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A Condition-Based Maintenance Policy and Input Parameters Estimation for Deteriorating Systems under Periodic Inspection

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Dissertação de mestrado submetida ao Programa de Pós-Graduação em Engenharia Elétrica da Universidade Federal de Minas Gerais, como requisito parcial à obtenção do Título de Mestre em Engenharia Elétrica.
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Orientador: Prof. Dr. Carlos A. Maia
Co-Orientador: Prof. Dr. Leonardo P. Santiago

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Abstract

We study the problem of proposing Condition-Based Maintenance policies for machines and equipments. Our approach combines an optimization model and input parameters estimation from empirical data.

The system deterioration is described by discrete states ordered from the state "as good as new" to the state "completely failed". At each periodic inspection, whose outcome might not be accurate, a decision has to be made between continuing to operate the system or stopping and performing its preventive maintenance. This decision-making problem is discussed and we tackle it by using an optimization model based on the Dynamic Programming and Optimal Control theory.

We then explore the problem of how to estimate the model input parameters, i.e., how to adequate the model inputs to the empirical data available. The literature has not explored the combination of optimization techniques and model input parameters, through historical data, for problems with imperfect information such as the one considered in this work. We develop our formulation using the Hidden Markov Model theory.

We illustrate our framework using empirical data provided by a mining company and the results show the applicability of our models. We conclude by pointing out some possible directions for future research on this field.

Resumo

O foco deste trabalho é a definição de políticas ótimas de manutenção preventiva em função da condição do equipamento. Propomos uma abordagem que combina um modelo de otimização com um modelo de estimação de parâmetros a partir dos dados de campo.

A condição do sistema é descrita por estados discretos ordenados do "tão bom quanto novo" até o estado "completamente falhado". A cada inspeção, cujo resultado pode ser impreciso, uma decisão é tomada: continuar a operação ou efetuar a manutenção preventiva. Este problema de tomada de decisão é analisado e propomos um algoritmo de otimização baseado em Programação Dinamica-Estocástica e Controle Ótimo.

Em seguida, exploramos o problema de como estimar as entradas do modelo, ou seja, como adequar os parâmetros de entrada em função dos dados disponíveis. Até o momento, a literatura não apresentou uma técnica que lida com otimização e estimação de parâmetros de entrada (usando dados históricos) para problemas com informação imperfeita como o considerado neste trabalho. Desenvolvemos nossa abordagem usando os Modelos Ocultos de Markov.

Ilustramos a aplicação dos modelos desenvolvidos com dados de campo fornecidos por uma empresa de mineração. Os resultados mostram a aplicabilidade da nossa abordagem. Concluímos o texto apresentando possíveis direções para pesquisa futura na área.

Resumo Estendido

Uma política de manutenção baseada na condição e estimação de parâmetros de entrada para sistemas sujeitos à deterioração e a inspeções periódicas.

Introdução

As atividades ligadas à manutenção de máquinas e equipamentos são essenciais ao bom funcionamento de uma indústria. Dentre essas atividades destacam-se os programas de manutenção preventiva que visam otimizar o uso e a operação dos equipamentos e máquinas (que serão referidos neste texto como "sistemas") através da realização de intervenções planejadas.

O objetivo destas intervenções é reparar os sistemas antes que os mesmos falhem[1], garantindo, portanto, o funcionamento regular e permanente da atividade produtiva. Se por um lado a necessidade da manutenção preventiva é clara, por outro a programação de tais intervenções não é tão evidente. Uma grande dificuldade reside na elaboração de um planejamento que determine quando realizar a Manutenção Preventiva (PM).

Manutenção Preventiva pode ser classificada em dois tipos: Manutenção Programada (SM) – ou manutenção baseada no tempo – e Manutenção Baseada na Condição (CBM) – ou manutenção preditiva. No primeiro caso assume-se que o sistema assume apenas dois estados – não-falhado e falhado – e a manutenção é realizada em intervalos de tempos pré-estabelecidos, embora não necessariamente iguais. Um exemplo deste tipo de política

[1] Entendemos falha como a incapacidade do sistema executar as operações as quais lhe foram designadas, em condições bem definidas.

de manutenção é a Manutenção Preventiva Programada. No segundo caso (CBM), procura-se usar a informação da condição do sistema, através da análise de sintomas e/ou de uma estimativa do estado de degradação, visando determinar o momento adequado de realizar a manutenção. Assim, a CBM considera que o sistema possui múltiplos estados de deterioração, indo do "tão bom quanto novo" até o falhado. Mais informações podem ser obtidas em (Bloom, 2006; Nakagawa, 2005; Wang and Pham, 2006; Pham, 2003; Smith, 1993; Moubray, 1993)

Propomos nesta dissertação um modelo para formular políticas CBM em sistemas cuja condição pode ser estimada. Esta estimação pode ser incerta (não perfeita), já que a hipótese de conhecimento da condição real do sistema quase sempre não é factível. Uma política dita a forma com que as ações devem ser escolhidas ao longo do tempo em função das informações coletadas. Exploramos este problema usando cadeias de Markov e Programação Dinâmica-Estocástica (SDP).

Além do modelo de otimização, propõe-se uma técnica para estimação dos parâmetros de entrada (do modelo de CBM). Isto é feito usando a teoria dos Modelos de Markov Ocultos (HMM). A combinação da técnica de estimação com o modelo de otimização apresenta certa novidade pois, dentro da bibliografia consultada, a grande maioria dos modelos de CBM não discute como calcular seus parâmetros a partir dos dados de campo.

Assim, a principal contribuição deste trabalho situa-se na junção de um modelo de CBM com um modelo de inferência dos parâmetros de entrada, enfoque que ainda não foi explorado na literatura. Esta contribuição torna-se clara no exemplo de aplicação fornecido.

Um Modelo de Manutenção Baseada na Condição

Assumindo que a condição do sistema pode ser discretizada em estados, associamos cada estado a um nível de degradação. Periodicamente, obtém-se uma estimação da condição, sendo que esta estimação pode ser imperfeita (diferente do verdadeiro estado do sistema). Nossas outras hipóteses são:

1. O sistema é colocado em serviço no tempo 0 no estado "tão bom quanto novo";

2. Todos reparos são perfeitos, ou seja, após o reparo o sistema volta à condição "tão bom quanto novo";

3. O tempo é discreto com relação a um período fixo T, ou seja: $T_{k+1} = T_k + T$, onde $k = 1, 2, \ldots$ representa k-ésimo tempo amostrado. Representaremos o instante de tempo T_k por k;

4. No instante k, o sistema é inspecionado a fim de medir sua condição. Isto pode ser feito medindo uma variável do sistema como vibração ou temperatura. Assume-se que a variável monitorada é diretamente relationada com o modo de falha que é analisado;

5. No instante k, uma ação u_k é tomada: ou $u_k = C$ (continuar a operação do sistema) ou $u_k = S$ (parar e realizar a manutenção). Assim, o espaço de decisão é $U = \{C, S\}$;

6. Falhas não são imediatamente detectadas. Ou seja, se o sistema falha em $[k-1, k)$, isto será detectado apenas no instante k.

Nós consideramos dois horizontes:

- Horizonte de curto prazo: desde o início da operação do sistema ($k = 0$) até a parada do sistema ($u_k = S$).

- Horizonte de longo prazo: definido como os horizontes de curto prazo acumulados ao longo do tempo.

Como assume-se reparo perfeito, otimizar no curto prazo garante a otimização em longo prazo. Assim, nós reiniciamos k toda vez que o sistema é parado ($u_k = S$). Após o reparo, o sistema volta à condição "tão bom quanto novo", k é "setado" em 0 e o sistema volta a operar. Nosso foco consiste então em otimizar o horizonte de curto prazo.

Considere que o sistema possua vários estágios de deterioração $1, 2, \ldots, L$, ordenados do estado "tão bom quanto novo" (1) até o estado completamente falhado (L). A evolução ao longo do tempo da condição do equipamento segue um processo estocástico. Se nenhuma ação é tomada e sob a hipótese de que o estado futuro depende apenas do estado presente (i.e., o passado encontra-se "embutido" no presente), esta evolução caracteriza um processo estocástico markoviano.

Seja então $\{X_k\}_{k\geq 0}$ uma cadeia de Markov onde X_k denota o estado do sistema no instante k e $\{X_k\}$ modela a deterioração do sistema ao longo do tempo. Assim, o espaço de estado de X_k é $X = \{1, 2, \ldots, L\}$ o qual associamos uma distribuição de probabilidade definida como

$$a_{ij} = \Pr[X_{k+1} = j | X_k = i,\ u_k = C] = \Pr[X_1 = j | X_0 = i,\ u_0 = C],$$

sendo que $\sum_{j=i}^{L} a_{ij} = 1,\ \forall i, j$. Vamos expressar essas probabilidades na forma matricial. Para tal, seja: $A \equiv [a_{ij}]$.

Seja $g(\cdot)$ uma função de custo do sistema definida como o custo a ser pago no instante k caso o sistema se encontre no estado x_k e caso a ação tomada for u_k. Esta função representa o custo operacional do sistema, o custo esperado em caso de indisponibilidade devido a falhas (lucro cessante), além dos custos de manutenção preventiva e corretiva. Logo:

- Para $x_k \in 1, \ldots, L-1$ tem-se:
  - $u_k = C$ (continuar a operar): $g(x_k, u_k)$ representa o custo operacional, que pode ser escrito em função do estado do sistema;
  - $u_k = S$ (parar e efetuar a manutenção preventiva): g denota o custo esperado da manutenção preventiva (incluindo o lucro cessante), que também pode ser escrito em função do estado do sistema;
- Para $x_k = L$ (falhado):

- $u_k = S$: $g(\cdot)$ descreve o custo esperado de manutenção corretiva incluindo o custo de indisponibilidade durante o reparo;
- $u_k = C$: $g(\cdot)$ representa o custo de indisponibilidade no período $[k, k+1)$, geralmente uma decisão não ótima pois implica em não mais operar o sistema.

Utilizando os conceitos acima enunciamos a Definição 1, que descreve as características de um problema dito bem-definido. Assumiremos que o problema satisfaz esta definição. A Fig. 3.4 mostra a cadeia de Markov de um problema bem-definido. Pierskalla and Voelker (1976) provaram que sempre existe uma regra de reparo ótima. Entretanto, para calculá-la é necessário conhecer o estado do sistema X_k a qualquer instante. Como assumimos que temos apenas uma leitura da condição do sistema, precisamos utilizar esta informação para estimar X_k.

Assim, definimos uma medida de condição Z_k que tem distribuição de probabilidade condicionada em X_k (veja a Fig. 3.5). Denotamos o espaço de estados de Z_k por $Z = \{\underline{1}, \underline{2}, \ldots, \underline{L}\}$, onde a condição observada $\underline{1}$ representa "o sistema parece estar no estado 1" e etc.. Seja $b_x(z)$ a probabilidade $\Pr[Z_k = z | X_k = x]$. Por conveniência, expressaremos essas probabilidades na forma matricial: $B \equiv [b_x(z)]$. Nota-se que Z_k representa a ligação entre o estado do sistema e a(s) variável(is) que monitora(m) o sistema. Conseqüentemente, uma etapa de classificação é necessária convertendo cada valor de medida a um valor de Z. No Capítulo 5 apresentamos exemplos de classificação.

Definimos um vetor de informação I_k que armazena a condição estimada (Z_k) desde o início da operação até o instante k. Logo, I_k tem tamanho k e pode ser escrito como

$$I_1 = z_1 \quad (1)$$

$$I_k = (I_{k-1}, z_k), \qquad k = 2, 3, \ldots. \quad (2)$$

Usando este vetor nós podemos criar um estimador para X_k em qualquer instante k. Este estimador é escrito como

$$\widehat{X}_k = \arg\max_{x \in X} \Pr[X_k = x | I_k]. \tag{3}$$

Uma maneira de calcular a probabilidade apresenta acima é indicada na Eq. 4.1. Vamos chamar os parâmetros do modelo (A, B) por Ψ. Logo, nesta dissertação, qualquer sistema pode ser integralmente representado pelo seu Ψ e sua função de custo $g(\cdot)$.

A política CBM ótima μ é um mapeamento entre a informação disponível e a ação a ser tomada, ou seja,

$$u_k = \mu(I_k) = \begin{cases} C, & \text{if } \widehat{X}_k < r, \\ S, & \text{if } \widehat{X}_k \geq r, \end{cases}$$

sendo $r \in X$ o estado limite de operação (ou a regra de reparo). Determinar este limite consiste em resolver o horizonte de curto-prazo. Para tal, precisamos de um algoritmo que minimize o custo acumulado, que é resultado da soma dos custos em cada estágio k e influenciado pelas decisões tomadas. Resolvemos este problema usando Programação Dinâmica Estocástica (SDP).

Para encontrar a regra de reparo ótima r, vamos primeiro definir J como o custo total de operação do sistema até a sua parada, ou seja, a soma de $g(\cdot)$ até $u_k = S$. Sob uma dada política μ, J é escrito como

$$J_\mu = \lim_{N \to \infty} \mathrm{E}\left[\sum_{k=1}^{N} \alpha^k g(X_k, u_k)\right], \tag{4}$$

sendo $\alpha \in [0, 1]$ o fator de desconto usado para descontar os custos futuros. O caso mais complexo é quando temos $\alpha = 1$ pois a soma da Eq. 4 pode não convergir. Entretanto, na Proposição 2 mostramos que isso não acontece se o problema for bem definido e, logo, sempre teremos uma solução. Para encontrar a política ótima, decompomos a Eq. 4 na seguinte equação de SDP

$$J_k(I_k) = \min_{u_k} \Big\{ \mathrm{E}\big[g(X_k, u_k)|I_k, u_k\big] + \alpha \mathrm{E}\big[J_{k+1}(I_k, Z_{k+1})|I_k, u_k\big] \Big\}, \quad k = 1, 2, \ldots . \tag{5}$$

Usamos na equação acima um procedimento chamado de redução de um problema de informação imperfeita em um problema de informação perfeita. Isso é possível usando o estimador definido na Eq. 3.

Resolvemos a Eq. 5 utilizando um algoritmo de iteração de valor (VI). Bertsekas (2005); Sutton and Barto (1998); Puterman (1994) descrevem com detalhes este procedimento. O Algoritmo 1 apresenta os passos do VI. Como saída temos a regra de reparo r que, combinado com o estimador $\widehat{X}_k$, representa a política CBM ótima.

Como descrito anteriormente, como assumimos reparo perfeito (hipótese 2), resolvendo o horizonte de curto prazo de forma ótima garante que otimizamos também o horizonte de longo prazo, já que o último é resultado dos horizontes de custo prazo acumulados ao longo do tempo.

A segunda parte desta dissertação se dedica a estimar os parâmetros do modelo de otimização, ou seja, Ψ.

Inferência dos Parâmetros do Modelo

Busca-se agora adequar os parâmetros de entrada do modelo CBM para sua aplicação em um dado sistema. Assim, estamos interessados em encontrar uma técnica que utiliza os dados disponíveis e nos dê a melhor estimação possível.

A motivação para o uso dos Modelos de Markov Ocultos (HMM) vem da habilidade deles de diferenciar mudanças na leitura da condição do sistema que podem ser causadas por alterações no sistema (exemplo: degradação) ou flutuações na medição (exemplo: precisão da medição). Além disso, existem métodos computacionais eficientes para o cálculo das verossimilhanças devido, em particular, ao maduro uso dos HMMs em processamento de sinais. Mais informações a respeito podem ser encontradas em (Rabiner, 1989; Ephraim and Merhav, 2002; Dugad and Desai, 1996; Baum et al., 1970).

Considere O o conjunto de toda informação disponível sobre o sistema. O pode ser visto como um conjunto de M seqüências de observação, ou seja, $O = \{O_1, O_2, \ldots, O_M\}$, onde O_m representa uma seqüência de leitura da condição do sistema e pode ser escrita como $O_m = \{z_1, z_2, \ldots, z_N\}$, onde N é o tamanho da seqüência e z_n é a condição do sistema observada no instante n.

A estimação dos parâmetros de entrada $\Psi = (A, B)$ é um problema no qual, dado a informação disponível O, deseja-se definir Ψ como uma função destes dados. Ou seja, desejamos encontrar Ψ que maximiza a $\Pr[O|\Psi]$. Este problema é conhecido na literatura de HMM como problema 3. Para resolvê-lo, assume-se que temos uma estimativa inicial ("palpite") sobre Ψ, que chamaremos de Ψ_0. Este problema pode ser resolvido numericamente aplicando um conjunto de fórmulas conhecidas como fórmulas de Baum-Welch em homenagem a seus autores.

Primeiro, definimos as seguintes variáveis:

- forward: $\alpha_n(x) = \Pr[Z_1 = z_1, Z_2 = z_2, \ldots, Z_n = z_n, X_n = x|\Psi]$;
- backward: $\beta_n(x) = \Pr[Z_{n+1} = z_{n+1}, Z_{n+2} = z_{n+2}, \ldots, Z_N = z_N, X_N = x|\Psi]$.

Seja $\gamma_n(x)$ a probabilidade do sistema estar no estado x no instante n dado a seqüência de observações O_n, ou seja, $\gamma_n(x) = \Pr[X_n = x|O_n, \Psi]$. Usando a regra de Bayes tem-se

$$\gamma_k(x) = \Pr[X_k = x|O_n, \Psi] = \frac{\Pr[X_k = x, O_n|\Psi]}{\Pr[O_n|\Psi]} = \frac{\alpha_k(x)\beta_k(x)}{\Pr[O_n|\Psi]}.$$

Seja agora $\xi_k(i, j)$ a probabilidade de o sistema estar no estado i no instante k e realizar a transição para j em $k+1$, ou seja, $\xi_k(i, j) = \Pr[X_k = i, X_{k+1} = j|O_n, \Psi]$, o que implica em (usando a regra de Bayes):

$$\xi_k(i,j) = \frac{\Pr[X_k = i, X_{k+1} = j, O_n|\Psi]}{\Pr[O_n|\Psi]} = \frac{\alpha_k(i)a_{ij}b_j(Z_{k+1})\beta_{k+1}(j)}{\Pr[O_n|\Psi]}.$$

Finalmente, sejam $\bar{a}_{ij}$ e $\bar{b}_x(z)$ os estimadores de a_{ij} e $b_x(z)$ respectivamente. Podemos escrever estes estimadores como:

- $\bar{a}_{ij} = \sum_{k=1}^{K-1} \xi_k(i,j) / \sum_{k=1}^{K-1} \gamma_k(i)$
- $\bar{b}_x(z) = \sum_{\substack{k=1 \\ z_k=z}}^{K} \gamma_k(x) / \sum_{k=1}^{K} \gamma_k(x)$

Através da aplicação das fórmulas de Baum-Welch, Ψ é ajustado de forma a aumentar a $\Pr[O|\Psi]$ até alcançar um valor máximo. Isto é feito da seguinte maneira:

1. Usando o palpite inicial Ψ_0, aplicamos as fórmulas de Baum-Welch para a primeira seqüência de dados O_1. Como resultado, obtém-se as estimações $\bar{a}_{ij}$ e $\bar{b}_x(z)$, que chamaremos de Ψ_1.

2. Voltamos ao passo 1 usando agora como entrada a estimação dos parâmetros atual (Ψ_1) e a próxima seqüência de dados (O_2).

O Algoritmo 2 apresenta a aplicação sucessiva das fórmulas de Baum-Welch como descrito acima. Ao final, $\Pr[O|\Psi]$ terá seu valor máximo. Este máximo representa o máximo da função de verossimilhança e pode ser local ou global, sendo que no último caso temos a melhor estimação possível com os dados disponíveis.

Um Exemplo de Aplicação

Para ilustrar a metodologia apresentada neste trabalho, aplicamos as técnicas discutidas usando dados de campo. O equipamento estudado é movido a energia elétrica e o principal modo de falha consiste em uma degradação interna que afeta a produtividade do processo. Esta falha pode ocorrer se a corrente elétrica consumida ultrapassa um valor fixado pelo fabricante. Em caso de ocorrência da falha em estudo, o equipamento pode até funcionar em modo degradado mas a degradação terá sido grande e um reparo complexo será necessário para rejuvenescer o equipamento.

Vamos definir o estado falhado (L) como o estado onde será necessário executar o reparo complexo para colocar o sistema no estado 1 ("tão bom quanto novo"). A falha analisada pode ser vista como oculta pois ela não implica necessariamente em parada do sistema. Assume-se que outros modos de falha não são relevantes para este estudo. A partir da análise do equipamento e tendo em vista limitações técnicas, foi definido que a corrente elétrica é o parâmetro monitorado, que será medido todo dia (período de amostragem T).

Os dados de campos foram obtidos a partir do histórico de funcionamento de 3 equipamentos distintos mas em condições de operação similares. Os dados são compostos por um total de 11 séries de leitura de corrente ao longo do tempo, todas se iniciando com o sistema no estado "tão bom quanto novo". Duas destas séries terminam com o sistema sofrendo uma manutenção preventiva (como discutido na Fig. 3.1a) e as demais séries terminam com a falha do equipamento.

A função de custo $g(x_k, u_k)$ é apresentada na Tab. 5.2. Lembramos que ela representa o custo a ser pago por estar no estado x_k e tomar a decisão u_k, em cada época de decisão k. A Fig. 5.5 apresenta os dados de campo e a Tab. 5.1 mostra o passo de classificação, onde transformamos o valor do parâmetro de controle (θ_k) em medida de condição (Z_k). O resultado é apresentado na Fig. 5.6.

A discussão completa do exemplo de aplicação é apresentada no Cap. 5. Nele discutimos passo a passo as etapas das técnicas discutidas do trabalho e ilustramos seus pontos chaves.

Conclusão e Pesquisa Futura

Neste trabalho discutimos a formulação de políticas de manutenção baseada na condição (CBM) para sistemas sujeitos à deterioração e a inspeções periódicas. O sistema é representado por um processo de Markov com estados discretos e levou-se em consideração que a estimação da condição do sistema pode não ser perfeita. Apresentamos também uma discussão sobre a estimação dos parâmetros tanto de um ponto de vista teórico quanto prático.

O resultado principal da dissertação é uma técnica que combina um modelo de otimização e um modelo de inferência a partir dos dados históricos do sistema. Este fato foi ilustrado com a aplicação da metodologia proposta em um problema industrial, no qual discutimos passo a passo as etapas apresentadas. Os resultados sugerem uma aplicação industrial viável que reflete a realidade encontrada pelos gestores responsáveis pela tomada de decisão em manutenção.

Um ponto que acreditamos relevante do nosso trabalho é que conseguimos realizar a estimação dos parâmetros do modelo de forma consistente. Alguns artigos na literatura já tinham apontado uma potencial aplicação dos Modelos de Markov Ocultos (HMM) para modelar a evolução da condição dos sistemas em manutenção. Nós expandimos esta idéia propondo um modelo de otimização via Programação Dinâmica Estocástica (SDP) que combina uma etapa de estimação de parâmetros usando os HMMs. Acreditamos que esta combinação é interessante e pode motivar mais pesquisas na área.

Existem extensões e refinamentos que provavelmente merecem ser explorados. Como trabalhos futuros, acreditamos que nossa metodologia pode ser melhorada considerando algumas sofisticações como uso de reparos intermediários além do reparo perfeito, renovação estocástica e uso de inspeção aleatória ou seqüencial. Naturalmente, estes refinamentos podem aumentar a complexidade e assim podem requerer uma forma diferente de processar os dados históricos a fim de estimar os parâmetros do modelo.

Contents

List of Figures

List of Tables

List of Abbreviations

CBM:	Condition Based Maintenance
CM:	Corrective Maintenance
HHM:	Hidden Markov Model
LP:	Linear Programming
MDP:	Markov Decision Process
PI:	Policy Iteration
PM:	Preventive Maintenance
RCM:	Reliability Centered Maintenance
RTF:	Run To Failure
SDP:	Stochastic Dynamic Programming
SM:	Scheduled Maintenance
TBM:	Time Based Maintenance
VI:	Value Iteration

List of Symbols

Chapter 1

Introduction

1.1 Background

Maintenance plays a key role in industry competitiveness. The activities of maintaining military equipments, transportation systems, manufacturing systems, electric power generation plants, etc., often incur high costs and demand high service quality. Consequently, the study of Preventive Maintenance (PM) has received considerable attention in the literature in the past decades. PM means to maintain an equipment or a machine (hereafter denoted as "system") on a preventive maintenance basis rather than a "let-it-fail-then-fix-it" basis, commonly known as run-to-failure (RTF).

Preventive Maintenance can be classified into two categories: Scheduled Maintenance (SM) (also known as time-based maintenance) and Condition-Based Maintenance (CBM) (or predictive maintenance). The first category considers the system as having two states: non-failed and failed, while the second considers a multi-state deteriorating system. The aim of a SM policy is to derive a statistically fixed "optimal" interval, at which one should intervene in the system (Wang et al., 2008).

Barlow and Hunter (1960) published one of the first papers on SM and since then a large theory has been developed in this field. For example, we can cite the Reliability Centered Maintenance, which is an optimized way to formulate and apply SM policies (Bloom, 2006). However, a SM policy may not take into account variations in environmental conditions and applications of individual systems, which might not follow population-based distributions. Moreover, the SM policy does not include the system condition's status when the latter is available.

The CBM approach, which is growing in popularity since the 1990s, highlights the importance of maintenance policies that rely on the conditions (past and present) of systems. In fact, the term CBM denotes monitoring for the purpose of determining the current "health status" of a system's internal components and predicting its remaining operating life. In other words, on a CBM policy, we try to assess the system's condition and use this information to propose a more accurate maintenance policy.

1.2 The Problem Addressed

This dissertation proposes a CBM policy and input parameters estimation for deteriorating systems under periodic inspection. Thus, we assume that the system deterioration is described by discrete states ordered from the state "as good as new" to the state "completely failed". At each periodic inspection, whose outcome might not be accurate, a decision has to be made between continuing to operate the system or stopping and performing its preventive maintenance.

This problem is modeled using the Markov chains theory that, in combination with Stochastic-Dynamic Programming, leads to an optimal solution. An optimal solution is a rule determining when the system should be maintained, based on its inspection result, in order to minimize its operation cost. We consider that the preventive repair is perfect, that is, it brings back the system to the state "as good as new". In order to apply our optimization model, we have formulated an estimation technique using the Hidden Markov Models. This estimation allows us to infer about the optimization model parameters using the historical data of the system.

In this context, this dissertation develops a framework combining an optimization model and input parameters estimation from empirical data.

1.3 Contributions of this Dissertation

The aim of this dissertation is threefold: i) to propose a model to formulate optimal CBM policies; ii) to develop a procedure for model parameters estimation; and iii) to illustrate our approach with an empirical example.

Our main contribution lies in the fact that we combine an optimization model and a technique for estimating the model input parameters based on system historical data. We believe our approach fills a commonly noticed gap in the literature namely, the fact that most of CBM models do not discuss the model input parameters. Hence, the literature has not explored the combination of optimization techniques and model input parameters, through historical data, for problems with imperfect information such as the one considered in this dissertation.

We argue that our approach is more realistic as far as the estimation of model inputs parameters is concerned. This dissertation also provides a practical discussion using empirical data.

1.4 An Overview of the Dissertation

This dissertation is organized into five chapters. Chapter 2 includes a brief survey of the vast literature in the field of optimal maintenance. This chapter highlights the diversity of approaches proposed to tackle maintenance problems. We also provide in this chapter a brief discussion of the mathematical concepts used in this dissertation.

In Chapter 3 we formally state the problem we are concerned with and we propose an algorithm for CBM policies construction. In Chapter 4 we discuss the estimation of the input parameters of the model developed in the previous chapter by using Hidden Markov Models and we present an algorithm to estimate these input data. In Chapter 5 we provide an application example of this study using data provided by a zinc mining company. Finally, in Chapter 6, we conclude this dissertation by discussing the "pros" and "cons" of our methodology and pointing out some suggestions for future research.

Chapter 2

Basic Concepts

This chapter provides a briefly description of the maintenance optimization. Different maintenance policies are presented. We also introduce some mathematical concepts used in this dissertation.

2.1 Reliability and Maintenance

Reliability engineering studies the application of mathematical tools, specially statistics and probability, for product and process improvement. In this context, researchers and industries are interested in investigating the systems deterioration and how to tackle this phenomenon in order to optimize some quantity.

Any system can be classified as repairable and nonrepairable: a nonrepairable system being a system that fails only once and is then discarded. This work addresses to repairable systems. Since the system can be repaired, it may be wise to plan these repairs, i.e, to plan the maintenance actions.

We are interested in certain quantities for analyzing reliability and maintenance models. In general, for a given system, we aim to consider three: reliability, availability and maintainability.

Reliability: is defined as the probability that a system will satisfactorily perform its intended function under given circumstances for a specified period of time. Usually, the reliability of a repairable system is measured by its failure intensity function, which is defined as follows

$$\lim_{\Delta t \to 0} \frac{\Pr[\text{Number of failures in } (t, t+\Delta t] \geq 1]}{\Delta t}.$$

With this function we can obtain the mean time between failures (MTBF) representing the expected time that the next failure will be observed.

Availability: it means the proportion of time a given system is in a functioning condition. The straightforward representation for availability is as a ratio of the expected uptime value to the expected values of the uptime plus downtime, i.e.,

$$A = \frac{\text{E[Uptime]}}{\text{E[Uptime]} + \text{E[Downtime]}}.$$

Maintainability: is defined as the probability of performing a successful perfect repair within a given time. The mean time to repair (MTTR) is a common measure of the maintainability of a system and it is equals to E[Downtime] under some assumptions.

For additional information on this topic, the reader is referred to (Bloom, 2006; Nakagawa, 2005; Wang and Pham, 2006; Pham, 2003).

2.2 Optimal Maintenance Models

The rise of optimal maintenance studies is closely correlated to the beginning of Operations Research in general which was developed during the Second World War. For example, during this time, a researcher called Weibull focused on approximating probability distributions to model the failure mechanics of materials and introduced the well-known Weibull distribution for use in modeling component lifetimes.

The literature about optimal maintenance models (continuous or discrete time) can be classified as follows (Sherif and Smith, 1981):

1. Deterministic models

2. Stochastic models

1 Under risk

2 Under uncertainty

a Simple (or single-unit) system

b Complex (or multi-unit) system

i Preventive Maintenance (periodic[1], sequential[2])

ii Preparedness Maintenance (periodic, sequential, opportunistic[3])

Several Applied Mathematics and Computational techniques, such as Operations Research, Optimization and Artificial Intelligence, have been employed for analyzing maintenance problems and obtaining optimal maintenance policies. We can cite the Linear Programming, Nonlinear Programming, Mixed-Integer Programming, Dynamic Programming, Search techniques and Heuristic approaches.

We define now the concepts of corrective and preventive maintenance. After that we introduce some optimal maintenance models.

2.2.1 Corrective and Preventive Maintenance

Maintenance can be classified into two main categories: corrective and preventive (Wang and Pham, 2006). Corrective Maintenance (CM) is the maintenance that occurs when the system fails. CM means all actions performed as a result of failure, to restore an item to a specified condition. Some texts refer to CM only as repair. Obviously, CM is performed at unpredictable time points since the system's failure time is not known.

Preventive maintenance (PM) is the maintenance that occurs when the system is operating. PM means all actions performed in an attempt to retain an item in specified condition by providing systematic inspection, detection, and prevention of incipient failures.

[1]In a periodic PM policy, the maintenance is performed at fixed intervals.

[2]A sequential PM policy means to maintain the system at different intervals.

[3]Opportunistic Maintenance explores the occurrence of an unscheduled failure or repair to maintain the system.

Both CM and PM can be classified according to the degree to which the system's operating condition is restored by maintenance action in the following way (Wang and Pham, 2006):

1. Perfect repair or perfect maintenance: maintenance actions which restore a system operating condition to "as good as new". That is, upon a perfect maintenance, a system has the same lifetime distribution and failure intensity function as a new one.

2. Minimal repair or minimal maintenance: maintenance actions which restore a system to the same level of the failure intensity function as it had when it failed. The system operating state after the minimal repair is often called "as bad as old" in the literature.

3. Imperfect repair or imperfect maintenance: maintenance actions which make a system not "as good as new" but younger. Usually, it is assumed that imperfect maintenance restores the system operating state to somewhere between "as good as new" and "as bad as old".

We briefly present now the characteristics of each optimal maintenance model family.

2.2.2 Deterministic Models

These models are developed under some assumptions, we cite the followings:

- The outcome of every PM is not random and it restores the system to its original state.

- The system's purchase price and the salvage value are function of the system age.

- Degradation (aging, wear and tear) increases the system operation cost.

- All failures are observed instantaneously.

The optimal policy for deterministic models is a periodic policy (Sherif and Smith, 1981) and hence the times between PMs are equal. Charng (1981) presents a short discussion on this kind of model, which is is also known as age-dependent deterministic continuous deterioration.

2.2.3 Stochastic Models Under Risk

Risk is a time-dependent property that is measured by probability. For a system subject to failure, it is impossible to predict the exact time of failure. However, it is possible to model the stochastically behavior of the system (e.g. the distribution of the time to failure).

Some of these models will be presented in Section 2.3.

2.2.4 Stochastic Models Under Uncertainty

We deal here with failing systems under uncertainty, i.e., neither the exact time to failure nor the distribution of that time is known. These problems are harder since less information is available. Literature reports methods such as (Sherif and Smith, 1981):

- Minimax techniques: applied when the system is new or failure data are not known;
- Chebychev-type bounds: applied when partial information about the system (such as failure rate) is known;
- Bayesian techniques: applied when subjective beliefs about the system failure and non-quantitative information are available.

Since this dissertation does not deal with this type of problem, this topic will not be covered.

2.2.5 Single-unit and Multi-unit Systems

A simple (single-unit) system is a system which can not be separated into independent parts and has to be considered as a whole. However, in practice, a system may consist of several components, i.e., the system is compounded by a number of subsystems.

In terms of reliability and maintenance, a complex (multi-unit) system can be assumed to be a single-unit system only if there exists neither economic dependence, failure dependence nor structural dependence. If there is dependence, then this has to be considered when modeling the system. For example, the failure of one subsystem results in the possible opportunity to undertake maintenance on other subsystems (opportunistic maintenance).

This dissertation considers only single-unit systems or multi-unit systems which can be analyzed as a single-unit one.

2.2.6 Preventive Maintenance Under Risk

In this case we are interested in modeling the system deterioration in order to diagnose the best time to carry out a preventive maintenance. Since this is the focus of this work, we will dedicate the Section 2.3 to cover this theme.

2.2.7 Preparedness Maintenance Under Risk

In preparedness maintenance, a system is placed in storage and it replaces the original system only if a specific but unpredictable event occurs. Some maintenance actions may be taken while the system is in storage and the objective is to choose the sequence of maintenance actions resulting in the highest level of system "preparedness for field use".

For instance, when the system is in storage, it can be submitted to a long-term cold standby and an objective would be to choose the maintenance actions providing the best level of preparedness (or readiness to use).

2.3 Related Research on Preventive Maintenance Under Risk for Single-unit Systems

Firstly we wish to classify the models into two categories: the first considers the system as having two states: non-failed and failed; the second considers a multi-state deteriorating system.

For each category, we discuss the five families of maintenance strategies according to Lam and Yeh (1994):

1. Failure maintenance (Run-To-Failure): no inspection is performed. The system is maintained or replaced only when it is in the failed-state.

2. Age maintenance: the system is subject to maintenance or replacement at age t (regardless the system state) or when it is in the failed-state, whichever occurs first. The block replacement policy is an example.

3. Sequential inspection: the system is inspected sequentially: the information gathered during inspection is used to determine if the system is maintained or the system is scheduled for a new inspection to be performed some time later.

4. Periodic inspection: a special case of sequential, when the period of inspection is constant.

5. Continuous inspection: the system is monitored continuously and whenever some threshold is reached the system is maintained.

2.3.1 Main Strategies for Two-states Systems

This family of models usually utilizes the following assumptions:

- The time to failure is a random variable with known distribution.

- The system is either operating or failed and failure is an absorbing state: the system only can be regenerated if a maintenance action is performed.

- The intervals between successive regeneration points are independent random variables, i.e., the time between failures are independent.

- The cost of an maintenance action is higher if it is undertaken after failure than before.

Sherif and Smith (1981); Wang and Pham (2006) indicate that failure maintenance is the optimal policy recommended for systems with a constant failure intensity function (exponential). On the other hand, for a system with increasing failure intensity function (weibull or gamma for some parameters) should be maintained or not in function of its age.

The act of using reliability models to plane the maintenance actions is the essence of the Reliability-Centered Maintenance (RCM) (Smith, 1993; Moubray, 1993; Bloom, 2006).

2.3.2 Main Strategies for Multi-states Systems

For multi-state degrading systems, the system is considered to be subject to failure processes which increase the system degradation and random shocks. In this case, we are not only interested in obtaining the system reliability model but also to obtain expressions about the system states by calculating probabilities (Wang and Pham, 2006).

The most common approach is using Markov chains (continuous and discrete time) to describe the system. In such approach, the system condition is classified by a finite number of discrete states, such as in Refs. (Kawai et al., 2002; Bloch-Mercier, 2002; Chen et al., 2003; Chen and Trivedi, 2005; Gong and Tang, 1997; Gürler and Kaya, 2002; Chiang and Yuan, 2001; Ohnishi et al., 1994). The purpose of these models is to determine an action to be carried out at each state of the system (repaired/replaced) in order to obtain a minimum expected maintenance cost.

In terms of the condition measurement, the literature considers sequential checking such as studied in (Bloch-Mercier, 2002; Gürler and Kaya, 2002), or periodic inspection, such as in (Chen et al., 2003; Gong and Tang, 1997; Chen and Trivedi, 2005; Ohnishi et al., 1994); perfect inspection, such as in (Bloch-Mercier, 2002; Chen and Trivedi, 2005; Chen et al., 2003; Chiang and Yuan, 2001) or imperfect inspection, such as in (Gong and Tang, 1997; Ohnishi et al., 1994).

The act of considering the system to be multi-state is related to Condition-Based Maintenance (CBM) since these models generally deals with multi-state systems (Valdez-Flores and Feldman, 1989).

2.4 Mathematical Tools

In this section we briefly introduce some mathematical concepts used in this dissertation. We do not aim to cover these topics comprehensively but just provide a short introduction to them.

2.4.1 Reliability and Statistics

We introduce in this section some statistical terminology for common used in reliability engineering. For the purposes of this section, let T be a non-negative continuous random variable which denotes the first failure time of the system. T has a given probability distribution $f(t)$; the cumulative distribution $F(t)$ is called the failure distribution and it describes the probability of failure prior the time t, i.e.,

$$F(t) = \Pr[T \leq t]. \tag{2.1}$$

The reliability function is defined as $R(t) = 1 - F(t)$, which is the probability that the system will continue working past time t. The failure rate (or hazard function) is defined as follows

$$\lambda(t) = \frac{R(t) - R(t + \Delta t)}{\Delta t \cdot R(t)} = \frac{\Pr[t < T \leq t + \Delta t | T > t]}{\Delta t}, \tag{2.2}$$

when $\Delta \to 0$. It can be shown that $\lambda(t) = \frac{f(t)}{R(t)}$.

This concepts are applied for systems called non-repairable as well as for repairable system when the repair is perfect or the system is simply replaced by a new one. We briefly discuss the two most common models for the failure time. For further information on this topic please check (Rigdon and Basu, 2000; Nakagawa, 2005; Wang and Pham, 2006).

Exponential model

In this model, T is assumed to follow an exponential distribution with a parameter θ. Hence, we have

$$f(t) = \frac{1}{\theta}\exp(-t/\theta) \text{ and } F(t) = 1 - \exp(-t/\theta).$$

This model has the main features:

1. The memoryless property, i.e., $\Pr[T \leq t + a | T \leq t] = \Pr[T \leq a] = \exp(-a/\theta)$;
2. A constant failure rate, i.e, $\lambda(t) = \dfrac{f(t)}{R(t)} = 1/\theta$;
3. The expected time to failure (MTTF) is $\mathrm{E}[T] = \theta$.

Weibull model

Here we assume that T a weibull distribution with the parameters η and α. Hence,

$$f(t) = \frac{\eta}{\alpha}\left(\frac{t}{\alpha}\right)^{\eta-1} \exp\left(-\left(\frac{t}{\alpha}\right)^{\eta}\right) \text{ and } F(t) = 1 - \exp\left(-\left(\frac{t}{\alpha}\right)^{\eta}\right).$$

The main properties of this model are:

1. The failure rate is $\lambda(t) = \dfrac{\eta}{\alpha}\left(\dfrac{t}{\alpha}\right)^{\eta-1}$;
2. The MTTF is $\mathrm{E}[T] = \alpha\Gamma\left(1 + \frac{1}{\eta}\right)$, where Γ is the gamma function defined as follows for a $a > 0$:

$$\Gamma(a) = \int_0^{\infty} x^{a-1}e^{-x}dx.$$

2.4.2 Markov Chains

Consider a system that can be in any one of a finite or countably infinite number of states. Let X denote this set of states. Without loss of generality, assume that X is a subset of the natural numbers ($\{1, 2, 3, \dots\}$). X is called the state space of the system. Let the system be observed at the discrete moments of time $k = 0, 1, 2, \dots$, and let X_k denote the state of the system at epoch k.

If the system is non-deterministic we can consider $\{X_k\}$ ($k \geq 0$) as random variables defined on a common probability space. The simplest way to manipulate $\{X_k\}$ is to suppose that X_k are independent random variables, i.e., future states of the system are independent of past and present states. However, in most system in the practice, this assumption does not hold.

There are systems that have the property that given the present state, the past states have no influence on the future. This property is called the Markov property and one of the most used stochastic processes having this property is called Markov chain. Formally, the Markov property is defined by the requirement that

$$\Pr[X_{k+1} = x_{k+1} | X_0 = x_0, \dots, X_k = x_k] = \Pr[X_{k+1} = x_{k+1} | X_k = x_k], \quad (2.3)$$

for every $k \geq 1$ and the states $x_0, \dots, x_{k+1}$ each in X. The conditional probabilities $\Pr[X_{k+1} = x_{k+1} | X_k = x_k]$ are called the transition probabilities of the chain.

We call the system's initial state as ω which is defined by

$$\omega(x) = \Pr[X_0 = x], \quad x \in X. \quad (2.4)$$

ω is hence the initial distribution of the chain. If the conditional probabilities depicted in equation 2.3 are constant in time, i.e.,

$$\Pr[X_{k+1} = j | X_k = i] = \Pr[X_1 = j | X_0 = i], \quad i, j \in X,\ k \geq 0. \quad (2.5)$$

the Markov chain is called time-homogeneous. Let us call these probabilities as

$$a_{ij} = \Pr[X_{k+1} = j | X_k = i], \quad i, j \in X,\ k \geq 0. \tag{2.6}$$

We define the matrix $A \equiv [a_{ij}]$ which is called the transition probabilities matrix of the (time-homogeneous) chain.

The Markov chains are well-known in the literature mainly because their study is worthwhile from two viewpoints. First, they have a rich theory and, secondly, there are a large number of systems that can be modeled by Markov chains. Further information on this topic can be found in (Hoel et al., 1972; Grimmett and Stirzaker, 2001; Cassandras and Lafortune, 2009).

2.4.3 Hidden Markov Models

In the previous section, we have assumed that we know the system's state (X_k) any time, i.e., the Markov chain is observable. Indeed, this assumption has allowed us to state the transition probabilities matrix A (equation 2.6) that, in combination with the initial distribution of the chain, allows us to predict future behavior of the system (e.g. the Chapman-Kolmogorov equation). However, this assumption may not be reasonable for some systems in the practice.

Actually, it is quite common to find a system in which we do not have the directly access to its state. In other words, X_k is unknown or hidden. In this case, we want to be able to handle this constraint by creating a way to assess X_k. For this purpose, there have been models which focus on estimating X_k based on observations.

A Hidden Markov Model (HMM) is a discrete-time finite-state homogeneous Markov chain ($\{X_k\}$ in our case) observed through a discrete-time memoryless invariant channel. Through this "channel", we observe a finite number of outcomes. Without loss of generality, let us assume that the number of outcomes and states of the Markov chain are the same. Let this number be L. Hence, each observation corresponds to a state of the system being modeled.

We denote the set of these observations as $Z = \{z_1, z_2, \ldots, z_L\}$ and we define the observation probability distribution as

$$b_x(z) = \Pr[Z_k = z | X_k = x], \quad z \in Z,\ x \in X,\ k \geq 0. \tag{2.7}$$

It is assumed that this distribution does not change over time, i.e., $b_x(z)$ is the same for every $k \geq 0$. These probabilities can be written in a the matrix form: $B \equiv b_x(z)$. Hence, a HMM is fully represented by its probability distributions A, B and ω. For convenience, we define the notation:

$$\Psi = (A, B, \omega). \tag{2.8}$$

There are three basic problems in HMM that are very useful in practical applications. These problems are:

Problem 1: Given the model $\Psi = (A, B, \omega)$ and the observation sequence $O = z_1, z_2, \cdots, z_k$, how to compute $\Pr[O|\Psi]$ (i.e., the probability of occurrence of O)?

Problem 2: Given the model $\Psi = (A, B, \omega)$ and the observation sequence $O = z_1, z_2, \cdots, z_k$, how to choose a state sequence $I = x_1, x_2, \cdots, x_k$ so that $\Pr[O, I|\Psi]$[4] is maximized (i.e., best "explain" the observations)?

Problem 3: Given the observation sequence $O = z_1, z_2, \cdots, z_k$, how do we adjust the HMM model parameters $\Psi = (A, B, \omega)$ so that $\Pr[O|\Psi]$ is maximized?

While problems 1 and 2 are analysis problems, problem 3 can be viewed as a synthesis (or model identification or inference) problem.

The HMMs have various applications and one is pattern recognition such as speech and handwriting. For example, there is a known technique in Signal Processing called the Viterbi Algorithm which efficiently tackles the problem 2. For additional information on this topic, the reader is referred to (Dugad and Desai, 1996; Ephraim and Merhav, 2002; Rabiner, 1989; Grate, 2006).

[4] Which represents the joint probability of the state sequence and observation sequence

2.4.4 Applying Control in Markov Chains: Markov Decision Process

So far, we have considered Markov chains having fixed transition probabilities. If we can change these probabilities then we will change the evolution of the chain. In some situations we might be interested in how to induce the system to follow some behavior in order to optimize some quantity. It can be performed if we can "control" the transition probability of the system.

We call the act of controlling the transition probability by applying control in Markov chains. Let us call the control applied at the time k by $u_k \in U$, U being the control space, i.e, the set of all possible actions. We rewrite Eq. 2.6 taking into account the control as follows

$$a_{ij}(u) = \Pr[X_{k+1} = j | X_k = i, u_k = u], \quad i, j \in X,\ u \in U,\ k \geq 0, \tag{2.9}$$

with $\sum_{j \in X} a_{ij}(u) = 1,\ i \in X,\ u \in U$. The control follows a policy denoted by π, which is nothing but the strategy or a plan of action. In general, a policy is developed using the feedback, i.e., the policy is a plan of actions for each state $x_k \in X$ at the time k. Hence, a policy can be written as

$$\pi = \{\mu_0, \mu_1, \ldots, \mu_k, \ldots\}, \tag{2.10}$$

where μ_k maps each state to an action, i.e., $\mu_k : x_k \mapsto u_k \Rightarrow u_k = \mu_k(x_k)$. In this case, it is assumed that the Markov chain is observable. If μ_k are all the same, π is called a stationary policy.

Different policies will lead to different probability distributions. In a optimal control context, we are interested in finding the best or optimal policy. To this end, we need to compare different policies, which can be done by specifying a cost function. Let us assume the cost function is additive. Thus, the total cost until $k = n$ is calculated as follows

$$\sum_{k=0}^{n} g(x_k, u_k), \tag{2.11}$$

where $g(x, u)$ is interpreted as the cost to be paid if $X_k = x$ and $u_k = u$ at the time k. $g(x, u)$ is referred to as immediate or one period cost. If the chain (or the system evolution) stops at the time N, there is no action to be taken when $k = N$ and we rewrite Eq. 2.11 as follows

$$\sum_{k=0}^{N-1} g(x_k, u_k) + g(x_N), \tag{2.12}$$

$g(x_N)$ is called the terminal cost.

Notice that X_k and the actions u_k all depend on the choice of the policy π. Furthermore, $g(x_k, u_k)$ is a random variable. If we deal with a finite horizon problem, for any horizon N, we write the system cost under the policy π as

$$J_\pi(x_0) = \mathrm{E}\left[\sum_{k=0}^{N-1} g(x_k, u_k) + g_N(x_N)\right], \tag{2.13}$$

where x_0 is the initial state. For an infinite horizon problem, we have

$$J_\pi(x_0) = \lim_{N\to\infty} \mathrm{E}\left[\sum_{k=0}^{N} g(x_k, u_k)\right]. \tag{2.14}$$

Let an optimization problem be the minimization of the expected cost J. Then, the optimal policy π^* is that one which minimizes J, i.e., $J_{\pi^*} = \min_\pi J_\pi$. This is a sequential decisions problem that can be tackled using Dynamic Programming (DP). We will use the term Stochastic Dynamic Programming (SDP) to reinforce the stochastic character of our problems. SDP decomposes a large problem into subproblems and it is based on the Principle of Optimality proposed by Bellman[5]. Under this result, the solution of the general problem is compounded by the solutions of the subproblems.

An explicit algorithm for determining an optimal policy can be developed using SDP. Let $J_k(x_k)$ be the cumulated cost at time k for all states x_k. Then,

$$J_k(x_k) = \min_{u_k \in U} \mathrm{E}\left[g(x_k, u_k) + J_{k+1}(x_{k+1})\right], \quad k = 0, 1, \ldots, \tag{2.15}$$

[5]In Proposition 1, for the purposes of this result, we recall the definition of this principle.

is the optimal cost-to-go from state x_k to state x_{k+1}. The function $J_k(x_k)$ is referred to as the cost-to-go function in the following sense: the equation states a single-step problem involving the present cost $g(x_k, u_k)$ and the future cost $J_{k+1}(x_{k+1})$. The optimal action u_k is then chosen and the problem is solved. Of course, $J_{k+1}(x_{k+1})$ is not known.

For convenience, let us assume a finite horizon problem. Hence, $J_N(x_N) = g(x_N)$ is known and it can be used to obtain $J_{N-1}(x_{N-1})$ by solving the minimization problem in Eq. 2.15 over u_{N-1}. Then $J_{N-1}(x_{N-1})$ is used to obtain $J_{N-2}(x_{N-2})$ and so on. Ultimately, we obtain $J_0(x_0)$ which will be optimal cost. This is a backward procedure.

Some attention is required when working with infinite horizon because the Eq. 2.11 can be infinite or undefined when $n \to \infty$. Under some circumstances, the infinite horizon problem does have solution. We illustrate some of such scenarios:

Optimal stopping time (stochastic shortest path): it is assumed that there exist state x and action u such that $g(x, u) = 0$.

Expected discounted cost: we use a discounting factor to discount future values. Hence, Eq. 2.14 is replaced by

$$J_\pi(x_0) = \lim_{N\to\infty} \mathrm{E}\left[\sum_{k=0}^{N} \alpha^k g(x_k, u_k)\right], \quad \alpha \in (0, 1). \tag{2.16}$$

Expected average cost: a policy is evaluated according to its average cost per unit of time. Hence, Eq. 2.14 is replaced by

$$J_\pi(x_0) = \lim_{N\to\infty} \frac{1}{N}\mathrm{E}\left[\sum_{k=0}^{N} g(x_k, u_k)\right]. \tag{2.17}$$

Further information on this topic can be found in (Bellman, 1957; Bertsekas, 2005; Sutton and Barto, 1998; Puterman, 1994).

Chapter 3

A Model for Condition-Based Maintenance

3.1 Introduction and Motivation

In this chapter we formulate an optimization model for CBM. Our goal is to determine a CBM policy for a given system in order to minimize its long-run operation cost.

3.2 Problem Statement

We assume that the system's condition can be discretized in states and that each state is associated with a possible degradation degree. Periodically, we have an estimation of the condition, which can be obtained by inspection or by the use of sensor(s) or other devices used to monitor the system. The estimation can be imperfect, that is to say, different from the true system condition. Other assumptions are:

1. The system is put into service in time 0 in the state "as good as new";
2. All repairs are perfect (for example, by replacing the system), that is, once repaired, the system turns back to the state "as good as new". We do not consider the minimal-repair possibility;

3. The time is discrete (sampled) with respect to a fixed period T, i.e., $T_{k+1} = T_k + T$, where $k = 1, 2, \ldots$ is the kth sample time. In order to shorten the notation, we denote hereafter the instant time T_k as k ;

4. At the instant k, the system is inspected in order for its condition to be estimated. This inspection may be performed by measuring a system's variable, e.g., vibration or temperature. The system's variable monitored is directly related to the failure mode being analyzed;

5. At the instant k, an action u_k is taken : either $u_k = C$ (continue the system's operation, i.e., it is left as it is) or $u_k = S$ (stop and perform the maintenance), i.e., the decision space is $U = \{C, S\}$;

6. Failures are not instantaneously detected. In other words, if the system fails in $[k-1, k)$, it will be detected only at the epoch k. This is not restrictive since we can overcome this assumption by choosing a small period T.

In our CBM framework, the model parameters are adjusted and the decisions are made based on the history of the system condition. The data set can be separated basically into two groups, distinguished by the kind of maintenance stop. We call each of these groups as a cycle of measurement. The first cycle consists of a sequence of condition records from the system's start up until the occurrence of a failure (marked as "b" in Fig. 3.1). The second cycle ("a" in the same figure) terminates at a preventive maintenance action. Since the corrective maintenance is carried out upon failure, they are considered in the first case.

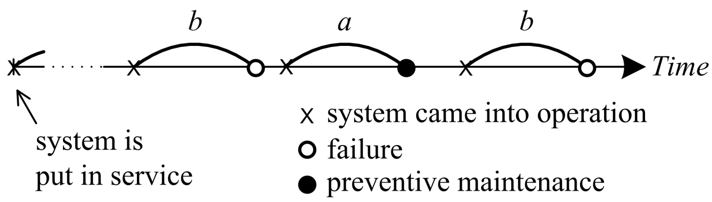

Figure 3.1: Condition measurement cycles: up to a preventive maintenance (a) and up to a failure (b).

The notion of cycles is particularly important in our approach. Each measurement cycle is compounded by the condition measurements gathered from system operation start-up until stop, upon maintenance action. We call that the short-run horizon. On the other hand, we have the long-run horizon, which is composed by the measurement cycles cumulated over time. Fig. 3.2 presents a diagram which illustrates our approach.

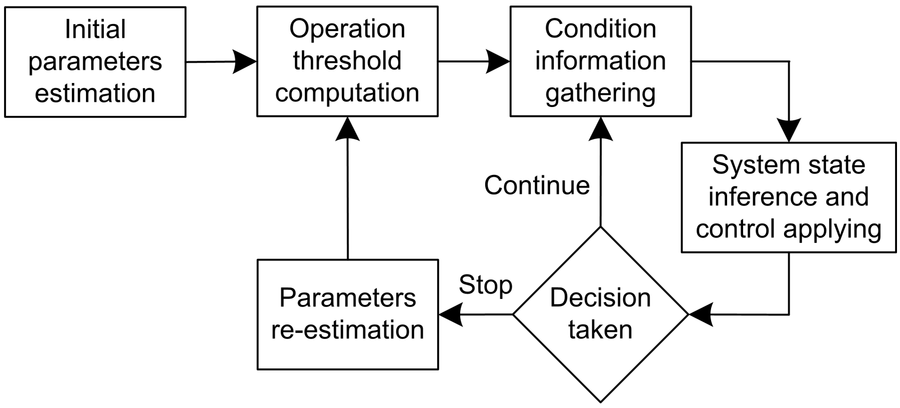

Figure 3.2: The CBM approach proposed in this paper.

We start by performing the initial parameters estimation and we compute the optimal operating threshold based on this estimation. As time evolves, at each decision epoch k, we have a condition measurement, which is used to estimate the system state, and an action is taken based on this estimation. If $u_k = S$, we end a cycle carrying out a parameters re-estimation and a new optimal threshold computation. k is reset to 0 and we start a new cycle.

3.2.1 Short-run and Long-run Optimization

As introduced in last paragraph, we consider a double time frame:

- Short-run horizon: from system start-up ($k = 0$) until system stop ($u_k = S$).
- Long-run horizon: defined as the short-run horizons cumulated over time.

Since we assume perfect repair (assumption 2, Section 3.2) by solving optimally the short-run model we also guarantee the long-run cost minimization. This is the subject of the following results.

Proposition 1 (The structure of the long-run horizon problem)**.** *The optimal solution of the long-run horizon problem can be divided into various optimal sub-solutions, each sub-solution being the optimal solution of the associated short-run horizon problem.*

Proof. Under the assumption of perfect repair, we are able to slice the long-run horizon problem in various short-run horizon problems, all short-run problems having the same structure (however they might have different Ψ's because of re-estimation step). Figure 3.3 illustrates that procedure. Notice that we reset k whenever a repair is carried out ($u_k = S$). Thus, the system is brought back to the state 1 ("as good as new") and we set $k = 0$ at the same time the system operation restarts.

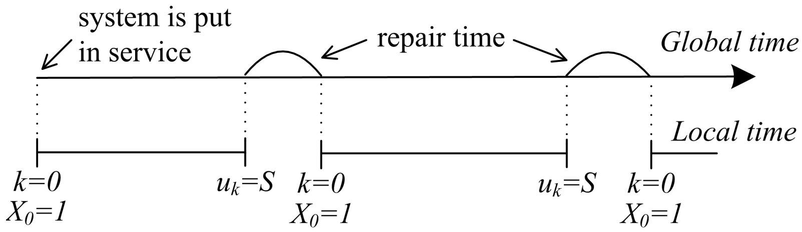

Figure 3.3: Long-run horizon optimization.

The Bellman's Principle of Optimality states that "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, 1957). In other words, given an optimal sequence of decisions, each subsequence must also be optimal. The principle of optimality applies to a problem (not an algorithm) and a problem satisfying this principle has the so-called Optimal Substructure.

The long-run horizon problem has the Optimal Substructure and, by the Bellman's Principle of Optimality, the optimal solution of the long-run necessarily contains optimal solutions to all subproblems (short-run).

□

Corollary 1 (Long-run horizon optimization). *Solving optimally (cost minimization) the short-run horizon implies in long-run optimization.*

Proof. Since all short-run horizon problem (subproblems of the long-run horizon problem) have the same structure, by finding the solution of the short-run problem we get the long-run solution.
Alternative proof:

Let $\{\mu_1, \mu_2, \cdots, \mu_m, \cdots\}$ be the optimal policy for the long-run horizon problem. Then μ_m is the optimal policy of the mth short-run horizon (Proposition 1).

If μ_m was not an optimal policy of the mth short-run horizon, we could then substitute it by the *optimal* policy for the mth short-run horizon, μ^*_m. The result is a *better* policy for the long-run horizon problem. This contradicts our assumption that $\{\mu_1, \mu_2, \cdots, \mu_m, \cdots\}$ is the optimal policy for the long-run horizon problem.

□

Thus, we focus the rest of the chapter on optimizing the short-run horizon.

3.3 Mathematical Formulation

Consider a multi-state deteriorating system subject to aging and sudden failures, with states in deteriorating order from 1 (as good as new) to L (completely failed) . If no action is taken the system is left as it is ($u_k = C$). We assume that the system condition evolution is a Markovian stochastic process and, since we consider periodic inspections, we can model the deterioration using a discrete-time Markov chain.

For this purpose, let $\{X_k\}_{k\geq 0}$ be the Markov chain in question, where X_k denotes the system condition at epoch k and $\{X_k\}$ models the system deterioration over time. The $\{X_k\}$ state space is $X = \{1, 2, \ldots, L\}$ with the associated probability transition $a_{ij}(u_k = C)$, simply denoted as a_{ij}, defined as

$$a_{ij} = \Pr[X_{k+1}=j|X_k=i,\ u_k=C] = \Pr[X_1=j|X_0=i,\ u_0=C],$$

subject to $\sum_{j=i}^{L} a_{ij} = 1,\ \forall i, j$. For convenience, we express these probabilities in matrix form: $A \equiv [a_{ij}]$.

Let $g(\cdot)$ be the cost of the system at each period, written as function of the system's state (X_k) and the decision taken (u_k). This function denotes the expected operational system's cost, the expected unavailability cost incurred upon failure and/or maintenance actions and the expected maintenance action costs themselves, as follows:

- For $x_k \in 1, \ldots, L-1$ we have:
  - $u_k = C$ (continue to operate, i.e., do nothing): $g(x_k, u_k)$ represents the operational system's cost, which can be written in terms of the system's state;
  - $u_k = S$ (stop the operation and perform the preventive maintenance): g symbolizes the expected preventive maintenance cost (including the unavailability cost), which can be written as a function of the system's state: in general, the poorer the condition the higher the cost;
- For $x_k = L$ (failed):
  - $u_k = S$: $g(\cdot)$ describes the expected corrective maintenance cost, including the unavailability cost carried out during the repair;
  - $u_k = C$: $g(\cdot)$ represents the unavailability cost over period $[k, k+1)$, generally a non-optimal decision, since it implies that the system no longer operates.

Now we introduce the following definition:

Definition 1 (Well-defined problem). *A problem is well-defined if it satisfies the following conditions:*

1. *the system condition can be improved only by a maintenance intervention. That is, A entries can be written as*

$$a_{ij} = \begin{cases} Pr[X_{k+1}=j|X_k=i,\ u_k=C], & \text{if } j \geq i, \\ 0, & \text{otherwise.} \end{cases} \tag{3.1}$$

2. *if no maintenance action is taken, there is a positive probability that the state L will be reached after p periods (L is reachable), i.e.,*

$$Pr[X_p = L|X_0 = 1] > 0, \quad \text{with } u_k = C, \forall k < p.$$

   This condition, together with the first, implies that L is also an absorbing state if $u_k = C, \forall k < p$.

3. *the unavailability cost incurred by the system non-operation is higher than the corrective maintenance cost, i.e.,*

$$g(X_k = L, u_k = C) > g(X_k = L, u_k = S).$$

Fig. 3.4 illustrates the Markov chain of a well-defined problem. It reflects the wear out deterioration. Once the system is deteriorated, its condition cannot be improved over time (unless by a maintenance intervention). Moreover, the probabilities a_{iL}, $\forall i \in \{1, \dots, L-2\}$, denotes the sudden failure. Hereafter, it is assumed that the problem is well-defined.

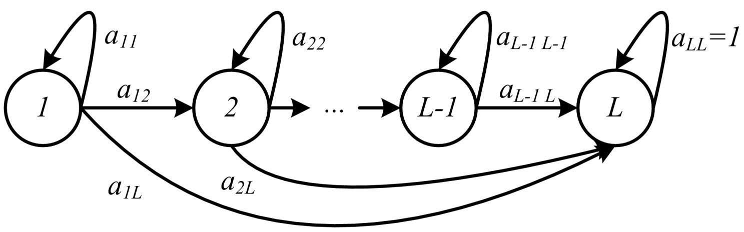

Figure 3.4: The Markov chain that denotes the system condition evolution (X_k). The probabilities $a_{ij:j>i+1,i<L}$ have been omitted for succinctness.

Under Definition 1, it can be shown that there exists an optimal repair rule (Pierskalla and Voelker, 1976). To calculate it, we need to know the system state X_k at any time. Since we do not have access to the information about the system condition (i.e., X_k is unobservable), we shall estimate it. This is the subject of the next subsection.

3.3.1 Modeling the incomplete information

We deal with this problem by creating a condition measurement Z_k , which has its probability distribution conditioned on X_k (see Fig. 3.5). We denote the space state Z_k by $Z = \{\underline{1}, \underline{2}, \ldots, \underline{L}\}$, i.e., the observed condition $\underline{1}$ represents "the system appears to be in the state 1", and so on. Let $b_x(z)$ be the probability $\Pr[Z_k = z|X_k = x]$. For convenience, we define the matrix $B \equiv [b_x(z)]$.

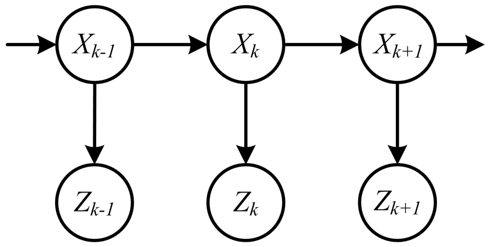

Figure 3.5: System state evolution (X_k) and its estimation (Z_k).

The computation of Z_k establishes a link between the system state and the system monitoring measurement(s). Consequently, a classification step is required to connect the measurement(s) to each value of Z. Hence, the choice would depend on the case being analyzed[1].

We define an information vector, I_k , which retains the condition estimation (Z_k), from the system start-up up to k, i.e., the condition measurement history of the actual cycle. Hence, I_k has size k and can be written recursively as follows

[1] In Chapter 5 we provide examples of classification.

$$\begin{aligned} I_1 &= z_1 \\ I_k &= (I_{k-1}, z_k), \qquad k = 2, 3, \ldots. \end{aligned} \tag{3.2}$$

Using this vector, we are able to estimate[2] the system state X_k at any time k. A straightforward estimator is the following:

$$\widehat{X}_k = \arg\max_{x \in X} \Pr[X_k = x | I_k]. \tag{3.3}$$

An approach to compute this probability will be presented in Section 4.2 (Eq. 4.1). In order to homogenize the notation for HMM, let's denote the model parameters (A, B) by Ψ[3] . Hence, in this dissertation, any system can be fully represented by Ψ and its cost function $g(\cdot)$.

The optimal CBM policy μ is a mapping from available information to the actions, i.e.,

$$u_k = \mu(I_k) = \begin{cases} C, & \text{if } \widehat{X}_k < r, \\ S, & \text{if } \widehat{X}_k \geq r, \end{cases}$$

where $r \in X$ is the optimal threshold state (or repair rule). This is a sequential problem where each action will result in an immediate cost $g(\cdot)$, carried out along the period $[k, k+1)$ and, in addition, the chosen action impacts the evolution of the system over time. We call this subproblem as short-run model. As discussed in Section 3.2, this model can be viewed as a cycle, which starts at the beginning of the operation ($k = 0$) and terminates whenever $u_k = S$.

Solving the short-run model requires an algorithm to minimize the cumulative cost resulting from a sequence of decisions. We solve this problem by

[2]In the Control Theory, this estimation is known as system state estimator.

[3]In HMM theory, Ψ is defined as (A, B, ω), where ω represents the probability distribution of the state X_0. Because of assumption 1, we have $\Pr[X_0 = 1] = 1$ and so we can ignore ω.

using the Stochastic Dynamic Programming (SDP). Similar results can be obtained by using a partially observable Markov Decision Process approach.

3.3.2 Finding the optimal threshold state (short-run model)

Let J be the system operation cost over time until stop, i.e., the sum of $g(\cdot)$ until $u_k = S$. We intend to find the CBM policy μ which minimizes J, i.e., $J^* = \min_\mu J_\mu$, where J_μ represents the system expected cost under the policy μ and can be written as follows

$$J_\mu = \lim_{N\to\infty} \mathrm{E}\left[\sum_{k=1}^{N} \alpha^k g(X_k, u_k)\right], \tag{3.4}$$

where $\alpha \in [0, 1]$ is the discount factor, used to discount future costs. The infinite horizon model considered in Eq. 3.4 can be, recalling that $\mu_k : I_k \mapsto u_k$, decomposed into the following SDP equation

$$\begin{aligned} J_k(I_k) = \min_{u_k} \Big\{ & \mathrm{E}\big[g(X_k, u_k)|I_k, u_k\big] + \\ & \alpha \mathrm{E}\big[J_{k+1}(I_k, Z_{k+1})|I_k, u_k\big]\Big\}, \quad k = 1, 2, \ldots. \end{aligned} \tag{3.5}$$

A "basic" SDP equation would require the knowledge of the system state which, in our case, is unknown. Nevertheless, by combining the system estimator presented in equation 3.3, we can write the SDP equation as pointed out in equation 3.5. This procedure is called reduction an imperfect information problem to a perfect one.

Two algorithms are well known to solve infinite horizon SDP: value iteration (VI) and policy iteration (PI). In fact, VI is a special formulation of the PI algorithm (Bertsekas, 2005; Sutton and Barto, 1998; Puterman, 1994). If we are considering a short period T (e.g., a day), the time value of money is not relevant, and the discount factor (α) can be fixed in 1. In this case, the VI method may require an infinite number of iterations. However, the method converges finitely under special circumstances. Under definition 1 our problem can be reduced to a Stochastic Shortest-Path (SSP) problem and, using this fact, we can guarantee the convergence of the VI algorithm, for $\alpha = 1$, which will be discussed in the proposition 2. For $\alpha < 1$, the convergence is straightforward.

Proposition 2 (Convergence of the value iteration algorithm)**.** *Suppose that the problem is well-defined. Then, the VI algorithm converges with probability 1, since the action* $u_k = S$ *will be taken with finite* k*.*

Proof. Under definition 1, we are able to write the problem as a Stochastic Shortest-Path (SSP) problem (Bertsekas, 2005; Bertsekas and Tsitsiklis, 1991), where the system states are nodes. A SSP problem is a problem in which the state space $\{1, 2, \ldots, n, t\}$ is such that the node t is a goal (target) state that is absorbing and cost-free (i.e., $\Pr[X_{k+1} = t | X_k = t] = 1$ and $g(t, u) = 0, \ \forall u$), and the discount factor $\alpha = 1$.

In order to write our problem as a SSP problem, let the states $1, \ldots, L$ be the nodes $1, \ldots, n$ and create the target node t. This target can be reached from each state whenever the action $u = S$ is taken (see Fig. 3.6).

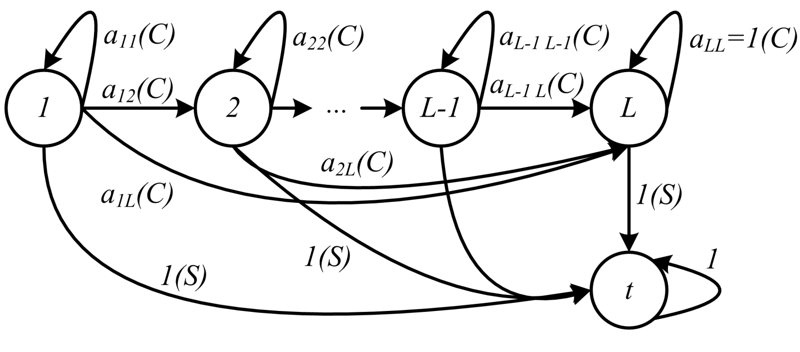

Figure 3.6: Stochastic Shortest-Path associated with the problem.

This SSP problem has all cost positives (i) and, with probability 1, the node t will be reached (ii). Under these two points, it can be shown that the VI algorithm converges to an optimal and stationary policy (Bertsekas and Tsitsiklis, 1991; Sutton and Barto, 1998; Bertsekas, 2005), in terms of a SDP model.

□

This algorithm can be used to calculate the optimal repair rule, i.e., determine the operation threshold r.

3.3.3 The value iteration algorithm

The VI algorithm consists in calculating successive approximations for J^* through the recursive equations

$$\begin{aligned} J_{k+1}(i) &= \min_u \mathrm{E}\Big[g(X_{k+1}, u) + J_k(X_{k+1}) | X_k = i, u_k = u\Big] \\ &= \min_u \sum_{j \in X} a_{ij}(u)(g(j, u) + J_k(j)), \end{aligned} \tag{3.6}$$

for all $i \in X$. For an arbitrary value of J_0, the sequence $\{J_k\}_{k \geq 1}$ converges to J^*, i.e., $|J_{k+1}(i) - J_k(i)| \to 0$, $\forall i \in X$, under the same conditions that guarantee the existence of J^* (Sutton and Barto, 1998; Bertsekas, 2005).

Algorithm 1 Value iteration algorithm

$J(i) \leftarrow 0, \quad \forall i \in X$ ▷ begin with an arbitrary J
repeat
  $\Delta \leftarrow 0$
  for all $i \in X$ **do** ▷ for all states
    $v \leftarrow J(i)$
    $J(i) \leftarrow \min_{u_k} \sum_{j \in X} p_{ij}(u)(g(j, u) + J_k(j))$
    $\Delta \leftarrow \max(\Delta, |v - J(i)|)$
until $\Delta < \epsilon$ ▷ precision shall
return $\mu(i) = \arg\min_u \sum_{j \in X} p_{ij}(u)(g(j, u) + J(j))$

In addition to the optimal operation threshold r, VI algorithm outputs the expected cost to termination ($J(i)$) which, in a SSP context, means that the state t was reached. In our CBM problem, this cost indicates the expected cost until the end of a cycle.

3.4 Model Analytical Properties

In this section we discuss some analytical properties of our CBM model.

Proposition 3 (Our CBM model contemplates both constant and increasing failure rate in the short-run horizon)**.** *In the short-run horizon, if no control is applied ($u_k = C$, $\forall k$), the system failure rate is constant or increases over time.*

Proof. If the probabilities $a_{1x} = 0$ for $2 \leq x \leq L{-}1$ we have a constant probability to failure (a_{1L}) and then a constant failure rate, which is a particular case. The general case (increasing failure rate) has been proved in 1965 by Barlow and Proschan[4].

□

In Chapter 5 we provide an illustration of this proposition (page 46). Despite the Proposition 3 and considering the long-run horizon, the average uptime is constant: this is the subject of the following result.

Proposition 4 (In the long-run horizon, the number of failures follows a Renewal Process)**.** *In the long-run horizon, if no control is applied (except that the perfect repair is performed whenever the system fails), the number of failures follows a Renewal Process.*

Proof. Since we consider that the system is left as it is (no control is applied) and the perfect repair is performed after each failure, the times between failures are independent and identically distributed (i.i.d.).

Let $\tau(i, j)$ be the first passage time in going from state i to state j in a Markov chain, i.e., the mean length of time required to go from i to j. $\{X_k\}$ is not ergodic (because of Definition 1 it is not possible to go from state $i+i$ to state i) but we only aim to compute the first passage time from state 1 (which is ergodic) since the system starts at this state. $\tau(1, L)$ represents then the time to failure.

[4]Barlow, R. and Proschan, F. (1965). *Mathematical Theory of Reliability.* New York: Wiley.

Let $\tau^m(1, L)$ be the time to failure in the mth short-run horizon. $\tau^m(1, L)$ are i.i.d. discrete random variables: since no control is applied, there is no re-estimation of Ψ after each short-run horizon. $\tau^m(1, L)$ have the following probability distribution:

n	$p_n = \Pr[\tau^m(1, L) = n]$
1	a_{1L}
2	$a_{11}a_{1L} + a_{12}a_{2L}$
3	$a_{11}^2 a_{1L} + a_{11}a_{12}a_{2L} + a_{12}a_{22}a_{2L} + a_{12}a_{23}a_{3L}$
$\vdots$	$\vdots$

$p_n \to 0$ as $n \to \infty$ because the state L is an absorbing state. Hence, $\mathrm{E}[\tau^m(1, L)]$ is finite and has the same value for all short-run horizon.

□

In Chapter 5 we also provide an illustration of this result (page 48)

Proposition 5 (A SM policy is a particular case of our CBM model)**.** *For the same system, a SM (Scheduled Maintenance) policy is a particular case of our CBM policy. Moreover, the SM policy cost is greater or equals to the CBM policy cost.*

Proof. A SM policy can be written as follows:

$$u_k = \begin{cases} C, & \text{if } k < n, \\ S, & \text{if } k = n, \end{cases}$$

where n is the period of the scheduled preventive repair. Let W be the system cost under the SM policy. Since we can write the SM policy as a particular CBM policy and the CBM policy's cost is optimal (see Eq. 3.5) we have

$$W \geq J,$$

which demonstrates this result.

□

Notice that the information about the system required to apply any CBM policy may have a cost. We have not considered this case but in practice one should take it into account when planning the CBM approach.

Proposition 6 (Our CBM policy improves the system availability)**.** *The system availability under our CBM model is greater than or equal to the availability under a SM policy.*

Proof. Recalling the definition of availability (Section 2.1) and fixing the MTTR (mean time to repair) since it is not concerned (it is assumed to be the same in both cases), the availability can be increased as the MTTF (mean time to failure = E[Uptime]) increases.

In order to demonstrate this result, let us set the cost function g as follows: $g(x < L, u = C) = -1$, $g(x < L, u = S) = 1$, $g(L, u_k = S) = 1$ and $g(L, u_k = C) = 2$. Note that this satisfies Definition 1.

Our Stochastic-Dynamic Programming equation (Eq. 3.5) is designed to minimize the cost to go from stage k to stage $k+1$. With g as defined above, Eq. 3.5 is now an algorithm to maximize the expected number of transitions of $\{X_k\}$.

Let W and J be the expected number of transitions under a SM and a CBM policy, respectively. From Proposition 5 we have

$$W \geq J,$$

which demonstrates this result since both W and J are expected to be negative numbers (as consequence of cost function g).

□

Chapter 4

Inference of Model Parameters

4.1 Introduction and Motivation

This chapter addresses to the problem of fitting the CBM model parameters to apply the model to a given system. That means we are looking for a technique that uses the available data and leads to the best parameters estimation.

The motivation for using Hidden Markov Models (HMM for short) for parameters estimation comes from its ability to differentiate between changes in the system measurement which are due to regular system changes (e.g., changes of operating conditions), and condition measurement changes which are due to changes in the measuring instrument and/or measurement precisions. Also, there exist computationally efficient methods for computing likelihoods, in particular in the signal processing literature. Additional information on this topic can be found in (Rabiner, 1989; Ephraim and Merhav, 2002; Dugad and Desai, 1996).

HMM has already been used to tackle maintenance problems. For instance, Tai et al. (2009) use HMM to deal with machine's production conformity in order to detect machine/equipment failures. In line with that work, Bunks et al. (2000) consider vibration measures to assess the status of helicopter gearboxes. However, different from our approach, those papers do not aim to propose CBM policies. Furthermore, we are also interested expanding the use of CBM policies through HMM by offering an approach for estimating the input parameters.

4.2 Model Parameters Estimation

We deal with the problem of parameters estimation by modeling the available data for estimating the parameters of our model (Ψ) in order to construct the statistical inference for Ψ.

Let O represent the historical information about the system, i.e., the historical data. O can be viewed as a set of M observation sequences, or observed cycles (as defined in Fig. 3.1). That is $O = \{O_1, O_2, \ldots, O_M\}$, where O_m represents a cycle, and it can be written as $O_m = \{z_1, z_2, \ldots, z_N\}$, where N is the cycle's length and z_n is the system condition observed in n.

The estimation of $\Psi = (A, B)$ can be viewed as follows: given the available historical information O, we want to define Ψ as a function of the data available. In other words, the problem is to find Ψ that maximizes $\Pr[O|\Psi]$[1]. We assume that an initial guess of Ψ is available, which will be denoted by Ψ_0.

Unfortunately, there is no known analytical approach to solve $\max_\Psi \Pr[O|\Psi]$ (Rabiner, 1989). Nonetheless, this problem can be solved numerically using an iterative procedure. The most successful method, widely cited in the literature, is the Baum-Welch re-estimation formulas (Baum et al., 1970), which will be discussed below.

4.2.1 The Baum-Welch Algorithm

"Baum-Welch algorithm" is an algorithm based on successive applications of the Baum-Welch re-estimation formulas (Baum et al., 1970). The model $\Psi = (A, B)$ is adjusted so as to increase $\Pr[O|\Psi]$ until a maximum value is reached. First, we offer an intuitive explanation of the formulas.

Let $\bar{a}_{ij}$ and $\bar{b}_x(z)$ be an estimation of a_{ij} and $b_x(z)$, respectively. These estimators are based on the relative frequency principle, as follows:

[1]This problem is called in HMM literature as "problem 3".

- $\bar{a}_{ij} = \dfrac{\text{expected number (e.n.) of transition from state } i \text{ to } j}{\text{e.n. of transition leaving } i}$;

- $\bar{b}_x(z) = \dfrac{\text{e.n. of times in } x \text{ where } z \text{ was observed}}{\text{e.n. of times in } x}$.

In order to describe the procedure for re-estimation for each observation sequence O_n we define the following variables:

- forward: $\alpha_n(x) = \Pr[Z_1 = z_1, Z_2 = z_2, \ldots, Z_n = z_n, X_n = x|\Psi]$;

- backward: $\beta_n(x) = \Pr[Z_{n+1} = z_{n+1}, Z_{n+2} = z_{n+2}, \ldots, Z_N = z_N, X_N = x|\Psi]$.

In the previous Chapter, we considered the question of estimating the system state (X_k) from the information vector (I_k), as described in equation 3.3. The computation of $\Pr[X_k = x|I_k]$ can be efficiently performed using the forward variable:

$$\Pr[X_k = x|I_k] = \frac{\Pr[X_k = x, I_k]}{\Pr[I_k]} = \frac{\alpha_k(x)}{\sum_{i \in X} \alpha_k(i)}. \tag{4.1}$$

Now let $\gamma_n(x)$ be the probability of being in state x at time n given O_n and Ψ, i.e., $\gamma_n(x) = \Pr[X_n = x|O_n, \Psi]$. The use of Bayes rule yields

$$\gamma_k(x) = \Pr[X_k = x|O_n, \Psi] = \frac{\Pr[X_k = x, O_n|\Psi]}{\Pr[O_n|\Psi]} = \frac{\alpha_k(x)\beta_k(x)}{\Pr[O_n|\Psi]}.$$

Finally, let $\xi_k(i,j)$ be the probability of being in the state i at epoch k and making a transition to j at $k+1$, i.e., $\xi_k(i,j) = \Pr[X_k = i, X_{k+1} = j|O_n, \Psi]$. Using the Bayes rule yields

$$\xi_k(i,j) = \frac{\Pr[X_k = i, X_{k+1} = j, O_n|\Psi]}{\Pr[O_n|\Psi]} = \frac{\alpha_k(i)a_{ij}b_j(Z_{k+1})\beta_{k+1}(j)}{\Pr[O_n|\Psi]}.$$

It can be shown that $\sum_{k=1}^{K-1} \gamma_k(x)$ is the expected number of transitions from state x and $\sum_{k=1}^{K-1} \xi_k(i,j)$ is the expected number of transitions from state i to state j (Rabiner, 1989; Dugad and Desai, 1996).
Now, we present the Baum-Welch re-estimation formulas:

- $\bar{a}_{ij} = \sum_{k=1}^{K-1} \xi_k(i,j) / \sum_{k=1}^{K-1} \gamma_k(i)$
- $\bar{b}_x(z) = \sum_{\substack{k=1 \\ z_k=z}}^{K} \gamma_k(x) / \sum_{k=1}^{K} \gamma_k(x)$

We can use these formulas to enhance the estimation quality using the historical data. That is to say, we start with an initial estimation of Ψ and we are able to improve this estimation by using field data, which makes our approach clearly more applicable to real contexts. The algorithm 2 is proposed based on these formulas. It involves collecting a set of data sequences, computing the posterior distribution over hidden variables (X_k) at all times given that sequences, and, finally, updating the parameters according to the statistics of the posterior distribution.

Algorithm 2 Baum-Welch algorithm

1: $\Psi_0 \leftarrow (A, B)$ ▷ a priori estimate for Ψ
2: **for all** $n \in 1..N$ **do** ▷ for each observation sequence
3: $\quad \bar{a}_{ij} \leftarrow \sum_{k=1}^{K-1} \xi_k(i,j) / \sum_{k=1}^{K-1} \gamma_k(i), \quad \forall i, j \in X$
4: $\quad \bar{b}_x(z) \leftarrow \sum_{\substack{k=1 \\ z_k=z}}^{K} \gamma_k(x) / \sum_{k=1}^{K} \gamma_k(x), \quad \forall x \in X, \quad \forall z \in Z$
5: $\quad \Psi_n \leftarrow (\bar{a}_{ij}, \bar{b}_x(z))$
6: **return** Ψ_N

Applying this algorithm at the end of each cycle (as described in Fig. 3.2) yields a systematic updating of Ψ, which is suitable for computer implementation as part of a maintenance software, for example. Consequently, we are able to get a more accurate estimation of Ψ as soon as more data are available.

4.3 Estimation Properties

The Baum-Welch algorithm is a computationally efficient iterative algorithm for local maximization of the log-likelihood function. For more details, please check (Ephraim and Merhav, 2002). It is an expectation-maximization algorithm and it was developed and proved to local converge by Baum et al. (1970).

In this section, we present some characteristics of our inference model based on the Baum-Welch algorithm. To this end, let us define the following terms for a given system[2]:

- Ψ: the system's real parameters;
- $\widehat{\Psi}_0$: initial guess of the system's parameters;
- $\widehat{\Psi}_N$: the estimation of the system's parameters yielded by the Baum-Welch algorithm after running N observation sequences.

We can use Ψ to generate N random samples in order to simulate Baum-Welch algorithm and evaluate its fitting quality. Formally, this is called statistical simulation and it means an artificial data generation process, driven by model design and parameter settings. The output, a synthetic sample, can be used to validate the inference process.

For the purpose of evaluate the bias between Ψ and $\widehat{\Psi}_N$, let us create the following fitness measure

$$\rho = \sum_{i,j \in X} (\widehat{a}_{ij} - a_{ij})^2 + \sum_{x \in X, z \in Z} (\widehat{b}_x(z) - b_x(z))^2. \tag{4.2}$$

Hence, ρ is the sum of the squared residual deviations between Ψ and $\widehat{\Psi}_N$ for both transition (A) and observation (B) probability matrices.

Proposition 7 (Baum-Welch algorithm convergence depends on the initial parameters guess)**.** *The Baum-Welch algorithm gives an estimation that corresponds to the local maximum of the likelihood function. However, there*

[2]Please recall that the system is assumed to be well-defined (Definition 1).

exists at least one $\widehat{\Psi}_0$ which garantees the convergence (global maximum of the likelihood function) to the real system's parameters as the number of empirical data increases. That is

$$\exists\widehat{\Psi}_0 : \rho \overset{N\to\infty}{\longrightarrow} 0. \tag{4.3}$$

Proof. Finding a $\widehat{\Psi}_0$ that results in the algorithm convergence is straightforward: Baum-Welch will always converge if we make $\widehat{\Psi}_0 = \Psi$ for a sufficiently larger N.

Indeed, the key question is how to choose the initial guess $\widehat{\Psi}_0$ so that the local maximum is the global maximum of the likelihood function. If it does not happen, the algorithm yields a suboptimal solution. This fact is due to the Baum-Welch formulas structure (Ephraim and Merhav, 2002; Rabiner, 1989; Baum et al., 1970) and basically there is no simple answer. We provide below examples showing optimal and suboptimal convergences. □

Let us illustrate the Proposition 7. For this purpose, we performed the following experiment:

Algorithm 3 Baum-Welch convergence test: unbiased $\widehat{\Psi}_0$ case

1: generate L as a discrete uniform distribution in $[2, 10]$
2: create Ψ as follows: (it assures that all probabilities add to 1 when needed and Ψ will be well-defined)
- $a_{ii} = 0.94$ for $1 \leq i < L-1$, $a_{L-1L-1} = 0.95$ and $a_{LL} = 1$
- $a_{ii+1} = 0.05$ for $1 \leq i < L$
- $b_1(\underline{1}) = 0.9$, $b_x(z) = 0.8$ for $1 < x < L, \underline{1} < z < \underline{L}$ and $b_L(\underline{L}) = 0.9$
- $b_{i\underline{i+1}} = 0.1$ for $1 \leq i < L$ and $b_{i\underline{i-1}} = 0.1$ for $1 < i \leq L$

3: let $\widehat{\Psi}_0 = \Psi$, i.e., a completely unbiased prior estimation
4: generate N random data series from Ψ. Each series corresponds to a complete simulation of the system running to failure, i.e., the series ends whenever L is reached
5: for each series $n \in N$, we use Baum-Welch to get $\widehat{\Psi}_n$ and then we evaluate ρ according to Eq. 4.2.

The Fig. 4.1 presents the results: it shows $\rho \to 0$ when N grows.

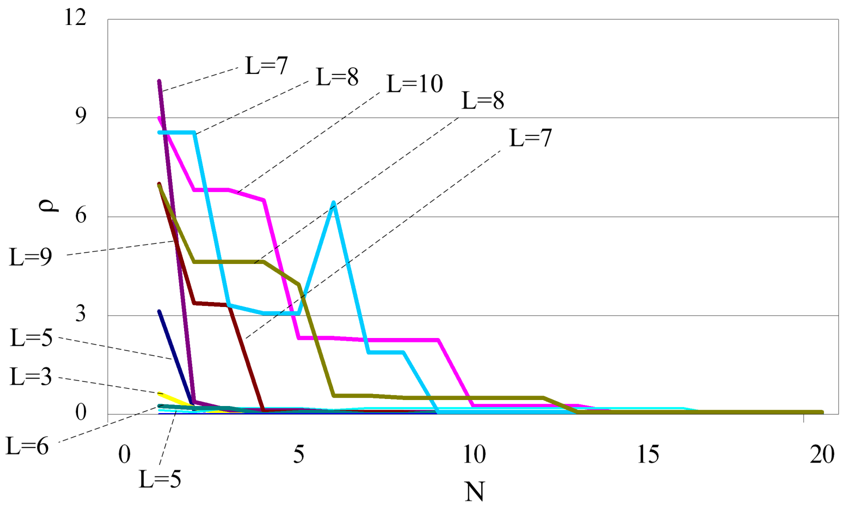

Figure 4.1: Baum-Welch optimal convergence for some random data (10 runs).

This result shows a cleary convergence of the method. Let us now run our experiment considering a "uniform" prior system parameters estimation ($\widehat{\Psi}_0$). We replace then line 3 in Alg. 3 by:

Algorithm 4 Baum-Welch convergence test: uniform $\widehat{\Psi}_0$ case

---

create $\widehat{\Psi}_0$ as follows:
for $i = 1$ to L **do**
  for $j = i$ to L **do** ▷ starts from i to ensure "well-definedness"
    $a_{ij} = \dfrac{1}{L-i+1}$ ▷ uniform distribution
for $x = 1$ to L **do**
  for $z = \underline{1}$ to $\underline{L}$ **do**
    $b_x(z) = \dfrac{1}{L}$ ▷ uniform distribution

This piece of pseudo-code produces an "uniform" $\widehat{\Psi}_0$. Running again our experiment produces the Fig. 4.2. We have limited the number of generated data series (N) to 50 due the computational effort required to run the Baum-Welch algorithm. In one case we have an optimal estimation ($L = 3$).

The Fig. 4.3 shows a long-run using an "uniform" $\widehat{\Psi}_0$ converging to a non-

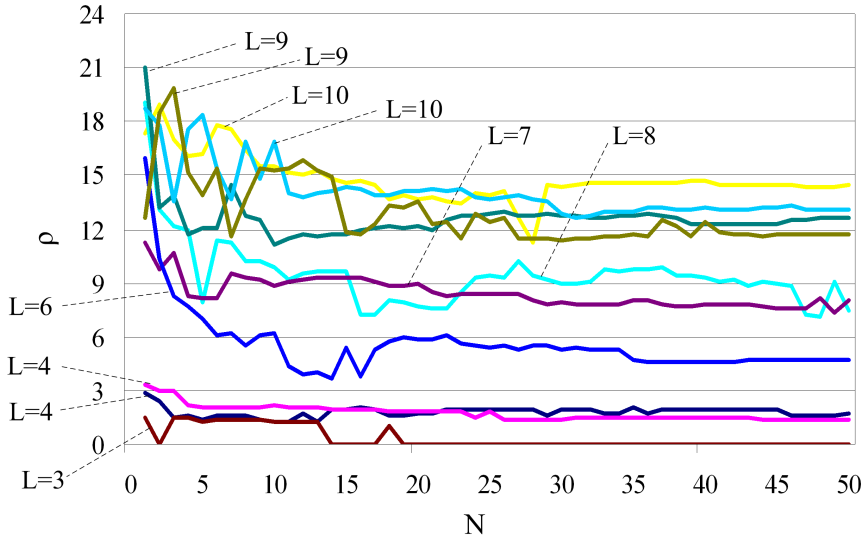

Figure 4.2: Baum-Welch suboptimal convergence for some random data (10 runs).

optimal estimation since $\rho \nrightarrow 0$. This test required about 3hs of machine[3].

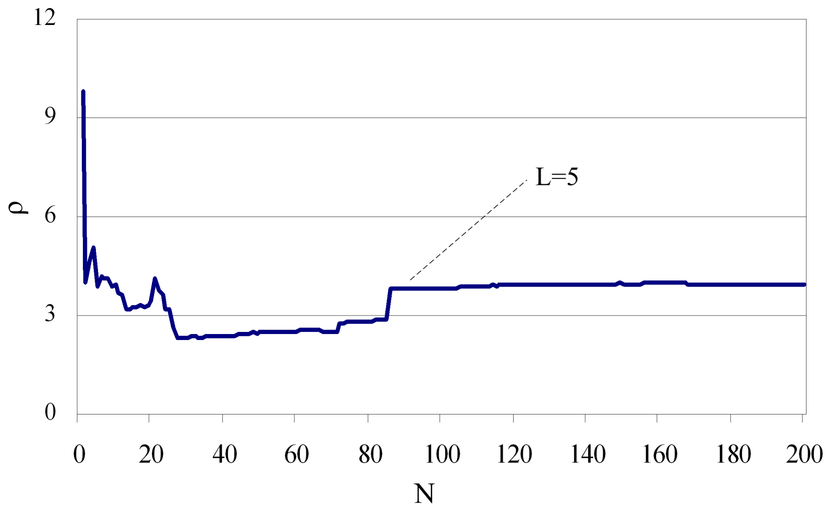

Figure 4.3: Baum-Welch suboptimal convergence for some random data (1 run).

Rabiner (1989) points out that the initial guess of the matrix B is specially

[3]The computational environment is described in the beginning of the Chapter 5.

critical. To illustrate this fact, let us replace the part of Alg. 4 that generates B by the following:

Algorithm 5 Baum-Welch convergence test: "semi-unbiased" B-matrix case

for $x = 1$ to L **do**
  for $z = \underline{1}$ to $\underline{L}$ **do**
    if observation z corresponds to x (as in $\underline{1} = 1$) **then**
      $b_x(z) = 0.5$
    else
      $b_x(z) = \dfrac{0.5}{L-1}$ ▷ "uniform" distribution

Hence, we give a half of the probability mass to the correct guess. The rest of B is left uniformly random. The results are sensibily differents: all estimations have optimally converged (see Fig. 4.4).

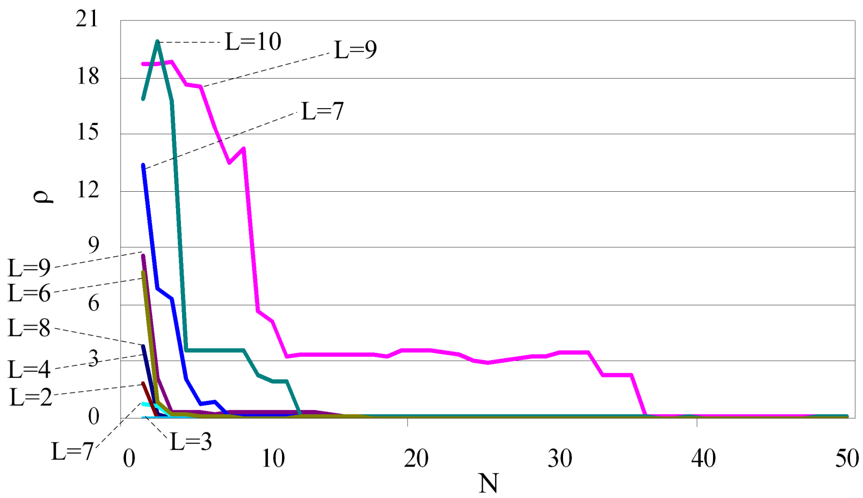

Figure 4.4: Baum-Welch optimal convergence for some random data (10 runs).

This section highlights the role of the initial parameters guess in the parameters estimation as a whole. Rabiner (1989) says that there is no general answer but there exist some techniques useful, though, helping to avoid suboptimals estimation regardless the initial guess. For the purposes of this dissertation, this topic will not be covered.

Chapter 5

An Application Example

This chapter discusses the application of our methodology using empiric data. The algorithms were coded using Matlab v6.5 and the tests were performed on a 2.0 GHz dual core machine with 3Gb RAM running Microsoft Windows XP. All computation times took no more than 10 seconds.

5.1 Preliminaries

Our application example uses data from a piece of equipment, provided by a zinc mining company. For confidentiality reasons, we have had to disguise the input data[1]. This system is electric-powered, it runs continuously, and its main failure mode is a serious internal degradation, affecting the product quality and the process productivity. This failure can happen whenever the electric current is higher than a given design-fixed value, which is supposed to be the maximum value for a correct operation. The perfect repair is performed by replacing some internal components. Upon failure, even while the piece of equipment is still running, the degradation is generalized, requiring a more complex repair.

We wish to define the state failed (L) as the state where we have to carry out the complex repair in order to bring back the system to state 1. Therefore, in some sense, the event failure is hidden since it does not necessarily imply

[1]It has been performed by multiplying all values by some constant in such a way that, despite the changes, the data remain coherent.

that the piece of equipment is broken or not operating. Because of equipment design and structure, as well as operational limitations, the electric current is the monitored parameter. It is assumed that other failure modes are not relevant and that they have no correlation with the mode being analyzed. Hence, let θ_k denote the current measured (in Ampere) at epoch k, sampled every day (the period T). We set $L = 5$ and Table 5.1 shows the classification adopted in our study, translating the parameter measurement into condition measurement.

Parameter value	z_k
$\theta_k < 27.5$	$\underline{1}$
$27.5 \leq \theta_k < 29.0$	$\underline{2}$
$29.0 \leq \theta_k < 30.5$	$\underline{3}$
$30.5 \leq \theta_k < 32.0$	$\underline{4}$
$\theta_k \geq 32.0$	$\underline{5}$ (system seems failed)

Table 5.1: Classification of the parameter measurement.

The cost function $g(x_k, u_k)$ considered is showed in Table 5.2. Notice that "stop and perform the preventive maintenance" ($g(x_k, u_k)$ with $x_k = \{1, 2, 3, 4\}, u_k = S$) is 50 times more expensive than operate the system in the condition "as good as new" ($g(x_k, u_k)$ with $x_k = 1, u_k = C$). For reference purposes, let us call this function as g_A.

	Action taken (u_k)	
System real condition (x_k)	Continue	Stop
1	1	50
2	1.1	50
3	1.2	50
4	1.3	50
5 (failed)	1000	200

Table 5.2: The cost function g_A.

5.2 Initial Parameters Estimation

We discuss now the model parameters (Ψ) estimation. As discussed, we need an initial guess of Ψ, showed in Fig. 5.1. This estimation considers the wear-out process and the possibility of failures due to shocks as well.

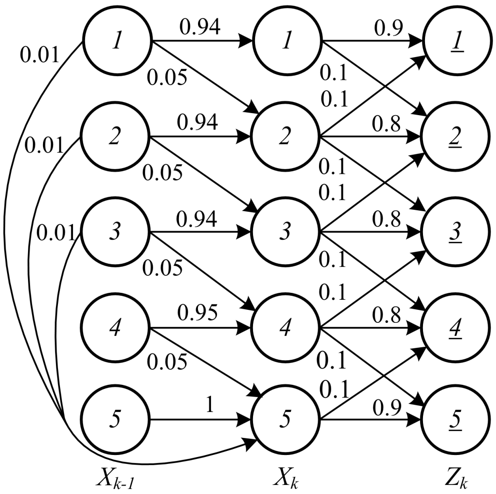

Figure 5.1: The initial guess (Ψ_0) of the transition and observation matrices.

Let us illustrate some properties of our model. If the system is left "as it is" (no control is applied), we expect that the system will fail in a finite time interval due to wear-out and shock failures. We can check this fact by computing the reliability and failure rate functions using Ψ_0, as in Figs. 5.2 and 5.3.

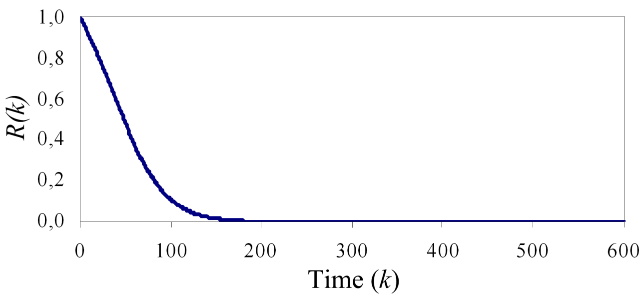

Figure 5.2: The reliability function for the system (Ψ_0) if no control is applied.

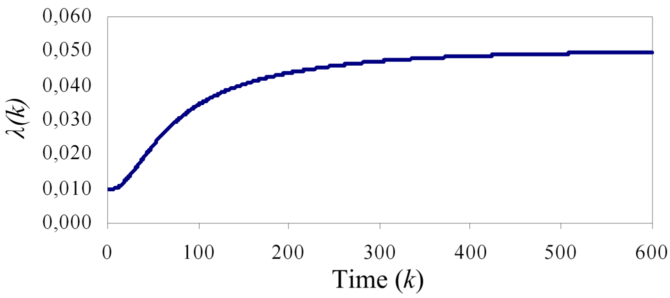

Figure 5.3: The failure rate function for the system (Ψ_0) if no control is applied.

We have an increasing failure rate, which is the general case by systems modeled using Markov chains, as discussed in Proposition 3. Since we consider discrete-time Markov chain, we have computed the failure rate $\lambda(k)$ with respect to a fixed interval equal to 1 (a day, the period T). This system deterioration can be verified by computing the n-step transition probability matrices, as shown in Table 5.3.

	0.5386	0.2865	0.0686	0.0099	0.0964
	0	0.5386	0.2865	0.0706	0.1043
A^{10}:	0	0	0.5386	0.3006	0.1608
	0	0	0	0.5987	0.4013
	0	0	0	0	1.0000
	0.0021	0.0109	0.0288	0.0661	0.8922
	0	0.0021	0.0109	0.0420	0.9450
A^{100}:	0	0	0.0021	0.0193	0.9786
	0	0	0	0.0059	0.9941
	0	0	0	0	1.0000
	0.0000	0.0000	0.0000	0.0000	1.0000
	0	0.0000	0.0000	0.0000	1.0000
A^{500}:	0	0	0.0000	0.0000	1.0000
	0	0	0	0.0000	1.0000
	0	0	0	0	1.0000

Table 5.3: Some n-step transition probability matrices if no control is applied.

Still considering that no control is applied, we can compute the distribution of the time to failure ($\tau(1, L)$, Proposition 4). For this purpose, we use

Ψ_0 to calculate $\tau(1, L)$, as in Fig 5.4.

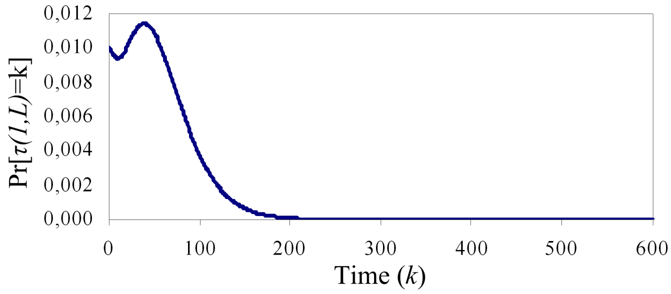

Figure 5.4: The distribution of the time to failure ($\tau(1, L)$) for the system (Ψ_0) if no control is applied.

The data set used for initial parameters estimation consists of 3 distinct systems that are in the same operating condition, in a total of 11 series. According to our definition of failure, two of these series are of the "up to preventive maintenance" type (Fig. 3.1a), while the others are of the "through failure" type. The Fig. 5.5 illustrates, for each series, the behavior of the electric current over time and Fig. 5.6 shows the data after classification, performed according to Table 5.1.

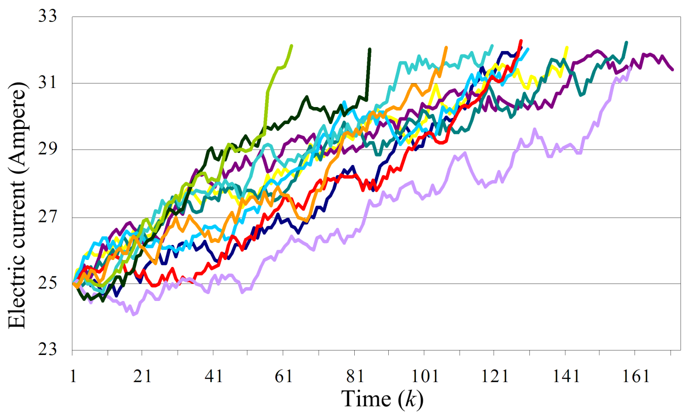

Figure 5.5: The data series (total: 11) used in our application example.

We use these data series in our Hidden Markov Model approach to construct the statistical inference for Ψ, providing a better estimation given the historical data available.

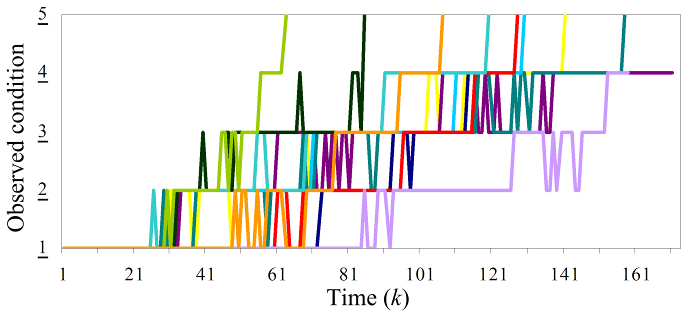

Figure 5.6: The same data of Fig. 5.5 after discretization.

Subsequently, we apply the Baum-Welch algorithm, obtaining the result Ψ, as shown in Table 5.4. For referencing purposes, let us call these parameters as Ψ_A.

0.9797	0.0203	0	0	0.0000
0	0.9603	0.0397	0	0.0000
0	0	0.9703	0.0297	0.0000
0	0	0	0.9824	0.0176
0	0	0	0	1

0.9682	0.0318	0	0	0
0.0309	0.9426	0.0265	0	0
0	0.0447	0.8915	0.0637	0
0	0	0.0087	0.9626	0.0287
0	0	0	0.1166	0.8834

Table 5.4: Ψ_A: the initial estimation of the matrices A (above) and B (below).

This result shows how the historical data processing works, as we pointed out in Section 4.2. As can be seen, our algorithm changed the probabilities, fitting our initial guess (Fig. 5.1) as a function of the empirical data. We can say that Ψ_A is optimal (stochastically speaking) given the initial guess and the historical data.

For illustration purposes, if we compute again the reliability and failure rate functions, and the distribution of the time to failure, always considering no control, we get the Fig. 5.7.

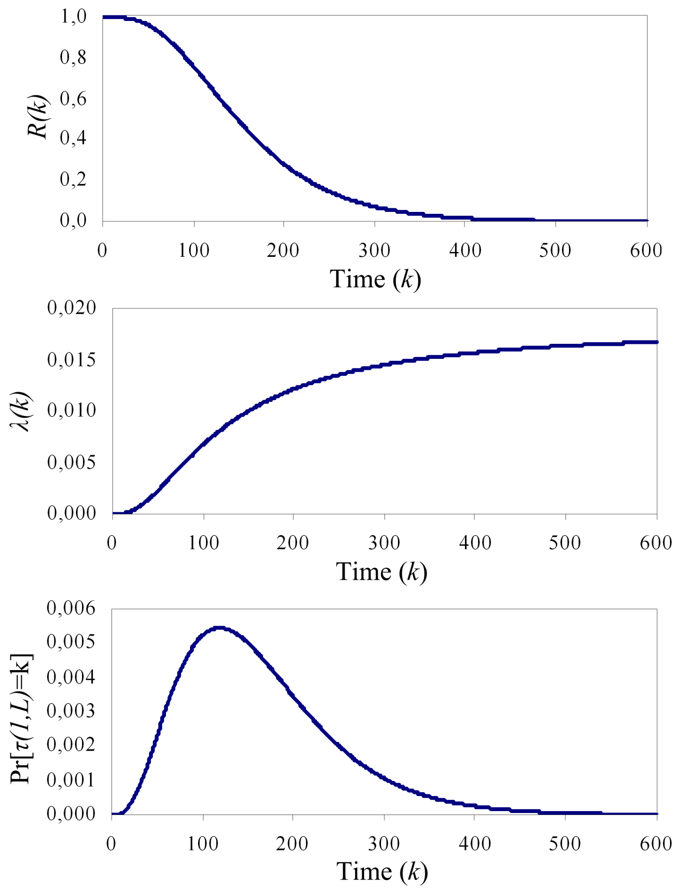

Figure 5.7: The reliability (top) and failure rate (middle) functions, and the distribution of time to failure (bottom) for Ψ_A if no control is applied.

Now that we have Ψ_A, we are able to calculate the CBM policy μ, we are interested in computing the threshold state (r) by solving the Eq. 3.5 as discussed in Section 3.3. The algorithm result, depicted in Table 5.5, indicates the state 4 as the threshold state. The expected cost until stop (preventive or corrective) is 168.8720.

State (X_k)	1	2	3	4(=r)	5
Action (u_k)	C	C	C	S	S

Table 5.5: Threshold state (r) computation and optimal action.

We are now prepared to apply our CBM policy μ. We illustrate its application via the following scenarios that illustrate how our proposed approach works.

5.3 Testing-Scenarios

5.3.1 Scenario 1: A Typical Scenario

We begin by analyzing the scenario illustrated in Fig. 5.8. This figure shows the behavior of μ: while $k < 85$, $\widehat{X}_k < r = 4$ hence $u_k = C$. At $k = 85$ we have $\widehat{X}_k = 4$ and hence $u_{85} = S$.

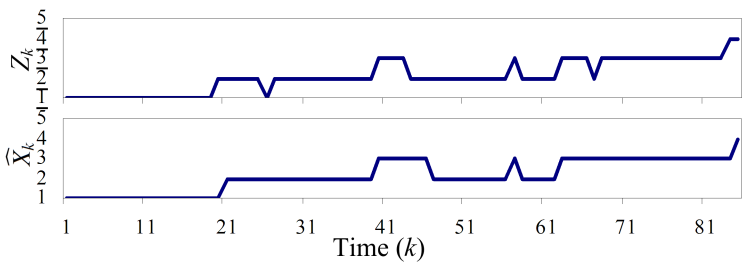

Figure 5.8: Scenario 1: condition observed (top) and state estimation (bottom).

Let us illustrate how Eq. 3.3 works: until $k = 19$ we have $Z_k = \underline{1}$ and (x, $\Pr[X_k = x|I_k]$) as follows: (1, 0.9993), (2, 0.0007); At $k = 20$, $Z_k = \underline{2}$ and (1, 0.6116), (2, 0.3883), (3, 0.0001); At $k = 21$, $Z_k = \underline{2}$ and (1, 0.0497), (2, 0.9485), (3, 0.0018); ...; At $k = 25$, $Z_k = \underline{2}$ and (2, 0.9979), (3, 0.0021); At $k = 26$, $Z_k = \underline{1}$ and (1, 0.0001), (2, 0.9999); At $k = 25$, $Z_k = \underline{2}$ and (2, 0.9980), (3, 0.0020).

Since we stopped the system at $k = 85$, the actual measurement cycle has just been closed. We wish to update Ψ_A using the newest condition measurement series, as described in Fig. 3.2. To this end, we apply Baum-Welch with the recently updated historical data (Fig. 5.6 plus Fig. 5.8) giving a new up to now parameters estimation Ψ_B shown in Table 5.6.

0.9786	0.0214	0	0	0
0	0.9625	0.0375	0	0.0000
0	0	0.9696	0.0304	0.0000
0	0	0	0.9824	0.0176
0	0	0	0	1

0.9693	0.0307	0	0	0
0.0299	0.9308	0.0393	0	0
0	0.0445	0.8945	0.0605	0
0	0	0.0092	0.9625	0.0283
0	0	0	0.1216	0.8784

Table 5.6: Ψ_B: the matrices A (above) and B (below) updated (Ψ_A + Fig. 5.8).

5.3.2 Scenario 2: Changes in the Cost $g(\cdot)$

In this scenario we assume operational costs that are higher than those considered in g_A, as follows: $g(1, u_k\!=\!C) = 1$, $g(2, u_k\!=\!C) = 1.5$, $g(3, u_k\!=\!C) = 2$, $g(4, u_k\!=\!C) = 3$ and $g(5, u_k\!=\!C) = 1000$; and $g(i, u_k\!=\!S) = 50$ (for $i \leq 4$) and $g(5, u_k\!=\!S) = 200$. Let us call it as $g_B(.)$.

Re-computing the CBM policy μ (using the Ψ_A, i.e., Table 5.4) yields, with this new cost function ($g_B(.)$), the state 3 as the threshold state. The expected cost until stop is 172.7884. As we had expected, this cost is higher than the cost of the previous scenario.

Hence, it is expected that $u_k = S$ will be carried out earlier, as far as system degradation is concerned, than the state defined in the previous scenario, reflecting the fact that operating in one of degraded states is more expensive than before.

5.3.3 Scenario 3: Incidence of Shocks

Let us consider g_A and Ψ_A. Hence, we have μ as in Table 5.5. So far, in the scenarios we have considered, we did not note the occurrence of shocks. Let us now analyze the scenario illustrated in Fig. 5.9. As can be seen, there has been a shock at $k = 14$ forcing a corrective repair. Therefore, we conducted a Ψ update, which produces Table 5.7.

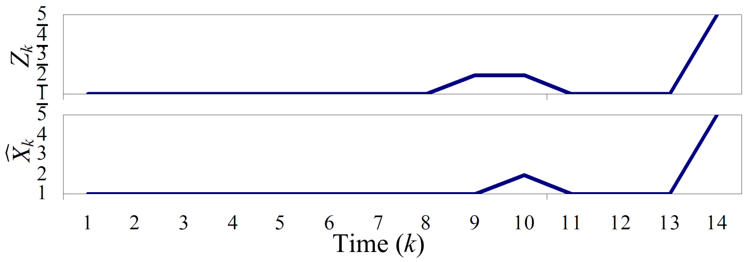

Figure 5.9: Scenario 3: condition observed (top) and state estimation (bottom).

0.9784	0.0198	0	0	0.0018
0	0.9603	0.0397	0	0
0	0	0.9703	0.0297	0
0	0	0	0.9823	0.0177
0	0	0	0	1

0.9651	0.0349	0	0	0
0.0304	0.9431	0.0265	0	0
0	0.0447	0.8915	0.0637	0
0	0	0.0088	0.9627	0.0285
0	0	0	0.0325	0.9675

Table 5.7: Updating Ψ after occurrence of shock (A above and B below).

This scenario illustrates the ability to take into account the incidence of shocks. One may see this result as counterintuitive since, as we can observe in Table 5.7, the probability a_{15} has decreased from 0.01 to 0.0018, instead of increasing. However, due to the Markovian property, the Baum-Welch algorithm works as follows: a_{ij} are updated dividing the expected number of transition from i to j ($\xi_k(i,j)$) by the expected number of all transition from i ($\gamma_k(i)$). Therefore, despite the occurrence of $\underline{1} \rightarrow \underline{5}$, we have from the historical data (Fig. 5.6) hundreds of transitions from $\underline{1}$ to $\underline{1}$. Hence, the impact of the shock is not as big as one might expect.

5.3.4 Scenario 4: Changing L and Classification Step

Let us now illustrate a scenario in which the discretization of the system condition is refined - that is, a scenario where we take into account higher values for L. Recall that L is the number of system states between "as good as new" and "completely failed". Let's set $L = 10$. Fig. 5.10 shows the same data presented in Fig. 5.5 after new classification, performed according to Table 5.8.

Parameter value	z_k
$\theta_k < 25.25$	$\underline{1}$
$25.25 \leq \theta_k < 26.00$	$\underline{2}$
(...)	
$31.25 \leq \theta_k < 32.0$	$\underline{9}$
$\theta_k \geq 32.0$	$\underline{10}$ (system seems failed)

Table 5.8: Classification of the parameter measurement.

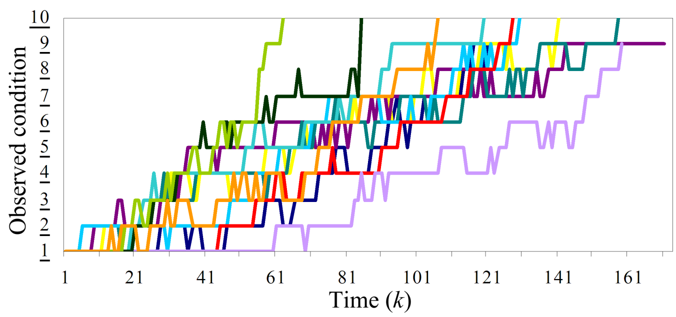

Figure 5.10: The same data of Figure 5.5 after new discretization (Table 5.8).

We consider an initial guess Ψ and a cost function g similar to the ones used in scenario 1, just adapted to this scenario where $L = 10$. Hence, for A we have $a_{ii:i\leq 8} = 0.94$, $a_{99} = 0.95$, $a_{1010} = 1$, $a_{ij:j=i+1,i\leq 9} = 0.05$ and $a_{i10:i\leq 8} = 0.01$. For B we have $b_1(\underline{1}) = 0.9$, $b_i(\underline{i}) = 0.8$ (for $i \leq 9$) and $b_{10}(\underline{10}) = 0.9$; $b_i(\underline{i+1}) = 0.1$ (for $i \leq 9$) and $b_{i+1}(\underline{i}) = 0.1$ (for $i \leq 9$). The cost function is the following: $g(1, u_k = C) = 1$, $g(2, u_k = C) = 1.1$, $g(3, u_k = C) = 1.2$, ..., $g(9, u_k = C) = 1.8$ and $g(10, u_k = C) = 1000$; and $g(i, u_k = S) = 50$ (for $i \leq 9$) and $g(10, u_k = S) = 200$.

Applying Baum-Welch for parameters estimation yields the state 7 as threshold state. The expected cost until stop is 272.6329. In Fig. 5.11 we present the same data used in scenario 1 using the new classification. In this case, we stop the system at $k = 71$ since $\widehat{X}_k = 7$. As should be expected, setting $L = 10$ implies different results from the model using $L = 5$.

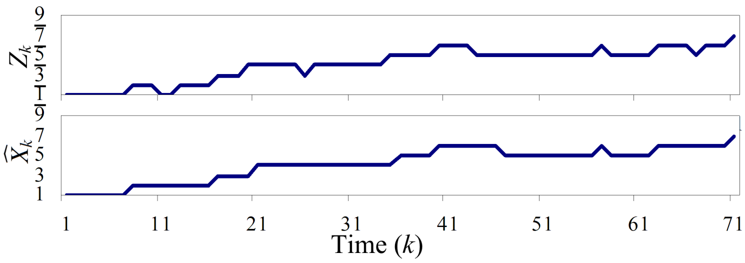

Figure 5.11: Scenario 4: condition observed and state estimation.

Chapter 6

Conclusion, Suggestions for Future Research

In this dissertation, we have studied the problem of constructing Condition-Based Maintenance policy for systems under periodic inspection. The system state is represented by a discrete-state Markov process and we take into account that the condition measurement may be not perfect. Also, we have discussed the model parameters inference from a both theoretical and practical point of view.

The main result of this dissertation is a framework that combines optimization and model parameter computation from historical data. We illustrated the application of our methodology in a industrial case and provided a step by step discussion of our approach, using four different scenarios, illustrating its key points. The result suggests an approach suitable for industrial applications, which can help managers to improve their decisions for cases similar to the one that we tackled.

Few papers in the literature have pointed out that the Hidden Markov Model theory is efficient to model historical data concerning condition measurement. This dissertation expands the use of HMMs in reliability and maintenance field by combining an optimization model with input parameters estimation from empirical data. We believe our work can increase the use of such models as well as motivate more research in this area.

An important issue of our approach concerns the parameters' estimation robustness, since this step critically determines the CBM model outputs as well as the performance and effectiveness of the model outcome. We have chosen HMM because of its solid mathematical structure and theoretical basis, as well as the computational efficiency to compute the estimations. However, some aspects of HMM were not explored – for instance, a formal discussion concerning the quality of the parameter estimates provided by the Baum-Welch algorithm.

There are a couple of extensions, refinements, and alternative formulations that might be worth exploring. As a future work, one can plan to improve on our approach by considering some sophistication, we suggest some possible directions:

1. Use of intermediate repairs, such as the minimal one. For instance, a minimal repair will bring the system back to the state it was just after failure.

2. Consider the stochastic rejuvenation. In this case, the repair action outcome is stochastic, i.e., the state that the system will be brought is a probability distribution.

3. Use random or sequential inspection (periodic inspection is a particular case). One should also consider the cost do obtain information about the system condition through inspection or any other mean (such as on-line monitoring using sensors).

Naturally, these refinements would increase the complexity and thus would require a different approach to process historical data for estimation of model parameters.

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