}(X_2^{-\top}, X_2^{-\top})$ on the left and its transpose on the right, replacing (7.29), and performing the exchange of variables $\widetilde{K_l} = K_l X_2$, and $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$ results in

$$\begin{bmatrix} P_2 - \epsilon \mathrm{He}(X_2^{-\top}) & \epsilon(X_2^{-\top}A_h + \bar{\lambda}_m X_2^{-\top}B_h K_l) \\ \star & (\sigma-1)P_2 \end{bmatrix}. \tag{7.34}$$

Multiplying (7.34) by $[A_h^\top + \bar{\lambda}_m K_l^\top B_h^\top \;\; I_{n_x}]$ on the left and its transpose in the right, it results in

$$\Phi_{hl\mathbf{m}} = (A_h + \bar{\lambda}_m B_h K_l)^\top P_2\, (A_h + \bar{\lambda}_m B_h K_l) + (\sigma-1)P_2,$$

from which it is possible to conclude that due to the convex properties of time-varying parameters, the LMIs (7.24)-(7.25) are sufficient to guarantee that $V(\delta(k\ +\ 1))\ +\ (\sigma\ -\ 1)\ V(\delta(k)) < 0$.

Combining the previous steps, it is possible to see that the LMIs (7.22)-(7.25) are sufficient to guarantee

$$V(e(k\ +\ 1))\ +\ (\sigma\ -\ 1)\ V(e(k)) < 0,$$

with a Lyapunov function $V(e(k)) = V(\delta(k)) + V(z(k))$, as defined in (7.16). Thus, if all the conditions of Theorem 7.1 hold, the trajectories of the error system (7.11) converge exponentially to the origin. This concludes the proof. □

Assuming that the compensation signals are properly designed, the presented Theorem 7.1 provides sufficient conditions to guarantee the exponential stability of the origin of the error system (7.11). If the input matrices $B(\rho_i(k))$ are invertible, the suitable design of the compensation gains can be directly performed by considering

$$\begin{aligned} M(\rho_i(k)) &= B(\rho_i(k))^{-1}\Big(I_{n_x} - A(\rho_i(k))\Big), \\ R(\rho_i(k)) &= B(\rho_i(k))^{-1}\Big(A(\rho_s(k)) - A(\rho_i(k))\Big). \end{aligned}$$

Moreover, if the matrices $B(\rho_i(k))$ are not invertible the Moore-Penrose pseudo-inverse $B(\rho_i(k))^\dagger$ may be applied for the computation of $M(\rho_i(k))$, and $R(\rho_i(k))$. In this case, the conditions (7.30) and (7.31) will be satisfied if

$$f_i \in \ker\Big(A(\rho_i(k)) - I + B(\rho_i(k))M(\rho_i(k))\Big), \quad \forall\, i \in \mathcal{V},$$
$$s(k) \in \ker\Big(A(\rho_i(k)) - A(\rho_s(k)) + B(\rho_i(k))R(\rho_i(k))\Big), \quad \forall\, i \in \mathcal{V}.$$

Note that, if the compensation gains $M(\rho_i(k))$ can be designed independently of the formations f_i, it is possible to extend the range of feasible desired formations, as discussed in Remark 7.2 and in [260].

7.2.3 Bounded-consensus design conditions

In the previous approach, if the design of the compensation gains is not possible, the leader-following consensus (7.3) cannot be achieved. Although suitable formations can be defined to ensure $(A(\rho_i(k) - I)f_i = 0$, $\forall i \in \mathbb{N}_{\leq N}$, the difference among the scheduling parameters will prevent the exact consensus. In this case, the goal is to design the consensus law (7.14), which guarantees that the error system (7.11) is bounded. To solve Problem 7.2, we present the design conditions in Theorem 7.2.

Theorem 7.2

Consider the augmented error dynamics (7.11) obtained with discrete-time LPV MAS (7.1), the gain-scheduling consensus protocol (7.14), the observer (7.7), and the rewritten internal perturbations (7.12)-(7.13). Given positive scalars $\sigma \in (0,1)$, $\epsilon \in \mathbb{R}^+$, and the eigenvalues $\bar{\lambda}_{\mathbf{m}}$ of $\bar{\mathcal{L}}$, $\forall\, \mathbf{m} = 1, \dots, N$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{P}_2 \in \mathbb{R}^{n_x \times n_x}$, matrices $X_1 \in \mathbb{R}^{n_x \times n_x}$, $X_2 \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{K}_l \in \mathbb{R}^{n_u \times n_x}$, and $\widetilde{L}_l \in \mathbb{R}^{n_x \times n_y}$, and a positive scalar $\gamma \in \mathbb{R}^+$, such that inequalities (7.22)-(7.23) hold together with

$$\Upsilon_{hh\mathbf{m}} < 0, \qquad \text{if } h = l,\ \forall\, \mathbf{m} = 1, \dots, N, \tag{7.35}$$
$$\Upsilon_{hl\mathbf{m}} + \Upsilon_{lh\mathbf{m}} < 0, \qquad \text{if } h < l,\ \forall\, \mathbf{m} = 1, \dots, N, \tag{7.36}$$
$$\begin{bmatrix} \widetilde{P} & \star \\ \bar{X}_2 & \gamma I \end{bmatrix} < 0, \tag{7.37}$$

for all $h, l \in \mathbb{N}_{\leq N_v}$, where $\widetilde{P} = \mathrm{diag}(P_1, \widetilde{P}_2)$, $\bar{X}_2 = \mathrm{diag}(I_{n_x}, X_2)$,

$$\Upsilon_{hl\mathbf{m}} = \begin{bmatrix} \widetilde{P}_2 - \epsilon\,\mathrm{He}(X_2) & \star & \star \\ \epsilon\,\widetilde{\Gamma}_{hl\mathbf{m}}^\top & (\sigma - 1)\widetilde{P}_2 & \star \\ \epsilon I & 0 & -\sigma\gamma I \end{bmatrix}, \tag{7.38}$$

and $\widetilde{\Gamma}_{hl\mathbf{m}}$ as in (7.29). Then, the error dynamics (7.11) is ℓ_∞ stable with performance index γ. The gains of the observer (7.7), the matrices of the Lyapunov function (7.16), and the gains of the consensus protocol (7.14) are given, respectively, by $L_l = X_1^{-1}\widetilde{L}_l$, $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$, P_1, $K_l = \widetilde{K}_l X_2^{-1}$.

Proof. Firstly, from the error system (7.11) with the rewritten internal perturbations (7.12)-(7.13), it is possible to see that the internal perturbations do not affect the state-estimation. Therefore, the separation principle can be once again employed, and the ℓ_∞ stability of the augmented error system (7.11) can be evaluated considering the following subsystems

$$z(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \alpha_{hl}(\rho_k) \otimes (A_h + L_l C_h) z(k), \tag{7.39}$$

$$\delta(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} (\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes A_h + \bar{\mathcal{L}} \otimes B_h K_l)\delta(k) + w(k). \tag{7.40}$$

Recall that it was already shown in the previous proof, that if the inequalities (7.22)-(7.23) hold, the trajectories of the estimation error subsystem (7.39) converges exponentially to the origin. Therefore, we only need to prove that subsystem (7.40) is ℓ_∞ stable.

The remainder of the proof follows the same steps performed in the proof of Theorem 7.1. Notice that considering (7.40), the condition

$$V(\delta(k+1)) + (\sigma - 1)V(\delta(k)) - \sigma\gamma\, w(k)^\top w(k) < 0,$$

can be equivalently written as

$$\widetilde{\delta}(k+1)^\top (I_N \otimes P_2)\widetilde{\delta}(k+1) + (\sigma-1)\widetilde{\delta}(k)^\top (I_N \otimes P_2)\,\widetilde{\delta}(k) - \widetilde{w}(k)^\top (I_N \otimes \sigma\gamma I_{n_x})\widetilde{w}(k) < 0, \tag{7.41}$$

with $V(\delta(k))$ as in (7.33), and $\widetilde{w}(k) = (\bar{T}^{-1} \otimes I_{n_x}) w(k)$.

Replacing (7.29) in (7.38), multiplying (7.38) on the left by $\mathrm{diag}(X_2^{-\top}, X_2^{-\top}, I_{n_x})$ and its transpose on the right, and performing the exchange of variables $\widetilde{K_l} = K_l X_2$, and $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$, results in

$$\begin{bmatrix} P_2 - \epsilon \mathrm{He}(X_2^{-\top}) & \epsilon(X_2^{-\top}A_h + \bar{\lambda}_m X_2^{-\top} B_h K_l) & \epsilon X_2^{-\top} \\ \star & (\sigma-1)P_2 & 0_{n_x \times n_x} \\ \star & \star & -\sigma\gamma I_{n_x} \end{bmatrix}. \tag{7.42}$$

Multiplying, (7.42) by

$$\mathcal{B}^\top = \begin{bmatrix} A_h^\top + \bar{\lambda}_{\mathbf{m}} K_l^\top B_h^\top & I_{n_x} & 0_{n_x \times n_x} \\ I_{n_x} & 0_{n_x \times n_x} & I_{n_x} \end{bmatrix},$$

on the left, and its transpose on the right, we obtain

$$\begin{bmatrix} (A_h^\top + \bar{\lambda}_{\mathbf{m}} K_l^\top B_h^\top) P_2 (A_h + \bar{\lambda}_{\mathbf{m}} B_h K_l) + (\sigma-1)P_2 & \star \\ P_2(A_h + \bar{\lambda}_{\mathbf{m}} B_h K_l) & P_2 - \sigma\gamma I_{n_x} \end{bmatrix}. \tag{7.43}$$

Moreover, performing a dimension adjustment, and then multiplying (7.43) by $[\widetilde{\delta}(k)^\top \; \widetilde{w}(k)^\top]$ on the left, and its transpose on the right, results in

$$\widetilde{\delta}(k)^\top \Big(I_N \otimes (A_h^\top + \bar{\lambda}_{\mathbf{m}} K_l^\top B_h^\top) P_2 (A_h + \bar{\lambda}_{\mathbf{m}} B_h K_l)\Big)\widetilde{\delta}(k) + \widetilde{w}(k)^\top \Big(I_N \otimes (P_2 - \sigma\gamma I_{n_x})\Big)\widetilde{w}(k)$$
$$+ \widetilde{\delta}(k)^\top \Big((\sigma-1)(I_N \otimes P_2)\Big)\widetilde{\delta}(k) + \mathrm{He}\left(\widetilde{\delta}(k)^\top (I_N \otimes P_2(A_h + \bar{\lambda}_{\mathbf{m}} B_h K_l))\widetilde{w}(k)\right),$$

from which it is possible to conclude that due to the convex properties of time-varying parameters, the LMIs (7.35)-(7.36) are sufficient to guarantee (7.41). Therefore, we have that the LMIs (7.22)-(7.25) and (7.35)-(7.36) are sufficient to guarantee (7.17) with a Lyapunov function as defined in (7.16).

Finally, multiplying (7.37) by $\mathrm{diag}(I_{n_x}, X_2^{-\top}, I_{n_x}, I_{n_x})$ on the left, and its transpose on the right, and performing the exchange of variables $P_2 = X_2^{-\top}\tilde{P}_2 X_2^{-1}$, it is possible to see that (7.37) is equivalent to (7.18). Thus, all the conditions of Lemma 7.1 are satisfied. This concludes the proof. □

Remark 7.4

As discussed in [264], the upper bound of the error norm in (7.19) *can be reduced with minimization of* γ.

7.3 Numerical Examples

In this section, the main results of this Chapter are illustrated by two numerical examples. The first example is a numerical system, while the second is the model of an angular positioning MAS. The proposed consensus protocol is discussed, highlighting its advantages and limitations.

7.3.1 Example 1

Consider the LPV MAS, adapted from [241], composed of $N = 4$ agents with communication topology described by

$$\mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}, \quad \eta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Initially we take $\eta_i = 1$, $\forall\, i \in \mathbb{N}_{\leq 4}$, as defined in Assumption 7.1. The vertices of the continuous-time system are

$$\bar{A}_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \ \bar{A}_2 = R_p^{-1} \begin{bmatrix} 0 & 1+p \\ -1 & 0 \end{bmatrix} R_p, \ R_p = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}, \ \beta = \arctan(p),$$

$$\bar{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \bar{B}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1+10p \end{bmatrix}, \ C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1+10p & 0 \end{bmatrix}, \ p = 0.5.$$

By employing the Euler discretization with $T_s = 0.01s$, we have

$$A_1 = I + T_s\bar{A}_1 = \begin{bmatrix} 1.000 & 0.010 \\ -0.010 & 1.000 \end{bmatrix}, \ A_2 = I + T_s\bar{A}_2 = \begin{bmatrix} 1.002 & 0.014 \\ -0.011 & 0.998 \end{bmatrix},$$

$$B_1 = T_s\bar{B}_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \ B_2 = T_s\bar{B}_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.06 \end{bmatrix}, \ C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ C_2 = \begin{bmatrix} 6 & 0 \end{bmatrix}.$$

To ensure that conditions (7.30)-(7.31) hold, we define

$$M(\rho_i(k)) = B(\rho_i(k))^{-1}(I_{n_x} - A(\rho_i(k)), \quad R(\rho_i(k)) = B(\rho_i(k))^{-1}(A(\rho_s(k) - A(\rho_i(k)),$$

as the gains of the compensation signals. Then, the design of the observer-based consensus protocol can be carried out by invoking Theorem 7.1. By considering $\epsilon = 1$ and $\sigma = 0.01$, we obtain the following gains

$$K_1 = \begin{bmatrix} -27.8777 & -0.2284 \\ 0.1051 & -10.4342 \end{bmatrix}, \; K_2 = \begin{bmatrix} -27.9406 & -0.4216 \\ 0.0494 & -4.5015 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} -0.3941 \\ -0.3084 \end{bmatrix}, \; L_2 = \begin{bmatrix} -0.1582 \\ -0.1234 \end{bmatrix},$$

and Lyapunov matrices

$$P_1 = \begin{bmatrix} 1.2098 & -0.3890 \\ -0.3890 & 0.4823 \end{bmatrix}, \; P_2 = \begin{bmatrix} 1.3220 & -0.0047 \\ -0.0047 & 1.3213 \end{bmatrix}.$$

To validate the designed consensus protocol, we perform a simulation of the with initial conditions

$$x_1(0) = \begin{bmatrix} 3 \\ 10 \end{bmatrix}, \; x_2(0) = \begin{bmatrix} -7 \\ -3 \end{bmatrix}, \; x_3(0) = \begin{bmatrix} 10 \\ 1 \end{bmatrix}, \; x_4(0) = \begin{bmatrix} -5 \\ -8 \end{bmatrix},$$
$$\hat{x}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_3(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_4(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; s(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix},$$

exogenous time-varying scheduling parameters,

$$\begin{aligned}
\alpha_1(\rho_1(k)) &= \frac{1 + \sin(2t(k))}{2} & \alpha_2(\rho_1(k)) &= 1 - \alpha_1(\rho_1(k)), \\
\alpha_1(\rho_2(k)) &= \frac{1 + \cos(t(k))}{2} & \alpha_2(\rho_2(k)) &= 1 - \alpha_1(\rho_2(k)), \\
\alpha_1(\rho_3(k)) &= \frac{1 + \sin(0.05t(k))}{2} & \alpha_2(\rho_3(k)) &= 1 - \alpha_1(\rho_3(k)), \\
\alpha_1(\rho_4(k)) &= \frac{1 + \cos(0.05t(k))}{2} & \alpha_2(\rho_4(k)) &= 1 - \alpha_1(\rho_4(k)), \\
\alpha_1(\rho_s(k)) &= \frac{1 + \cos(5t(k))}{2} & \alpha_2(\rho_s(k)) &= 1 - \alpha_1(\rho_s(k)).
\end{aligned}$$

and the desired formations,

$$f_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \; f_2 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}, \; f_3 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \; f_4 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

Considering the simulation time of 10s and the sampling time $T_s = 0.01s$, the closed-loop trajectories of LPV MAS (7.1) equipped with the proposed consensus protocol (7.4) are depicted in Figure 6.1. Furthermore, the trajectories of the estimation and consensus errors are presented in Figures 7.2 and 7.3, respectively.

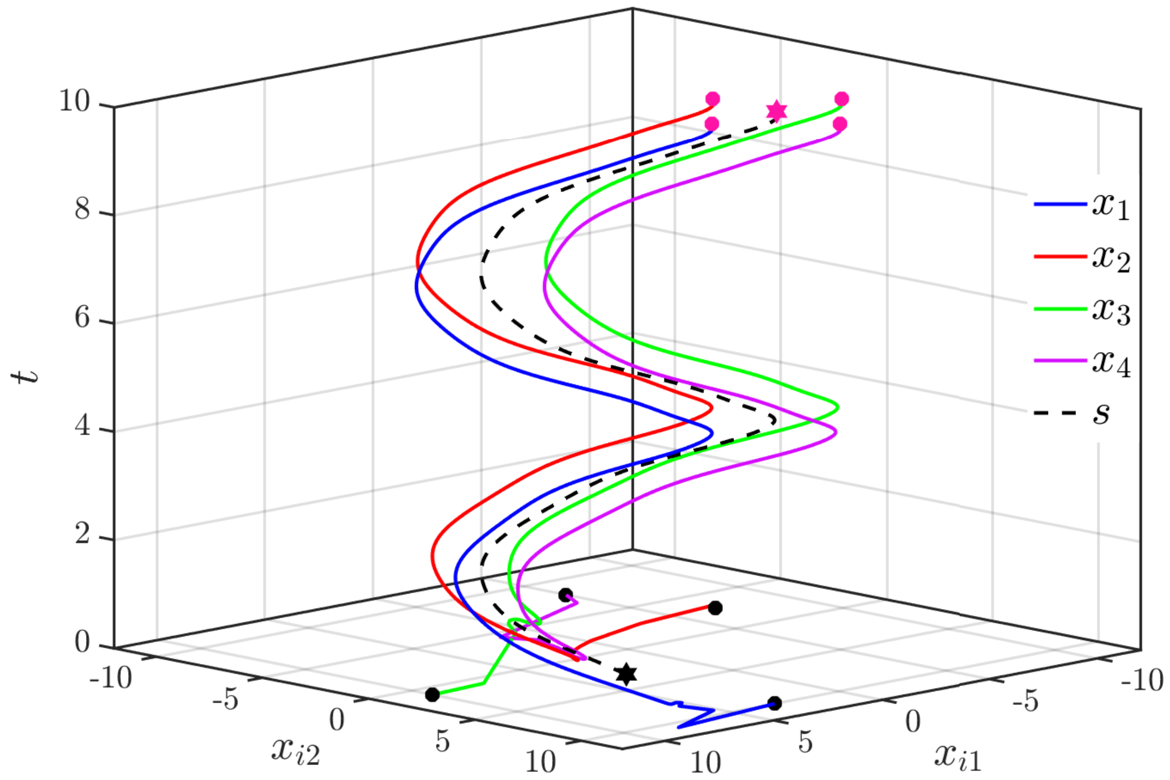

Figure 7.1 – Trajectories of the leader and following agents equipped with the proposed formation consensus protocol (7.4) designed with Theorem 7.1 - Example 1.

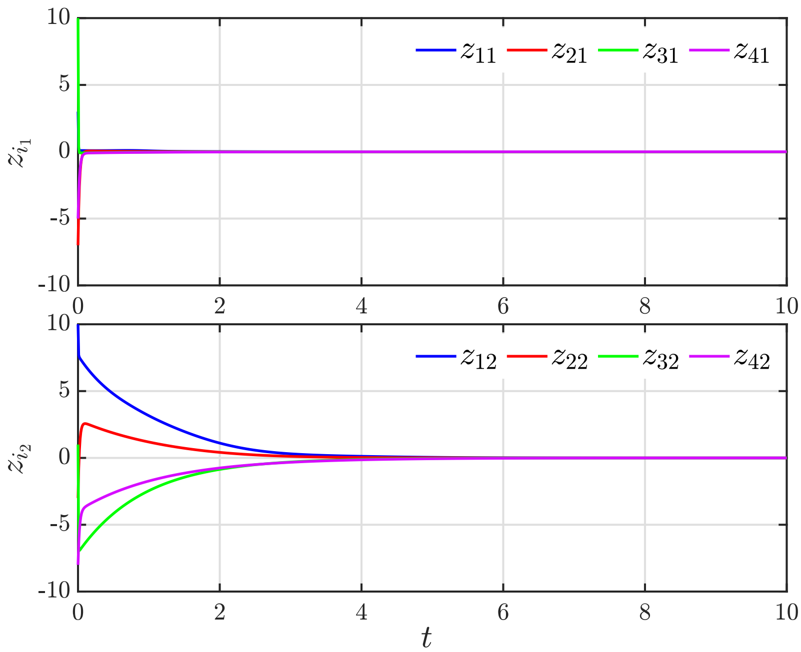

Figure 7.2 – Trajectories of the estimation error obtained with the observer designed with Theorem 7.1 - Example 1.

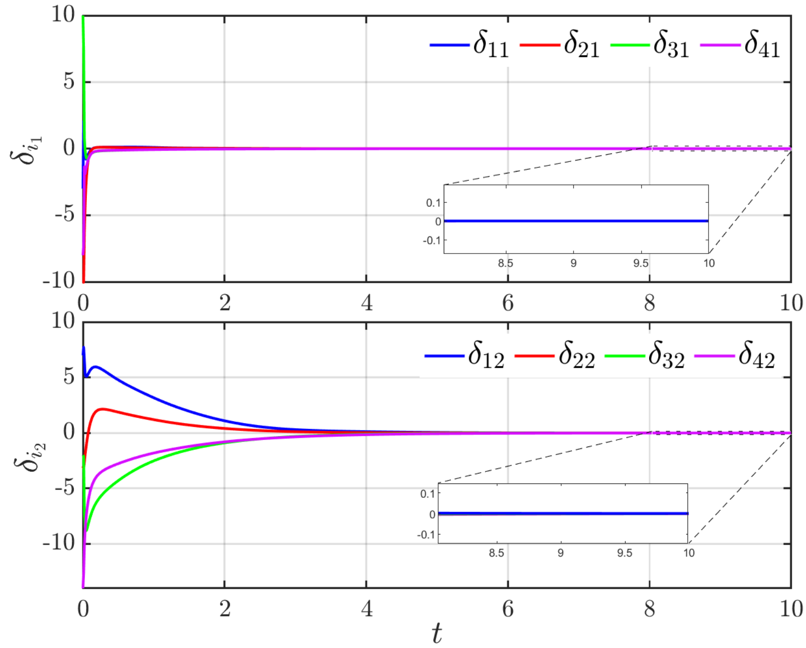

Figure 7.3 – Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 7.1 - Example 1.

The initial conditions of the leader and the following agents in the x-plane are highlighted by the black hexagram ✸ and the circles ●, respectively. Similarly, the position of the leader and the following agents in the x-plane at $t = 10s$ are highlighted by the magenta hexagram ✸ and the circles ●, respectively. From the results of Figure 7.1, it is possible to see that the following agents successfully track the leader in (- - - -) while maintaining the specified formation.

Notice that both errors converge exponentially to the origin. Therefore, it is clear that the leader-following consensus (7.3) is properly achieved, illustrating the effectiveness of the proposed observer-based consensus protocol in dealing with the formation tracking problem of LPV MASs with different time-varying scheduling parameters.

As previously discussed in Remark 7.2, the exact leader-following consensus obtained in the simulation results depicted in Figures 7.1-7.3 requires the proper design of the signals (7.5)-(7.6) to compensate for the internal perturbations. To illustrate the importance of the proposed compensation, especially when dealing with different scheduling parameters, consider a second scenario where information about the leader is unavailable to all agents. In this case, only agent x_1 will receive leader information, that is, $\eta_i = 0$, for $i = 2, 3, 4$.

Recall that without leader information, local controllers cannot be designed with the compensation signals (7.6). Therefore, since the condition of the Assumption 7.1 does not hold for all agents, the consensus protocol must be designed by invoking Theorem 7.2. Then, by considering $\epsilon = 1.8$ and $\sigma = 0.03$ we obtain $\gamma = 34.62$, and

$$K_1 = \begin{bmatrix} -36.9253 & 2.9426 \\ -0.5473 & -13.2470 \end{bmatrix}, \; K_2 = \begin{bmatrix} -27.6660 & -5.3710 \\ 2.4110 & -2.4536 \end{bmatrix},$$
$$P_1 = \begin{bmatrix} 105.7084 & -27.9799 \\ -27.9799 & 15.2488 \end{bmatrix}, \; P_2 = \begin{bmatrix} 0.0879 & -0.0146 \\ -0.0146 & 0.0325 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} -0.4186 \\ -0.7467 \end{bmatrix}, \; L_2 = \begin{bmatrix} -0.1644 \\ -0.2909 \end{bmatrix},$$

Considering the same initial conditions, formations, and scheduling parameters, the trajectories of the closed-loop following agents and the consensus error of the second scenario are depicted in Figures 7.4–7.5. It can be seen from the results of Figure 7.4 that even without compensations $r_i(k)$, the following agents can track the leader dynamics. However, as shown in Figure 7.5, the agents do not achieve the exact desired formation and consensus errors oscillate around the origin due to the internal perturbation $w_s(k)$ in (7.13). This result shows the contributions of the proposed approach, demonstrating that dealing with the formation perturbation $w_f(k)$ is not sufficient to achieve the exact formation (7.3) for LPV MASs in the form of (7.1).

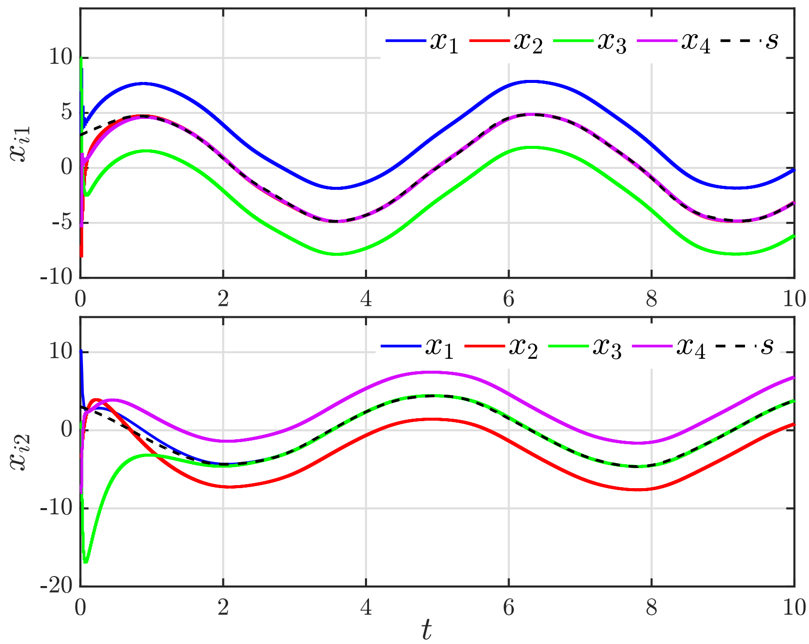

Figure 7.4 – Trajectories of the leader and following agents equipped with the proposed formation consensus protocol designed with Theorem 7.2 - Example 1.

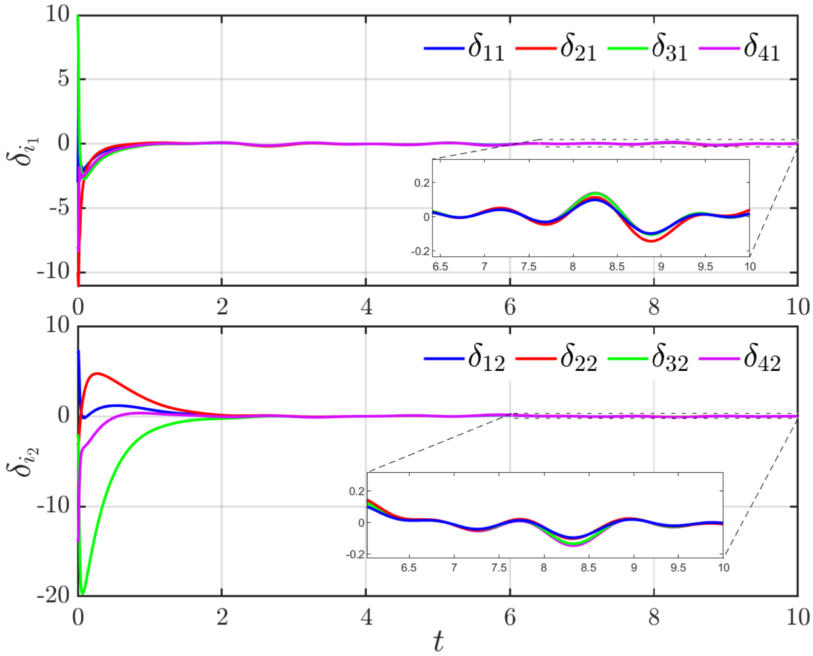

Figure 7.5 – Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 7.2 - Example 1.

Moreover, to evaluate the boundedness guarantees provided by the conditions of Theorem 7.2, consider a scenario where none of the compensation signals (7.5)–(7.6) are designed. The behavior of the combined internal perturbation $w(k)$, with the same initial conditions and time-varying scheduling parameters, is depicted in Figure 7.6. With the compute of $||w||_{\ell_\infty}$, together with the previously presented values of σ, γ, P_1, and P_2, obtained with Theorem 7.2, we can compute $\varphi(e(0), ||w||_{\ell_\infty})$ as defined in (7.19).

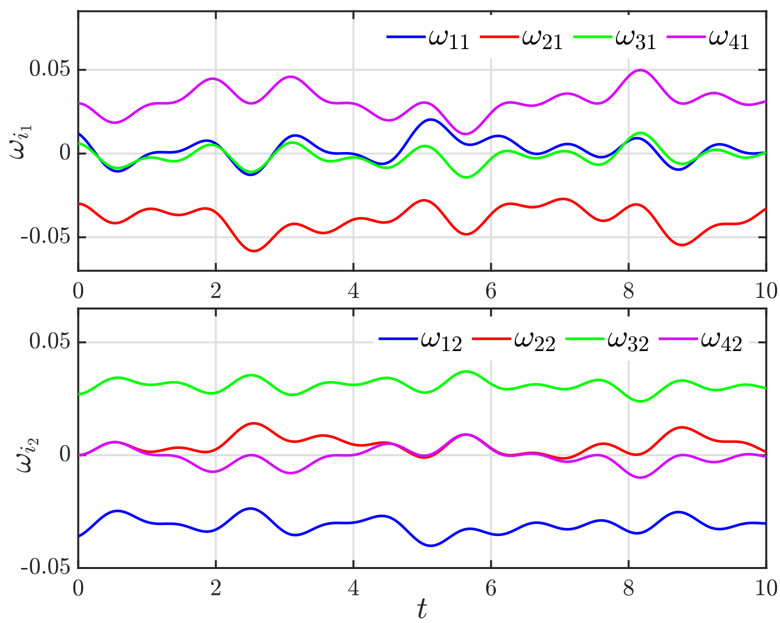

Figure 7.6 – Internal perturbations of the closed-loop consensus error - Example 1.

A comparison of the norm of the augmented error system (7.11), together with the obtained $\varphi(e(0), ||w||_{\ell_\infty})$, is presented in Figure 7.7. The exact leader-following consensus can be achieved when considering both compensations (depicted in ——). In other cases, when only one type of compensation is considered (only $r_i(k)$ depicted in —— and only $\nu_i(k)$ depicted in ——), or when the compensations are neglected (depicted in ——), the norm of the augmented error system is bounded by $\varphi(e(0), ||w||_{\ell_\infty})$ (depicted in - - - -). As expected, the consensus law without compensation yields the worst results, again accentuating the benefits of the proposed approach.

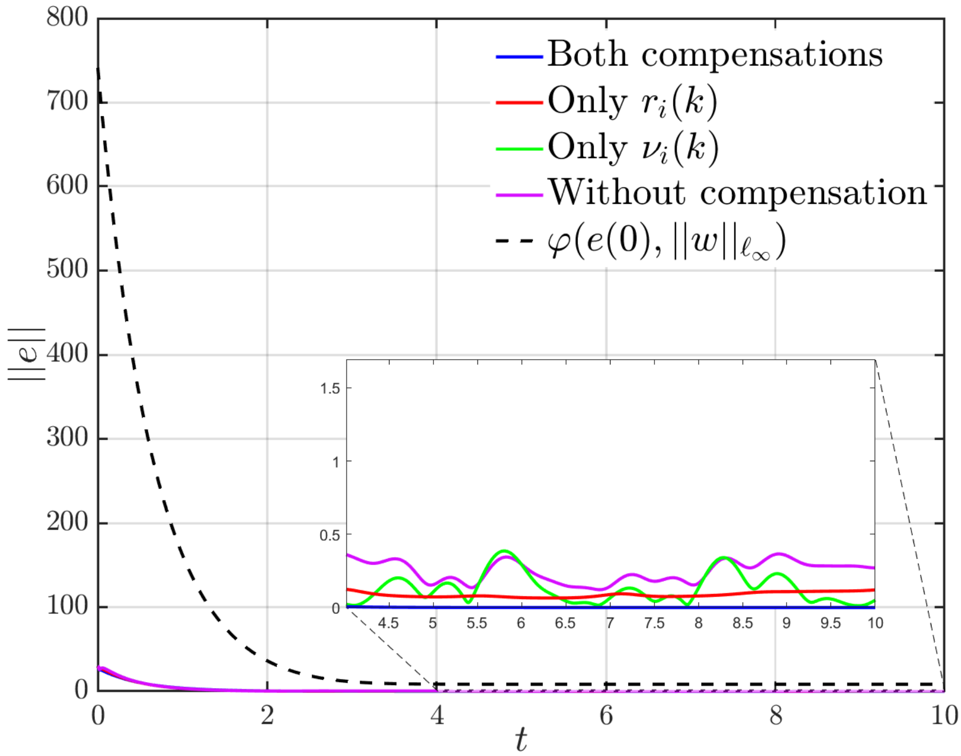

Figure 7.7 – Comparison of the norm of the augmented error system - Example 1.

7.3.2 Example 2

Consider an LPV MAS angular positioning inspired by the model presented in [265, 266]. The classical angular position system (APS) comprises a rotating antenna driven by an electric motor. The control goal is to drive the motor to rotate the antenna so that it points in the direction of a moving target. In this paper, we assume that the target is a MAS composed of a leader-following formation, and our goal is to design a distributed gain-scheduled observer-based consensus such that the formation of the angular positioning LPV MAS matches the formation of the targets, as illustrated in Figure 7.8.

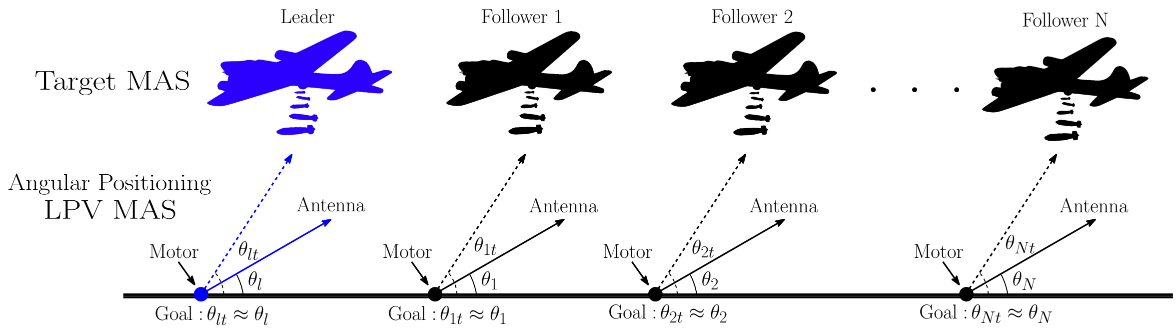

Figure 7.8 – Setup of an angular positioning MASs.

The time-varying dynamics of the APS is given by

$$x_i(k+1) = \begin{bmatrix} 1 & 0.1 \\ 0 & 1-0.1\rho_i(k) \end{bmatrix} x_i(k) + \begin{bmatrix} 0 \\ 0.1\kappa \end{bmatrix} u_i(k),$$
$$y_i(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_i(k),$$

where $x_i = [\theta_i^\top \ \dot{\theta}_i^\top]^\top$, θ_i $[\mathrm{rad}]$ is the angular position, $\dot{\theta}_i$ $[\mathrm{rad}/s]$ is the angular velocity, $0.1 \ \mathrm{s}^{-1} \leq \rho_i(k) \leq 10 \ \mathrm{s}^{-1}$ is proportional to the coefficient of viscous friction in the rotation parts of the antenna, and $\kappa = 0.787 \, \mathrm{rad}^{-1}\mathrm{V}^{-1}s^{-2}$ is a given constant. Similarly to [265, 266], $\rho_i(k)$ is arbitrarily time-varying in the indicated range of variation.

We assume that the angular position of the leader-target θ_{lt} $[\mathrm{rad}]$, and the formation of its followers is measurable and available. Considering a simulation time of $t = 10$ s with a sampling period of $T_s = 0.01$s, the position of the leader-target is given by

$$\theta_{lt}(k) = \begin{cases} \frac{\pi}{3} & 0 \leq k \leq 300, \\ \frac{\pi}{3} + (k-300)\frac{\pi}{6} & 300 < k \leq 400, \\ \frac{\pi}{2} & 300 < k \leq 700, \\ \frac{\pi}{2} + (k-700)\frac{\pi}{6} & 700 < k \leq 800, \\ \frac{2\pi}{3} & 800 < k \leq 1000. \end{cases} \tag{7.44}$$

Moreover, we assume the target system comprises one leader and three followers with the same distance of $\pi/12$. Therefore, the desired formation can be written as

$$f_1 = \begin{bmatrix} -\frac{\pi}{12} \\ 0 \end{bmatrix}, \quad f_2 = \begin{bmatrix} -\frac{\pi}{6} \\ 0 \end{bmatrix}, \quad f_3 = \begin{bmatrix} -\frac{\pi}{4} \\ 0 \end{bmatrix}.$$

Notice that a leader of the APS in the open-loop form of (7.2) cannot track the position of the leader-target described in (7.44). In this case, it is necessary to consider a controlled leader in the form

$$s(k+1) = A(\rho_s(k))s(k) + B(\rho_s(k))u_s(k).$$

where the leader control input $u_s(k)$ is properly designed to track the leader-target. By the definition of the consensus error, notice that we now have

$$\delta_i(k+1) = A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k) - A(\rho_s(k))s(k) - B(\rho_s(k))u_s(k) - f_i.$$

Similarly to [259], we assume that the leader control input is known by all followers, and the consensus law is modified to $u_i(k) = u_i(k) + u_s(k)$ resulting in

$$\delta_i(k+1) = A(\rho_i(k))x_i(k)+B(\rho_i(k))u_i(k)-A(\rho_s(k))s(k)-f_i+\Big(B(\rho_i(k))-B(\rho_s(k))\Big)u_s(k).$$

Since the input matrix of the APS systems is parameter-independent, we have $B(\rho_i(k)) = B(\rho_s(k)) = B$, and the augmented error system (7.11) remains unchanged. However, in the more general case, the consensus law can be modified to $u_i(k) = u_i(k) + \bar{u}_s(k)$ where employing the same strategy considered in the compensation signals, $\bar{u}_s(k)$ is defined to satisfy condition $B(\rho_i(k))\bar{u}_s(k) - B(\rho_s(k))u_s(k) = 0$.

Notice that the desired formations satisfy $\Big(A(\rho_i(k)) - I\Big)f_i = 0$. Therefore, we only design the compensation signals $r_i(k)$ considering the Moore-Penrose pseudoinverse to obtain

$$R(\rho_i(k)) = B(\rho_i(k))^{\dagger}(A(\rho_s(k)) - A(\rho_i(k)).$$

Consider the communication among the agents of the APS is given by

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad \eta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

With $\epsilon = 1$, and $\sigma = 0.025$, we solve the conditions of Theorem 7.1 to obtain the following gain vertices:

$$K_1 = \begin{bmatrix} -3.4843 & -4.8451 \end{bmatrix}, \; K_2 = \begin{bmatrix} -3.6610 & -0.3055 \end{bmatrix}, \; L_1 = \begin{bmatrix} -1.0372 \\ -0.3919 \end{bmatrix}, \; L_2 = \begin{bmatrix} -1.0412 \\ 0.0359 \end{bmatrix},$$

and Lyapunov matrices

$$P_1 = \begin{bmatrix} 0.8620 & -0.2865 \\ -0.2865 & 0.7248 \end{bmatrix}, \; P_2 = \begin{bmatrix} 2.6946 & 0.6532 \\ 0.6532 & 0.7807 \end{bmatrix}.$$

To validate the designed consensus protocol, we perform a simulation of the closed-loop system with trajectories starting from the initial conditions

$$s(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; x_1(0) = \begin{bmatrix} \frac{\pi}{10} \\ 0 \end{bmatrix}, \; x_2(0) = \begin{bmatrix} \frac{\pi}{12} \\ 0 \end{bmatrix}, \; x_3(0) = \begin{bmatrix} \frac{\pi}{14} \\ 0 \end{bmatrix},$$
$$\hat{x}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_3(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

and with the exogenous time-varying scheduling parameters of the followers and leader defined as

$$\alpha_1(\rho_1(k)) = \frac{1+\sin(3t(k))}{2} \qquad \alpha_2(\rho_1(k)) = 1-\alpha_1(\rho_1(k)),$$
$$\alpha_1(\rho_2(k)) = \frac{1+\cos(4t(k))}{2} \qquad \alpha_2(\rho_2(k)) = 1-\alpha_1(\rho_2(k)),$$
$$\alpha_1(\rho_3(k)) = \frac{1+\sin(0.01t(k))}{2} \qquad \alpha_2(\rho_3(k)) = 1-\alpha_1(\rho_3(k)),$$
$$\alpha_1(\rho_s(k)) = \frac{1+\cos(7t(k))}{2} \qquad \alpha_2(\rho_s(k)) = 1-\alpha_1(\rho_s(k)).$$

The closed-loop trajectories of the angular positioning LPV MAS are depicted in Figure 7.9. As shown in Figure 7.9, with the designed consensus protocol, the angular positioning system can successfully track the formation of the target. Notice that due to compensation signals $r_i(k)$, the consensus error depicted in Figure 7.10 converges exponentially to the origin, and the formation is maintained even during the transient period when the angular velocities are not null and $w_s(k)$ is actively disturbing the augmented error system.

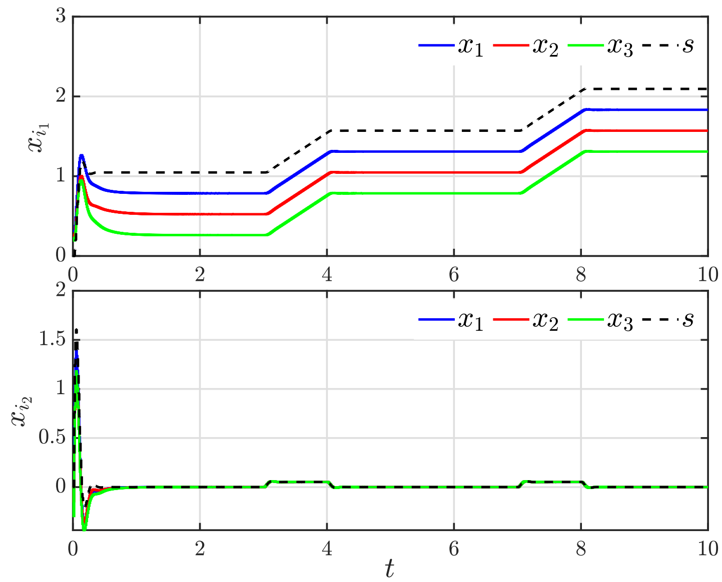

Figure 7.9 – Trajectories of the leader and following agents equipped with the proposed formation consensus protocol designed with Theorem 7.1- Example 2.

Furthermore, we show by the consensus error depicted in Figure 7.11 that, if the compensation signals $r_i(k)$ are neglected, the angular positioning system loses its desired formation during transition times, highlighting the importance of the proposed method.

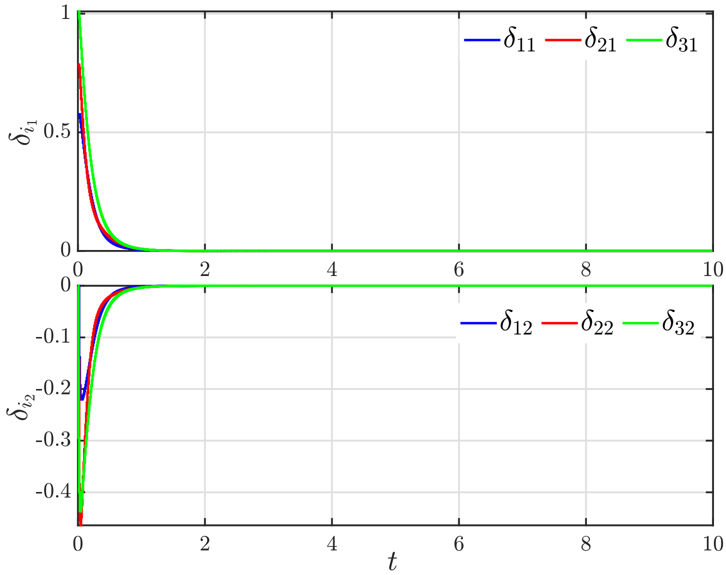

Figure 7.10 – Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 7.1 - Example 2 .

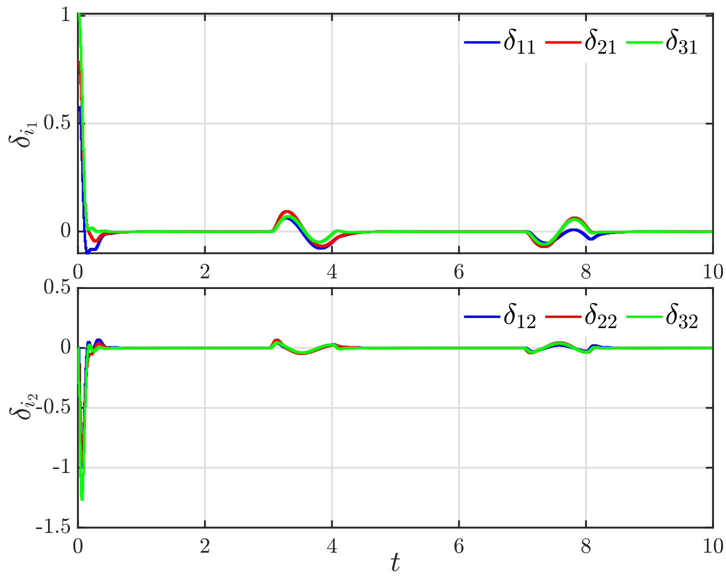

Figure 7.11 – Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 7.2 without the compensation signals $r_i(k)$. - Example 2 .

7.4 Conclusions

This chapter has addressed the problem of leader-following formation consensus for discrete-time LPV multi-agent systems. Compensation signals are introduced to properly cancel closed-loop terms that cannot be directly embedded into the error dynamics due to the dependency on the desired formation or mismatches among the time-varying scheduling parameters. For the design of the observer and the remaining gains of the proposed consensus protocol, conditions in the form of LMIs were developed based on Lyapunov stability arguments. Moreover, for the cases where the compensation signal cannot be designed, a ℓ_∞ analysis condition was presented that provides an upper bound for the disturbed augmented error system. The proposed approaches have been tested in numerical examples that highlight the effectiveness of the consensus protocol in achieving the exact leader-following consensus even when the mismatches among the time-varying scheduling parameters introduce heterogeneity into the discrete-time LPV MAS.

Part IV

Concluding Remarks and Further Steps

8 CONCLUDING REMARKS

This thesis has addressed multiple challenges in interconnected and multi-agent systems with advances in the design of distributed control and consensus approaches. The developed frameworks provide systematic solutions to stabilize continuous-time nonlinear interconnected delayed systems, save communication resources in the networked control of continuous-time nonlinear interconnected systems, achieve the practical leaderless state-consensus of continuous-time LPV MASs, and maintain the leader-following formation consensus of discrete-time LPV MASs.

In Chapter 3, sufficient conditions were proposed for designing distributed control laws that guarantee the local stability of the origin of continuous-time nonlinear interconnected systems subject to time-delays in its physical dynamics. The conditions were obtained employing Lyapunov-Krasovskii stability arguments and written in the form of LMIs. A set of admissible initial conditions has been defined to ensure that the state trajectories of the closed-loop system do not leave the validity domain of the N-TS fuzzy model representing the subsystems. Moreover, to increase the estimate of this set, an optimization procedure has also been proposed.

Chapter 4 investigated the asynchronous ETC problem for continuous-time interconnected nonlinear systems, proposing a novel distributed ETM to counteract mismatches from asynchronous premise variables. The exclusion of Zeno behavior has been formally proven, thus enabling the implementation of the distributed ETC scheme. Moreover, a procedure for estimating the domain of attraction has been provided. To enlarge this estimate and reduce the number of transmissions, a convex multi-objective optimization problem has been presented.

Chapter 6 investigated the practical state consensus of continuous-time MASs with agents described by LPV models. Sufficient conditions were presented for the design of a gain-scheduled observer-based consensus protocol. The main novelty of the proposed method is that, by treating parameter mismatches as internal disturbances, the approach ensured exponential convergence to a bounded error region.

Moreover, Chapter 7 extended the consensus investigation to the leader-following formation of discrete-time LPV-MAS. Similarly to Chapter 6, mismatches in scheduling parameters were considered in system modeling. To solve this problem, a distributed gain-scheduled observer-based consensus protocol composed of the combination of the relative information concerning neighboring agents and compensation signals is designed. Compensation signals are introduced to cope with closed-loop terms that emerge due to the dependence on the desired formation or mismatches among the time-varying scheduling parameters.

8.1 Further Steps

The possible next steps for this doctoral research are listed as follows:

(i) Explore different alternatives for the distributed ETC setup:

Notice that in the proposed setup, the continuous monitoring of state measurements by the local ETM may not be practical for digital implementations. To overcome this limitation, an alternative approach is to introduce a sampler into the setup, where the system measurements are made available to the ETM at fixed sample intervals of h, as depicted in Figure 8.1.

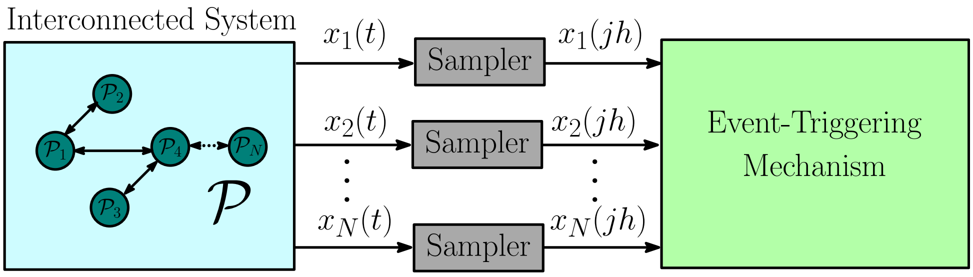

Figure 8.1 – Illustrating the inclusion of the sampler in the ETC setup

Notice that in this case, the measurements of the subsystems states are unavailable between two consecutive sampling intervals. As a result, the proposed cancellation-based scheme, which relies on continuous state measurements, is no longer feasible. Therefore, dealing with this setup without assuming deviation bounds, state-space partitions, or linear controllers is a very challenging task. Based on this discussion, a possible future direction is to investigate the development of **a distributed event-triggered impulsive approach for continuous-time nonlinear interconnected systems.**

With an impulsive approach, the distributed control laws will be implemented in the subsystems only at the transmission instants. This implies that the control actions will be executed exclusively at specific points where the measurements of the system are available and the asynchronism does not occur. Recent approaches that deal with event-triggered impulsive control have been proposed in [267–271].

(ii) Consider an ETC setup for the consensus of LPV MASs

Recall that in the context of LPV MASs, gain-scheduling controllers depend on exogenous parameters, and there is no mismatch among the parameters of the local consensus protocols and agents. Therefore, a possible future direction is to investigate the development of a **distributed periodic event-triggered consensus approach for the consensus of LPV MASs.** Moreover, in a scenario where not all states are available for measurements (as addressed in this thesis in the context of LPV MASs), another possible future direction is to investigate the development of a **distributed observer-based periodic event-triggered consensus approach for the consensus of LPV MASs.**

(iii) Consider effects of network-induced phenomena in the analysis:

In a networked-control setup, the system is subject to the effects of networked-induced phenomena even with an ETC approach reducing the use of communication [272–274]. Therefore, another possible future direction is **to derive conditions for distributed control or consensus approaches considering network-induced phenomena, such as packet dropouts, cyber-attacks, and network-induced delays.** More specifically, in the context of cyber-attacks, it is possible to address **the problem of distributed control for interconnected nonlinear systems or the design of distributed consensus approaches for LPV MASs subject to denial of service attacks (DoS), deception attacks, or a hybrid combination of both.**

The deception attack is also known as the false data injection attack. The main goal of this type of attack is to manipulate the packets transmitted in the communications channels compromising their data integrity. Normally, the modeling of deception attacks follows a stochastic process as considered in [275]. Meanwhile, the DoS attacks are capable of closing up the data exchange among controllers, sensors, and actuators, consuming the resources of the communication channels and consequently disrupting the data availability [265, 276–278]. An illustration of these effects is depicted in Figure 8.2.

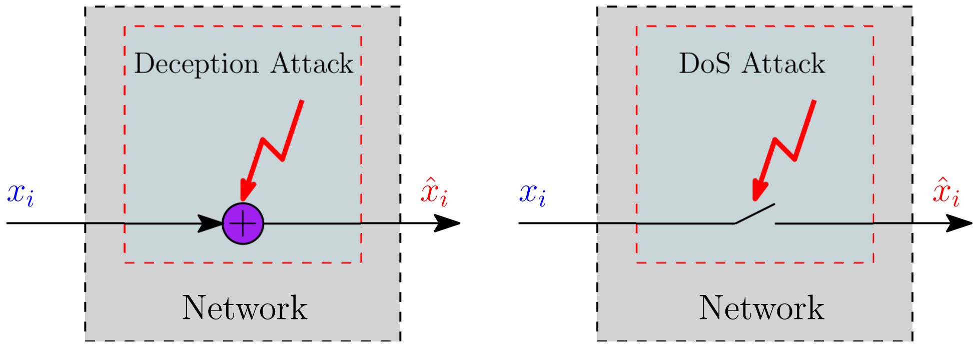

Figure 8.2 – Illustrating the effects of cyber-attacks in the Network

(iv) Manage unmeasurable states and premise variables on the control of Interconnected systems

Unlike the proposed approaches for LPV MASs in Chapters 6 and 7, the methods developed in Chapters 3 and 4 considers full-state information. However, in several practical applications, some of the systems states might not be available for measurement. In this scenario, the considered N-TS fuzzy modeling might present unmeasurable premise variables, impairing the design of distributed fuzzy controllers that cannot be defined in function of these variables. Based on this discussion, another possible future direction is to investigate the development of **distributed observer-based control approaches for nonlinear interconnected systems represented by N-TS fuzzy models**, capable of efficiently dealing with unmeasurable premise variables.

(v) Manage the presence of actuator and sensor faults

Similarly to the network-induced phenomena, it also known that the possible occurrences of sensor and actuator faults in modern control systems can lead to closed-loop performance degradation or even instability [279–281]. Therefore, another possible future direction is **to derive conditions for fault-tolerant distributed control or consensus approaches.**

8.2 Publications

During the period in which this doctoral research was developed, contributions related to the topics of distributed control have been made. The publications related to the specific topic of this thesis are listed below:

a) **PESSIM, P. S. P.**; COUTINHO, P. H. S.; LACERDA, M. J.; PALHARES, R. M. Distributed control of time-delay interconnected nonlinear systems. **Journal of the Franklin Institute**, Elsevier, v. 360, p. 9637-9662, 2023. doi: <https://doi.org/10.1016/j.jfranklin.2023.07.001>

b) **PESSIM, P. S. P.**; COUTINHO, P. H. S.; LACERDA, M. J.; PALHARES, R. M. Distributed event-triggered fuzzy control for nonlinear interconnected systems. **Chaos, Solitons** & **Fractals**, Elsevier, v. 177, p. 114276, 2023. doi: <https://doi.org/10.1016/j.chaos.2023.114276>

c) **PESSIM, P. S. P.**; COUTINHO, P. H. S.; LACERDA, M. J.; PUIG, V.; PALHARES, R. M. "A distributed gain-scheduled observer-based approach for the practical consensus of LPV multi-agent systems". **Automatica**, Elsevier, v. 177, p. 112353, 2025. doi: <https://doi.org/10.1016/j.automatica.2025.112353>

d) **PESSIM, P. S. P.**; COUTINHO, P. H. S.; LACERDA, M. J.; PUIG, V.; PALHARES, R. M. "Distributed leader-following formation of discrete-time multi-agent LPV systems". **International Journal of Robust and Nonlinear Control**, 1–15, 2025. doi: <https://doi.org/10.1002/rnc.70091>

e) **PESSIM, P. S. P.**; COUTINHO, P. H. S.; BESSA, I. V.; PEIXOTO, M. L. C.; LACERDA, M. J.; PALHARES, R. M. Controle Distribuído para Sistemas Não Lineares Interconectados Sujeitos a Retardos Variantes no Tempo nas Interconexões. In: XXIV Congresso Brasileiro de Automática. Fortaleza, 2022, p. 1–6. doi: <https://doi.org/10.20906/CBA2022/3435>.

f) **PESSIM, P. S. P.**; COUTINHO, P. H. S.; BESSA, I. V.; PEIXOTO, M. L. C.; LACERDA, M. J.; PALHARES, R. M. Controle Distribuído com Acionamento por Eventos para Sistemas Não Lineares Interconectados. In: XVI Simpósio Brasileiro de Automação Inteligente. Manaus, 2023, p. 1–6. url: <https://sba.org.br/open_journal_systems/index.php/sbai/article/view/3873>.

During the period in which this doctoral research was developed, additional related topics have been researched, and new results have also been obtained in collaboration with other members of the D!FCOM. The works published (or accepted for publication) related to other topics are listed below:

a) PEIXOTO, M. L. C.; COUTINHO, P. H. S.; BESSA, I. V.; **PESSIM, P. S. P.**; PALHARES, R. M. Event-triggered control of Takagi-Sugeno fuzzy systems under deception attacks. **International Journal of Robust and Nonlinear Control**, Wiley Online Library, v. 33, n. 13, p. 7471-7487, 2023.
doi: <https://doi.org/10.1002/rnc.6760>

b) COUTINHO, P. H. S.; BESSA, I. V.; **PESSIM, P. S. P.**; PALHARES, R. M. A switching approach to event-triggered control systems under denial-of-service attacks, **Nonlinear Analysis: Hybrid Systems**, Elsevier, v. 50, p.101383, 2023.
doi: <https://doi.org/10.1016/j.nahs.2023.101383>

c) PEIXOTO, M. L. C.; **PESSIM, P. S. P.**; COUTINHO, P. H. S.; BESSA, I. V.; PALHARES, R. M. Event-triggered control for LPV systems under hybrid cyber-attacks **Journal of Control, Automation and Electrical Systems**, Springer, v. 35, n. 2, p. 252-265, 2024.
doi: <https://doi.org/10.1007/s40313-024-01073-1>.

d) PEIXOTO, M. L. C.; **PESSIM, P. S. P.**; PALHARES, R. M. A new approach for dynamic output-feedback control design of time-delayed nonlinear systems. **European Journal of Control**, v. 77, p. 100993, 2024.
doi: <https://doi.org/10.1016/j.ejcon.2024.100993>

e) PEIXOTO, M. L. C.; **PESSIM, P. S. P.**; MOREIRA, P. M.; PIRES, P.O.F; COUTINHO, P. H. S.; XIE, X, R. M. Event-triggered dynamic output-feedback control for LPV systems. **International Journal of Systems Science**, v. 55, n. 14 p. 2952-2962, 2024.
doi: <https://doi.org/10.1080/00207721.2024.2364290>

f) PEIXOTO, M. L. C.; de OLIVEIRA, P. M.; BESSA, I. V.; COUTINHO, P. H. S.; **PESSIM, P. S. P.**;, PUIG, V.; PALHARES, R. M..Fault-tolerant dynamic output-feedback control of LPV systems via fault hiding. **Automatica**, 174, 112191. doi: <https://doi.org/10.1016/j.automatica.2025.112191>

g) **PESSIM, P. S. P.**; SILVA, F. A.; PEIXOTO, M. L. C.; PALHARES, R. M. LACERDA, M. J. State-feedback control for discrete-time uncertain positive linear systems under denial of service attacks. In: 2024 UKACC 14^{th} International Conference on Control. Winchester, United Kingdom, 2024, pp. 43-48,
doi: <https://doi.org/10.1109/CONTROL60310.2024.10532009>.

h) PEIXOTO, M. L. C.; COUTINHO, P. H. S.; **PESSIM, P. S. P.**; LACERDA, M. J.; D'ANGELO, M.F.S.V.; PALHARES, R. M. Static output-feedback control of time-delayed LPV systems under actuator saturation. In: XVI Simpósio Brasileiro de Automação Inteligente, Manaus, 2023, p. 1–6. URL: <https://sba.org.br/open_journal_systems/index.php/sbai/article/view/3806>

i) PEIXOTO, M. L. C.; COUTINHO, P. H. S.; **PESSIM, P. S. P.**; BESSA, I. V.; PALHARES, R. M. Controle em Rede com Acionamento por Eventos para Sistemas Sujeitos a Ataques Cibernéticos. In: XXIV Congresso Brasileiro de Automática, Fortaleza, 2022, p. 1–6. doi: <https://doi.org/10.20906/CBA2022/3504>

j) COUTINHO, P. H. S.; BESSA, I. V.; PEIXOTO, M. L.; **PESSIM, P. S. P.**; PIRES, P.O.F; PALHARES, R. M. Controle com Acionamento por Eventos Resiliente a Ataques de Negaçao de Serviço In: XXIV Congresso Brasileiro de Automática, Fortaleza, 2022, p. 1–6. doi: <https://doi.org/10.20906/CBA2022/3504>

k) BRENAG, E. C.; **PESSIM, P. S. P.**; de OLIVEIRA, P. M.; PALHARES, R. M "Controle Baseado em Dados para Sistemas Lineares Sujeitos a Retardo no Tempo na Entrada". Accepted for publication in the XVII Simpósio Brasileiro de Automação Inteligente, São João del-Rei, 2025

l) **PESSIM, P. S. P.**; PIRES, P.O.F; PEIXOTO, M. L. C; D'ANGELO, M.F.S.V.; PALHARES, R. M. "Consenso prático de sistemas multiagentes LPV sob ataques de injeção de dados falsos nos atuadores". Accepted for publication in the XVII Simpósio Brasileiro de Automação Inteligente, São João del-Rei, 2025

Finally, the current papers are under revision

a) COUTINHO, P. H. S.; **PESSIM, P. S. P.**; BESSA, I. V.; PEIXOTO, M. L.; PALHARES, R. M. "Handling Asynchronous Scheduling Functions in Periodic Event-Triggered Gain-Scheduled Control with Guaranteed Polytopic Inclusion".

b) BRENAG, E. C.; **PESSIM, P. S. P.**; de OLIVEIRA, P. M.; PALHARES, R. M "Quasi-Data-Driven Static Output-feedback Control of Linear Systems with Input and State delays"

c) BRENAG, E. C.; **PESSIM, P. S. P.**; de OLIVEIRA, P. M.; PALHARES, R. M. "Input saturated control of time-delay systems: a data-driven approach"

d) PEIXOTO, M. L. C.; **PESSIM, P. S. P.**; GUERRA, T. M.; PALHARES, R. M. "Static output-feedback control design for descriptor fuzzy systems under input saturation and unmeasured nonlinearities"

BIBLIOGRAPHY

[1] BERNSTEIN, D.S.; BUSHNELL, L.G. The history of control: from idea to technology. **IEEE Control Systems Magazine**, v. 22, n. 2, p. 21–23, 2002. Page 21.

[2] ELDEN, L. L. Notes on operation of large interconnected systems. **Transactions of the American Institute of Electrical Engineers**, XL, p. 1121–1126, 1921. Page 21.

[3] ANTONELLI, Gianluca. Interconnected dynamic systems: An overview on distributed control. **IEEE Control Systems Magazine**, v. 33, n. 1, p. 76–88, 2013. Page 21.

[4] CHEN, Fei; REN, Wei. On the control of multi-agent systems: A survey. **Foundations and Trends® in Systems and Control**, v. 6, n. 4, p. 339–499, 2019. ISSN 2325-6818. Pages 21, 28, 31, 32, and 84.

[5] LI, Zhongkui; ZHISHENG, Duan. **Cooperative Control of Multi-Agent Systems: A Consensus Region Approach**. [S.l.]: CRC Press, 2015. Page 21.

[6] AN, Baizheng; HUANG, Bomin; ZOU, Yao; CHEN, Fei; MENG, Ziyang. Distributed optimization for uncertain nonlinear interconnected multi-agent systems. **Systems & Control Letters**, v. 168, p. 105364, 2022. ISSN 0167-6911. Page 22.

[7] TURAN, Mustafa Sahin; XU, Liang; FERRARI-TRECATE, Giancarlo. On consensusability of linear interconnected multiagent systems and simultaneous stabilization. **IEEE Transactions on Control of Network Systems**, v. 9, n. 1, p. 403–414, 2022. Page 22.

[8] MALAN, Albertus Johannes; PFEIFER, Martin; HOHMANN, Sören. Distributed coordination of physically-interconnected multi-agent systems with actuated and unactuated agents. **European Journal of Control**, v. 68, p. 100673, 2022. ISSN 0947-3580. Page 22.

[9] YE, Hefu; WEN, Changyun; SONG, Yongduan. Decentralized and distributed control of large-scale interconnected multi-agent systems in prescribed time. **IEEE Transactions on Automatic Control**, p. 1–16, 2024. Page 22.

[10] ARAÚJO, Rodrigo Farias; TORRES, Leonardo A. B.; PALHARES, Reinaldo Martínez. Plug-and-Play distributed control of large-scale nonlinear systems. **IEEE Transactions on Cybernetics**, v. 53, n. 4, p. 2062–2073, 2023. Pages 22 and 77.

[11] ARAÚJO, Rodrigo Farias. **Distributed Control of Large-Scale Systems: A Takagi-Sugeno Fuzzy Model-Based Approach**. Tese (Doutorado) — Universidade Federal de Minas Gerais, Brazil, 2019. Pages 22 and 28.

[12] DEMIR, Ozan; LUNZE, Jan. Optimal and event-based networked control of physically interconnected systems and multi-agent systems. **International Journal of Control**, v. 87, n. 1, p. 169–185, 2013. ISSN 1366-5820. Page 22.

[13] KHALIL, H. K. **Nonlinear Systems**. 3rd. ed. Upper Saddle River, NJ: Prentice Hall, 2002. Page 27.

[14] ADVANCES and Applications in Nonlinear Control Systems. [S.l.]: Springer International Publishing, 2016. ISSN 1860-9503. ISBN 9783319301693. Page 27.

[15] PEIXOTO, Márcia Luciana da Costa. **Static Output-Feedback Control Design for Nonlinear Systems - Polytopic Based Approaches**. Tese (Doutorado) — Universidade Federal de Minas Gerais, Brazil, 2023. Pages 27, 28, and 33.

[16] KIM, Han Sol; PARK, Jin Bae; JOO, Young Hoon. Decentralized sampled-data tracking control of large-scale fuzzy systems: An exact discretization approach. **IEEE Access**, v. 5, p. 12668–12681, 2017. Page 28.

[17] KOO, Geun Bum; PARK, Jin Bae; JOO, Young Hoon. Decentralized fuzzy observer-based output-feedback control for nonlinear large-scale systems: An lmi approach. **IEEE Transactions on Fuzzy Systems**, v. 22, n. 2, p. 406–419, 2014. Page 28.

[18] ARAÚJO, Rodrigo Farias; TORRES, Leonardo A. B.; PALHARES, Reinaldo Martinez. Distributed control of networked nonlinear systems via interconnected Takagi–Sugeno fuzzy systems with nonlinear consequent. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 8, p. 4858–4867, 2021. Pages 28, 41, 50, 54, 75, 161, and 164.

[19] DONG, Jiuxiang; WANG, Youyi; YANG, Guang-Hong. Control synthesis of continuous-time t-s fuzzy systems with local nonlinear models. **IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)**, v. 39, n. 5, p. 1245–1258, 2009. Pages 28 and 164.

[20] DONG, Jiuxiang; WANG, Youyi; YANG, Guang Hong. $\mathcal{H}_\infty$ and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control of discrete-time T-S fuzzy systems with local nonlinear models. **Fuzzy Sets and Systems**, Elsevier, v. 164, n. 1, p. 1–24, 2011. Pages 28 and 164.

[21] GE, Xiaohua; HAN, Qing-Long; DING, Derui; ZHANG, Xian-Ming; NING, Boda. A survey on recent advances in distributed sampled-data cooperative control of multi-agent systems. **Neurocomputing**, v. 275, p. 1684–1701, 2018. ISSN 0925-2312. Page 28.

[22] TAKAGI, Tomohiro; SUGENO, Michio. Fuzzy identification of systems and its applications to modeling and control. **IEEE Transactions on Systems, Man, and Cybernetics**, SMC-15, n. 1, p. 116–132, 1985. Page 28.

[23] SHAMMA, Jeff S. **Analysis and design of gain scheduled control systems**. Tese (Doutorado) — Massachusetts Institute of Technology, 1988. Page 28.

[24] WANG, Hua O; TANAKA, Kazuo. **Fuzzy control systems design and analysis: a linear matrix inequality approach**. [S.l.]: John Wiley & Sons, 2004. Pages 28 and 159.

[25] ROTONDO, Damiano; PUIG, Vicenç; NEJJARI, Fatiha; WITCZAK, Marcin. Automated generation and comparison of takagi–sugeno and polytopic quasi-LPV models. **Fuzzy Sets and Systems**, v. 277, p. 44–64, 2015. ISSN 0165-0114. Page 28.

[26] NGUYEN, A.; TANIGUCHI, T.; ECIOLAZA, L.; CAMPOS, V.; PALHARES, R.; SUGENO, M. Fuzzy control systems: Past, present and future. **IEEE Computational Intelligence Magazine**, v. 14, n. 1, p. 56–68, 2019. Page 28.

[27] COUTINHO, Pedro Henrique Silva. **Enhanced nonquadratic stabilization of discrete-time Takagi-Sugeno fuzzy models**. Dissertação (Mestrado) — Universidade Federal de Minas Gerais, Brazil, 2019. Page 28.

[28] RUGH, W. J.; SHAMMA, J. S. Research on gain scheduling. **Automatica**, v. 36, n. 10, p. 1401–1425, 2000. Page 28.

[29] ATOUI, H.; SENAME, O.; MILANES, V.; MARTINEZ-MOLINA, J.J. Toward switching/interpolating LPV control: A review. **Annual Reviews in Control**, v. 54, p. 49–67, 2022. ISSN 1367-5788. Page 28.

[30] ERNEUX, Thomas. **Applied Delay Differential Equations**. [S.l.]: Springer New York, 2009. Page 29.

[31] RICHARD, Jean-Pierre. Time-delay systems: an overview of some recent advances and open problems. **Automatica**, v. 39, n. 10, p. 1667–1694, 2003. Page 29.

[32] FRIDMAN, Emilia. **Introduction to Time-Delay Systems**. Cham: Springer International Publishing, 2014. (Systems & Control: Foundations & Applications). Page 29.

[33] LIU, Kun; SELIVANOV, Anton; FRIDMAN, Emilia. Survey on time-delay approach to networked control. **Annual Reviews in Control**, v. 48, p. 57–79, 2019. Pages 29 and 47.

[34] BIRS, Isabela; MURESAN, Cristina; NASCU, Ioan; IONESCU, Clara. A survey of recent advances in fractional order control for time delay systems. **IEEE Access**, v. 7, p. 30951–30965, 2019. Page 29.

[35] FRIDMAN, Emilia. Tutorial on Lyapunov-based methods for time-delay systems. **European Journal of Control**, v. 20, n. 6, p. 271–283, 2014. Pages 29 and 47.

[36] ZHANG, Xian-Ming; HAN, Qing-Long; GE, Xiaohua; DING, Derui; DING, Lei; YUE, Dong; PENG, Chen. Networked control systems: a survey of trends and techniques. **IEEE/CAA Journal of Automatica Sinica**, v. 7, n. 1, p. 1–17, 2020. Page 30.

[37] GE, Xiaohua; YANG, Fuwen; HAN, Qing-Long. Distributed networked control systems: A brief overview. **Information Sciences**, v. 380, p. 117–131, 2017. Page 30.

[38] DING, Derui; HAN, Qing-Long; XIANG, Yang; GE, Xiaohua; ZHANG, Xian-Ming. A survey on security control and attack detection for industrial cyber-physical systems. **Neurocomputing**, v. 275, p. 1674 – 1683, 2018. Page 30.

[39] ZHANG, Dan; SHI, Peng; WANG, Qing-Guo; YU, Li. Analysis and synthesis of networked control systems: A survey of recent advances and challenges. **ISA Transactions**, v. 66, p. 376–392, 2017. Page 30.

[40] ZHANG, Xian-Ming; HAN, Qing-Long; YU, Xinghuo. Survey on recent advances in networked control systems. **IEEE Transactions on Industrial Informatics**, v. 12, n. 5, p. 1740–1752, 2016. Page 30.

[41] COUTINHO, Pedro Henrique Silva; PALHARES, Reinaldo Martínez. Dynamic periodic event-triggered gain-scheduling control co-design for quasi-lpv systems. **Nonlinear Analysis: Hybrid Systems**, v. 41, p. 101044, 2021. ISSN 1751-570X. Page 30.

[42] COUTINHO, Pedro H.S.; PEIXOTO, Márcia L.C.; BESSA, Iury; PALHARES, Reinaldo Martinez. Dynamic event-triggered gain-scheduling control of discrete-time quasi-lpv systems. **Automatica**, v. 141, p. 110292, 2022. ISSN 0005-1098. Page 30.

[43] COUTINHO, Pedro Henrique S.; BESSA, Iury; PESSIM, Paulo S.P.; PALHARES, Reinaldo Martínez. A switching approach to event-triggered control systems under denial-of-service attacks. **Nonlinear Analysis: Hybrid Systems**, v. 50, p. 101383, 2023. ISSN 1751-570X. Page 30.

[44] XU, Yao; JIANG, Zhuozhen; XIE, Xiangpeng; LI, Wenxue; WU, Yongbao; PALHARES, Reinaldo Martínez. Bipartite synchronization of fractional-order t–s fuzzy signed networks via event-triggered intermittent control. **IEEE Transactions on Fuzzy Systems**, v. 32, n. 12, p. 6979–6989, 2024. Page 30.

[45] COUTINHO, Pedro H.S.; BESSA, Iury; PEIXOTO, Márcia L.C.; PALHARES, Reinaldo M. A co-design condition for dynamic event-triggered feedback linearization control. **Systems & Control Letters**, v. 183, p. 105678, jan. 2024. ISSN 0167-6911. Page 30.

[46] LIU, Yang; XIE, Xiangpeng; PALHARES, Reinaldo Martinez; SUN, Jiayue. Dual channels event-triggered asymptotic consensus control for fractional-order nonlinear multiagent systems. **IEEE Transactions on Cybernetics**, v. 54, n. 11, p. 6780–6791, 2024. ISSN 2168-2275. Page 30.

[47] PEIXOTO, Márcia L. C.; PESSIM, Paulo S. P.; OLIVEIRA, Pedro M.; PIRES, Pedro O. F.; COUTINHO, Pedro H. S.; XIE, Xiangpeng; PALHARES, Reinaldo M. Event-triggered dynamic output-feedback control for LPV systems. **International Journal of Systems Science**, Informa UK Limited, v. 55, n. 14, p. 2952–2962, jul. 2024. ISSN 1464-5319. Page 30.

[48] PENG, Chen; LI, Fuqiang. A survey on recent advances in event-triggered communication and control. **Information Sciences**, v. 457-458, p. 113–125, 2018. Page 30.

[49] COUTINHO, Pedro Henrique Silva. **Dynamic event-triggered control of nonlinear systems: a quasi-LPV approach**. Tese (Doutorado) — Universidade Federal de Minas Gerais, Brazil, 2021. Page 30.

[50] SHI, Peng; YAN, Bing. A survey on intelligent control for multiagent systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 1, p. 161–175, 2021. Pages 31, 32, and 84.

[51] QIN, J.; MA, Q.; SHI, Y.; WANG, L. Recent advances in consensus of multi-agent systems: A brief survey. **IEEE Transactions on Industrial Electronics**, v. 64, n. 6, p. 4972–4983, 2017. Pages 31, 32, and 84.

[52] BAO, Lifeng Ma Guangyan; YI, Xiaojian. Recent advances on cooperative control of heterogeneous multi-agent systems subject to constraints: a survey. **Systems Science & Control Engineering**, Taylor & Francis, v. 10, n. 1, p. 539–551, 2022. Pages 32, 33, and 84.

[53] GE, Xiaohua; HAN, Qing-Long; DING, Derui; ZHANG, Xian-Ming; NING, Boda. A survey on recent advances in distributed sampled-data cooperative control of multi-agent systems. **Neurocomputing**, v. 275, p. 1684–1701, 2018. ISSN 0925-2312. Pages 32 and 84.

[54] GAO, Chen; HE, Xiao; DONG, Hongli; LIU, Hongjian; LYU, Guangran. A survey on fault-tolerant consensus control of multi-agent systems: trends, methodologies and prospects. **International Journal of Systems Science**, v. 53, n. 13, p. 2800–2813, 2022. ISSN 1464-5319. Pages 32 and 84.

[55] DOU, Yihang; XING, Guansheng; MA, Aohua; ZHAO, Guanwu. A review of event-triggered consensus control in multi-agent systems. **Journal of Control and Decision**, p. 1–23, 2024. ISSN 2330-7714. Pages 32 and 84.

[56] THUMMALAPETA, Mourya; LIU, Yen-Chen. Survey of containment control in multi-agent systems: concepts, communication, dynamics, and controller design. **International Journal of Systems Science**, v. 54, n. 14, p. 2809–2835, 2023. ISSN 1464-5319. Pages 32 and 84.

[57] DORRI, Ali; KANHERE, Salil S.; JURDAK, Raja. Multi-agent systems: A survey. **IEEE Access**, v. 6, p. 28573–28593, 2018. Pages 32 and 84.

[58] HE, Wangli; XU, Wenying; GE, Xiaohua; HAN, Qing-Long; DU, Wenli; QIAN, Feng. Secure control of multiagent systems against malicious attacks: A brief survey. **IEEE Transactions on Industrial Informatics**, v. 18, n. 6, p. 3595–3608, 2022. Pages 32 and 84.

[59] NOWZARI, Cameron; GARCIA, Eloy; CORTéS, Jorge. Event-triggered communication and control of networked systems for multi-agent consensus. **Automatica**, v. 105, p. 1–27, 2019. ISSN 0005-1098. Pages 32 and 84.

[60] DING, L.; ZHENG, W. X.; GUO, G. Network-based practical set consensus of multi-agent systems subject to input saturation. **Automatica**, v. 89, p. 316–324, 2018. ISSN 0005-1098. Page 33.

[61] DING, L.; ZHENG, W. X. Consensus tracking in heterogeneous nonlinear multi-agent networks with asynchronous sampled-data communication. **Systems & Control Letters**, v. 96, 2016. Page 33.

[62] DING, L.; ZHENG, W. X. Network-based practical consensus of heterogeneous nonlinear multiagent systems. **IEEE Transactions on Cybernetics**, v. 47, n. 8, p. 1841–1851, 2017. Pages 33, 90, and 91.

[63] OLIVEIRA, Pedro M.; COUTINHO, Pedro H.S.; BESSA, Iury; LACERDA, Márcio J.; PALHARES, Reinaldo M. Static output-feedback control design from data. **European Journal of Control**, Elsevier BV, v. 85, p. 101260, 2025. ISSN 0947-3580. Page 33.

[64] LUO, Yiping; XIAO, Xing; CAO, Jinde; LI, Anping; LIN, Guohan. Event-triggered guaranteed cost consensus control for second-order multi-agent systems based on observers. **Information Sciences**, v. 546, p. 283–297, 2021. ISSN 0020-0255. Page 33.

[65] PESSIM, Paulo S.P.; COUTINHO, Pedro Henrique Silva; LACERDA, Márcio J.; PALHARES, Reinaldo Martínez. Distributed control of time-delay interconnected nonlinear systems. **Journal of the Franklin Institute**, v. 360, n. 13, p. 9637–9662, set. 2023. ISSN 0016-0032. Page 35.

[66] PESSIM, Paulo S.P.; COUTINHO, Pedro Henrique Silva; LACERDA, Márcio J.; PALHARES, Reinaldo Martínez. Distributed event-triggered fuzzy control for nonlinear interconnected systems. **Chaos, Solitons** & **Fractals**, v. 177, p. 114276, dez. 2023. ISSN 0960-0779. Page 35.

[67] PESSIM, Paulo S.P; COUTINHO, Pedro Henrique Silva; LACERDA, Márcio J.; PUIG, Vicenç; PALHARES, Reinaldo Martinez. A distributed gain-scheduled observer-based approach for the practical consensus of LPV multi-agent systems. **Automatica**, v. 177, p. 112353, jul. 2025. ISSN 0005-1098. Page 35.

[68] PESSIM, Paulo S. P.; COUTINHO, Pedro H. S.; LACERDA, Márcio J.; PUIG, Vicenç; PALHARES, Reinaldo M. Distributed leader-following formation of discrete-time multi-agent LPV systems. **International Journal of Robust and Nonlinear Control**, p. 1–15, 2025. Page 36.

[69] SHEN, Qixia; LIN, Wenshuai; CAO, Liang; ZHAO, Meng. Dynamic event-triggered-based decentralized fault-tolerant control for interconnected nonlinear time-delay systems with unknown control directions. **Asian Journal of Control**, n/a, n. n/a. Page 38.

[70] CHEN, Ning; LUO, Zeng; LI, Binyan; LUO, Biao; CHEN, Jiayao; YANG, Chunhua; GUI, Weihua. ADP-Based decentralized controller design for nonlinear time-delay interconnected systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 54, n. 8, p. 5035–5047, 2024. Page 38.

[71] YANG, Yang; YUE, Dong; XUE, Yusheng. Decentralized adaptive neural output feedback control of a class of large-scale time-delay systems with input saturation. **Journal of the Franklin Institute**, v. 352, n. 5, p. 2129–2151, 2015. Page 38.

[72] XI, Changjiang; ZHAI, Ding; LI, Xiaojian; ZHANG, Qingling. Decentralized adaptive delay-dependent neural network control for a class of large-scale interconnected nonlinear systems. **Applied Mathematics and Computation**, v. 311, p. 148–163, 2017. Page 38.

[73] WANG, Yue; ZHANG, Xiuyu; WANG, Shunjiang; LI, Zhi; CHEN, Xinkai; SU, Chun-Yi. Decentralized implicit inverse control for large-scale hysteretic nonlinear time-delay systems and its application on triple-axis giant magnetostrictive actuators. **IEEE Transactions on Cybernetics**, 2023. Pages 38 and 39.

[74] ZHANG, Liuliu; ZHU, Lingchen; HUA, Changchun; QIAN, Cheng. Decentralised state-feedback prescribed performance control for a class of interconnected nonlinear full-state time-delay systems with strong interconnection. **International Journal of Systems Science**, v. 52, n. 12, p. 2580–2596, 2021. Pages 38 and 39.

[75] LI, Hanfeng; ZHANG, Xianfu; HOU, Ting. Output feedback control of large-scale nonlinear time-delay systems with unknown measurement sensitivities. **IET Control Theory & Applications**, v. 13, n. 13, p. 2122–2127, 2019. Pages 38 and 39.

[76] HUA, Changchun; SUN, Zhonghua; CHEN, Zihan. Exponential stabilization for time-delay nonlinear interconnected systems with unknown control directions and unmodeled dynamics. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, 2023. Pages 38 and 39.

[77] MA, Yuechao; JIN, Shujie; GU, Nannan. Delay-dependent decentralised control for a class of uncertain similar interconnected systems with state delay and input delay. **International Journal of Systems Science**, Taylor & Francis, v. 46, n. 16, p. 2887–2896, 2015. Pages 38 and 39.

[78] MA, Yuechao; JIN, Shujie; GU, Nannan. Decentralised memory static output feedback control for the nonlinear time-delay similar interconnected systems. **International Journal of Systems Science**, Taylor & Francis, v. 47, n. 10, p. 2487–2498, 2016. Pages 38 and 39.

[79] YU, Zhongming; CHEN, Keyu; SUN, Yue; DAI, Xin. Decentralized control for a class of interconnected delayed systems with nonlinear disturbance and control input saturation.

International Journal of Robust and Nonlinear Control, v. 33, n. 14, p. 8769–8790, 2023. Pages 38 and 39.

[80] WANG, Ke; SUN, Chao; HUANG, Shengjuan; WU, Libing; YI, Suhuan. Robust fault-tolerant cooperative control for a class of interconnected systems with actuator faults based on dynamic event-triggered. **International Journal of Robust and Nonlinear Control**, v. 34, n. 11, p. 7424–7459, 2024. Pages 38 and 40.

[81] HSIAO, Feng-Hsiag; CHEN, Cheng-Wu; LIANG, Yew-Wen; XU, Sheng-Dong; CHIANG, Wei-Ling. T-S fuzzy controllers for nonlinear interconnected systems with multiple time delays. **IEEE Transactions on Circuits and Systems I: Regular Papers**, v. 52, n. 9, p. 1883–1893, 2005. Pages 38 and 40.

[82] HSIAO, Feng-Hsiag; HWANG, Jung-Dong; CHEN, Cheng-Wu; TSAI, Zhi-Ren. Robust stabilization of nonlinear multiple time-delay large-scale systems via decentralized fuzzy control. **IEEE Transactions on Fuzzy Systems**, v. 13, n. 1, p. 152–163, 2005. Pages 38 and 40.

[83] OU, Ou; ZHANG, Hongbin; YU, Guanjie; GUO, Xiansheng; DANG, Chuangyin. Stability analysis and decentralized $\mathcal{H}_\infty$ control for time-delay fuzzy interconnected systems via fuzzy Lyapunov-krasovskii functional. **Journal of Intelligent & Fuzzy Systems**, v. 26, n. 4, p. 1731–1744, 2014. Pages 38 and 40.

[84] LI, Yuting; LI, Junmin. Decentralized stabilization of fractional order t-s fuzzy interconnected systems with multiple time delays. **Journal of Intelligent & Fuzzy Systems**, IOS Press, v. 30, n. 1, p. 319–331, 2016. Pages 38 and 40.

[85] SARBAZ, Mohammad; ZAMANI, Iman; MANTHOURI, Mohammad; IBEAS, Asier. Hierarchical optimization-based model predictive control for a class of discrete fuzzy large-scale systems considering time-varying delays and disturbances. **International Journal of Fuzzy Systems**, v. 24, n. 4, p. 2107–2130, 2022. Pages 38 and 40.

[86] SHANMUGAM, Lakshmanan; JOO, Young Hoon. Investigation on stability of delayed t-s fuzzy interconnected systems via decentralized memory-based sampled-data control and validation through interconnected power systems with dfig-based wind turbines. **Information Sciences**, v. 580, p. 934–952, 2021. ISSN 0020-0255. Pages 38 and 40.

[87] ZHONG, Zhixiong; YU, Jinyong; HE, Yidong; HAYAT, Tasawar; ALSAADI, Fuad. Fuzzy-model-based decentralized dynamic-output-feedback h_∞ control for large-scale nonlinear systems with time-varying delays. **Neurocomputing**, v. 173, p. 1054–1065, 2016. ISSN 0925-2312. Pages 38 and 40.

[88] HSIAO, Feng-Hsiag. Delay-dependent t–s fuzzy control for nonlinear interconnected systems: Using dithers as auxiliaries. **Applied Soft Computing**, v. 13, n. 11, p. 4442–4459, 2013. ISSN 1568-4946. Pages 38 and 40.

[89] MANI, Prakash; JOO, Young Hoon. Fuzzy logic-based integral sliding mode control of multi-area power systems integrated with wind farms. **Information Sciences**, v. 545, p. 153–169, 2021. ISSN 0020-0255. Page 38.

[90] XUE, Xiaojuan; XU, Huiling; XU, Li. Distributed filtering and control for time delay systems interconnected over an undirected graph. **International Journal of Control**, v. 93, n. 8, p. 1839–1858, 2020. Pages 38 and 40.

[91] LI, Li; ZHANG, Yaofeng. Distributed preview control for large-scale systems with time-varying delay. **ISA Transactions**, v. 109, p. 22–33, 2021. ISSN 0019-0578. Pages 38 and 40.

[92] PHAM, Thanh Ngoc; TRINH, Hieu; OO, Amanullah Maung Than. Distributed control of hvdc links for primary frequency control of time-delay power systems. **IEEE Transactions on Power Systems**, v. 34, n. 2, p. 1301–1314, 2019. Pages 38 and 40.

[93] JIA, Xianglei; XU, Shengyuan; SHI, Xiaocheng; LU, Junwei; DU, Baozhu. Adaptive output feedback control for large-scale time-delay systems with output-dependent uncertain growth rate. **International Journal of Adaptive Control and Signal Processing**, v. 36, n. 4, p. 965–979, 2022. Pages 38 and 39.

[94] ZHOU, Jing. Decentralized adaptive control for large-scale time-delay systems with dead-zone input. **Automatica**, v. 44, n. 7, p. 1790–1799, 2008. Pages 38 and 39.

[95] ZHANG, Xianfu; ZHANG, Chenghui; WANG, Yuzhen. Decentralized output feedback stabilization for a class of large-scale feedforward nonlinear time-delay systems. **International Journal of Robust and Nonlinear Control**, v. 24, n. 17, p. 2628–2639, 2014. Pages 38 and 39.

[96] ZHANG, Xianfu; LIU, Lu; FENG, Gang; ZHANG, Chenghui. Output feedback control of large-scale nonlinear time-delay systems in lower triangular form. **Automatica**, v. 49, n. 11, p. 3476–3483, 2013. Pages 38 and 39.

[97] ZHANG, Xu; LIN, Yan. Adaptive output feedback control for a class of large-scale nonlinear time-delay systems. **Automatica**, v. 52, p. 87–94, 2015. Pages 38 and 39.

[98] YAN, Xing-Gang; SPURGEON, Sarah K; EDWARDS, Christopher. Decentralised stabilisation for nonlinear time delay interconnected systems using static output feedback. **Automatica**, v. 49, n. 2, p. 633–641, 2013. Pages 38 and 39.

[99] YOO, Sung Jin; PARK, Jin Bae. Neural-network-based decentralized adaptive control for a class of large-scale nonlinear systems with unknown time-varying delays. **IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)**, v. 39, n. 5, p. 1316–1323, 2009. Pages 38 and 39.

[100] YE, Xudong. Decentralized adaptive stabilization of large-scale nonlinear time-delay systems with unknown high-frequency-gain signs. **IEEE Transactions on Automatic Control**, v. 56, n. 6, p. 1473–1478, 2011. Pages 38 and 39.

[101] THANH, Nguyen T; PHAT, Vu N. Decentralized h_∞ control for large-scale interconnected nonlinear time-delay systems via lmi approach. **Journal of Process Control**, v. 22, n. 7, p. 1325–1339, 2012. Pages 38 and 39.

[102] MIRKIN, Boris; GUTMAN, Per-Olof; SHTESSEL, Yuri. Decentralized continuous MRAC with local asymptotic sliding modes of nonlinear delayed interconnected systems. **Journal of the Franklin Institute**, v. 351, n. 4, p. 2076–2088, 2014. Pages 38 and 39.

[103] LIU, Ye; FANG, Zhijun; LI, Yuanyuan. Global decentralized output feedback control for interconnected time delay systems with saturated pi hysteresis. **Journal of the Franklin Institute**, v. 357, n. 9, p. 5460–5484, 2020. ISSN 0016-0032. Pages 38 and 39.

[104] CHOI, Yun Ho; YOO, Sung Jin. Minimal-approximation-based decentralized backstepping control of interconnected time-delay systems. **IEEE Transactions on Cybernetics**, v. 46, n. 12, p. 3401–3413, 2016. Pages 38 and 39.

[105] HUA, Changchun; LI, Kuo; LI, Yafeng; GUAN, Xinping. Decentralized adaptive tracking quantized control for interconnected pure feedback time delay nonlinear systems. **Journal of the Franklin Institute**, v. 355, n. 5, p. 2313–2328, 2018. ISSN 0016-0032. Pages 38 and 39.

[106] CHOI, Yun Ho; YOO, Sung Jin. Decentralized adaptive output-feedback control of interconnected nonlinear time-delay systems using minimal neural networks. **Journal of the Franklin Institute**, v. 355, n. 1, p. 81–105, 2018. ISSN 0016-0032. Pages 38 and 39.

[107] HE, Chao; LI, Junmin; ZHANG, Lin. Decentralized adaptive control of nonlinear large-scale pure-feedback interconnected systems with time-varying delays. **International Journal of Adaptive Control and Signal Processing**, v. 29, n. 1, p. 24–40, 2015. Pages 38 and 39.

[108] WU, Qiuye; ZHAO, Bo; LIU, Derong. Adaptive dynamic programming-based decentralised control for large-scale nonlinear systems subject to mismatched interconnections with unknown time-delay. **International Journal of Systems Science**, Taylor & Francis, v. 51, n. 15, p. 2883–2898, 2020. Pages 38 and 39.

[109] GUO, Tao; CHEN, Weisheng. Adaptive fuzzy decentralised fault-tolerant control for uncertain non-linear large-scale systems with unknown time-delay. **IET Control Theory & Applications**, Institution of Engineering and Technology, v. 10, p. 2437–2446, 2016. Pages 38 and 39.

[110] HUA, Changchun; ZHANG, Liuliu; GUAN, Xinping. Decentralized output feedback controller design for nonlinear interconnected systems with unknown control direction and time-varying delays. **International Journal of Adaptive Control and Signal Processing**, v. 28, n. 11, p. 1160–1173, 2014. Pages 38 and 39.

[111] CHOI, Yun Ho; YOO, Sung Jin. Decentralized approximation-free control for uncertain large-scale pure-feedback systems with unknown time-delayed nonlinearities and control directions. **Nonlinear Dynamics**, v. 85, n. 2, p. 1053–1066, 2016. Pages 38 and 39.

[112] ZHANG, Zhengqiang; YANG, Chen; CHEN, Xinkai. Decentralized model reference adaptive control for systems with time delays and dead zones. **IEEE Transactions on Automatic Control**, p. 1–8, 2022. Pages 38 and 39.

[113] GHOSH, Sandip; DAS, Sarit K.; RAY, Goshaidas. Decentralized adaptive robust stabilization of uncertain interconnected time-delay systems. **International Journal of Adaptive Control and Signal Processing**, v. 26, n. 1, p. 13–29, 2012. Pages 38 and 39.

[114] HUA, Changchun; GUAN, Xinping; SHI, Peng. Adaptive fuzzy control for uncertain interconnected time-delay systems. **Fuzzy Sets and Systems**, v. 153, n. 3, p. 447–458, 2005. ISSN 0165-0114. Pages 38 and 39.

[115] HSIAO, Feng-Hsiag. Optimal h_∞ fuzzy control for nonlinear multiple time-delay interconnected systems: Using dithers as auxiliaries. **Journal of the Franklin Institute**, v. 351, n. 7, p. 3932–3964, 2014. ISSN 0016-0032. Pages 38 and 39.

[116] HSIAO, Feng-Hsiag; XU, Sheng-Dong; LIN, Chia-Yen; TSAI, Zhi-Ren. Robustness design of fuzzy control for nonlinear multiple time-delay large-scale systems via neural-network-based approach. **IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)**, v. 38, n. 1, p. 244–251, 2008. Pages 38 and 39.

[117] MAO, Zhi-Zhong; XIAO, Xiao-Shi. Decentralized adaptive tracking control of nonaffine nonlinear large-scale systems with time delays. **Information Sciences**, v. 181, n. 23, p. 5291–5303, 2011. ISSN 0020-0255. Pages 38 and 39.

[118] GHOREISHEE, Aref; SHAHROKHI, Mohammad; MOHIT, Mohammaderfan. Fuzzy observer-based control of mimo interconnected systems subject to state delay, input nonlinearities, quantized input and output and sensor and actuator faults. **European Journal of Control**, Elsevier BV, v. 77, p. 100964, maio 2024. ISSN 0947-3580. Pages 38 and 39.

[119] YOO, Sung Jin; PARK, Bong Seok. Quantized-output-feedback practical prescribed-time design strategy for decentralized tracking of a class of interconnected nonlinear systems with unknown interaction delays. **ISA Transactions**, v. 147, p. 202–214, 2024. ISSN 0019-0578. Pages 38 and 39.

[120] CHOI, Yun Ho; YOO, Sung Jin. Quantized-state-based decentralized neural network control of a class of uncertain interconnected nonlinear systems with input and interaction time delays. **Engineering Applications of Artificial Intelligence**, Elsevier BV, v. 125, p. 106759, out. 2023. ISSN 0952-1976. Pages 38 and 39.

[121] ZHU, Lingchen; ZHANG, Liuliu; HUA, Changchun. Improved full-error constrained control for unknown interconnected time-delay nonlinear systems with input saturation. **IEEE Transactions on Fuzzy Systems**, v. 32, n. 8, p. 4496–4509, 2024. Pages 38 and 39.

[122] LIU, Guopin; ZHANG, Yu; HUA, Changchun. Adaptive output feedback control for nonlinear interconnected time-delay systems subject to global performance constraint. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 54, n. 12, p. 7670–7682, 2024. Pages 38 and 39.

[123] PARK, Ju H; JUNG, Ho Y; PARK, Jung I; LEE, Suk G. Decentralized dynamic output feedback controller design for guaranteed cost stabilization of large-scale discrete-delay systems. **Applied Mathematics and Computation**, v. 156, n. 2, p. 307–320, 2004. Pages 38 and 40.

[124] OUCHERIAH, Said. Decentralized stabilization of large scale systems with multiple delays in the interconnections. **International Journal of Control**, v. 73, n. 13, p. 1213–1223, 2000. Pages 38 and 40.

[125] HU, Zhongzhi. Decentralized stabilization of large scale interconnected systems with delays. **IEEE Transactions on Automatic Control**, v. 39, n. 1, p. 180–182, 1994. Pages 38 and 40.

[126] FERNANDO, TL; PHAT, VN; TRINH, HM. Decentralized stabilization of large-scale systems with interval time-varying delays in interconnections. **International Journal of Adaptive Control and Signal Processing**, v. 26, n. 6, p. 541–554, 2012. Pages 38 and 40.

[127] LEE, TN; RADOVIC, UL. Decentralized stabilization of linear continuous and discrete-time systems with delays in interconnections. **IEEE Transactions on Automatic Control**, v. 33, n. 8, p. 757–761, 1988. Pages 38 and 40.

[128] WANG, Wen-June; MAU, Lih-Gwo. Stabilization and estimation for perturbed discrete time-delay large-scale systems. **IEEE Transactions on Automatic Control**, v. 42, n. 9, p. 1277–1282, 1997. Pages 38 and 40.

[129] OUCHERIAH, Said. Decentralized adaptive control for a class of large-scale systems with multiple delays in the interconnections. **International Journal of Adaptive Control and Signal Processing**, v. 19, n. 1, p. 1–11, 2005. Pages 38 and 40.

[130] SHYU, Kuo-Kai; LIU, Wen-Jeng; HSU, Kou-Cheng. Design of large-scale time-delayed systems with dead-zone input via variable structure control. **Automatica**, v. 41, n. 7, p. 1239–1246, 2005. ISSN 0005-1098. Pages 38 and 40.

[131] WU, Hansheng. Decentralised robust stabilisation of uncertain large-scale interconnected time-delay systems with unknown upper bounds of uncertainties. **International Journal of Systems Science**, Taylor & Francis, v. 47, n. 12, p. 2816–2826, 2016. Pages 38 and 40.

[132] DOU, Chun Xia; DUAN, Zhi Sheng; JIA Xing-Bei; NIU, Pei-Feng. Study of delay-independent decentralized guaranteed cost control for large scale systems. **International Journal of Control, Automation and Systems**, v. 9, n. 3, p. 478–488, 2011. Pages 38 and 40.

[133] WANG, Rong-Jyue. Nonlinear decentralized state feedback controller for uncertain fuzzy time-delay interconnected systems. **Fuzzy Sets and Systems**, v. 151, n. 1, p. 191–204, 2005. ISSN 0165-0114. Pages 38 and 40.

[134] ZHONG, Zhixiong; WAI, Rong-Jong; SHAO, Zhenhua; XU, Min. Reachable set estimation and decentralized controller design for large-scale nonlinear systems with time-varying delay and input constraint. **IEEE Transactions on Fuzzy Systems**, v. 25, n. 6, p. 1629–1643, 2017. Pages 38 and 40.

[135] ZHANG, Hang; WANG, Xia; WANG, Yi-Jun; SUN, Zhao-Mei. Decentralized control of uncertain fuzzy large-scale system with time delay and optimization. **Journal of Applied Mathematics**, Hindawi Limited, v. 2012, p. 1–19, 2012. Pages 38 and 40.

[136] YE, Xin; SUN, Shaoxin; JIANG, Tao; SU, Xiaojie. Finite-time control for multiple time-delayed fuzzy large-scale systems against state and input constraints. **IEEE Transactions on Fuzzy Systems**, v. 30, n. 12, p. 5390–5404, 2022. Pages 38 and 40.

[137] WANG, Rong-Jyue. Control law for quadratic stabilization of perturbed fuzzy time-delay large-scale systems via lmi. **Asian Journal of Control**, v. 8, n. 4, p. 359–371, 2006. Pages 38 and 40.

[138] LIU, Zidong; ZHAO, Dongya; FENG, Jiehua; ZHANG, Ye; SPURGEON, Sarah K. Distributed robust partial state consensus control for chain interconnected delay systems. **IEEE Transactions on Control Systems Technology**, v. 31, n. 2, p. 977–984, 2023. Pages 38 and 40.

[139] RONG, Nannan; WANG, Zhanshan; ZHANG, Huaguang. Finite-time stabilization for discontinuous interconnected delayed systems via interval type-2 T–S fuzzy model approach. **IEEE Transactions on Fuzzy Systems**, v. 27, n. 2, p. 249–261, 2019. Pages 38, 40, and 41.

[140] YANG, Chen; XU, Shengyuan; ZHANG, Zhengqiang. Decentralized model reference adaptive control for interconnected systems with time-varying delays and unknown dead-zone inputs. **International Journal of Robust and Nonlinear Control**, v. 34, n. 6, p. 3679–3696, 2024. Pages 38 and 39.

[141] DENG, Chao; YANG, Guang-Hong; ER, Meng Joo. Decentralized fault-tolerant MRAC for a class of large-scale systems with time-varying delays and actuator faults. **Journal of Process Control**, v. 75, p. 171–186, 2019. Pages 38 and 39.

[142] HUA, Chang-Chun; LENG, Jing; GUAN, Xin-Ping. Decentralized MRAC for large-scale interconnected systems with time-varying delays and applications to chemical reactor systems. **Journal of Process Control**, v. 22, n. 10, p. 1985–1996, 2012. Pages 38 and 39.

[143] MAHMOUD, Magdi S. Decentralized output-feedback stabilization for interconnected discrete systems with unknown delays. **Optimal Control Applications and Methods**, v. 31, n. 6, p. 529–545, 2010. Pages 38 and 39.

[144] YAN, Xing-Gang; SPURGEON, Sarah K. Decentralised delay-dependent static output feedback variable structure control. **Journal of the Franklin Institute**, v. 351, n. 4, p. 2033–2047, 2014. Pages 38 and 39.

[145] YU, Zhongming; SUN, Yue; DAI, Xin; SU, Xiaojie. Decentralized time-delay control using partial variables with measurable states for a class of interconnected systems with time delays. **IEEE Transactions on Cybernetics**, v. 52, n. 10, p. 10882–10894, 2022. Pages 38 and 39.

[146] MAHMOUD, Magdi S. Decentralized stabilization of interconnected systems with time-varying delays. **IEEE Transactions on Automatic Control**, v. 54, n. 11, p. 2663–2668, 2009. Pages 38 and 39.

[147] HUA, Changchun; GUAN, Xinping. Output feedback stabilization for time-delay nonlinear interconnected systems using neural networks. **IEEE Transactions on Neural Networks**, v. 19, n. 4, p. 673–688, 2008. Pages 38 and 39.

[148] HUA, Chang-Chun; WANG, Qing-Guo; GUAN, Xin-Ping. Exponential stabilization controller design for interconnected time delay systems. **Automatica**, v. 44, n. 10, p. 2600–2606, 2008. Pages 38 and 40.

[149] MAHMOUD, Magdi S.; QURESHI, Amin ud-din. Decentralized sliding-mode output-feedback control of interconnected discrete-delay systems. **Automatica**, v. 48, n. 5, p. 808–814, 2012. ISSN 0005-1098. Pages 38 and 40.

[150] HASHEMIPOUR, Seyed Hamid; VASEGH, Nastaran; SEDIGH, Ali Khaki. Control of large scale interconnected systems with input and state delays using decentralized adaptive state observers. **Asian Journal of Control**, v. 22, n. 4, p. 1458–1470, 2020. Pages 38 and 40.

[151] MAHMOUD, Magdi S. Improved stability and stabilization approach to linear interconnected time-delay systems. **Optimal Control Applications and Methods**, v. 31, n. 2, p. 81–92, 2010. Pages 38, 39, and 40.

[152] MAHMOUD, Magdi S; XIA, Yuanqing. A generalized approach to stabilization of linear interconnected time-delay systems. **Asian Journal of Control**, v. 14, n. 6, p. 1539–1552, 2012. Pages 38 and 40.

[153] GUANS, Zhi-Hong; CHEN, Guanrong; YU, Xinghuo; QIN, Yi. Robust decentralized stabilization for a class of large-scale time-delay uncertain impulsive dynamical systems. **Automatica**, v. 38, n. 12, p. 2075–2084, 2002. ISSN 0005-1098. Pages 38 and 40.

[154] DOU, Chun-Xia; DUAN, Zhi-Sheng; JIA, Xing-Bei; LI, Xiao-Gang; YANG, Jin-Zhao; GUI, Ting; BI, Ye-Fei. Delay-dependent robust stabilization for nonlinear large systems via decentralized fuzzy control. **Mathematical Problems in Engineering**, v. 2011, p. 1–20, 2011. Pages 38, 40, and 41.

[155] XIE, Hao; LIAO, Fucheng; WU, Jiang; LI, Li. Design of distributed preview tracking controller for nonlinear interconnected systems with time-delay. **International Journal of Robust and Nonlinear Control**, v. 34, n. 1, p. 3–26, 2024. Pages 38 and 39.

[156] BIJAMI, Ehsan; FARSANGI, Malihe M; LEE, Kwang Y. Distributed control of networked wide-area systems: A power system application. **IEEE Transactions on Smart Grid**, IEEE, v. 11, n. 4, p. 3334–3345, 2019. Pages 38 and 40.

[157] WANG, Hui; XU, Huiling; CHEN, Xuefeng; GE, Shuzhi Sam. Distributed control for spatially interconnected time-varying delay systems under input saturation. **Journal of the Franklin Institute**, Elsevier, v. 360, n. 4, p. 3137–3161, 2023. Pages 38 and 40.

[158] SHI, Shengnan; LI, Yuan-Xin; TONG, Shaocheng. Event-based decentralized adaptive finite-time tracking control of interconnected nonlinear time-varying systems. **Nonlinear Dynamics**, v. 111, n. 4, p. 3479–3495, 2023. Page 41.

[159] XU, Hao; YU, Dengxiu; SUI, Shuai; CHEN, C. L. Philip. An event-triggered predefined time decentralized output feedback fuzzy adaptive control method for interconnected systems. **IEEE Transactions on Fuzzy Systems**, v. 31, n. 2, p. 631–644, 2023. Page 41.

[160] CHENG, Yan; NIU, Ben; ZHAO, Xudong; ZONG, Guangdeng; AHMAD, Adil M. Event-triggered adaptive decentralised control of interconnected nonlinear systems with bouc-wen hysteresis input. **International Journal of Systems Science**, v. 54, n. 6, p. 1275–1288, 2023. Page 41.

[161] ZHAO, Yanwei; NIU, Ben; ZONG, Guangdeng; XU, Ning; AHMAD, Adil M. Event-triggered optimal decentralized control for stochastic interconnected nonlinear systems via adaptive dynamic programming. **Neurocomputing**, v. 539, p. 126163, 2023. Page 41.

[162] WU, Qiuye; ZHAO, Bo; LIU, Derong; POLYCARPOU, Marios M. Event-triggered adaptive dynamic programming for decentralized tracking control of input constrained unknown nonlinear interconnected systems. **Neural Networks**, v. 157, p. 336–349, 2023. ISSN 0893-6080. Page 41.

[163] LIU, Shihui; NIU, Ben; XU, Ning; ZHAO, Xudong. Zero-sum game-based decentralized optimal control for saturated nonlinear interconnected systems via a data and event driven approach. **IEEE Systems Journal**, v. 18, n. 1, p. 758–769, 2024. Page 41.

[164] LIU, Chong; CHU, Zhousheng; DUAN, Zhongxing; ZHANG, Huaguang; MA, Zongfang. Decentralized event-triggered tracking control for unmatched interconnected systems via particle swarm optimization-based adaptive dynamic programming. **IEEE Transactions on Cybernetics**, v. 54, n. 11, p. 6895–6908, 2024. Page 41.

[165] LUO, Qiliang; XUE, Shan; LIU, Derong. Adaptive critic designs for decentralised robust control of nonlinear interconnected systems via event-triggering mechanism. **International Journal of Systems Science**, v. 53, n. 5, p. 1031–1047, 2022. Page 41.

[166] YANG, Xiong; HE, Haibo. Adaptive critic learning and experience replay for decentralized event-triggered control of nonlinear interconnected systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 50, n. 11, p. 4043–4055, 2020. Page 41.

[167] LUAN, Xinyang; SU, Hanguang; ZHANG, Huaguang; LIANG, Xiaodong; LIANG, Yuling; WANG, Jiawei. Dynamic event-triggered-based online irl algorithm for the decentralized control of the input and state constrained large-scale unmatched interconnected system. **Neurocomputing**, v. 568, p. 127042, 2024. ISSN 0925-2312. Page 41.

[168] SU, Hanguang; LUAN, Xinyang; ZHANG, Huaguang; LIANG, Xiaodong; YANG, Jinzhu; WANG, Jiawei. Decentralized optimal control of large-scale partially unknown nonlinear mismatched interconnected systems based on dynamic event-triggered control. **Neurocomputing**, v. 568, p. 127013, 2024. ISSN 0925-2312. Page 41.

[169] HUO, Yu; WANG, Ding; LI, Menghua; QIAO, Junfei. Decentralized event-triggered asymmetric constrained control through adaptive critic designs for nonlinear interconnected systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 54, n. 1, p. 391–402, 2024. Page 41.

[170] WANG, Tengda; WANG, Huanqing; XU, Ning; ZHANG, Liang; ALHARBI, Khalid H. Sliding-mode surface-based decentralized event-triggered control of partially unknown interconnected nonlinear systems via reinforcement learning. **Information Sciences**, v. 641, p. 119070, set. 2023. ISSN 0020-0255. Page 41.

[171] LIU, Jinliang; ZHANG, Nan; ZHA, Lijuan; XIE, Xiangpeng; TIAN, Engang. Reinforcement learning-based decentralized control for networked interconnected systems with communication and control constraints. **IEEE Transactions on Automation Science and Engineering**, v. 21, n. 3, p. 4674–4685, 2024. Page 41.

[172] LI, Min; AHN, Choon Ki; XIANG, Zhengrong. Decentralized adaptive fuzzy finite-time event-triggered control for interconnected nonlinear systems subject to input saturation. **IEEE Systems Journal**, v. 17, n. 1, p. 1648–1659, 2023. Page 41.

[173] JIANG, Feng; CAO, Liang; LIN, Wenshuai; XIAO, Wenbin. Event-based adaptive active disturbance rejection control for nonlinear large-scale systems with input delay. **International Journal of Systems Science**, p. 1–16, 2024. ISSN 1464-5319. Page 41.

[174] ZHANG, Weihai; LIU, Mengru. Error transformation-based event-triggered adaptive fuzzy control for interconnected nonlinear systems with output constraints. **IEEE Transactions on Fuzzy Systems**, v. 32, n. 5, p. 3132–3141, 2024. Page 41.

[175] LI, Yongming; TONG, Shaocheng. Event-triggered output-feedback adaptive control of interconnected nonlinear systems: A cyclic-small-gain approach. **IEEE Transactions on Cybernetics**, v. 54, n. 5, p. 3239–3250, 2024. Page 41.

[176] ZHU, Zhechen; ZHU, Quanxin. Adaptive neural network dynamic event-triggered control for strong interconnected stochastic nonlinear systems with output constraint. **International Journal of Robust and Nonlinear Control**, v. 33, n. 2, p. 899–918, 2023. Page 41.

[177] CUI, Yahui; SUN, Haibin; HOU, Linlin. Decentralized event-triggered adaptive neural network control for nonstrict-feedback nonlinear interconnected systems with external disturbances against intermittent dos attacks. **Neurocomputing**, v. 517, p. 133–147, 2023. ISSN 0925-2312. Page 41.

[178] CUI, Yahui; SUN, Haibin; HOU, Linlin; SHI, Kaibo. Dynamic event-triggered adaptive neural network decentralized output-feedback control for nonlinear interconnected systems with hybrid cyber attacks and its application. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 54, n. 4, p. 2149–2158, 2024. Page 41.

[179] LIU, Shihui; NIU, Ben; XU, Ning; ZHAO, Xudong. Zero-sum game-based decentralized optimal control for saturated nonlinear interconnected systems via a data and event driven approach. **IEEE Systems Journal**, v. 18, n. 1, p. 758–769, 2024. Page 41.

[180] BEY, Oussama; CHEMACHEMA, Mohamed. Finite-time event-triggered output-feedback adaptive decentralized echo-state network fault-tolerant control for interconnected pure-feedback nonlinear systems with input saturation and external disturbances: A fuzzy control-error approach. **Information Sciences**, v. 669, p. 120557, 2024. ISSN 0020-0255. Page 41.

[181] CAO, Liang; REN, Hongru; LI, Hongyi; LU, Renquan. Event-triggered output-feedback control for large-scale systems with unknown hysteresis. **IEEE Transactions on Cybernetics**, v. 51, n. 11, p. 5236–5247, 2021. Page 41.

[182] YAO, Yangang; TAN, Jieqing; WU, Jian; ZHANG, Xu. Decentralized adaptive neural safe tracking control for nonlinear systems with conflicted output constraints. **ISA Transactions**, 2023. ISSN 0019-0578. Pages 41 and 42.

[183] CHOI, Yun Ho; YOO, Sung Jin. Event-triggered decentralized adaptive fault-tolerant control of uncertain interconnected nonlinear systems with actuator failures. **ISA Transactions**, v. 77, p. 77–89, 2018. ISSN 0019-0578. Pages 41 and 42.

[184] XU, Lin-Xing; WANG, Yu-Long; WANG, Xiaofan; PENG, Chen. Decentralized event-triggered adaptive control for interconnected nonlinear systems with actuator failures. **IEEE Transactions on Fuzzy Systems**, v. 31, n. 1, p. 148–159, 2023. Pages 41 and 42.

[185] ZHANG, Lili; DENG, Chao; CHE, Wei-Wei; AN, Liwei. Adaptive backstepping control for nonlinear interconnected systems with prespecified-performance-driven output triggering. **Automatica**, v. 154, p. 111063, ago. 2023. ISSN 0005-1098. Pages 41 and 42.

[186] YANG, Wenjing; XIA, Jianwei; GUO, Xiaoxiao; YU, Miao; ZHANG, Na. Adaptive decentralized event-triggered tracking control for large-scale strongly interconnected nonlinear system with global performance. **International Journal of Control, Automation and Systems**, v. 21, n. 5, p. 1547–1559, 2023. ISSN 2005-4092. Pages 41 and 42.

[187] CHEN, Guangjun; DONG, Jiuxiang. Adaptive prescribed time fuzzy control of interconnected nonlinear systems and its applications: A compensation-based approach. **IEEE Transactions on Automation Science and Engineering**, p. 1–10, 2024. Pages 41 and 42.

[188] YU, Hao; HAO, Fei; CHEN, Tongwen. A uniform analysis on input-to-state stability of decentralized event-triggered control systems. **IEEE Transactions on Automatic Control**, v. 64, n. 8, p. 3423–3430, 2019. Pages 41 and 42.

[189] LIU, Tengfei; JIANG, Zhong-Ping; ZHANG, Pengpeng. Decentralized event-triggered control of large-scale nonlinear systems. **International Journal of Robust and Nonlinear Control**, v. 30, n. 4, p. 1451–1466, 2020. Pages 41 and 42.

[190] TALLAPRAGADA, Pavankumar; CHOPRA, Nikhil. Decentralized event-triggering for control of nonlinear systems. **IEEE Transactions on Automatic Control**, v. 59, n. 12, p. 3312–3324, 2014. Pages 41 and 42.

[191] WANG, Jing; WU, Jiacheng; SHEN, Hao; CAO, Jinde; RUTKOWSKI, Leszek. A decentralized learning control scheme for constrained nonlinear interconnected systems based on dynamic event-triggered mechanism. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, p. 1–10, 2023. Pages 41 and 42.

[192] NARAYANAN, Vignesh; MODARES, Hamidreza; JAGANNATHAN, Sarangapani. Event-triggered control of input-affine nonlinear interconnected systems using multiplayer game. **International Journal of Robust and Nonlinear Control**, v. 31, n. 3, p. 950–970, 2021. Pages 41 and 42.

[193] NARAYANAN, Vignesh; MODARES, Hamidreza; JAGANNATHAN, Sarangapani; LEWIS, Frank L. Event-driven off-policy reinforcement learning for control of interconnected systems. **IEEE Transactions on Cybernetics**, v. 52, n. 3, p. 1936–1946, 2022. Pages 41 and 42.

[194] RONG, Nannan; WANG, Zhanshan. Event-based fixed-time control for interconnected systems with discontinuous interactions. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 52, n. 8, p. 4925–4936, 2022. Pages 41 and 43.

[195] CAI, Jianping; YU, Rui; WANG, Binrui; MEI, Congli; SHEN, Lujuan. Decentralized event-triggered control for interconnected systems with unknown disturbances. **Journal of the Franklin Institute**, v. 357, n. 3, p. 1494–1515, 2020. ISSN 0016-0032. Page 41.

[196] GHORBANI, Saeid; SAFAVI, Ali Akbar; NAGHAVI, S. Vahid. Decentralized event-triggered robust MPC for large-scale networked lipchitz non-linear control systems. **IET Control Theory & Applications**, v. 15, n. 18, p. 2316–2333, 2021. Pages 41 and 42.

[197] PENG, Chen; TIAN, Engang; ZHANG, Jin; DU, Dajun. Decentralized event-triggering communication scheme for large-scale systems under network environments. **Information Sciences**, v. 380, p. 132–144, 2017. ISSN 0020-0255. Pages 41 and 42.

[198] HEEMELS, W.P.M.H.; DONKERS, M.C.F. Model-based periodic event-triggered control for linear systems. **Automatica**, v. 49, n. 3, p. 698–711, 2013. ISSN 0005-1098. Pages 41, 42, and 43.

[199] ZHOU, Jing; ZHU, Jun-Wei; ZHANG, Wen-An; YU, Li. Event-triggered dynamic output feedback tracking control for large-scale interconnected systems with disturbances. **Journal of the Franklin Institute**, v. 356, n. 17, p. 10547–10563, 2019. ISSN 0016-0032. Pages 41, 42, and 43.

[200] LIU, Dan; YANG, Guang-Hong. Decentralized event-triggered output feedback control for a class of interconnected large-scale systems. **ISA Transactions**, v. 93, p. 156–164, 2019. ISSN 0019-0578. Pages 41, 42, and 43.

[201] GU, Zhou; PARK, Ju H.; YUE, Dong; WU, Zheng-Guang; XIE, Xiangpeng. Event-triggered security output feedback control for networked interconnected systems subject to cyber-attacks. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 10, p. 6197–6206, 2021. Pages 41, 42, and 43.

[202] ZHANG, Dan; CHEN, Xuefeng; WANG, Hui; REN, Hangli; XU, Huiling. Event-triggered H_∞ control for networked dynamic system with nonideal interconnection. **Applied Mathematics and Computation**, v. 463, p. 128373, 2024. ISSN 0096-3003. Pages 41, 42, and 43.

[203] SONG, Wenting; TONG, Shaocheng. Fuzzy decentralized output feedback event-triggered control for interval type-2 fuzzy systems with saturated inputs. **Information Sciences**, v. 575, p. 639–653, 2021. ISSN 0020-0255. Pages 41, 43, and 44.

[204] RONG, Nannan; WANG, Zhanshan. Fixed-time stabilization for it2 t–s fuzzy interconnected systems via event-triggered mechanism: An exponential gain method. **IEEE Transactions on Fuzzy Systems**, v. 28, n. 2, p. 246–258, 2020. Pages 41 and 43.

[205] ZHONG, Zhixiong; LIN, Chih-Min; SHAO, Zhenhua; XU, Min. Decentralized event-triggered control for large-scale networked fuzzy systems. **IEEE Transactions on Fuzzy Systems**, v. 26, n. 1, p. 29–45, 2018. Pages 41, 43, 44, and 70.

[206] ZHONG, Zhixiong; ZHU, Yanzheng. Observer-based output-feedback control of large-scale networked fuzzy systems with two-channel event-triggering. **Journal of the Franklin Institute**, v. 354, n. 13, p. 5398–5420, 2017. ISSN 0016-0032. Pages 41, 43, and 44.

[207] ZHONG, Zhixiong; ZHU, Yanzheng; KARIMI, Mohammad. Dynamic output feedback fuzzy control of large-scale nonlinear networked systems: A two-channel triggering approach. **IEEE Access**, v. 5, p. 12428–12438, 2017. Pages 41, 43, and 44.

[208] WANG, Huimin; YANG, Guang-Hong. Decentralized event-triggered h_∞ control for affine fuzzy large-scale systems. **IEEE Transactions on Fuzzy Systems**, v. 27, n. 11, p. 2215–2226, 2019. Pages 41, 43, 44, and 70.

[209] ZHOU, Yuanqiang; LI, Dewei; XI, Yugeng; GAO, Furong. Event-triggered distributed robust model predictive control for a class of nonlinear interconnected systems. **Automatica**, v. 136, p. 110039, 2022. ISSN 0005-1098. Pages 41 and 42.

[210] NARAYANAN, Vignesh; JAGANNATHAN, Sarangapani. Event-triggered distributed control of nonlinear interconnected systems using online reinforcement learning with exploration. **IEEE Transactions on Cybernetics**, v. 48, n. 9, p. 2510–2519, 2018. Pages 41 and 42.

[211] NARAYANAN, Vignesh; SAHOO, Avimanyu; JAGANNATHAN, Sarangapani; GEORGE, Koshy. Approximate optimal distributed control of nonlinear interconnected systems using event-triggered nonzero-sum games. **IEEE Transactions on Neural Networks and Learning Systems**, v. 30, n. 5, p. 1512–1522, 2019. Pages 41 and 42.

[212] CUI, Lili; XIE, Xiangpeng; GUO, Hongyan; LUO, Yanhong. Dynamic event-triggered distributed guaranteed cost ftc scheme for nonlinear interconnected systems via adp approach. **Applied Mathematics and Computation**, v. 425, p. 127082, 2022. ISSN 0096-3003. Pages 41 and 42.

[213] CUI, Lili; ZHANG, Yong; WANG, Xiaowei; XIE, Xiangpeng. Event-triggered distributed self-learning robust tracking control for uncertain nonlinear interconnected systems. **Applied Mathematics and Computation**, v. 395, p. 125871, 2021. ISSN 0096-3003. Pages 41 and 42.

[214] TAN, Luy Nguyen. Event-triggered distributed H_∞ constrained control of physically interconnected large-scale partially unknown strict-feedback systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 4, p. 2444–2456, 2021. Pages 41 and 42.

[215] SUN, Haibin; LIU, Yujie; JIAO, Ticao; HOU, Linlin. Distributed extended state observer design and dual-side dynamic event-triggered output feedback anti-disturbance control for nonlinear interconnected systems with quantization. **Journal of the Franklin Institute**, v. 361, n. 9, p. 106847, jun. 2024. ISSN 0016-0032. Pages 41 and 42.

[216] GUO, Bin; DIAN, Songyi; ZHAO, Tao. Event- driven-observer-based adaptive distributed sliding mode fault compensation control for interconnected nonlinear systems. **International Journal of Robust and Nonlinear Control**, v. 34, n. 3, p. 2013–2035, 2023. ISSN 1099-1239. Pages 41 and 42.

[217] LIU, Yujie; SUN, Haibin; HOU, Linlin. Distributed extended state observer design and output feedback anti-disturbance control for nonlinear interconnected systems: A dynamic memory event-triggered mechanism. **Asian Journal of Control**, v. 26, n. 1, p. 227–245, 2023. ISSN 1934-6093. Pages 41 and 42.

[218] LI, Hongli; SU, Baili; GUO, Benshuai. Observer-based event-triggered distributed model predictive control for a class of nonlinear interconnected systems. **International Journal of Robust and Nonlinear Control**, v. 34, n. 7, p. 4719–4735, 2024. ISSN 1099-1239. Pages 41 and 42.

[219] ZHANG, Meng; DONG, Shanling; SHI, Peng; CHEN, Guanrong; GUAN, Xiaohong. Distributed observer-based event-triggered load frequency control of multiarea power systems under cyber attacks. **IEEE Transactions on Automation Science and Engineering**, v. 20, n. 4, p. 2435–2444, 2023. Pages 41, 42, and 43.

[220] GUAN, Yanpeng; PING, Guonan; ZHENG, Wei Xing; YAO, Huijuan. Distributed event-triggered scheduling in networked interconnected systems with sparse connections. **Neurocomputing**, v. 400, p. 420–428, 2020. ISSN 0925-2312. Pages 41, 42, and 43.

[221] SHI, Ting; TANG, Tingting; BAI, Jianjun. Distributed event-triggered control co-design for large-scale systems via static output feedback. **Journal of the Franklin Institute**, v. 356, n. 17, p. 10393–10404, 2019. ISSN 0016-0032. Pages 41, 42, and 43.

[222] WANG, Xin; SUN, Jian; WANG, Gang; ALLGÖWER, Frank; CHEN, Jie. Data-driven control of distributed event-triggered network systems. **IEEE/CAA Journal of Automatica Sinica**, v. 10, n. 2, p. 351–364, 2023. Pages 41, 42, and 43.

[223] MI, Xiaoxiao; ZOU, Yuanyuan; LI, Shaoyuan. Event-triggered MPC design for distributed systems toward global performance. **International Journal of Robust and Nonlinear Control**, v. 28, n. 4, p. 1474–1495, 2018. Pages 41 and 42.

[224] GUINALDO, María; DIMAROGONAS, Dimos V.; JOHANSSON, Karl H.; SáNCHEZ, José; DORMIDO, Sebástian. Distributed event-based control strategies for interconnected linear systems. **IET Control Theory & Applications**, v. 7, n. 6, p. 877–886, 2013. Pages 41 and 42.

[225] GUINALDO, M.; SáNCHEZ, J.; DORMIDO, R.; DORMIDO, S. Distributed control for large-scale systems with adaptive event-triggering. **Journal of the Franklin Institute**, v. 353, n. 3, p. 735–756, 2016. ISSN 0016-0032. Pages 41 and 42.

[226] MA, Aoyun; LIU, Kun; ZHANG, Qirui; LIU, Tao; XIA, Yuanqing. Event-triggered distributed MPC with variable prediction horizon. **IEEE Transactions on Automatic Control**, v. 66, n. 10, p. 4873–4880, 2021. Pages 41 and 42.

[227] QIAN, Yang-Yang; WAN, Yan; LIN, Zongli; SHAMASH, Yacov A.; DAVOUDI, Ali. Distributed dynamic event-triggered control of power buffers in DC microgrids. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 52, n. 12, p. 7748–7759, 2022. Pages 41 and 42.

[228] ZHANG, Taixin; YU, Hao; HAO, Fei. A novel distributed event-triggered control with time-varying thresholds. **Journal of the Franklin Institute**, v. 357, n. 7, p. 4132–4153, 2020. ISSN 0016-0032. Pages 41 and 42.

[229] ZHOU, Siyu; QIAN, Yangyang; WAN, Yan; LIN, Zongli; SHAMASH, Yacov A.; DAVOUDI, Ali. Model-based dynamic event-triggered distributed control of linear physically interconnected systems and application to power buffers. **IEEE Internet of Things Journal**, v. 10, n. 24, p. 22320–22330, 2023. Pages 41 and 42.

[230] LI, Xiehuan; YE, Dan. Event-based distributed fuzzy load frequency control for multiarea nonlinear power systems with switching topology. **IEEE Transactions on Fuzzy Systems**, v. 30, n. 10, p. 4262–4272, 2022. Pages 41, 43, 44, and 70.

[231] ZHONG, Zhixiong; ZHU, Yanzheng; BASIN, Michael V.; LAM, Hak-Keung. Event-based multirate control of large-scale distributed nonlinear systems subject to time-driven zero order holds. **Nonlinear Analysis: Hybrid Systems**, v. 36, p. 100864, 2020. ISSN 1751-570X. Pages 41, 43, and 44.

[232] ZHONG, Zhixiong; ZHU, Yanzheng; LAM, Hak-Keung. Asynchronous piecewise output-feedback control for large-scale fuzzy systems via distributed event-triggering schemes. **IEEE Transactions on Fuzzy Systems**, v. 26, n. 3, p. 1688–1703, 2018. Pages 41, 43, 44, and 70.

[233] FRIDMAN, Emilia. **Introduction to Time-Delay Systems**. Cham: Springer International Publishing, 2014. (Systems & Control: Foundations & Applications). Page 47.

[234] COUTINHO, P. H. S.; ARAÚJO, R. F.; NGUYEN, Anh-Tu; PALHARES, R. M. A Multiple-Parameterization Approach for local stabilization of constrained Takagi-Sugeno fuzzy systems with nonlinear consequents. **Information Sciences**, Elsevier Inc., v. 506, p. 295–307, 2020. Pages 58 and 164.

[235] SADAMOTO, Tomonori; ISHIZAKI, Takayuki; IMURA, Jun; SANDBERG, Henrik; JOHANSSON, Karl Henrik. Retrofitting state feedback control of networked nonlinear systems based on hierarchical expansion. In: **IEEE 55th Conference on Decision and Control (CDC)**. [S.l.: s.n.], 2016. p. 3432–3437. Page 62.

[236] COUTINHO, Pedro Henrique Silva; PALHARES, Reinaldo Martínez. Codesign of dynamic event-triggered gain-scheduling control for a class of nonlinear systems. **IEEE Transactions on Automatic Control**, v. 67, n. 8, p. 4186–4193, 2022. Pages 70, 72, and 76.

[237] TABUADA, Paulo. Event-triggered real-time scheduling of stabilizing control tasks. **IEEE Transactions on Automatic Control**, v. 52, n. 9, p. 1680–1685, 2007. Page 72.

[238] MAVROTAS, G. Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. **Applied mathematics and computation**, Elsevier, v. 213, n. 2, p. 455–465, 2009. Page 76.

[239] CHEN, Y.; HUANG, X.; ZHAN, J. Gain-scheduled robust control for multi-agent linear parameter-varying systems with communication delays. **International Journal of Robust and Nonlinear Control**, v. 32, n. 2, p. 792–806, 2021. ISSN 1099-1239. Pages 84 and 85.

[240] CHEN, J.; ZHANG, W.; CAO, Y.-Y.; CHU, H. Observer-based consensus control against actuator faults for linear parameter-varying multiagent systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 47, n. 7, p. 1336–1347, 2017. Pages 84, 85, and 98.

[241] ROTONDO, D.; PONSART, J.-C.; THEILLIOL, D. Gain-scheduled observer-based consensus for linear parameter varying multi-agent systems. **Automatica**, v. 135, p. 109979, 2022. ISSN 0005-1098. Pages 9, 84, 85, 86, 89, 93, 94, 95, 96, 97, and 113.

[242] ROTONDO, D.; THEILLIOL, D.; PONSART, J.-C. Virtual actuator and sensor fault tolerant consensus for homogeneous linear multi-agent systems. **IEEE Transactions on Circuits and Systems I: Regular Papers**, p. 1–11, 2023. ISSN 1558-0806. Pages 84 and 85.

[243] TREJO, Jesus A. Vazquez; PONSART, Jean-Christophe; ADAM-MEDINA, Manuel; VALENCIA-PALOMO, Guillermo. Fault-tolerant observer-based leader-following consensus control for LPV multi-agent systems using virtual actuators. **International Journal of Systems Science**, p. 1–18, dez. 2024. ISSN 1464-5319. Pages 84 and 85.

[244] TREJO, Jesus A. Vazquez; PONSART, Jean-Christophe; ADAM-MEDINA, Manuel; VALENCIA-PALOMO, Guillermo; THEILLIOL, Didier. Distributed observer-based leader-following consensus control for LPV multi-agent systems: Application to multiple VTOL-UAVs formation control. In: **2023 International Conference on Unmanned Aircraft Systems (ICUAS)**. [S.l.: s.n.], 2023. p. 1316–1323. Pages 84 and 85.

[245] GHANAVATI, Ali Karami; ASEMANI, Mohammad Hassan. Observer-based consensus control of multi-agent linear parameter varying systems. **Journal of the Franklin Institute**, v. 357, n. 17, p. 12125–12142, nov. 2020. ISSN 0016-0032. Pages 84 and 85.

[246] YUAN, Chengzhi. Leader-following consensus of parameter-dependent networks via distributed gain-scheduling control. **International Journal of Systems Science**, v. 48, n. 10, p. 2013–2022, mar. 2017. ISSN 1464-5319. Pages 84 and 85.

[247] MESBAHI, Afshin; VELNI, Javad Mohammadpour. Distributed observer-based cooperative control for output regulation in multi-agent linear parameter-varying systems. **IET Control Theory & Applications**, v. 11, n. 9, p. 1394–1403, 2017. Pages 84 and 85.

[248] FU, Chengcheng; ZHANG, Hao; HUANG, Chao; WANG, Zhuping; YAN, Huaicheng. Fully distributed cooperative output regulation for heterogeneous linear parameter-varying systems with directed graphs. **IEEE Transactions on Control of Network Systems**, v. 10, n. 3, p. 1350–1361, 2023. Pages 84 and 85.

[249] FU, Chengcheng; ZHANG, Hao; WANG, Zhuping. Fully distributed event-triggered cooperative output regulation of linear parameter-varying systems. In: **2021 40th Chinese Control Conference (CCC)**. [S.l.: s.n.], 2021. p. 5631–5636. Pages 84 and 85.

[250] LI, Shanglin; CHEN, Yangzhou; LIU, Peter Xiaoping. Finite-frequency fault estimation and adaptive event-triggered fault-tolerant consensus for LPV multiagent systems. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 54, n. 12, p. 7871–7883, 2024. Pages 84 and 85.

[251] SAADABADI, Hamideh; WERNER, Herbert. Event-triggered ℓ_2-optimal formation control for agents modeled as LPV systems. In: **2021 60th IEEE Conference on Decision and Control (CDC)**. [S.l.: s.n.], 2021. p. 1256–1262. Pages 84 and 85.

[252] GEBHARDT, Gerald; SAADABADI, Hamideh; WERNER, Herbert. Event-triggered ℓ_2-Optimal formation control with state-estimation for agents modeled as LPV systems. In: **2022 IEEE 61st Conference on Decision and Control (CDC)**. [S.l.: s.n.], 2022. p. 1740–1745. Pages 84 and 85.

[253] REHMAN, Ateeq ur; REHAN, Muhammad; IQBAL, Naeem; AHN, Choon Ki. LPV scheme for robust adaptive output feedback consensus of lipschitz multiagents using lipschitz nonlinear protocol. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 51, n. 11, p. 7040–7050, 2021. Page 86.

[254] REHMAN, Ateeq ur; SHI, Peng; WU, Xiaotai; HUANG, Tingwen; ZHANG, Wenbing. Robust leaderless consensus control of multiagent systems with lipschitz nonlinearity via LPV scheme under fixed and switching topologies. **IEEE Systems Journal**, v. 18, n. 1, p. 379–390, 2024. Page 86.

[255] REHMAN, Ateeq ur; HUANG, Tingwen; REHAN, Muhammad; WU, Xiaotai; ZHANG, Wenbing. New results on cooperative optimal consensus control of multiagents using LPV approach for lipschitz nonlinear systems under digraphs. **IEEE Transactions on Systems, Man, and Cybernetics: Systems**, v. 54, n. 9, p. 5655–5666, 2024. Page 86.

[256] REHMAN, Ateeq ur; REHAN, Muhammad; IQBAL, Naeem; AHN, Choon Ki. Toward the LPV approach for adaptive distributed consensus of lipschitz multi-agents. **IEEE Transactions on Circuits and Systems II: Express Briefs**, v. 66, n. 1, p. 91–95, 2019. Page 86.

[257] REHAN, Muhammad; TUFAIL, Muhammad; AHMED, Shakeel. Leaderless consensus control of nonlinear multi-agent systems under directed topologies subject to input saturation using adaptive event-triggered mechanism. **Journal of the Franklin Institute**, v. 358, n. 12, p. 6217–6239, ago. 2021. ISSN 0016-0032. Page 86.

[258] FRIDMAN, E.; DAMBRINE, M. Control under quantization, saturation and delay: An LMI approach. **Automatica**, v. 45, n. 10, p. 2258–2264, 2009. Pages 90 and 91.

[259] LIU, Zhongxin; LI, Yangbo; WANG, Fuyong; CHEN, Zengqiang. Reduced-order observer-based leader-following formation control for discrete-time linear multi-agent systems. **IEEE/CAA Journal of Automatica Sinica**, v. 8, n. 10, p. 1715–1723, 2021. Pages 102 and 121.

[260] HE, Quangui; LIU, Wei. Formation control for linear multi-agent systems with asynchronously sampled outputs. **Information Sciences**, v. 658, p. 119992, 2024. ISSN 0020-0255. Pages 102, 103, 105, and 111.

[261] ZHU, Guoliang; LIU, Kexin; GU, Haibo; LUO, Weilin; Lü, Jinhu. Observer-based event-triggered formation control of multi-agent systems with switching directed topologies. **IEEE Transactions on Circuits and Systems I: Regular Papers**, v. 69, n. 3, p. 1323–1332, 2022. Page 103.

[262] DONG, Xiwang; HU, Guoqiang. Time-varying formation tracking for linear multiagent systems with multiple leaders. **IEEE Transactions on Automatic Control**, v. 62, n. 7, p. 3658–3664, 2017. Page 103.

[263] GE, Xiaohua; HAN, Qing-Long. Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism. **IEEE Transactions on Industrial Electronics**, v. 64, n. 10, p. 8118–8127, 2017. Page 103.

[264] NGUYEN, Anh-Tu; CAMPOS, Víctor; GUERRA, Thierry-Marie; PAN, Juntao; XIE, Wenbo. Takagi–Sugeno fuzzy observer design for nonlinear descriptor systems with unmeasured premise variables and unknown inputs. **International Journal of Robust and Nonlinear Control**, v. 31, n. 17, p. 8353–8372, 2021. Pages 107 and 113.

[265] PESSIM, Paulo S. P.; LACERDA, Márcio J. State-feedback control for cyber-physical LPV systems under dos attacks. **IEEE Control Systems Letters**, v. 5, n. 3, p. 1043–1048, 2021. Pages 119, 120, and 128.

[266] KOTHARE, Mayuresh V.; BALAKRISHNAN, Venkataramanan; MORARI, Manfred. Robust constrained model predictive control using linear matrix inequalities. **Automatica**, v. 32, n. 10, p. 1361–1379, 1996. ISSN 0005-1098. Pages 119 and 120.

[267] RONG, Nannan; WANG, Zhanshan. Event-based impulsive control of it2 t–s fuzzy interconnected system under deception attacks. **IEEE Transactions on Fuzzy Systems**, v. 29, n. 6, p. 1615–1628, 2021. Page 127.

[268] PAN, Weihao; ZHANG, Xianfu; LI, Hanfeng; FAN, Debao. Event-triggered impulsive control of lower-triangular large-scale nonlinear systems based on gain scaling technique. **Nonlinear Analysis: Hybrid Systems**, v. 49, p. 101362, 2023. ISSN 1751-570X. Page 127.

[269] LI, Xiaodi; PENG, Dongxue; CAO, Jinde. Lyapunov stability for impulsive systems via event-triggered impulsive control. **IEEE Transactions on Automatic Control**, v. 65, n. 11, p. 4908–4913, 2020. Page 127.

[270] TAN, Xuegang; CAO, Jinde; LI, Xiaodi. Consensus of leader-following multiagent systems: A distributed event-triggered impulsive control strategy. **IEEE Transactions on Cybernetics**, v. 49, n. 3, p. 792–801, 2019. Page 127.

[271] XU, Yao; ZHANG, Ziming; XIE, Xiangpeng; PALHARES, Reinaldo Martínez; LI, Wenxue; WU, Yongbao. Consensus of fractional-order fuzzy multi-agent systems: Exploring impulsive security protocols. **IEEE Transactions on Control of Network Systems**, p. 1–12, 2025. Page 127.

[272] PEIXOTO, Márcia L. C.; PESSIM, Paulo S. P.; COUTINHO, Pedro H. S.; BESSA, Iury; PALHARES, Reinaldo M. Event-triggered control for lpv systems under hybrid cyberattacks. **Journal of Control, Automation and Electrical Systems**, v. 35, n. 2, p. 252–265, 2024. ISSN 2195-3899. Page 128.

[273] COUTINHO, Pedro Henrique S.; BESSA, Iury; PALHARES, Reinaldo M. Resilient dynamic event-triggered fuzzy control against dos attacks on cyber-physical systems. **Journal of Control, Automation and Electrical Systems**, v. 36, n. 3, p. 402–412, 2025. ISSN 2195-3899. Page 128.

[274] YANG, Fuyi; XIE, Xiangpeng; WANG, Yan-Wu; PALHARES, Reinaldo Martínez. Hppd-type fuzzy state-fdi estimation and resilient control of cyber-physical dc microgrids with hybrid attacks. **IEEE Transactions on Fuzzy Systems**, v. 32, n. 12, p. 6684–6694, 2024. Page 128.

[275] PEIXOTO, Márcia L. C.; COUTINHO, Pedro H. S.; BESSA, Iury; PESSIM, Paulo S. P.; PALHARES, Reinaldo M. Event-triggered control of takagi-sugeno fuzzy systems under deception attacks. **International Journal of Robust and Nonlinear Control**, 2023. Page 128.

[276] PESSIM, Paulo S.P.; PEIXOTO, Márcia L.C.; PALHARES, Reinaldo M.; LACERDA, Márcio J. Static output-feedback control for cyber-physical LPV systems under dos attacks. **Information Sciences**, v. 563, p. 241–255, 2021. ISSN 0020-0255. Page 128.

[277] PESSIM, Paulo S.P.; LACERDA, Márcio J. On the robustness of cyber-physical LPV systems under dos attacks. **Journal of the Franklin Institute**, v. 359, n. 2, p. 677–696, 2022. ISSN 0016-0032. Page 128.

[278] OLIVEIRA, Pedro M.; PALMA, Jonathan M.; LACERDA, Márcio J. Control design for cyber-physical uncertain systems under unreliable markovian network susceptible to denial-of-service attacks. **IEEE Transactions on Circuits and Systems II: Express Briefs**, v. 71, n. 2, p. 732–736, fev. 2024. ISSN 1558-3791. Page 128.

[279] PEIXOTO, Márcia L.C.; COUTINHO, Pedro H.S.; NGUYEN, Anh-Tu; GUERRA, Thierry-Marie; PALHARES, Reinaldo M. Fault estimation for nonlinear parameter-varying time-delayed systems. **Applied Mathematics and Computation**, v. 465, p. 128405, mar. 2024. ISSN 0096-3003. Page 129.

[280] BESSA, Iury; PEIXOTO, Márcia L. C.; COUTINHO, Pedro H. S.; PUIG, Vicenç; PALHARES, Reinaldo M. Fault hiding of nonlinear parameter varying systems. **IEEE Control Systems Letters**, v. 9, p. 889–894, 2025. ISSN 2475-1456. Page 129.

[281] PEIXOTO, Márcia L.; OLIVEIRA, Pedro M.; BESSA, Iury; COUTINHO, Pedro H. S.; PESSIM, Paulo S.P.; PUIG, Vicenç; PALHARES, Reinaldo M. Fault-tolerant dynamic output feedback control of LPV systems via fault hiding. **Automatica**, v. 174, p. 112191, 2025. ISSN 0005-1098. Page 129.

[282] OLIVEIRA, Maurício C. de; SKELTON, Robert E. Stability tests for constrained linear systems. In: ______. **Perspectives in robust control**. [S.l.]: Springer London, 2001. p. 241–257. Page 165.

[283] Gomes da Silva Jr., J. M.; TARBOURIECH, S. Antiwindup design with guaranteed regions of stability: an LMI-based approach. **IEEE Transactions on Automatic Control**, IEEE, v. 50, n. 1, p. 106–111, 2005. Page 165.

[284] WANG, Yan; RAJAMANI, Rajesh; BEVLY, David M. Observer design for parameter varying differentiable nonlinear systems, with application to slip angle estimation. **IEEE Transactions on Automatic Control**, v. 62, n. 4, p. 1940–1945, 2017. Page 166.

[285] SEURET, Alexandre; GOUAISBAUT, Frédéric. Stability of linear systems with time-varying delays using bessel–legendre inequalities. **IEEE Transactions on Automatic Control**, v. 63, n. 1, p. 225–232, 2018. Page 166.

[286] SEURET, Alexandre; GOUAISBAUT, Frédéric. Delay-dependent reciprocally convex combination lemma for the stability analysis of systems with a fast-varying delay. In: **Delays and Interconnections: Methodology, Algorithms and Applications**. [S.l.]: Springer International Publishing, 2019. p. 187–197. Page 166.

[287] ZHANG, Xian-Ming; HAN, Qing-Long; SEURET, Alexandre; GOUAISBAUT, Frédéric. An improved reciprocally convex inequality and an augmented Lyapunov–krasovskii functional for stability of linear systems with time-varying delay. **Automatica**, v. 84, p. 221–226, 2017. Page 166.

[288] PEIXOTO, Márcia L.C.; PESSIM, Paulo S.P.; PALHARES, Reinaldo M. A new approach for dynamic output feedback control design of time-delayed nonlinear systems. **European Journal of Control**, v. 77, p. 100993, 2024. ISSN 0947-3580. Page 166.

[289] PARK, PooGyeon; KO, Jeong Wan; JEONG, Changki. Reciprocally convex approach to stability of systems with time-varying delays. **Automatica**, v. 47, n. 1, p. 235–238, jan. 2011. Page 166.

[290] SEURET, A.; GOUAISBAUT, F. Wirtinger-based integral inequality: Application to time-delay systems. **Automatica**, Elsevier BV, v. 49, n. 9, p. 2860–2866, 2013. Page 167.

Appendix

APPENDIX A – GENERAL CONCEPTS OF GRAPH THEORY

In this thesis, the interconnections among subsystems and interactions among agents are represented with undirected graphs. In the graph theory, a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is composed by a vertex set $\mathcal{V} = \{1, 2, \dots, N\}$ that corresponds to the set of indexes of the subsystems, agents or controllers, and a set of edges $\mathcal{E} = \{e_{ij} = (i, j) \in \mathcal{V} \times \mathcal{V}\}$, where the first argument of $e_{ij} = (i, j)$ is the parent node and the second argument is the child node. The graph is called undirected once the interaction between two vertices is mutual, i.e., for all $(i, j) \in \mathcal{E}$, the correspondent (j, i) also belongs to set edges.

The edges of the graph can be represented with a *adjacency matrix* $\mathcal{A} = [a_{ij}]$, $i, j \in \mathcal{V}$. As usual, the elements of $\mathcal{A}$, are defined as

$$a_{ij} = \begin{cases} 0, & \text{if } i = j \text{ or } \; e_{ij} \notin \mathcal{E}, \\ 1, & \text{if } e_{ij} \in \mathcal{E}. \end{cases}$$

The degree matrix $\mathcal{D}$ associated with the graph $\mathcal{G}$ is the diagonal matrix $\mathcal{D} = \mathrm{diag}(d_1, \dots, d_N)$ that represents the number of edges attached to each vertex. Its elements are defined as

$$d_i = \sum_{j=1}^{N} a_{ij} \quad \forall i \in \mathcal{V}.$$

The neighborhood set of the subsystem $\mathcal{P}_i$ is defined as $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. Notice that the cardinality of $\mathcal{N}_i$ is d_i, and it can be rewritten as $\mathcal{N}_i = \{k_{i1}, k_{i2}, \dots, k_{id_i}\}$. For illustration purposes, consider the undirected graph depicted in Figure 1.6. In this case, the neighborhood set of each subsystem is:

- $\mathcal{N}_1 = \{3, 4\}$; $\mathcal{N}_2 = \{4, 5\}$; $\mathcal{N}_3 = \{1, 4, 6\}$; $\mathcal{N}_4 = \{1, 2, 3, 5\}$;
- $\mathcal{N}_5 = \{2, 4, 7\}$; $\mathcal{N}_6 = \{3, 7, 8\}$; $\mathcal{N}_7 = \{5, 6, 8\}$; $\mathcal{N}_8 = \{6, 7\}$.

Moreover, the corresponding adjacency and degree matrices are given by

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \quad \mathcal{D} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

From the Adjacency and Degree matrices one can compute the Laplacian matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Moreover, in a leader-following framework of multi-agent systems, the communication among the leader and the following agents is represented by $\eta = \mathrm{diag}(\eta_1, \ldots, \eta_N)$, where the pinning parameters η_i indicate whether the i-th follower has access to the leader dynamics ($\eta_i = 1$) or not ($\eta_i = 0$). Therefore, the overall communication can be represented by $\bar{\mathcal{L}} = \mathcal{L} + \eta$.

The matrices $\mathcal{L}$ and $\bar{\mathcal{L}}$ can be written in terms of its spectral decomposition, such that $\mathcal{L} = T\Lambda T^{-1}$, and $\bar{\mathcal{L}} = \bar{T}\bar{\Lambda}\bar{T}^{-1}$, where the orthogonal matrices $T, \bar{T} \in \mathbb{R}^{N\times N}$ constitutes the eigenvectors of $\mathcal{L}$ and $\bar{\mathcal{L}}$, and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N) \in \mathbb{R}^{N\times N}$ and $\bar{\Lambda} = \mathrm{diag}(\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_N) \in \mathbb{R}^{N\times N}$ are diagonal matrices with the eigenvalues of $\mathcal{L}$ and $\bar{\mathcal{L}}$ ordered as $\lambda_1 < \lambda_2 \leq \ldots \leq \lambda_N$, and $\bar{\lambda}_1 < \bar{\lambda}_2 \leq \ldots \leq \bar{\lambda}_N$.

APPENDIX B – TS AND N-TS FUZZY MODELING

Standard TS Fuzzy and N-TS Fuzzy modeling

The TS fuzzy is known as an appropriate way to represent input-affine nonlinear systems

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t),$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector, and $f(\cdot) \in \mathbb{R}^{n_x}$ and $g(\cdot) \in \mathbb{R}^{n_x \times n_u}$ are smooth nonlinear function defined over a compact region $\mathcal{D} \subset \mathbb{R}^n$ containing the origin, as the following set of fuzzy rules

$$\begin{aligned} \textbf{Model rule } i: \quad & \text{If } z_1(t) \in \mathcal{M}_1^i \text{ and } \ldots \text{ and } z_p(t) \in \mathcal{M}_p^i \\ & \text{Then } \dot{x}(t) = A_i x(t) + B_i u(t), \quad i \in \mathcal{I}_r, \end{aligned}$$

where $z_i(t)$ is the i-th element of the vector of premise variables $z(t) \in \mathbb{R}^p$, $A_i \in \mathbb{R}^{n\times n}$, and $B_i \in \mathbb{R}^{n\times m}$ are the matrices of the local dynamics, $\mathcal{M}_j^i$ is the fuzzy set related to the j-th premise variable $z_j(t)$, $\mathcal{I}_r = \{1, 2, \ldots, r\}$, and r is the number of fuzzy rules. The advantage of this approach is that after the center-of-gravity method for defuzzification procedure, the nonlinear dynamics can be expressed as a convex sum of local linear models weighted by the normalized membership grades. The inferred TS fuzzy model is described by

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z_i)\left(A_i x(t) + B_i u(t)\right), \tag{B.1}$$

where the normalized membership functions are given by,

$$\alpha_i(z_i) = \frac{\prod_{j=1}^{p} M_j^i(z_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_j^i(z_j(t))},$$

being $M_j^i(z_j(t)) \in [0, 1]$ the membership degree of $z_j(t)$ with respect to the fuzzy set $\mathcal{M}_j^i$. Moreover, the normalized membership functions $\alpha_i(z_i)$ satisfy the convex sum property

$$\alpha_i(z_i) \geq 0, \quad \sum_{i=1}^{r} \alpha_i(z_i) = 0.$$

Examples of the modeling process of (B.1) are presented in [24]. Usually, when considering the sector nonlinearity approach for the TS fuzzy modeling, the nonlinearities of the system are chosen as premise variables. If the nonlinear system is complex (as in the case of interconnected systems), the number of fuzzy rules required to construct the TS fuzzy model might lead to the curse of dimensionality. An alternative to overcome this drawback is to consider the N-TS fuzzy approach. In this case, it is assumed that the function $f(x(t))$ of the input-affine nonlinear system can be rewritten as

$$f(x(t)) = f_a(x(t)) + f_b(x(t))\phi(x(t)),$$

being $\phi(x(t))$ a sector-bounded nonlinear function. Finally, the inferred N-TS fuzzy model is described as

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z_i) \left(A_i x(t) + B_i u(t) + G_i \phi(x(t))\right), \tag{B.2}$$

where $G_i \in \mathbb{R}^{n \times n_\phi}$ is a matrix that is also considered to describe the local dynamics.

In this thesis, the subsystems of the interconnected system are represented in a polytopic embedding constructed with the same reasoning of the N-TS fuzzy modeling presented to obtain (B.2). Due to the interconnections, and the presence of time-varying delays in the system dynamics, the obtained model differs from (B.2), and each case has its own particularities. Therefore, it is presented in the sequel, the specific N-TS fuzzy model employed in this work for the cases of non-delayed and time-delay interconnected nonlinear systems.

Interconnected N-TS Fuzzy systems

Consider a continuous-time interconnected nonlinear system, composed by N interacting subsystems described as:

$$\mathcal{P}_i: \ \dot{x}_i(t) = A_i(x_i)x_i(t) + B_i(x_i)u_i(t) + \sum_{j \in \mathcal{N}_i} g_{ij}(x_i(t), x_j(t)), \tag{B.3}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state of the i-th subsystem, $u_i \in \mathbb{R}^{m_i}$ is the i-th control input, and

$$g_{ij}(x_i, x_j) = A_{ij}x_j + G_{ij}(x_i)\phi_{ij}(x_i, x_j), \tag{B.4}$$

is the function that models the interconnection between $\mathcal{P}_i$ and the subsystem $\mathcal{P}_j$ in the neighborhood $\mathcal{N}_i$ of $\mathcal{P}_i$. The functions ϕ_{ij} are sector-bounded functions representing the nonlinear interconnections, and A_{ij} represent the linear interconnections.

The following convex polytope defines the validity domain of the subsystems

$$\mathscr{D}_i = \{x_i \in \mathbb{R}^{n_i} : h_{i\ell_i}^\top x_i \leq 1, \ \ell_i \in \mathbb{N}_{\leq n_{fi}}\}, \tag{B.5}$$

where $h_{i\ell_i} \in \mathbb{R}^{n_i}$, $\ell_i \in \mathbb{N}_{\leq n_{fi}}$, and n_{fi} is the number of half-spaces defining the polytope.

The premise variables are the non-constant terms in (B.3). Let $z_i = (z_{i1}, z_{i2}, \ldots, z_{ip_i}) \in \mathbb{R}^{p_i}$ be the vector of state-dependent premise variables $z_{ik} : \mathbb{R}^{n_i} \to \mathbb{R}$, $k \in \mathbb{N}_{\leq p_i}$. Regarding the sector nonlinearity approach, for $x_i \in \mathscr{D}_i$, each premise variable z_{ik} can be written as

$$z_{ik} = w_0^{ik}(z_{ik})z_{ik}^0 + w_1^{ik}(z_{ik})z_{ik}^1,$$

where $z_{ik}^0 = \inf_{x_i \in \mathscr{D}_i} z_{ik}$, $z_{ik}^1 = \sup_{x_i \in \mathscr{D}_i} z_{ik}$, the weighting functions are

$$w_0^{ik}(z_{ik}) = \frac{z_{ik}^1 - z_{ik}}{z_{ik}^1 - z_{ik}^0}, \quad w_1^{ik}(z_{ik}) = 1 - w_0^{ik}(z_{ik}), \tag{B.6}$$

and the state-dependent membership functions that satisfy the convex sum property are defined as:

$$\alpha_{\mathbf{i}_i}(z_i) = \prod_{k=1}^{p_i} w_{i_k}^{ik}(z_{ik}), \quad \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) = 1, \ \text{and} \ \alpha_{\mathbf{i}_i}(z_i) \geq 0, \quad \forall \mathbf{i}_i \in \mathbb{B}^{p_i}. \tag{B.7}$$

By defining the collection of nonlinearities $\phi_{ik_{i\ell}}$ for $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\leq d_i}$, as

$$\phi_i(x) = (\phi_{ik_{i1}}(x_i, x_{k_1}), \ldots, \phi_{ik_{id_i}}(x_i, x_{k_{d_i}})) \in \mathbb{R}^{d_i}, \tag{B.8}$$

and

$$G_i(x_i) = \begin{bmatrix} G_{ik_1}(x_i) & \cdots & G_{ik_{d_i}}(x_i) \end{bmatrix}, \tag{B.9}$$

with $x = (x_1, \ldots, x_N) \in \mathbb{R}^n$, the i-th subsystem $\mathcal{P}_i$ in (B.3) can be equivalently written as the following N-TS fuzzy model:

$$\dot{x}_i(t) = \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) \left(A^i_{\mathbf{i}_i} x_i(t) + B^i_{\mathbf{i}_i} u_i(t) + G^i_{\mathbf{i}_i} \phi_i(x(t)) \right) + \sum_{j \in \mathcal{N}_i} A_{ij} x_j(t), \tag{B.10}$$

once the matrix-valued functions $A_i(x_i)$, $B_i(x_i)$, and $G_i(x_i)$ of the i-th subsystem $\mathcal{P}_i$ in (B.3) are given by

$$A_i(x_i) = \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) A^i_{\mathbf{i}_i}, \quad B_i(x_i) = \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) A^i_{\mathbf{i}_i}, \quad G_i(x_i) = \sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) G^i_{\mathbf{i}_i}, \tag{B.11}$$

for all $x_i \in \mathscr{D}_i$, where $\mathbf{i}_i$ is a p_i-dimensional multi-index. A modeling example is presented in the sequel.

Example: Network of interconnected pendulums

Consider the network of inverted interconnected pendulums proposed in [18] where the linear springs are replaced by hardening springs. The nonlinear model of each pendulum is given by:

$$\begin{aligned}
\dot{x}_{i1}(t) &= x_{i2}(t), \\
\dot{x}_{i2}(t) &= \frac{g}{l_i} \sin(x_{i1}(t)) + \frac{1}{m_i l_i^2} u_i(t) - \frac{ka^2}{m_i l_i^2} \sum_{j \in \mathcal{N}_i} (x_{i1}(t) - x_{j1}(t)) \\
&\quad - \frac{ka^2 \gamma^2}{m_i l_i^2} \sum_{j \in \mathcal{N}_i} (x_{i1}(t) - x_{ij}(t))^3,
\end{aligned}$$

where $x_{i1}(t)$ is the rod angle with respect to the upright position, $x_{i2}(t)$ is the angular velocity, $u_i(t)$ is the torque applied to the base of the i-th pendulum, g is the gravitational acceleration, m_i is the mass, and l_i the rod length of the i-th pendulum. The linear and nonlinear elastic coefficients are k and γ, respectively, and a is the height of the their connection at the pendulum's rods. Defining $z_{i1}(t) = \sin(x_{i1}(t))$, and imposing the state constraints $\mathscr{D}_i = \{x_i \in \mathbb{R} : |x_i| \leq \bar{\theta}\}$, one can compute that:

$$z^0_{i1} = \inf_{x_1 \in \mathscr{D}_1} z_{11} = \frac{sin(\bar{\theta})}{\bar{\theta}}, \quad z^1_{i1} = \sup_{x_1 \in \mathscr{D}_1} z_{11} = 1,$$

and, consequently $z_{i1}(t) = w^{i1}_0(z_{i1}(t)) z^0_{i1} + w^{i1}_1(z_{i1}(t)) z^1_{i1}$ with

$$\begin{aligned}
&w^{i1}_0(z_{i1}(t)) = \frac{z^1_{i1} - z_{i1}(t)}{z^1_{i1} - z^0_{i1}}, \quad w^{i1}_1(z_{i1}(t)) = 1 - w^{i1}_0(z_{i1}(t)), \\
&\alpha_0(z_i) = w^{i1}_0(z_1(t)), \quad \alpha_1(z_i) = w^{i1}_1(z_1(t)),
\end{aligned}$$

since $p_1 = 1$.

By gathering the nonlinear interconnections $\phi_{ij}(x_i, x_j) = (x_{i1}(t) - x_{ij}(t))^3$ for all $j \in \mathcal{N}_i$, it is possible to construct $\phi_i(x_i)$ as in (B.8) with $x_i(t) = [x_{i1}(t) \;\; x_{i2}(t)]^\top$. Furthermore, the local state-space matrices are:

$$
\begin{aligned}
A_i(x_i) &= \alpha_0(z_i) \begin{bmatrix} 0 & 1 \\ \frac{g}{l_i}\frac{\sin(\bar{\theta})}{\bar{\theta}} - \frac{d_i k a^2}{m_i l_i^2} & 0 \end{bmatrix} + \alpha_1(z_i) \begin{bmatrix} 0 & 1 \\ \frac{g}{l_i} - \frac{d_i k a^2}{m_i l_i^2} & 0 \end{bmatrix}, \\
B_i(x_i) &= \alpha_0(z_i) \begin{bmatrix} 0 \\ \frac{1}{m_i l_i^2} \end{bmatrix} + \alpha_1(z_i) \begin{bmatrix} 0 \\ \frac{1}{m_i l_i^2} \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{m_i l_i^2} & 0 \end{bmatrix}, \\
G_i(x_i) &= \alpha_0(z_i) \begin{bmatrix} 0 & \dots & 0 \\ -\frac{ka^2\gamma^2}{m_i l_i^2} & \dots & -\frac{ka^2\gamma^2}{m_i l_i^2} \end{bmatrix} + \alpha_1(z_i) \begin{bmatrix} 0 & \dots & 0 \\ -\frac{ka^2\gamma^2}{m_i l_i^2} & \dots & -\frac{ka^2\gamma^2}{m_i l_i^2} \end{bmatrix}.
\end{aligned}
$$

Note that the dimension of $G_i(x_i)$ as defined in (B.9) depends on the total number of interconnections of each subsystem.

Time-Delay Interconnected Fuzzy-Systems

Consider the i-th nonlinear time-delay subsystem, as :

$$\mathcal{P}_i\colon \dot{x}_i(t) = A_i(x_i)x_i(t) + A_{di}(x_i)x_i(t - h_i(t)) + B_i(x_i)u_i(t) + \sum_{j\in\mathcal{N}_i} g_{ij}(x_i(t), x_j(t - \tau_{ij}(t))), \quad \text{(B.12)}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}^{m_i}$ is the input, $g_{ij}(x_i, x_j)$ as in (B.4) is the function that models the interconnection between the subsystems $\mathcal{P}_j$ and $\mathcal{P}_i$, $\phi_{ij} : \mathbb{R}^{n_i} \times \mathbb{R}^{n_j} \to \mathbb{R}^{n_{\phi,i}}$, $h_i(t)$ are the internal time-varying delays of the subsystem's dynamics, and $\tau_{ij}(t)$ is the time-varying delay induced from the connection between $\mathcal{P}_i$ and $\mathcal{P}_j$.

The matrix-value functions $A_i(x_i)$, $B_i(x_i)$, $G_i(x_i)$, and $A_{di}(x_i)$, are defined in the validity domain represented by the convex polytope (B.5), and with a similar procedure of (B.6)-(B.7) it is possible to represent (B.12) as

$$
\begin{aligned}
\dot{x}_i(t) = &\sum_{\mathbf{i}_i \in \mathbb{B}^{p_i}} \alpha_{\mathbf{i}_i}(z_i) \Big(A^i_{\mathbf{i}_i} x_i(t) + A^i_{\mathrm{d}\mathbf{i}_i} x_i(t - h(t)) + B^i_{\mathbf{i}_i} u_i(t) + G^i_{\mathbf{i}_i} \phi_i(\chi_i(t - \tau(t))\Big) && \text{(B.13)} \\
&+ \sum_{j\in\mathcal{N}_i} A_{ij} x_j(t - \tau(t)), && \text{(B.14)}
\end{aligned}
$$

where the collection of time-delay nonlinearities $\phi_{ik_{i\ell}}$ for $k_{i\ell} \in \mathcal{N}_i$, $\ell \in \mathbb{N}_{\le d_i}$ is given by

$$\phi_i\left(\chi_i(t - \tau(t)\right) = \Big(\phi_{ik_{i1}}(x_i(t), x_{k_1}(t - \tau(t))), \dots, \phi_{ik_{id_i}}(x_i(t), x_{k_{d_i}}(t - \tau(t)))\Big) \in \mathbb{R}^{d_i},$$

for all $x_i \in \mathscr{D}_i$. The time-delay in the interconnections affects the states of the neighboring subsystems. Therefore, the overall gathering of nonlinear time-delayed interconnections is written in terms of

$$\chi_i(t - \tau(t)) = (\underline{x}_i(t - \tau(t)), x_i(t), \bar{x}_i(t - \tau(t))) \in \mathbb{R}^n,$$

$\underline{x}_i = (x_1, \ldots, x_{i-1}) \in \mathbb{R}^{\underline{n}_i}$, $\bar{x}_i = (x_{i+1}, \ldots, x_N) \in \mathbb{R}^{\bar{n}_i}$, $\underline{n}_i \triangleq n - \sum_{k=i}^{N} n_k$, and $\bar{n}_i \triangleq n - \sum_{k=1}^{i} n_k$,. Notice that $\chi_i(t) = (\underline{x}_i(t), x_i(t), \bar{x}_i(t)) = x(t)$. A modeling example is presented in the sequel.

Example: Time-delay interconnected system

Consider the following interconnected system with two subsystems:

$$\begin{aligned}\dot{x}_1(t) =& x_1(t) + 2x_1^3(t)\cos(2x_1(t)) + 3x_1(t-h(t)) - 3x_2(t-\tau(t)) \\ &+ 0.5\arctan^5(x_1(t) - x_2(t-\tau(t))) + u_1(t), \\ \dot{x}_2(t) =& x_2(t) + 2x_2^3(t) - 3x_1(t-\tau(t)) - 3x_2(t-h(t)) \\ &+ 0.5\arctan^5(x_2(t) - x_1(t-\tau(t))) + x_2^2(t)u_2(t).\end{aligned} \tag{B.15}$$

Defining $z_1(t) = \cos(2x_1(t))x_1^2(t)$, and $z_2(t) = x_2^2(t)$ as premise variables, and

$$\phi_{12} = \arctan^5(x_1(t) - x_2(t-\tau(t)),$$

$$\phi_{21} = \arctan^5(x_2(t) - x_1(t-\tau(t))),$$

as the sector-bounded nonlinear interconnections it is possible to rewrite the system as:

$$\begin{aligned}\dot{x}_1(t) &= (1+2z_1(t))x_1(t) + 3x_1(t-h(t)) - 3x_2(t-\tau(t)) \\ &\quad + 0.5\phi_{12}(x_1(t), x_2(t-\tau(t))) + u_1(t), \\ \dot{x}_2(t) &= (1+2z_2(t))x_2(t) - 3x_1(t-\tau(t)) - 3x_2(t-h(t)) \\ &\quad + 0.5\phi_{21}(x_1(t-\tau(t)), x_2(t)) + z_2(t)u_2(t).\end{aligned}$$

The region of validity of each subsystem is defined as $\mathscr{D}_i = \{x_i \in \mathbb{R} : |x_i| \leq 2\pi\}$. Therefore, one can compute that:

$$\begin{aligned}z_{11}^0 &= \inf_{x_1 \in \mathscr{D}_1} z_{11} = -22.6977, \quad z_{11}^1 = \sup_{x_1 \in \mathscr{D}_1} z_{11} = 39.4784, \\ z_{21}^0 &= \inf_{x_2 \in \mathscr{D}_2} z_{21} = 0, \quad z_{21}^1 = \sup_{x_2 \in \mathscr{D}_2} z_{21} = 39.4784,\end{aligned}$$

and, consequently

$$z_1(t) = w_0^{11}(z_1(t))z_{11}^0 + w_1^{11}(z_1(t))z_{11}^1, \quad z_2(t) = w_0^{21}(z_2(t))z_{21}^0 + w_1^{21}(z_2(t))z_{21}^1,$$

with

$$\begin{aligned}\alpha_0(z_1) &= w_0^{11}(z_1(t)) = \frac{z_{11}^1 - z_1(t)}{z_{11}^1 - z_{11}^0}, \quad \alpha_1(z_1) = w_1^{11}(z_1(t)) = 1 - w_0^{11}(z_1(t)), \\ \alpha_0(z_2) &= w_0^{21}(z_2(t)) = \frac{z_{21}^1 - z_2(t)}{z_{21}^1 - z_{21}^0}, \quad \alpha_1(z_2) = w_1^{21}(z_2(t)) = 1 - w_0^{21}(z_2(t)),\end{aligned}$$

and the matrices are given by:

$$
\begin{aligned}
&A_1(x_1) = \alpha_0(z_1) \cdot (-44.3959) + \alpha_1(z_1) \cdot 79.9588, \\
&A_2(x_2) = \alpha_0(z_2) \cdot 1 + \alpha_1(z_2) \cdot 79.9588, \\
&A_{d1}(x_1) = \alpha_0(z_1) \cdot 3 + \alpha_1(z_1) \cdot 3 = 3, \\
&A_{d2}(x_2) = \alpha_0(z_2) \cdot (-3) + \alpha_1(z_2) \cdot (-3) = -3, \\
&B_1(x_1) = \alpha_0(z_1) \cdot 1 + \alpha_1(z_1) \cdot 1 = 1, \\
&B_2(x_2) = \alpha_0(z_2) \cdot 0 + \alpha_1(z_2) \cdot 39.4784, \\
&G_1(x_1) = \alpha_0(z_1) \cdot 0.5 + \alpha_1(z_1) \cdot 0.5 = 0.5, \\
&G_2(x_2) = \alpha_0(z_2) \cdot 0.5 + \alpha_1(z_2) \cdot 0.5 = 0.5, \\
&A_{12} = A_{21} = -3.
\end{aligned}
$$

Notice that in the standard T-S fuzzy modeling, one should define the nonlinear interconnections $\arctan^5(x_1(t) - x_2(t-\tau(t))$ and $\arctan^5(x_2(t) - x_1(t-\tau(t)))$ as premise variables, which increases the number of fuzzy rules and the complexity of the T-S fuzzy model. As highlighted in [234], the N-TS fuzzy models can reduce not only the numerical complexity for control design and implementation but also the conservativeness of the results. More information and examples regarding the benefits of N-TS fuzzy modeling are presented in the works [18–20].

APPENDIX C – USEFUL LEMMAS

This appendix presents technical lemmas that are considered for obtaining the results of this thesis.

Lemma C.1: Finsler's Lemma [282]

Let $x \in \mathbb{R}^n$, $\mathcal{Q} \in \mathbb{S}^n$ and $\mathcal{B} \in \mathbb{R}^{m \times n}$ such that $\operatorname{rank}(\mathcal{B}) < n$, and $\mathcal{B}^{\perp}$ denote the basis for the null-space of $\mathcal{B}$, that is, $\mathcal{B}\mathcal{B}^{\perp} = 0$. The following statements are equivalent:

i) $x^{\top}\mathcal{Q}x < 0, \quad \forall x$ such that $\mathcal{B}x = 0, \quad x \neq 0$.

ii) $\mathcal{B}^{\perp\top}\mathcal{Q}\mathcal{B}^{\perp} \prec 0$.

iii) $\exists\, \mu \in \mathbb{R} : \mathcal{Q} - \mu\mathcal{B}^{\top}\mathcal{B} \prec 0$.

iv) $\exists\, \mathcal{X} \in \mathbb{R}^{n \times m} : \mathcal{Q} + \mathcal{X}\mathcal{B} + \mathcal{B}^{\top}\mathcal{X}^{\top} \prec 0$.

The next Lemma C.2 presents the generalized sector condition that is considered to design the proposed distributed control laws for interconnected systems.

Lemma C.2: Sector Condition [283]

If the nonlinearities $\phi_i : \mathbb{R}^n \to \mathbb{R}^{d_i}$, $\forall i \in \mathcal{V}$, belongs to the sector $[0, \Omega_i]$ on a compact domain $\mathscr{D} \subset \mathbb{R}^n$, then the following condition

$$\mathcal{S}_i(x, \Lambda_i) \triangleq \phi_i^{\top}(x)\Lambda_i^{-1}(\phi_i(x) - \Omega_i x) < 0, \quad \forall x \in \mathscr{D}, \tag{C.1}$$

holds for any diagonal matrix $\Lambda_i > 0 \in \mathbb{R}^{d_i \times d_i}$, and $\Omega_i = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \cdots & \Omega_{iN} \end{bmatrix} \in \mathbb{R}^{d_i \times n}$, with $\Omega_{ij} \in \mathbb{R}^{d_i \times n_j}$ and $\Omega_{ij} = 0\ \forall j \notin \mathcal{N}_i$.

Moreover, a second sector condition is also considered for the cases evolving the partial derivatives of the nonlinearities

Lemma C.3: Sector Condition for partial derivatives [284]

If the partial derivatives of the nonlinearities $\phi_i(x) : \mathbb{R}^n \to \mathbb{R}^{d_i}$, $\forall i \in \mathbb{N}_{\leq N}$, satisfy

$$0 \leq \frac{\partial \phi_i(x)}{\partial x} \leq J_i,$$

then the following conditions

$$\mathcal{H}_i(e, x, \Lambda_{h_i}) \triangleq \rho_i^\top(e, x)\Lambda_{h_i}^{-1}(\rho_i(e, x) - J_i e) < 0, \tag{C.2}$$

holds for any diagonal matrices $\Lambda_{h_i} > 0 \in \mathbb{R}^{d_i \times d_i}$, and

$$J_i = \begin{bmatrix} J_{i1} & J_{i2} & \cdots & J_{iN} \end{bmatrix} \in \mathbb{R}^{d_i \times n},$$

with $e = \hat{x} - x$, $\rho_i(e, x) = \phi_i(\hat{x}) - \phi_i(x)$, $J_{ij} \in \mathbb{R}^{d_i \times n_j}$ and $J_{ij} = 0\ \forall j \notin \mathcal{N}_i$, $\forall x, \hat{x} \in \mathscr{D}$.

The delay-dependent reciprocally convex inequality, previously considered in works such as [285–288], is considered to construct delay-dependent conditions that generally lead to less conservative results. The inequality is presented in the following Lemma.

Lemma C.4: Delay-dependent reciprocally convex inequality [285]

Let $n \in \mathbb{N}$, and R_1, $R_2 \in \mathbb{R}^{n \times n}$ be symmetric positive definite matrices. If there exist symmetric matrices X_1, $X_2 \in \mathbb{R}^{n \times n}$ and matrices Y_1, $Y_2 \in \mathbb{R}^{n \times n}$ such that

$$\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} - \varrho \begin{bmatrix} X_1 & Y_1 \\ Y_1^\top & 0 \end{bmatrix} - (1 - \varrho) \begin{bmatrix} 0 & Y_2 \\ Y_2^\top & X_2 \end{bmatrix} \geq 0,$$

holds for all $\varrho \in \mathbb{B}$. Then, the following inequality

$$\begin{bmatrix} \frac{R_1}{\varrho} & 0 \\ 0 & \frac{R_2}{1-\varrho} \end{bmatrix} \geq \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} + (1 - \varrho) \begin{bmatrix} X_1 & Y_2 \\ Y_2^\top & 0 \end{bmatrix} + \varrho \begin{bmatrix} 0 & Y_1 \\ Y_1^\top & X_2 \end{bmatrix}$$

holds for all $\varrho \in (0, 1) \subset \mathbb{R}$.

If the parameter ϱ is defined in terms of the time-varying delays of the system, the aforementioned Lemma C.4 makes possible the development of delay-dependent conditions. Moreover, by imposing $X_1 = R_1 - Y_1 R_2^{-1} Y_1^\top$, and $X_2 = R_2 - Y_2^\top R_1^{-1} Y_2$, the number of decision variables is reduced. With the particular choices $X_1 = X_2 = 0$, and $Y_1 = Y_2$, the results of Lemma C.4 are reduced to the standard version presented in [289].

Furthermore, the Wirtinger-based integral inequality as presented in the sequel is also considered in the development of the proposed design conditions.

Lemma C.5: Wirtinger-based integral inequality [290]

Let $n \in \mathbb{N}$, for any symmetric positive definite matrix S_+^n, the following inequality holds for all continuously differentiable function $\omega \in [a, b] \to \mathbb{R}^n$:

$$(b-a)\int_a^b \dot{\omega}^\top(s)\, R\, \dot{\omega}(s)ds \geq \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}^\top \begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix} \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix},$$

where

$$\theta_1 = \omega(b) - \omega(a), \ \text{ and } \ \theta_2 = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(s)ds.$$