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Education Projections using Age-Period-Cohort Models: Classical and Bayesian Approaches

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History and societies do not crawl. They make jumps.
They go from fracture to fracture, with a few vibrations in between.
Yet we (and historians) like to believe in the predictable,
small incremental progression.

Nassim Nicholas Taleb, The Black Swan

Abstract

The APC framework for modeling and forecasting the education profile of Brazilian males and females is considered from both classical and Bayesian perspectives. For a classical analysis, I calculate maximum likelihood estimates of APC parameters. For the Bayesian analysis, I estimate posterior means and credible intervals. Both methods are simple and computationally efficient. Results show that both classical and Bayesian methods are able to provide very good forecasts in the short term. However, the Bayesian method performed best for in-sample and out-of-sample forecasts. On the other hand, in a Bayesian setting, uncertainty indeed becomes an issue for long-term forecasts because of the rapidly increasing width of the intervals as the length of the projection increases. A number of enhancements of the classical and Bayesian methods proposed here are suggested for a future research agenda. Foremost is an investigation into an integrated approach to account for uncertainty in the classical multinomial APC model and refined ways of eliciting prior information in the Bayesian framework.

Keywords: Age-period-cohort models; Forecasting; Classical Statistics; Bayesian Statistics.

Resumo

O arcabouço idade-período-coorte (IPC) para modelar e prever o perfil educacional de homens e mulheres brasileiras é considerado nas perspectivas clássica e Bayesiana. Na análise segundo a estatística clássica, calcularam-se estimativas de máxima verossimilhança dos parâmetros do IPC. Na análise Bayesiana, estimaram-se médias *a posteriori* e intervalos de credibilidade. Ambos os métodos são simples e computacionalmente eficientes. Os resultados mostram que tanto os métodos clássicos quanto Bayesianos são capazes de fornecer predições excelentes no curto prazo. Contudo, o modelo Bayesiano teve uma melhor performance para previsões dentro e fora da amostra. Por outro lado, na perspectiva Bayesiana, a incerteza se torna uma questão importante para previsões de longo prazo, devido à largura do intervalo, que cresce consideravelmente quando o horizonte de projeção aumenta. Aperfeiçoamentos nos métodos clássico e Bayesiano propostos aqui são sugeridos para uma agenda futura de pesquisa. Dentre eles, destaca-se a investigação de uma abordagem integrada para lidar com a incerteza no modelo IPC multinomial clássico e formas aprimoradas de elicitar a informação a priori no arcabouço Bayesiano.

Palavras-chave: Modelo Idade-período-coorte; Previsões; Estatística Clássica; Estatística Bayesiana.

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Introduction

Demographers have historically attempted to project a range of indicators into the future: population, mortality, fertility, migration. These future predictions help tremendously in public policy efforts, and hence many consumers have historically demanded demographic forecasts (Alho and Spencer, 2005). Despite their value, it was only recently that some demographers began forecasting a range of demographic outcomes along with associated probabilities (Tuljapurkar, 1997). Lee and colleagues pioneered this effort aiming to project mortality trends with their respective uncertainty by extrapolating time series parameters (Lee and Carter, 1992). Also, demographers of the IIASA[1] team derived future population estimated based on expert judgment and established scenarios with alternative conditions of fertility and mortality in 2050, with their attached probabilities (Lutz et al., 1998).

The traditional approach in the studies above to demographic forecasting has been to use classical statistical inference, with an emphasis on maximum likelihood and method-of-moments estimators. Recently, there has been increased interest in Bayesian methods as a result of the development of analytical methods and the advancement of computational techniques (Girosi and King, 2008; Alkema, 2008; Bijak, 2011) that allows to handle complex models in an easier and more efficient way. Girosi and King (2008) employed the Bayesian paradigm to predict mortality rates using information pooling from similar cross-sections (i.e., age groups, countries). Bijak (2011) applied the Bayesian paradigm to model and project the path of international migration in Europe. Finally, the United Nations Population Division started in 2010 to develop Bayesian probabilistic projections for the total fertility rate (TFR) and life expectancy in their 2010 Revision of the World Population Prospects (Chunn et al., 2010; Raftery et al., 2012).

It is not the intent of this dissertation to bridge the philosophical and theoretical debate between Bayesian and classical statistics (Gelman, 2008), as divisions between these frameworks have resisted decades of efforts to bring them together (Efron, 2005).

Putting it simple, a bayesian statistician assumes the existence of a probability distribution depicting the prior knowledge or belief of a researcher with respect to the possible values of the parameters, unconditional on the empirical evidence avail-

[1] International Institute for Applied Systems Analysis

able from data. Such prior information is updated by the data using the Bayes Theorem generating the posterior knowledge about the parameter. Hence, the essence of Bayesian inference is to transform prior beliefs and uncertainty about the parameters to the posterior knowledge, by incorporating empirical evidence (Paulino et al., 2003). In contrast, a classical statistician only consider the available data information for doing inference and predictions.

Thus, analyses of demographic forecasting exercises require distinct designs whether conducted in a classical or Bayesian framework. Demographers who propose the use of classical or Bayesian methods in forecasting vary in their motivations. In this study, however, I employ both classical and Bayesian methods for demographic forecasting.

Demographic Forecasting using APC Models

In order to derive accurate forecasts of a demographic indicator of interest, age-period-cohort (henceforth APC) models are appealing. APC models were first introduced by demographers and sociologists beginning in the 1970s and provide a simple and elegant accounting framework of age, period, and cohort effects on observed changes in a demographic condition (Mason et al., 1973; Rodgers, 1982; Fienberg and Mason, 1985; Mason and Smith, 1985; Berzuini et al., 1993; Yang and Land, 2013a).

Forecasting the Brazilian Education Profile

In this study the demographic indicator of interest is the education profile of Brazilian males and females aged 20-59 years. In this study the demographic indicator of interest is the education profile of Brazilian males and females aged 20-59 years. The education profile is defined here as the proportion of individuals in each educational group at a given time. In other words, the education profile is the distribution of the population at a certain year according to education groups.

To illustrate the substantive appropriateness of APC models to this study, consider this example. Assume that, in Brazil, the proportion of the population aged 30-34 years in 2008 that had some tertiary education (12 years of schooling or more) was particularly high. This fact might be explained by the simultaneous operation of three factors: first, that this age group was particularly predisposed to achieve tertiary education (*age effect*); second, that the achievement of some higher education was higher in 2008 than in the other periods due to an economic downturn (*period effect*); and third, that the cohort born between 1984 and 1988 was particularly willing to achieve some higher education because people born or socialized at that time were more likely to value tertiary education (*cohort effect*). Hence, these three factors may be independently operating to explain changes in the education profile. The goal of the APC approach is, then, to separate age, period, and cohort effects, providing practical advantages for the current and prospective study of the education profile.

The relevance of the study of future trends in the Brazilian education profile is clear: it is of foremost substantive interest for a large array of social, economic and health-related issues in the country. Accordingly, changes toward higher skill levels of the Brazilian labor force may directly impact its productivity levels, the health status of its population and, by consequence, its economic growth (Barro and Lee, 1993, 2001; Mankiw et al., 1992; Krueger and Lindahl, 2000; Lutz et al., 2008; Cuaresma et al., 2013).

The Identification Problem in APC Models

Although age-groups, periods, and cohorts may all be related to a demographic indicator independently, it is not possible to uniquely estimate these independent effects: the APC model is not identified. This is the so-called identification problem in the APC framework (Mason et al., 1973; Fienberg and Mason, 1985; Mason and Smith, 1985). The identification problem in the APC parameter estimation has been studied since the 1970s, but plausible alternatives to address this problem are still being debated (Bray, 2002; O'Brien, 2011b; Yang and Land, 2013a). Developments designed to overcome the identification problem include: i. the use of prior information to impose parameter constraints (Mason et al., 1973; Fienberg and Mason, 1985; Smith, 2004); ii. the use of non-informative constraints (Yang et al., 2004, 2008; Yang and Land, 2013a); iii. the use of causal models to approximate for age, period and cohort effects (Winship and Harding, 2008); iv. the use of estimable (invariant) functions of the APC parameters (Rodgers, 1982; Holford, 1983; Kupper et al., 1985; Clayton and Schifflers, 1987; O'Brien, 2012); and v. the application of Bayesian smoothing models (Berzuini and Clayton, 1994; Bray, 2002; Held and Rainer, 2001).

Even though there have been clear advances in estimation methods, it is hard to determine which identification strategy should be selected because results depend largely on the specification imposed (Fienberg and Mason, 1985; O'Brien, 2011b). Furthermore, each one of the constrained models, some of which use prior information and some of which do not, fits the data equally well. Therefore, the validity of the chosen constraint cannot be judged from the model fit (Yang et al., 2004; O'Brien, 2011b). As a result, the debate on the identification problem in the APC framework is still very much alive in Demography, as can be demonstrated by a series of articles, reviews and replies accepted for publication in the journal *Demography* in 2013, in which the existence (or not) of a solution to the problem of identification is debated (Luo, 2013a;

Yang and Land, 2013b; Held and Riebler, 2013; Fienberg, 2013; Luo, 2013b).

However, the identification of the true APC parameters that generated the data process is not a critical issue in this study. Here I am particularly interested in the APC parameters for forecasting purposes. Hence, I consider several conditions that ensure that the identification problem has no bearing on the forecasts I produce.

APC Model-Based Demographic Forecasting

Another important advantage of the APC accounting framework is its applicability to forecasting. For a given set of estimated age, period, and cohort effects, forecasts of a demographic indicator of interest Y are easily implemented through a recombination of a set of extrapolated values of the period and cohort coefficients.

The major challenge that arises when forecasting from APC models is the definition of criteria for the extrapolation of the period and cohort effects, as there is not a unique way of determining for a given data set how best to extrapolate period and cohort coefficients into their future values (Bray, 2002). Also, it has been demonstrated that some extrapolation functions do not yield invariant forecasts: hence, forecasts may depend on arbitrary identification constraints (Kuang et al., 2008). Hence, many approaches have been developed to address this challenge.

In APC forecasting exercises under the classical statistics paradigm, researchers usually obtain unknown values of period and cohort coefficients by extrapolating the linear trends in the period and cohort effects observed from recent data using either i. a regression or a deterministic approach (Osmond, 1985), or ii. time series models (Yang and Land, 2013a). This strategy is often criticized because it requires strong parametric assumptions (Held and Rainer, 2001; Bray, 2002). Also, it is only recently that researchers in the classical APC approach became concerned with the choice of a particular extrapolation function that leads to invariant forecasts under different identification constraints (Kuang et al., 2008).

On the other hand, Bayesian APC forecasting formulations assume a degree of smoothness of age, period and cohort effects in order to improve estimation and facilitate prediction. In contrast to non-Bayesian approaches, these forecasts do not rely on strong parametric assumptions of the trends in future values of cohort and period effects, and identification of the APC model is not required (Berzuini et al., 1993; Berzuini and Clayton, 1994; Besag et al., 1995; Held and Rainer, 2001; Bray, 2002).

In this study I take advantage of APC forecasting methods (in both classical and Bayesian perspectives) in which the identification problem does not affect the point forecasts I obtain. In the classical APC model proposed here, the identification problem has no bearing on the forecasts because I adopt invariant extrapolation functions for period and cohort effects. In the Bayesian APC model, period and cohort parameters are extrapolated using an autoregressive model in such a way that the identification problem is avoided.

Given the discussion above, it is clear that APC models have an enormous potential for the projection of the education profile and of any demographic indicator of interest. Moreover, improvements and advances in these models have been increasingly discussed in the literature, especially in the last decade, placing APC models on the frontier of research in demography.

That being said and exploiting a substantial amount of information already available from Brazilian Household Surveys, the purpose of this dissertation is to present and compare two different statistical perspectives (classical and Bayesian) to model and forecast the Brazilian education profile using APC models. In this study I deal with model construction, computation and validation issues.

I expect that this dissertation will benefit both Brazilian policy for education and the demographic scholarly community. For the former, the availability of accurate educational forecasts with their respective uncertainty is desirable to assist in planning and policy formulation. For demographers, the implications of this research may positively impact demographic modeling using APC models. Demographers are recurrently challenged to model rates using APC models. However, the choice of an appropriate APC model is a classic problem. I expect that this dissertation will offer a succinct and comprehensive appraisal of the advantages and challenges that lie behind different APC models.

This dissertation is organized as follows. Chapter 1 assesses the literature on APC models, focusing on the contributions of scholars to the solution of the identification problem and forecasting strategies. In Chapter 2 the data used in this dissertation is described in detail. A classical and a Bayesian APC multinomial model are constructed for estimating and forecasting purposes in Chapter 3. Results are presented in Chapter 4, in which an internal and external validity analysis is also conducted. Finally, I conclude this study addressing the potential advantages and limitations of the two APC approaches considered here, in which researchers might gain confidence from addressing uncertainty in one or other framework. That discussion is followed by a future research agenda on APC modeling and forecasting.

1 Age-Period-Cohort Modeling and Forecasting: a review

In this chapter I provide an assessment of the literature pertaining to age-period-cohort (APC) models. I review the most relevant articles that framed the literature on cohort analysis in demography and the social sciences over the past five decades. First, I introduce the formal aspects of the APC framework in section 1.1. In section 1.2, I address the solutions in the literature to the identification problem in APC analysis. Finally, in section 1.3, forecast formulations in the APC framework are presented.

1.1 The APC Accounting Model

For more than a half-century, demographers and social scientists have presented a profound interest in modeling and separating age, period and cohort effects on a demographic condition of a certain population. This attempt may have substantive motivations: an age effect could entail that the risk of a demographic condition to be more likely as an individual gets older; a cohort effect could suggest that people born or socialized at a certain period are more likely to present the demographic condition throughout their life; finally, a period effect could suggest that the demographic condition is more likely during a particular time period (such as an economic downturn) that affects all individuals, irrespective of their age.

The APC framework is well represented by an age-by-period, age-by-cohort, or period-by-cohort tabulation of a demographic indicator Y, as illustrated in Table 1. In this example, I is the number of age groups, J denotes the number of time periods and L gives the number of cohorts. Thus, y_{ijl} denotes the characteristic of interest for one individual in the ith age group, $i = 1,\ldots,I$, in the jth time period, $j = 1,\ldots,J$, and in the lth cohort, where $l = (I - i + j)$ and $l = 1,\ldots,L$. The prevalence or rates of a demographic condition are expressed in the cells of the table.

For a statistical analysis of Table 1, an analysis of variance model (ANOVA) is appropriate because it provides estimates of the effects of rows (age), columns (period), and row-column combinations (interaction effects - cohort) on changes in a demographic indicator Y (Smith, 2004). This model belongs to the class of generalized linear models (GLM), that estimates the variation in Y as a function of age, period and

Table 1: Age-by-period contingency table for APC analysis

Age i	Period j			
	1	2	...	J
1	I	...	...	$I-1+j$
2	$\vdots$	$\ddots$	...	$\vdots$
$\vdots$	$\vdots$	...	$\ddots$	$\vdots$
I	1	...	...	J

cohort effects provided in a design matrix X as follows

$$Y = \beta X, \tag{1.1}$$

where β is the vector of age, period and cohort effects, that is, the parameter of interest.

Since there is a linear dependency between age, period and cohort effects, that is, $period = cohort - age$, it is not possible to derive a single solution for β (the design matrix X is singular). This is the so-called *identification problem* in the APC framework. Next, I further elaborate on this issue.

1.2 The Identification Problem

1.2.1 Formalization

For the sake of concreteness, I give now an illustrative example of the identification problem (and of the singularity of the X matrix). For the sake of simplicity, consider $I = 4$ age groups, $J = 4$ periods and $L = (I + J - 1) = 7$ cohorts. Assume that we aim to model a demographic indicator Y as a function of age, period and cohort effects, as stated in Equation 1.1. This can be represented by a 4×4 age by period contingency table, in which cohorts are followed in each cell (Table 2).

Table 2: 4×4 Age by period table

Age	Period			
	1	2	3	4
1	4	5	6	7
2	3	4	5	6
3	2	3	4	5
4	1	2	3	4

The desired model expressed in Equation 1.1 can only be estimated through centralizing the parameters or requiring that one category of each age, period and cohort independent variable is chosen as reference (Yang et al., 2008; Fienberg and Mason, 1985). Assume that the oldest age group ($age = 4$), the latest period ($period = 4$) and the latest cohort ($cohort = 7$) are chosen as reference categories.

The effect coding X matrix of independent variables (including the intercept) is as follows (Table 3). The dimension of the matrix X is $IJ \times 2(I+J)-3$ or 16×13. The first column of the matrix represents the intercept and contains IJ elements, being all ones. The first row codes the youngest age group ($age1 = 1$, $age2 = 0$, $age3 = 0$) in the earliest period ($per1 = 1$, $per2 = 0$, $per3 = 0$), and therefore, the fourth cohort ($coh1 = 0$, $coh2 = 0$, $coh3 = 0$, $coh4 = 1$, $coh5 = 0$, $coh6 = 0$), as presented in Table 2). The fourth row codes the oldest age group (the fourth age group which serves as the reference category) as -1 in each of the three age group columns ($age1 = -1$, $age2 = -1$, $age3 = -1$) in the first period ($per1 = 1$, $per2 = 0$, $per3 = 0$) and first cohort ($coh1 = 1$, $coh2 = 0, \ldots, coh6 = 0$).

Table 3: Effect coding for the X matrix based on a 4×4 age by period table, assuming age = 4, period = 4 and cohort = 7 as reference categories

intercept	age1	age2	age3	per1	per2	per3	coh1	coh2	coh3	coh4	coh5	coh6
1	1	0	0	1	0	0	0	0	0	1	0	0
1	0	1	0	1	0	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0	1	0	0	0	0
1	-1	-1	-1	1	0	0	1	0	0	0	0	0
1	1	0	0	0	1	0	0	0	0	0	1	0
1	0	1	0	0	1	0	0	0	0	1	0	0
1	0	0	1	0	1	0	0	0	1	0	0	0
1	-1	-1	-1	0	1	0	0	1	0	0	0	0
1	1	0	0	0	0	1	0	0	0	0	0	1
1	0	1	0	0	0	1	0	0	0	0	1	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	-1	-1	-1	0	0	1	0	0	1	0	0	0
1	1	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	0	1	0	-1	-1	-1	0	0	0	0	0	1
1	0	0	1	-1	-1	-1	0	0	0	0	1	0
1	-1	-1	-1	-1	-1	-1	0	0	0	1	0	0

If X were of full column rank, that is, if each of the columns of X were linearly independent, we could obtain a unique least squares solution b for the unknown vector of the age, period and cohort coefficients (represented by β) as follows:

$$b = (X'X)^{-1}X'Y. \tag{1.2}$$

The problem is that the inverse of $X'X$ does not exist, since the linear dependency among the columns of X prevents a unique solution for b in Equation 1.1: that is, there are infinite solutions for b . Following O'Brien (2012), this linear dependency provides that there exists a vector v consisting of a set of coefficients that, when multiplied by the columns of X, produces a column vector of zeros, that is

$$Xv = 0, \tag{1.3}$$

where X is the $IJ \times 2(I+J)-3$ design matrix, v is the so-called null vector of dimension $2(I+J)-3 \times 1$. The null vector is defined as follows (O'Brien, 2012):

$$v' = \left[0, 1-\frac{(I+1)}{2}, 2-\frac{(I+1)}{2}, \ldots, (I-1)-\frac{(I+1)}{2}, \frac{(J+1)}{2}-1, \frac{(J+1)}{2}-2, \right. \\ \left. \ldots, \frac{(J+1)}{2}-(J-1), 1-\frac{(I+J)}{2}, 2-\frac{(I+J)}{2}, \ldots, (I+J-2)-\frac{(I+J)}{2}\right]. \tag{1.4}$$

In our 4×4 age by period example, the null vector is:

$$v' = [0, -1.5, -0.5, 0.5, 1.5, 0.5, -0.5, -3, -2, -1, 0, 1, 2]. \tag{1.5}$$

It can be shown that the result in Equation 1.3 holds for our example in Table 3. Therefore, if the researcher is willing to estimate age, period and cohort effects on a demographic indicator Y, it is not possible to uniquely estimate the independent coefficients, which means that the APC model is not identified. However, it is possible to find a solution to b by assuming some identifying constraint in the design matrix X (Mason and Smith, 1985). However, all just-identified constrained models will produce the same levels of goodness-of-fit to the data, making the use of model fit approach not appropriate as a criterion for selecting the best constrained model (Rodgers, 1982; Yang et al., 2004)

1.2.2 Approaches to the Identification Problem

In order to address the identification problem, there has been a significant ongoing debate in the literature since the 1970s. I focus here on studies on the macro-level APC framework in which the dependent variable Y is measured in the aggregate-level. Recent studies employ microdata in the form of repeated cross-sectional surveys, in which respondents are nested in two higher-level dimensions: survey years (periods) and birth cohorts, to allow for random variation in age, period and cohort effects and hence avoiding the identification problem (Yang and Land, 2006, 2013a). This dissertation, however, is confined to macro-level APC models.

With that in mind, the solutions proposed by researchers to address the identification problem that employ macro-level data can be divided into four categories, as shown in Table 4: *constrained estimators, mechanism-based APC, estimable functions* and *bayesian smoothing*.

Table 4: Solutions to the Identification Problem in APC models

	Constrained Estimators	Mechanism-based APC	Estimable Functions	Bayesian smoothing
Description	Constraints are imposed on parameter vector based on prior knowledge (Mason) or are imposed uniquely according to the design matrix (Yang)	Use of intervening variables for age, period and cohort effects, based on causal models (Winship, Heckman)	Use of estimable functions of the age, period and cohort parameters, that avoid the identification problem. For instance, non linear APC effects (Holford); the relationship between APC slopes (Rodgers), second differences of the parameters (Clayton \& Schiffers).	Assumes that changes in age, period and cohort coefficients occurs gradually.
Drawbacks	Parameter estimates depend upon the constraint adopted. Constraint is not easily verifiable and interpretable (Luo, 2013)	Do not estimate the full set of age, period and cohort coefficients. Require appropriate data, in this case a relatively rich set of potential mechanism variables.	Not easily interpretable and do not solve the identification problem.	Identifiability is not achieved.

1.2.2.1 Constrained Estimators

In the first category, Mason et al. (1973); Fienberg and Mason (1985) pioneered in proposing a parameter constraint in the APC model to solve the identification problem using prior knowledge. According to the authors, the matrix $X'X$ becomes invertible by setting two of the coefficients associated with age, period, or cohort to be equal (for instance, age1=age2 or period3=period4). Hence, for a given constraint c, we have a modified design matrix X_c and a solution b_c associated with this constraint:

$$b_c = (X_c'X_c)^{-1}X_c'Y. \tag{1.6}$$

It should be stressed that for each constraint c we derive an estimate b_c for β. Hence, different constraints are associated with different estimates for the APC parameter vector: model effect coefficients are sensitive to the arbitrary choice of the identifying constraint, and therefore, this may lead to erroneous conclusions (Rodgers, 1982; Yang et al., 2004). Also, critics of the Mason and colleagues solution to the APC model point out that the theoretical information used to impose constraints often does not exist or cannot easily be verified. These drawbacks led the researchers to become skeptical about the use of APC models for substantive interpretation of phenomena using Mason and colleagues' solution.

As a response to what would be a decisive limitation of the APC analysis - the arbitrariness in the imposition of a constraint - Fu, Yang and colleagues proposed the Intrinsic Estimator (IE) (Yang et al., 2004, 2008; Yang and Land, 2013a). The IE relies on a constraint imposed in the geometric orientation of the solution b. I provide now a brief formal explanation of this estimator. Yang and colleagues show that each of the infinite solutions for the APC model obtained by imposing an identification constraint in the parameter vector β, as proposed by Mason et al. (1973); Fienberg and Mason (1985), can be decomposed into two parts that are orthogonal or independent:

$$b_c = B + sv, \tag{1.7}$$

where b_c is one of the infinite constrained estimators given in equation 1.6, B is the intrinsic estimator (IE), s is a scalar corresponding to a the specific constrained solution in the sense of Mason et al. (1973); Fienberg and Mason (1985), and v is the null vector, which, as I described previously, does not depend upon the demographic indicator Y, but is uniquely determined by the design matrix X (Yang et al., 2004, 2008). Hence, to solve the identification problem, the intrinsic estimator B imposes a constraint on the geometric orientation of the parameter vector β in parameter space (Yang et al., 2004, 2008). In more formal terms, the constraint associated with the IE is that the solution B must be orthogonal to the null vector v.

The IE is computed by deriving an initial constrained estimate $\hat{b}_c$ in the sense of Mason et al. (1973); Fienberg and Mason (1985), and then geometrically projecting it to the intrinsic estimator B by removing the component in the direction of the null vector v, that is, imposing that s=0, that is:

$$B = (I - vv')\hat{b}_c. \tag{1.8}$$

Hence, although the IE is considered a constrained estimator, proponents argue that it presents at least three advantages: i. the identification restriction is not set by the researcher (Yang et al., 2008; O'Brien, 2011b); ii. it can be viewed as an average of the constrained estimators (Smith, 2004); and iii. among the constrained estimators, the IE is unbiased and consistent (Yang et al., 2004; Fu et al., 2011).

However, the IE approach is not without criticism. I stated above that, in the IE framework, the identification restriction is entirely fixed by the design matrix. To the extent that this assumption does not hold in the true mechanism that generated the data, the estimates associated with this constraint will be biased (O'Brien, 2011b; Luo, 2013a). In regards to the unbiasedness property of the IE, O'Brien (2011a) clarifies that it provides an unbiased estimate of the parameters only if the assumption that B must be orthogonal to the null vector v holds (O'Brien, 2011a, 436). In a recent study, Luo (2013a) argues against the use of the IE in APC research. She states that the IE assumptions to solve the identification problem "not only depends on the number of age, period, and cohort groups [that defines the design matrix X and, hence, the null vector v], but also is extremely difficult, if not impossible, to verify in empirical research" (Luo, 2013a, 18). Using simulation analysis, she demonstrates that, if there is prior knowledge on the demographic phenomenon of interest, the constrained estimator of Mason and colleagues performs better. Under prior ignorance, her results do not benefit the IE as compared to Mason and colleagues constrained estimator: both present poor performance (Luo, 2013a).

In sum, the constrained estimator category of solutions to the identification problem in the APC framework still faces significant challenges. Reliable estimates of the true age, period, and cohort effects are only provided if constraint assumptions are met (Rodgers, 1982; O'Brien, 2011a). However, researchers usually have limited information or do not know what are those true parameters. Besides, they are not able to find them based on the fit of the model since many sets of parameters generate the same best fitting solution. Still, in terms of substantive research, it is the parameters that are of most interest to the demographers and social scientists. I move now to the second category of solutions in the literature that attempt to address this puzzle.

1.2.2.2 Mechanism-based APC models

As a response to the impossibility of identifying the full set of age, period and cohort effects in the analysis of a demographic indicator Y, proponents of the *Mechanism-based APC models* solution do not intend variance decomposition or reparameterization as the standard approach in the APC models. Instead, their goal is to identify substantive variables that operationalized the age, period and cohort effects (Heckman and Robb, 1985; Winship and Harding, 2008). In short, the mechanism-based APC model employs sets of measured variables that intervene in the age, period and cohort effects, and thus identifies the model.

The mechanism-based APC approach is advantageous for at least two reasons: first, one proxy variable may be associated with one or more age, period and cohort dimensions; second, the plausibility of competitive constraints restrictions can be tested statistically using the model goodness of fit, unlike the traditional APC framework with constrained estimators (Smith, 2004). On the other hand, limitations also face this approach: it is not always possible to specify all the mechanisms through which the APC variables affect the outcome variable (Winship and Harding, 2008). Besides, this approach does not provide the full set of age, period, and cohort coefficients (O'Brien, 2012). This drawback is particularly severe for researchers with the aim to forecast a demographic indicator using APC models, as in the case of this dissertation.

1.2.2.3 Estimable Functions

In the third category of solutions to the APC identification problem, studies focused on the use of *estimable functions* of the parameters to solve the unidentifiability issue (Rodgers, 1982; Holford, 1983; Kupper et al., 1985; Clayton and Schifflers, 1987; O'Brien, 2012). Estimable functions consist of linear combinations of the APC parameters, which are invariant across the infinite solutions for β. In other words, estimable functions are invariant to the selection of constraints on the parameters. Besides, these estimable functions are also unbiased for the parameters that generated the outcome variables (O'Brien, 2012).

Several estimable functions are derived in the literature, and some examples include the second differences of parameters and the relationship between age, period and cohort slopes Holford (1983); Clayton and Schifflers (1987); Kupper et al. (1985);

Berzuini and Clayton (1994); O'Brien (2012). Of particular interest in this dissertation is the estimable function that ensures that the predicted values of the demographic indicator of interest Y are *estimable*, that is, they are the same across all of the infinite solutions (or estimates) b, the same for the generating parameters β, and the same for estimates based on constraints that may be not consistent with the generating parameters $b_c \neq \beta$ (O'Brien, 2012). This is why the fit of APC models is the same for all of the constraints that only identify APC model.

Based on the explanation provided by O'Brien (2012), I present now formally the proof that the predicted values of Y are estimable in the APC framework. For one specific constrained solution of the APC model b_c, we can rewrite Equation 1.1 as:

$$(X_c'X_c)b_c = X_c'Y. \tag{1.9}$$

Also, from the linear dependency between age, period and cohort, we see that $Xv = 0$ (Eq. 1.3). For a given solution b_c, the null-vector is multiplied by a scalar s (Eq. 1.7). Hence, it is also true that:

$$(X_c'X_c)sv = 0. \tag{1.10}$$

Replacing Equation 1.10 in Equation 1.9, it follows that:

$$(X_c'X_c)b_c + (X_c'X_c)sv = X_c'Y, \tag{1.11}$$

$$(X_c'X_c)(b_c + sv) = X_c'Y. \tag{1.12}$$

Hence, all constrained solutions to the APC model (expressed by $b_c + sv$) will produce the same observed values. This result is key for this dissertation, as my ultimate goal is to obtain predicted values of Y, and do not interpret the age, period and cohort effects.

Hence, the *estimable functions* approach provides that there are some functions of the age-period-cohort model that are unique. However, it should be noted that solving for estimable functions is not the same as solving the identification problem. Also, some authors argue that estimable functions are not easy to interpret.

1.2.2.4 Bayesian Smoothing

Bayesian formulations to the APC model were pioneered by Berzuini et al. (1993); Berzuini and Clayton (1994); Besag et al. (1995). The statistical model is the same as the one implemented in a classical statistics setting. However, the main singularity of this approach is that it allows researchers to incorporate prior belief about the smoothness of the age, period, and cohort effects to improve estimation and facilitate prediction (Besag et al., 1995; Bray et al., 2001; Held and Rainer, 2001; Bray, 2002). In a seminal paper that introduces the Bayesian APC approach, (Berzuini et al., 1993, 151) points out that the imposition of prior distributions for the APC parameters imposes smoothness by relating the demographic rates Y to each other by "an autoregressive process over the Lexis plane." Therefore, the demographic indicator Y estimated by the Bayesian model in each cell from the contingency table "borrows strength" from information in adjacent cells.

Formally, consider a general APC model for a demographic indicator Y. Let Y_{ijl} be such indicator for the age group i, the time period j and the cohort k, which is a function of age effects θ_i, $i = 1,\dots,I$, period effects ϕ_j, $J = 1,\dots,J$, and cohort effects ψ_l, $l = 1,\dots,(I-j+1)$, as follows:

$$Y_{ijl} = \theta_i + \phi_j + \psi_l. \tag{1.13}$$

Bayesian APC models usually assume that *a priori* the second differences of age, period or cohort parameters in model 1.13 are independent. However, it assumes that for each of these effects the second differences are dependent and are Gaussian random variables (Berzuini et al., 1993; Held and Rainer, 2001; Bray, 2002). For example, for the θ age effects, a smoothing prior based on second differences is given by:

$$p(\theta|\kappa) \sim N\left(2\theta_{.i-1} - \theta_{.i-2}, \frac{1}{\tau_\theta}\right), 3 \le i \le I, \tag{1.14}$$

where κ is a precision parameter which determines the smoothness of the age effects and $\theta_{.i-1}$ and $\theta_{.i-2}$ are the two preceding effects of the age effect $\theta_{.i}$. In sum, the prior distribution for each age effect $\theta_{.i}$ depends on the two preceding age effects. Similar distributions are used for the period and cohort effects.

The Bayesian APC model described above, then, implies the posterior distribution for all unknown parameters: θ, ϕ, ψ and predicted values of Y. To sample from this posterior distribution, Markov Chain Monte Carlo (MCMC) methods are implemented.

Unfortunately, as in the other set of solutions to the APC model, the unidentifiability of APC parameters is not solved in the Bayesian approach. As (Besag et al., 1995, 15) points out, the APC parameters may be unidentifiable in the prior and in the likelihood. Hence, posterior probability statements may only be made about the predicted demographic indicator Y.

1.2.3 Discussion

Summing up the existing literature, it has been proven that different constraints to overcome the identification problem in APC models yield different solutions. Researchers have debated methodological solutions, such as new constrained estimators (Yang et al., 2004, 2008; Yang and Land, 2013a), mechanism-based APC models (Heckman and Robb, 1985; Winship and Harding, 2008), estimable functions (Rodgers, 1982; Holford, 1983; Kupper et al., 1985; Clayton and Schifflers, 1987; O'Brien, 2012) and Bayesian models, but there is no consensus on the best solution to the unidentifiability issue. In fact, in the latter two approaches, the identification problem is ignored. Even though it is the APC effects that are of most interest to the demographers in terms of substantive research, research has demonstrated that it is not possible to disentangle APC effects in a meaningful way (Mason and Wolfinger, 2002).

Happily, the problems of identifiability discussed above have no consequences for the predictions obtained by the APC model. In the classical approach, the predicted values of Y are the same regardless of the constraint used to identify the model (Smith, 2004; Yang et al., 2004). In the Bayesian framework, the predicted values of Y are also the same because identification of parameters is not required (Besag et al., 1995). This is a crucial result for this dissertation, guaranteeing that my findings are not impacted by a particular identification solution.

1.3 APC Model-Based Forecasting

Another important advantage of the APC accounting framework is its applicability to forecasting. Since future trends in a demographic indicator Y may be strongly influenced by past and current trends (period), and hence somewhat related to the trends observed in successive cohorts, the incorporation of this information in a single model may allow accurate estimation of future trends. Scholars believe that the APC forecasting framework is preferable to other methods of data extrapolation because it accounts for cohort variations in the demographic indicator Y unlike, for instance, the

Lee-Carter Model (Lee and Carter, 1992), which accounts only for period and age variation (Osmond, 1985; Bray, 2002; Yang and Land, 2013a). I review now approaches to forecasting using APC models.

1.3.1 Formalization

The procedure that lies behind forecasting using APC models is quite simple and intuitive. First, a set of age, period and cohort coefficients for model 1.13 is estimated. Next, age effects θ_i are combined with predicted period and cohort effects t units ahead, ϕ_{j+t} and ψ_{l+t}. Age effects are generally not extrapolated because no extension to other ages is required. Finally, after extrapolating the parameter values for the purpose of forecast, the coefficients may be recombined to produce estimates of the future demographic indicator Y, for a given age group i, t units ahead:

$$Y_{i,j+t,k+t} = \theta_i + \phi_{j+t} + \psi_{l+t}. \tag{1.15}$$

Despite the fact that forecast exercises using APC models are quite easy to implement, the challenge arises when determining how to extrapolate period and cohort trends into the future, that is, to project beyond the range of existing data. Challenges involve two issues: i. the choice of a function to be used to extrapolate effect parameters; and ii. the assurance that forecasts are invariant to the restriction to solve the identification problem. It has been demonstrated that forecasted values Y^t obtained by a combination of extrapolations of effects are invariant to the solution adopted in the APC model. In other words, when extrapolating the period and cohort by linear extension of the coefficients for the purpose of projection, recombination of forecasted values will produce unique estimates of future rates. That is, forecasts are independent of the particular solution to the identification problem chosen (Osmond, 1985; Holford, 1985). In the Bayesian APC model, period and cohort effects are extrapolated using a smoothing specification in such a way that the identification problem is avoided: in fact, APC parameters are not identified at all in the likelihood. Thus, forecasts from the Bayesian APC model are always invariant.

I demonstrate now how forecasts in the classical approach are independent of the particular solution to the identification problem chosen based on Holford (1985). Assume that a constrained solution $(\hat{\theta}_i, \hat{\phi}_j, \hat{\psi}_l)$ is adopted to the APC model given in equation 1.13. Thus, the predicted value for Y_{ijk} is given by:

$$Y_{ijk} = \hat{\theta}_i + \hat{\phi}_j + \hat{\psi}_l. \tag{1.16}$$

Next, linear extrapolations of period and cohort effects are performed. Following the same age group over the next period and cohort, the forecasted value of Y is given by:

$$Y_{i,j+1,k+1} = \hat{\theta}_i + \hat{\phi}_{j+1} + \hat{\psi}_{l+1}. \tag{1.17}$$

The difference between $Y_{i,j+1,k+1}$ and Y_{ijk} is given by:

$$\hat{Y}_{i,j+1,k+1} - \hat{Y}_{ijk} = (\hat{\phi}_{j+1} - \hat{\phi}_j) + (\hat{\psi}_{l+1} - \hat{\psi}_l). \tag{1.18}$$

The difference between period and cohort coefficients as expressed in 1.18 is an estimable function and, hence, invariant to the constraint for identification chosen (Holford, 1985; O'Brien, 2011a). Therefore, forecasted values Y^t obtained by a combination of linear extrapolation of effects are invariant to the solution adopted in the APC model.

Other conditions under which invariant forecasts are obtained by a forecasting model that involves a particular identification of θ_i, ϕ_j and ψ_l are derived by Kuang et al. (2008). The authors formally demonstrated that a regression model only involving the second differences of the APC parameters would suffice for invariance to the solutions. However, time series models that appear to involve first differences or even levels can also be used as long as they eliminate any linear trend behaviour in the effects. Table 5 summarizes Kuang and colleagues findings for invariance in APC forecasting.

Table 5: Invariance properties of various forecasting models to a APC identification solution

APC parameters' order of integration	Invariant forecasts	Non-invariant forecasts
No differentiation	$\phi_t = a + bt + e_t$ - Linear trend $\phi_t = \phi_{t-1} + a + bt + e_t$ - Random walk with drift	$\phi_t = a + e_t$ - Constant level $\phi_t = c\phi_{t-1} + a + e_t$ - Random walk without drift
First differences	$\Delta\phi_t = a + e_t$	$\Delta\phi_t = e_t$
Second differences	$\Delta^2\phi_t = e_t$ $\Delta^2\phi_t = c\Delta^2\phi_{t-1} + e_t$	

Source: Kuang et al. (2008)

1.3.2 Approaches to the APC Model-Based Forecasting

1.3.2.1 Classical Approach

Several authors employed APC models to forecast demographic indicators in the classical statistics paradigm. In this approach for forecasting, unknown period and cohort effects for future periods are obtained by setting parameters to be constant during the projection horizon, or by applying linear regression or non linear time-series models (such as ARIMA) to estimated effects on each scale or to a subset of coefficients. Estimated age effects generally do not need to be extrapolated.

Osmond (1985) pioneered the classical forecasting applications using APC models to investigate future trends in lung cancer mortality rates. The author employed linear regression to predict cohort and period effects and tested for the sensibility of the effects by considering different numbers of past coefficients. In conclusion, the author advocated for the use of weighted linear regression models, as the researcher may wish that more recent periods are weighted more heavily in the extrapolation. Inspired by the work of Osmond, Negri et al. (1990) considered alternative methods of extrapolating APC parameters based on theoretical grounds to analyze future trends in death rates by neoplasms. Their extrapolation assumptions included setting future periods to be equal to the last one observed in the sample and linear regression extrapolation. Even though the authors admitted that different extrapolation methods often produce such different projections as to be of "little practical value" (p. 213), they conclude that qualitative indications may, nonetheless, be derived from forecasting exercises using APC models.

After a few years with no contributions to forecasting APC models in the classical paradigm, Yang and Land (2013a) employed the APC framework using the intrinsic estimator (IE) for forecasting mortality rates. They compared the performance of forecasts based on extrapolations of IE coefficients via linear and time series regressions (ARIMA model). To account for uncertainty in future estimates, the authors derived prediction intervals using a bootstrap residual resampling scheme. The authors concluded that different specifications of the weighted linear regression, which considered different period coefficients, led to considerable differences in predicting mortality rates. When comparing the coverage of the true values, the authors concluded that the linear extrapolation method produced better forecasts, but the ARIMA extrapolation provided wider confidence intervals that covered the true values when the forecasts were biased.

One important limitation of the classical APC model for forecasting is that forecasts are not invariant to the restriction to solve the identification problem, as I argued in section 1.3.1. Although Yang and Land (2013a) do not state this clearly, they recognize that "when using nonlinear extrapolations such as the ARIMA, the predictions using CGLIM [the constrained estimator] and IE coefficients will be somewhat different, however" (p. 172). Hence, the classical applications for forecast using APC models reviewed here that do not rely on invariant extrapolation functions (complied by Kuang et al. (2008) may be misleading.

Overall, projections based on classical APC require the analyst to make strong parametric assumptions. Despite the arbitrariness in the choice of past values to use and the type of regression applied, I recognize that there are strengths in this approach. Studies demonstrated that this approach yields good predictions and meaningful qualitative indications (Negri et al., 1990; Yang and Land, 2013a). Also, experimentation with different numbers of points for the regressions, weighted regression and smoothing of points before applying regression may all improve the performance of this method. However, in a classical framework, there is no way of knowing for a given data set how best to extrapolate period and cohort effects. Also, it was only recently that uncertainty in forecasts classical APC model were explored by Yang and Land (2013a), in which the authors derived prediction intervals using a bootstrap residual resampling scheme. However, the classical framework does not provide an integrated approach to account for uncertainty in future estimates. Furthermore, not all the extrapolation functions are invariant to the constraints imposed to identify the model. When extrapolating dimensions, invariant functions presented in the literature may be used for this purpose (Kuang et al., 2008).

1.3.2.2 Bayesian Approach

The previous applications of forecasting indicators using the APC models are framed within the classical statistics framework. As I discussed earlier, these studies rely on parametric assumptions about the future trends in APC model parameters.

A series of studies took advantage of the Bayesian framework to account for prior belief about the smoothness of the APC parameters and to rely on fewer parametric assumptions for forecasting purposes (Berzuini and Clayton, 1994; Besag et al., 1995; Held and Rainer, 2001; Bray et al., 2001; Bray, 2002). Bayesian APC forecasting models are inspired by Bayesian non-parametric density estimations, in which information contained in past periods and cohorts flows into the estimates of the future parameters through the adoption of an autoregressive prior distribution (Berzuini and Clayton, 1994). In other words, the Bayesian approach resembles a non-parametric analysis, in which unknown demographic rates in adjacent bands are directly related via an autoregressive process, therefore preventing rate estimates from differing too much from each other (Berzuini and Clayton, 1994; Besag et al., 1995). As a result, through the repeated application of the autoregressive prior structure of the APC parameters, the researcher is able to sample for future values of the APC parameters (and the demographic indicator of interest) by just including in the age by period table of input data empty columns for future periods and cohorts (Berzuini and Clayton, 1994).

Among the forecasting applications using the Bayesian APC model, Berzuini and Clayton (1994) predicted lung cancer mortality rates among Italian males. Besag et al. (1995) fitted a Bayesian logistic regression to cancer mortality rates in the United States. For some time, these applications turned out to be computationally intensive. In the 2000s, advances in computation power and the adoption of Markov Chain Monte Carlo (MCMC) methods as a standard for Bayesian inference enabled the emergence of new studies in the area. Held and Rainer (2001) proposed a Bayesian APC model to study future trends in lung cancer mortality in West Germany. The authors proposed a more reliable and efficient algorithm to sample from the posterior distribution of the APC parameters. Also, the authors advanced by specifying a model with random effect parameters that accounted for unstructured heterogeneity (variation not explained by age, period and cohort effects). They conclude that the Bayesian APC model produces quite reliable estimates of lung cancer mortality data. Bray et al. (2001); Bray (2002) compares the performance of classical and Bayesian APC models to project chronic diseases. Even though the author faced difficulties in modeling very low rates of incidence in both classical and Bayesian models, the Bayesian specification outperformed all classical estimates.

In sum, the reviewed studies suggest that fewer parametric assumptions are required for forecasting procedures using the Bayesian APC models in contrast to classical applications. In the Bayesian version of the APC model, the most appropriate degree of smoothing can be learned from the data. In terms of the uncertainty in future rates, the Bayesian APC model provides an integrated approach, in which both the uncertainty associated with the choice of model and the uncertainty associated with projecting beyond the range of the data are explored.

1.3.3 Discussion

Summing up the literature on forecasting using APC models, I argue that this approach is appealing for demographic research for at least two reasons: first, because of its substantive interpretation, as it takes into account cohort variation, unlike traditional demographic forecasting models, such as the Lee-Carter; second, because the implementation of forecasts is quite straightforward - it is only necessary to extrapolate the APC parameters and recombine them to produce future estimates of a demographic indicator Y.

However, I showed that challenges arise when choosing an appropriate extrapolation function to forecast age, period and cohort effects into the future. In a classical framework, assumptions have to be made in regards to the format of the extrapolation function and the choice of past values to use (Osmond, 1985; Negri et al., 1990; Yang and Land, 2013a). Besides, the researcher interested in APC models for forecasting must be aware that not all the extrapolation functions are invariant to the constraints imposed to identify the model. When extrapolating dimensions, invariant functions presented in the literature may be used for this purpose (Kuang et al., 2008).

In contrast, fewer assumptions for forecast purposes are necessary in a Bayesian setting: identifiability of the APC model in the likelihood and in the prior is not required for estimation and future estimates are derived naturally according to the autoregressive prior structure of the model. The disadvantage of Bayesian APC models is that the posterior distribution is usually difficult to evaluate (Besag et al., 1995). However, recent advances in MCMC techniques have overcome this problem (Held and Rainer, 2001; Bray et al., 2001).

To contribute for this debate, I apply and compare in this dissertation the classical and Bayesian statistics paradigms for APC modeling. In the next chapter, I first present the data used for this purpose.

2 Data and Descriptive Analysis

In this chapter I present the data used for the purpose of this dissertation and also portray the changes in education profile for Brazilian males and females since the 1980s.

The education profile of a population is usually measured by four groups according to education attainment, which are meant to follow the international standard classification of education (ISCED) (UNESCO, 2011): no education (less than one year of education); primary education (at least one year of education, but less than nine years); secondary education (at least nine years of education, but less than twelve years); and tertiary education (at least twelve years of education).

In this dissertation I adopt the classification presented in Table 6. Note that this categorization differs from the one proposed by ISCED only in the low levels of schooling. This criterion was proposed by Crespo and Reis (2006), is used at Cedeplar/UFMG in their projects, and provides a more comprehensive definition of the education profile required in the Brazilian labor market. It has been shown that the earnings return to education in Brazil is non linear to years of schooling and attached to the completion of degrees (Crespo and Reis, 2006). Hence, in my classification, the low schooling level (P1) refers to the non completion of a former degree of the Brazilian educational system: the *primário* level (conclusion of the Grade 4). The completion of the *primário* level was largely considered a minimum requirement for jobs in the Brazilian labor market in the past (Crespo and Reis, 2006). Hence, individuals who had not completed the *primário* level were similar to non educated individuals in terms of labor market opportunities.

Table 6: Definition of the education categories in this study

Category	Definition
Low schooling (P1)	Zero to three years of education (0-3)
Primary education (P2)	At least four years of education, but less than nine years (4-8)
Secondary education (P3)	At least nine years of education, but less than twelve years (9-11)
Tertiary education (P4)	At least twelve years of education (P4)

With that said, this study follows five-year age-groups over five-year time periods, and thus thirteen cohorts, with the aim of forecasting the education profile for males and females aged 20-59 years in the categories presented in Table 6. Data were selected from the National Household Sample Survey in Brazil (PNAD), which provides nationally representative data from 1983 to 2008. PNAD is collected annually by the Brazilian Bureau of Geography and Statistics (IBGE) using a stratified and clustered sample design. The units of analysis are the residential units and their inhabitants. The sampling design follows a random selection of units in three stages: primary sampling units (municipalities), second sampling units (census areas) and third sampling units (households). This ensures the representative coverage of urban and rural areas in all Brazilian states, except for the rural parts of some Northern states, which were excluded until 2003. The results reported throughout this dissertation use the frequency weights provided by IBGE to expand the sample to the national population in each educational and age group [1].

The education profile is provided for females (Table 7) and males (Table 8).

[1]Counts of cases by educational group are indexed by age groups and time periods are provided in the Appendix

Table 7: Observed education profile by age group and period. Brazil, Females

Age group	Category	Period 1983	1988	1993	1998	2003	2008
20-24	P1	0.262	0.216	0.185	0.144	0.089	0.048
	P2	0.449	0.454	0.480	0.414	0.300	0.231
	P3	0.213	0.249	0.254	0.333	0.462	0.522
	P4	0.076	0.080	0.081	0.108	0.149	0.199
	Total	1.000	1.000	1.000	1.000	1.000	1.000
25-29	P1	0.325	0.249	0.221	0.177	0.135	0.084
	P2	0.426	0.434	0.438	0.431	0.355	0.259
	P3	0.164	0.219	0.232	0.278	0.356	0.443
	P4	0.086	0.098	0.109	0.114	0.154	0.214
	Total	1.000	1.000	1.000	1.000	1.000	1.000
30-34	P1	0.410	0.300	0.255	0.214	0.164	0.125
	P2	0.385	0.424	0.426	0.428	0.391	0.327
	P3	0.128	0.172	0.203	0.236	0.307	0.366
	P4	0.077	0.104	0.116	0.121	0.138	0.183
	Total	1.000	1.000	1.000	1.000	1.000	1.000
35-39	P1	0.487	0.393	0.309	0.242	0.197	0.142
	P2	0.363	0.381	0.415	0.412	0.403	0.365
	P3	0.094	0.138	0.161	0.213	0.261	0.328
	P4	0.056	0.087	0.115	0.132	0.138	0.165
	Total	1.000	1.000	1.000	1.000	1.000	1.000
40-44	P1	0.540	0.471	0.388	0.295	0.234	0.187
	P2	0.350	0.363	0.378	0.407	0.396	0.338
	P3	0.070	0.098	0.130	0.175	0.230	0.286
	P4	0.040	0.068	0.104	0.123	0.141	0.188
	Total	1.000	1.000	1.000	1.000	1.000	1.000
45-49	P1	0.603	0.536	0.466	0.365	0.281	0.212
	P2	0.317	0.340	0.356	0.386	0.394	0.390
	P3	0.055	0.077	0.100	0.138	0.190	0.243
	P4	0.025	0.046	0.077	0.111	0.135	0.155
	Total	1.000	1.000	1.000	1.000	1.000	1.000
50-54	P1	0.625	0.587	0.543	0.455	0.359	0.275
	P2	0.304	0.328	0.330	0.362	0.376	0.382
	P3	0.052	0.057	0.076	0.103	0.150	0.208
	P4	0.019	0.029	0.051	0.080	0.115	0.135
	Total	1.000	1.000	1.000	1.000	1.000	1.000
55-59	P1	0.652	0.624	0.588	0.527	0.446	0.353
	P2	0.288	0.300	0.317	0.343	0.358	0.353
	P3	0.046	0.054	0.057	0.079	0.113	0.168
	P4	0.014	0.022	0.038	0.051	0.082	0.126
	Total	1.000	1.000	1.000	1.000	1.000	1.000

Source: Pesquisa Nacional por Amostra de Domicílios, 1983 to 2008 (IBGE)

Table 8: Observed education profile by age group and period. Brazil, Males

Age group	Category	1983	1988	1993	1998	2003	2008
20-24	P1	0.286	0.258	0.248	0.199	0.130	0.079
	P2	0.466	0.470	0.491	0.449	0.350	0.291
	P3	0.191	0.216	0.206	0.278	0.414	0.492
	P4	0.057	0.057	0.055	0.074	0.105	0.139
	Total	1.000	1.000	1.000	1.000	1.000	1.000
25-29	P1	0.319	0.265	0.246	0.221	0.176	0.114
	P2	0.446	0.450	0.465	0.460	0.392	0.307
	P3	0.158	0.203	0.207	0.231	0.316	0.417
	P4	0.077	0.081	0.082	0.088	0.116	0.162
	Total	1.000	1.000	1.000	1.000	1.000	1.000
30-34	P1	0.388	0.313	0.277	0.258	0.208	0.170
	P2	0.417	0.435	0.424	0.428	0.419	0.357
	P3	0.118	0.164	0.199	0.213	0.265	0.331
	P4	0.076	0.088	0.100	0.101	0.107	0.142
	Total	1.000	1.000	1.000	1.000	1.000	1.000
35-39	P1	0.466	0.372	0.306	0.269	0.233	0.199
	P2	0.387	0.421	0.423	0.418	0.413	0.391
	P3	0.085	0.125	0.164	0.202	0.245	0.291
	P4	0.062	0.082	0.108	0.111	0.109	0.119
	Total	1.000	1.000	1.000	1.000	1.000	1.000
40-44	P1	0.527	0.456	0.373	0.296	0.255	0.233
	P2	0.364	0.382	0.391	0.416	0.409	0.351
	P3	0.064	0.095	0.132	0.174	0.222	0.261
	P4	0.045	0.066	0.105	0.114	0.115	0.155
	Total	1.000	1.000	1.000	1.000	1.000	1.000
45-49	P1	0.555	0.515	0.440	0.353	0.281	0.237
	P2	0.347	0.359	0.374	0.389	0.409	0.383
	P3	0.063	0.073	0.091	0.140	0.188	0.247
	P4	0.034	0.053	0.095	0.118	0.122	0.133
	Total	1.000	1.000	1.000	1.000	1.000	1.000
50-54	P1	0.589	0.544	0.507	0.426	0.339	0.268
	P2	0.329	0.349	0.338	0.362	0.393	0.393
	P3	0.054	0.072	0.071	0.101	0.150	0.215
	P4	0.029	0.035	0.084	0.111	0.118	0.124
	Total	1.000	1.000	1.000	1.000	1.000	1.000
55-59	P1	0.637	0.596	0.539	0.502	0.413	0.346
	P2	0.292	0.311	0.336	0.345	0.367	0.372
	P3	0.047	0.059	0.064	0.077	0.119	0.163
	P4	0.024	0.034	0.061	0.076	0.101	0.120
	Total	1.000	1.000	1.000	1.000	1.000	1.000

Source: Pesquisa Nacional por Amostra de Domicílios, 1983 to 2008 (IBGE)

2.1 Changes in the Education Profile of Brazilian Males and Females

In this section I first describe the changes in education profile by sex from 1983 to 2008 in Brazil. Next, I evaluate these changes by age groups, periods and cohorts. Scholars argue that the graphical inspection of data arrayed by age and period should be the first step in APC analysis (Mason and Smith, 1985; Smith, 2004). Hence, before proceeding with the age-period-cohort statistical analysis, I begin with the analysis from an exploratory and visual perspective.

Table 9 provides a summary of education developments in Brazil since the 1980s. The comparison of the proportion of people in education categories across time shows the tremendous impact of school expansion on educational attainment for Brazilian males and females. In the country, universalization of the former "primário" level (Grade 1 to Grade 4) and later of all grades of primary education started to be introduced in the 1980s. Since then, a rapid expansion of enrollments has been documented (Rios-Neto et al., 2010; Rios-Neto and Guimaraes, 2010; Oliveira, 2007). As a result of expansion policies, the proportion of males and females with low education (P1) decreased dramatically between 1983 and 2008. The trend for the proportion of people with 4-8 years of schooling (P2) increased initially at this time, and then decreased with time as a result of further expansion.

In regards to the other education categories, secondary education in Brazil (P3) did not become a mass phenomenon until long after the introduction of primary education. Expansion of secondary education and more recently of tertiary education have also occurred rapidly in the country, but at a slower rate than observed for primary education (Rios-Neto et al., 2010). As a result, prevalence of these levels of schooling in the population has increased since 1983, but levels were still very low in 2008 if compared to other Latin American countries similar to Brazil, especially for P4 (Rios-Neto and Guimaraes, 2013).

Table 9: Education profile by sex and period. Brazil, 1983 to 2008

Category	1983		1988		1993		1998		2003		2008	
	Females	Males	Females	Males	Females	Males	Females	Males	Females	Males	Females	Males
P1	0.488	0.471	0.422	0.415	0.369	0.367	0.303	0.316	0.238	0.254	0.178	0.206
P2	0.360	0.381	0.378	0.397	0.393	0.405	0.398	0.408	0.372	0.394	0.331	0.355
P3	0.103	0.098	0.133	0.126	0.152	0.142	0.194	0.177	0.259	0.240	0.320	0.302
P4	0.049	0.050	0.067	0.062	0.086	0.086	0.105	0.099	0.132	0.112	0.171	0.137
Total	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Source: PNAD data (IBGE)

I now analyze trends in schooling by age groups, periods and cohorts. As Smith (2004) points out, graphical inspection of data provide initial and meaningful evidence of APC effects. Cohort effects are observed whenever there are different trends in age-specific schooling proportions over a range of cohorts; age effects are observed if age-specific schooling proportions vary by age for the same birth cohort or period. The period effects are observed if schooling proportions for all age groups change by period.

Figures 1-4 array age-specific proportions of males and females by education category, period and cohort, respectively. Levels of educational attainment are in general higher for women for all periods, in agreement with documented educational differentials by sex in Brazil (Beltrão, 2002). However, trends are similar for males and females whether considering a cohort or period perspective. Therefore, I analyze trends for Brazilians indistinctly.

Figures 1 and 2 reveal that schooling proportions changed from 1983 to 2008 for all age groups in Brazil and provide clear evidence of the existence of period effects. Additionally, age-specific schooling proportions do vary by age for a given period, confirming the existence of age effects. Finally, figures reveal that period changes in educational attainment in Brazil were not neutral by age-groups: young individuals (20-39 years) were more responsive to expansion of all education levels (P1, P2, P3 and P4) than older individuals (40-59), probably as a result of weaker labor market commitments.

Changes in educational attainment by age-groups within cohorts, as revealed by Figures 3 and 4, indicates the existence of cohort effects: there is considerable variation in age-specific schooling proportions over a range of cohorts for all education categories.

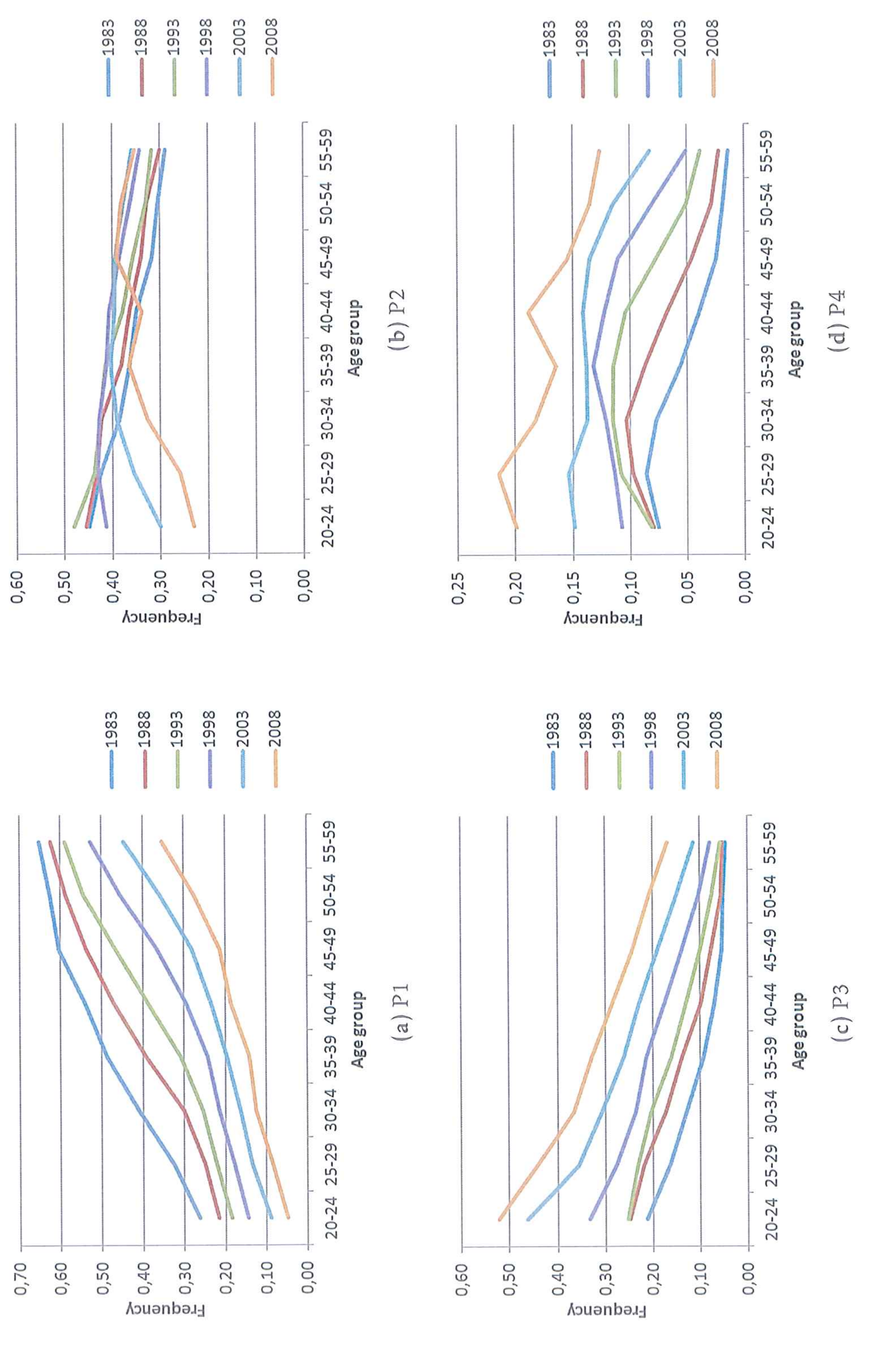
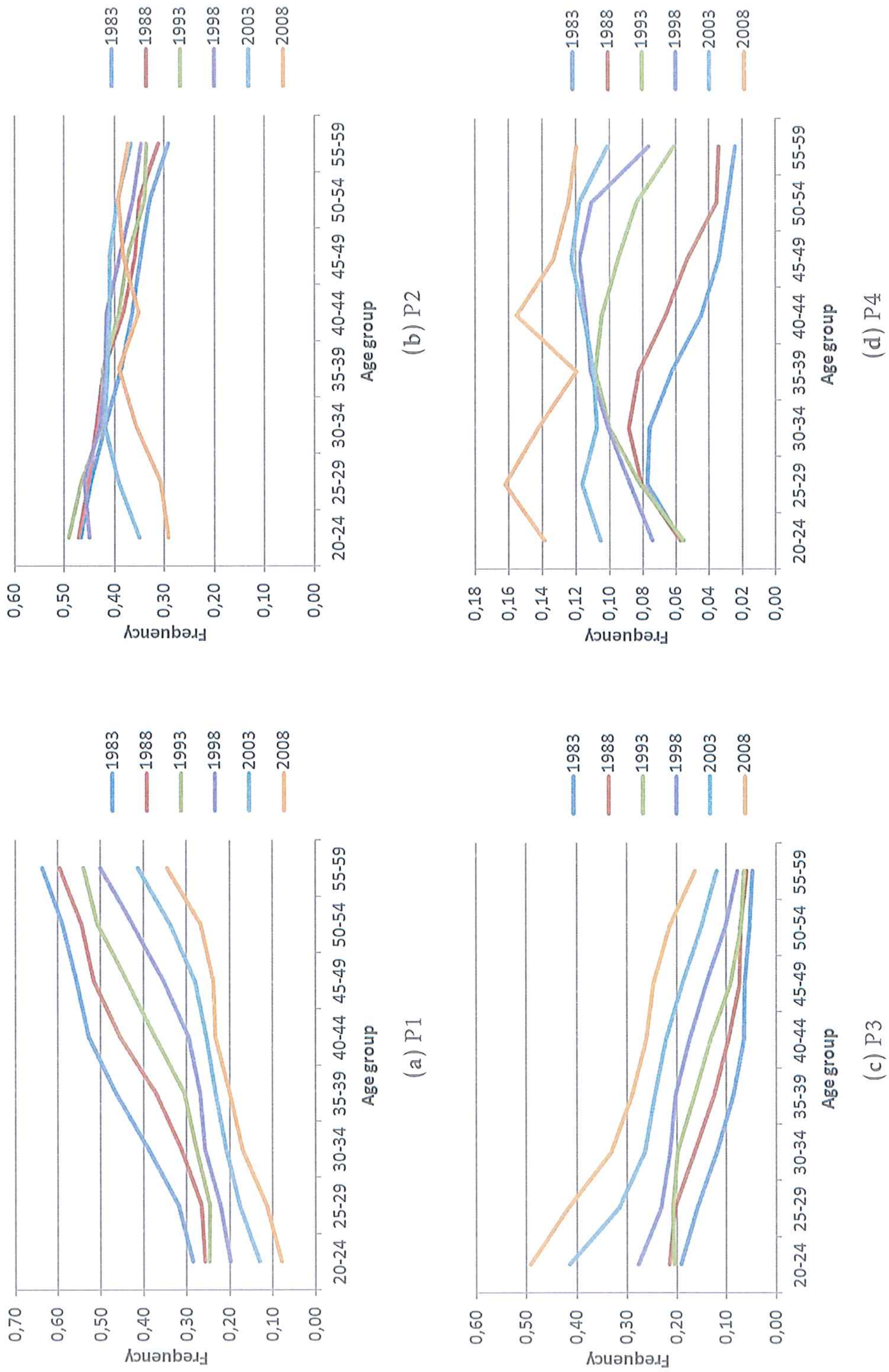


Figure 1: Proportion of females by education category, age group and period. Brazil, 1983-2008

Figure 2: Proportion of males by education category, age group and period. Brazil, 1983-2008



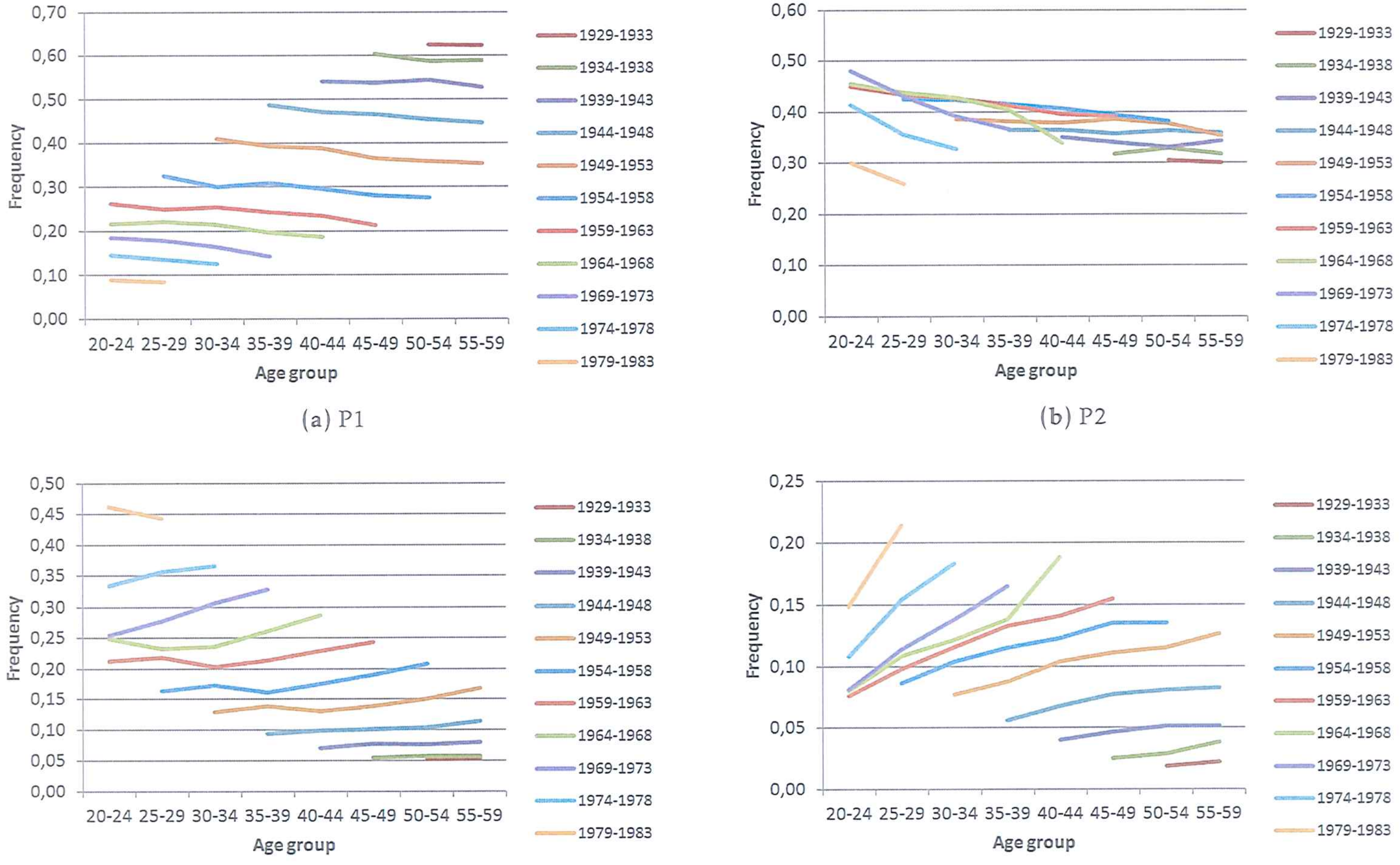
(a) P1

(b) P2

(c) P3

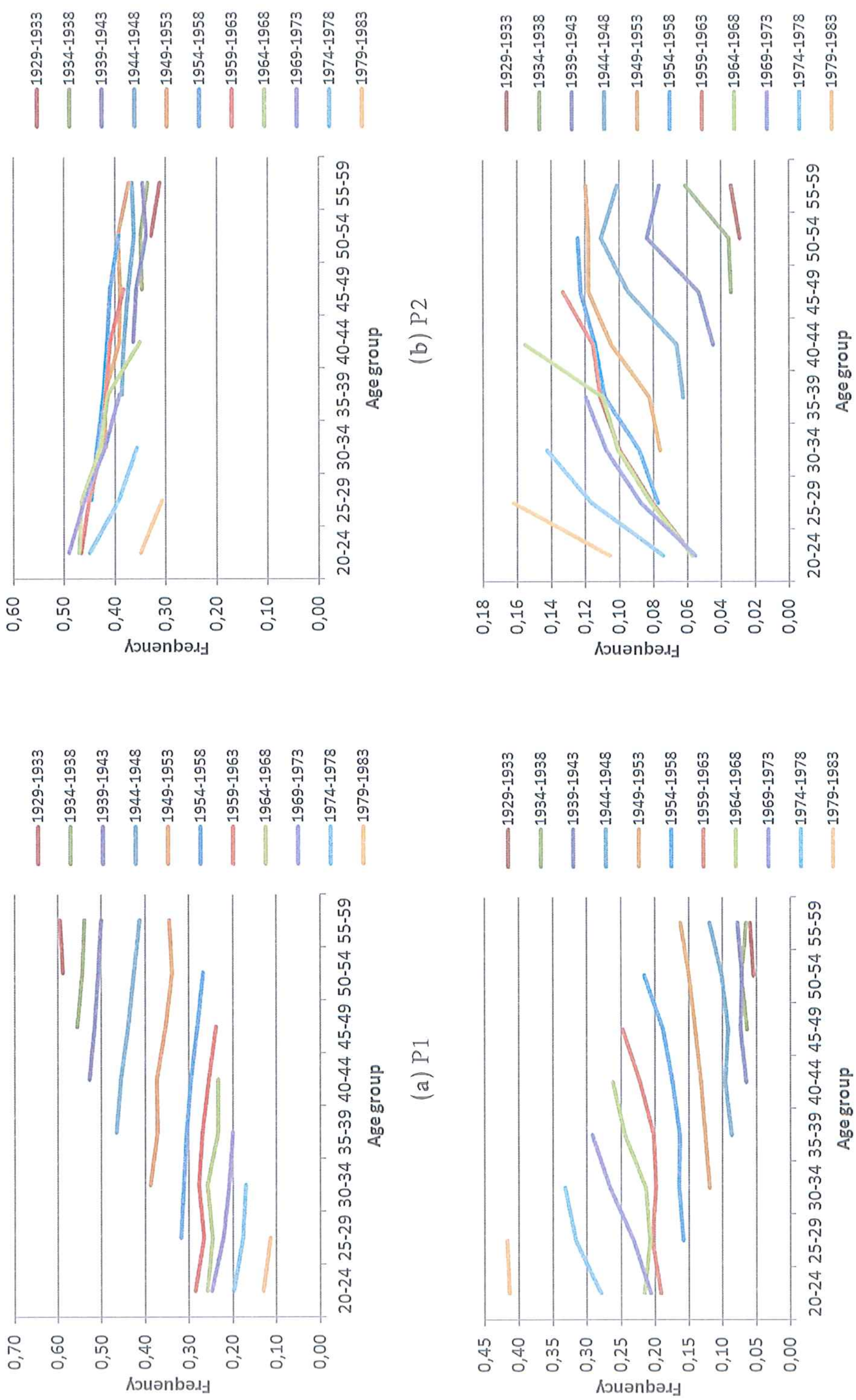
(d) P4

Figure 3: Proportion of females by education category, age group and cohort. Brazil, 1983-2008



(a) P1 (b) P2 (c) P3 (d) P4

Figure 4: Proportion of males education category, age group and cohort. Brazil, 1983-2008



3 The Multinomial APC Model: classical and Bayesian perspectives

In this chapter I introduce the age-period-cohort model for education profile. The response variable is a vector composed by the number of individuals in each category of schooling. I first develop in section 3.1 the multinomial APC model, which provide the basis for the bulk of work in this dissertation. This multinomial version of the APC model extends the works of Besag et al. (1995) and is derived based on the formulation of a bayesian multinomial logistic model presented in Agresti (2002) and considered by Paulino et al. (2013). In section 3.2, I present the development of the model and forecasting procedures under the classical perspective. Finally, I consider in section 3.3 the multinomial APC model from a Bayesian point of view.

3.1 Formulae for the Multinomial APC Model

Suppose that a sample of n individuals was independently observed and each of them is classified in a education category. Let y_{ijk} be the number of individuals in the age group i, period j and cohort $l = [ij]$ who fall into category k of schooling, where $i = 1,...,I$, $j = 1,...,J$ and $l = I - i + j$ and $L = I - 1 + J$.

Define p_{ijk} as the probability that an individual from age group i in period j and cohort l falls into the k-th category of schooling. Let also n_{ij} be the total number of individuals in the age group i, period j and cohort l. Assume also $k = 4$.

By considering these hypothesis, the uncertainty about y is given by the following multinomial model:

$$P(\underset{\sim}{y} = y|\underset{\sim}{p}) = \prod_{j=1}^{J}\prod_{i=1}^{I}\frac{n_{ij}!}{\prod_{k=1}^{4} y_{ijk}!}\prod_{k=1}^{4} p_{ijk}^{y_{ijk}}. \tag{3.1}$$

Taking the education category ordering into account, such multinomial distribution can be factored in a product of binomial distributions, and, then, the likelihood function is as follows:

$$
\begin{aligned}
l(\underset{\sim}{p}|\underset{\sim}{y}) = & \prod_{j=1}^{J}\prod_{i=1}^{I}\binom{n_{ij}}{y_{ij1}}\left(p_{ij1}\right)^{y_{ij1}}\left(1-p_{ij1}\right)^{n_{ij}-y_{ij1}} \\
& \binom{n_{ij}-y_{ij1}}{y_{ij2}}\left(\frac{p_{ij2}}{1-p_{ij1}}\right)^{y_{ij2}}\left(\frac{1-p_{ij1}-p_{ij2}}{1-p_{ij1}}\right)^{n_{ij}-y_{ij1}-y_{ij2}} \\
& \binom{n_{ij}-y_{ij1}-y_{ij2}}{y_{ij3}}\left(\frac{p_{ij3}}{1-p_{ij1}-p_{ij2}}\right)^{y_{ij3}} \\
& \left(\frac{1-p_{ij1}-p_{ij2}-p_{ij3}}{1-p_{ij1}-p_{ij2}}\right)^{n_{ij}-y_{ij1}-y_{ij2}-y_{ij3}}.
\end{aligned} \tag{3.2}
$$

One of the goals of the binomial APC model introduced by Besag et al. (1995) is to decompose the continuation-ratio logit L_{ijk}, into an overall effect of the category k, denoted by μ_k; the age group effects for the category k, denoted by θ_{ki}; the period effects for the category k, denoted by ϕ_{kj}; and the cohort effects for the category k, denoted by ψ_{kl}. Notice that the likelihood of the multinomial APC model introduced in Equation 3.2 depends on products of binomial distributions. Such way of presenting the likelihood greatly facilitate the representation of it in terms of logit functions and, consequently, in terms of the age, period and cohort effects.

Let me consider the continuation-ratio logit L_{ijk}, $k = 1,\ldots,3$, associated with each binomial distribution involved in the construction of the likelihood function in equation 3.2, which are respectively given by:

$$L_{ij1} = \ln\left(\frac{p_{ij1}}{1-p_{ij1}}\right) = \ln\left(\frac{p_{ij1}}{p_{ij2}+p_{ij3}+p_{ij4}}\right), \tag{3.3}$$

$$L_{ij2} = \ln\left(\frac{p_{ij2}}{1-p_{ij1}-p_{ij2}}\right) = \ln\left(\frac{p_{ij2}}{p_{ij3}+p_{ij4}}\right), \tag{3.4}$$

$$L_{ij3} = \ln\left(\frac{p_{ij3}}{1-p_{ij1}-p_{ij2}-p_{ij3}}\right) = \ln\left(\frac{p_{ij3}}{p_{ij4}}\right). \tag{3.5}$$

The goal of the APC model is to decompose the continuation-ratio logit L_{ijk}, k=1,2,3 into an overall effect of the category k, denoted by μ_k; age group effects for the category k, denoted by θ_{ki}; period effects for the category k, denoted by ϕ_{kj}; and cohort effects for the category k, denoted by ψ_{kl}:

$$L_{ij1} = \mu_1 + \theta_{i1} + \phi_{j1} + \psi_{l1}, \tag{3.6}$$

$$L_{ij2} = \mu_2 + \theta_{i2} + \phi_{j2} + \psi_{l2}, \tag{3.7}$$

$$L_{ij3} = \mu_3 + \theta_{i3} + \phi_{j3} + \psi_{l3}. \tag{3.8}$$

Substituting equations 3.6, 3.7 and 3.8 into 3.2, the likelihood function associated to the multinomial APC model in terms of the continuation-ratio logit L_{ijk} becomes:

$$\begin{aligned} l(\underset{\sim}{\mu}, \underset{\sim}{\theta}, \underset{\sim}{\phi}, \underset{\sim}{\psi}, |\underset{\sim}{y}) = & \prod_{j=1}^{J}\prod_{i=1}^{I}\binom{n_{ij}}{y_{ij1}}\binom{n_{ij}-y_{ij1}}{y_{ij2}}\binom{n_{ij}-y_{ij1}-y_{ij2}}{y_{ij3}} \\ & \left[\frac{\exp\{L_{ij1}\}}{1+\exp\{L_{ij1}\}}\right]^{y_{ij1}}\left[\frac{\exp\{L_{ij2}\}}{\left(1+\exp\{L_{ij2}\}\right)\left(1+\exp\{L_{ij1}\}\right)}\right]^{y_{ij2}} \\ & \left[\frac{\exp\{L_{ij3}\}}{\left(1+\exp\{L_{ij3}\}\right)\left(1+\exp\{L_{ij2}\}\right)\left(1+\exp\{L_{ij1}\}\right)}\right]^{y_{ij3}} \\ & \left[\frac{1}{\left(1+\exp\{L_{ij3}\}\right)\left(1+\exp\{L_{ij2}\}\right)\left(1+\exp\{L_{ij1}\}\right)}\right]^{n_{ij}-y_{ij1}-y_{ij2}-y_{ij3}}. \end{aligned} \tag{3.9}$$

This derivation was inspired in the formulation proposed in Paulino et al. (2013) for the trinomial model. As for the binomial APC model, I assume a linear structure on the age, period and cohort effects for the logit functions given in equations 3.6, 3.7 and 3.8.

Besides restrictions on the probability space, so that, $\sum_{k=1}^{4} p_{ijk} = 1$, an identifiability condition must be imposed on each block of age, period, and cohort parameters for a given education category k, because there is more than one solution to the parameters that leads to the same probability p_{ijk} in model 3.10. There are two alternatives to address this issue. One possibility is to achieve identification by imposing a mean constraint (Berzuini and Clayton, 1994):

$$\sum_{i=1}^{I} \theta_{\cdot i} = 0, \text{ for all } k; \tag{3.10}$$

$$\sum_{i=1}^{J} \phi_{\cdot j} = 0, \text{ for all } k; \tag{3.11}$$

$$\sum_{i=1}^{L} \psi_{\cdot l} = 0, \text{ for all } k. \tag{3.12}$$

Another possibility is to identify the model by setting one of the APC parameters for a given category to be zero. In this case, the coefficients will differ because they have different interpretations, but the predicted probabilities p_{ijk} will still be the same (Maddala, 1986):

$$\theta_{k\cdot} = 0, \text{ for } k{=}1 \text{ or } k{=}2 \text{ or } k{=}3 \text{ or } k{=}4 \text{ and all } i; \tag{3.13}$$

$$\phi_{k\cdot} = 0, \text{ for } k{=}1 \text{ or } k{=}2 \text{ or } k{=}3 \text{ or } k{=}4 \text{ and all } j; \tag{3.14}$$

$$\psi_{k\cdot} = 0, \text{ for } k{=}1 \text{ or } k{=}2 \text{ or } k{=}3 \text{ or } k{=}4 \text{ and all } l. \tag{3.15}$$

(3.16)

3.1.1 The Identification Problem

If i indexes the age group and j indexes the period, then it follows that the cohort index k is deterministically derived, as follows:

$$K = I - i + j, \tag{3.17}$$

where i is the total of age groups. Hence, there is a linear relationship between age, period and cohort. As has been demonstrated elsewhere (Held and Rainer, 2001), for any value of α, the linear transformations

$$\theta_{ik} \to \theta_{ik} + \alpha\left(i - \frac{I+1}{2}\right), \tag{3.18}$$

$$\phi_{jk} \to \phi_{jk} + \alpha\left(j - \frac{J+1}{2}\right), \tag{3.19}$$

$$\psi_{lk} \to \psi_{lk} + \alpha\left(l - \frac{L+1}{2}\right), \tag{3.20}$$

still satisfy equations required for identifiability in the multinomial model (for instance, equations 3.10, 3.11 and 3.12) and leave ?? unchanged for all possible combinations of i and j. Therefore, there is no single set of parameter estimates that maximizes the likelihood function. Thus, in absence of prior information, these models are not identifiable, so that it may be necessary to impose additional constraints on the parameters.

Next, I present classical and Bayesian solutions to this multinomial APC modeling and forecasting.

3.2 A Classical Multinomial APC Model

There are many potential versions of the classical multinomial APC model, depending on assumptions regarding the unidentifiability issue and the functional form of the extrapolation function. Given my interest in forecasting exercises, I opt here for a classical multinomial APC model in which the set of parameters θ_{ik}, ϕ_{jk} and ψ_{lk} are derived. Therefore, identification using proxy variables (Mechanism-based APC) or the use of estimable functions of the APC parameters are not considered.

With the aim of retrieving all APC parameters for forecasting purposes, I first decide on which function will be used to extrapolate the parameters. The choice of the extrapolation function comes first because, in the case invariant extrapolation functions are chosen, the choice of a particular identification constraint is irrelevant, as I presented in section 1.3.1. Therefore, I choose here extrapolations based on the linear trend.

As I choose an invariant extrapolation function of the APC parameters, the particular choice of a constraint to solve the identification problem will not influence my forecasts (Kuang et al., 2008). Hence, the classical multinomial APC model that I propose relies on a informative constraint for the cohort parameters in APC model ??:

$$\psi_{1k} = \psi_{2k}, \text{ for all } k. \tag{3.21}$$

3.2.1 The Choice of a Reference Category in the Multinomial Logistic Model

As I presented before, in a multinomial model, restrictions are also necessary to identify the probability space (see equation 3.10). In this proposed classical model, I adopt a normalization rule setting one of the APC parameters for a category one (P1), and in another exercise category 4 (P4), to be zero. As is well known, it is inconsequential which category we pick as the reference cell because we can always convert from one formulation to another. However, as I explain next, the choice of a certain reference category may imply different uncertainty levels for future values of the period and cohort coefficients, and therefore, I opt here for two exercises.

In the first classical APC model, I choose the first category ($k = 1$) as the base outcome, that is, I measure the change relative to the low schooling group:

$$\theta_{1\cdot} = 0, \text{ for all } i; \quad (3.22)$$
$$\phi_{1\cdot} = 0, \text{ for all } j; \quad (3.23)$$
$$\psi_{1\cdot} = 0, \text{ for all } l. \quad (3.24)$$
$$(3.25)$$

In the second classical APC model, I choose the last category ($k = 4$) as the base outcome, that is, I measure the change relative to the tertiary education group:

$$\theta_{4\cdot} = 0, \text{ for all } i; \quad (3.26)$$
$$\phi_{4\cdot} = 0, \text{ for all } j; \quad (3.27)$$
$$\psi_{4\cdot} = 0, \text{ for all } l. \quad (3.28)$$
$$(3.29)$$

After setting the restrictions above, the likelihood 3.10 is now identifiable. Interpretation of the APC parameters θ_{ik}, ϕ_{jk} and ψ_{lk} will vary according to which education category is normalized to have zero coefficient. In other words, the APC effects vary according to the response paired with the baseline (Agresti, 2002). However, any normalization rule will automatically yield the same adjusted probabilities p_{ij1} (Maddala, 1986; Cameron and Trivedi, 2005).

3.2.2 Forecasts and Uncertainty

Once the full set of parameters θ_{ik}, ϕ_{jk} and ψ_{lk} is derived, it is necessary to extrapolate period and cohort effects into the future (age effects are held constant). I employ in this dissertation an invariant extrapolation function to retrieve parameter estimates N periods ahead based on the linear trend:

$$\phi_{kt} = a + bt + e_t, t = 1, \ldots, N; \tag{3.30}$$
$$\psi_{kt} = c + dt + e_t, t = 1, \ldots, N. \tag{3.31}$$
(3.32)

Finally, the forecasted education profile N periods ahead are then obtained through a recombination of the extrapolated period and cohort effects (age effects and global effects of the category are held constant) according to:

$$L_{i(j+t)k} = \mu_k + \theta_{ik} + \phi_{(j+N)k} + \psi_{(l+N)k}, t = 1, \ldots, N. \tag{3.33}$$

3.2.2.1 Assessing uncertainty in a classical framework: combining probabilistic and scenario approach

Forecasts should be provided with their respective uncertainty measures. In this classical APC model, uncertainty is associated with APC parameters and their extrapolated values, which, in turn, lead to uncertainty in the future estimates of the education profile. In this study I propose a two-step procedure to account for uncertainty in forecasts using the classical APC model, which combines probabilistic and scenario components.

First, in order to measure uncertainty for the APC parameters, I derive 95% prediction intervals for the coefficients and their extrapolated values using the bootstrap method with 1000 replications. The bootstrap is a method for estimating the distribution of an estimator in a classical framework by resampling a model estimated from the data. For variance stabilized parameters, the bootstrap method provides approximations to coverage probabilities of confidence intervals (Efron and Tibshirani, 1994). This strategy was inspired by the work of Yang and Land (2013a).

Second, uncertainty for the education profile is derived using a scenario approach. I propose three alternative scenarios: i. *estimated scenario*, obtained through the forecasts for future period and cohort coefficients; ii. *pessimistic scenario*, obtained through the lower bound of the 95% confidence interval for both period and cohort effects; and iii. *optimistic scenario*, obtained through the upper bound of the 95% confidence interval for period and cohort effects. Age effects are held constant in the forecast. These three scenarios aim to span the range of plausible futures of schooling, and therefore, to provide self-consistent stories about the future: the prevalence in the future of the most likely low/high (pessimist/optimist) values of period and cohort coefficients. Taken as a set, the scenarios should be compelling and acceptable to all the individuals interested in future trends in education profile.

There are limitations in the scenario approach for the classical APC model proposed here. First, a clear limitation resides in the lack of an integrated assessment between uncertainty in APC parameters and uncertainty in estimated probabilities. Second, the interval between the pessimistic and optimistic scenario for education profile has no probabilistic meaning; therefore, users may not interpret that the interval would contain the actual future values for the education profile. Third, the assumptions implicit in the scenario construction embody an element of rigidity: period and cohort coefficients are high or low for the duration of the forecast rather than varying within the range of probable values over time.

3.2.2.2 The choice of a Reference Category and their impact on forecast scenarios

As I argued before, a normalization rule is required in the classical APC model to identify the likelihood. For prediction and fit purposes, it is inconsequential which category we pick as the reference cell because we can always convert from one formulation to another and, therefore, fitted and extrapolated probabilities will be the same regardless of which normalization rule is imposed. However, in the scenario approach I am proposing here, prediction confidence intervals for the APC coefficients and their extrapolated values using the bootstrap method may not be the same depending on the reference category chosen. Each specification (either using category one or category four as reference) provides a set of estimated period and cohort coefficients, for which different bootstrapped confidence intervals (and hence low and Optimistic Scenarios) are derived. Therefore, uncertainty level may differ from one normalization rule to another, and I illustrate this in the Results section. However, point estimate predicted probabilities into the future will be the same whichever reference category is chosen.

Hence, along with a lack of an integrated assessment of uncertainty in this proposed classical APC model, a clear limitation regards the uncertainty in the forecasted education profile, which may be a function of the reference category chosen as normalization rule.

3.3 A Bayesian Multinomial APC Model

Now let us consider the multinomial APC model from a Bayesian point of view. As I discussed in Chapter 1, the Bayesian model works by relating temporal changes in a demographic indicator by an autoregressive process over the lexis plane. Accordingly, the predicted education profile borrows information from adjacent bins, as appropriate for forecasting exercises Besag et al. (1995).

3.3.1 Specification of the Prior Distributions

In the Bayesian paradigm, I follow the strategy proposed by Held and Rainer (2001) and Bray (2002) by setting a Bayesian age-period-cohort model to smooth age, period and cohort trends and to extrapolate N future periods and cohorts. This smoothing model is implemented in a Bayesian setting by the imposition of prior distribution of the parameters. I use a flat prior for μ_k, that is, a normal distribution with mean zero and variance 1000. For the APC parameters, I assume that the second differences of age, period or cohort parameters are independent. I also assume that for each of these effects the second differences are dependent and Gaussian random variables. This model was termed by Held and Rainer (2001) a *second-order random walk model*, or simply RW2. The RW2 formulation penalizes deviations from a linear trend.

Hence, for each parameter vector $\underset{\sim}{\theta}_k$, $\underset{\sim}{\phi}_k$ and $\underset{\sim}{\psi}_k$, I assume an RW2 model in which the first two parameters for age, period and cohort effects are given non-informative priors (including a term for the hyperparameter to provide the correct likelihood). Accordingly, parameters are needed which control how much smoothness is present in the estimated probability surface. Precision hyperparameters τ_θ, τ_ϕ and τ_ψ are defined to control the smoothness of age, period and cohort effects, respectively. This requires also the incorporation in the model a subjective hyperprior distribution for these hyperparameters, and the choice of the hyperprior is crucial to forecasting applications. Here I assign non-informative but proper gamma distributions to hyperparameters.

In sum, for i age effects, I define

$$\theta_{.1} \sim N\left(0, 10^6 \frac{1}{\tau_\theta}\right), \tag{3.34}$$

$$\theta_{.2} \sim N\left(0, 10^6 \frac{1}{\tau_\theta}\right), \tag{3.35}$$

$$\theta_{.i}|\theta_{.1},...,\theta_{.i-1} \sim N\left(2\theta_{.i-1} - \theta_{.i-2}, \frac{1}{\tau_\theta}\right), 3 \le i \le I. \tag{3.36}$$

For $J+N$ period effects, I consider

$$\phi_{\cdot 1} \sim N\left(0, 10^6 \frac{1}{\tau_\phi}\right), \tag{3.37}$$

$$\phi_{\cdot 2} \sim N\left(0, 10^6 \frac{1}{\tau_\phi}\right), \tag{3.38}$$

$$\phi_{\cdot j} | \phi_{\cdot 1}, \ldots, \phi_{\cdot j-1} \sim N\left(2\phi_{\cdot j-1} - \phi_{\cdot j-2}, \frac{1}{\tau_\phi}\right), 3 \leq j \leq J+N. \tag{3.39}$$

For $L+N$ cohort effects, the prior specifications are

$$\psi_{\cdot 1} \sim N\left(0, 10^6 \frac{1}{\tau_\psi}\right), \tag{3.40}$$

$$\psi_{\cdot 2} \sim N\left(0, 10^6 \frac{1}{\tau_\psi}\right), \tag{3.41}$$

$$\psi_{\cdot l} | \psi_{\cdot 1}, \ldots, \psi_{\cdot l-1} \sim N\left(2\psi_{\cdot l-1} - \psi_{\cdot l-2}, \frac{1}{\tau_\psi}\right), 3 \leq l \leq L+N. \tag{3.42}$$

I also define the following flat hyperpriors for the precision parameters. These hyperparameters are, therefore, estimated solely from the data:

$$\tau_{\theta\cdot} \sim G(0.001, 0.001), \tag{3.43}$$

$$\tau_{\phi\cdot} \sim G(0.001, 0.001), \tag{3.44}$$

$$\tau_{\psi\cdot} \sim G(0.001, 0.001). \tag{3.45}$$

The problem is that in the RW2 model, the parameters μ_k, θ_{ik}, ϕ_{jk} and ψ_{lk} are still unidentifiable as the condition in Equation 3.17 still follows. However, identifiability of the likelihood is not required for estimation in this Bayesian model, but the estimated probabilities are (Besag et al., 1995).

Now, an important question is whether the joint prior distribution for APC the parameters $(\mu, \theta, \phi, \psi)$ is proper. An improper prior distribution can cause problems in implementing and making inferences in the Bayesian APC model. If the model does not include the global effect μ, the RW2 prior distribution for (θ, ϕ, ψ) is proper whenever the sum of each of these parameters is equal zero (see (Assunção et al., 2002) for the proof). For the case study presented in this paper, it makes sense to consider the global effect. However, in this study, the prior distribution I elicited for μ is proper: a normal distribution with mean zero and variance 1000. Thus, it turns out that the joint prior distribution is also proper.

In order to improve numeric stability and mixing of the MCMC algorithm, I impose a linear trend constraint for the age effects, as suggested by the literature (Holford, 1983; Berzuini and Clayton, 1994; Bray, 2002; Held and Rainer, 2001; Baker and Bray, 2005). Following the demonstration presented in Holford (1983), consider the age effects $\theta_{i\cdot}$ given that condition 3.17 holds. In this case, the linear trend in age effects can be described by the contrast:

$$\theta_{L\cdot} = C \sum_{i=1}^{I} c_i \theta_{i\cdot}, \tag{3.46}$$

where:

$$c_i = i - \frac{I+1}{2}, \tag{3.47}$$

and:

$$C = \sum_{i=1}^{I} c_i^2. \tag{3.48}$$

The curvature component is, in turn, given by the age effects with the linear trend removed, as follows:

$$\tilde{\theta}_{i\cdot} = \theta_{i\cdot} - c_i \theta_{L\cdot}. \tag{3.49}$$

3.3.2 Forecasts and Uncertainty

Once posterior distributions for the μs and for the age, period and cohort effects are obtained through the MCMC algorithm, the forecasted education profile N periods ahead are then obtained through the posterior predictive distribution. Such posterior distribution is approximated via the MCMC algorithm. Similarly to the classical model, I consider a recombination of the extrapolated period and cohort effects (age effects and global effects of the category are held constant) according to:

$$L_{i(j+t)k} = \mu_k + \theta_{ik} + \phi_{(j+N)k} + \psi_{(l+N)k},\ t = 1, \ldots, N, \tag{3.50}$$

and assuming Equation 3.50, I generate the projected education profile N periods ahead.

Uncertainty in the Bayesian APC framework is derived in terms of a posterior distribution. The MCMC techniques are particularly attractive because they allow the uncertainty associated with functions of the APC parameters - in this study, the education profile - to be readily explored, in contrast with the classical approach. The credible intervals presented encompass both uncertainty associated with the choice of model and uncertainty associated with projecting beyond the range of the data.

In the next chapter, I present the results of the classical and Bayesian multinomial APC models.

4 Results

In this chapter I present results of the fitted and forecasted education profile for Brazilian males and females using the APC framework under two different approaches: classical and Bayesian. First, I report results on the fitted and extrapolated APC parameters, and I give the fitted and projected education profile in each approach. Next, I provide an inter-comparison of the models and a substantive perspective on the results, which is of interest to policy planners and analysts. Finally, I validate the forecast models using out-of-sample projections, in order to assess and compare the predictive quality of the classical and Bayesian APC approaches.

4.1 Classical APC Model

4.1.1 Computation

The classical APC model was estimated using the *mlogit* command in Stata software (StataCorp, 2009b). Estimation of the parameters of the APC model proceeds by maximization of the likelihood expressed in equation 3.10. This usually requires numerical procedures. In *mlogit*, Newton-Raphson maximum likelihood is used (StataCorp, 2009a).

4.1.2 Classical APC-Model Parameters and Forecasts

4.1.2.1 Results when P1 is chosen as reference category

Tables 10 and 11 provide the estimated age (θ), period (ϕ) and cohort (ψ) parameters and their respective standard errors. As in any non linear model, regression parameters in APC multinomial models as provided here are difficult to interpret. Given the fact that in equation 3.10 one needs a normalization rule, we cannot compare the absolute values of the coefficients in the different educational groups. The sign of θ_{ik}, ϕ_{jk} and ψ_{lk} is not necessarily the sign of the response p_{ijk}. To the researcher interested in meaningful interpretation of the parameters, methods may be employed to compare the change in the average predicted p_{ijk} as regressors change (Maddala, 1986). However, this is not the focus of this work. I am mainly interested in the set of estimated APC parameters for forecasting purposes.

Although APC parameters are not easily interpretable in a multinomial framework per se, age and temporal trends of these parameters are. Hence, I now analyze

Table 10: Estimated (Standard error) for the female group using the classical approach for the APC multinomial model, Reference Category is P1, Full sample, Brazil

	P2	P3	P4
θ_2	-0.1261	-0.1703	-0.1176
	(0.0021)	(0.0042)	(0.0072)
θ_3	-0.1975	-0.3103	-0.4120
	(0.0040)	(0.0084)	(0.0143)
θ_4	-0.2560	-0.4335	-0.7254
	(0.0060)	(0.0125)	(0.0214)
θ_5	-0.3360	-0.5782	-1.0351
	(0.0080)	(0.0167)	(0.0285)
θ_6	-0.3562	-0.7199	-1.4488
	(0.0100)	(0.0208)	(0.0356)
θ_7	-0.4290	-0.8651	-1.8976
	(0.0120)	(0.0250)	(0.0428)
θ_8	-0.5032	-1.0086	-2.2697
	(0.0138)	(0.0290)	(0.0498)
ϕ_2	0.1052	0.2393	0.5068
	(0.0020)	(0.0042)	(0.0071)
ϕ_3	0.1556	0.2967	0.9500
	(0.0040)	(0.0083)	(0.0142)
ϕ_4	0.2481	0.5446	1.4200
	(0.0059)	(0.0124)	(0.0213)
ϕ_5	0.3070	0.8410	1.9287
	(0.0079)	(0.0166)	(0.0285)
ϕ_6	0.3599	1.1004	2.5020
	(0.0099)	(0.0208)	(0.0356)
ψ_3	0.0416	-0.0173	0.0020
	(0.0028)	(0.0058)	(0.0094)
ψ_4	0.1382	0.2120	0.0891
	(0.0047)	(0.0096)	(0.0161)
ψ_5	0.2722	0.4618	0.2390
	(0.0066)	(0.0137)	(0.0231)
ψ_6	0.4602	0.8173	0.3614
	(0.0086)	(0.0178)	(0.0302)
ψ_7	0.7167	1.1442	0.3552
	(0.0105)	(0.0219)	(0.0373)
ψ_8	0.8746	1.4162	0.2511
	(0.0125)	(0.0261)	(0.0444)
ψ_9	0.9474	1.5430	0.0673
	(0.0145)	(0.0302)	(0.0515)
ψ_{10}	1.1045	1.7406	-0.1158
	(0.0165)	(0.0344)	(0.0586)
ψ_{11}	1.1110	1.9404	-0.1113
	(0.0185)	(0.0385)	(0.0657)
ψ_{12}	1.2071	2.4141	0.1527
	(0.0205)	(0.0427)	(0.0728)
ψ_{13}	1.5284	2.9372	0.5099
	(0.0225)	(0.0469)	(0.0800)
μ	-0.3129	-1.6487	-1.5876
	(0.0126)	(0.0261)	(0.0444)

Source: Calculations based on PNAD data
Obs.: Standard errors between parentesis

Table 11: Estimated (Standard error) for the male group using the classical approach for the APC multinomial model, Reference Category is P1, Full sample, Brazil

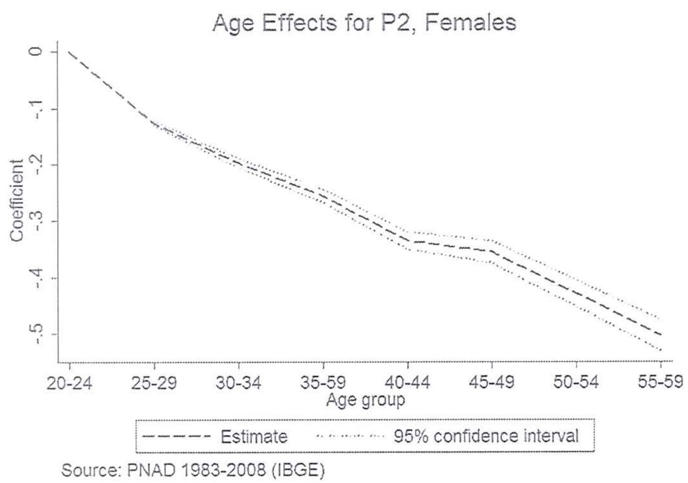
	P2	P3	P4
θ_2	-0.1261	-0.1193	0.1311
	(0.0022)	(0.0043)	(0.0058)
θ_3	-0.3362	-0.3295	-0.0617
	(0.0042)	(0.0085)	(0.0115)
θ_4	-0.4595	-0.4898	-0.3044
	(0.0062)	(0.0127)	(0.0173)
θ_5	-0.6331	-0.6894	-0.5195
	(0.0082)	(0.0169)	(0.0230)
θ_6	-0.7272	-0.8382	-0.7717
	(0.0103)	(0.0212)	(0.0287)
θ_7	-0.8533	-0.9867	-1.0324
	(0.0124)	(0.0255)	(0.0345)
θ_8	-1.0193	-1.1913	-1.3445
	(0.0143)	(0.0295)	(0.0401)
ϕ_2	0.1657	0.2793	0.3623
	(0.0020)	(0.0042)	(0.0057)
ϕ_3	0.2888	0.4168	0.8144
	(0.0041)	(0.0084)	(0.0115)
ϕ_4	0.4376	0.6802	1.1663
	(0.0061)	(0.0127)	(0.0172)
ϕ_5	0.6021	1.0431	1.5379
	(0.0082)	(0.0169)	(0.0229)
ϕ_6	0.6995	1.3232	1.9636
	(0.0102)	(0.0211)	(0.0286)
ψ_3	0.0152	0.0722	0.0373
	(0.0029)	(0.0058)	(0.0078)
ψ_4	-0.0297	0.0139	0.1733
	(0.0048)	(0.0098)	(0.0132)
ψ_5	0.0398	0.2587	0.3144
	(0.0068)	(0.0139)	(0.0188)
ψ_6	0.1618	0.5660	0.3567
	(0.0088)	(0.0181)	(0.0245)
ψ_7	0.2650	0.8357	0.3282
	(0.0109)	(0.0223)	(0.0302)
ψ_8	0.2484	0.9605	0.1827
	(0.0129)	(0.0265)	(0.0359)
ψ_9	0.1801	0.9077	-0.0187
	(0.0149)	(0.0307)	(0.0416)
ψ_{10}	0.1804	0.8960	-0.2730
	(0.0170)	(0.0350)	(0.0474)
ψ_{11}	0.1229	1.0697	-0.1674
	(0.0191)	(0.0392)	(0.0531)
ψ_{12}	0.1602	1.5063	0.1817
	(0.0211)	(0.0434)	(0.0588)
ψ_{13}	0.3678	1.9126	0.5308
	(0.0232)	(0.0477)	(0.0646)
μ	0.2385	-1.4055	-1.9309
	(0.0130)	(0.0266)	(0.0360)

Source: Calculations based on PNAD data
Obs.: Standard errors between parentesis

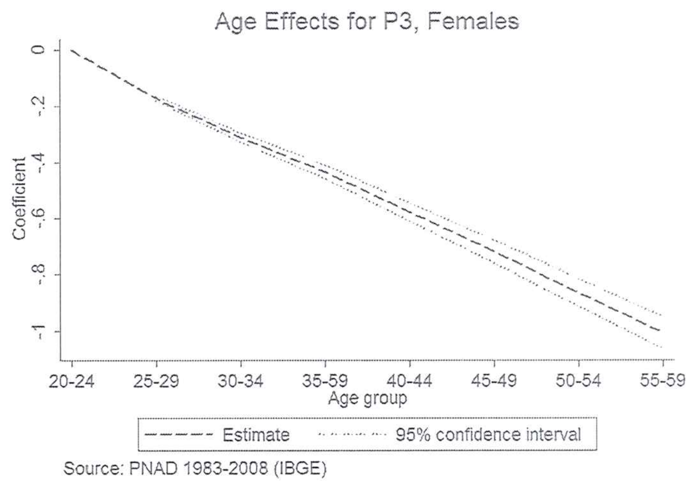
age, period and cohort effects model parameters in order to discern what trends might be present. Figures 5-10 allow this description. APC effects for P1 are assumed to be zero as a result of the normalization rule. Hence, figures display the APC coefficients when the low schooling category (listed as P1) is the reference category.

The age effects exhibit a similar pattern for males and females and education category: age effects are larger for younger age groups and smaller for older age groups. The period effects have an increasing slope, as well as the cohort effects for P2 and P3. The cohort effects for P4 have more variation than age, period and cohort effects for P2 and P3. For both sexes there is a distinct peak in cohort effects for P4 for birth cohorts born around 1949 and 1958, a sharp decreasing slope for people born between 1959 and 1968, a distinct valley for people born between 1969 and 1978, and then an increasing slope for cohorts born after 1979. Note that, for both sexes, the uncertainty of the estimates increases by a large amount only for cohort effects for P4.

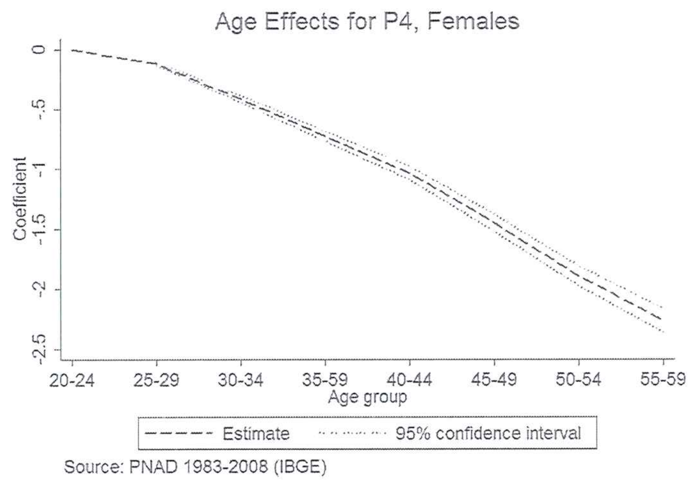
Figures 11-14 display extrapolated period and cohort effects for P2, P3 and P4 and for males and females using a linear trend model. 95% prediction intervals for future period and cohort coefficients were provided by a bootstrap simulation as described in section 3.2.2.1. Linear trend extrapolation assumes that the observed trend in the period and cohort effects remains constant over the period of extrapolation. Trends are similar for males and females, so I interpret them indistinctly. Period effects for all education categories and cohort effects for P2 and P3 for females exhibit an upward trend, and hence a long-term increase in the parameters is expected. Cohort effects for P2 and P3 for males and for P4, both sexes, do not exhibit a clear trend. Therefore, forecasted cohort effects may more accurately reflect long-term conditions of the parameters.



(a) P2

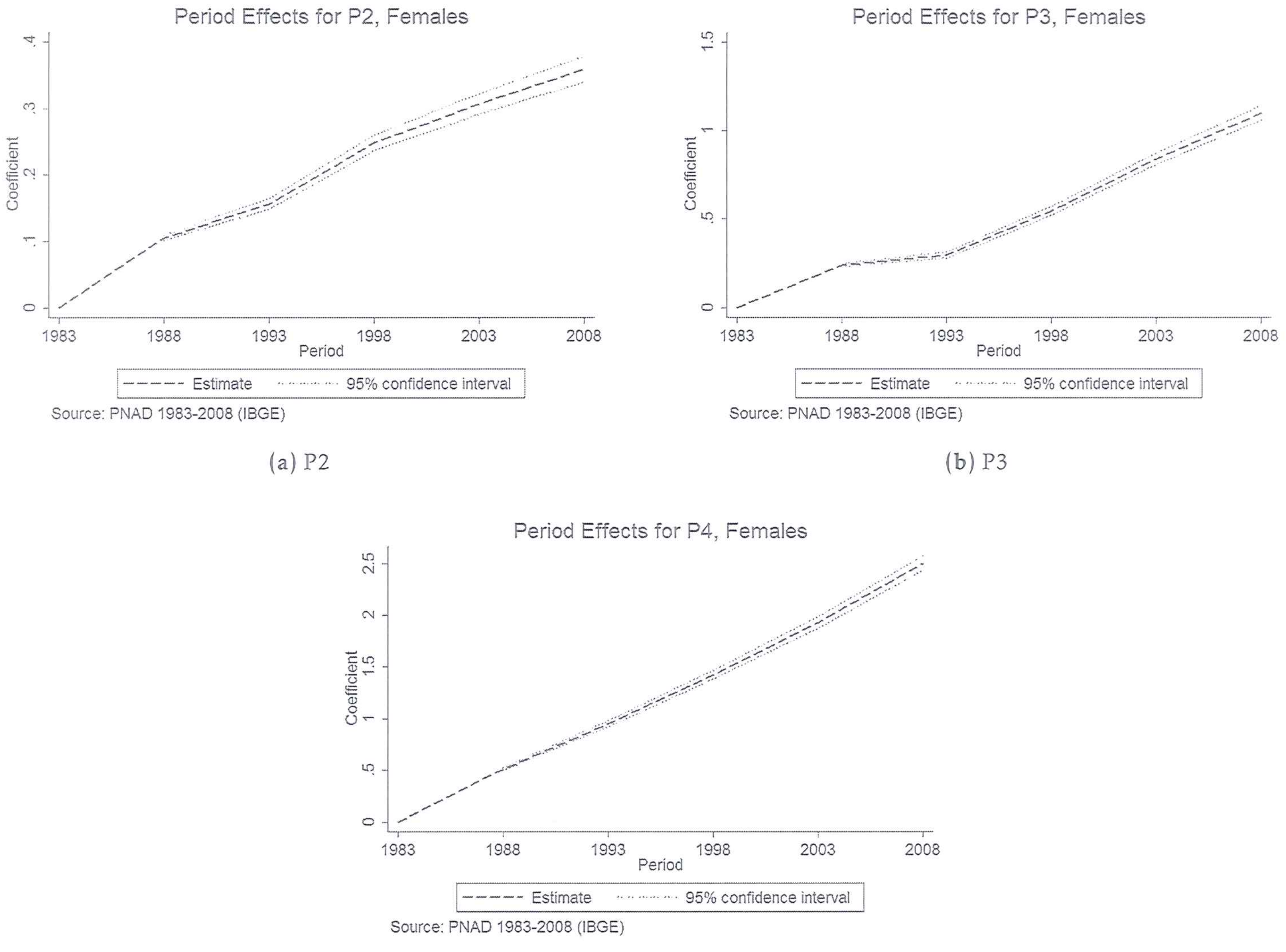


(b) P3



(c) P4

Figure 5: Age effects (Reference category is P1) for the classical APC model by education category. Females

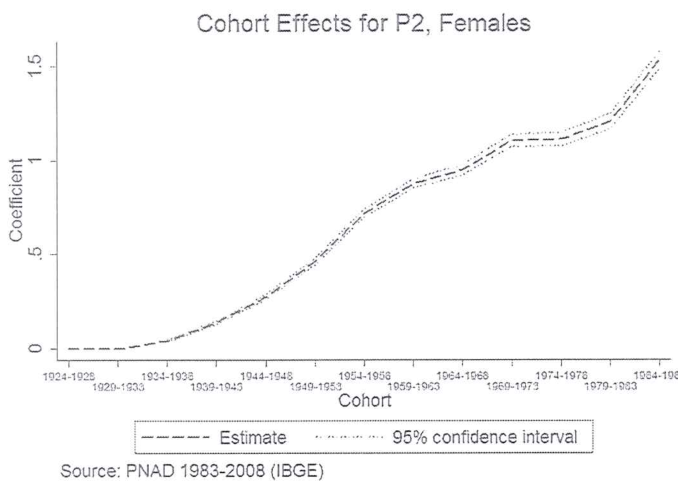


(a) P2

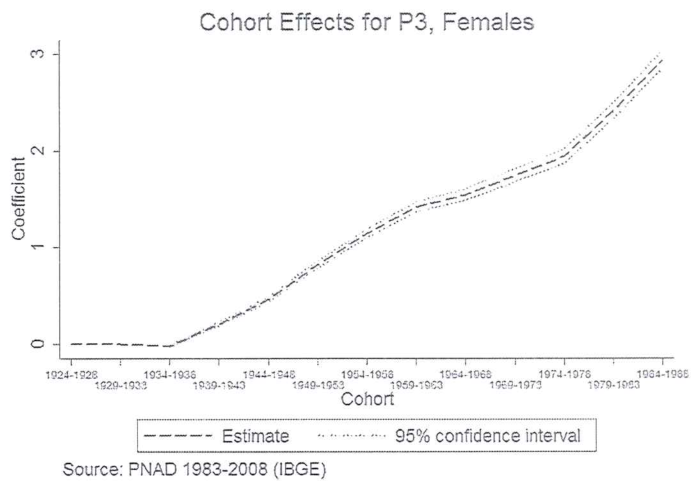
(b) P3

(c) P4

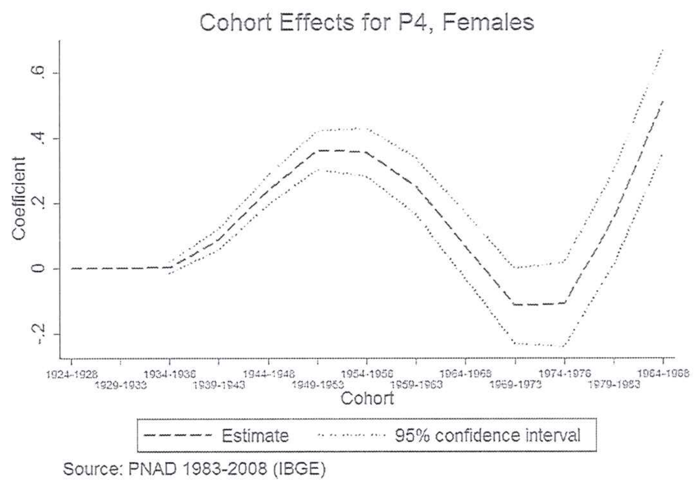
Figure 6: Period effects (Reference category is P1) for the classical APC model by education category. Females



(a) P2

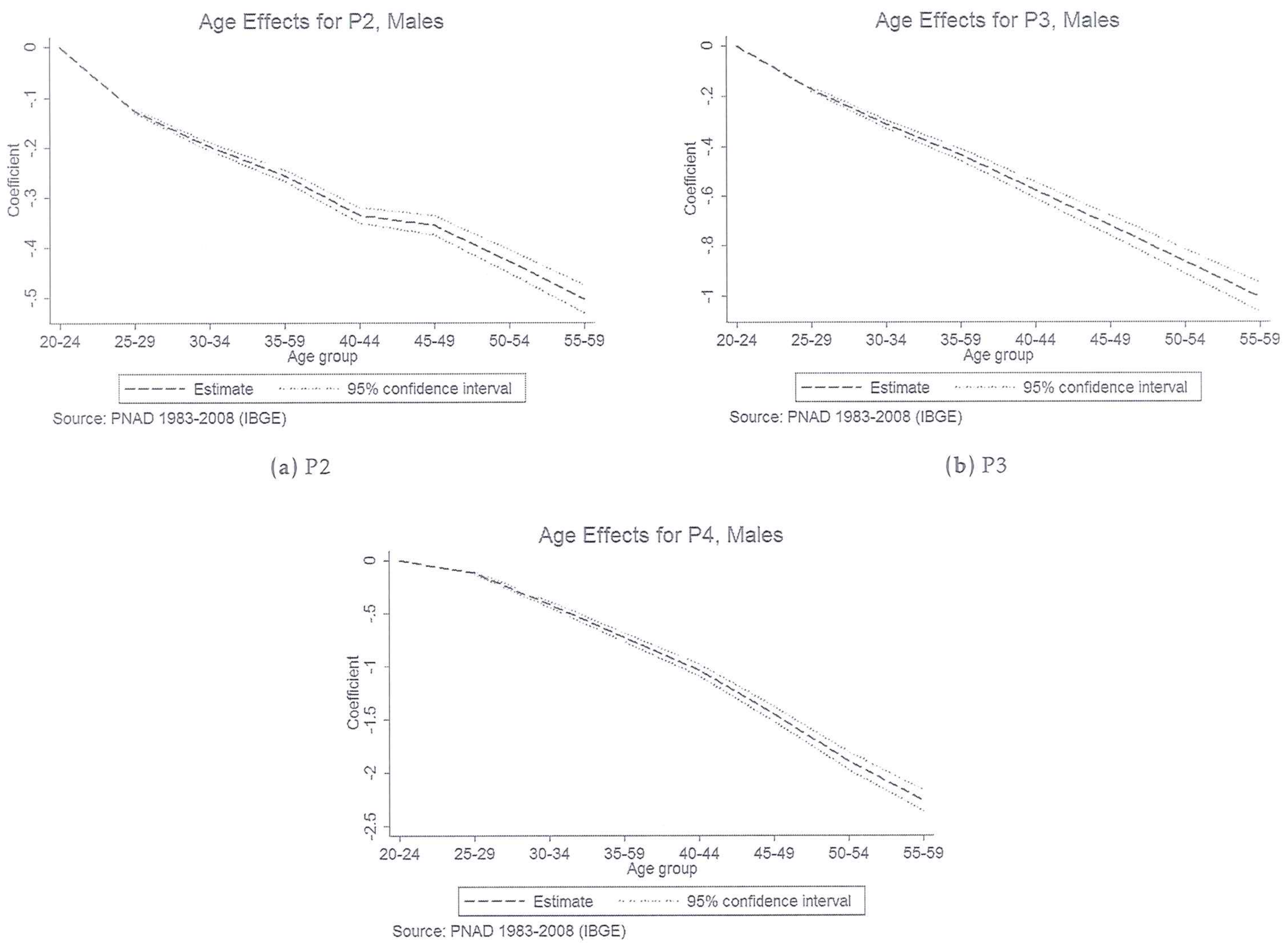


(b) P3



(c) P4

Figure 7: Cohort effects (Reference category is P1) for the classical APC model by education category. Females

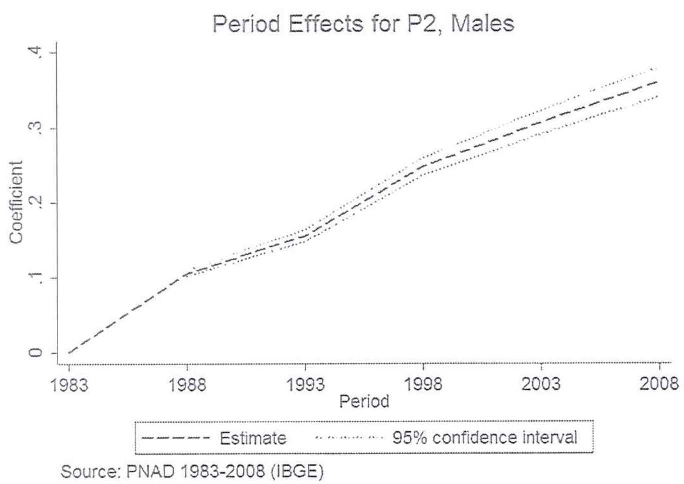


(a) P2

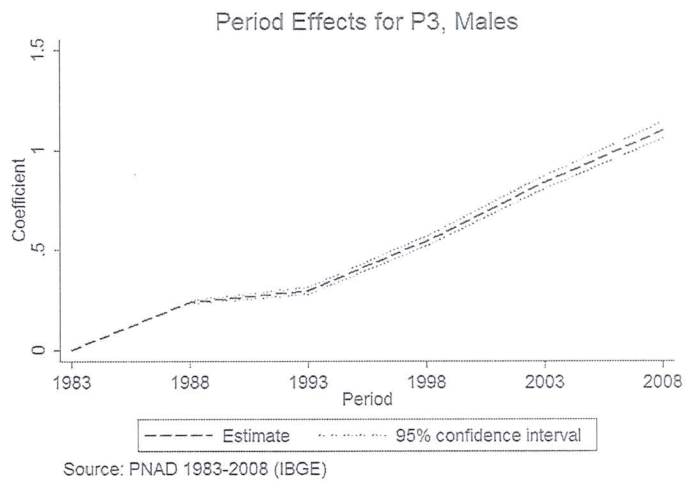
(b) P3

(c) P4

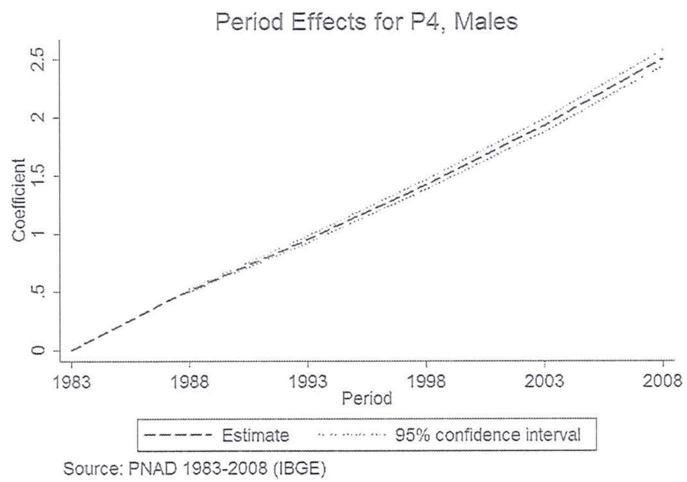
Figure 8: Age effects (Reference category is P1) for the classical APC model by education category. Males



(a) P2

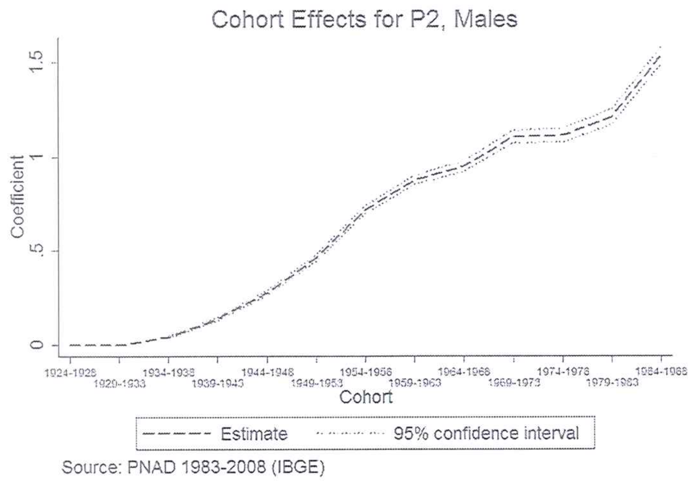


(b) P3

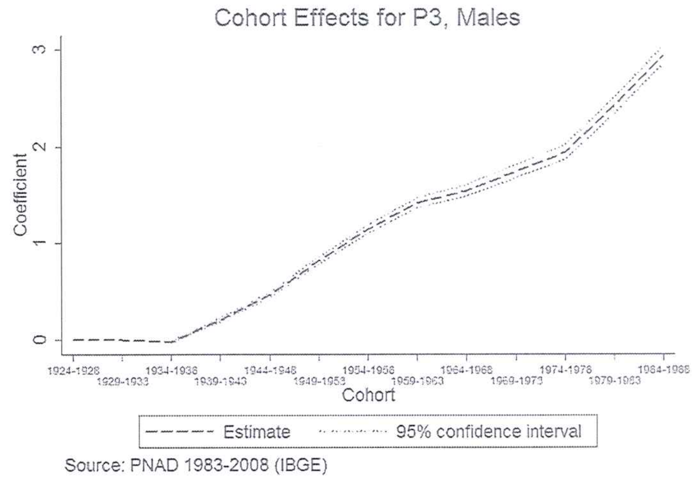


(c) P4

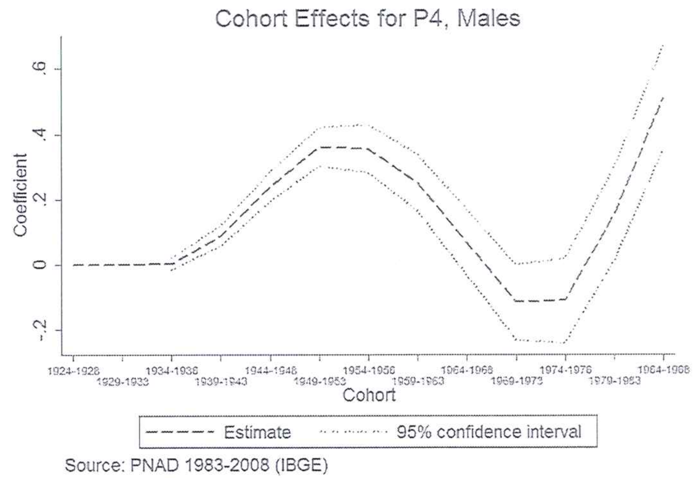
Figure 9: Period effects (Reference category is P1) for the classical APC model by education category. Males



(a) P2

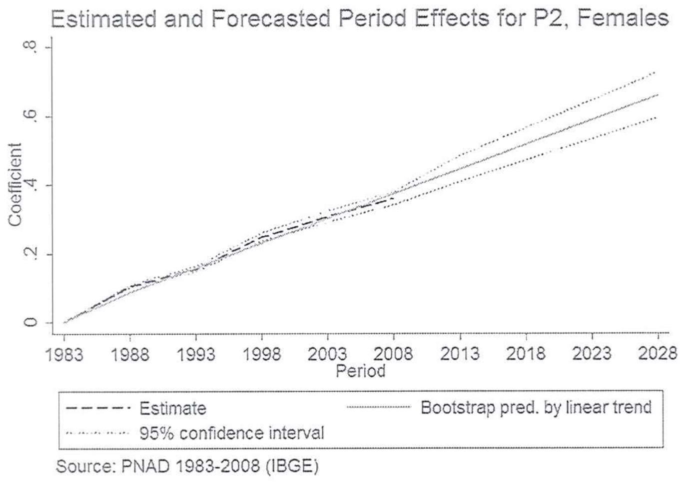


(b) P3

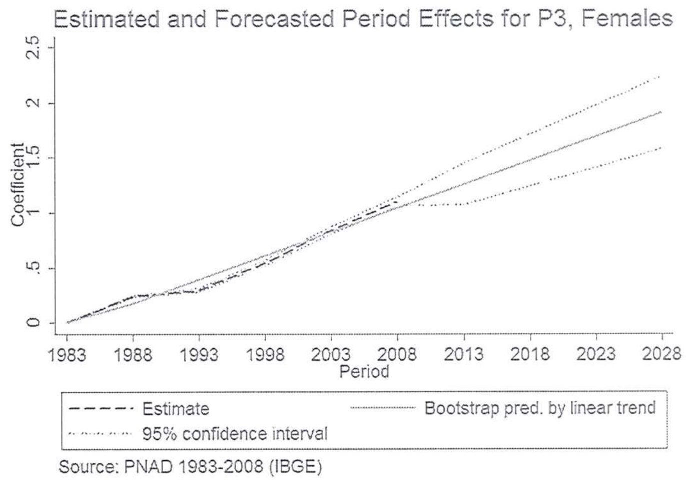


(c) P4

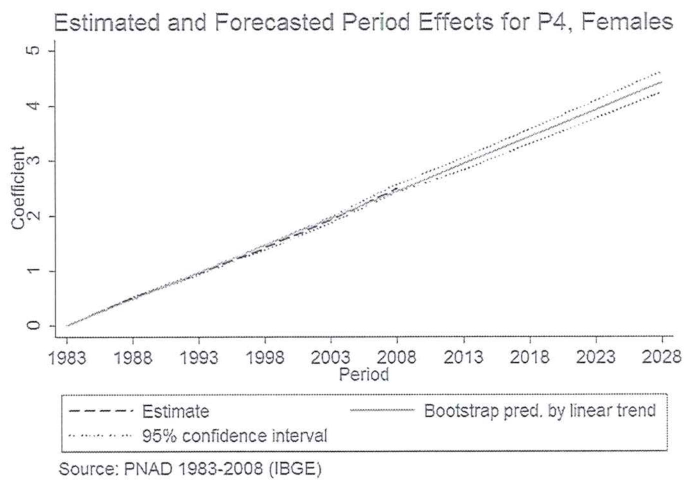
Figure 10: Cohort effects (Reference category is P1) for the classical APC model by education category. Males



(a) P2



(b) P3



(c) P4

Figure 11: Linear trend extrapolation of period effects (Reference category is P1) by education category. Females

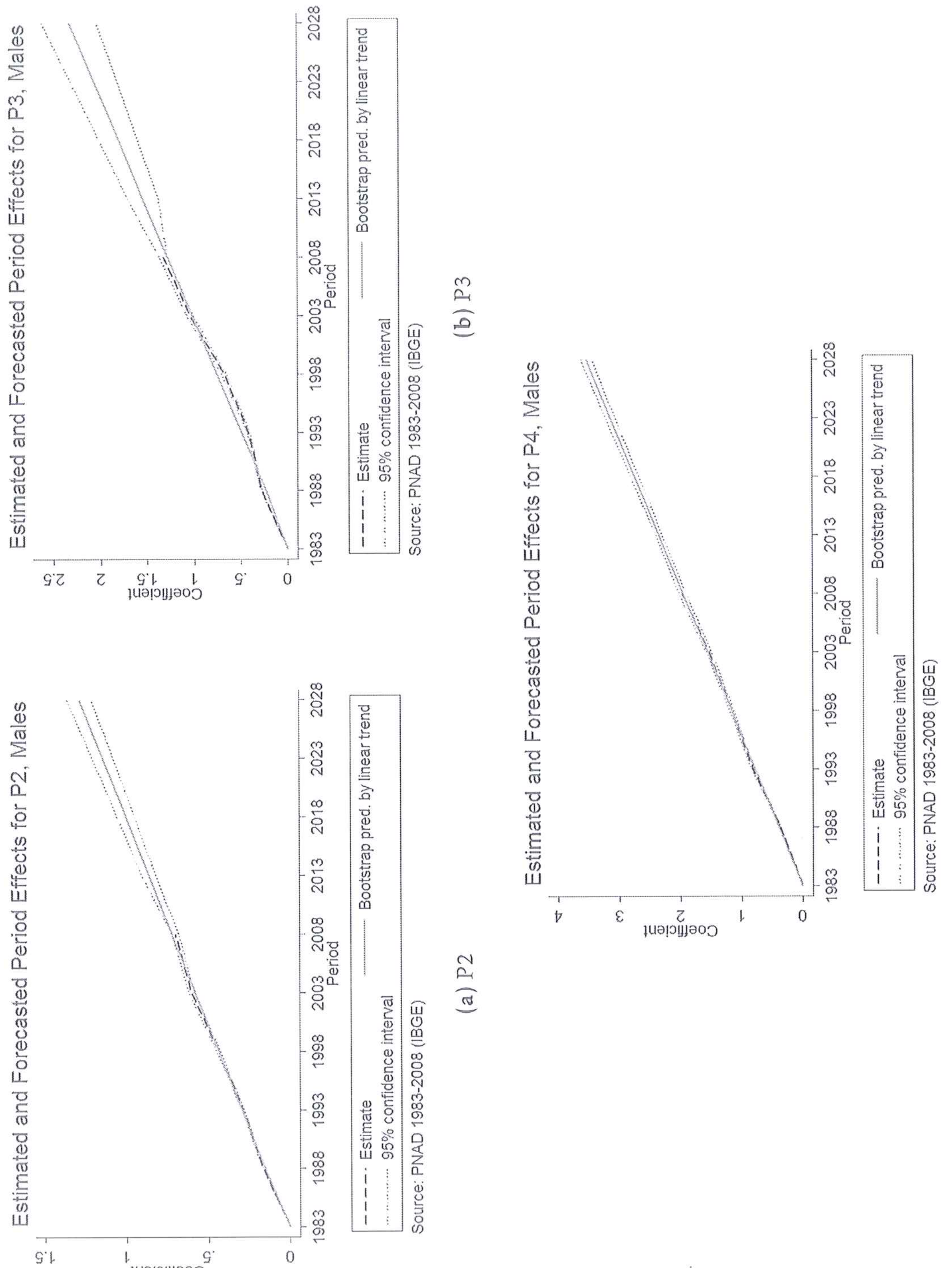
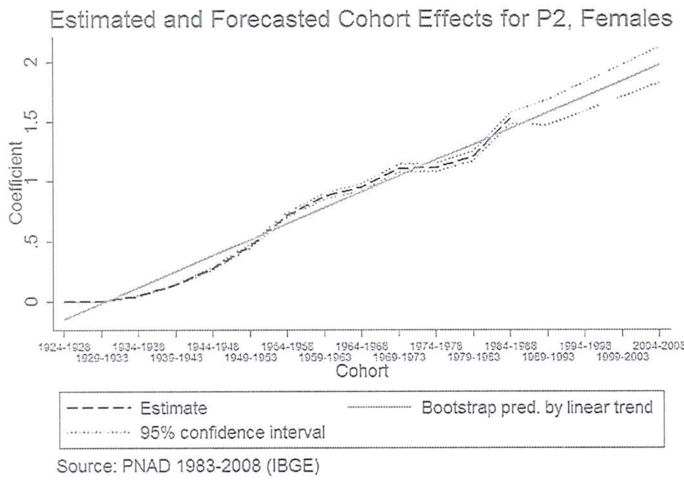
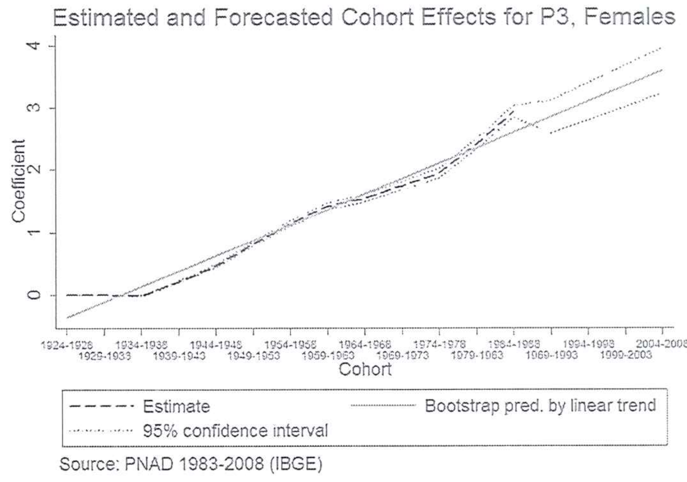


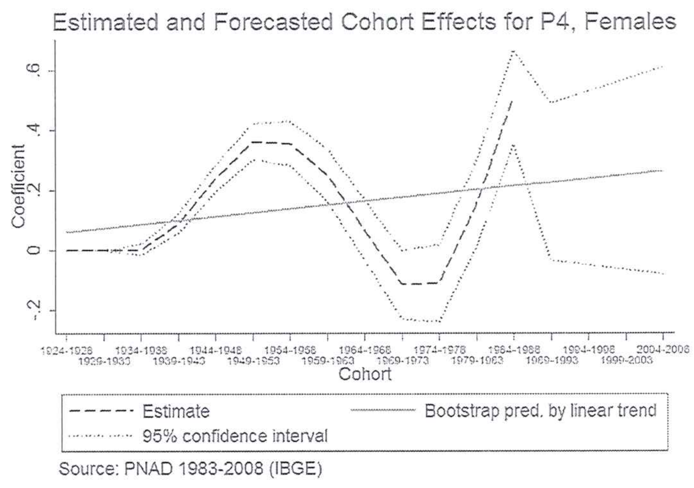
Figure 12: Linear trend extrapolation of period effects (Reference category is P1) by education category. Males



(a) P2

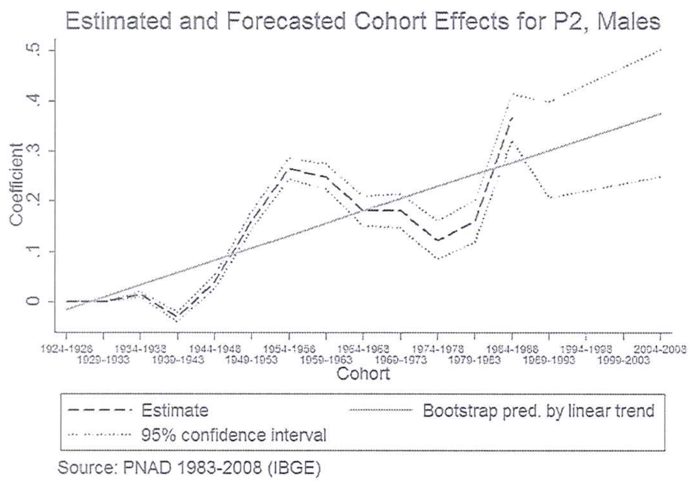


(b) P3

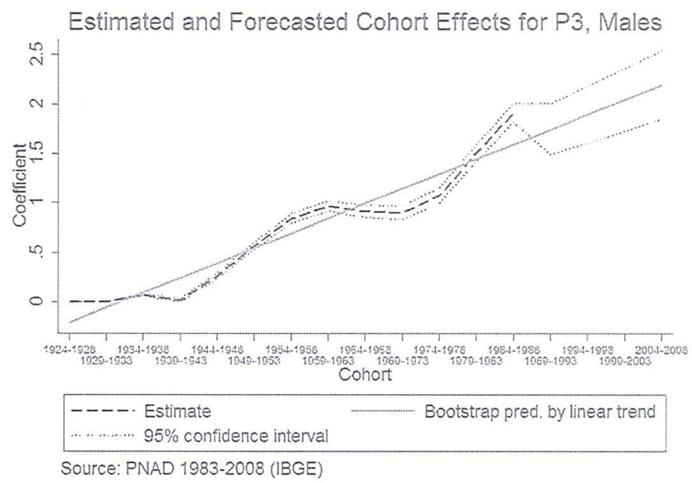


(c) P4

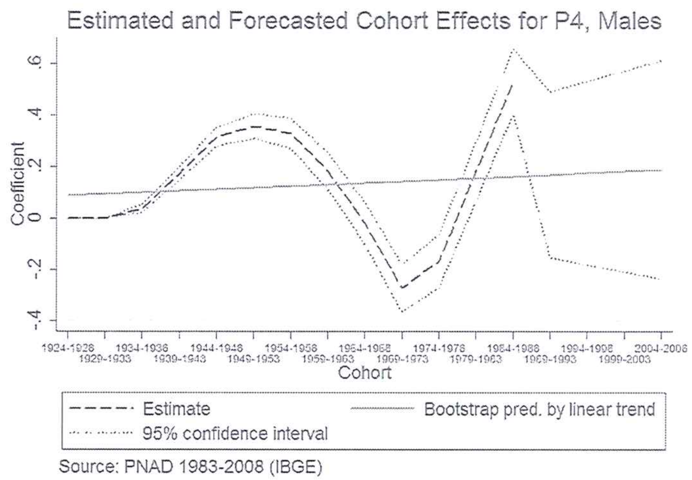
Figure 13: Linear trend extrapolation of cohort effects (Reference category is P1) by education category. Females



(a) P2



(b) P3



(c) P4

Figure 14: Linear trend extrapolation of cohort effects (Reference category is P1) by education category. Males

4.1.2.2 Results when P4 is chosen as reference category

Tables 12 and 13 provide the estimated age (θ), period (ϕ) and cohort (ψ) parameters and their respective standard errors when P4 is used as normalization rule.

Figures 15-20 display estimated age, period and cohort effects for P1, P2 and P3 and for males and females. APC effects for P4 are assumed to be zero as a result of the normalization rule. Hence, figures display the APC coefficients when the tertiary education category (listed as P4) is the reference category.

The age effects exhibit a similar pattern observed when P1 is chosen as reference category: age effects are larger for younger and smaller for older age groups. However, period effects for both sexes have a decreasing slope as a result of a different normalization rule. Cohort effects for P3 also reveal a similar pattern for males and females with an increasing trend. Cohort effects for P2 (males and females) are close to zero for individuals born until 1953, then reveal an increasing slope for cohorts born until 1973, a distinct peak for or birth cohorts born around 1974 and 1979, and a slightly decreasing slope for people born after 1979. Cohort effects for P1 (males and females) do not clearly exhibit a linear trend. For both sexes there is a distinct valley in cohort effects for P1 for cohorts born around 1944 and 1958, a sharp increasing slope for people born between 1959 and 1968, a distinct peak for people born between 1969 and 1973, and then a decreasing slope for cohorts born after 1979. Note that, for both sexes, the uncertainty of the estimates increases by a large amount only for cohort effects for P1.

Figures 21-24 display extrapolated period and cohort effects for P1, P2 and P3 and for males and females using a linear trend model. 95% confidence intervals are provided by a bootstrap simulation for future period and cohort coefficients. Period effects (males and females) for all education categories and cohort effects for P2 and P3 for females exhibit a downward trend, and hence a long-term decrease in the parameters is expected. For cohort effects for P2 and P3 (males and females), an upward trend is expected. For cohort effects for P1, both sexes, the trend is not constant, and forecasted coefficients are likely to reflect long-term conditions of the parameters.

Table 12: Estimated (Standard error) for the female group using the classical approach for the APC multinomial model, Reference Category is P4, Full sample, Brazil

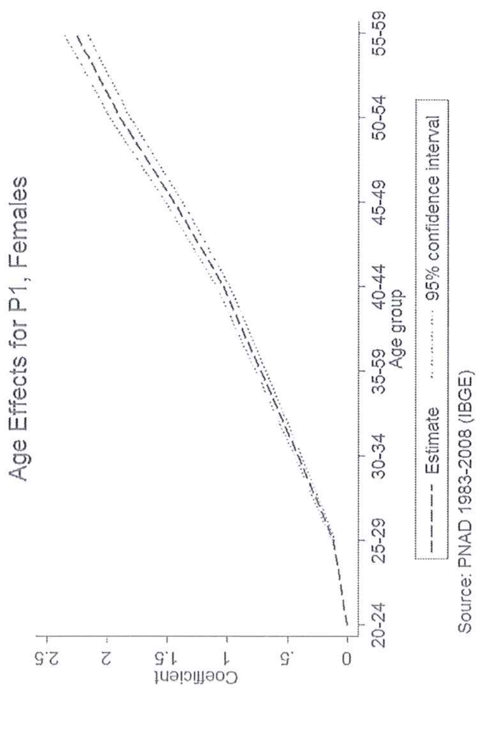
	P1	P2	P3
θ_2	0.1176	-0.0085	-0.0527
	(0.0072)	(0.0073)	(0.0081)
θ_3	0.4120	0.2146	0.1017
	(0.0143)	(0.0145)	(0.0162)
θ_4	0.7254	0.4695	0.2920
	(0.0214)	(0.0217)	(0.0243)
θ_5	1.0351	0.6991	0.4569
	(0.0285)	(0.0289)	(0.0324)
θ_6	1.4488	1.0925	0.7289
	(0.0356)	(0.0361)	(0.0405)
θ_7	1.8976	1.4686	1.0325
	(0.0428)	(0.0434)	(0.0486)
θ_8	2.2697	1.7665	1.2611
	(0.0498)	(0.0505)	(0.0565)
ϕ_2	-0.5068	-0.4016	-0.2675
	(0.0071)	(0.0072)	(0.0081)
ϕ_3	-0.9500	-0.7944	-0.6533
	(0.0142)	(0.0144)	(0.0162)
ϕ_4	-1.4200	-1.1718	-0.8754
	(0.0213)	(0.0216)	(0.0242)
ϕ_5	-1.9287	-1.6217	-1.0877
	(0.0285)	(0.0288)	(0.0323)
ϕ_6	-2.5020	-2.1421	-1.4017
	(0.0356)	(0.0361)	(0.0404)
ψ_3	-0.0020	0.0396	-0.0193
	(0.0094)	(0.0095)	(0.0108)
ψ_4	-0.0891	0.0491	0.1229
	(0.0161)	(0.0164)	(0.0184)
ψ_5	-0.2390	0.0332	0.2228
	(0.0231)	(0.0234)	(0.0263)
ψ_6	-0.3614	0.0988	0.4559
	(0.0302)	(0.0306)	(0.0344)
ψ_7	-0.3552	0.3615	0.7890
	(0.0373)	(0.0378)	(0.0424)
ψ_8	-0.2511	0.6235	1.1651
	(0.0444)	(0.0450)	(0.0505)
ψ_9	-0.0673	0.8801	1.4757
	(0.0515)	(0.0522)	(0.0585)
ψ_{10}	0.1158	1.2203	1.8564
	(0.0586)	(0.0594)	(0.0666)
ψ_{11}	0.1113	1.2223	2.0517
	(0.0657)	(0.0666)	(0.0747)
ψ_{12}	-0.1527	1.0544	2.2615
	(0.0728)	(0.0738)	(0.0828)
ψ_{13}	-0.5099	1.0185	2.4273
	(0.0800)	(0.0810)	(0.0909)
μ	1.5876	1.2747	-0.0611
	(0.0444)	(0.0450)	(0.0505)

Source: Calculations based on PNAD data
Obs.: Standard errors between parentesis

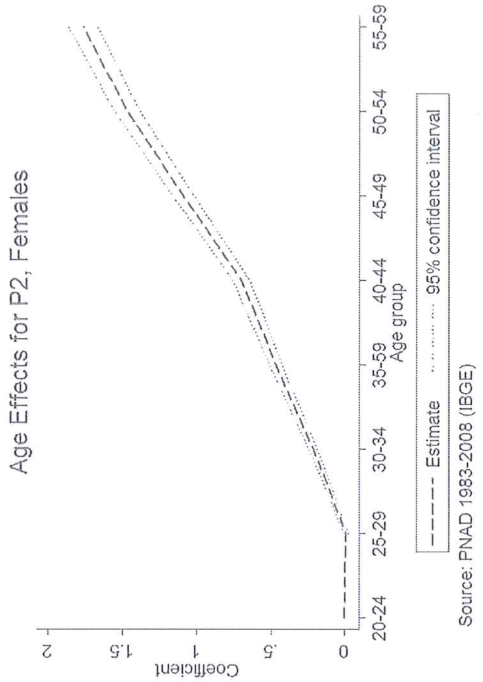
Table 13: Estimated (Standard error) for the male group using the classical approach for the APC multinomial model, Reference Category is P4, Full sample, Brazil

	P1	P2	P3
θ_2	-0.1311	-0.2573	-0.2505
	(0.0058)	(0.0059)	(0.0070)
θ_3	0.0617	-0.2744	-0.2678
	(0.0115)	(0.0118)	(0.0139)
θ_4	0.3044	-0.1551	-0.1854
	(0.0173)	(0.0176)	(0.0208)
θ_5	0.5195	-0.1136	-0.1699
	(0.0230)	(0.0235)	(0.0277)
θ_6	0.7717	0.0446	-0.0665
	(0.0287)	(0.0293)	(0.0347)
θ_7	1.0324	0.1791	0.0457
	(0.0345)	(0.0352)	(0.0417)
θ_8	1.3445	0.3251	0.1532
	(0.0401)	(0.0409)	(0.0484)
ϕ_2	-0.3623	-0.1967	-0.0830
	(0.0057)	(0.0059)	(0.0069)
ϕ_3	-0.8144	-0.5256	-0.3976
	(0.0115)	(0.0117)	(0.0138)
ϕ_4	-1.1663	-0.7287	-0.4861
	(0.0172)	(0.0175)	(0.0207)
ϕ_5	-1.5379	-0.9358	-0.4948
	(0.0229)	(0.0234)	(0.0277)
ϕ_6	-1.9636	-1.2641	-0.6404
	(0.0286)	(0.0292)	(0.0346)
ψ_3	-0.0373	-0.0221	0.0349
	(0.0078)	(0.0079)	(0.0094)
ψ_4	-0.1733	-0.2030	-0.1594
	(0.0132)	(0.0134)	(0.0159)
ψ_5	-0.3144	-0.2746	-0.0557
	(0.0188)	(0.0192)	(0.0227)
ψ_6	-0.3567	-0.1950	0.2093
	(0.0245)	(0.0250)	(0.0296)
ψ_7	-0.3282	-0.0632	0.5075
	(0.0302)	(0.0308)	(0.0364)
ψ_8	-0.1827	0.0657	0.7777
	(0.0359)	(0.0366)	(0.0434)
ψ_9	0.0187	0.1988	0.9264
	(0.0416)	(0.0425)	(0.0503)
ψ_{10}	0.2730	0.4534	1.1690
	(0.0474)	(0.0483)	(0.0572)
ψ_{11}	0.1674	0.2903	1.2371
	(0.0531)	(0.0542)	(0.0641)
ψ_{12}	-0.1817	-0.0215	1.3245
	(0.0588)	(0.0601)	(0.0710)
ψ_{13}	-0.5308	-0.1631	1.3817
	(0.0646)	(0.0659)	(0.0780)
μ	1.9309	2.1695	0.5255
	(0.0360)	(0.0367)	(0.0434)

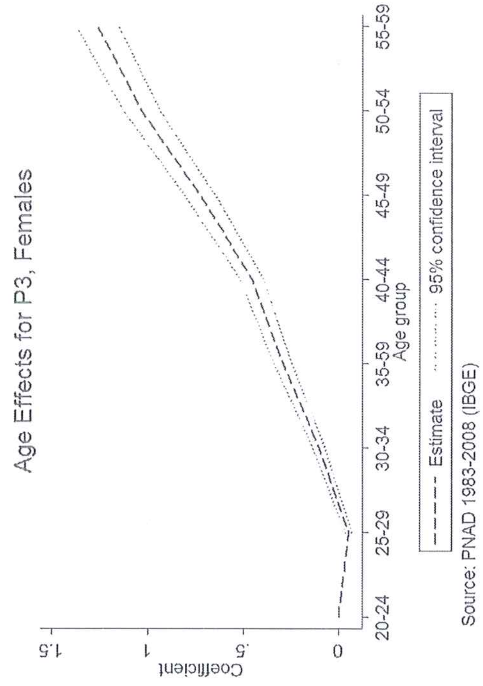
Source: Calculations based on PNAD data
Obs.: Standard errors between parentesis



(a) P1

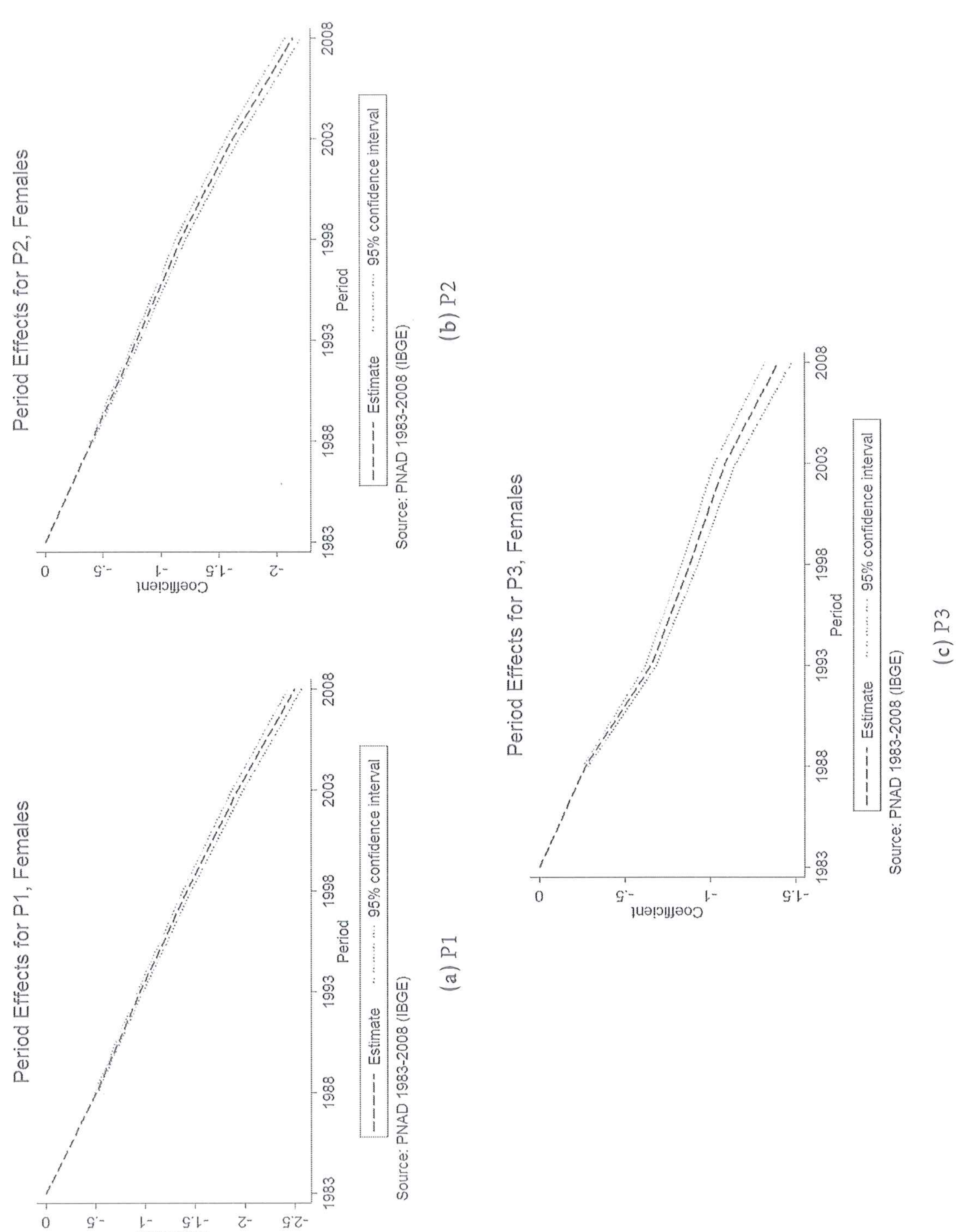


(b) P2



(c) P3

Figure 15: Age effects (Reference category is P4) for the classical APC model by education category. Females

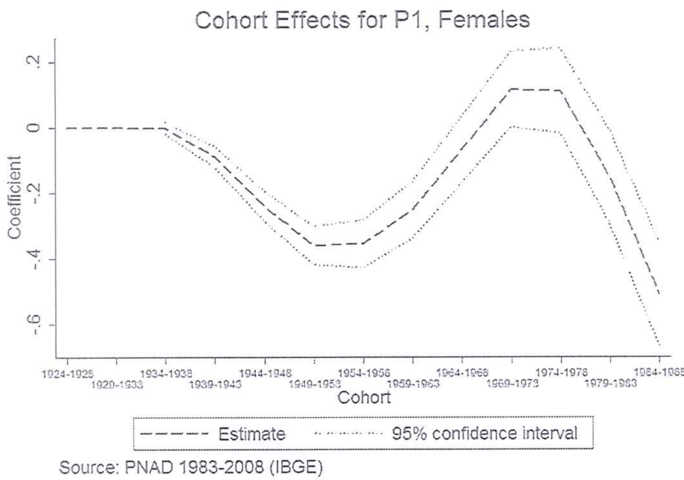


(a) P1

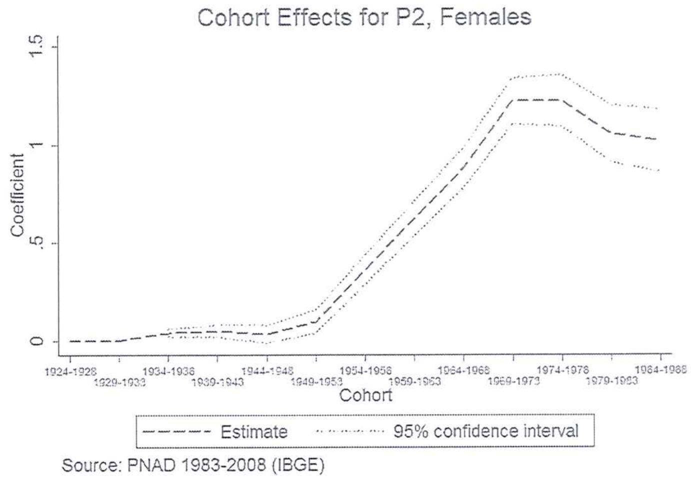
(b) P2

(c) P3

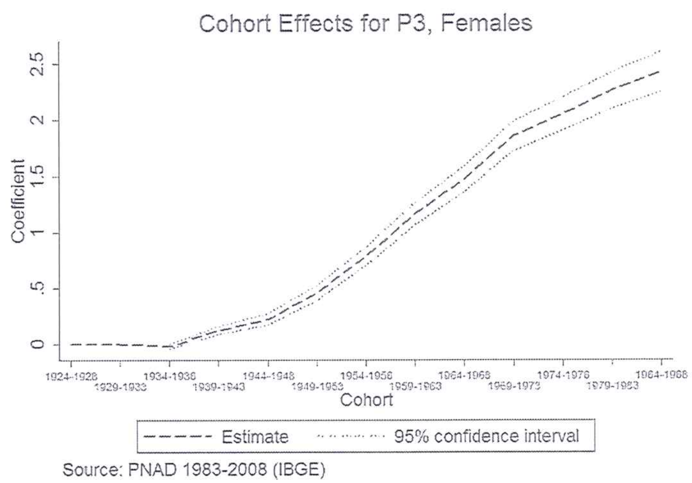
Figure 16: Period effects (Reference category is P4) for the classical APC model by education category. Females



(a) P1

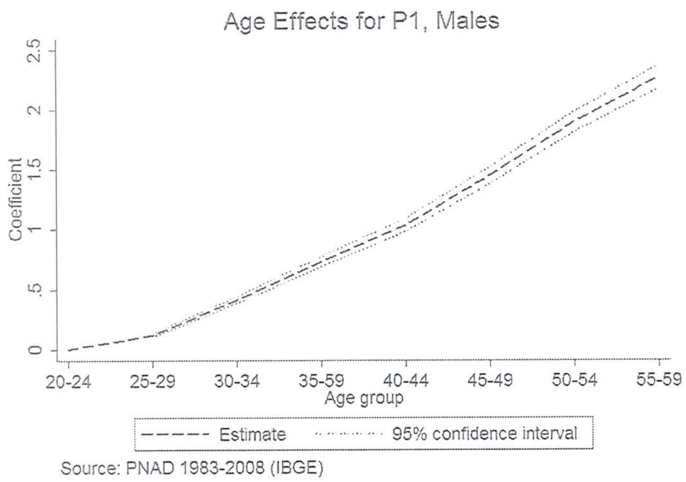


(b) P2

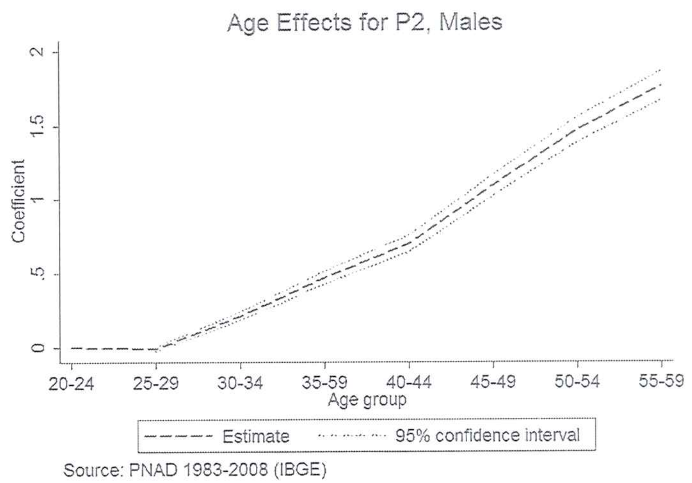


(c) P3

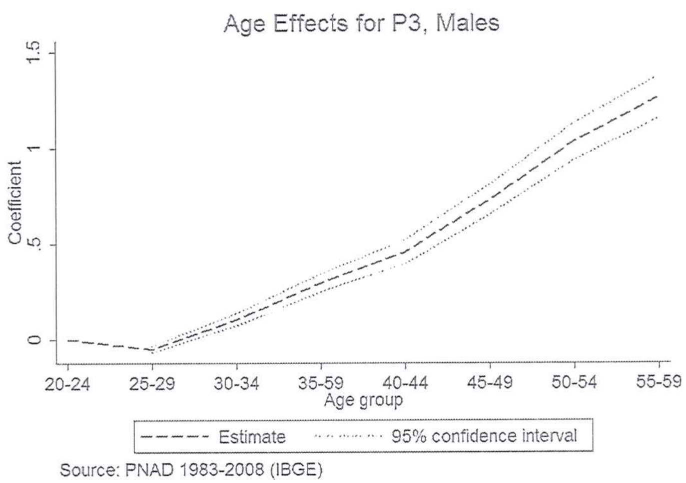
Figure 17: Cohort effects (Reference category is P4) for the classical APC model by education category. Females



(a) P1

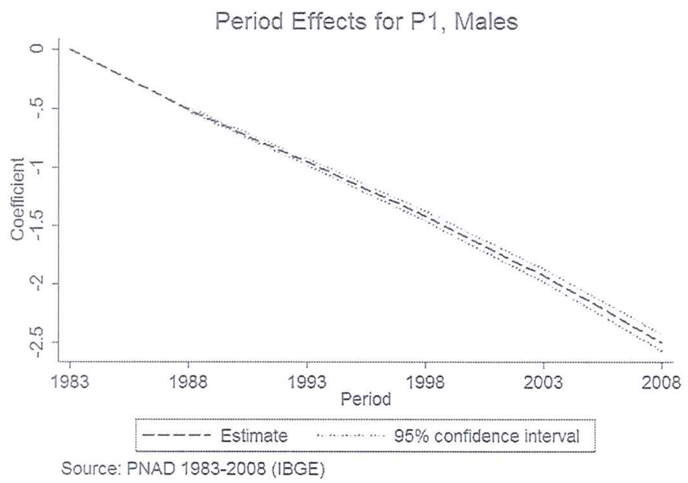


(b) P2

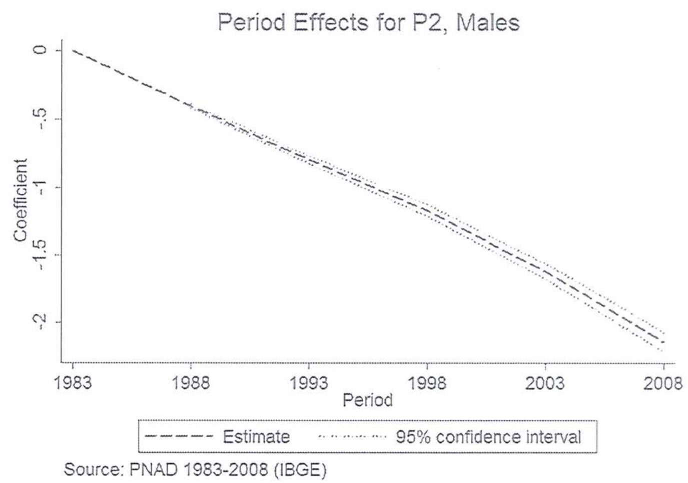


(c) P3

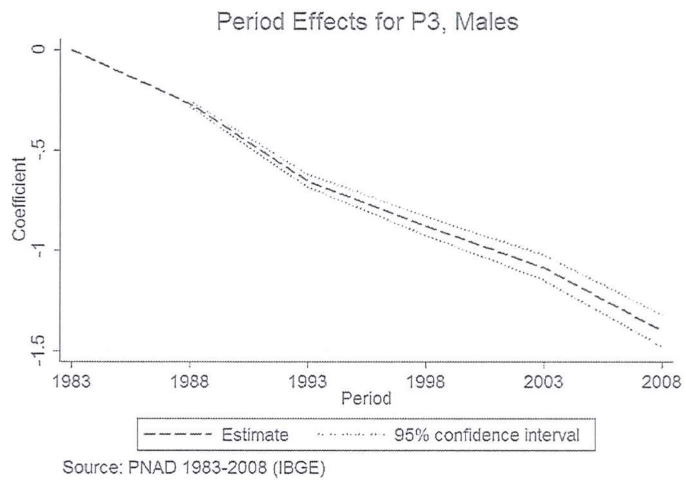
Figure 18: Age effects (Reference category is P4) for the classical APC model by education category. Males



(a) P1

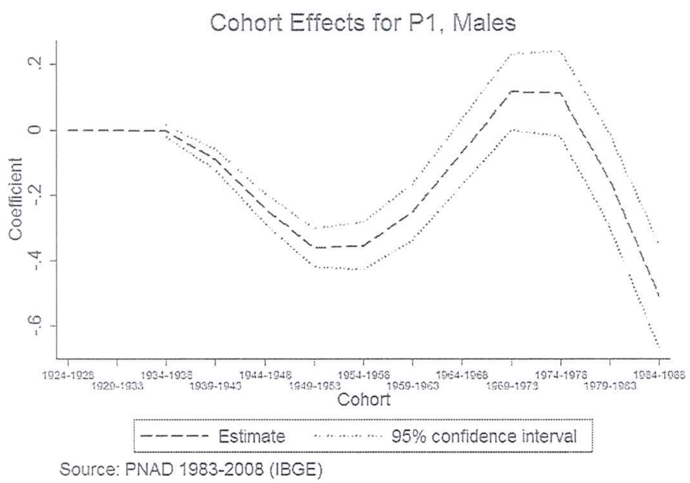


(b) P2

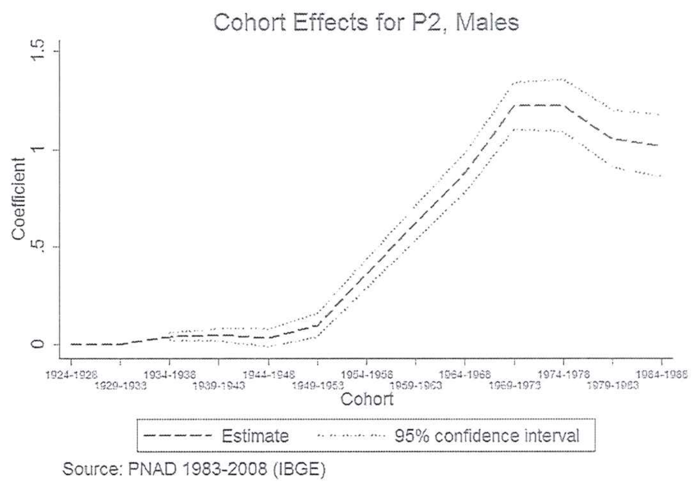


(c) P3

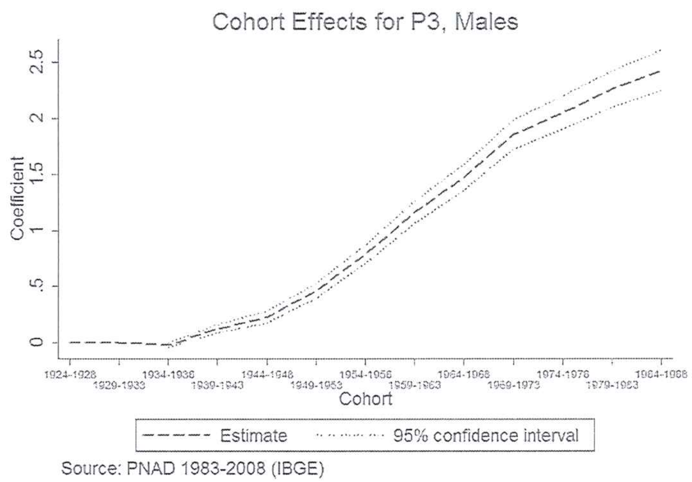
Figure 19: Period effects (Reference category is P4) for the classical APC model by education category. Males



(a) P1

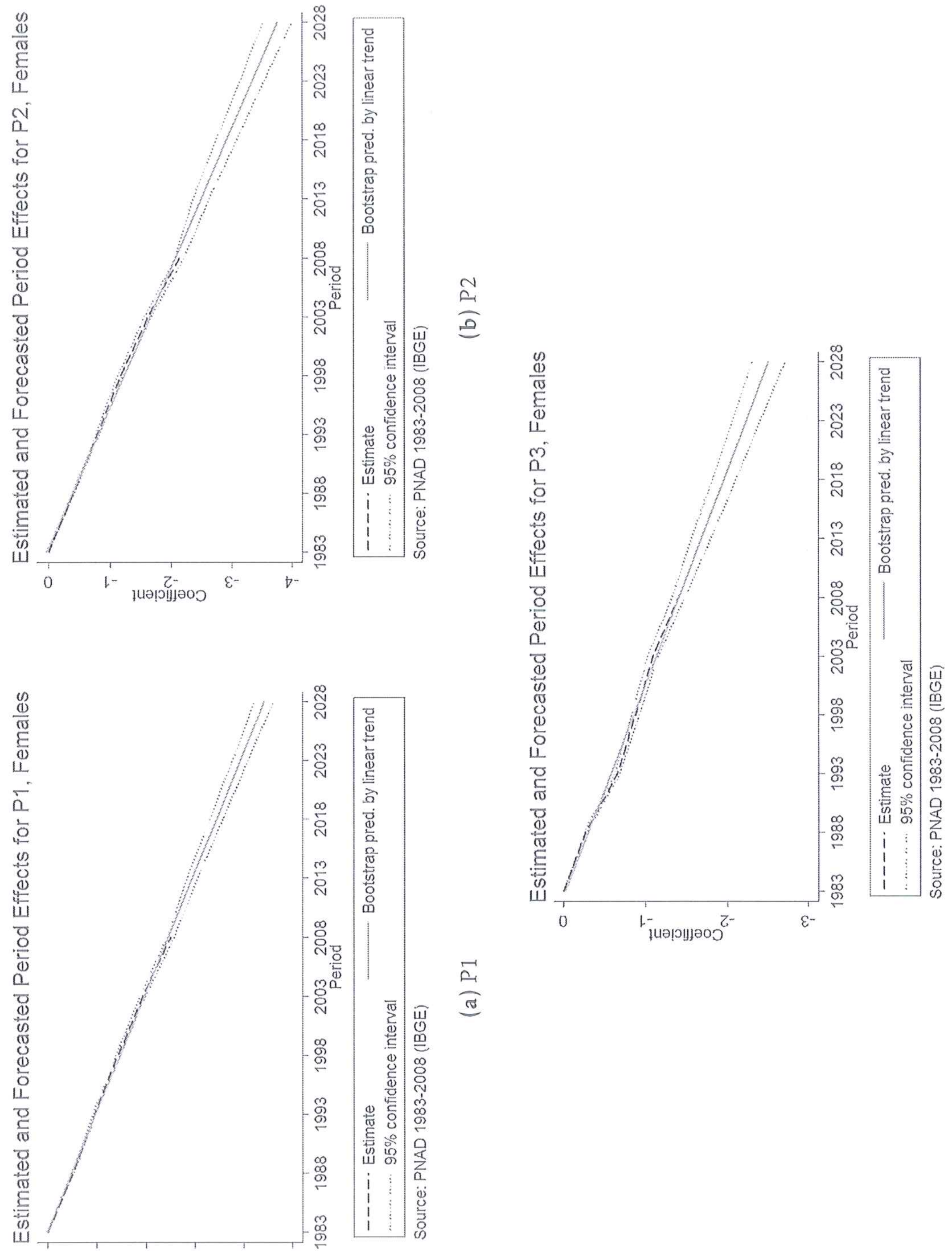


(b) P2



(c) P3

Figure 20: Cohort effects (Reference category is P4) for the classical APC model by education category. Males

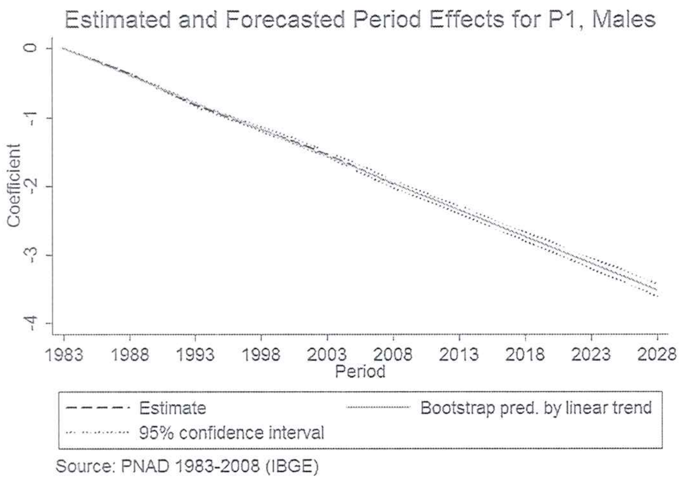


(a) P1

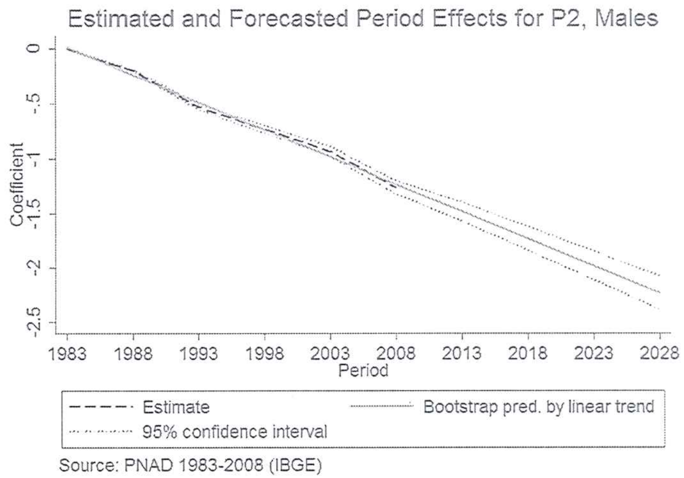
(b) P2

(c) P3

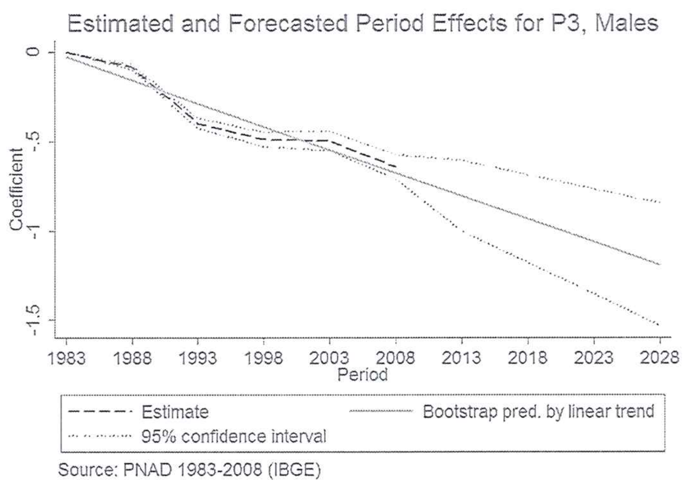
Figure 21: Linear trend extrapolation of period effects (Reference category is P4) by education category. Females



(a) P1

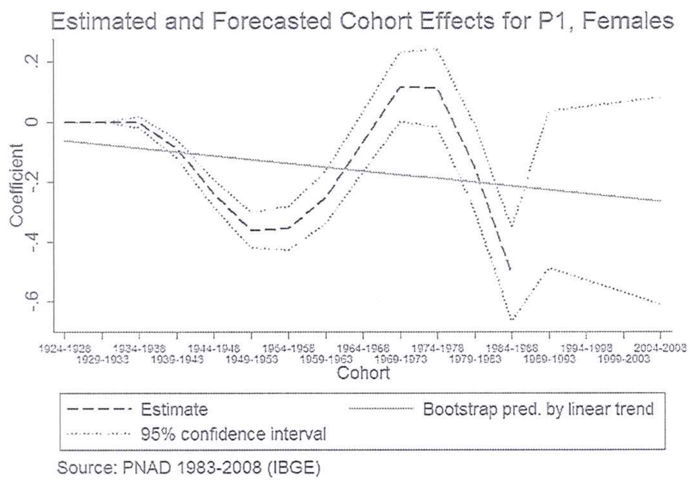


(b) P2

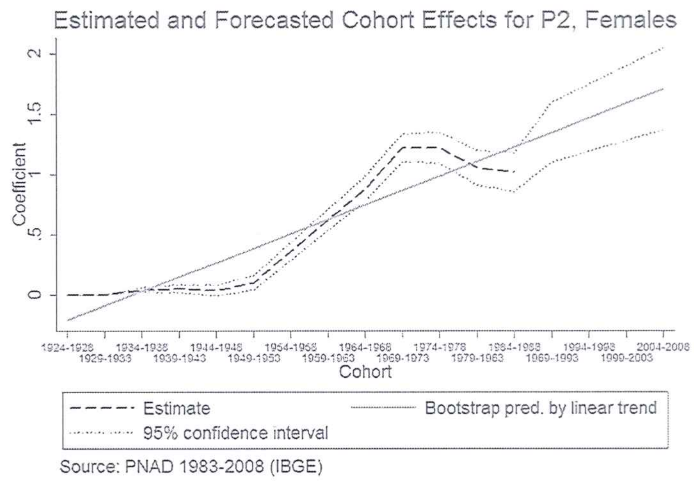


(c) P3

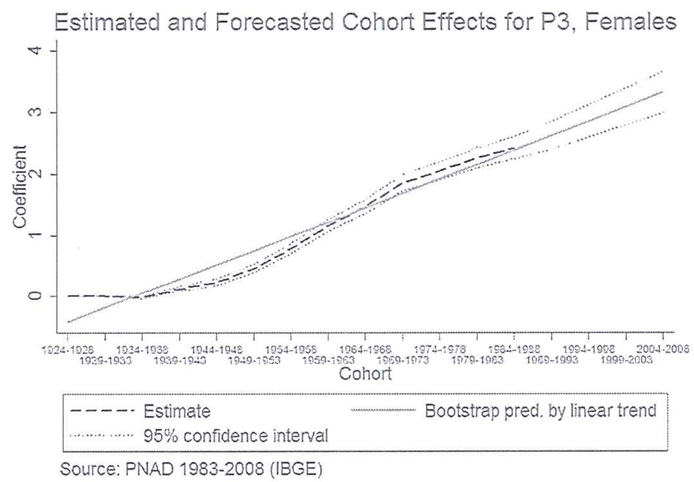
Figure 22: Linear trend extrapolation of period effects (Reference category is P4) by education category. Males



(a) P1

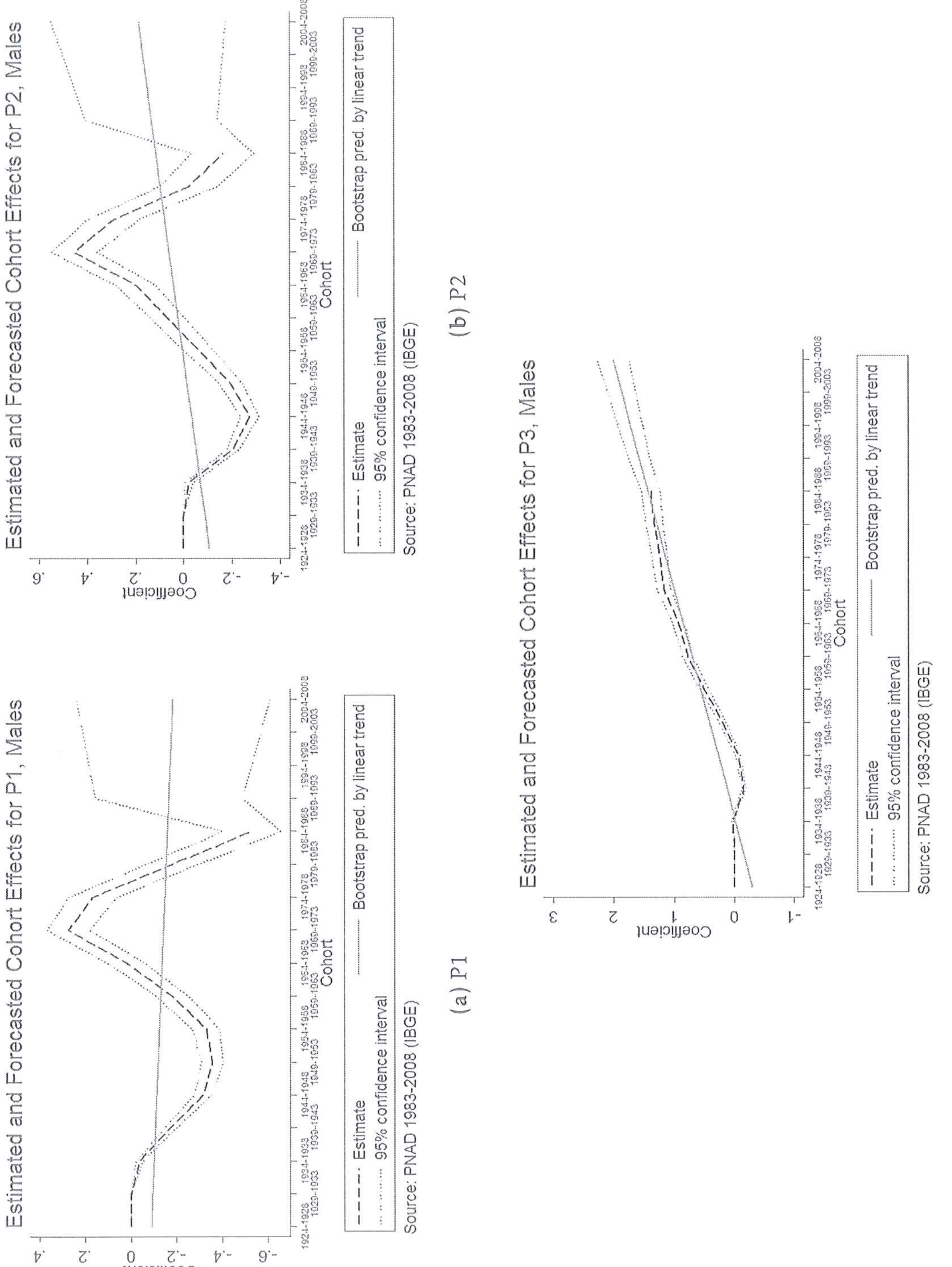


(b) P2



(c) P3

Figure 23: Linear trend extrapolation of cohort effects (Reference category is P4) by education category. Females



(a) P1

(b) P2

(c) P3

Figure 24: Linear trend extrapolation of cohort effects (Reference category is P4) by education category. Males

4.1.3 Comparison of the Classical Prediction Intervals under Different Normalization Assumptions

Now I compare the prediction intervals for period and cohort effects when using different reference categories. As I argued before, in the scenario approach I propose here, prediction confidence intervals for the APC coefficients and their extrapolated values using the bootstrap method may not be the same depending on the reference category chosen. I illustrate this fact in Table 14, which arrays period effects for the probability of falling into the category 9-11 years of schooling (P3) for females using P1 and P4 as reference categories.

As explained previously, parameter estimates in Table 14 are not readily comparable and interpretable in a multinomial setting. As I extrapolate linear trends based on actual values of parameters and derive confidence intervals for future parameter values using a bootstrap procedure, uncertainty levels under different normalization rules may not be the same.

Table 15 illustrates this conclusion for P3 period effects when using P1 and P4 as reference categories. I provide in this table future parameter estimates and the length of their 95% bootstrapped confidence intervals. It is clear that intervals for P3 future period effects are wider when P1 is used as reference category than when P4 is used. Therefore, scenarios built under different normalization rules may not be the same, although point estimates for estimated probabilities in the future are.

Table 14: Period effects for P3 and their standard errors according to reference category. Brazil, Females

Parameter	Reference Category	
	P1	P4
ϕ_2	0.2393	-0.2675
	(0.0042)	(0.0081)
ϕ_3	0.2967	-0.6533
	(0.0083)	(0.0162)
ϕ_4	0.5446	-0.8754
	(0.0124)	(0.0242)
ϕ_5	0.8410	-1.0877
	(0.0166)	(0.0323)
ϕ_6	1.1004	-1.4017
	(0.0208)	(0.0404)

Source: Calculations based on PNAD data.
Obs.: Standard errors between parenthesis.

4.1.4 Education Profile: Fit and Empirical Projections

I now present results for the fitted education profile (1983-2008) and empirical forecasts (2013-2028) for Brazilian males and females using the classical APC model

Table 15: Forecasted period effects for P3, their bootstrapped confidence intervals and range of intervals according to reference category. Brazil, Females

	P1 as reference category				P4 as reference category			
	ϕ	Lower 95% CI	Upper 95% CI	Range	ϕ	Lower 95% CI	Upper 95% CI	Range
2013	1.259	1.071	1.448	0.377	-1.683	-1.801	-1.566	0.236
2018	1.475	1.240	1.710	0.469	-1.960	-2.106	-1.815	0.291
2023	1.691	1.409	1.973	0.564	-2.237	-2.411	-2.063	0.348
2028	1.907	1.577	2.236	0.659	-2.514	-2.717	-2.311	0.406

Source: Calculations based on PNAD data (1983-2008)

under two different normalization rules: when P1 or P4 is chosen as reference category. It is worth mentioning that there are no differences between the point estimates of the education profile using different normalization rules of the multinomial APC model (that is, choosing either category P1 or P4 as reference). Only low and Optimistic Scenarios may differ between specifications, as I argued in section 3.2.2.2.

4.1.4.1 Results when P1 is chosen as Reference Category

Results when P1 is chosen as reference category in the multinomial APC model are displayed in Tables 16-21. For the period 1983-2008, tables provide the observed profile, point estimates and their respective uncertainty (low and Optimistic Scenarios). Results suggest that observed education profile scatter about the fitted profile for 1983 to 1998 with no suggestion of any systematic departure in any of the age groups, confirming the goodness of fit of the model.

I also present here graphics for the average education profile and the optimistic and pessimistic scenarios across all age groups with the aim of summarizing the evidence presented in the tables. Figures 25-26 illustrate these results. For a given education category, graphs are provided at the same scale for males and females. I provide a substantive interpretation of these results later in section . As I cautioned in section 3.2.2.1, scenarios derived from this classical APC model have no probabilistic meaning. As a result, the reader should interpret these scenarios as possible trajectories of the lower/upper bound of the 95% confidence interval for both period and cohort effects combined.

Table 16: Estimated and observed education profile (1983-1993) using the classical APC model (Reference category is P1). Brazil, Females

		1983				1988				1993			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.2625	0.2787	0.2927	0.2810	0.2165	0.2204	0.2437	0.2243	0.1847	0.1688	0.1894	0.1740
	4-8	0.4494	0.4441	0.4449	0.4482	0.4543	0.4384	0.4508	0.4489	0.4803	0.4117	0.4291	0.4241
	9-11	0.2125	0.2110	0.2003	0.2112	0.2494	0.2556	0.2271	0.2546	0.2539	0.3108	0.2812	0.3097
	12+	0.0756	0.0663	0.0621	0.0596	0.0798	0.0857	0.0785	0.0722	0.0811	0.1087	0.1003	0.0921
25-29	0-3	0.3248	0.3404	0.3569	0.3398	0.2490	0.2746	0.3003	0.2771	0.2208	0.2145	0.2372	0.2198
	4-8	0.4256	0.4188	0.4165	0.4225	0.4342	0.4217	0.4287	0.4311	0.4383	0.4039	0.4161	0.4153
	9-11	0.1635	0.1698	0.1600	0.1742	0.2187	0.2099	0.1856	0.2129	0.2321	0.2603	0.2355	0.2618
	12+	0.0861	0.0710	0.0666	0.0634	0.0981	0.0937	0.0854	0.0788	0.1088	0.1213	0.1112	0.1031
30-34	0-3	0.4097	0.4007	0.4205	0.3963	0.3000	0.3309	0.3591	0.3299	0.2550	0.2648	0.2893	0.2682
	4-8	0.3848	0.4020	0.3964	0.4064	0.4240	0.4143	0.4158	0.4226	0.4263	0.4066	0.4138	0.4159
	9-11	0.1285	0.1358	0.1256	0.1431	0.1719	0.1718	0.1498	0.1783	0.2032	0.2184	0.1964	0.2228
	12+	0.0770	0.0615	0.0574	0.0541	0.1040	0.0830	0.0753	0.0691	0.1156	0.1102	0.1004	0.0931
35-39	0-3	0.4870	0.4600	0.4840	0.4524	0.3933	0.3885	0.4201	0.3835	0.3093	0.3186	0.3457	0.3186
	4-8	0.3635	0.3812	0.3720	0.3872	0.3809	0.4018	0.3980	0.4104	0.4145	0.4041	0.4064	0.4122
	9-11	0.0936	0.1078	0.0970	0.1165	0.1384	0.1394	0.1188	0.1484	0.1611	0.1815	0.1612	0.1893
	12+	0.0560	0.0510	0.0470	0.0439	0.0875	0.0704	0.0632	0.0577	0.1150	0.0957	0.0868	0.0800
40-44	0-3	0.5399	0.5240	0.5519	0.5142	0.4706	0.4525	0.4880	0.4441	0.3875	0.3805	0.4113	0.3766
	4-8	0.3502	0.3510	0.3379	0.3586	0.3634	0.3783	0.3690	0.3883	0.3784	0.3901	0.3871	0.3984
	9-11	0.0703	0.0830	0.0723	0.0919	0.0984	0.1098	0.0906	0.1200	0.1300	0.1466	0.1272	0.1569
	12+	0.0396	0.0421	0.0379	0.0353	0.0675	0.0594	0.0524	0.0477	0.1041	0.0828	0.0744	0.0681
45-49	0-3	0.6028	0.5765	0.6076	0.5654	0.5365	0.5082	0.5467	0.4969	0.4664	0.4380	0.4723	0.4300
	4-8	0.3171	0.3314	0.3140	0.3401	0.3400	0.3646	0.3494	0.3757	0.3562	0.3854	0.3766	0.3945
	9-11	0.0553	0.0619	0.0519	0.0699	0.0775	0.0836	0.0665	0.0934	0.1004	0.1144	0.0962	0.1254
	12+	0.0248	0.0302	0.0264	0.0246	0.0460	0.0435	0.0374	0.0340	0.0770	0.0622	0.0549	0.0500
50-54	0-3	0.6252	0.6298	0.6368	0.6302	0.5867	0.5716	0.6124	0.5585	0.5432	0.5044	0.5423	0.4934
	4-8	0.3041	0.2999	0.2962	0.3004	0.3280	0.3340	0.3135	0.3457	0.3299	0.3614	0.3468	0.3715
	9-11	0.0520	0.0510	0.0491	0.0516	0.0568	0.0636	0.0484	0.0724	0.0761	0.0891	0.0721	0.0998
	12+	0.0187	0.0193	0.0179	0.0178	0.0285	0.0309	0.0256	0.0234	0.0507	0.0452	0.0388	0.0353
55-59	0-3	0.6521	0.6521	0.6598	0.6523	0.6239	0.6245	0.6406	0.6230	0.5877	0.5672	0.6077	0.5548
	4-8	0.2883	0.2883	0.2839	0.2888	0.2997	0.3018	0.2948	0.3051	0.3167	0.3304	0.3110	0.3415
	9-11	0.0457	0.0457	0.0437	0.0464	0.0541	0.0524	0.0458	0.0536	0.0575	0.0678	0.0526	0.0775
	12+	0.0138	0.0138	0.0126	0.0125	0.0223	0.0213	0.0188	0.0182	0.0381	0.0346	0.0287	0.0262

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 17: Estimated and observed education profile (1998-2008) using the classical APC model (Reference category is P1). Brazil, Females

		1998				2003				2008			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.1445	0.1253	0.1450	0.1307	0.0894	0.0902	0.1098	0.0952	0.0479	0.0633	0.0820	0.0675
	4-8	0.4142	0.3747	0.3990	0.3901	0.2997	0.3310	0.3643	0.3499	0.2314	0.2846	0.3267	0.3061
	9-11	0.3332	0.3662	0.3321	0.3676	0.4623	0.4190	0.3769	0.4257	0.5219	0.4664	0.4159	0.4819
	12+	0.1081	0.1338	0.1239	0.1116	0.1487	0.1597	0.1490	0.1292	0.1989	0.1857	0.1754	0.1444
25-29	0-3	0.1774	0.1620	0.1840	0.1685	0.1349	0.1184	0.1409	0.1251	0.0845	0.0840	0.1061	0.0902
	4-8	0.4311	0.3740	0.3932	0.3894	0.3554	0.3353	0.3635	0.3554	0.2589	0.2917	0.3291	0.3156
	9-11	0.2776	0.3121	0.2833	0.3143	0.3558	0.3623	0.3259	0.3687	0.4425	0.4080	0.3632	0.4227
	12+	0.1138	0.1518	0.1395	0.1278	0.1539	0.1840	0.1697	0.1508	0.2142	0.2163	0.2017	0.1715
30-34	0-3	0.2143	0.2048	0.2286	0.2109	0.1638	0.1531	0.1780	0.1605	0.1248	0.1108	0.1361	0.1182
	4-8	0.4279	0.3857	0.3999	0.3991	0.3914	0.3536	0.3770	0.3726	0.3271	0.3137	0.3472	0.3378
	9-11	0.2364	0.2682	0.2428	0.2713	0.3070	0.3183	0.2855	0.3231	0.3655	0.3657	0.3240	0.3766
	12+	0.1215	0.1413	0.1287	0.1187	0.1378	0.1750	0.1595	0.1438	0.1826	0.2099	0.1927	0.1674
35-39	0-3	0.2421	0.2527	0.2784	0.2567	0.1971	0.1935	0.2206	0.2004	0.1421	0.1431	0.1715	0.1512
	4-8	0.4123	0.3930	0.4022	0.4040	0.4034	0.3690	0.3873	0.3859	0.3651	0.3348	0.3636	0.3578
	9-11	0.2132	0.2286	0.2056	0.2343	0.2612	0.2779	0.2481	0.2830	0.3281	0.3264	0.2877	0.3347
	12+	0.1324	0.1258	0.1138	0.1050	0.1383	0.1596	0.1440	0.1308	0.1647	0.1957	0.1772	0.1563
40-44	0-3	0.2948	0.3096	0.3385	0.3105	0.2336	0.2432	0.2732	0.2485	0.1871	0.1844	0.2161	0.1925
	4-8	0.4072	0.3893	0.3932	0.3989	0.3957	0.3751	0.3877	0.3895	0.3384	0.3486	0.3718	0.3697
	9-11	0.1753	0.1894	0.1679	0.1984	0.2296	0.2363	0.2091	0.2437	0.2864	0.2843	0.2489	0.2925
	12+	0.1228	0.1117	0.1004	0.0921	0.1411	0.1454	0.1300	0.1183	0.1881	0.1827	0.1633	0.1454
45-49	0-3	0.3648	0.3664	0.3984	0.3627	0.2808	0.2965	0.3291	0.2978	0.2122	0.2317	0.2661	0.2373
	4-8	0.3860	0.3953	0.3934	0.4045	0.3939	0.3924	0.3987	0.4045	0.3903	0.3760	0.3927	0.3938
	9-11	0.1382	0.1520	0.1315	0.1629	0.1901	0.1953	0.1698	0.2051	0.2429	0.2424	0.2091	0.2511
	12+	0.1110	0.0863	0.0767	0.0699	0.1351	0.1157	0.1024	0.0927	0.1545	0.1499	0.1321	0.1178
50-54	0-3	0.4550	0.4334	0.4695	0.4254	0.3585	0.3612	0.3978	0.3576	0.2753	0.2910	0.3293	0.2925
	4-8	0.3619	0.3808	0.3726	0.3904	0.3762	0.3892	0.3886	0.4001	0.3816	0.3846	0.3937	0.3992
	9-11	0.1027	0.1215	0.1017	0.1333	0.1504	0.1608	0.1360	0.1725	0.2079	0.2057	0.1738	0.2166
	12+	0.0805	0.0643	0.0562	0.0509	0.1149	0.0889	0.0776	0.0698	0.1352	0.1187	0.1032	0.0916
55-59	0-3	0.5275	0.4987	0.5385	0.4880	0.4461	0.4264	0.4672	0.4189	0.3529	0.3533	0.3959	0.3503
	4-8	0.3426	0.3563	0.3424	0.3670	0.3581	0.3736	0.3665	0.3851	0.3526	0.3796	0.3813	0.3933
	9-11	0.0791	0.0947	0.0763	0.1062	0.1134	0.1285	0.1051	0.1412	0.1682	0.1691	0.1388	0.1821
	12+	0.0509	0.0504	0.0428	0.0388	0.0824	0.0714	0.0611	0.0548	0.1263	0.0981	0.0840	0.0743

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 18: Forecasted education profile (2013-2028) using the classical APC model (Reference category is P1). Brazil, Females

		2013			2018			2023			2028		
		Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS
20-24	0-3	0.0433	0.0603	0.0468	0.0291	0.0436	0.0318	0.0192	0.0312	0.0212	0.0125	0.0220	0.0140
	4-8	0.2390	0.2880	0.2618	0.1966	0.2498	0.2195	0.1590	0.2135	0.1811	0.1269	0.1802	0.1474
	9-11	0.5070	0.4490	0.5341	0.5401	0.4762	0.5810	0.5657	0.4972	0.6221	0.5844	0.5123	0.6573
	12+	0.2107	0.2027	0.1573	0.2343	0.2304	0.1677	0.2561	0.2581	0.1757	0.2762	0.2855	0.1814
25-29	0-3	0.0580	0.0784	0.0633	0.0391	0.0570	0.0434	0.0259	0.0408	0.0293	0.0169	0.0288	0.0194
	4-8	0.2470	0.2920	0.2734	0.2044	0.2543	0.2316	0.1660	0.2179	0.1927	0.1327	0.1841	0.1580
	9-11	0.4474	0.3950	0.4740	0.4795	0.4208	0.5208	0.5041	0.4405	0.5622	0.5219	0.4543	0.5981
	12+	0.2476	0.2346	0.1893	0.2770	0.2679	0.2041	0.3040	0.3008	0.2158	0.3286	0.3329	0.2245
30-34	0-3	0.0778	0.1021	0.0846	0.0532	0.0751	0.0590	0.0356	0.0542	0.0403	0.0234	0.0386	0.0270
	4-8	0.2701	0.3126	0.2979	0.2267	0.2757	0.2564	0.1861	0.2387	0.2161	0.1500	0.2034	0.1789
	9-11	0.4077	0.3579	0.4290	0.4430	0.3862	0.4780	0.4708	0.4087	0.5223	0.4915	0.4251	0.5612
	12+	0.2443	0.2275	0.1885	0.2771	0.2630	0.2066	0.3075	0.2983	0.2214	0.3351	0.3329	0.2329
35-39	0-3	0.1025	0.1306	0.1106	0.0713	0.0974	0.0785	0.0484	0.0712	0.0544	0.0321	0.0512	0.0369
	4-8	0.2940	0.3330	0.3220	0.2508	0.2982	0.2820	0.2087	0.2615	0.2412	0.1700	0.2252	0.2022
	9-11	0.3712	0.3237	0.3874	0.4100	0.3549	0.4383	0.4417	0.3806	0.4855	0.4660	0.4002	0.5277
	12+	0.2323	0.2127	0.1800	0.2679	0.2495	0.2011	0.3013	0.2867	0.2189	0.3318	0.3234	0.2332
40-44	0-3	0.1349	0.1673	0.1442	0.0956	0.1267	0.1046	0.0659	0.0939	0.0738	0.0443	0.0683	0.0508
	4-8	0.3129	0.3471	0.3403	0.2720	0.3161	0.3041	0.2298	0.2812	0.2645	0.1896	0.2450	0.2248
	9-11	0.3305	0.2861	0.3436	0.3720	0.3193	0.3949	0.4069	0.3476	0.4438	0.4346	0.3702	0.4887
	12+	0.2216	0.1995	0.1720	0.2604	0.2379	0.1965	0.2974	0.2773	0.2179	0.3316	0.3165	0.2357
45-49	0-3	0.1747	0.2105	0.1831	0.1273	0.1629	0.1367	0.0899	0.1233	0.0989	0.0617	0.0914	0.0696
	4-8	0.3477	0.3762	0.3725	0.3107	0.3510	0.3418	0.2690	0.3194	0.3046	0.2265	0.2840	0.2644
	9-11	0.2902	0.2478	0.3005	0.3357	0.2841	0.3520	0.3764	0.3167	0.4035	0.4104	0.3444	0.4525
	12+	0.1874	0.1655	0.1439	0.2263	0.2020	0.1695	0.2648	0.2406	0.1930	0.3014	0.2802	0.2135
50-54	0-3	0.2263	0.2664	0.2323	0.1698	0.2107	0.1786	0.1231	0.1630	0.1328	0.0865	0.1232	0.0957
	4-8	0.3667	0.3875	0.3874	0.3374	0.3710	0.3651	0.2999	0.3459	0.3338	0.2585	0.3144	0.2964
	9-11	0.2540	0.2132	0.2645	0.3026	0.2521	0.3155	0.3484	0.2885	0.3682	0.3887	0.3212	0.4206
	12+	0.1530	0.1328	0.1158	0.1902	0.1662	0.1408	0.2286	0.2026	0.1651	0.2663	0.2411	0.1873
55-59	0-3	0.2827	0.3273	0.2851	0.2181	0.2640	0.2251	0.1623	0.2082	0.1720	0.1167	0.1604	0.1272
	4-8	0.3725	0.3852	0.3904	0.3524	0.3780	0.3768	0.3216	0.3607	0.3531	0.2836	0.3349	0.3210
	9-11	0.2148	0.1763	0.2276	0.2632	0.2153	0.2765	0.3110	0.2534	0.3279	0.3551	0.2888	0.3805
	12+	0.1300	0.1112	0.0969	0.1663	0.1427	0.1216	0.2051	0.1778	0.1470	0.2445	0.2160	0.1713

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 19: Estimated and observed education profile (1983-1993) using the classical APC model (Reference category is P1). Brazil, Males

		1983				1988				1993			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.2863	0.3107	0.3251	0.2964	0.2575	0.2597	0.2839	0.2360	0.2481	0.2121	0.2353	0.1898
	4-8	0.4658	0.4610	0.4615	0.4597	0.4698	0.4602	0.4679	0.4497	0.4908	0.4436	0.4568	0.4279
	9-11	0.1913	0.1769	0.1665	0.1876	0.2155	0.2164	0.1908	0.2440	0.2057	0.2669	0.2388	0.2963
	12+	0.0567	0.0514	0.0469	0.0563	0.0572	0.0637	0.0573	0.0703	0.0554	0.0774	0.0691	0.0860
25-29	0-3	0.3191	0.3449	0.3597	0.3302	0.2655	0.2908	0.3153	0.2667	0.2461	0.2395	0.2631	0.2168
	4-8	0.4459	0.4403	0.4395	0.4404	0.4504	0.4433	0.4480	0.4362	0.4649	0.4311	0.4409	0.4190
	9-11	0.1576	0.1500	0.1413	0.1590	0.2034	0.1851	0.1635	0.2083	0.2074	0.2303	0.2069	0.2548
	12+	0.0774	0.0647	0.0594	0.0704	0.0807	0.0808	0.0732	0.0888	0.0816	0.0991	0.0892	0.1095
30-34	0-3	0.3881	0.4054	0.4215	0.3892	0.3127	0.3475	0.3731	0.3220	0.2771	0.2913	0.3159	0.2671
	4-8	0.4173	0.4093	0.4066	0.4114	0.4348	0.4191	0.4197	0.4162	0.4244	0.4147	0.4199	0.4073
	9-11	0.1185	0.1230	0.1149	0.1314	0.1641	0.1543	0.1355	0.1747	0.1986	0.1953	0.1751	0.2166
	12+	0.0761	0.0623	0.0571	0.0680	0.0884	0.0792	0.0717	0.0871	0.0999	0.0988	0.0891	0.1089
35-39	0-3	0.4658	0.4518	0.4697	0.4337	0.3720	0.3931	0.4196	0.3665	0.3056	0.3351	0.3600	0.3104
	4-8	0.3866	0.3936	0.3889	0.3975	0.4210	0.4090	0.4064	0.4096	0.4226	0.4116	0.4131	0.4081
	9-11	0.0854	0.1005	0.0924	0.1091	0.1246	0.1280	0.1111	0.1466	0.1636	0.1647	0.1469	0.1838
	12+	0.0622	0.0542	0.0491	0.0597	0.0823	0.0699	0.0629	0.0772	0.1082	0.0886	0.0799	0.0978
40-44	0-3	0.5269	0.5083	0.5288	0.4873	0.4562	0.4494	0.4777	0.4204	0.3728	0.3897	0.4161	0.3633
	4-8	0.3641	0.3632	0.3563	0.3693	0.3820	0.3835	0.3777	0.3874	0.3910	0.3927	0.3908	0.3926
	9-11	0.0644	0.0797	0.0716	0.0885	0.0955	0.1031	0.0878	0.1204	0.1316	0.1350	0.1190	0.1524
	12+	0.0447	0.0489	0.0433	0.0550	0.0663	0.0640	0.0568	0.0718	0.1047	0.0826	0.0741	0.0917
45-49	0-3	0.5555	0.5479	0.5710	0.5239	0.5155	0.4906	0.5210	0.4593	0.4398	0.4318	0.4598	0.4035
	4-8	0.3473	0.3478	0.3384	0.3563	0.3585	0.3720	0.3630	0.3789	0.3744	0.3865	0.3817	0.3894
	9-11	0.0633	0.0637	0.0556	0.0726	0.0730	0.0835	0.0693	0.0999	0.0913	0.1109	0.0960	0.1275
	12+	0.0339	0.0407	0.0350	0.0471	0.0529	0.0540	0.0468	0.0619	0.0946	0.0707	0.0625	0.0795
50-54	0-3	0.5887	0.5939	0.6013	0.5863	0.5438	0.5369	0.5696	0.5026	0.5074	0.4794	0.5098	0.4482
	4-8	0.3288	0.3212	0.3174	0.3249	0.3494	0.3502	0.3379	0.3604	0.3381	0.3691	0.3612	0.3750
	9-11	0.0536	0.0543	0.0523	0.0564	0.0716	0.0677	0.0546	0.0835	0.0708	0.0914	0.0771	0.1076
	12+	0.0289	0.0307	0.0290	0.0324	0.0352	0.0452	0.0379	0.0536	0.0837	0.0601	0.0519	0.0692
55-59	0-3	0.6368	0.6368	0.6449	0.6286	0.5959	0.5940	0.6102	0.5773	0.5385	0.5372	0.5698	0.5031
	4-8	0.2917	0.2917	0.2872	0.2961	0.3112	0.3171	0.3100	0.3240	0.3360	0.3420	0.3305	0.3514
	9-11	0.0475	0.0475	0.0454	0.0496	0.0589	0.0558	0.0494	0.0629	0.0644	0.0718	0.0587	0.0872
	12+	0.0241	0.0241	0.0225	0.0257	0.0340	0.0331	0.0304	0.0359	0.0611	0.0490	0.0409	0.0583

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 20: Estimated and observed education profile (1998-2008) using the classical APC model (Reference category is P1). Brazil, Males

		1998				2003				2008			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.1987	0.1691	0.1929	0.1469	0.1304	0.1317	0.1566	0.1093	0.0788	0.1002	0.1257	0.0785
	4-8	0.4490	0.4176	0.4377	0.3949	0.3505	0.3839	0.4131	0.3523	0.2910	0.3449	0.3841	0.3043
	9-11	0.2782	0.3214	0.2877	0.3559	0.4137	0.3780	0.3352	0.4208	0.4916	0.4344	0.3815	0.4860
	12+	0.0742	0.0918	0.0818	0.1022	0.1054	0.1064	0.0951	0.1176	0.1385	0.1205	0.1087	0.1312
25-29	0-3	0.2214	0.1926	0.2169	0.1697	0.1759	0.1511	0.1771	0.1275	0.1144	0.1158	0.1427	0.0925
	4-8	0.4595	0.4092	0.4254	0.3906	0.3924	0.3791	0.4037	0.3520	0.3069	0.3429	0.3771	0.3070
	9-11	0.2314	0.2796	0.2514	0.3085	0.3157	0.3314	0.2949	0.3682	0.4170	0.3834	0.3373	0.4290
	12+	0.0877	0.1185	0.1063	0.1312	0.1160	0.1384	0.1244	0.1523	0.1618	0.1578	0.1429	0.1715
30-34	0-3	0.2583	0.2382	0.2641	0.2134	0.2084	0.1900	0.2183	0.1636	0.1698	0.1477	0.1779	0.1209
	4-8	0.4275	0.4004	0.4113	0.3871	0.4189	0.3769	0.3955	0.3556	0.3567	0.3461	0.3739	0.3156
	9-11	0.2134	0.2412	0.2167	0.2667	0.2653	0.2905	0.2580	0.3238	0.3312	0.3412	0.2989	0.3837
	12+	0.1007	0.1202	0.1080	0.1328	0.1074	0.1426	0.1282	0.1570	0.1423	0.1650	0.1492	0.1798
35-39	0-3	0.2690	0.2789	0.3050	0.2535	0.2328	0.2263	0.2554	0.1988	0.1989	0.1791	0.2109	0.1500
	4-8	0.4185	0.4044	0.4109	0.3956	0.4132	0.3874	0.4009	0.3711	0.3908	0.3619	0.3842	0.3363
	9-11	0.2017	0.2070	0.1854	0.2297	0.2446	0.2537	0.2246	0.2840	0.2909	0.3031	0.2643	0.3431
	12+	0.1109	0.1097	0.0987	0.1212	0.1094	0.1325	0.1191	0.1461	0.1194	0.1560	0.1407	0.1706
40-44	0-3	0.2963	0.3303	0.3574	0.3034	0.2546	0.2730	0.3034	0.2436	0.2329	0.2198	0.2538	0.1881
	4-8	0.4156	0.3928	0.3952	0.3883	0.4088	0.3833	0.3915	0.3722	0.3509	0.3644	0.3806	0.3447
	9-11	0.1741	0.1728	0.1538	0.1930	0.2216	0.2157	0.1900	0.2428	0.2612	0.2623	0.2275	0.2988
	12+	0.1140	0.1041	0.0936	0.1153	0.1150	0.1281	0.1150	0.1414	0.1550	0.1535	0.1381	0.1685
45-49	0-3	0.3530	0.3720	0.4000	0.3441	0.2810	0.3129	0.3437	0.2828	0.2371	0.2566	0.2912	0.2237
	4-8	0.3893	0.3931	0.3921	0.3919	0.4089	0.3903	0.3943	0.3835	0.3827	0.3779	0.3890	0.3632
	9-11	0.1402	0.1443	0.1269	0.1632	0.1877	0.1833	0.1603	0.2081	0.2472	0.2271	0.1956	0.2606
	12+	0.1176	0.0906	0.0809	0.1009	0.1223	0.1134	0.1016	0.1256	0.1330	0.1384	0.1242	0.1525
50-54	0-3	0.4260	0.4196	0.4497	0.3892	0.3390	0.3590	0.3913	0.3270	0.2681	0.2997	0.3356	0.2650
	4-8	0.3623	0.3814	0.3772	0.3833	0.3934	0.3853	0.3854	0.3823	0.3925	0.3797	0.3858	0.3698
	9-11	0.1011	0.1208	0.1042	0.1391	0.1501	0.1560	0.1346	0.1795	0.2152	0.1967	0.1679	0.2281
	12+	0.1107	0.0783	0.0689	0.0884	0.1175	0.0997	0.0887	0.1112	0.1241	0.1238	0.1107	0.1371
55-59	0-3	0.5015	0.4787	0.5113	0.4452	0.4129	0.4177	0.4523	0.3828	0.3459	0.3561	0.3940	0.3186
	4-8	0.3450	0.3597	0.3518	0.3653	0.3669	0.3706	0.3664	0.3718	0.3720	0.3729	0.3738	0.3683
	9-11	0.0772	0.0966	0.0811	0.1143	0.1191	0.1273	0.1075	0.1497	0.1626	0.1639	0.1375	0.1934
	12+	0.0762	0.0650	0.0558	0.0752	0.1011	0.0844	0.0738	0.0957	0.1195	0.1070	0.0946	0.1198

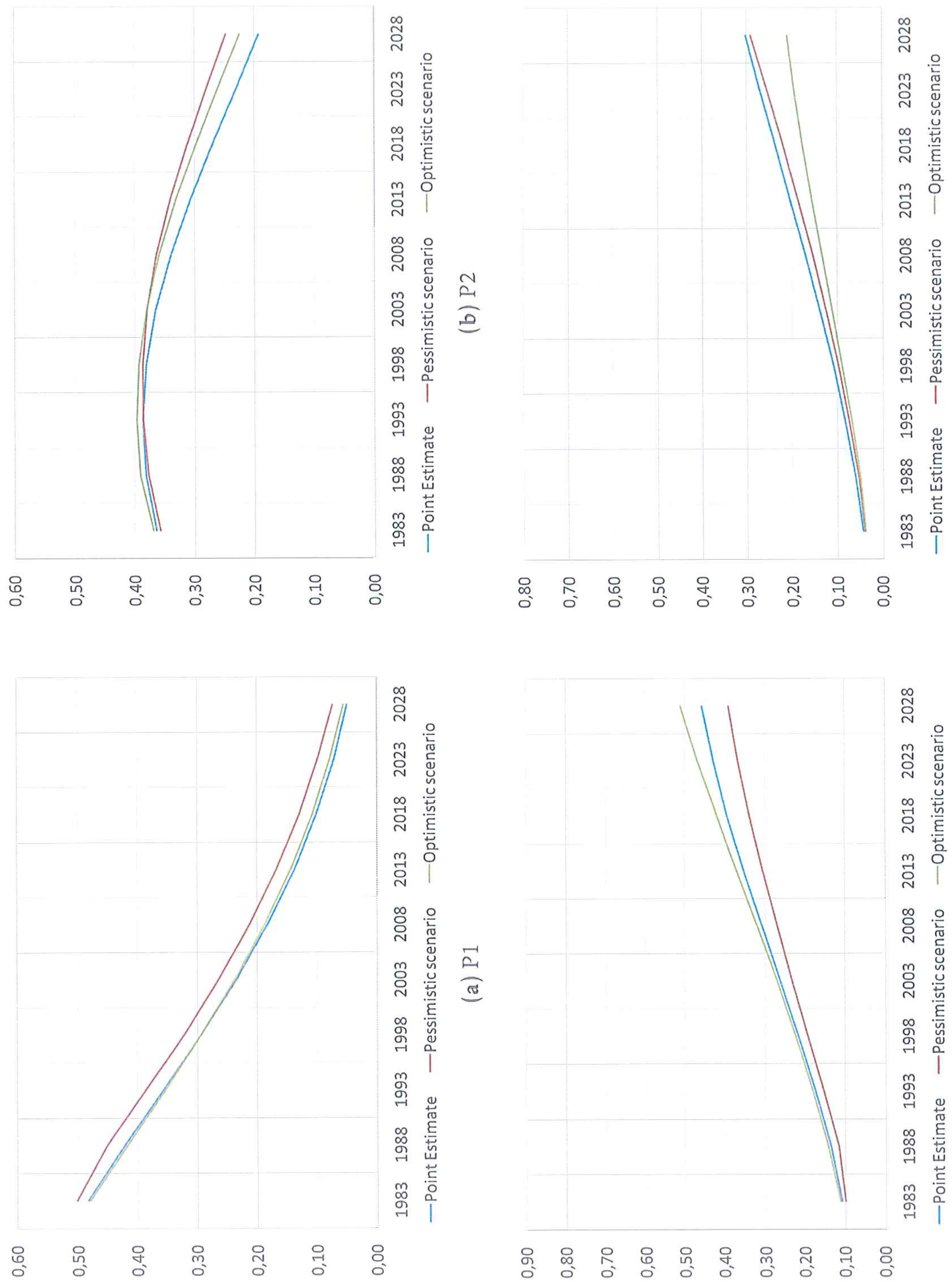
Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 21: Forecasted education profile (2013-2028) using the classical APC model (Reference category is P1). Brazil, Males

		2013			2018			2023			2028		
		Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS
20-24	0-3	0.0746	0.0995	0.0547	0.0545	0.0777	0.0372	0.0391	0.0599	0.0248	0.0277	0.0457	0.0162
	4-8	0.3033	0.3519	0.2555	0.2615	0.3179	0.2092	0.2216	0.2834	0.1679	0.1851	0.2496	0.1325
	9-11	0.4886	0.4262	0.5475	0.5389	0.4687	0.6027	0.5843	0.5082	0.6507	0.6241	0.5442	0.6913
	12+	0.1335	0.1224	0.1424	0.1451	0.1357	0.1508	0.1550	0.1485	0.1567	0.1631	0.1605	0.1600
25-29	0-3	1.0000	0.1134	0.0651	1.0000	0.0888	0.0446	1.0000	0.0686	0.0299	1.0000	0.0523	0.0197
	4-8	0.0868	0.3468	0.2599	0.0637	0.3141	0.2145	0.0459	0.2807	0.1733	0.0326	0.2476	0.1376
	9-11	0.3034	0.3784	0.4873	0.2630	0.4174	0.5405	0.2239	0.4537	0.5874	0.1877	0.4866	0.6275
	12+	0.4339	0.1615	0.1877	0.4811	0.1797	0.2004	0.5240	0.1971	0.2094	0.5618	0.2134	0.2152
30-34	0-3	0.1759	0.1428	0.0864	0.1922	0.1129	0.0600	0.2062	0.0879	0.0407	0.2179	0.0675	0.0270
	4-8	1.0000	0.3475	0.2714	1.0000	0.3178	0.2270	1.0000	0.2862	0.1854	1.0000	0.2543	0.1484
	9-11	0.1122	0.3392	0.4424	0.0833	0.3778	0.4969	0.0607	0.4141	0.5457	0.0434	0.4473	0.5880
	12+	0.3102	0.1705	0.1998	0.2720	0.1915	0.2161	0.2338	0.2118	0.2283	0.1975	0.2310	0.2365
35-39	0-3	0.3912	0.1714	0.1093	0.4387	0.1370	0.0772	0.4824	0.1078	0.0531	0.5214	0.0836	0.0357
	4-8	0.1864	0.3617	0.2948	0.2060	0.3347	0.2507	0.2231	0.3048	0.2077	0.2376	0.2734	0.1682
	9-11	1.0000	0.3040	0.4027	1.0000	0.3430	0.4598	1.0000	0.3802	0.5119	1.0000	0.4148	0.5578
	12+	0.1382	0.1629	0.1931	0.1041	0.1853	0.2122	0.0768	0.2073	0.2273	0.0556	0.2282	0.2383
40-44	0-3	0.3296	0.2088	0.1400	0.2932	0.1689	0.1008	0.2553	0.1344	0.0704	0.2181	0.1052	0.0479
	4-8	0.3532	0.3632	0.3083	0.4019	0.3403	0.2670	0.4477	0.3133	0.2245	0.4892	0.2838	0.1842
	9-11	0.1790	0.2657	0.3573	0.2007	0.3037	0.4149	0.2203	0.3406	0.4688	0.2372	0.3755	0.5172
	12+	1.0000	0.1623	0.1944	1.0000	0.1871	0.2174	1.0000	0.2116	0.2363	1.0000	0.2354	0.2507
45-49	0-3	0.1725	0.2427	0.1705	0.1320	0.1989	0.1254	0.0987	0.1603	0.0892	0.0722	0.1270	0.0617
	4-8	0.3376	0.3765	0.3322	0.3049	0.3577	0.2937	0.2691	0.3337	0.2516	0.2325	0.3060	0.2097
	9-11	0.3108	0.2324	0.3178	0.3591	0.2699	0.3762	0.4054	0.3070	0.4326	0.4482	0.3427	0.4845
	12+	0.1792	0.1484	0.1795	0.2040	0.1735	0.2048	0.2269	0.1991	0.2266	0.2471	0.2243	0.2441
50-54	0-3	1.0000	0.2832	0.2068	1.0000	0.2351	0.1555	1.0000	0.1917	0.1129	1.0000	0.1537	0.0795
	4-8	0.2050	0.3789	0.3458	0.1596	0.3650	0.3122	0.1213	0.3450	0.2726	0.0901	0.3203	0.2309
	9-11	0.3565	0.2033	0.2830	0.3276	0.2401	0.3411	0.2938	0.2772	0.3991	0.2576	0.3136	0.4539
	12+	0.2740	0.1346	0.1644	0.3221	0.1599	0.1911	0.3695	0.1861	0.2154	0.4144	0.2124	0.2357
55-59	0-3	0.1645	0.3377	0.2558	0.1906	0.2846	0.1977	0.2154	0.2357	0.1472	0.2380	0.1918	0.1059
	4-8	1.0000	0.3737	0.3532	1.0000	0.3663	0.3272	1.0000	0.3521	0.2926	1.0000	0.3321	0.2532
	9-11	0.2438	0.1707	0.2444	0.1932	0.2061	0.3009	0.1491	0.2426	0.3595	0.1124	0.2794	0.4169
	12+	0.3647	0.1178	0.1465	0.3411	0.1430	0.1742	0.3108	0.1695	0.2006	0.2765	0.1967	0.2240

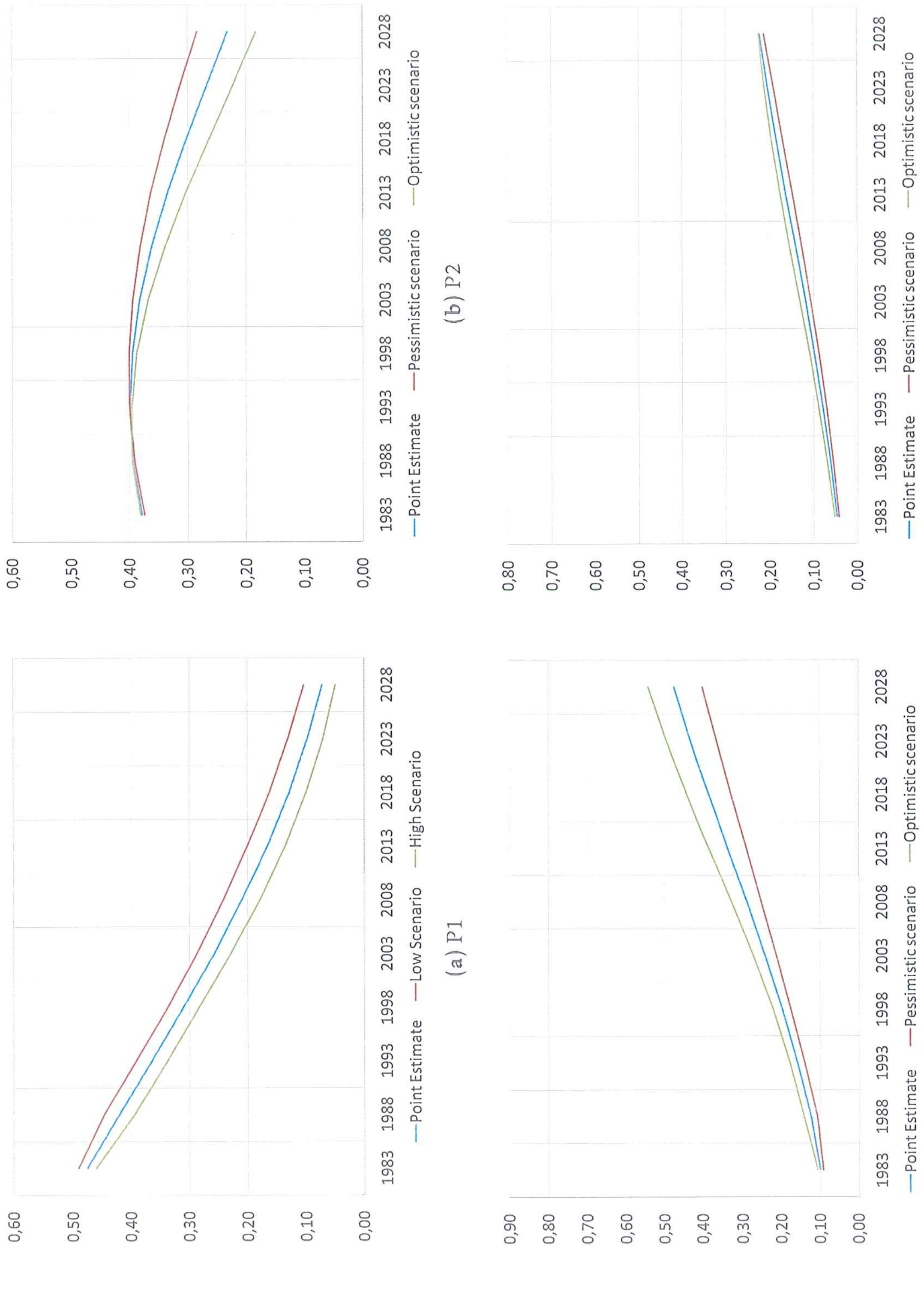
Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Figure 25: Fitted and projected probabilities for the Classical APC model when P1 is the reference category. Brazil, Females, 1983-2028



(a) P1 (b) P2 (c) P3 (d) P4

Figure 26: Fitted and projected probabilities for the Classical APC model when P1 is the reference category. Brazil, Males, 1983-2028



4.1.4.2 Results when P4 is chosen as reference category

Again, Tables 22-27 show that the observed education profile scatter about the fitted profile with no suggestion of any systematic departure in any of the age groups, confirming the goodness of fit of the model. As I mentioned above, there are no differences between the point estimates and forecasts of the education profile using different normalization rules of the multinomial APC model (that is, choosing either category P1 or P4 as reference). Only low and Optimistic Scenarios differ between specifications, as I argued in section 3.2.2.2.

Again, I also present here graphics for the average education profile and the optimistic and pessimistic scenarios across all age groups with the aim of summarizing the evidence presented in the tables. Figures 27-28 illustrate these results. For a given education category, graphs are provided at the same scale for males and females. The reader should be aware that, when interpreting scenarios for a given category of schooling, the adjusted probabilities in the pessimistic scenario may be higher than those obtained through the optimistic scenario. This is expected because the scenarios are built taking into account future trends in period and cohort effects for the education profile as a whole.

Table 22: Estimated and observed education profile (1983-1993) using the classical APC model (Reference category is P4). Brazil, Females

		1983				1988				1993			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.2625	0.2787	0.2761	0.2625	0.2165	0.2204	0.2160	0.2173	0.1847	0.1688	0.1631	0.1810
	4-8	0.4494	0.4441	0.4397	0.4604	0.4543	0.4384	0.4269	0.4554	0.4803	0.4117	0.3983	0.4667
	9-11	0.2125	0.2110	0.2106	0.2081	0.2494	0.2556	0.2558	0.2484	0.2539	0.3108	0.3107	0.2670
	12+	0.0756	0.0663	0.0735	0.0690	0.0798	0.0857	0.1013	0.0789	0.0811	0.1087	0.1279	0.0852
25-29	0-3	0.3248	0.3404	0.3406	0.3238	0.2490	0.2746	0.2714	0.2529	0.2208	0.2145	0.2085	0.2239
	4-8	0.4256	0.4188	0.4146	0.4274	0.4342	0.4217	0.4113	0.4343	0.4383	0.4039	0.3914	0.4349
	9-11	0.1635	0.1698	0.1653	0.1649	0.2187	0.2099	0.2063	0.2147	0.2321	0.2603	0.2579	0.2286
	12+	0.0861	0.0710	0.0795	0.0839	0.0981	0.0937	0.1110	0.0981	0.1088	0.1213	0.1422	0.1125
30-34	0-3	0.4097	0.4007	0.4045	0.4057	0.3000	0.3309	0.3308	0.3078	0.2550	0.2648	0.2606	0.2555
	4-8	0.3848	0.4020	0.3970	0.3859	0.4240	0.4143	0.4048	0.4204	0.4263	0.4066	0.3962	0.4297
	9-11	0.1285	0.1358	0.1287	0.1295	0.1719	0.1718	0.1650	0.1731	0.2032	0.2184	0.2133	0.1998
	12+	0.0770	0.0615	0.0698	0.0788	0.1040	0.0830	0.0994	0.0987	0.1156	0.1102	0.1299	0.1150
35-39	0-3	0.4870	0.4600	0.4669	0.4839	0.3933	0.3885	0.3922	0.3860	0.3093	0.3186	0.3176	0.3081
	4-8	0.3635	0.3812	0.3746	0.3596	0.3809	0.4018	0.3919	0.3846	0.4145	0.4041	0.3949	0.4173
	9-11	0.0936	0.1078	0.0995	0.0957	0.1384	0.1394	0.1304	0.1384	0.1611	0.1815	0.1734	0.1622
	12+	0.0560	0.0510	0.0591	0.0608	0.0875	0.0704	0.0855	0.0910	0.1150	0.0957	0.1141	0.1124
40-44	0-3	0.5399	0.5240	0.5325	0.5516	0.4706	0.4525	0.4592	0.4689	0.3875	0.3805	0.3829	0.3898
	4-8	0.3502	0.3510	0.3427	0.3310	0.3634	0.3783	0.3671	0.3574	0.3784	0.3901	0.3805	0.3771
	9-11	0.0703	0.0830	0.0748	0.0736	0.0984	0.1098	0.1000	0.1020	0.1300	0.1466	0.1364	0.1281
	12+	0.0396	0.0421	0.0500	0.0438	0.0675	0.0594	0.0737	0.0718	0.1041	0.0828	0.1002	0.1050
45-49	0-3	0.6028	0.5765	0.5862	0.5974	0.5365	0.5082	0.5176	0.5298	0.4664	0.4380	0.4442	0.4666
	4-8	0.3171	0.3314	0.3220	0.3189	0.3400	0.3646	0.3524	0.3461	0.3562	0.3854	0.3748	0.3665
	9-11	0.0553	0.0619	0.0547	0.0550	0.0775	0.0836	0.0746	0.0779	0.1004	0.1144	0.1040	0.0933
	12+	0.0248	0.0302	0.0370	0.0287	0.0460	0.0435	0.0555	0.0462	0.0770	0.0622	0.0770	0.0736
50-54	0-3	0.6252	0.6298	0.6293	0.6298	0.5867	0.5716	0.5826	0.5870	0.5432	0.5044	0.5133	0.5376
	4-8	0.3041	0.2999	0.2993	0.2999	0.3280	0.3340	0.3213	0.3237	0.3299	0.3614	0.3500	0.3434
	9-11	0.0520	0.0510	0.0504	0.0510	0.0568	0.0636	0.0556	0.0593	0.0761	0.0891	0.0792	0.0724
	12+	0.0187	0.0193	0.0210	0.0193	0.0285	0.0309	0.0406	0.0299	0.0507	0.0452	0.0575	0.0466
55-59	0-3	0.6521	0.6521	0.6519	0.6521	0.6239	0.6245	0.6256	0.6191	0.5877	0.5672	0.5772	0.5931
	4-8	0.2883	0.2883	0.2878	0.2883	0.2997	0.3018	0.2983	0.3041	0.3167	0.3304	0.3183	0.3194
	9-11	0.0457	0.0457	0.0451	0.0457	0.0541	0.0524	0.0513	0.0552	0.0575	0.0678	0.0591	0.0550
	12+	0.0138	0.0138	0.0152	0.0138	0.0223	0.0213	0.0248	0.0217	0.0381	0.0346	0.0453	0.0325

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 23: Estimated and observed education profile (1998-2008) using the classical APC model (Reference category is P4). Brazil, Females

		1998				2003				2008			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.1445	0.1253	0.1195	0.1447	0.0894	0.0902	0.0848	0.0913	0.0479	0.0633	0.0584	0.0479
	4-8	0.4142	0.3747	0.3580	0.4119	0.2997	0.3310	0.3105	0.3036	0.2314	0.2846	0.2610	0.2314
	9-11	0.3332	0.3662	0.3630	0.3339	0.4623	0.4190	0.4088	0.4554	0.5219	0.4664	0.4452	0.5219
	12+	0.1081	0.1338	0.1595	0.1095	0.1487	0.1597	0.1959	0.1497	0.1989	0.1857	0.2353	0.1989
25-29	0-3	0.1774	0.1620	0.1548	0.1737	0.1349	0.1184	0.1110	0.1352	0.0845	0.0840	0.0771	0.0825
	4-8	0.4311	0.3740	0.3574	0.4331	0.3554	0.3353	0.3136	0.3598	0.2589	0.2917	0.2657	0.2548
	9-11	0.2776	0.3121	0.3083	0.2769	0.3558	0.3623	0.3529	0.3538	0.4425	0.4080	0.3882	0.4495
	12+	0.1138	0.1518	0.1795	0.1163	0.1539	0.1840	0.2225	0.1512	0.2142	0.2163	0.2690	0.2132
30-34	0-3	0.2143	0.2048	0.1980	0.2119	0.1638	0.1531	0.1449	0.1625	0.1248	0.1108	0.1024	0.1244
	4-8	0.4279	0.3857	0.3710	0.4203	0.3914	0.3536	0.3328	0.4000	0.3271	0.3137	0.2875	0.3250
	9-11	0.2364	0.2682	0.2638	0.2409	0.3070	0.3183	0.3110	0.3028	0.3655	0.3657	0.3504	0.3668
	12+	0.1215	0.1413	0.1672	0.1269	0.1378	0.1750	0.2113	0.1348	0.1826	0.2099	0.2597	0.1839
35-39	0-3	0.2421	0.2527	0.2475	0.2409	0.1971	0.1935	0.1854	0.1974	0.1421	0.1431	0.1338	0.1502
	4-8	0.4123	0.3930	0.3805	0.4191	0.4034	0.3690	0.3503	0.3918	0.3651	0.3348	0.3095	0.3676
	9-11	0.2132	0.2286	0.2219	0.2133	0.2612	0.2779	0.2709	0.2670	0.3281	0.3264	0.3145	0.3207
	12+	0.1324	0.1258	0.1500	0.1267	0.1383	0.1596	0.1934	0.1438	0.1647	0.1957	0.2422	0.1615
40-44	0-3	0.2948	0.3096	0.3073	0.2955	0.2336	0.2432	0.2364	0.2285	0.1871	0.1844	0.1746	0.1853
	4-8	0.4072	0.3893	0.3781	0.4053	0.3957	0.3751	0.3586	0.3892	0.3384	0.3486	0.3251	0.3580
	9-11	0.1753	0.1894	0.1799	0.1725	0.2296	0.2363	0.2274	0.2355	0.2864	0.2843	0.2734	0.2810
	12+	0.1228	0.1117	0.1347	0.1266	0.1411	0.1454	0.1775	0.1467	0.1881	0.1827	0.2269	0.1757
45-49	0-3	0.3648	0.3664	0.3683	0.3717	0.2808	0.2965	0.2934	0.2795	0.2122	0.2317	0.2240	0.2140
	4-8	0.3860	0.3953	0.3846	0.3865	0.3939	0.3924	0.3782	0.3984	0.3903	0.3760	0.3555	0.3767
	9-11	0.1382	0.1520	0.1411	0.1358	0.1901	0.1953	0.1849	0.1905	0.2429	0.2424	0.2316	0.2481
	12+	0.1110	0.0863	0.1060	0.1060	0.1351	0.1157	0.1436	0.1317	0.1545	0.1499	0.1889	0.1612
50-54	0-3	0.4550	0.4334	0.4393	0.4579	0.3585	0.3612	0.3625	0.3632	0.2753	0.2910	0.2869	0.2706
	4-8	0.3619	0.3808	0.3696	0.3669	0.3762	0.3892	0.3761	0.3725	0.3816	0.3846	0.3671	0.3782
	9-11	0.1027	0.1215	0.1102	0.1014	0.1504	0.1608	0.1489	0.1544	0.2079	0.2057	0.1937	0.2067
	12+	0.0805	0.0643	0.0809	0.0737	0.1149	0.0889	0.1125	0.1099	0.1352	0.1187	0.1523	0.1444
55-59	0-3	0.5275	0.4987	0.5069	0.5275	0.4461	0.4264	0.4313	0.4481	0.3529	0.3533	0.3530	0.3507
	4-8	0.3426	0.3563	0.3441	0.3432	0.3581	0.3736	0.3601	0.3536	0.3526	0.3796	0.3631	0.3521
	9-11	0.0791	0.0947	0.0839	0.0788	0.1134	0.1285	0.1162	0.1156	0.1682	0.1691	0.1556	0.1674
	12+	0.0509	0.0504	0.0650	0.0504	0.0824	0.0714	0.0924	0.0827	0.1263	0.0981	0.1282	0.1298

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 24: Forecasted education profile (2013-2028) using the classical APC model (Reference category is P4). Brazil, Females

		2013			2018			2023			2028		
		Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS
20-24	0-3	0.0433	0.0392	0.0468	0.0291	0.0258	0.0318	0.0192	0.0166	0.0212	0.0125	0.0105	0.0140
	4-8	0.2390	0.2135	0.2618	0.1966	0.1706	0.2195	0.1590	0.1337	0.1811	0.1269	0.1030	0.1474
	9-11	0.5070	0.4711	0.5341	0.5401	0.4865	0.5810	0.5657	0.4923	0.6221	0.5844	0.4899	0.6573
	12+	0.2107	0.2761	0.1573	0.2343	0.3171	0.1677	0.2561	0.3575	0.1757	0.2762	0.3966	0.1814
25-29	0-3	0.0580	0.0520	0.0458	0.0391	0.0341	0.0434	0.0259	0.0219	0.0292	0.0169	0.0138	0.0194
	4-8	0.2470	0.2183	0.2186	0.2044	0.1746	0.2312	0.1660	0.1366	0.1923	0.1327	0.1049	0.1576
	9-11	0.4474	0.4131	0.4992	0.4795	0.4274	0.5188	0.5041	0.4321	0.5600	0.5219	0.4286	0.5958
	12+	0.2476	0.3167	0.2363	0.2770	0.3639	0.2066	0.3040	0.4094	0.2184	0.3286	0.4526	0.2272
30-34	0-3	0.0778	0.0701	0.0799	0.0532	0.0466	0.0422	0.0356	0.0302	0.0401	0.0234	0.0192	0.0269
	4-8	0.2701	0.2400	0.2572	0.2267	0.1945	0.2026	0.1861	0.1536	0.2151	0.1500	0.1188	0.1781
	9-11	0.4077	0.3797	0.4483	0.4430	0.3983	0.4967	0.4708	0.4069	0.5181	0.4915	0.4066	0.5567
	12+	0.2443	0.3102	0.2146	0.2771	0.3606	0.2585	0.3075	0.4093	0.2267	0.3351	0.4554	0.2384
35-39	0-3	0.1025	0.0932	0.1197	0.0713	0.0630	0.0737	0.0484	0.0413	0.0385	0.0321	0.0265	0.0367
	4-8	0.2940	0.2634	0.3302	0.2508	0.2169	0.2418	0.2087	0.1736	0.1886	0.1700	0.1356	0.2007
	9-11	0.3712	0.3490	0.3697	0.4100	0.3730	0.4540	0.4417	0.3864	0.4982	0.4660	0.3902	0.5211
	12+	0.2323	0.2943	0.1805	0.2679	0.3471	0.2306	0.3013	0.3987	0.2747	0.3318	0.4477	0.2416
40-44	0-3	0.1349	0.1241	0.1466	0.0956	0.0852	0.1131	0.0659	0.0567	0.0687	0.0443	0.0368	0.0356
	4-8	0.3129	0.2828	0.3710	0.2720	0.2371	0.3115	0.2298	0.1925	0.2249	0.1896	0.1520	0.1737
	9-11	0.3305	0.3124	0.3209	0.3720	0.3414	0.3758	0.4069	0.3596	0.4552	0.4346	0.3676	0.4945
	12+	0.2216	0.2807	0.1615	0.2604	0.3363	0.1996	0.2974	0.3912	0.2513	0.3316	0.4437	0.2963
45-49	0-3	0.1747	0.1642	0.1801	0.1273	0.1158	0.1389	0.0899	0.0790	0.1069	0.0617	0.0523	0.0646
	4-8	0.3477	0.3196	0.3816	0.3107	0.2759	0.3721	0.2690	0.2298	0.3116	0.2265	0.1854	0.2242
	9-11	0.2902	0.2760	0.2808	0.3357	0.3130	0.3278	0.3764	0.3397	0.3829	0.4104	0.3556	0.4621
	12+	0.1874	0.2402	0.1575	0.2263	0.2952	0.1612	0.2648	0.3515	0.1986	0.3014	0.4067	0.2490
50-54	0-3	0.2263	0.2175	0.2132	0.1698	0.1582	0.1756	0.1231	0.1108	0.1349	0.0865	0.0750	0.1034
	4-8	0.3667	0.3424	0.3905	0.3374	0.3055	0.3739	0.2999	0.2618	0.3634	0.2585	0.2165	0.3030
	9-11	0.2540	0.2407	0.2532	0.3026	0.2845	0.2942	0.3484	0.3201	0.3423	0.3887	0.3450	0.3982
	12+	0.1530	0.1994	0.1431	0.1902	0.2518	0.1563	0.2286	0.3074	0.1593	0.2663	0.3635	0.1954
55-59	0-3	0.2827	0.2767	0.2667	0.2181	0.2074	0.2060	0.1623	0.1490	0.1690	0.1167	0.1031	0.1293
	4-8	0.3725	0.3508	0.3874	0.3524	0.3234	0.3787	0.3216	0.2851	0.3611	0.2836	0.2413	0.3496
	9-11	0.2148	0.2002	0.2091	0.2632	0.2459	0.2634	0.3110	0.2871	0.3048	0.3551	0.3192	0.3534
	12+	0.1300	0.1723	0.1369	0.1663	0.2233	0.1519	0.2051	0.2788	0.1652	0.2445	0.3363	0.1677

Source: Calculations based on PNAD data (IBGE)

Table 25: Estimated and observed education profile (1983-1993) using the classical APC model (Reference category is P4). Brazil, Males

		1983				1988				1993			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.2863	0.3107	0.3032	0.3180	0.2575	0.2597	0.2554	0.2634	0.2481	0.2121	0.2043	0.2195
	4-8	0.4658	0.4610	0.4584	0.4631	0.4698	0.4602	0.4556	0.4637	0.4908	0.4436	0.4343	0.4518
	9-11	0.1913	0.1769	0.1808	0.1729	0.2155	0.2164	0.2128	0.2197	0.2057	0.2669	0.2688	0.2643
	12+	0.0567	0.0514	0.0576	0.0459	0.0572	0.0637	0.0761	0.0532	0.0554	0.0774	0.0926	0.0645
25-29	0-3	0.3191	0.3449	0.3397	0.3500	0.2655	0.2908	0.2880	0.2928	0.2461	0.2395	0.2324	0.2460
	4-8	0.4459	0.4403	0.4392	0.4410	0.4504	0.4433	0.4394	0.4460	0.4649	0.4311	0.4229	0.4381
	9-11	0.1576	0.1500	0.1487	0.1512	0.2034	0.1851	0.1768	0.1933	0.2074	0.2303	0.2273	0.2325
	12+	0.0774	0.0647	0.0724	0.0578	0.0807	0.0808	0.0959	0.0680	0.0816	0.0991	0.1174	0.0834
30-34	0-3	0.3881	0.4054	0.4023	0.4081	0.3127	0.3475	0.3471	0.3468	0.2771	0.2913	0.2860	0.2957
	4-8	0.4173	0.4093	0.4092	0.4090	0.4348	0.4191	0.4164	0.4204	0.4244	0.4147	0.4092	0.4189
	9-11	0.1185	0.1230	0.1183	0.1276	0.1641	0.1543	0.1427	0.1662	0.1986	0.1953	0.1882	0.2020
	12+	0.0761	0.0623	0.0703	0.0553	0.0884	0.0792	0.0938	0.0666	0.0999	0.0988	0.1166	0.0834
35-39	0-3	0.4658	0.4518	0.4502	0.4528	0.3720	0.3931	0.3952	0.3897	0.3056	0.3351	0.3327	0.3364
	4-8	0.3866	0.3936	0.3936	0.3930	0.4210	0.4090	0.4068	0.4098	0.4226	0.4116	0.4082	0.4136
	9-11	0.0854	0.1005	0.0943	0.1069	0.1246	0.1280	0.1148	0.1421	0.1636	0.1647	0.1542	0.1753
	12+	0.0622	0.0542	0.0620	0.0473	0.0823	0.0699	0.0833	0.0584	0.1082	0.0886	0.1048	0.0746
40-44	0-3	0.5269	0.5083	0.5065	0.5091	0.4562	0.4494	0.4523	0.4446	0.3728	0.3897	0.3894	0.3886
	4-8	0.3641	0.3632	0.3629	0.3629	0.3820	0.3835	0.3805	0.3850	0.3910	0.3927	0.3898	0.3941
	9-11	0.0644	0.0797	0.0734	0.0863	0.0955	0.1031	0.0900	0.1176	0.1316	0.1350	0.1226	0.1481
	12+	0.0447	0.0489	0.0572	0.0417	0.0663	0.0640	0.0772	0.0528	0.1047	0.0826	0.0983	0.0692
45-49	0-3	0.5555	0.5479	0.5454	0.5492	0.5155	0.4906	0.4935	0.4855	0.4398	0.4318	0.4329	0.4290
	4-8	0.3473	0.3478	0.3476	0.3473	0.3585	0.3720	0.3684	0.3738	0.3744	0.3865	0.3836	0.3879
	9-11	0.0633	0.0637	0.0579	0.0698	0.0730	0.0835	0.0714	0.0971	0.0913	0.1109	0.0982	0.1248
	12+	0.0339	0.0407	0.0491	0.0337	0.0529	0.0540	0.0666	0.0436	0.0946	0.0707	0.0853	0.0583
50-54	0-3	0.5887	0.5939	0.5933	0.5943	0.5438	0.5369	0.5389	0.5324	0.5074	0.4794	0.4806	0.4761
	4-8	0.3288	0.3212	0.3204	0.3219	0.3494	0.3502	0.3465	0.3522	0.3381	0.3691	0.3659	0.3707
	9-11	0.0536	0.0543	0.0535	0.0551	0.0716	0.0677	0.0572	0.0799	0.0708	0.0914	0.0793	0.1047
	12+	0.0289	0.0307	0.0328	0.0287	0.0352	0.0452	0.0574	0.0355	0.0837	0.0601	0.0742	0.0485
55-59	0-3	0.6368	0.6368	0.6363	0.6372	0.5959	0.5940	0.5984	0.5891	0.5385	0.5372	0.5375	0.5345
	4-8	0.2917	0.2917	0.2910	0.2923	0.3112	0.3171	0.3134	0.3207	0.3360	0.3420	0.3387	0.3436
	9-11	0.0475	0.0475	0.0467	0.0483	0.0589	0.0558	0.0510	0.0609	0.0644	0.0718	0.0615	0.0834
	12+	0.0241	0.0241	0.0260	0.0223	0.0340	0.0331	0.0372	0.0294	0.0611	0.0490	0.0623	0.0384

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 26: Estimated and observed education profile (1998-2008) using the classical APC model (Reference category is P4). Brazil, Males

		1998				2003				2008			
		Observed	Prediction	PS	OS	Observed	Prediction	PS	OS	Observed	Prediction	PS	OS
20-24	0-3	0.1987	0.1691	0.1604	0.1775	0.1304	0.1317	0.1239	0.1390	0.0788	0.1002	0.0941	0.1056
	4-8	0.4490	0.4176	0.4043	0.4295	0.3505	0.3839	0.3686	0.3973	0.2910	0.3449	0.3294	0.3575
	9-11	0.2782	0.3214	0.3238	0.3177	0.4137	0.3780	0.3741	0.3794	0.4916	0.4344	0.4193	0.4455
	12+	0.0742	0.0918	0.1116	0.0753	0.1054	0.1064	0.1333	0.0844	0.1385	0.1205	0.1572	0.0914
25-29	0-3	0.2214	0.1926	0.1837	0.2010	0.1759	0.1511	0.1426	0.1591	0.1144	0.1158	0.1085	0.1222
	4-8	0.4595	0.4092	0.3963	0.4206	0.3924	0.3791	0.3629	0.3932	0.3069	0.3429	0.3251	0.3576
	9-11	0.2314	0.2796	0.2780	0.2799	0.3157	0.3314	0.3244	0.3361	0.4170	0.3834	0.3654	0.3977
	12+	0.0877	0.1185	0.1420	0.0985	0.1160	0.1384	0.1702	0.1117	0.1618	0.1578	0.2010	0.1224
30-34	0-3	0.2583	0.2382	0.2301	0.2456	0.2084	0.1900	0.1812	0.1979	0.1698	0.1477	0.1396	0.1547
	4-8	0.4275	0.4004	0.3904	0.4088	0.4189	0.3769	0.3629	0.3889	0.3567	0.3461	0.3292	0.3599
	9-11	0.2134	0.2412	0.2363	0.2451	0.2653	0.2905	0.2822	0.2970	0.3312	0.3412	0.3237	0.3557
	12+	0.1007	0.1202	0.1432	0.1004	0.1074	0.1426	0.1738	0.1162	0.1423	0.1650	0.2076	0.1297
35-39	0-3	0.2690	0.2789	0.2732	0.2836	0.2328	0.2263	0.2191	0.2324	0.1989	0.1791	0.1715	0.1851
	4-8	0.4185	0.4044	0.3977	0.4096	0.4132	0.3874	0.3765	0.3963	0.3908	0.3619	0.3472	0.3735
	9-11	0.2017	0.2070	0.1985	0.2151	0.2446	0.2537	0.2434	0.2629	0.2909	0.3031	0.2862	0.3179
	12+	0.1109	0.1097	0.1307	0.0918	0.1094	0.1325	0.1610	0.1084	0.1194	0.1560	0.1951	0.1235
40-44	0-3	0.2963	0.3303	0.3269	0.3322	0.2546	0.2730	0.2675	0.2768	0.2329	0.2198	0.2132	0.2246
	4-8	0.4156	0.3928	0.3881	0.3960	0.4088	0.3833	0.3750	0.3894	0.3509	0.3644	0.3520	0.3738
	9-11	0.1741	0.1728	0.1608	0.1848	0.2216	0.2157	0.2021	0.2287	0.2612	0.2623	0.2440	0.2794
	12+	0.1140	0.1041	0.1242	0.0870	0.1150	0.1281	0.1553	0.1050	0.1550	0.1535	0.1909	0.1222
45-49	0-3	0.3530	0.3720	0.3713	0.3710	0.2810	0.3129	0.3105	0.3134	0.2371	0.2566	0.2526	0.2584
	4-8	0.3893	0.3931	0.3894	0.3950	0.4089	0.3903	0.3844	0.3940	0.3827	0.3779	0.3684	0.3842
	9-11	0.1402	0.1443	0.1305	0.1589	0.1877	0.1833	0.1672	0.1999	0.2472	0.2271	0.2068	0.2471
	12+	0.1176	0.0906	0.1089	0.0751	0.1223	0.1134	0.1380	0.0927	0.1330	0.1384	0.1722	0.1102
50-54	0-3	0.4260	0.4196	0.4203	0.4168	0.3390	0.3590	0.3592	0.3565	0.2681	0.2997	0.2987	0.2981
	4-8	0.3623	0.3814	0.3778	0.3831	0.3934	0.3853	0.3804	0.3877	0.3925	0.3797	0.3725	0.3836
	9-11	0.1011	0.1208	0.1065	0.1362	0.1501	0.1560	0.1382	0.1750	0.2152	0.1967	0.1743	0.2200
	12+	0.1107	0.0783	0.0954	0.0639	0.1175	0.0997	0.1222	0.0808	0.1241	0.1238	0.1545	0.0983
55-59	0-3	0.5015	0.4787	0.4795	0.4754	0.4129	0.4177	0.4194	0.4133	0.3459	0.3561	0.3577	0.3513
	4-8	0.3450	0.3597	0.3559	0.3616	0.3669	0.3706	0.3658	0.3729	0.3720	0.3729	0.3668	0.3757
	9-11	0.0772	0.0966	0.0836	0.1111	0.1191	0.1273	0.1100	0.1464	0.1626	0.1639	0.1411	0.1887
	12+	0.0762	0.0650	0.0810	0.0519	0.1011	0.0844	0.1049	0.0674	0.1195	0.1070	0.1345	0.0844

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Table 27: Forecasted education profile (2013-2028) using the classical APC model (Reference category is P4). Brazil, Males

		2013			2018			2023			2028		
		Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS	Forecast	PS	OS
20-24	0-3	0.0746	0.0701	0.0782	0.0545	0.0513	0.0565	0.0391	0.0370	0.0400	0.0277	0.0263	0.0278
	4-8	0.3033	0.2888	0.3135	0.2615	0.2486	0.2685	0.2216	0.2106	0.2252	0.1851	0.1759	0.1854
	9-11	0.4886	0.4587	0.5122	0.5389	0.4919	0.5763	0.5843	0.5184	0.6357	0.6241	0.5384	0.6891
	12+	0.1335	0.1824	0.0962	0.1451	0.2082	0.0987	0.1550	0.2340	0.0991	0.1631	0.2594	0.0977
25-29	0-3	0.0868	0.0809	0.0915	0.0637	0.0591	0.0668	0.0459	0.0425	0.0477	0.0326	0.0301	0.0334
	4-8	0.3034	0.2851	0.3169	0.2630	0.2453	0.2742	0.2239	0.2073	0.2321	0.1877	0.1726	0.1928
	9-11	0.4339	0.4008	0.4613	0.4811	0.4297	0.5239	0.5240	0.4521	0.5831	0.5618	0.4681	0.6374
	12+	0.1759	0.2332	0.1303	0.1922	0.2658	0.1352	0.2062	0.2980	0.1371	0.2179	0.3292	0.1363
30-34	0-3	0.1122	0.1052	0.1176	0.0833	0.0776	0.0871	0.0607	0.0561	0.0630	0.0434	0.0399	0.0446
	4-8	0.3102	0.2918	0.3240	0.2720	0.2532	0.2844	0.2338	0.2155	0.2438	0.1975	0.1802	0.2047
	9-11	0.3912	0.3598	0.4180	0.4387	0.3898	0.4807	0.4824	0.4132	0.5414	0.5214	0.4300	0.5979
	12+	0.1864	0.2432	0.1404	0.2060	0.2794	0.1478	0.2231	0.3152	0.1518	0.2376	0.3498	0.1527
35-39	0-3	0.1382	0.1312	0.1433	0.1041	0.0981	0.1079	0.0768	0.0718	0.0792	0.0556	0.0516	0.0568
	4-8	0.3296	0.3125	0.3423	0.2932	0.2748	0.3053	0.2553	0.2366	0.2656	0.2181	0.1999	0.2259
	9-11	0.3532	0.3247	0.3782	0.4019	0.3576	0.4409	0.4477	0.3842	0.5030	0.4892	0.4041	0.5622
	12+	0.1790	0.2317	0.1362	0.2007	0.2695	0.1459	0.2203	0.3074	0.1522	0.2372	0.3444	0.1551
40-44	0-3	0.1725	0.1654	0.1772	0.1320	0.1253	0.1359	0.0987	0.0928	0.1014	0.0722	0.0673	0.0738
	4-8	0.3376	0.3216	0.3491	0.3049	0.2865	0.3171	0.2691	0.2495	0.2804	0.2325	0.2127	0.2420
	9-11	0.3108	0.2833	0.3359	0.3591	0.3178	0.3966	0.4054	0.3463	0.4586	0.4482	0.3682	0.5191
	12+	0.1792	0.2296	0.1377	0.2040	0.2703	0.1504	0.2269	0.3115	0.1596	0.2471	0.3519	0.1651
45-49	0-3	0.2050	0.1998	0.2076	0.1596	0.1538	0.1623	0.1213	0.1156	0.1233	0.0901	0.0850	0.0912
	4-8	0.3565	0.3430	0.3654	0.3276	0.3109	0.3383	0.2938	0.2748	0.3045	0.2576	0.2375	0.2670
	9-11	0.2740	0.2468	0.3000	0.3221	0.2840	0.3579	0.3695	0.3159	0.4190	0.4144	0.3415	0.4806
	12+	0.1645	0.2104	0.1269	0.1906	0.2513	0.1416	0.2154	0.2937	0.1532	0.2380	0.3360	0.1611
50-54	0-3	0.2438	0.2411	0.2434	0.1932	0.1890	0.1936	0.1491	0.1443	0.1498	0.1124	0.1075	0.1127
	4-8	0.3647	0.3541	0.3706	0.3411	0.3268	0.3491	0.3108	0.2935	0.3200	0.2765	0.2572	0.2853
	9-11	0.2417	0.2132	0.2704	0.2891	0.2519	0.3256	0.3371	0.2870	0.3849	0.3836	0.3163	0.4464
	12+	0.1498	0.1916	0.1156	0.1766	0.2323	0.1317	0.2029	0.2752	0.1454	0.2275	0.3189	0.1557
55-59	0-3	0.2960	0.2962	0.2919	0.2396	0.2379	0.2368	0.1888	0.1855	0.1871	0.1449	0.1407	0.1437
	4-8	0.3659	0.3575	0.3696	0.3496	0.3380	0.3549	0.3252	0.3101	0.3321	0.2948	0.2768	0.3023
	9-11	0.2057	0.1765	0.2367	0.2515	0.2145	0.2894	0.2992	0.2517	0.3463	0.3469	0.2849	0.4067
	12+	0.1323	0.1697	0.1018	0.1594	0.2096	0.1189	0.1868	0.2527	0.1345	0.2134	0.2976	0.1474

Source: Calculations based on PNAD data (IBGE). Obs: PS = Pessimistic scenario; OS = Optimistic scenario.

Figure 27: Fitted and projected probabilities for the Classical APC model when P4 is the reference category. Brazil, Females, 1983-2028

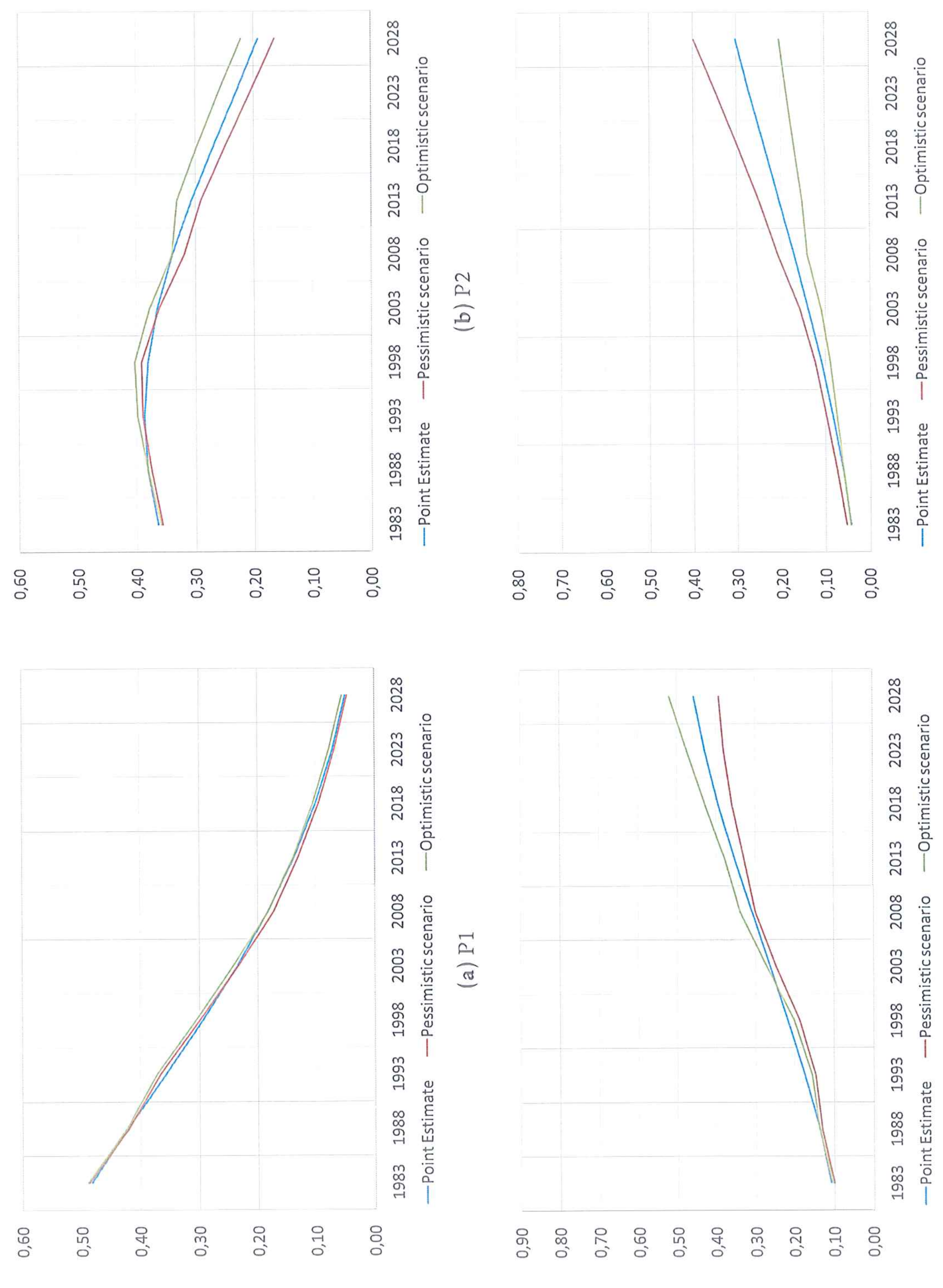
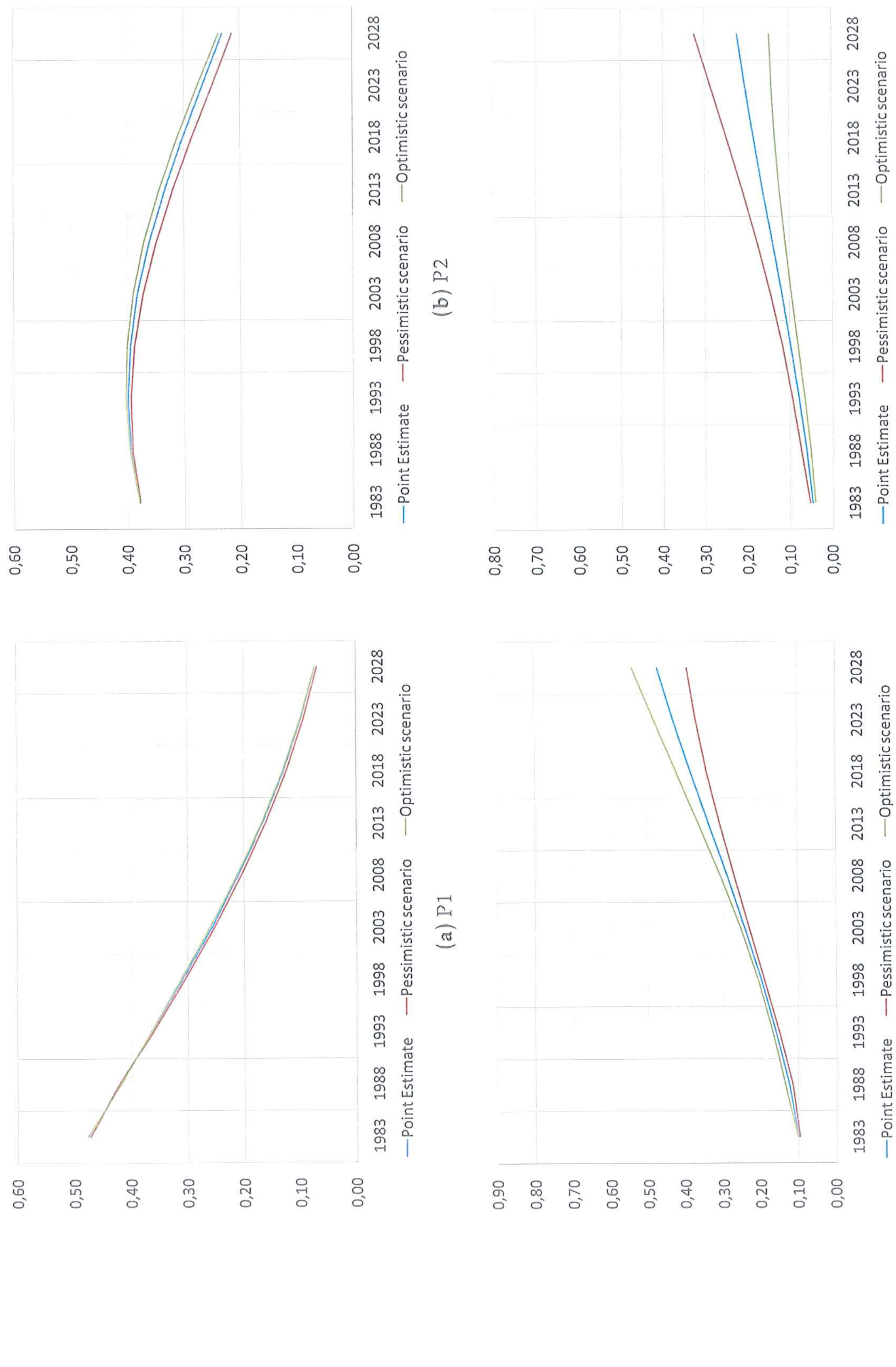


Figure 28: Fitted and projected probabilities for the Classical APC model when P4 is the reference category. Brazil, Males, 1983-2028



(a) P1

(b) P2

(c) P3

(d) P4

4.2 Bayesian APC Model

4.2.1 Computation

The Bayesian APC model was estimated using *rjags* package (Plummer, 2011). *Rjags* provides an interface from R R Core Team (2013) to the *JAGS* library for Bayesian data analysis. *JAGS* uses Markov Chain Monte Carlo (MCMC) to generate a sequence of dependent samples from the posterior distribution of the parameters. For a detailed description of the program see Plummer (2003).

Parameter inference from posterior samples is only valid when the MCMC chains, associated with the given parameter of interest, have converged. I test convergence of the parameters using Heidelberg and Welch diagnostic to accept or reject the null hypothesis that the Markov Chain is from a stationary distribution Heidelberger and Welch (1983).

Briefly, Heidelberg and Welch diagnostic test consists of two parts. First, a chain of N iterations of the parameter of interest is generated. Given a level of significance (I adopt here 5% level), the null hypothesis that the chain is from a stationary distribution is accepted or rejected. If the null hypothesis is rejected, the first 10% of the chain is discarded. The test statistic is calculated again for the new chain and acceptance of the null hypothesis is checked. This process is repeated until null hypothesis is accepted. If the test still rejects the null hypothesis, then it is recommended to generate a longer chain.

If the chain passes the first part, the second part of the Heidelberg and Welch diagnostic test - *halfwidth test* - is performed. Based on the chain not discarded from the first part of the test, the halfwidth test calculates half the width of the (1 - α)% credible interval around the mean. If the ratio of the halfwidth and the mean is lower than some ϵ, then the chain passes the test. Otherwise, the chain must be run out longer (Heidelberger and Welch, 1983).

Table 28 presents results of Heidelberg and Welch diagnostic test for the APC parameters and estimated probabilities. Tests were performed using significance level of 5% and 10%, and the results were the same. From the 108 estimated APC parameters for female and male groups, the chains converged for, respectively, 86 and 88 after 10,000 iterations and therefore passed in the first part of the test. In the second part of the test, only 41 of the 86 parameters passed the test in the females group and 31 of the 88 parameters passed the test in the males group. I demonstrated before that age, period and cohort parameters are fully unidentifiable in the likelihood, and hence the diagnostic test results confirm that there is dependency between the samples for age, period and cohort parameters. However, samples for p_{ij} remain stable and very close to independence: from the 320 estimated parameters, 302 passed in both the first and the second part of the Heidelberg and Welch diagnostic test in the females group and 319 passed both tests in the males group.

Table 28: Results of the Heidelberg & Welch diagnostic test by sample

	Females		Males	
	APC parameters	Estimated probabilities	APC parameters	Estimated probabilities
Total Parameters	108	320	108	320
Passed Stationarity Test	86	302	88	319
Passed Halfwidth Test	41	302	31	319

Source: Calculations based on PNAD data (1983-2008)

Figures 29 and 30 illustrate the posterior samples from 150,000 MCMC iterations for selected parameters and the trace plot, which displays iterations against sampled values for each variable in the chain and allows inspection of convergence in the posterior samples. For the sake of brevity, I next present one figure for selected APC parameters and one for the estimated probabilities, and I omit the remaining figures. After eliminating the burn-in iterations (10,000), the remaining draws can be judged to constitute a representative sample for the posterior distribution of the model parameters; these are then used for posterior inference of the APC models. Therefore, I use iterations 10001-150000 for the calculations that are presented next.

When evaluating trace plots, a chain is considered stationary if the distribution of points is not changing as the chain progresses, that is, when parameters present a relatively constant mean and variance. For most of the APC parameters, which are fully unidentifiable, there is a strong drift among individual model parameters as the chain progresses (Figure 29). This is due to unidentifiability in the likelihood. However, my interest lies in the probabilities p_{ij}, which are all independent, as Figure 30 demonstrates. Note that the center of the chain for estimated probabilities seems to be around a fixed mean, with very small fluctuations. This indicates that the chain has reached the right distribution. This pattern recurs for all probabilities.

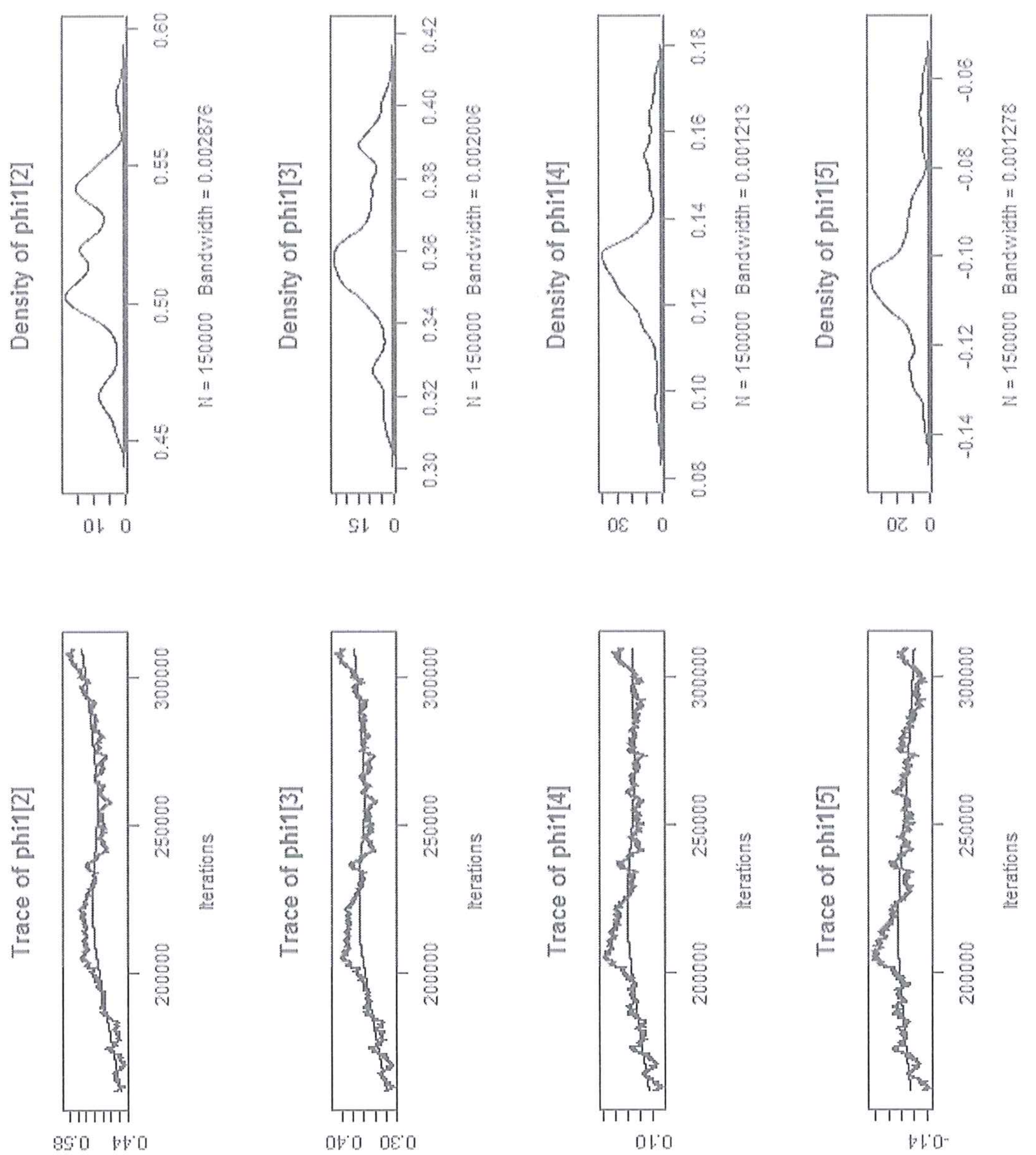


Figure 29: Trace and density of the posterior samples for selected period effects. Females

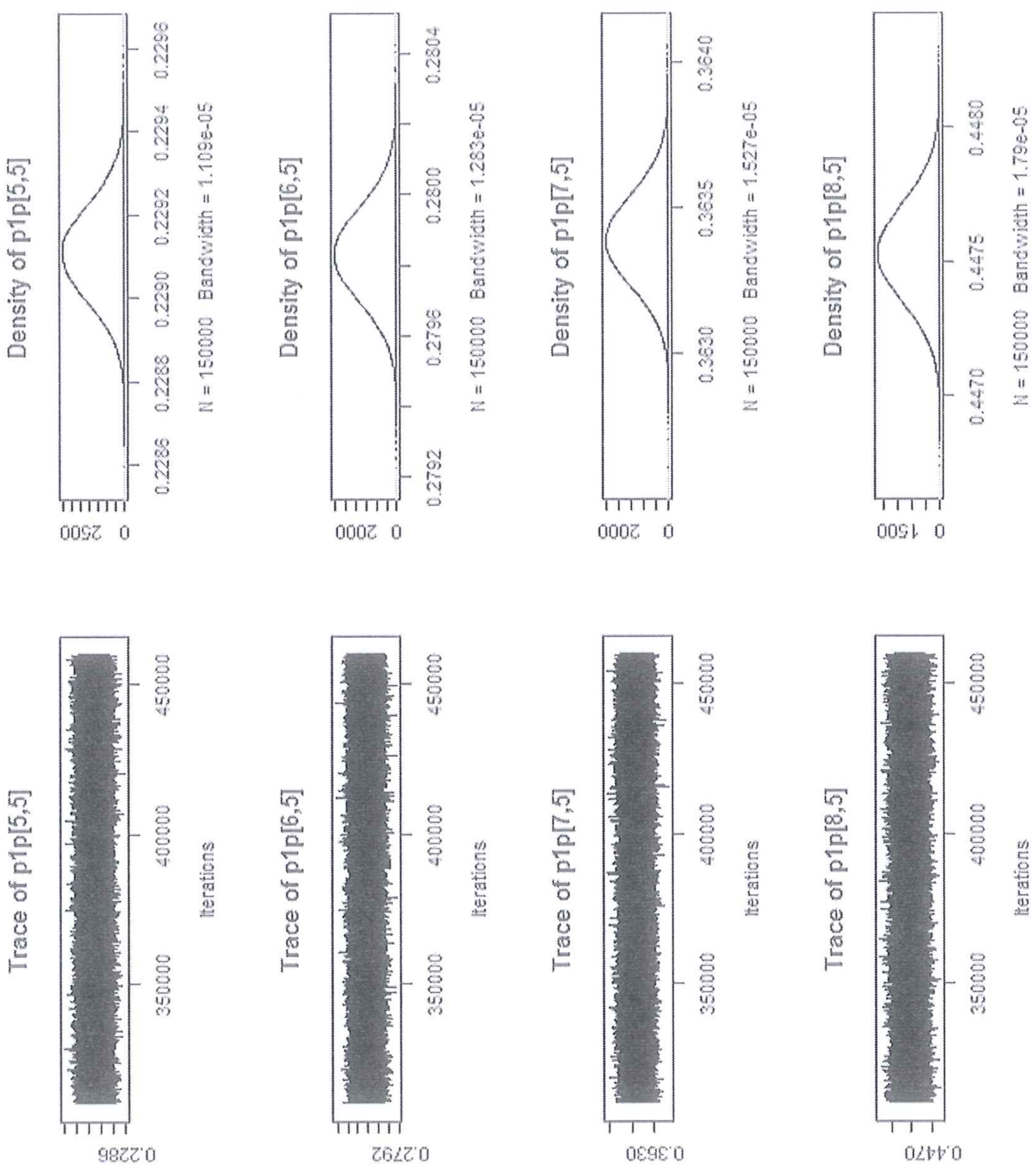


Figure 30: Trace and density of the posterior samples for selected probabilities. Females

4.2.2 Bayesian approach for the APC-Model Parameters and Forecasts

Unlike the classical APC model, in the Bayesian framework it is not possible to make inferences about model parameters describing the age period and cohort effects in order to discern what trends might be present because the model is not identified (Held and Rainer, 2001). Hence, I do not interpret here figures displaying estimated age, period and cohort parameters nor extrapolation trends. The estimated effects and their respective standard errors are presented in the Appendix A.2for the interested reader.

4.2.3 Education Profile: Fit and Empirical Forecasts

Results for the fitted education profile (1983-2008) and empirical forecasts (2013-2028) using the Bayesian APC model are presented in Tables 32-34. For the period 1983-2008, tables provide the observed profile, the posterior mean of each parameter and their respective 95% Highest Posterior Density (HPD) intervals. The widths of the 95% HPD intervals, relative to the fitted rates, indicate more precise estimates for the estimated sample (1983-2008), that is, more posterior certainty about the true value of the parameter. The increasing uncertainty that is associated with making empirical forecasts for future periods (2013-2028) is reflected by widening credible intervals. Not surprisingly, given the number of parameters in the model, there is good agreement between the observed values and the posterior means for 1983-2008.

Again, I present here graphics for the average education profile and their 95% credible intervals across all age groups with the aim of summarizing the evidence presented in the tables. Figures 31-32 illustrate these results. For a given education category, graphs are provided at the same scale for males and females. The increasing uncertainty that is associated with making empirical forecasts for future periods (2013-2028) again is reflected by widening credible intervals.

Table 29: Estimated and observed education profile (1983-1993) for females using the Bayesian approach for the APC model, Brazil.

		1983				1988				1993			
		Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper
20-24	0-3	0.2625	0.2647	0.2644	0.2649	0.2165	0.2188	0.2186	0.2191	0.1847	0.1811	0.1809	0.1813
	4-8	0.4494	0.4593	0.4590	0.4596	0.4543	0.4555	0.4553	0.4558	0.4803	0.4666	0.4663	0.4668
	9-11	0.2125	0.2067	0.2065	0.2069	0.2494	0.2471	0.2469	0.2473	0.2539	0.2679	0.2676	0.2681
	12+	0.0756	0.0693	0.0692	0.0695	0.0798	0.0785	0.0784	0.0787	0.0811	0.0844	0.0843	0.0846
25-29	0-3	0.3248	0.3240	0.3237	0.3242	0.2490	0.2527	0.2525	0.2530	0.2208	0.2223	0.2220	0.2225
	4-8	0.4256	0.4266	0.4263	0.4269	0.4342	0.4346	0.4344	0.4349	0.4383	0.4351	0.4348	0.4353
	9-11	0.1635	0.1648	0.1646	0.1650	0.2187	0.2142	0.2139	0.2144	0.2321	0.2303	0.2301	0.2305
	12+	0.0861	0.0846	0.0844	0.0847	0.0981	0.0985	0.0983	0.0986	0.1088	0.1124	0.1122	0.1126
30-34	0-3	0.4097	0.4054	0.4050	0.4057	0.3000	0.3081	0.3078	0.3084	0.2550	0.2544	0.2542	0.2546
	4-8	0.3848	0.3856	0.3853	0.3859	0.4240	0.4200	0.4197	0.4202	0.4263	0.4301	0.4298	0.4304
	9-11	0.1285	0.1296	0.1294	0.1298	0.1719	0.1730	0.1728	0.1732	0.2032	0.2004	0.2002	0.2007
	12+	0.0770	0.0794	0.0793	0.0796	0.1040	0.0990	0.0988	0.0991	0.1156	0.1151	0.1149	0.1153
35-39	0-3	0.4870	0.4833	0.4830	0.4836	0.3933	0.3859	0.3856	0.3862	0.3093	0.3083	0.3080	0.3085
	4-8	0.3635	0.3603	0.3599	0.3606	0.3809	0.3847	0.3844	0.3850	0.4145	0.4178	0.4175	0.4180
	9-11	0.0936	0.0955	0.0953	0.0956	0.1384	0.1383	0.1381	0.1385	0.1611	0.1622	0.1620	0.1624
	12+	0.0560	0.0610	0.0608	0.0611	0.0875	0.0911	0.0909	0.0913	0.1150	0.1118	0.1116	0.1119
40-44	0-3	0.5399	0.5491	0.5487	0.5495	0.4706	0.4674	0.4671	0.4677	0.3875	0.3903	0.3900	0.3906
	4-8	0.3502	0.3329	0.3325	0.3332	0.3634	0.3581	0.3578	0.3584	0.3784	0.3773	0.3771	0.3776
	9-11	0.0703	0.0738	0.0736	0.0740	0.0984	0.1024	0.1023	0.1026	0.1300	0.1283	0.1281	0.1285
	12+	0.0396	0.0442	0.0441	0.0443	0.0675	0.0720	0.0719	0.0722	0.1041	0.1041	0.1039	0.1043
45-49	0-3	0.6028	0.5964	0.5960	0.5968	0.5365	0.5291	0.5287	0.5295	0.4664	0.4678	0.4675	0.4682
	4-8	0.3171	0.3199	0.3195	0.3203	0.3400	0.3469	0.3465	0.3472	0.3562	0.3664	0.3661	0.3667
	9-11	0.0553	0.0550	0.0548	0.0552	0.0775	0.0780	0.0778	0.0781	0.1004	0.0933	0.0931	0.0935
	12+	0.0248	0.0287	0.0286	0.0288	0.0460	0.0461	0.0459	0.0462	0.0770	0.0725	0.0723	0.0727
50-54	0-3	0.6252	0.6300	0.6296	0.6305	0.5867	0.5865	0.5860	0.5869	0.5432	0.5393	0.5390	0.5397
	4-8	0.3041	0.2998	0.2994	0.3003	0.3280	0.3243	0.3239	0.3246	0.3299	0.3421	0.3418	0.3425
	9-11	0.0520	0.0509	0.0506	0.0511	0.0568	0.0594	0.0592	0.0596	0.0761	0.0724	0.0722	0.0726
	12+	0.0187	0.0193	0.0191	0.0194	0.0285	0.0299	0.0298	0.0300	0.0507	0.0462	0.0460	0.0463
55-59	0-3	0.6521	0.6522	0.6515	0.6528	0.6239	0.6188	0.6183	0.6193	0.5877	0.5948	0.5944	0.5952
	4-8	0.2883	0.2883	0.2877	0.2890	0.2997	0.3041	0.3036	0.3045	0.3167	0.3179	0.3175	0.3183
	9-11	0.0457	0.0457	0.0454	0.0460	0.0541	0.0553	0.0551	0.0556	0.0575	0.0551	0.0549	0.0553
	12+	0.0138	0.0138	0.0136	0.0139	0.0223	0.0218	0.0216	0.0219	0.0381	0.0322	0.0321	0.0324

Source: Calculations based on PNAD data (IBGE)

Table 30: Estimated and observed education profile (1998-2008) for females using the Bayesian approach for the APC model, Brazil.

		1998				2003				2008			
		Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper
20-24	0-3	0.1445	0.1430	0.1428	0.1432	0.0894	0.0900	0.0898	0.0901	0.0479	0.0479	0.0477	0.0480
	4-8	0.4142	0.4122	0.4119	0.4125	0.2997	0.3041	0.3039	0.3044	0.2314	0.2314	0.2311	0.2316
	9-11	0.3332	0.3353	0.3350	0.3355	0.4623	0.4557	0.4555	0.4560	0.5219	0.5219	0.5215	0.5222
	12+	0.1081	0.1095	0.1093	0.1097	0.1487	0.1502	0.1500	0.1504	0.1989	0.1989	0.1986	0.1992
25-29	0-3	0.1774	0.1732	0.1730	0.1734	0.1349	0.1355	0.1353	0.1357	0.0845	0.0839	0.0838	0.0840
	4-8	0.4311	0.4334	0.4332	0.4337	0.3554	0.3600	0.3597	0.3603	0.2589	0.2543	0.2541	0.2546
	9-11	0.2776	0.2775	0.2773	0.2778	0.3558	0.3533	0.3530	0.3535	0.4425	0.4487	0.4485	0.4490
	12+	0.1138	0.1159	0.1157	0.1161	0.1539	0.1512	0.1510	0.1514	0.2142	0.2130	0.2128	0.2133
30-34	0-3	0.2143	0.2111	0.2109	0.2113	0.1638	0.1628	0.1626	0.1630	0.1248	0.1256	0.1254	0.1257
	4-8	0.4279	0.4205	0.4202	0.4207	0.3914	0.4004	0.4001	0.4006	0.3271	0.3246	0.3244	0.3249
	9-11	0.2364	0.2415	0.2412	0.2417	0.3070	0.3023	0.3020	0.3025	0.3655	0.3662	0.3659	0.3664
	12+	0.1215	0.1270	0.1268	0.1272	0.1378	0.1345	0.1343	0.1347	0.1826	0.1836	0.1834	0.1839
35-39	0-3	0.2421	0.2406	0.2404	0.2409	0.1971	0.1978	0.1976	0.1980	0.1421	0.1502	0.1500	0.1504
	4-8	0.4123	0.4194	0.4192	0.4197	0.4034	0.3921	0.3918	0.3924	0.3651	0.3680	0.3677	0.3682
	9-11	0.2132	0.2129	0.2127	0.2131	0.2612	0.2662	0.2660	0.2665	0.3281	0.3206	0.3203	0.3209
	12+	0.1324	0.1270	0.1268	0.1272	0.1383	0.1439	0.1437	0.1441	0.1647	0.1612	0.1610	0.1614
40-44	0-3	0.2948	0.2964	0.2962	0.2967	0.2336	0.2291	0.2289	0.2293	0.1871	0.1857	0.1855	0.1859
	4-8	0.4072	0.4046	0.4043	0.4049	0.3957	0.3881	0.3878	0.3883	0.3384	0.3571	0.3569	0.3574
	9-11	0.1753	0.1726	0.1723	0.1728	0.2296	0.2352	0.2350	0.2355	0.2864	0.2814	0.2811	0.2816
	12+	0.1228	0.1264	0.1262	0.1266	0.1411	0.1476	0.1474	0.1478	0.1881	0.1758	0.1756	0.1760
45-49	0-3	0.3648	0.3731	0.3728	0.3734	0.2808	0.2798	0.2796	0.2801	0.2122	0.2128	0.2126	0.2130
	4-8	0.3860	0.3861	0.3858	0.3864	0.3939	0.3982	0.3979	0.3984	0.3903	0.3774	0.3771	0.3777
	9-11	0.1382	0.1358	0.1356	0.1360	0.1901	0.1904	0.1902	0.1906	0.2429	0.2483	0.2480	0.2485
	12+	0.1110	0.1051	0.1049	0.1053	0.1351	0.1316	0.1314	0.1318	0.1545	0.1615	0.1613	0.1617
50-54	0-3	0.4550	0.4595	0.4591	0.4598	0.3585	0.3634	0.3631	0.3637	0.2753	0.2688	0.2686	0.2691
	4-8	0.3619	0.3657	0.3653	0.3660	0.3762	0.3718	0.3715	0.3721	0.3816	0.3792	0.3789	0.3795
	9-11	0.1027	0.1014	0.1012	0.1016	0.1504	0.1546	0.1544	0.1548	0.2079	0.2071	0.2069	0.2074
	12+	0.0805	0.0734	0.0732	0.0736	0.1149	0.1102	0.1100	0.1104	0.1352	0.1449	0.1447	0.1451
55-59	0-3	0.5275	0.5293	0.5289	0.5297	0.4461	0.4475	0.4472	0.4479	0.3529	0.3492	0.3489	0.3495
	4-8	0.3426	0.3418	0.3414	0.3421	0.3581	0.3536	0.3533	0.3540	0.3526	0.3532	0.3529	0.3536
	9-11	0.0791	0.0786	0.0784	0.0788	0.1134	0.1158	0.1155	0.1160	0.1682	0.1674	0.1671	0.1677
	12+	0.0509	0.0503	0.0501	0.0504	0.0824	0.0831	0.0829	0.0833	0.1263	0.1302	0.1299	0.1304

Source: Calculations based on PNAD data (IBGE)

Table 31: Forecasted education profile (2013-2028) for females using the Bayesian approach for the APC model, Brazil.

		2013			2018			2023			2028		
		Mean	HPD 95% Lower	HPD 95% Upper	Mean	HPD 95% Lower	HPD 95% Upper	Mean	HPD 95% Lower	HPD 95% Upper	Mean	HPD 95% Lower	HPD 95% Upper
20-24	0-3	0.0259	0.0131	0.0403	0.0156	0.0017	0.0350	0.0114	0.0000	0.0344	0.0108	0.0000	0.0386
	4-8	0.1697	0.0951	0.2507	0.1299	0.0210	0.2755	0.1103	0.0002	0.3236	0.1049	0.0000	0.4107
	9-11	0.5557	0.4429	0.6653	0.5555	0.3081	0.7932	0.5328	0.1587	0.8862	0.5021	0.0427	0.9214
	12+	0.2487	0.1508	0.3493	0.2990	0.0845	0.5367	0.3455	0.0258	0.7095	0.3822	0.0000	0.8294
25-29	0-3	0.0455	0.0272	0.0655	0.0264	0.0046	0.0536	0.0181	0.0001	0.0497	0.0154	0.0000	0.0529
	4-8	0.1907	0.1240	0.2604	0.1439	0.0356	0.2753	0.1176	0.0003	0.3085	0.1072	0.0000	0.3828
	9-11	0.4917	0.3967	0.5848	0.5019	0.2814	0.7178	0.4887	0.1427	0.8241	0.4657	0.0420	0.8852
	12+	0.2721	0.1863	0.3589	0.3277	0.1286	0.5435	0.3757	0.0555	0.7116	0.4117	0.0032	0.8222
30-34	0-3	0.0790	0.0481	0.1122	0.0457	0.0093	0.0891	0.0299	0.0002	0.0781	0.0236	0.0000	0.0786
	4-8	0.2245	0.1513	0.3036	0.1721	0.0501	0.3137	0.1378	0.0034	0.3356	0.1206	0.0000	0.3963
	9-11	0.4464	0.3536	0.5362	0.4714	0.2658	0.6755	0.4693	0.1444	0.7885	0.4530	0.0422	0.8545
	12+	0.2501	0.1700	0.3316	0.3108	0.1223	0.5067	0.3630	0.0643	0.6822	0.4028	0.0115	0.8009
35-39	0-3	0.1175	0.0740	0.1655	0.0781	0.0183	0.1505	0.0502	0.0005	0.1288	0.0374	0.0000	0.1242
	4-8	0.2927	0.2045	0.3814	0.2037	0.0630	0.3614	0.1640	0.0052	0.3828	0.1401	0.0000	0.4337
	9-11	0.3755	0.2880	0.4627	0.4366	0.2391	0.6340	0.4472	0.1346	0.7553	0.4395	0.0438	0.8313
	12+	0.2143	0.1421	0.2869	0.2816	0.1066	0.4653	0.3386	0.0583	0.6451	0.3831	0.0018	0.7595
40-44	0-3	0.1428	0.0910	0.1990	0.1170	0.0287	0.2191	0.0847	0.0014	0.2123	0.0610	0.0000	0.2018
	4-8	0.3306	0.2366	0.4225	0.2622	0.0969	0.4477	0.1889	0.0116	0.4299	0.1610	0.0000	0.4763
	9-11	0.3322	0.2492	0.4171	0.3716	0.1874	0.5618	0.4135	0.1141	0.7134	0.4159	0.0233	0.7859
	12+	0.1944	0.1299	0.2651	0.2492	0.0887	0.4164	0.3129	0.0421	0.5958	0.3622	0.0000	0.7273
45-49	0-3	0.1742	0.1126	0.2396	0.1396	0.0369	0.2587	0.1223	0.0041	0.2973	0.0967	0.0000	0.3114
	4-8	0.3437	0.2493	0.4361	0.3177	0.1342	0.5196	0.2571	0.0221	0.5344	0.1951	0.0000	0.5399
	9-11	0.2920	0.2118	0.3716	0.3350	0.1551	0.5132	0.3621	0.0799	0.6407	0.3934	0.0138	0.7475
	12+	0.1901	0.1238	0.2558	0.2077	0.0669	0.3584	0.2585	0.0246	0.5131	0.3148	0.0000	0.6692
50-54	0-3	0.2062	0.1355	0.2800	0.1750	0.0494	0.3165	0.1488	0.0068	0.3530	0.1385	0.0000	0.4228
	4-8	0.3540	0.2602	0.4469	0.3206	0.1329	0.5157	0.2987	0.0399	0.5932	0.2486	0.0000	0.6186
	9-11	0.2647	0.1883	0.3406	0.3018	0.1317	0.4737	0.3347	0.0649	0.6024	0.3512	0.0000	0.6864
	12+	0.1752	0.1129	0.2374	0.2026	0.0647	0.3493	0.2178	0.0147	0.4509	0.2618	0.0000	0.5948
55-59	0-3	0.2592	0.1762	0.3457	0.2049	0.0628	0.3647	0.1827	0.0092	0.4172	0.1642	0.0000	0.4826
	4-8	0.3534	0.2601	0.4436	0.3257	0.1379	0.5202	0.2962	0.0351	0.5814	0.2800	0.0000	0.6546
	9-11	0.2195	0.1517	0.2901	0.2711	0.1120	0.4369	0.2984	0.0492	0.5582	0.3219	0.0000	0.6523
	12+	0.1680	0.1089	0.2287	0.1983	0.0654	0.3441	0.2228	0.0149	0.4529	0.2339	0.0000	0.5525

Source: Calculations based on PNAD data (IBGE)

Table 32: Estimated and observed education profile (1983-1993) for males using the Bayesian approach for the APC model, Brazil.

		1983				1988				1993			
		Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper
20-24	0-3	0.2863	0.2921	0.2918	0.2923	0.2575	0.2648	0.2645	0.2650	0.2481	0.2396	0.2393	0.2398
	4-8	0.4658	0.4724	0.4721	0.4727	0.4698	0.4722	0.4719	0.4725	0.4908	0.4829	0.4826	0.4832
	9-11	0.1913	0.1843	0.1841	0.1845	0.2155	0.2091	0.2089	0.2093	0.2057	0.2188	0.2186	0.2191
	12+	0.0567	0.0512	0.0511	0.0513	0.0572	0.0539	0.0538	0.0540	0.0554	0.0588	0.0586	0.0589
25-29	0-3	0.3191	0.3132	0.3129	0.3135	0.2655	0.2676	0.2674	0.2679	0.2461	0.2501	0.2499	0.2504
	4-8	0.4459	0.4573	0.4570	0.4576	0.4504	0.4550	0.4548	0.4553	0.4649	0.4496	0.4493	0.4499
	9-11	0.1576	0.1566	0.1564	0.1568	0.2034	0.2002	0.2000	0.2005	0.2074	0.2083	0.2081	0.2086
	12+	0.0774	0.0730	0.0728	0.0731	0.0807	0.0771	0.0769	0.0772	0.0816	0.0920	0.0918	0.0921
30-34	0-3	0.3881	0.3881	0.3878	0.3885	0.3127	0.3117	0.3115	0.3120	0.2771	0.2751	0.2749	0.2754
	4-8	0.4173	0.4145	0.4141	0.4148	0.4348	0.4356	0.4353	0.4359	0.4244	0.4288	0.4285	0.4290
	9-11	0.1185	0.1206	0.1204	0.1208	0.1641	0.1671	0.1668	0.1673	0.1986	0.1944	0.1942	0.1946
	12+	0.0761	0.0768	0.0767	0.0770	0.0884	0.0856	0.0854	0.0858	0.0999	0.1017	0.1016	0.1019
35-39	0-3	0.4658	0.4594	0.4590	0.4597	0.3720	0.3729	0.3726	0.3732	0.3056	0.3075	0.3072	0.3077
	4-8	0.3866	0.3831	0.3828	0.3834	0.4210	0.4139	0.4136	0.4142	0.4226	0.4288	0.4285	0.4290
	9-11	0.0854	0.0893	0.0891	0.0895	0.1246	0.1305	0.1303	0.1307	0.1636	0.1618	0.1616	0.1620
	12+	0.0622	0.0682	0.0681	0.0684	0.0823	0.0827	0.0825	0.0829	0.1082	0.1020	0.1018	0.1021
40-44	0-3	0.5269	0.5306	0.5302	0.5310	0.4562	0.4515	0.4512	0.4519	0.3728	0.3759	0.3756	0.3762
	4-8	0.3641	0.3481	0.3477	0.3484	0.3820	0.3749	0.3745	0.3752	0.3910	0.3977	0.3974	0.3980
	9-11	0.0644	0.0662	0.0660	0.0664	0.0955	0.0959	0.0957	0.0960	0.1316	0.1236	0.1234	0.1238
	12+	0.0447	0.0552	0.0550	0.0553	0.0663	0.0777	0.0776	0.0779	0.1047	0.1027	0.1026	0.1029
45-49	0-3	0.5555	0.5535	0.5530	0.5539	0.5155	0.5095	0.5091	0.5098	0.4398	0.4416	0.4413	0.4420
	4-8	0.3473	0.3452	0.3448	0.3456	0.3585	0.3584	0.3580	0.3587	0.3744	0.3771	0.3768	0.3775
	9-11	0.0633	0.0628	0.0626	0.0630	0.0730	0.0732	0.0730	0.0733	0.0913	0.0920	0.0918	0.0921
	12+	0.0339	0.0386	0.0384	0.0387	0.0529	0.0590	0.0589	0.0592	0.0946	0.0893	0.0891	0.0895
50-54	0-3	0.5887	0.5935	0.5930	0.5940	0.5438	0.5386	0.5382	0.5390	0.5074	0.5056	0.5052	0.5060
	4-8	0.3288	0.3216	0.3211	0.3221	0.3494	0.3494	0.3490	0.3498	0.3381	0.3544	0.3540	0.3548
	9-11	0.0536	0.0543	0.0540	0.0545	0.0716	0.0704	0.0702	0.0706	0.0708	0.0714	0.0712	0.0716
	12+	0.0289	0.0306	0.0304	0.0308	0.0352	0.0416	0.0414	0.0418	0.0837	0.0686	0.0684	0.0688
55-59	0-3	0.6368	0.6368	0.6361	0.6375	0.5959	0.5909	0.5904	0.5914	0.5385	0.5469	0.5465	0.5474
	4-8	0.2917	0.2917	0.2910	0.2924	0.3112	0.3187	0.3182	0.3192	0.3360	0.3386	0.3382	0.3391
	9-11	0.0475	0.0475	0.0471	0.0478	0.0589	0.0582	0.0580	0.0585	0.0644	0.0667	0.0665	0.0669
	12+	0.0241	0.0241	0.0238	0.0243	0.0340	0.0322	0.0320	0.0323	0.0611	0.0477	0.0476	0.0479

Source: Calculations based on PNAD data (IBGE)

Table 33: Estimated and observed education profile (1998-2008) for males using the Bayesian approach for the APC model, Brazil.

		1998				2003				2008			
		Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper	Observed	Mean	HPD 95% Lower	HPD 95% Upper
20-24	0-3	0.1987	0.1971	0.1969	0.1974	0.1304	0.1288	0.1286	0.1289	0.0788	0.0788	0.0787	0.0790
	4-8	0.4490	0.4417	0.4414	0.4420	0.3505	0.3555	0.3552	0.3558	0.2910	0.2910	0.2907	0.2913
	9-11	0.2782	0.2831	0.2828	0.2833	0.4137	0.4098	0.4095	0.4101	0.4916	0.4917	0.4913	0.4920
	12+	0.0742	0.0781	0.0780	0.0783	0.1054	0.1059	0.1058	0.1061	0.1385	0.1385	0.1383	0.1387
25-29	0-3	0.2214	0.2202	0.2200	0.2205	0.1759	0.1738	0.1736	0.1740	0.1144	0.1161	0.1159	0.1163
	4-8	0.4595	0.4581	0.4578	0.4584	0.3924	0.4017	0.4014	0.4019	0.3069	0.3017	0.3014	0.3020
	9-11	0.2314	0.2333	0.2330	0.2335	0.3157	0.3117	0.3114	0.3119	0.4170	0.4208	0.4205	0.4211
	12+	0.0877	0.0884	0.0882	0.0886	0.1160	0.1129	0.1127	0.1131	0.1618	0.1614	0.1612	0.1616
30-34	0-3	0.2583	0.2511	0.2509	0.2514	0.2084	0.2135	0.2133	0.2137	0.1698	0.1733	0.1731	0.1735
	4-8	0.4275	0.4228	0.4226	0.4231	0.4189	0.4234	0.4231	0.4237	0.3567	0.3550	0.3547	0.3553
	9-11	0.2134	0.2182	0.2179	0.2184	0.2653	0.2604	0.2601	0.2606	0.3312	0.3307	0.3304	0.3309
	12+	0.1007	0.1079	0.1077	0.1080	0.1074	0.1028	0.1026	0.1029	0.1423	0.1410	0.1408	0.1412
35-39	0-3	0.2690	0.2648	0.2645	0.2650	0.2328	0.2332	0.2330	0.2335	0.1989	0.2034	0.2032	0.2036
	4-8	0.4185	0.4210	0.4207	0.4213	0.4132	0.4075	0.4072	0.4078	0.3908	0.3965	0.3962	0.3968
	9-11	0.2017	0.2053	0.2051	0.2056	0.2446	0.2451	0.2448	0.2453	0.2909	0.2817	0.2815	0.2820
	12+	0.1109	0.1089	0.1087	0.1091	0.1094	0.1142	0.1140	0.1144	0.1194	0.1184	0.1182	0.1186
40-44	0-3	0.2963	0.3033	0.3031	0.3036	0.2546	0.2524	0.2521	0.2526	0.2329	0.2282	0.2280	0.2284
	4-8	0.4156	0.4117	0.4114	0.4120	0.4088	0.3959	0.3956	0.3961	0.3509	0.3710	0.3707	0.3713
	9-11	0.1741	0.1695	0.1693	0.1698	0.2216	0.2295	0.2292	0.2297	0.2612	0.2621	0.2618	0.2623
	12+	0.1140	0.1155	0.1153	0.1157	0.1150	0.1223	0.1221	0.1225	0.1550	0.1387	0.1385	0.1389
45-49	0-3	0.3530	0.3591	0.3588	0.3594	0.2810	0.2790	0.2788	0.2793	0.2371	0.2374	0.2371	0.2376
	4-8	0.3893	0.4002	0.3999	0.4005	0.4089	0.4063	0.4060	0.4066	0.3827	0.3770	0.3768	0.3773
	9-11	0.1402	0.1323	0.1320	0.1325	0.1877	0.1937	0.1934	0.1939	0.2472	0.2484	0.2481	0.2486
	12+	0.1176	0.1084	0.1082	0.1086	0.1223	0.1210	0.1208	0.1212	0.1330	0.1372	0.1370	0.1375
50-54	0-3	0.4260	0.4299	0.4295	0.4302	0.3390	0.3380	0.3377	0.3383	0.2681	0.2678	0.2675	0.2681
	4-8	0.3623	0.3741	0.3737	0.3744	0.3934	0.3908	0.3904	0.3911	0.3925	0.3814	0.3811	0.3817
	9-11	0.1011	0.1004	0.1002	0.1006	0.1501	0.1552	0.1550	0.1555	0.2152	0.2134	0.2131	0.2136
	12+	0.1107	0.0956	0.0954	0.0958	0.1175	0.1161	0.1158	0.1163	0.1241	0.1374	0.1372	0.1376
55-59	0-3	0.5015	0.5059	0.5055	0.5063	0.4129	0.4191	0.4187	0.4195	0.3459	0.3363	0.3359	0.3366
	4-8	0.3450	0.3470	0.3466	0.3473	0.3669	0.3641	0.3637	0.3644	0.3720	0.3664	0.3660	0.3667
	9-11	0.0772	0.0752	0.0749	0.0754	0.1191	0.1151	0.1149	0.1154	0.1626	0.1667	0.1664	0.1670
	12+	0.0762	0.0720	0.0718	0.0722	0.1011	0.1017	0.1015	0.1019	0.1195	0.1307	0.1304	0.1310

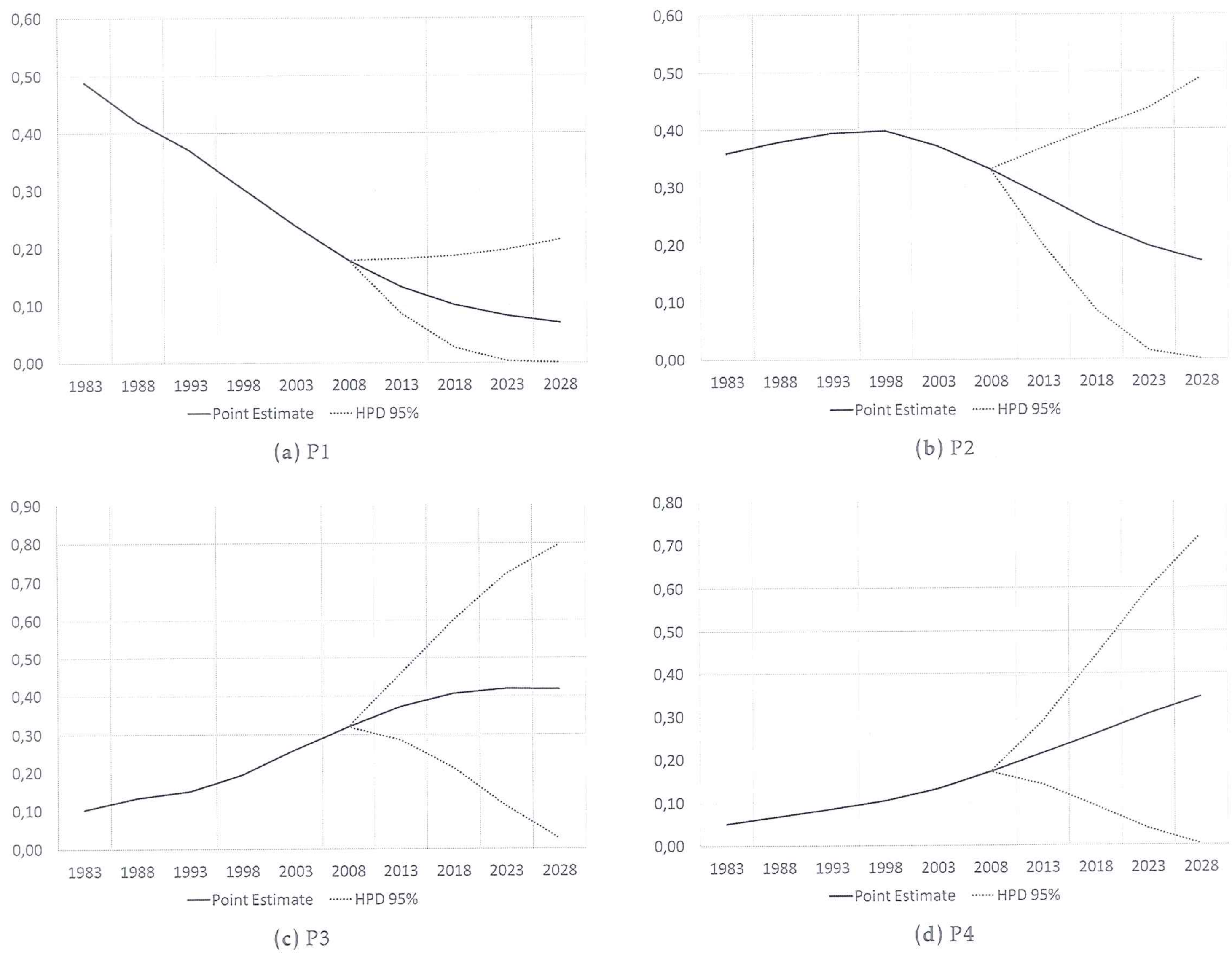
Source: Calculations based on PNAD data (IBGE)

Table 34: Forecasted education profile (2013-2028) for males using the Bayesian approach for the APC model, Brazil.

		2013			2018			2023			2028		
		Mean	HPD 95% Lower	HPD 95% Upper	Mean	HPD 95% Lower	HPD 95% Upper	Mean	HPD 95% Lower	HPD 95% Upper	Mean	HPD 95% Lower	HPD 95% Upper
20-24	0-3	0.0488	0.0254	0.0748	0.0331	0.0042	0.0734	0.0267	0.0000	0.0813	0.0262	0.0000	0.0996
	4-8	0.2285	0.1344	0.3272	0.1832	0.0337	0.3686	0.1581	0.0018	0.4342	0.1485	0.0000	0.5316
	9-11	0.5494	0.4344	0.6604	0.5706	0.3188	0.8039	0.5608	0.1695	0.8994	0.5350	0.0562	0.9442
	12+	0.1733	0.0901	0.2631	0.2130	0.0320	0.4305	0.2543	0.0020	0.6151	0.2903	0.0000	0.7719
25-29	0-3	0.0721	0.0440	0.1018	0.0474	0.0095	0.0946	0.0359	0.0006	0.0989	0.0325	0.0000	0.1166
	4-8	0.2412	0.1651	0.3204	0.1914	0.0527	0.3474	0.1604	0.0045	0.3967	0.1459	0.0000	0.4797
	9-11	0.4823	0.3873	0.5754	0.5130	0.2907	0.7271	0.5141	0.1514	0.8388	0.4979	0.0464	0.9025
	12+	0.2045	0.1287	0.2852	0.2482	0.0678	0.4518	0.2897	0.0117	0.6229	0.3237	0.0000	0.7669
30-34	0-3	0.1176	0.0740	0.1638	0.0774	0.0192	0.1484	0.0562	0.0007	0.1445	0.0474	0.0000	0.1620
	4-8	0.2594	0.1804	0.3409	0.2095	0.0697	0.3667	0.1734	0.0064	0.3980	0.1532	0.0000	0.4676
	9-11	0.4274	0.3358	0.5191	0.4700	0.2627	0.6728	0.4835	0.1466	0.7952	0.4767	0.0436	0.8666
	12+	0.1955	0.1240	0.2731	0.2432	0.0692	0.4284	0.2870	0.0195	0.5968	0.3227	0.0000	0.7369
35-39	0-3	0.1669	0.1079	0.2282	0.1186	0.0309	0.2207	0.0850	0.0014	0.2107	0.0682	0.0000	0.2254
	4-8	0.3261	0.2372	0.4151	0.2375	0.0830	0.4043	0.1982	0.0141	0.4373	0.1721	0.0000	0.4921
	9-11	0.3473	0.2628	0.4348	0.4274	0.2276	0.6224	0.4535	0.1344	0.7566	0.4573	0.0426	0.8365
	12+	0.1598	0.0966	0.2239	0.2165	0.0594	0.3868	0.2634	0.0128	0.5484	0.3024	0.0000	0.6961
40-44	0-3	0.2011	0.1325	0.2717	0.1713	0.0524	0.3108	0.1304	0.0039	0.3107	0.1017	0.0000	0.3239
	4-8	0.3577	0.2657	0.4475	0.2904	0.1174	0.4753	0.2158	0.0174	0.4631	0.1879	0.0000	0.5145
	9-11	0.2974	0.2169	0.3790	0.3494	0.1670	0.5364	0.4082	0.0969	0.6956	0.4225	0.0229	0.7871
	12+	0.1438	0.0864	0.2034	0.1889	0.0505	0.3422	0.2456	0.0129	0.5153	0.2879	0.0000	0.6636
45-49	0-3	0.2165	0.1443	0.2912	0.1968	0.0640	0.3518	0.1763	0.0089	0.4020	0.1436	0.0000	0.4325
	4-8	0.3498	0.2591	0.4388	0.3350	0.1522	0.5302	0.2738	0.0341	0.5474	0.2111	0.0000	0.5502
	9-11	0.2791	0.2017	0.3604	0.3080	0.1388	0.4877	0.3456	0.0662	0.6207	0.3895	0.0030	0.7331
	12+	0.1547	0.0955	0.2175	0.1603	0.0397	0.2975	0.2043	0.0069	0.4440	0.2558	0.0000	0.6138
50-54	0-3	0.2295	0.1542	0.3074	0.2155	0.0712	0.3789	0.2043	0.0135	0.4538	0.1914	0.0000	0.5357
	4-8	0.3484	0.2584	0.4372	0.3205	0.1392	0.5089	0.3074	0.0480	0.5891	0.2560	0.0000	0.6089
	9-11	0.2680	0.1901	0.3459	0.2920	0.1237	0.4659	0.3114	0.0547	0.5804	0.3361	0.0000	0.6714
	12+	0.1541	0.0955	0.2167	0.1720	0.0448	0.3135	0.1769	0.0023	0.3946	0.2165	0.0000	0.5464
55-59	0-3	0.2684	0.1838	0.3540	0.2364	0.0805	0.4093	0.2299	0.0170	0.4965	0.2253	0.0000	0.5971
	4-8	0.3527	0.2630	0.4410	0.3188	0.1376	0.5046	0.2941	0.0419	0.5684	0.2845	0.0002	0.6385
	9-11	0.2256	0.1555	0.2989	0.2746	0.1129	0.4451	0.2894	0.0385	0.5460	0.3008	0.0000	0.6292
	12+	0.1534	0.0956	0.2145	0.1702	0.0446	0.3099	0.1867	0.0056	0.4089	0.1895	0.0000	0.4982

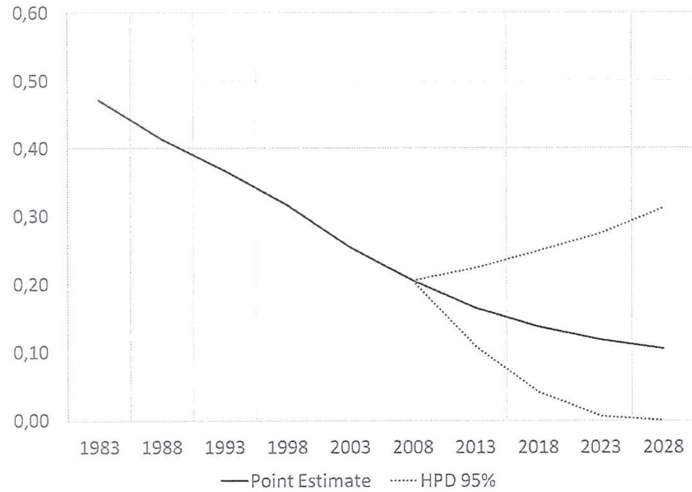
Source: Calculations based on PNAD data (IBGE)

Figure 31: Fitted and projected probabilities for the Bayesian APC model. Brazil, Females, 1983-2028

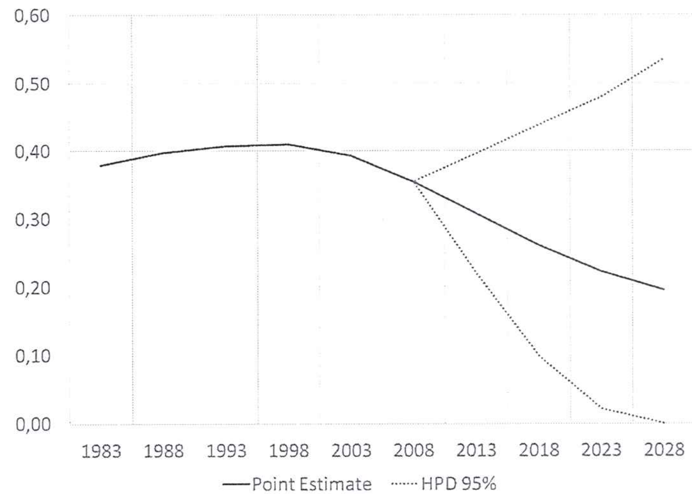


(a) P1 (b) P2 (c) P3 (d) P4

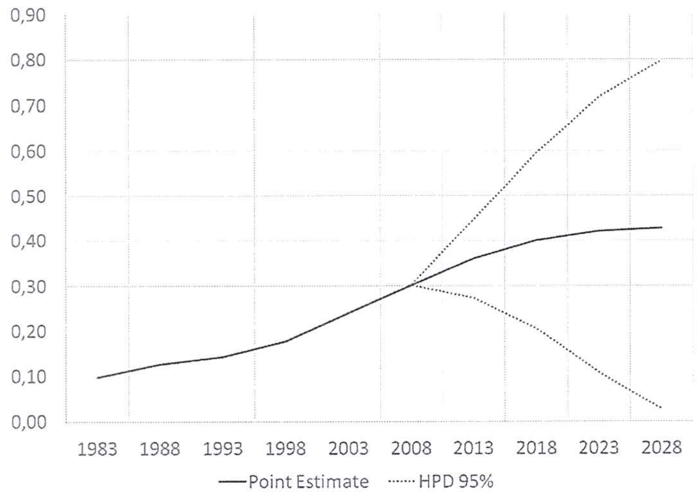
Figure 32: Fitted and projected probabilities for the Bayesian APC model. Brazil, Males, 1983-2028



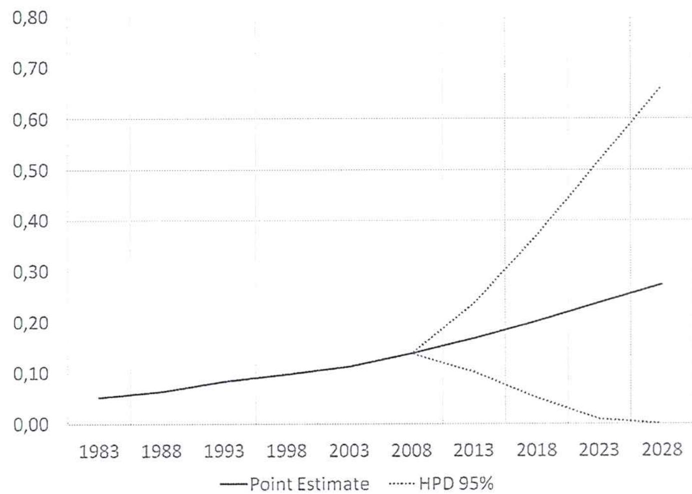
(a) P1



(b) P2



(c) P3



(d) P4

4.3 Inter-comparison of Results and Substantive Findings

In this section I first compare the performance of the classical and Bayesian APC model for in-sample predictions. In this study, I compared age-by-period probabilities obtained from applying classical and Bayesian APC methods with the observed proportions. Then, residuals were calculated and the sum of the squared residuals (SSR) was used to assess the accuracy of the fitted probabilities. Results are displayed in Table 35. Considering all time periods, the SSR for the classical APC model is about 11 times higher than that for from the Bayesian APC model for males and about 17 times higher for females. Hence, I conclude that the Bayesian APC model fits the observed data better.

Table 35: Sum of squared residuals (SSR) by APC model and year.

	Classical		Bayesian	
	Males	Females	Males	Females
1983	0.004	0.004	0.001	0.001
1998	0.005	0.006	0.001	0.000
1993	0.016	0.019	0.002	0.001
1998	0.019	0.018	0.001	0.000
2003	0.013	0.013	0.001	0.001
2008	0.016	0.014	0.002	0.001
Average SSR	0.012	0.013	0.001	0.001

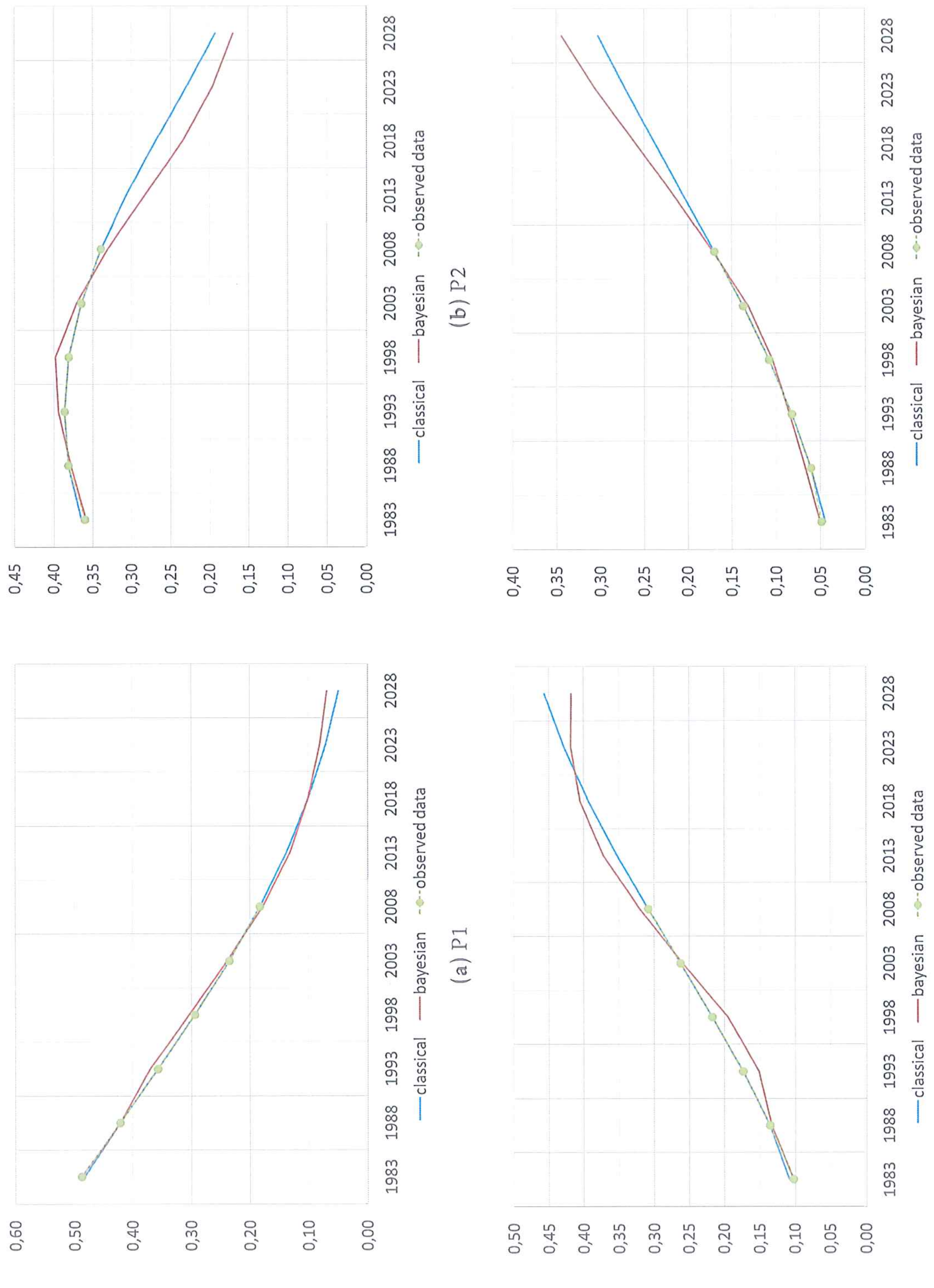
Source: Calculations based on PNAD data (1983-2008)

From now on I provide a substantive analysis of the results. With the aim to reduce dimensionality, I computed the average education profile across all age groups, which provides a relevant summary of past and future trends of schooling in Brazil. In regards to the uncertainty in projected education profile in classical and Bayesian estimates, I refrain here from comparing uncertainty measures because scenarios (classical approach) and credible intervals (Bayesian framework) are not comparable (see sections 3.2.2.1 and 3.3.2).

Figures 33 and 34 display fitted and extrapolated trends in the education profile according to the classical and the Bayesian APC models. As expected by the presence of schooling differentials by sex, I observe differences in level between males and females, especially for P1 and P4. Overall, trends derived from classical and Bayesian models are similar for P1. For P2, P3 and P4, the classical and the Bayesian APC model disagree somehow, and the Bayesian APC model gives more optimistic predictions regarding increases in education attainment than the classical model. Finally, considering future trends, Bayesian projections tend to be more optimistic than the classical ones in regards to increases in the proportion of males and females with some college education (P4) from 2013-2028.

As there is not much disagreement in general trends between classical and Bayesian estimates, I interpret them from now on indistinctly. Overall, projections suggest good news regarding schooling of Brazilians, but sex differentials in educational attainment may be anticipated as well: the country will experience a considerable increase in the proportion of individuals with high schooling (P3 and P4 levels) and a decrease of the share of the population in low schooling levels (P1 and P2). The proportion of people with 0-3 years of schooling (P1) will go from 49 percent for females and 47 percent for males in 1983 to 6 percent for females and 9 percent for males in 2028. Meanwhile, the proportion of people with some college education (P4) will go from 5 percent for females and males in 1983 to 32 percent for females and 20 percent for males in 2028.

Despite the expected improvement in the education profile of the Brazilian population, results indicate that the proportion of individuals with some college education (P4) in 2028 will still be low compared to those observed in the present in Latin American countries with similar level of development in the present. As an example, Table 36 presents the proportion of females aged 20-59 with some college education (P4). In 2028, projected P4 in Brazil is 32 percent. This forecast is lower than the observed P4 in 2001 in Argentina (42 percent), and similar to the observed P4 in 2010 in Mexico (33 percent) and in 2006 in Uruguay (32 percent). This finding has clear implications for the future capacity of Brazil to reach advanced stages of development (Barro, 1991; Mankiw et al., 1992; Krueger and Lindahl, 2000), considering that the country may not achieve in the near future high levels of highly educated individuals.



(a) P1 (b) P2 (c) P3 (d) P4

Figure 33: Fitted and projected probabilities by APC model. Brazil, Females, 1983-2028

Figure 34: Fitted and projected probabilities by APC model. Brazil, Males, 1983-2028

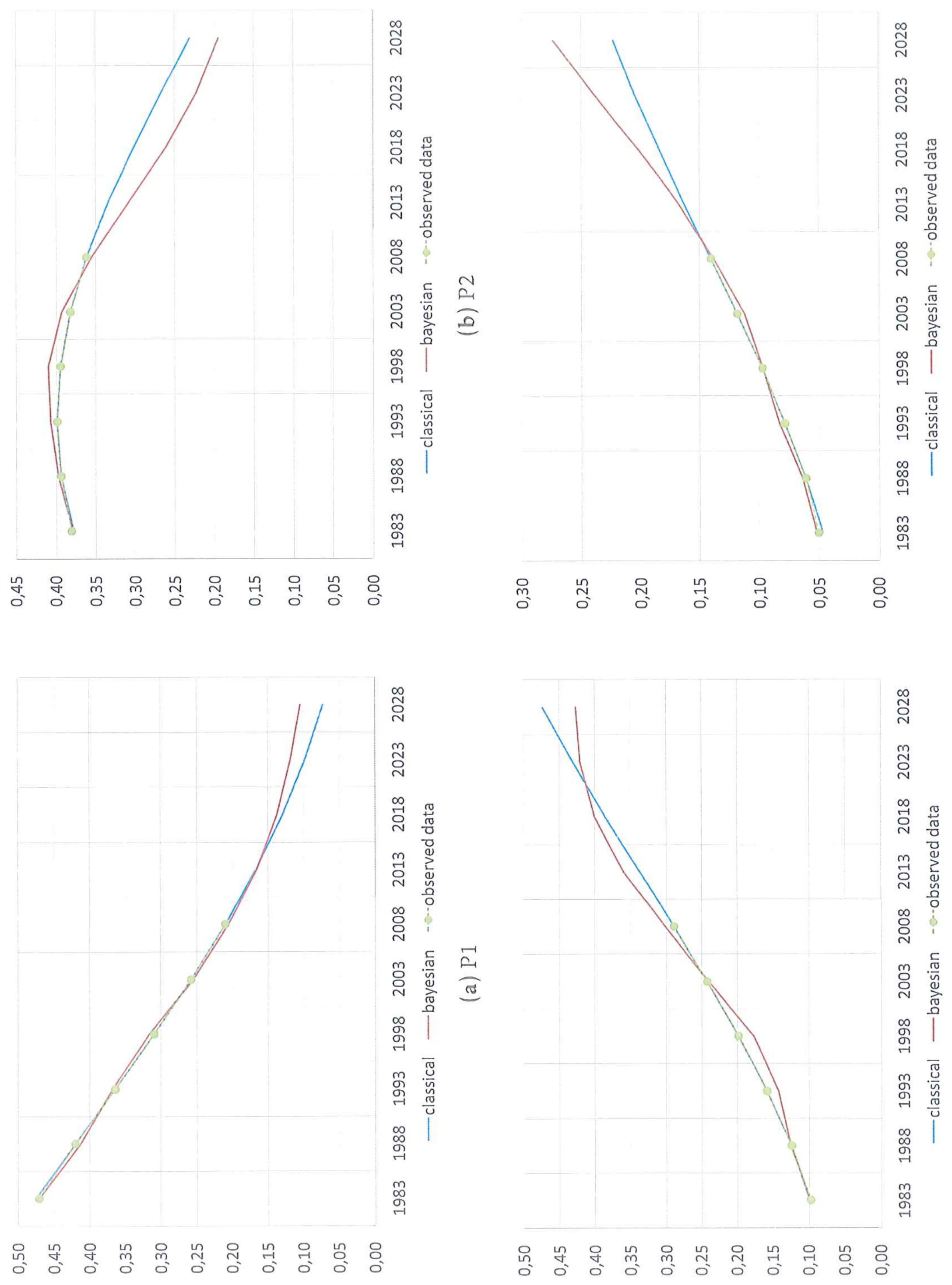


Table 36: Education profile of selected Latin American countries. Females (20-59 years)

Category	Brazil, 2028	Mexico, 2010	Argentina, 2001	Uruguay, 2006
P1	0.049	0.096	0.054	0.019
P2	0.192	0.243	0.337	0.342
P3	0.457	0.331	0.189	0.323
P4	0.302	0.331	0.420	0.316

Obs.: Average proportion for 20-29 age groups.
Source: *Brazil: Calculations based on PNAD data: classical APC model, Point Estimate
*Mexico, Argentina and Uruguay: Demographic Censuses (IPUMS International)

4.4 Model Validation

In this section I examine how well the classical and Bayesian APC models performed in forecasting the education profile. This retrospective analysis was composed by four steps. First, I withdrew the education profile data for the more recent two periods in the PNAD series: 2003 and 2008. Next, I fitted the classical and Bayesian APC models with the remaining dataset excluding the removed part. Third, I used the fitted model to forecast the removed part (2003 and 2028). Finally, I computed the sum of squared errors (SSE)[1] of each model to describe average model-performance.

Results of the internal validation analysis are displayed in Tables 37-34. Tables provide the observed profile in 2003 and 2008, classical forecasts and their respective low and Optimistic Scenarios [2], Bayesian forecasts (given by the mean of the MCMC posterior sample) and their 95% Highest Posterior Density (HPD) intervals, and the sum of squared errors (SSE) of the models. When comparing the SSEs from classical and Bayesian specifications, it is possible to conclude that, overall, the Bayesian projections performed better. Considering all time periods, the SSE for the classical APC model is about 1.4 times higher than that for from the Bayesian APC model for males and about 2.5 times higher for females.

[1]It involves summing the magnitudes (squared values) of the forecast errors (observed - forecast) to obtain the "total error".

[2]The same forecast estimates are obtained in classical APC models using different normalization rules (see section 3.2.2.2).

Table 37: Internal validation analysis of the classical APC model (Reference category is P1). Brazil, Females

		2003 Observed	2003 Forecast	2003 Error	2008 Observed	2008 Forecast	2008 Error
20-24	0-3	0.0894	0.1031	-0.0137	0.0479	0.0757	-0.0279
	4-8	0.2997	0.3842	-0.0846	0.2314	0.3482	-0.1169
	9-11	0.4623	0.3512	0.1111	0.5219	0.3839	0.1380
	12+	0.1487	0.1615	-0.0128	0.1989	0.1921	0.0068
25-29	0-3	0.1349	0.1320	0.0029	0.0845	0.0978	-0.0134
	4-8	0.3554	0.3821	-0.0267	0.2589	0.3493	-0.0904
	9-11	0.3558	0.3092	0.0466	0.4425	0.3409	0.1016
	12+	0.1539	0.1767	-0.0228	0.2142	0.2120	0.0022
30-34	0-3	0.1638	0.1686	-0.0048	0.1248	0.1272	-0.0024
	4-8	0.3914	0.4009	-0.0095	0.3271	0.3732	-0.0461
	9-11	0.3070	0.2661	0.0409	0.3655	0.2988	0.0667
	12+	0.1378	0.1643	-0.0266	0.1826	0.2008	-0.0182
35-39	0-3	0.1971	0.2115	-0.0144	0.1421	0.1627	-0.0205
	4-8	0.4034	0.4080	-0.0046	0.3651	0.3870	-0.0220
	9-11	0.2612	0.2328	0.0284	0.3281	0.2664	0.0617
	12+	0.1383	0.1477	-0.0094	0.1647	0.1839	-0.0192
40-44	0-3	0.2336	0.2559	-0.0223	0.1871	0.2008	-0.0137
	4-8	0.3957	0.4142	-0.0184	0.3384	0.4008	-0.0624
	9-11	0.2296	0.2009	0.0286	0.2864	0.2345	0.0519
	12+	0.1411	0.1290	0.0121	0.1881	0.1639	0.0242
45-49	0-3	0.2808	0.3070	-0.0261	0.2122	0.2459	-0.0337
	4-8	0.3939	0.4107	-0.0168	0.3903	0.4058	-0.0155
	9-11	0.1901	0.1718	0.0184	0.2429	0.2047	0.0382
	12+	0.1351	0.1106	0.0245	0.1545	0.1435	0.0111
50-54	0-3	0.3585	0.3676	-0.0091	0.2753	0.3018	-0.0265
	4-8	0.3762	0.4006	-0.0244	0.3816	0.4056	-0.0240
	9-11	0.1504	0.1421	0.0083	0.2079	0.1735	0.0344
	12+	0.1149	0.0897	0.0252	0.1352	0.1192	0.0161
55-59	0-3	0.4461	0.4299	0.0162	0.3529	0.3609	-0.0080
	4-8	0.3581	0.3805	-0.0224	0.3526	0.3941	-0.0415
	9-11	0.1134	0.1154	-0.0020	0.1682	0.1442	0.0240
	12+	0.0824	0.0742	0.0082	0.1263	0.1008	0.0255
			SSE	0.0328		SSE	0.0803

Source: Calculations based on PNAD data (IBGE)

Table 38: Internal validation analysis of the classical APC model (Reference category is P1). Brazil, Males

		2003			2008		
		Observed	Forecast	Error	Observed	Forecast	Error
20-24	0-3	0.1304	0.1473	-0.0169	0.0788	0.1166	-0.0378
	4-8	0.3505	0.4253	-0.0748	0.2910	0.3967	-0.1057
	9-11	0.4137	0.3169	0.0968	0.4916	0.3567	0.1350
	12+	0.1054	0.1105	-0.0051	0.1385	0.1300	0.0085
25-29	0-3	0.1759	0.1665	0.0094	0.1144	0.1324	-0.0180
	4-8	0.3924	0.4189	-0.0265	0.3069	0.3923	-0.0854
	9-11	0.3157	0.2806	0.0351	0.4170	0.3171	0.0999
	12+	0.1160	0.1340	-0.0180	0.1618	0.1583	0.0035
30-34	0-3	0.2084	0.2109	-0.0025	0.1698	0.1701	-0.0003
	4-8	0.4189	0.4146	0.0043	0.3567	0.3939	-0.0372
	9-11	0.2653	0.2421	0.0232	0.3312	0.2775	0.0537
	12+	0.1074	0.1324	-0.0250	0.1423	0.1586	-0.0163
35-39	0-3	0.2328	0.2459	-0.0131	0.1989	0.2011	-0.0021
	4-8	0.4132	0.4213	-0.0082	0.3908	0.4058	-0.0150
	9-11	0.2446	0.2100	0.0346	0.2909	0.2441	0.0468
	12+	0.1094	0.1227	-0.0133	0.1194	0.1490	-0.0296
40-44	0-3	0.2546	0.2866	-0.0320	0.2329	0.2374	-0.0045
	4-8	0.4088	0.4130	-0.0041	0.3509	0.4030	-0.0521
	9-11	0.2216	0.1891	0.0325	0.2612	0.2227	0.0386
	12+	0.1150	0.1114	0.0036	0.1550	0.1370	0.0180
45-49	0-3	0.2810	0.3245	-0.0435	0.2371	0.2719	-0.0348
	4-8	0.4089	0.4025	0.0064	0.3827	0.3972	-0.0145
	9-11	0.1877	0.1634	0.0243	0.2472	0.1946	0.0527
	12+	0.1223	0.1096	0.0128	0.1330	0.1364	-0.0034
50-54	0-3	0.3390	0.3653	-0.0264	0.2681	0.3093	-0.0412
	4-8	0.3934	0.3847	0.0088	0.3925	0.3837	0.0088
	9-11	0.1501	0.1397	0.0104	0.2152	0.1682	0.0471
	12+	0.1175	0.1103	0.0072	0.1241	0.1388	-0.0146
55-59	0-3	0.4129	0.4189	-0.0060	0.3459	0.3605	-0.0146
	4-8	0.3669	0.3661	0.0008	0.3720	0.3711	0.0010
	9-11	0.1191	0.1151	0.0040	0.1626	0.1407	0.0218
	12+	0.1011	0.0999	0.0012	0.1195	0.1277	-0.0082
			SSE	0.0262		SSE	0.0700

Source: Calculations based on PNAD data (IBGE)

Table 39: Internal validation analysis of the bayesian APC model. Brazil, Females

		2003			2008		
		Observed	Forecast	Error	Observed	Forecast	Error
20-24	0-3	0.0894	0.1165	-0.0271	0.0479	0.1045	-0.0566
	4-8	0.2997	0.3476	-0.0479	0.2314	0.2901	-0.0587
	9-11	0.4623	0.4010	0.0613	0.5219	0.4415	0.0804
	12+	0.1487	0.1350	0.0137	0.1989	0.1639	0.0350
25-29	0-3	0.1349	0.1427	-0.0078	0.0845	0.1242	-0.0397
	4-8	0.3554	0.3778	-0.0224	0.2589	0.3161	-0.0572
	9-11	0.3558	0.3383	0.0175	0.4425	0.3867	0.0558
	12+	0.1539	0.1413	0.0126	0.2142	0.1729	0.0413
30-34	0-3	0.1638	0.1734	-0.0096	0.1248	0.1492	-0.0244
	4-8	0.3914	0.4261	-0.0347	0.3271	0.3635	-0.0364
	9-11	0.3070	0.2755	0.0315	0.3655	0.3291	0.0364
	12+	0.1378	0.1250	0.0128	0.1826	0.1583	0.0243
35-39	0-3	0.1971	0.2065	-0.0094	0.1421	0.1791	-0.0370
	4-8	0.4034	0.4074	-0.0040	0.3651	0.4013	-0.0362
	9-11	0.2612	0.2501	0.0111	0.3281	0.2802	0.0479
	12+	0.1383	0.1360	0.0023	0.1647	0.1395	0.0252
40-44	0-3	0.2336	0.2315	0.0021	0.1871	0.2071	-0.0200
	4-8	0.3957	0.3997	-0.0040	0.3384	0.3885	-0.0501
	9-11	0.2296	0.2264	0.0032	0.2864	0.2559	0.0305
	12+	0.1411	0.1425	-0.0014	0.1881	0.1485	0.0396
45-49	0-3	0.2808	0.2811	-0.0003	0.2122	0.2331	-0.0209
	4-8	0.3939	0.3962	-0.0023	0.3903	0.3799	0.0104
	9-11	0.1901	0.1845	0.0056	0.2429	0.2331	0.0098
	12+	0.1351	0.1382	-0.0031	0.1545	0.1539	0.0006
50-54	0-3	0.3585	0.3635	-0.0050	0.2753	0.2850	-0.0097
	4-8	0.3762	0.3681	0.0081	0.3816	0.3784	0.0032
	9-11	0.1504	0.1495	0.0009	0.2079	0.1911	0.0168
	12+	0.1149	0.1188	-0.0039	0.1352	0.1455	-0.0103
55-59	0-3	0.4461	0.4464	-0.0003	0.3529	0.3624	-0.0095
	4-8	0.3581	0.3520	0.0061	0.3526	0.3515	0.0011
	9-11	0.1134	0.1115	0.0019	0.1682	0.1561	0.0121
	12+	0.0824	0.0901	-0.0077	0.1263	0.1300	-0.0037
			SSE	0.0110		SSE	0.0403

Source: Calculations based on PNAD data (IBGE)

Table 40: Internal validation analysis of the bayesian APC model. Brazil, Males

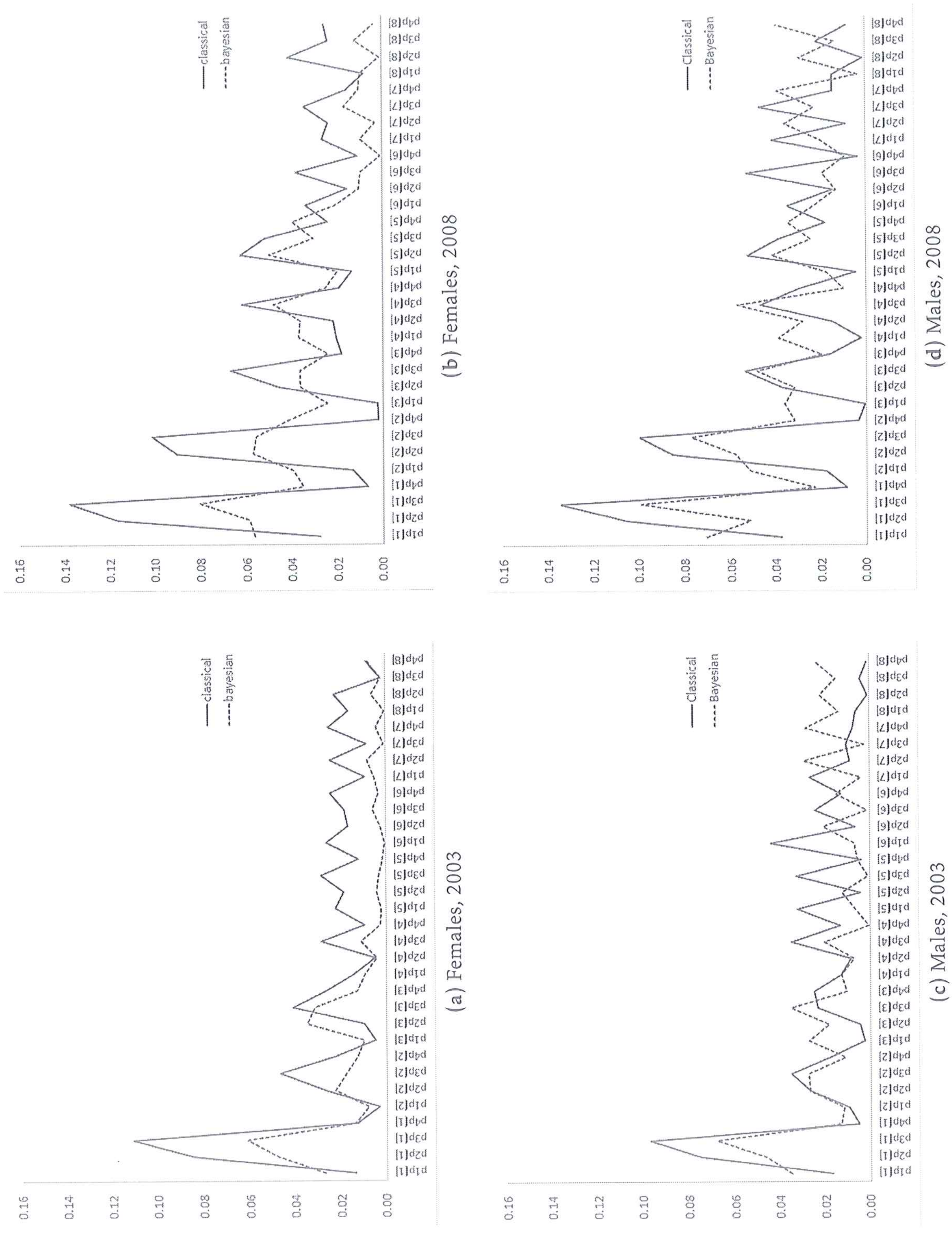
		2003			2008		
		Observed	Forecast	Error	Observed	Forecast	Error
20-24	0-3	0.1304	0.1654	-0.0350	0.0788	0.1497	-0.0709
	4-8	0.3505	0.3967	-0.0462	0.2910	0.3423	-0.0513
	9-11	0.4137	0.3455	0.0682	0.4916	0.3925	0.0991
	12+	0.1054	0.0924	0.0130	0.1385	0.1155	0.0230
25-29	0-3	0.1759	0.1875	-0.0116	0.1144	0.1656	-0.0512
	4-8	0.3924	0.4193	-0.0269	0.3069	0.3646	-0.0577
	9-11	0.3157	0.2881	0.0276	0.4170	0.3398	0.0772
	12+	0.1160	0.1051	0.0109	0.1618	0.1300	0.0318
30-34	0-3	0.2084	0.2356	-0.0272	0.1698	0.2058	-0.0360
	4-8	0.4189	0.4371	-0.0182	0.3567	0.3884	-0.0317
	9-11	0.2653	0.2302	0.0351	0.3312	0.2828	0.0484
	12+	0.1074	0.0971	0.0103	0.1423	0.1229	0.0194
35-39	0-3	0.2328	0.2455	-0.0127	0.1989	0.2377	-0.0388
	4-8	0.4132	0.4203	-0.0071	0.3908	0.4191	-0.0283
	9-11	0.2446	0.2245	0.0201	0.2909	0.2339	0.0570
	12+	0.1094	0.1097	-0.0003	0.1194	0.1094	0.0100
40-44	0-3	0.2546	0.2608	-0.0062	0.2329	0.2503	-0.0174
	4-8	0.4088	0.3967	0.0121	0.3509	0.3924	-0.0415
	9-11	0.2216	0.2222	-0.0006	0.2612	0.2368	0.0244
	12+	0.1150	0.1203	-0.0053	0.1550	0.1205	0.0345
45-49	0-3	0.2810	0.2878	-0.0068	0.2371	0.2604	-0.0233
	4-8	0.4089	0.3884	0.0205	0.3827	0.3693	0.0134
	9-11	0.1877	0.1865	0.0012	0.2472	0.2280	0.0192
	12+	0.1223	0.1373	-0.0150	0.1330	0.1424	-0.0094
50-54	0-3	0.3390	0.3428	-0.0038	0.2681	0.2880	-0.0199
	4-8	0.3934	0.3642	0.0292	0.3925	0.3567	0.0358
	9-11	0.1501	0.1476	0.0025	0.2152	0.1922	0.0230
	12+	0.1175	0.1455	-0.0280	0.1241	0.1632	-0.0391
55-59	0-3	0.4129	0.4264	-0.0135	0.3459	0.3494	-0.0035
	4-8	0.3669	0.3451	0.0218	0.3720	0.3426	0.0294
	9-11	0.1191	0.1040	0.0151	0.1626	0.1491	0.0135
	12+	0.1011	0.1245	-0.0234	0.1195	0.1588	-0.0393
			SSE	0.0169		SSE	0.0529

Source: Calculations based on PNAD data (IBGE)

I now analyze the forecasting error of classical and Bayesian APC models more carefully. Figure 35 presents the absolute error of the forecast according to age group and education category. The absolute error series reveal that the maximum value of the forecast error occurs in the P3 category in both classical and Bayesian models for the 20-24 age group. Peaks are also documented for P3 in the remaining age-groups, in which this education category has a larger absolute error compared to the others.

I hypothesize that this poor adjustment for P3 in both classical and Bayesian specifications results from a structural shift in the educational policy in Brazil for the secondary education level. This structural shift in policies for secondary education may be explained by two factors. First, since the 1996 Brazilian Law for Basic Education (LDB), the Brazilian government has strived to promote access to a "basic education" condition. The LDB defines "basic education" as composed of primary and secondary education. Thereafter, the Brazilian educational system experienced a great expansion of enrollment at the secondary level: in 2001, gross enrollment rates reached 80 percent, compared to 60 percent in the 1980s (Rios-Neto et al., 2010). Second, this structural shift could also have resulted from a school flow normalization in the primary level during the 1990s (Rios-Neto et al., 2010), which may have increased the demand for secondary education (Gomes, 1998). Hence, the classical and Bayesian APC models, which assume a smooth extrapolation of past trends into the future, were not able to anticipate this structural shift in P3 in their forecasts.

Figure 35: Absolute forecasting error by APC model, sex and year.



(a) Females, 2003

(b) Females, 2008

(c) Males, 2003

(d) Males, 2008

Conclusion

In this dissertation I have proposed two age-period-cohort model (APC) approaches to model and forecast the education profile of Brazilian males and females. Scant empirical literature directly addresses estimation and forecasting issues using the APC framework. I utilized both classical and Bayesian methods for demographic forecasting using APC models. Also, I provided a succinct and comprehensive appraisal of the challenges and advantages that lie behind different APC models.

The classical approach to forecasting using APC models is often criticized for its strong parametric assumptions. Here, I proposed a scenario approach to forecasting in a classical paradigm. I demonstrated that the scenario approach found its strengths (ease of computing) tempered by its inability to express uncertainty in a probabilistic fashion.

On the other hand, the Bayesian techniques applied here are particularly attractive because they allow the uncertainty associated with APC parameters and functions of the parameters (i.e., the probabilities) to be readily explored. Hence, in the Bayesian APC model, both uncertainty associated with the choice of model and uncertainty associated with projecting beyond the range of the data are encompassed by credible intervals.

My results show that both classical and Bayesian methods are able to provide very good forecasts in the short term. However, the Bayesian method performed best for in-sample and out-of-sample forecasts. On the other hand, in a Bayesian setting, uncertainty indeed becomes an issue for long-term forecasts because of the rapidly increasing width of the intervals as the length of the projection increases.

Although not the main focus of this study, the forecasts produced here could in practice provide a basis for educational policy analysis. I showed that there is good news regarding schooling of the Brazilians in the near future, but I also documented a slow growth of tertiary education prevalence in the long-run. This finding may compromise the future capacity of Brazil to reach advanced stages of development (Barro, 1991; Mankiw et al., 1992; Krueger and Lindahl, 2000), considering that the country is not expected to achieve high levels of highly educated individuals in the near future. Therefore, initiatives aimed at increasing student participation in higher education are urgent for Brazil.

A number of enhancements of the classical and Bayesian methods proposed here are suggested for a future research agenda. Foremost among these is an investigation into an integrated approach to account for uncertainty in the classical multinomial APC model. Also, refined ways of eliciting prior information in the Bayesian framework warrant attention.

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A Additional Resources

A.1 Counts of Cases for the Statistical Analysis

Table A1: Number of cases by education category, age group and period. Brazil, Females

Age group	Category	Period					
		1983	1988	1993	1998	2003	2008
20-24	P1	1,538,727	1,444,672	1,206,999	1,002,960	744,597	390,301
	P2	2,634,569	3,031,916	3,138,776	2,875,868	2,495,888	1,886,174
	P3	1,245,931	1,664,176	1,659,375	2,313,647	3,850,196	4,254,634
	P4	443,082	532,861	529,652	750,272	1,238,386	1,621,587
25-29	P1	1,640,130	1,472,255	1,383,947	1,109,395	984,775	697,701
	P2	2,148,776	2,567,507	2,747,428	2,696,199	2,593,985	2,138,303
	P3	825,634	1,293,382	1,454,776	1,736,181	2,596,719	3,655,532
	P4	434,659	580,330	681,962	711,982	1,122,896	1,769,104
30-34	P1	1,769,569	1,573,498	1,489,025	1,351,284	1,111,397	935,166
	P2	1,662,296	2,224,336	2,489,723	2,698,322	2,655,746	2,450,716
	P3	554,953	901,957	1,186,747	1,490,664	2,083,400	2,738,988
	P4	332,741	545,754	674,913	765,975	934,699	1,368,088
35-39	P1	1,747,764	1,790,841	1,626,852	1,407,388	1,284,583	995,236
	P2	1,304,387	1,734,318	2,180,203	2,396,749	2,629,088	2,555,934
	P3	335,824	630,232	847,486	1,239,538	1,702,256	2,296,941
	P4	200,803	398,293	605,068	769,978	901,344	1,153,449
40-44	P1	1,676,555	1,787,181	1,655,400	1,527,122	1,421,772	1,309,989
	P2	1,087,351	1,380,119	1,616,309	2,109,287	2,408,800	2,369,855
	P3	218,213	373,786	555,286	907,876	1,397,221	2,005,940
	P4	123,054	256,422	444,836	636,005	858,923	1,317,207
45-49	P1	1,561,922	1,643,467	1,598,855	1,570,645	1,495,999	1,319,439
	P2	821,642	1,041,702	1,221,283	1,661,647	2,098,345	2,426,842
	P3	143,202	237,344	344,229	594,993	1,012,777	1,510,283
	P4	64,374	140,993	264,006	478,034	719,768	960,796
50-54	P1	1,456,258	1,561,892	1,567,872	1,555,237	1,551,579	1,532,222
	P2	708,327	873,092	952,245	1,237,084	1,628,334	2,123,709
	P3	121,187	151,070	219,731	351,040	650,930	1,157,146
	P4	43,626	75,918	146,263	275,042	497,079	752,721
55-59	P1	1,243,436	1,387,943	1,436,659	1,511,134	1,549,578	1,539,580
	P2	549,763	666,699	774,205	981,393	1,243,888	1,538,431
	P3	87,211	120,279	140,551	226,530	393,887	733,699
	P4	26,266	49,623	93,171	145,835	286,051	550,938

Source: Pesquisa Nacional por Amostra de Domicílios, 1983 to 2008 (IBGE)

Table A2: Number of cases by education category, age group and period. Brazil, Males

Age group	Category	Period					
		1983	1988	1993	1998	2003	2008
20-24	P1	1,607,763	1,611,075	1,557,716	1,356,842	1,078,326	654,733
	P2	2,615,973	2,938,695	3,081,372	3,065,554	2,898,929	2,416,574
	P3	1,074,364	1,348,373	1,291,215	1,899,310	3,422,004	4,082,863
	P4	318,297	357,656	347,562	506,434	871,655	1,150,291
25-29	P1	1,489,742	1,440,659	1,443,012	1,299,270	1,206,804	900,957
	P2	2,081,581	2,444,107	2,725,630	2,697,099	2,692,062	2,416,668
	P3	735,984	1,103,638	1,216,214	1,358,504	2,165,915	3,283,973
	P4	361,299	438,156	478,245	514,753	795,791	1,274,030
30-34	P1	1,587,686	1,485,288	1,495,577	1,493,453	1,322,574	1,190,235
	P2	1,706,938	2,065,449	2,290,379	2,471,787	2,658,004	2,500,864
	P3	484,538	779,423	1,071,600	1,233,859	1,683,349	2,321,893
	P4	311,263	419,953	538,880	582,394	681,244	997,976
35-39	P1	1,554,545	1,546,319	1,489,883	1,457,860	1,424,907	1,277,828
	P2	1,290,294	1,750,149	2,060,207	2,268,242	2,528,623	2,510,311
	P3	285,011	518,059	797,447	1,093,338	1,497,153	1,868,214
	P4	207,690	342,245	527,693	600,888	669,397	766,864
40-44	P1	1,548,773	1,554,155	1,525,654	1,438,946	1,403,032	1,494,606
	P2	1,070,183	1,301,533	1,600,021	2,017,929	2,253,274	2,251,974
	P3	189,287	325,286	538,517	845,548	1,221,275	1,676,747
	P4	131,415	226,021	428,308	553,563	633,743	995,160
45-49	P1	1,358,938	1,465,688	1,411,666	1,388,838	1,335,216	1,337,456
	P2	849,730	1,019,337	1,201,721	1,531,968	1,942,691	2,158,773
	P3	154,828	207,698	293,058	551,474	891,939	1,394,545
	P4	82,916	150,525	303,649	462,587	581,303	750,010
50-54	P1	1,262,814	1,365,649	1,333,121	1,351,085	1,327,493	1,310,552
	P2	705,401	877,481	888,500	1,149,033	1,540,833	1,918,904
	P3	114,952	179,811	186,066	320,825	587,934	1,052,122
	P4	61,893	88,387	219,881	350,983	460,220	606,859
55-59	P1	1,136,168	1,233,513	1,155,288	1,260,609	1,232,676	1,331,873
	P2	520,425	644,313	720,921	867,125	1,095,239	1,432,517
	P3	84,661	121,968	138,073	194,115	355,507	625,954
	P4	42,950	70,291	130,999	191,600	301,874	460,337

Source: Pesquisa Nacional por Amostra de Domicílios, 1983 to 2008 (IBGE)

A.2 Bayesian APC Parameter Estimates

Table A3: Results of the bayesian APC multinomial model. Full sample. Brazil, Females.

	P1	P2	P3
θ_1	-0.3902	0.2007	0.7295
	(0.0273)	(0.0290)	(0.0834)
θ_2	-0.3559	0.0917	0.4646
	(0.0195)	(0.0263)	(0.0683)
θ_3	-0.3327	0.0600	0.4066
	(0.0159)	(0.0241)	(0.0545)
θ_4	-0.3176	0.0354	0.3848
	(0.0191)	(0.0224)	(0.0433)
θ_5	-0.2847	-0.0089	0.3400
	(0.0268)	(0.0214)	(0.0371)
θ_6	-0.2686	0.0530	0.4040
	(0.0362)	(0.0212)	(0.0385)
θ_7	-0.2132	0.0729	0.4919
	(0.0463)	(0.0218)	(0.0467)
θ_8	-0.1644	0.0629	0.5053
	(0.0567)	(0.0232)	(0.0590)
ϕ_1	0.0397	0.1132	0.1464
	(0.0465)	(0.0458)	(0.0755)
ϕ_2	-0.0570	0.0425	0.0955
	(0.0393)	(0.0433)	(0.0588)
ϕ_3	-0.0713	0.0544	-0.0685
	(0.0341)	(0.0411)	(0.0425)
ϕ_4	-0.1603	-0.0205	-0.0849
	(0.0320)	(0.0392)	(0.0276)
ϕ_5	-0.2574	-0.1728	-0.0906
	(0.0335)	(0.0377)	(0.0177)
ϕ_6	-0.3680	-0.3307	-0.1908
	(0.0383)	(0.0366)	(0.0216)
ψ_1	0.7611	1.3860	0.5762
	(0.0769)	(0.0563)	(0.1235)
ψ_2	0.7138	1.2511	0.3600
	(0.0662)	(0.0527)	(0.1077)
ψ_3	0.6273	1.1585	0.1273
	(0.0556)	(0.0494)	(0.0926)
ψ_4	0.4500	0.9169	0.0543
	(0.0451)	(0.0461)	(0.0785)
ψ_5	0.2190	0.6697	-0.0554
	(0.0349)	(0.0430)	(0.0660)
ψ_6	-0.0822	0.4234	-0.0352
	(0.0255)	(0.0402)	(0.0560)
ψ_7	-0.4114	0.3163	0.0840
	(0.0178)	(0.0375)	(0.0502)
ψ_8	-0.6634	0.1795	0.2446
	(0.0150)	(0.0352)	(0.0501)
ψ_9	-0.8174	0.0768	0.3492
	(0.0193)	(0.0332)	(0.0556)
ψ_{10}	-1.0393	0.0100	0.5214
	(0.0276)	(0.0316)	(0.0654)
ψ_{11}	-1.2320	-0.2721	0.5023
	(0.0373)	(0.0306)	(0.0778)
ψ_{12}	-1.6587	-0.7331	0.4991
	(0.0475)	(0.0301)	(0.0919)
ψ_{13}	-2.2240	-1.0222	0.4538
	(0.0580)	(0.0301)	(0.1069)
μ	-0.0079	0.0158	-0.0279
	(0.0228)	(0.0229)	(0.0214)

Source: Calculations based on PNAD data
Obs.: Standard errors between parentesis

Table A4: Results of the bayesian APC multinomial model. Full sample. Brazil, Males.

	P1	P2	P3
θ_1	-0.3919	0.1521	0.4666
	(0.0634)	(0.0315)	(0.0651)
θ_2	-0.3729	0.0744	0.1285
	(0.0576)	(0.0295)	(0.0544)
θ_3	-0.2392	0.0533	0.0217
	(0.0524)	(0.0285)	(0.0456)
θ_4	-0.1633	0.0983	0.0151
	(0.0480)	(0.0286)	(0.0399)
θ_5	-0.0546	0.1027	-0.0563
	(0.0447)	(0.0299)	(0.0388)
θ_6	0.0011	0.1698	-0.0360
	(0.0427)	(0.0323)	(0.0425)
θ_7	0.0816	0.2054	-0.0099
	(0.0422)	(0.0354)	(0.0501)
θ_8	0.2109	0.2566	-0.0010
	(0.0432)	(0.0392)	(0.0601)
ϕ_1	0.0745	0.4725	0.2254
	(0.0332)	(0.0337)	(0.0638)
ϕ_2	-0.0659	0.3492	0.2373
	(0.0261)	(0.0292)	(0.0509)
ϕ_3	-0.1617	0.2452	0.0372
	(0.0198)	(0.0253)	(0.0389)
ϕ_4	-0.2899	0.1228	0.0307
	(0.0153)	(0.0222)	(0.0287)
ϕ_5	-0.4634	-0.0562	0.0969
	(0.0145)	(0.0204)	(0.0231)
ϕ_6	-0.6004	-0.2638	0.0405
	(0.0178)	(0.0202)	(0.0253)
ψ_1	0.2793	0.7233	0.4477
	(0.0557)	(0.0686)	(0.1166)
ψ_2	0.2258	0.7009	0.3508
	(0.0499)	(0.0634)	(0.1029)
ψ_3	0.1423	0.6299	0.2918
	(0.0449)	(0.0585)	(0.0894)
ψ_4	0.1058	0.525	0.0065
	(0.0410)	(0.0537)	(0.0763)
ψ_5	-0.0706	0.3647	0.0219
	(0.0383)	(0.0491)	(0.0635)
ψ_6	-0.2872	0.2626	0.1973
	(0.0373)	(0.0448)	(0.0513)
ψ_7	-0.4837	0.189	0.403
	(0.0380)	(0.0409)	(0.0405)
ψ_8	-0.5647	0.1183	0.5822
	(0.0404)	(0.0375)	(0.0323)
ψ_9	-0.5601	0.1307	0.6454
	(0.0442)	(0.0347)	(0.0290)
ψ_{10}	-0.5983	0.2032	0.8048
	(0.0490)	(0.0328)	(0.0321)
ψ_{11}	-0.7192	-0.0268	0.7837
	(0.0547)	(0.0318)	(0.0403)
ψ_{12}	-1.0534	-0.4211	0.7828
	(0.0609)	(0.0319)	(0.0511)
ψ_{13}	-1.4626	-0.6141	0.7533
	(0.0675)	(0.0331)	(0.0632)
μ	-0.0033	-0.0467	0.0065
	(0.0138)	(0.0263)	(0.0316)

Source: Calculations based on PNAD data
Obs.: Standard errors between parentesis