UNIVERSIDADE FEDERAL DE MINAS GERAIS

PROGRAMA DE PÓS-GRADUAÇÃO EM NEUROCIÊNCIAS

ISABELLA STARLING ALVES

NONSYMBOLIC AND SYMBOLIC MAGNITUDES PROCESSING IN CHILDREN WITH MATHEMATICAL DIFFICULTIES

Belo Horizonte

2017

ISABELLA STARLING ALVES

NONSYMBOLIC AND SYMBOLIC MAGNITUDES PROCESSING IN CHILDREN WITH MATHEMATICAL DIFFICULTIES

Thesis committee:

Professor Renato Bortoloti (Advisor),

Professor Vitor Geraldi Haase (Co-advisor),

Professor Antônio Jaeger,

Professor Edward Hubbard

Submitted in partial fulfillment of the
requirements for the degree of
Master in Neurosciences

UNIVERSIDADE FEDERAL DE MINAS GERAIS

2017

043 Alves, Isabella Starling.

Nonsymbolic and symbolic magnitudes processing in children with mathematical difficulties [manuscrito] / Isabella Starling Alves. – 2017.

85 f. : il. ; 29,5 cm.

Orientador: Professor Renato Bortoloti. Co-orientador: Professor Vitor Geraldi Haase.

Dissertação (mestrado) – Universidade Federal de Minas Gerais, Instituto de Ciências Biológicas.

1. Neurociências - Teses. 2. Dificuldade de aprendizagem - Teses. 3. Desempenho nas crianças. 4. Matemática. I. Bortoloti, Renato. II. Haase, Vitor Geraldi. III. Universidade Federal de Minas Gerais. Instituto de Ciências Biológicas. IV. Título.

CDU: 612.8

Table of Contents

Abbreviations and Acronyms

ANS	Approximate number system
DD	Developmental Dyscalculia
MLD	Mathematical Learning Disability
MD	Mathematical Difficulties
TA	Typical achievement
TDE	Brazilian School Achievement Test
RT	Reaction time

Summary of Tables

Summary of Figures

Acknowledgements

This master thesis is a result of a research project of the Developmental Neuropsychology Lab – UFMG, of which I have the honor to participate since 2009. First, I would like to thank prof. Vitor Haase, for giving me the opportunity to learn about this wonderful world of neuropsychology, for always being ready to help me whenever I had questions on my research, and for inspiring me as a professional. I would also like to acknowledge prof. Renato Bortoloti, my thesis advisor, for his support and motivation with my writing. I am also grateful to Prof. Edward Hubbard and prof. Antônio Jaeger for being part of my thesis committee, and helping me to improve this work.

I would also like to thank Ricardo and Annelise for the feedback and collaboration on this work, and for their friendship. To the research assistants of this project, Luana, Drielle, Amanda, Ana Lídia, Manu, Babi, Jéssica, Mavi, Thayane, and Ailton: thank you! This thesis would not exist without your help. In addition, my gratitude to my lab colleagues, in special Giu, Malu, Júlia, Lívia, and Dessa. Thanks to Pedro, for the inspiring words whenever I felt a little lost, and to my dear friends Gui, Ju, Dani, Ti, and Debs.

I wish to express my sincere thanks to CAPES for the financial support to my research, and to the kids and families that took part in this study.

At last, but not least, I acknowledge my parents and my brother for their emotional support and unconditional love.

Abstract

There is a persistent debate in the literature on the mechanisms related to specific deficits in Mathematical Difficulties (MD). The core deficit hypothesis states that children with MD present difficulties to process and manipulate nonsymbolic magnitudes. On the other hand, the access deficit hypothesis proposes that children with MD present intact nonsymbolic magnitude processing abilities, but an impaired capacity to link symbolic numbers to their correspondent analog representations. This master's thesis investigated the performance of children with MD in nonsymbolic and symbolic magnitude processing, and the association of these abilities to math achievement. In the first study, children with MD were compared to typical achievement (TA) peers in nonsymbolic and symbolic magnitude comparison tasks, indexed by reaction time, accuracy, distance effect and Weber fraction. Children with MD presented difficulties in the nonsymbolic but not in the symbolic magnitude processing. Finally, both symbolic and nonsymbolic magnitude processing predicted the math achievement of TA, but not MD children. In the second study, children with MD and TA were compared regarding nonsymbolic magnitude processing and two-digit number processing and the relationship between these abilities and arithmetic and numerical transcoding skills was investigated. Children with MD presented difficulties in nonsymbolic and two-digit number processing, and both abilities were associated to math achievement and numerical transcoding skills. Overall, results of the first and the second studies support the core deficit hypothesis, and reinforce the importance of numerical processing for math achievement.

Keywords: mathematical difficulties, ANS, nonsymbolic magnitude comparison, symbolic magnitude comparison, distance effect, compatibility effect

Resumo

Há um debate persistente na literatura quanto aos mecanismos relacionados a déficits específicos das dificuldades na matemática (MD). Por um lado, a hipótese do déficit central (core deficit) assume que crianças com MD apresentam dificuldades em processar e manipular magnitudes não-simbólicas. Por outro lado, a hipótese do transtorno de acesso propõe que crianças com MD apresentam habilidades de processamento de magnitudes não-simbólicas preservadas, mas dificuldades em associar magnitudes simbólicas às suas representações analógicas correspondentes. Essa dissertação de mestrado investigou o desempenho de crianças com MD no processamento não-simbólico e simbólico de magnitudes, bem como a associação entre essas habilidades e o desempenho na matemática. No primeiro estudo, crianças com MD foram comparadas a pares com desempenho típico (TA) quanto a tempo de reação (RT), acurácia, efeito de distância e fração de Weber calculados em tarefas de comparação de magnitudes numéricas não-simbólicas e simbólicas. Crianças com MD apresentaram dificuldades no processamento de magnitudes não-simbólicas, mas não no simbólico. Por fim, tanto o processamento simbólico de magnitudes quanto o não-simbólico foram preditores do desempenho na matemática em crianças com TA, mas não com MD. No segundo estudo, crianças com MD e TA foram comparadas quanto ao processamento não-simbólico de magnitudes e ao processamento de numerais com dois dígitos. Além disso, a relação entre o processamento numérico e as habilidades de aritmética e transcodificação numérica foi investigada. Crianças com MD e TA apresentaram dificuldades no processamento de magnitudes não-simbólicas e de números de dois-dígitos, e ambas as tarefas foram associados ao desempenho aritmético e na transcodificação numérica. De forma geral, os resultados do primeiro e do segundo estudo corroboram a hipótese do déficit central, e reforçam a importância do processamento numérico para o desempenho na matemática.

Palavras-chave: dificuldades na matemática, ANS, comparação de magnitudes não-simbólicas, comparação de magnitudes simbólicas, efeito de distância, efeito de compatibilidade

Nonsymbolic and symbolic magnitudes processing in children with mathematical difficulties

Numerical skills are highly intertwined to our daily lives, from simple activities as dialing a phone number, making payments, and telling the hours, to application on high technology. Numerical skills correlate to higher employability (Parsons & Byners, 2005) and greater health outcomes (Reyna & Brainerd, 2007), and play a role in a nation's economic success (Hanushek, Woessmann, Jamison, & Jamison, 2008), especially on the advance of STEM (science, technology, engineering, and mathematics) disciplines. Thus, mathematical education has been a target of investments in diverse nations (White House, 2015; Ciências sem Fronteiras, 2016). In Brazil, in spite of its importance been recognized, the mathematical education is still inadequate: the frequency of students with low math achievement raised from 2.24% in 2011 to 4.94% in 2013 (INEP, 2016). Results from PISA (2012) indicated that 67.1% Brazilian students presented low scores in the mathematics literacy test.

The identification of underlying mechanisms of mathematical difficulties (MD) has attracted considerable research interest. Some studies suggest that an impairment in nonsymbolic magnitudes processing is the root of mathematical difficulties – the core deficit hypothesis (Piazza, et al., 2010, Mazzocco, Feigenson, & Halberda, 2011; Pinheiro-Chagas et al., 2014). The nonsymbolic magnitudes processing relies on the approximate number system (ANS), which is dedicated to representation of numbers in an analog fashion (Halberda & Feigenson, 2008), and allows nonsymbolic magnitude comparison, estimation and calculation (Dehaene, 2001). However, the core deficit hypothesis has been called into question by some studies showing that children with MD present an intact ANS, and difficulties in symbolic number processing (De Smedt & Gilmore, 2011; Rousselle & Noël, 2007). The access deficit hypothesis proposes that the main cause of MD is an impairment in symbolic processing due to difficulties in linking symbolic numbers to their correspondent nonsymbolic representation (Noël & Rousselle, 2011).

One difficulty in analyzing the core and the access deficit hypotheses through the available literature is that different methods are used. Some studies present only accuracy and reaction time (RT) in nonsymbolic and symbolic processing tasks (e.g. Landerl, Bevan, & Butterworth, 2004; De Smedt & Gilmore, 2011), while others use more refined measures (e.g. distance effect and Weber fraction; Ashkenazi et al., 2009; Rousselle & Noël, 2007; Mazzocco et al., 2011; Pinheiro-Chagas et al., 2014). Another inconstancy in the literature is the magnitude range used in numerical processing tasks – from smaller and easier numerosities (from 1 to 9; De Smedt & Gilmore, 2011; Mussolin, Mejias & Noël, 2010; Rousselle & Noël, 2007) to larger ones (two-digit numbers; Ashkenazi et al., 2009; Mazzocco et al., 2011; Pinheiro-Chagas et al., 2014).

In this thesis, the performance of children with MD in nonsymbolic and symbolic processing was investigated. In the first study, children with MD were compared to peers with typical achievement (TA) in a nonsymbolic and a symbolic magnitude processing task. In order to minimize the difficulty in interpreting results due to diverse indexes, RT, accuracy, distance effect and the Weber fraction were presented. In the second study, the nonsymbolic magnitude and the two-digit number processing were investigated in children with MD and TA. In both studies, the association between numerical processing and math achievement was also investigated.

If children with MD presented impairments in nonsymbolic and symbolic number processing, the core deficit hypothesis would be supported. Otherwise, if children with MD presented difficulties only in symbolic number processing, the access deficit hypothesis would be corroborated. A significant association between nonsymbolic and symbolic magnitude processing and math achievement was expected as well.

References

Ashkenazi, S., Mark-Zigdon, N., & Henik, A. (2009). Numerical distance effect in developmental dyscalculia. *Cognitive Development*, 24(4), 387-400. doi:10.1016/j.cogdev.2009.09.006

Ciências sem Fronteiras (2016). *O programa*. Retrieved from: http://www.cienciasemfronteiras.gov.br/web/csf/o-programa.

Dehaene, S. (2001). Précis of the number sense. *Mind & Language*, 16(1), 16-36. doi: 10.1111/1468-0017.00154

De Smedt, B. & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of experimental child psychology*, 108(2), 278-92. doi: 10.1016/j.jecp.2010.09.003

Halberda, J. & Feigenson, L. (2008). Developmental change in the acuity of the "Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental psychology*, 44(5), 1457-1465. doi: 10.1037/a0012682

Hanushek, E. A., Woessmann, L., Jamison, E. A., & Jamison, D. T. (2008). Education and economic growth. *Education Next*, 8(2).

INEP (2016). *Devolutivas pedagógicas*. Retrieved from: http://devolutivas.inep.gov.br/proficiencia.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: a study of 8-9-year-old students. *Cognition*, 93(2), 99-125. doi:10.1016/j.cognition.2003.11.004

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia). *Child Development*, 82(4), 1224-1237. doi: 10.1111/j.1467-8624.2011.01608.x

Mussolin, C., Mejias, S., & Noël, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition*, 115(1), 10-25. doi:10.1016/j.jecp.2009.02.003

Noël, M. P. & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: an explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience*. doi: 10.3389/fnhum.2011.00165

Parsons, S. & Bynner, J. (2005). *Does numeracy matter more?*. London: National Research and Development Centre for adult literacy and numeracy.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41. doi:10.1016/j.cognition.2010.03.012

Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., Willmes, K., & Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation?. *PloS ONE*, 9(11), e111155. doi: 10.1371/journal.pone.0111155

PISA (2012). *Results from PISA 2012*. Retrieved from: https://www.oecd.org/brazil/PISA-2012-results-brazil.pdf

Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, 17(2), 147-159. doi:10.1016/j.lindif.2007.03.010

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, 102(3), 361-95. doi:10.1016/j.cognition.2006.01.005

White House (2015). *Investing in America's future: Preparing students with STEM skills*. Retrieved from: https://www.whitehouse.gov/sites/default/files/microsites/ostp/stem_fact_sheet_2016_budget_0.pdf.

Core deficit or access hypothesis: which one rules in mathematical difficulties?

Isabella Starling-Alves[1], Ricardo J. Moura[2], Annelise Júlio-Costa[1], Vitor Geraldi Haase[1,3,4,5], and Renato Bortoloti[1,3,4,5]

1.Programa de Pós-Graduação em Neurociências, Universidade Federal de Minas Gerais, Brasil

2. Programa de Pós-Graduação em Ciências do Comportamento, Universidade de Brasília, Brasil

3. Departamento de Psicologia, Universidade Federal de Minas Gerais, Brasil

4. Programa de Pós-Graduação em Psicologia: Cognição e Comportamento, Universidade Federal de Minas Gerais, Brasil

5. Instituto Nacional de Ciência e Tecnologia: Comportamento, Cognição e Ensino

Abstract

There is a persistent debate in the literature on the mechanisms related to specific deficits in Mathematical Difficulties (MD). On the one hand, the core deficit hypothesis states that children with MD present difficulties to process and manipulate nonsymbolic magnitudes. On the other hand, the access deficit hypothesis proposes that children with MD present intact nonsymbolic magnitude processing, but an impaired capacity to link symbolic numbers to their correspondent magnitude representation. This study investigated nonsymbolic and symbolic magnitude processing in children with MD, and the relationship between these abilities and math achievement. We proposed a new index for symbolic magnitude processing, an internal symbolic Weber fraction. Children with MD presented difficulties in the nonsymbolic but not in the symbolic magnitude processing. Finally, both symbolic and nonsymbolic Weber fractions predicted the scores of TA but not MD children in addition, subtraction, and multiplication tasks. Results support the core deficit hypothesis.

Keywords: mathematical difficulties, ANS, nonsymbolic comparison, symbolic comparison

Core deficit or access hypothesis: which one rules in mathematical difficulties?

Developmental dyscalculia (DD) is a learning disorder characterized by persistent difficulties in mathematics, which cannot be attributed to intellectual disabilities, sensorial deficits, emotional and motivational reasons, or poor educational methods (Butterworth, 2005). The prevalence of DD has been estimated between 3 to 6% of age-school children (Shalev, Auerbach, Manor & Gross-Tsur, 2000).

Estimating the proper prevalence of DD is difficult, since there is no consensus on the best criteria for its identification. While some studies use a criterion of discrepancy between intelligence and math achievement scores (Lewis, Hitch, & Walker, 1994), others select a cut-off point in standardized math achievement tests (Badian, 1983). Besides the criteria diversity, different names have been used as synonyms for DD, for example Math Learning Disabilities (MLD; e.g. Mazzocco, Feigenson, & Halberda, 2011), Mathematical Disabilities (e.g. Geary, 1993), and Mathematical Difficulties (MD; e.g. Jordan, Hanich, & Kaplan, 2003). Mazzocco (2007) proposed a uniform terminology, with a liberal criterion (scores below the 25th percentile in math tests) assigned to identification of MD, and a more strict criterion (scores below the 5th percentile in math tests) for DD or MLD. The group with DD or MLD presents more specific and persistent difficulties with a constitutional etiology. In the group with MD, however, the math achievement can be influenced by socio-economical and motivational factors, and might improve over academic years. The term MD will be used hereinafter, since it is more inclusive.

The symptoms of MD depend on age, but a persistent difficulty in calculation is recurrent. Children with MD are unsuccessful to perform additions (Landerl, Bevan, & Butterworth, 2004; Ostad, 1997), subtractions (Landerl et al., 2004; Ostad, 1999), and multiplications (Berteletti, Prado, & Booth, 2014; Landerl et al., 2004). Such struggle is usually manifested by high reaction times (RT), low accuracy, and the use of immature strategies, as finger counting (Landerl et al., 2004; Geary, 1993; Hanich, Jordan, Kaplan, & Dick, 2001; Siegler & Robinson, 1982). Other difficulties usually observed in MD are reading analog clocks (Anderson, 2008; Burny, Valcke, & Desoete, 2012), as well as comparing and estimating quantities (Mazzocco, et al. 2011; Pinheiro-Chagas et al., 2014).

There are two main hypotheses to explain the distinctive difficulties of MD, the core deficit hypothesis and the access deficit hypothesis. The core deficit hypothesis assumes that MD present a deficit in the approximate number system (ANS; Piazza et al., 2010; Mazzocco et al., 2011). The ANS is a system dedicated to representation of magnitudes in an analog fashion (Halberda & Feigenson, 2008), and allows nonsymbolic comparison, estimation, and calculation (Dehaene, 2001). Evidences demonstrate that the ANS is spatially organized along a mental number line, which is oriented from left (smaller magnitudes) to right (greater magnitudes), and is logarithmically compressed (Dehaene, 2003; Hubbard, Piazza, Pinel, & Dehaene, 2005).

The distance effect and the size effect are evidences for the mental number line (Moyer & Landauer, 1967; Verguts & van Opstal, 2005). The distance effect accounts for higher RT and lower accuracy when numerically close magnitudes are compared, in contrast to lower RT and higher accuracy for the comparison of numerically distant magnitudes. For example, it is easier to distinguish 4 from 9 (distance 5) than 4 from 6 (distance 2). The size effect, in turn, refers to lower RT and higher accuracy to compare small pairs of numbers, in contrast to higher RT and lower accuracy to compare large pairs of the same distance. For example, it is easier to compare 1 to 2 (small pair – distance 1) than 6 to 7 (large pair – also distance 1).

The numerical representation in the mental number line is not exact and discrete, but noisy. Thus, each time a number is processed, an error occurs, and the representation of other magnitudes is activated as well – for example, when the number 5 is processed, the representation for 3, 4, 6, and 7 can be also activated (Gallistel & Gelman, 2000). The ability to distinguish different magnitudes depends on the ratio between them, with higher accuracy for increasing ratios, as stated by Weber's law, and indexed by the internal Weber fraction (Piazza, Izard, Pinel, LeBihan, & Dehaene, 2004). The Weber fraction accounts for the variability in the representation of a specific magnitude (Chesney, 2016), and the higher its value, the lower is the sensitivity to numerical differences, or, in other words, the worse is the ANS acuity (to see methods for Weber fraction calculation, see Piazza et al., 2004 and Halberda, Mazzocco, & Feigenson, 2008).

The ANS is present in different species, from invertebrates (Gross, Pahl, Si, Zhu, Tautz, & Zhang, 2009) to fishes (Agrillo, Dadda, Serena, & Bisazza, 2008), amphibians (Krusche, Uller, & Dicke, 2010), birds (Garland & Low, 2014), and mammals (Jordan, MacLean, & Brannon, 2008; McComb, Packer, & Pusey, 1994). The processing of magnitudes in these species might be important to find food, run from predators, live in groups, and find sexual partners (McComb, Packer, & Pusey, 1994.) Hence, an evolutionary function is attributed to the ANS (Dehaene, 2001). In humans, there is evidence that the ANS is already present in fetus (Schleger et al., 2014), and that neonates can differ quantities in the ratio of 3:1 (Izard, Sann, Spelke, & Streri, 2009). By 5 months of age, infants can accurately perceive manipulations in quantities, as addition or subtraction of objects in a scene (Wynn, 1992), and 6 month olds infants already differ quantities in the ratio of 2:1 (Xu, Spelke, & Goddard, 2005). Studies with infants evince that the ANS is independent of language and formal instruction, which is also corroborated by ethnology studies with indigenous (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004).

Pica et al. (2004) investigated the ANS in the Amazonian tribe Munduruku. In the Munduruku language, the numerical lexicon is restrict to small numbers, from one to five, and there are no words for large magnitudes. In Pica et al. study (2004), the Munduruku indigenous were instructed to compare two quantities, presented as sets of 20 to 80 dots. They were able to discriminate the quantities accurately, even in the lack of a symbolic system for magnitudes greater than five.

In spite of the innate and primitive character of the ANS, children with MD present low acuity in nonsymbolic magnitude comparisons (Landerl, Fussenegger, Moll, & Willburger, 2009; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007), estimations (Mazzocco et al., 2011; Mejias, Mussolin, Rousselle, Gregoire, & Noel, 2011; Pinheiro-Chagas et al., 2014), and calculation (De Smedt & Gilmore, 2011; Pinheiro-Chagas et al., 2014). Specially, MD children present a higher Weber fraction when compared to typical achievement children (TA) (Mazzocco et al., 2011; Costa et al., 2011; Olsson, Östergren, & Träff, 2016; Pinheiro-Chagas et al., 2014). Piazza et al. (2010), using a nonsymbolic magnitude comparison task, showed that Weber fractions of 10 years-old MD children were significantly higher when compared to TA children paired by age, but corresponded to the Weber fraction of 5-years olds. Our research team recently showed that MD children presented higher Weber fraction than TA, and did not engage the ANS in arithmetic operations and word problems (Pinheiro-Chagas et al., 2014). We also showed, through a single case study, that the Weber fraction is a stable measure, apparently insensitive to practice in MD children (Júlio-Costa, Starling-Alves, Lopes-Silva, Wood, & Haase, 2015).

Notwithstanding, there is no consensus on ANS impairments in children with MD (De Smedt & Gilmore, 2011; Kovas et al., 2009; Rousselle & Noël, 2007). The access deficit hypothesis proposes that a deficit in linking analog magnitudes to symbolic representations of number justifies the difficulties in MD, instead of ANS impairments (Noël & Rousselle 2011; see also De Smedt, Noël, Gilmore, & Ansari, 2013). In typical achievement, the symbolic representations of number activate the ANS, and respond to distance and size effects (Dehaene, 1992).

In a literature review, Noël and Rousselle (2011) showed that young children (from 6 to 9 years old) with MD do not present difficulties in nonsymbolic magnitude comparison, but in a symbolic version of the task (see Iuculano, Tang, Hall, & Butterworth, 2008; Landerl et al., 2004; Landerl & Kölle, 2009). On the other hand, later in the development, children from 10 to 14 years old present impairments in the ANS, as assessed by nonsymbolic magnitude comparison tasks (see Costa et al., 2011; Kucian, Loenneker, Dietrich, Dosch, Martin & von Aster, 2006; Kucian, Loenneker, Martin, & von Aster, 2011; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010). For this reason, the authors sustain that the main deficit in MD is in accessing the nonsymbolic magnitude through symbolic representations, in line with the access deficit hypothesis. As MD children do not engage the nonsymbolic magnitude in symbolic number tasks, the ANS would not develop properly. Consequently, ANS deficits could only be observed in older children.

Differences between MD and TA children in symbolic magnitude comparison tasks have been consistent across studies (De Smedt et al., 2013). Rousselle and Noël (2007) showed that 7 years old children with MD presented impairments in RT, accuracy and distance effect in a symbolic magnitude comparison task, with Arabic digits from one to nine. Using a similar experiment, Landerl et al. (2004) and Iuculano et al. (2008) also showed differences between MD and TA children in RT and accuracy, respectively.

Although there is still a debate on which mechanism is related to the deficits present in MD, both symbolic (Bugden, Price, McLean, & Ansari, 2012; Castronovo & Göbel, 2012; De Smedt, Verschaffel, & Ghesquière, 2009) and nonsymbolic (Halberda et al., 2008; Inglis, Attridge, Batchelor, & Gilmore, 2011; Paulsen, Woldorff, & Brannon, 2010) magnitude processing are stated as relevant for mathematical achievement. De Smedt, Verschaffel, and Ghesquière (2009) found a moderate correlation between RT and accuracy in a symbolic magnitude comparison task and a standardized math achievement test. There is evidence that the symbolic magnitude comparison task is correlated to simple math operations as well (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; Mundy & Gilmore, 2009).

On the other hand, the ANS is indicated as a start-up tool for the formal mathematics abilities (Piazza, 2010), and there is a significant correlation between nonsymbolic magnitude comparison and both standardized (Halberda et al., 2008; Landerl & Kölle, 2009; Desoete, Ceulemans, De Weerdt, & Pieters, 2012) and simple arithmetic tasks (Mundy & Gilmore, 2009; Piazza et al., 2010). The ANS is also correlated to math achievement when assessed through estimation (Booth & Siegler, 2006; Mejias et al., 2011; Castronovo & Göbel, 2012) and nonsymbolic calculation (Gilmore, McCarthy, & Spelke, 2010) tasks. In a meta-analysis, Chen and Li (2014) showed that the ANS prospectively predicts mathematics performance, and is likewise retrospectively correlated to early math achievement, with moderate effects.

The present study

Although several studies investigated whether the core deficit or the access deficit hypothesis accounts for impairments in MD, this debate is still unsolved. One limitation is the inconsistency derived from the variety of methods used in the literature, in a way that it becomes difficult to compare results. Some studies that supported the core deficit hypothesis used the Weber fraction as an index for the ANS in the nonsymbolic magnitude comparison task (Piazza et al., 2010; Pinheiro-Chagas et al., 2014), whereas others studies that supported the access deficit hypothesis used only RT, accuracy, or distance effect measures (Kovas et al., 2009; Rousselle & Noël, 2007). In addition, there are not many studies presenting an investigation of MD children nonsymbolic and symbolic magnitude processing (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Landerl et al., 2004; Landerl et al., 2009; Landerl & Kölle, 2009; Oliveira-Ferreira et al., 2012a; Olsson et al., 2016; Rousselle & Noël, 2007).

In the present study, the performance of MD children in nonsymbolic and symbolic magnitude comparison tasks is investigated, and contrasted to TA peers. If the core deficit hypothesis is more accurate to explain deficits in MD, differences between groups in the nonsymbolic magnitude comparison task are expected. Otherwise, if the access deficit hypothesis is correct, then differences between MD and TA children are expected only in the symbolic magnitude comparison task. In order to reduce methodological issues, RT, accuracy, distance effect, and Weber fraction were computed for both nonsymbolic and symbolic magnitude comparison. Usually, the Weber fraction is computed only in nonsymbolic magnitude comparison tasks, but, if the symbolic representations of number are attached to the ANS, possibly the symbolic magnitude processing responds to Weber's law as well. Therefore, we present an innovative internal Weber fraction as an index for a symbolic magnitude comparison task. Finally, the relationship between nonsymbolic and symbolic Weber fractions and arithmetic is investigated separately for MD and TA children.

Method

Participants

Three hundred and twenty-two children voluntarily took part in this study. Children were students from first to sixth grade of public and private schools of Belo Horizonte and Mariana, Brazil. First, the intelligence and the school achievement of all children were assessed in groups. Children with spelling difficulties (percentile < 25^{th} in a spelling test) or low intelligence (percentile < 10^{th} in an intelligence test), n = 100, were excluded from this study. Thus, 170 children with TA (scores above the 25th percentile in the scholar achievement task), and 52 with MD (scores under the 25th percentile in mathematics in the scholar achievement task) were selected to complete an individual assessment, comprising a task of basic arithmetic operations and magnitude comparison tasks. Twenty-six children were excluded from posterior analysis because they presented either a poor R^2 (< .20) in the fitting procedure to calculate the Weber fraction, or presented a value of Weber fraction that exceeded the discriminability of the tasks (> .60 in nonsymbolic magnitude comparison task, and >.80 in symbolic magnitude comparison task).

The final sample was composed by 159 children in the TA group (93 female, 66 male, age range: 8-12 years, *M* = 9.74, *SD* = 1.13), and 37 in the MD group (18 female, 19 male, age range: 8-12 years, *M* = 9.81, *SD* = 1.19). Groups were matched in age, $F(1,194) = .11$, $p = .74$, $\eta^2 = .001$, and sex, $X^2 = 1.18$, $p = .36$, $\varphi = .07$. Although both groups had normal intelligence, the TA group (*M* = .51, *SD* = .62) presented higher scores in Raven's colored progressive matrices when compared to the MD group (*M* = .08, *SD*= .74), $F(1,194) = 13.36$, $p<.001$, $\eta^2 = .64$. The descriptive data is presented in table 2.1.

Table 2.1: Descriptive data of the individual assessment sample.

	TA (n=159)		MD (n=37)		χ^2	*df*	*p*	φ
Sex (% female)	58.49		48.64		1.18	1	0.36	.07
	Mean	***SD***	***Mean***	***SD***	***F***	***df***	***p***	η^2
Age (years)	9.74	1.13	9.81	1.19	.11	1;194	0.74	.001
Raven (Z score)	.51	.62	.08	.74	13.36	1;194	<0.001	.064

Note. TA = typically achieving children; MD = children with mathematics difficulties; Z score (mean = 0, SD = 1).

Materials

Raven's colored progressive matrices. The Raven's Colored Progressive Matrices test was used to assess the general intelligence of participants, and z-scores were calculated for each participant according to the Brazilian norms (Angelini, Alves, Custódio, Duarte, & Duarte, 1999). In this test, children were instructed find the best option to complete a pattern with a missing part.

The Brazilian school achievement test. The Brazilian school achievement test (Teste de desempenho escolar, TDE, Stein, 1994) is the most widely used standardized test of school achievement with norms for the Brazilian population (see also Oliveira-Ferreira et al., 2012b). It comprises three subtests: arithmetic, single-word spelling, and single-word reading. In this study, the arithmetic and the single-word spelling subtests were used, and applied in groups up to five children. The arithmetic subtest is composed of three simple verbally presented word problems (i.g., "If you had three candies and received four, how many candies do you have now?") and 35 written arithmetic calculations of increasing complexity (i.g., very easy: 4−1; easy: 1230+150+1620; intermediate: 823×96; hard: 3/4+2/8). There is no time limit to complete the task, and children are instructed to work to the best of their ability. The single-word spelling subtest is composed by 36 items. In the first item, children might write their names, whereas in the other items they write a word presented orally. These subtests were used to characterize 2^{nd} to 7^{th} grade children's scholar achievement, and identify groups with math learning disabilities (percentile in arithmetic subtest < 25^{th}). Reliability coefficients (Cronbach's α) of TDE subtests are 0.89.

Basic Arithmetic Operations. The Basic Arithmetic Operation task (Pinheiro-Chagas et al., 2014) consisted of addition (27 items), subtraction (27 items) and multiplication (28 items) operations, which were printed on separate sheets of paper. Children were instructed to answer as fast and as accurate as they could. Arithmetic operations were organized in two levels of complexity and were presented to children in separate blocks: one consisted of simple arithmetic facts and the other of more complex ones. Simple additions were defined as those operations with the results below 10 (i.g., 3+5), while complex additions had results between 11 and 17 (i.g., 9+5). Tie problems (i.g., 4+4) were not used for addition. Simple subtraction comprised problems in which the operands were below 10 (i.g., 9-6), while in complex subtractions the first operand ranged from 11 to 17 (i.g., 16-9). No negative results were included in the subtraction problems. Simple multiplication consisted of operations with results below 25 or with the number 5 as one of the operands (i.g., 2*7, 5*6), while for the complex multiplication the result of operands ranged from 24 to 72 (i.g., 6*8). Tie problems were not used for multiplication. Time limit per block was set in 1 minute. For the analyses, the scores obtained in the simple and complex versions of addition, subtraction and multiplication tasks, respectively, were combined. The coefficients of these tasks were highly reliable (Cronbach's $\alpha > 0.90$).

Nonsymbolic Magnitude Comparison. In the nonsymbolic magnitude comparison task (Pinheiro-Chagas, 2014), participants were instructed to compare two sets of dots presented simultaneously, and to choose the larger numerosity by pressing a key congruent to its side (left or right) (Figure 2.1 - A). Black dots were presented on a white circle over a black background. On each trial, one of the two white circles contained 32 dots (reference numerosity) and the other one contained 20, 23, 26, 29, 35, 38, 41, or 44 dots. Each numerosity was presented 8 times, each time in a different configuration, with a total of 64 testing trials. Maximum stimulus presentation time was 4000 ms, and intertrial interval was 700 ms. Between each trial, a fixation point appeared on the screen for 500 ms - a cross, printed in white, with 3 cm in each line. To prevent the use of non-numerical cues, the sets of dots were designed and generated using a predefined MATLAB script (Dehaene, Izard, & Piazza, 2005), in such a way that on half of the trials, dot size remained constant so that the total dot area covaried positively with numerosity, and on the other half, total dot area was fixed so that dot size covaried negatively with numerosity. Data were trimmed in two interactive steps for each child to exclude responses ± 3 *SD* from the mean reaction time (RT). As a measure of the ANS acuity, the internal Weber fraction was calculated for each child based on the Log-Gaussian model of number representation (Dehaene, 2007), with the methods described by Piazza et al. (2004).

Symbolic Magnitude Comparison. In the symbolic magnitude comparison task (Oliveira-Ferreira, 2012a), children were instructed to judge if an Arabic number presented on the computer screen was greater or smaller than 5 (Figure 2.1 - B). The numbers presented on the screen were 1, 2, 3, 4, 6, 7, 8, or 9 (distances 1 to 4), printed in white over a black background. If the presented number was smaller than 5, children should press a predefined key on the left side of keyboard. Otherwise, if the presented number was greater than 5, children should press a key on the right side of the keyboard. The task comprised a total of 80 trials, 10 trials for each numerosity. The presented number was shown on the screen for 4000ms, and the time interval between trials was 700ms. Before the test trial, there was a fixation trial (a cross) with duration of 500ms. As in the nonsymbolic magnitude comparison task, responses ± 3 *SD* from the mean RT were excluded. To assure comparable indexes in nonsymbolic and symbolic magnitude comparison task, a symbolic version of the Weber fraction was calculated, based on Log-Gaussian model of number representation (Dehaene, 2007). As traditionally done in nonsymbolic magnitude comparisons, the error rates for each ratio were computed, and the Weber fraction was calculated with methods proposed by Piazza et al. (2004).

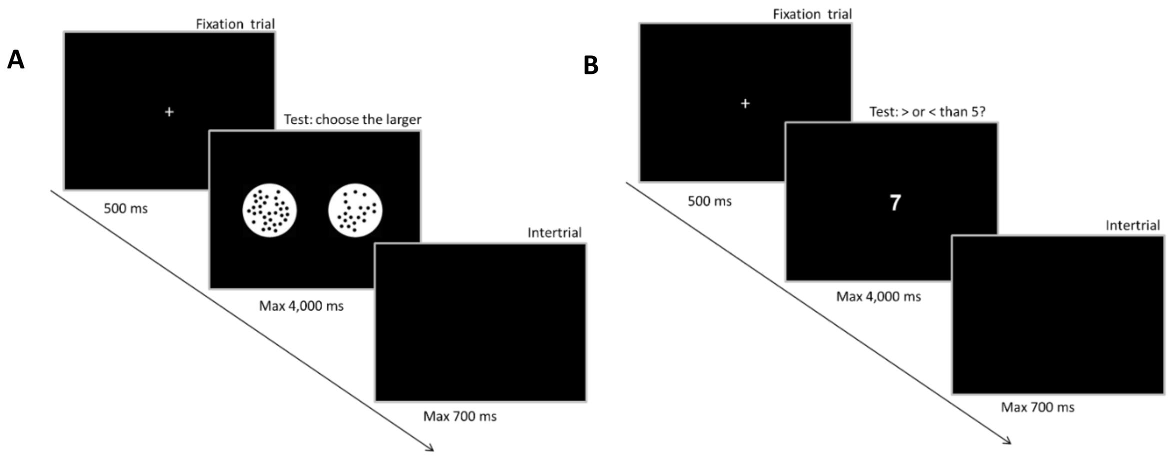

Figure 2.1. Illustration of experimental tasks, with arrows indicating the time curse of the tasks: fixation trial, experimental trial, and intertrial. a. Nonsymbolic magnitude comparison task; b. Symbolic magnitude comparison task.

Procedures

This study had approval from the IRB of Universidade Federal de Minas Gerais - Brazil. Children took part in the study only after they verbally agreed and their parents or legal guardians signed the informed consent form. Children's assessment occurred in quiet and comfortable rooms in their own schools. First, children completed the Raven's Colored

Progressive Matrices, followed by the spelling and arithmetic subtests of TDE, in groups with a maximum of five participants. Afterwards, children individually completed the Nonsymbolic Magnitude Comparison task, the Symbolic Magnitude Comparison task, and the Basic Arithmetic Operations task in a counterbalanced order.

To test the hypothesis that MD children present difficulties in nonsymbolic and symbolic magnitude processing, comparison analysis were run with group (between subjects) and numerical magnitude (within subjects) as independent variables. To investigate the relationship between nonsymbolic and symbolic magnitude processing and math achievement, linear regressions were run.

Results

Nonsymbolic magnitude comparison

There was no significant correlation between intelligence scores and RT (r =.01, p = .83), accuracy (r = -.07, p = .28), or Weber fraction (r = -.04, p = .55) in the nonsymbolic magnitude comparison task. Thus, Raven's z-scores were not added as covariate in posterior analysis.

A mixed-ANOVA, with magnitude (within-subjects, 8 levels: 20, 23, 26, 29, 35, 38, 41, and 44) and groups (between-subjects, 2 levels: TA and MD) as independent variables was conducted for accuracy and RT on the nonsymbolic magnitude comparison task. For RT, there was a main-effect of magnitude, F (1,194) = 20.69, p = <.001, η^2 =.096 (Figure 2.2 – A). *Post-hoc* analysis showed lower RT for magnitudes distant from the reference, with significant difference (p <.05) between 20 (M = 1159.99, SD = 26.16) and 23 (M = 1232.10, SD = 29.71), 20 and 26 (M = 1290.19, SD = 32.74), 20 and 29 (M = 1363.30, SD = 38.34), 20 and 35 (M = 1340.32, SD = 31.93), 20 and 41 (M = 1237.26, SD = 28.19), 20 and 44 (M = 1225.16, SD = 28.68), 23 and 29, 23 and 35, 26 and 38 (M = 1196.32, SD = 27.95), 26 and 44, 29 and 41, 29 and 44, 35 and 38, 35 and 41, and 35 and 44. There was no significant main-effect of groups, F (1,194) = 2.48, p = .12, η^2 = .013, although MD (M = 1298.19, SD = 48.46) presented higher RT than TA group (M = 1212.95, SD = 23.52). There was no significant interaction between factors, F (1,194) = 1.05, p = .40, η^2 = .005.

For accuracy, there was a main-effect of magnitude, F (1,194) = 140.08, p = <.001, η^2 = .419 (Figure 2.2 – B). *Post-hoc* analysis showed higher accuracy for magnitudes distant from the reference, with significant difference (p <.05) between 20 (M = .36, .SD = .04) and 23 (M = .93, SD = .08), 20 and 26 (M = 2.55, SD = .11), 20 and 29 (M = 3.61, SD = .12), 20 and 35 (M = 2.55, SD = .116), 20 and 38 (M = 1.18, SD = .10), 20 and 41 (M = 1.26, SD = .09), 20 and 44 (M = 1.02, SD = .08), 23 and 26, 23 and 29, 23 and 35, 26 and 29, 26 and 38, 26 and 41, 26 and 44, 29 and 35, 29 and 38, 29 and 41, 29 and 44, 35 and 38, 35 and 41, and 35 and 44. The MD group (M = 1.83, SD = .08) presented lower accuracy than TA (M = 1.54, SD = .04), as shown by a main-effect of groups, F (1,194) = 11.22, p <.01, η^2 = .055. There was no significant interaction between factors, F (1,194) = 1.16, p = .32, η^2 = .006.

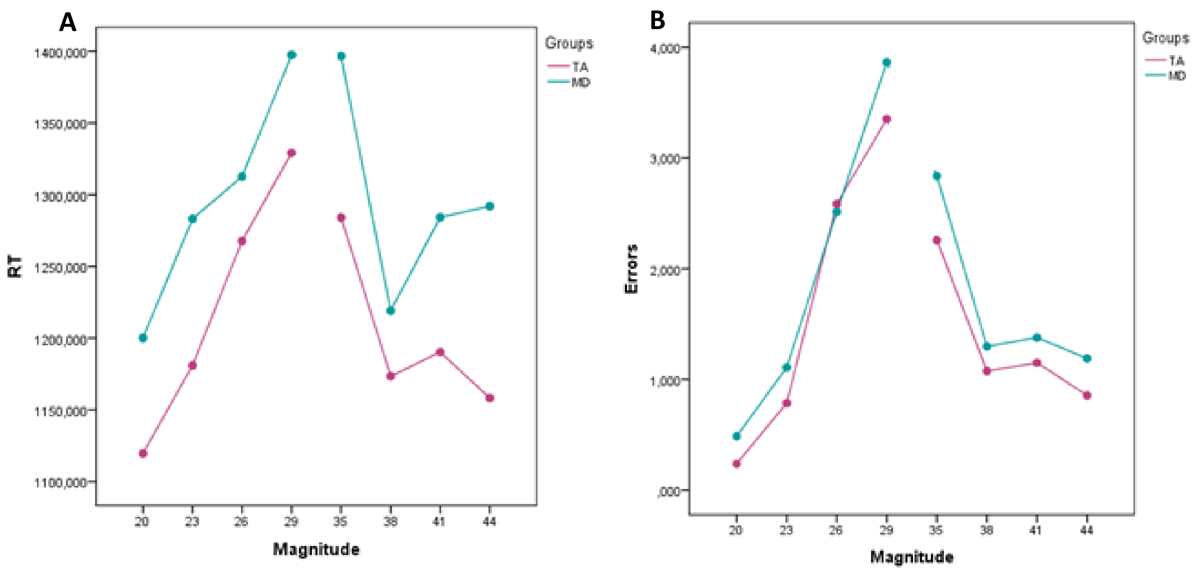

Figure 2.2. Distance effect in nonsymbolic magnitude comparison task. *a*. Reaction time – there was a significant distance effect for RT. *Post-hoc* analysis showed differences between 20 and 23, 20 and 26, 20 and 29, 20 and 35, 20 and 41, 20 and 44, 23 and 29, 23 and 35, 26 and 38, 26 and 44, 29 and 41, 29 and 44, 35 and 38, 35 and 41, and 35 and 44. There was no difference between MD and TA, and no interaction between magnitudes and groups. *b*. Accuracy – As in RT, there was no interaction between factors. The distance effect was significant, and the *Post-hoc* analysis showed differences between 20 and 23, 20 and 26, 20 and 29, 20 and 35, 20 and 38, 20 and 41, 20 and 44, 23 and 26, 23 and 29, 23 and 35, 26 and 29, 26 and 38, 26 and 41, 26 and 44, 29 and 35, 29 and 38, 29 and 41, 29 and 44, 35 and 38, 35 and 41, and 35 and 44. Children with MD presented lower accuracy when compared to TA.

To investigate differences between TA and MD groups in the Weber Fraction, a one-way analysis of variance (ANOVA) was conducted. There was a main-effect of group, F (1, 194) = 7.36, $p < .01$ $\eta^2 = .037$ (Figure 2.3), indicating higher Weber fraction values for the MD (M = .30, SD = .08) when compared to TA peers (M = .26, SD = .09).

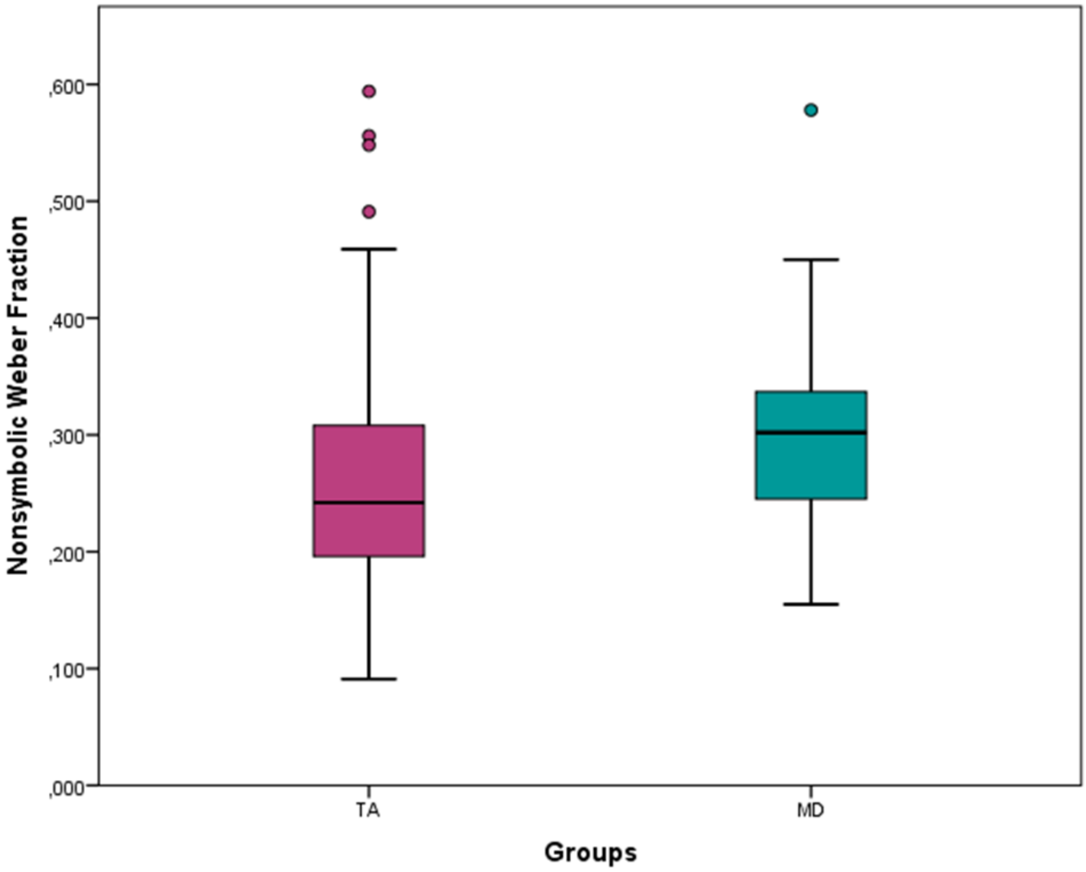

Figure 2.3. Boxplot of Weber fraction in the nonsymbolic magnitude comparison task. Children with MD presented a nonsymbolic Weber fraction significantly higher than TA peers.

Symbolic magnitude comparison

There was no significant correlation between intelligence scores and RT (r = .02, p = .73), accuracy (r = -.02, p = .73), or Weber fraction (r = .03, p = .67) in the symbolic magnitude comparison task. For this reason, as in nonsymbolic magnitude comparison analysis, Raven’s z-scores were not included as covariate.

A mixed-ANOVA, with numerical magnitude (within-subjects, 8 levels: 1, 2, 3, 4, 6, 7, 8, and 9) and groups (between-subjects, 2 levels: TA and MD) as independent variables was conducted for accuracy and RT on the symbolic magnitude comparison task. For RT, there was a main-effect of numerical magnitude, F (1,194) = 29.20, p = <.001, η^2 = .131(Figure 2.4 – A). *Post-hoc* analysis showed lower RT for magnitudes distant from the reference, with significant difference (p <.05) between 1 (M = 871.83, SD = 20.94) and 3 (M = 961.86, SD = 23.28), 1 and 4 (M = 992.79, SD = 23.27), 1 and 6 (M = 1011.51, SD = 25.07), 1 and 7 (M = 920.40, SD = 21.63), 1 and 8 (M = 928.89, SD = 21.68), 1 and 9 (M = 889.17, SD = 20.96), 2 (M = 904. 84, SD = 20.20) and 3, 2 and 4, 2 and 6, 3 and 6, 3 and 9, 4 and 7, 4 and 8, 4 and 9, and 8 and 9. There was no significant main-effect of groups, F (1,194) = 1.44, p = .23, η^2 = 007, although MD group (M = 959.73, SD = 36.85) presented higher RT than TA (M = 910.59, SD = 17.78). There was no significant interaction between factors, F (1,194) = .53, p = .81, η^2 = .003.

For accuracy, there was a main-effect of magnitude, F (1,194) = 11.66, p = <.001, η^2 = .057 (Figure 2.4 – B). *Post-hoc* analysis showed higher accuracy for magnitudes distant from the reference, with significant difference (p <.05) between 1(M = .62, SD = 0.7) and 2 (M = .96, SD = .09), 1 and 3 (M = .99, SD = .09), 1 and 4 (M = 1.13, SD =.10), 1 and 6 (M = 1.58, SD =.10), 1 and 7 (M = 1.02, SD =.09), 1 and 8 (M = 1.16, SD =.08), 1 and 9 (M = .69, SD =.08), 2 and 6, 3 and 6, 4 and 9, 6 and 7, 6 and 8, 6 and 9, and 8 and 9. There was no main-effect of groups, F (1,194) = .76, p = .38, η^2 = .004, but MD (M = 1.05, SD = .07) presented lower accuracy than TA group (M = .98, SD = .04). There was no significant interaction between factors, F (1,194) = 1.53, p = .15, η^2 = .008.

To investigate differences between TA and MD groups in the symbolic Weber Fraction, a one-way analysis of variance was conducted. There was no main effect of groups, *F* (1, 194) = 1.55, $p = .21$, $\eta^2 = .008$ (Figure 2.5). However, the MD group ($M = .21$, $SD = .15$) presented higher Weber fraction values than TA ($M = .18$, $SD = .12$).

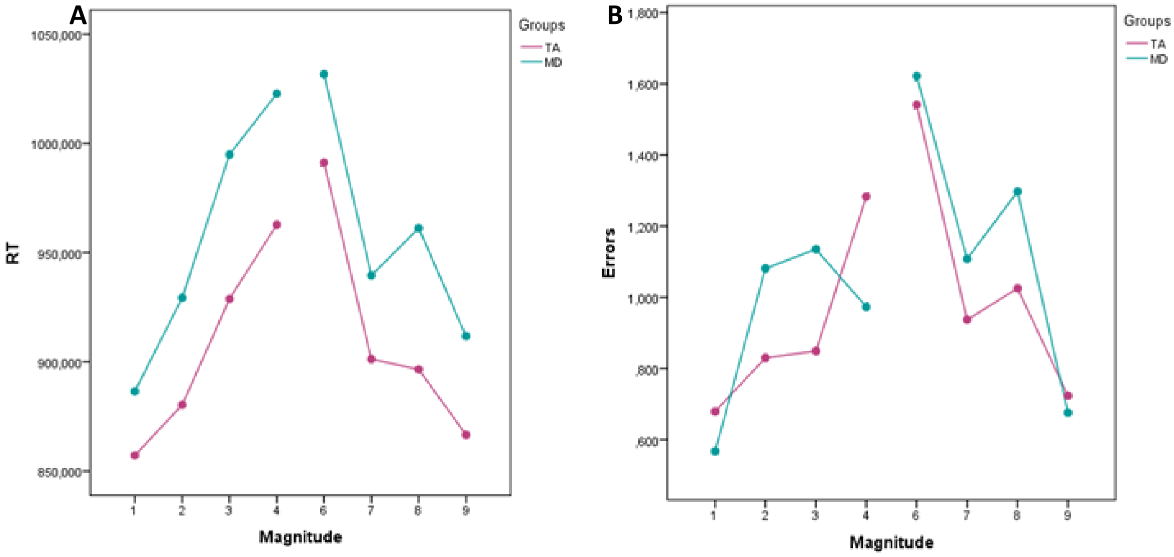

Figure 2.4. Distance effect in symbolic magnitude comparison task. *a.* Reaction time – there was a significant distance effect for RT. *Post-hoc* analysis showed differences between 1 and 3, 1 and 4, 1 and 6, 1 and 7, 1 and 8, 1 and 9, 2 and 3, 2 and 4, 2 and 6, 3 and 6, 3 and 9, 4 and 7, 4 and 8, 4 and 9, and 8 and 9. There was no difference between MD and TA, and no interaction between magnitudes and groups. *b.* Accuracy – As in RT, there was no difference between groups, or interaction between factors. The distance effect was significant, and the *Post-hoc* analysis showed differences between 1 and 2, 1 and 3, 1 and 4, 1 and 6, 1 and 7, 1 and 8, 1 and 9, 2 and 6, 3 and 6, 4 and 9, 6 and 7, 6 and 8, 6 and 9, and 8 and 9.

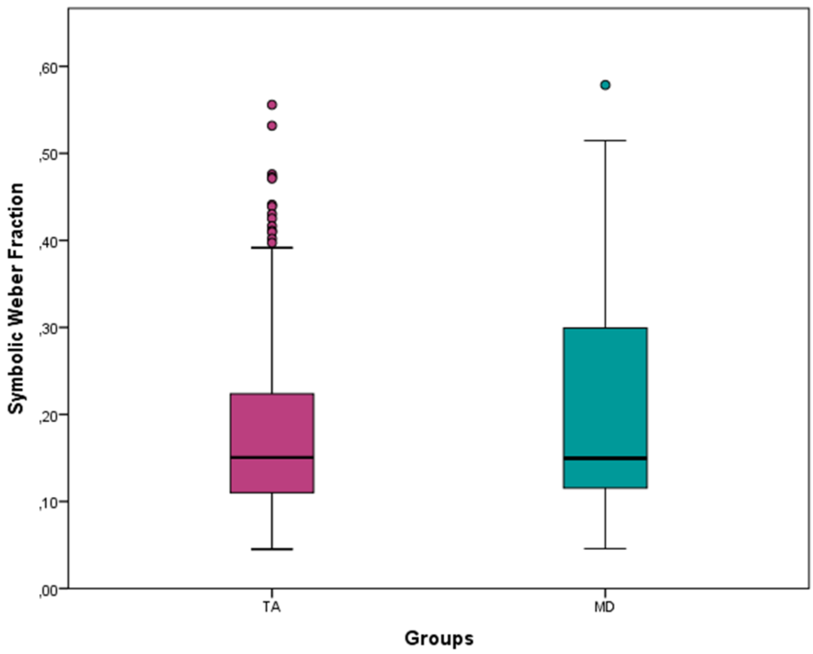

Figure 2.5. Boxplot of Weber fraction in the symbolic magnitude comparison task. Children with MD presented higher symbolic Weber fraction than TA peers, but the difference did not reach significance.

Number processing and math achievement

Groups were compared regarding their scores in Basic Arithmetic Operations. There was a significant difference in addition, F (1,194) = 19.15, p < .001, η^2 = .090, with higher accuracy for TA (M = 22.18, SD = 6.02) than MD (M = 18.29, SD = 6.02), and in subtraction, with higher accuracy for TA group (M = 16.59, SD = 5.99) than MD (M =11.16, SD = 5.99), F (1,194) = 24.63, p < .001, η^2 = .113. Finally, there was a significant difference between groups in multiplication, F (1,194) = 22.55, p < .001, η^2 = .103, with higher accuracy for TA (M = 14.63, SD = .66) when compared to MD (M = 7.51, SD = 1.36).

Regression analysis with addition, subtraction, and multiplication as dependent variables, and nonsymbolic and symbolic Weber fraction as independent variables were run separately for each group. First, results of nonsymbolic Weber fraction regressions are reported, followed by results of symbolic Weber fraction.

For nonsymbolic Weber fraction, in the TA group, a significant regression equation was found for addition, F (1,158) = 14.43; p<.001, adjusted R^2 of .08 (Figure 2.6 – A), subtraction, F (1,158) = 11.04; p <.01, adjusted R^2 = .06 (Figure 2.6 – B), and multiplication, F(1,158) = 6.19; p <.05, adjusted R^2 = .03 (Figure 2.6 – C). For MD children, no significant equation was found for addition, F (1,36) = .001; p = .97, adjusted R^2 = -.03 (Figure 2.6 – A), subtraction, F (1,36) = .04; p = .84, adjusted R^2 = -.03 (Figure 2.6 – B), or multiplication, F (1,36) = .15; p = .69, adjusted R^2 = -.02 (Figure 2.6 – C).

For symbolic Weber fraction, a significant regression equation was found for addition, F (1,158) = 14.32; p <.001, adjusted R^2 of .08 (Figure 2.7 – A), subtraction, F(1,158) = 5.57; p <.05, adjusted R^2 = .03 (Figure 2.7 – B), and multiplication, F(1,158) = 5.56; p <.05, adjusted R^2 = .03 (Figure 2.7 – C), for TA. For MD, no significant equation was found for addition, F (1,36) = .81; p = .37, adjusted R^2 = -.005 (Figure 2.7 – A), subtraction, F (1,36) =

1.11; p = .29, adjusted R^2 =.003 (Figure 2.7 – B), or multiplication, F (1,36) = .49; p = .48, adjusted R^2 = -.01 (Figure 2.7 – C).

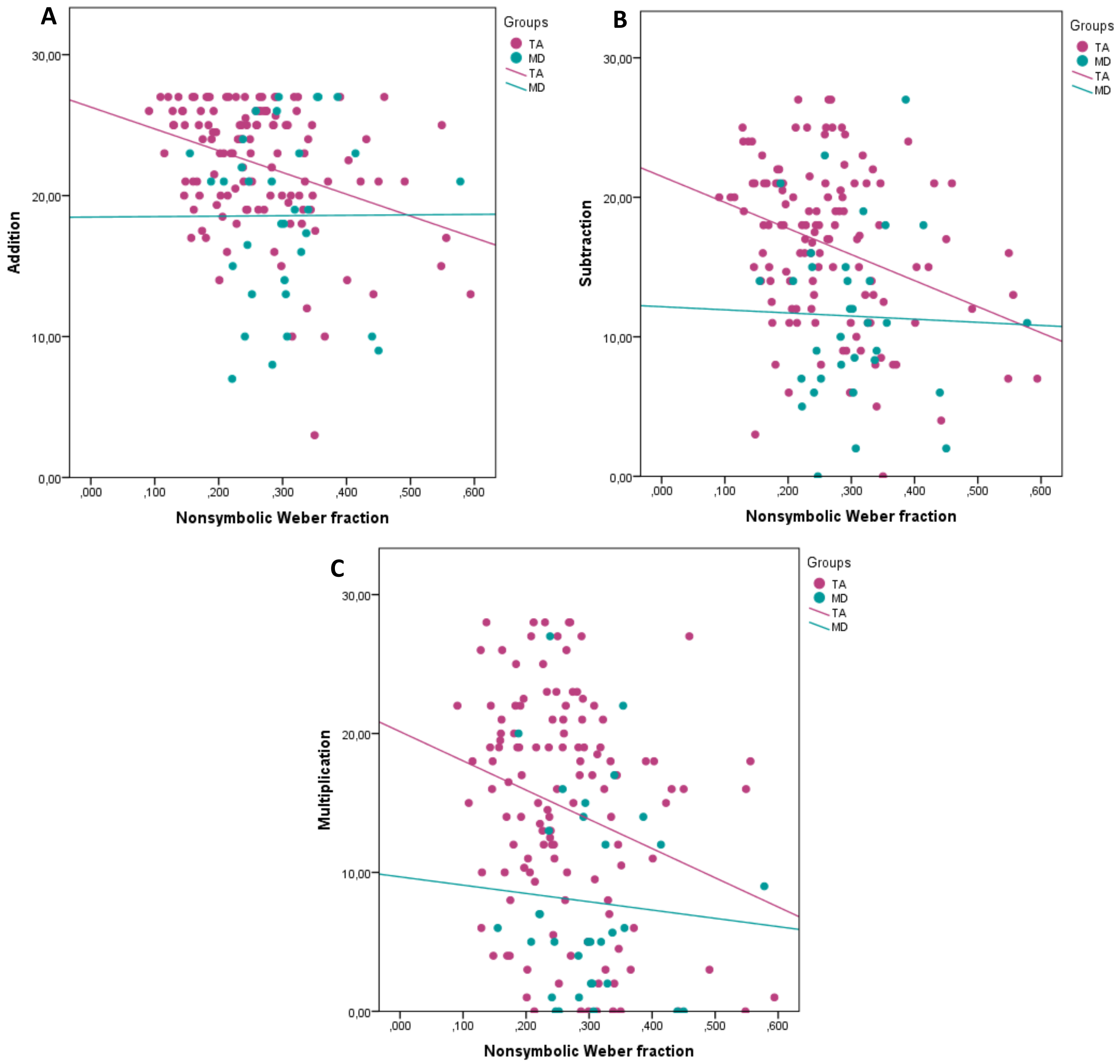

Figure 2.6. Linear regression between arithmetic operations and the Weber fraction indexed in a nonsymbolic magnitude comparison task. The regressions were significant for TA but not for MD *a*. Linear regression between nonsymbolic Weber fraction and addition; *b*. Linear regression between nonsymbolic Weber fraction and subtraction; *c*. Linear regression between nonsymbolic Weber fraction and multiplication.

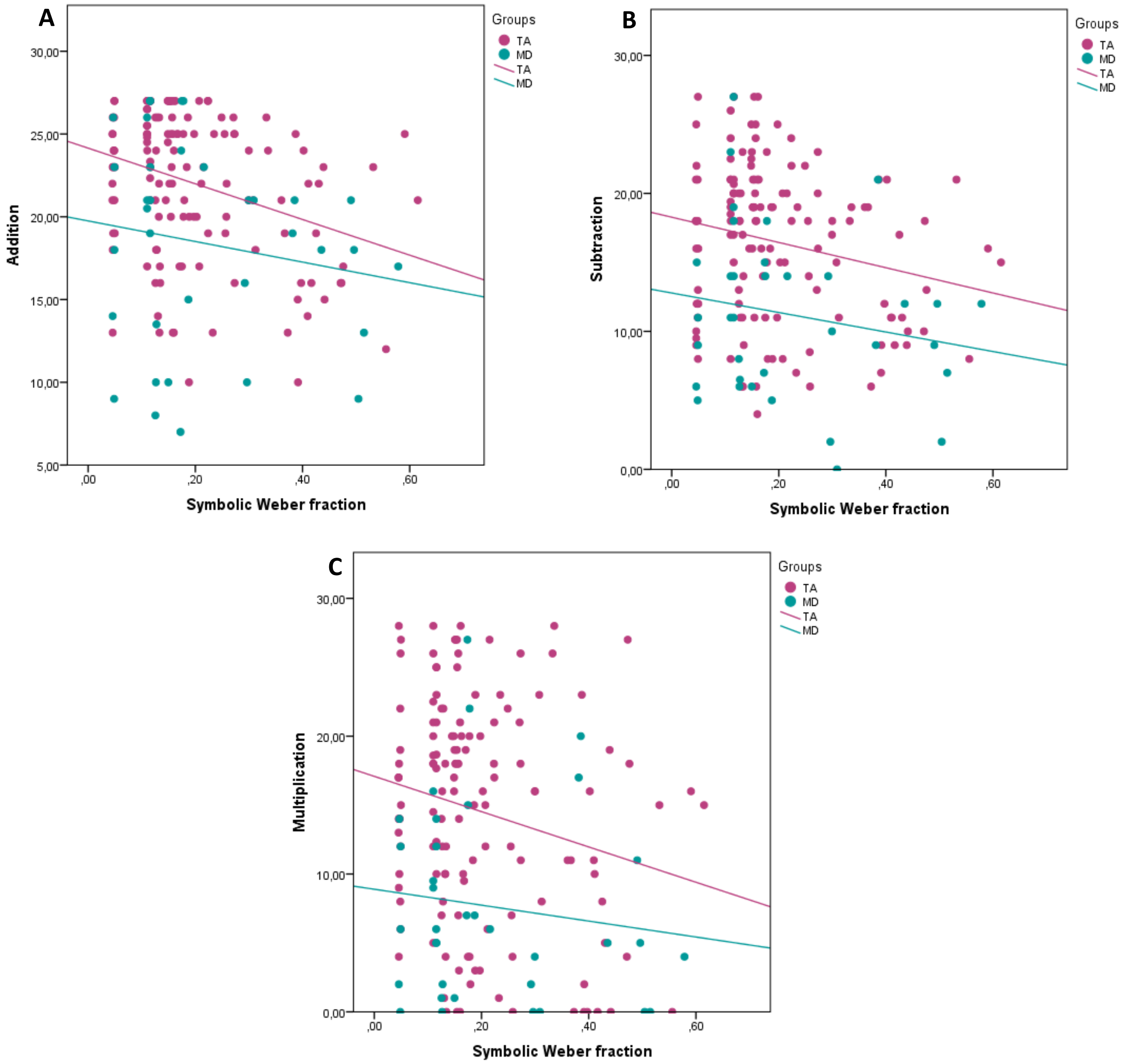

Figure 2.7. Linear regression between arithmetic operations and the Weber fraction indexed in a symbolic magnitude comparison task. The regressions were significant for TA but not for MD. *a.* Linear regression between symbolic Weber fraction and addition; *b.* Linear regression between symbolic Weber fraction and subtraction; *c.* Linear regression between symbolic Weber fraction and multiplication.

Discussion

In the present study, children with MD were compared to TA peers in nonsymbolic and symbolic magnitude processing. Overall, the distance effect was observed in nonsymbolic and symbolic magnitude comparison tasks. MD children presented lower accuracy and higher Weber fractions than TA in the nonsymbolic magnitude comparison task, but no differences were found between groups in any parameters of the symbolic magnitude comparison task.

These results are in line with the core deficit hypothesis. Finally, both nonsymbolic and symbolic magnitude processing acuity were related to addition, subtraction, and multiplication in the TA, but not in the MD group. Thereby, it is possible that children with MD do not engage basic number processing in arithmetic, and alternatively recruit other cognitive mechanisms.

Although there was a significant difference between groups in intelligence, no correlation was observed between nonsymbolic and symbolic magnitude comparison tasks and intelligence scores. Brankaer, Ghesquière, and De Smedt (2014) showed that numerical magnitude processing is independent of intelligence. They compared the magnitude processing of groups that presented MD in association with low intelligence, MD with normal intelligence, or TA in mathematics with normal intelligence. Children with MD, regardless of intelligence, presented lower magnitude processing acuity when compared to TA.

The distance effect was observed in both nonsymbolic and symbolic comparison tasks. However, there was no difference between groups in this domain, which is corroborated by previous studies (Ashkenazi, Mark-Zigdon, & Henik, 2009; Kovas et al., 2009; Kucian et al., 2011; Landerl & Köle, 2009). In the nonsymbolic magnitude comparison task, MD children presented lower accuracy and higher Weber fractions when compared to TA peers. Results suggesting an impairment in the ANS in children with MD have been replicated by most studies that presented the Weber fraction (Costa et al., 2011; Mazzocco et al., 2011; Olsson et al., 2016; Piazza et al., 2010; Pinheiro-Chagas et al., 2014). Thereby, the Weber fraction is probably a more sensitive index of the ANS acuity (Piazza et al., 2010; Schneider et al., 2016) when compared to RT and accuracy.

Since children with MD presented low ANS acuity, the core deficit hypothesis (Piazza et al., 2010) was corroborated. The core deficit hypothesis proposes that children with MD present an impairment in the ANS (Piazza et al., 2010, Pinheiro-Chagas et al., 2014), which leads to difficulties in the symbolic number processing and formal arithmetic skills. The ANS is responsible for an abstract representation of number in an analog fashion, with numbers logarithmic compressed along a mental number line (Dehaene, 2003; Halberda & Feigenson, 2008). The nonsymbolic magnitude comparison task is broadly used to investigate the ANS acuity (Halberda et al., 2008; Landerl et al., 2009; Mazzocco et al., 2011, Pinheiro-Chagas et al., 2014), but this method has been criticized, due the difficulty to control possible cofounds (Gebuis, & Reynvoet, 2011; Gilmore et al., 2013; Leibovich, Katzin, Harel, Henik, 2016; Szűcs, Nobes, Devine, Gabriel, & Gebuis, 2013).

Gilmore et al. (2013) suggested that choosing the greater numerosity in the nonsymbolic magnitude comparison task is influenced by executive functions. They presented children with congruent and incongruent trials in a nonsymbolic magnitude comparison task, with dots as stimuli. In the congruent trials, the greater array presented larger dots and a larger area, whereas on incongruent trials, the greater array presented smaller dots and a smaller area. They proposed that responses to incongruent trials might be a result of inhibition skills, instead of the ANS acuity. Other possible cofound in the nonsymbolic magnitude comparison task is the visual property of stimuli. Gebuis and Reynvoet (2011) investigated the interaction between number and visual properties using different versions of the nonsymbolic comparison task. They showed that the performance changed when manipulations on the visual properties of the stimuli were made, even when the ratio between numbers was constant. Accordingly, the authors proposed that the ANS measures might be compromised by visual cues that cannot be controlled (see also Szűcs et al., 2013). These studies suggest that nonsymbolic magnitudes processing might be actually a continuous property of the visual system, but this assumption has been taken into question (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016).

Anobile et al. (2016) showed that the numerosity processing is independent of other visual properties, as texture and density, in the nonsymbolic magnitude comparison task. The authors investigated the performance of children in two nonsymbolic magnitude comparison tasks, one with sparse stimuli, and another with very dense stimuli. The scores in the sparse stimuli condition were significantly associated to math achievement, but not the scores in the high-density condition. These results evince that numerosity and visual cues are perceived by independent systems. In addition, the analog magnitude processing follows Weber's law, corroborating the relevance of the ANS.

Other important evidence for the ANS is the theory of magnitude (ATOM: A Theory Of Magnitude; Walsh, 2003). Walsh (2003) proposed that time, space and number are part of generalized magnitude system. Skagerlund and Träff (2014) showed that children with MD suffer from a general magnitude processing deficit, since they presented lower acuity than TA in tasks assessing time, numerosity, and space magnitudes processing. Thus, there is evidence that the ANS is component of a multi-modal abstract model of magnitude representations.

As the symbolic magnitude processing arouse nonsymbolic representations (Dehaene, 1992), and relies on the ratio between magnitudes (Moyer & Landauer, 1967; Reynvoet, De Smedt, & van den Bussche, 2009), it would be plausible to assess the Weber fraction in a symbolic magnitude comparison task. We calculated a symbolic Weber fraction based on a Log-Gaussian model (Dehaene, 2007) with methods adapted from Piazza et al. (2004). However, no difference between MD and TA groups were observed in the symbolic magnitude comparison task, either in symbolic Weber fraction, accuracy or RT. Previous studies failed to find symbolic magnitude processing impairments in MD as well (Landerl & Kolle, 2009; Landerl et al., 2009; Mussolin et al., 2010).

One possibility is that the numerosity range used in the symbolic comparison task – from 1 to 9 – could have been not sensible enough to detect differences between groups, in opposition to the nonsymbolic magnitude comparison task, that presented a larger numerosity range – from 20 to 44. Landerl et al. (2009) failed to show differences between MD and TA in symbolic magnituce comparisons with single-digits, but found that MD present impairments in a two-digits number comparison. According to Skagerlund and Träff (2014), children are extensively presented with single-digit numbers in school, and two-digit number tasks might be more sensitive to assess symbolic magnitude processing. Nonsymbolic and symbolic magnitude comparison tasks with paired numerosity range should be used to investigate this subject, and our research group is working to clarify this in further studies.

Regarding arithmetic operations, children with MD presented difficulties in addition, subtraction and multiplication when compared to TA children, in line with previous findings (Landerl et al., 2004; Ostad, 1997; 1999). The association between symbolic and nonsymbolic magnitude processing and arithmetic was also investigated. Both symbolic and nonsymbolic Weber fractions predicted achievement in basic numerical operations for TA, but not for MD. Thus, to solve arithmetic problems, children with MD probably engage different cognitive resources instead of numerical processing, as proposed by Pinheiro-Chagas et al (2014). Geary et al. (2004) proposes that children with MD choose guessing as a strategy for solving arithmetic, since it is quick and require low effort (Geary et al., 2004). However, qualitative error analysis in the basic arithmetic operation task was beyond the scope of this current article.

This study investigated the performance of children with MD and TA in nonsymbolic and symbolic magnitude comparison tasks, and presented theoretical and methodological implications. The core deficit hypothesis was supported, even when different indexes of nonsymbolic and symbolic magnitude comparison tasks were presented: RT, acuity, distance effect, and Weber fraction. To our knowledge, this was the first study to index a symbolic

Weber fraction, which is a methodological innovation. However, new investigations are required in order to validate this measure.

Mathematical abilities are increasingly valued, especially with the interdisciplinary and applied approach of science, technology, engineering and mathematics (STEM) fields of study. Parsons and Bynner (2005) showed that a better mathematical achievement is related to higher employability rates and incomes. Thus, it is important to identify underlying mechanisms of MD in order to establish better diagnose criteria and consequently develop interventions techniques for a disorder that can negatively impact a significant number of school-age children.

References

Agrillo, C., Dadda, M., Serena, G., & Bisazza, A. (2008). Do fish count? Spontaneous discrimination of quantity in female mosquitofish. *Animal cognition*, 11(3), 495-503. doi:10.1007/s10071-008-0140-9

Andersson, U. (2008). Mathematical competencies in children with different types of learning difficulties. *Journal of Educational Psychology*, 100(1), 48-66. doi: 10.1037/0022-0663.100.1.48

Angelini, A. L., Alves, I. C. B., Custódio, E. M., Duarte, W. F., & Duarte, J. L. M. (1999). *Matrizes progressivas coloridas de Raven – escala especial*. São Paulo: Centro Editor de Testes e Pesquisas em Psicologia.

Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental psychology*, 52(8), 1206. doi: 10.1037/dev0000155

Ashkenazi, S., Mark-Zigdon, N., & Henik, A. (2009). Numerical distance effect in developmental dyscalculia. *Cognitive Development*, 24(4), 387-400. doi:10.1016/j.cogdev.2009.09.006

Badian, N. A. (1983). Dyscalculia and nonverbal disorders of learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (Vol. 5, pp. 235–264). New York: Stratton.

Berteletti, I., Prado, J., & Booth, J. R. (2014). Children with mathematical learning disability fail in recruiting verbal and numerical brain regions when solving simple multiplication problems. *Cortex*, 57, 143-155. doi: 10.1016/j.cortex.2014.04.001

Booth, J. L. & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 41(6), 189-201. doi: 10.1037/0012-1649.41.6.189

Brankaer, C., Ghesquière, P., & De Smedt, B. (2014). Numerical magnitude processing deficits in children with mathematical difficulties are independent of intelligence. *Research in developmental disabilities*, 35(11), 2603-2613. doi: 10.1016/j.ridd.2014.06.022

Bugden, S., Price, G. R., McLean, D. A., & Ansari, D. (2012). The role of the left intraparietal sulcus in the relationship between symbolic number processing and children's arithmetic competence. *Developmental Cognitive Neuroscience*, *2*(4), 448-457. doi:10.1016/j.dcn.2012.04.001

Burny, E., Valcke, M., & Desoete, A. (2012). Clock reading: An underestimated topic in children with mathematics difficulties. *Journal of learning disabilities,* 45(4), 351-360. doi: 10.1177/0022219411407773

Butterworth, B. (2005). Developmental dyscalculia. In J. I. D. Campbell (Ed.) *Handbook of Mathematical Cognition* (pp. 455-467). Hove: Psychology Press.

Castronovo, J. & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PLoS ONE*, 7(4), e33832. doi: 10.1371/journal.pone.0033832

Chen, Q. & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, 148, 163-172. doi:10.1016/j.actpsy.2014.01.016

Chesney, D. (2016). *The relationship between the numerical distance effect and approximate number system acuity is non-linear.* Available at: https://mindmodeling.org/cogsci2016/cogsci16_proceedings.pdf

Costa, A. J., Silva, J. B. L., Chagas, P. P., Krinzinger, H., Lonneman, J., Willmes,K., Wood, G., & Haase, V.G. (2011). A hand full of numbers: a role for offloading in arithmetics learning? *Frontiers in Psychology*. doi: 10.3389/fpsyg.2011.00368

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44(1), 1-42. doi: 10.1016/0010-0277(92)90049-N

Dehaene, S. (2001). Précis of the number sense. *Mind & Language*, 16(1), 16-36. doi: 10.1111/1468-0017.00154

Dehaene, S. (2003). The neural basis of the Weber–Fechner law: a logarithmic mental number line. *Trends in Cognitive Sciences*, 7(4), 145-147. doi:10.1016/S1364-6613(03)00055-X

Dehaene, S. (2007). Symbols and quantities in parietal cortex: elements of a mathematical theory of number representation and manipulation. In P. Haggard, Y. Rossetti, & M. Kawato (Eds.), *Sensorimotor Foundations of Higher Cognition: Attention and Performance* (pp. 527-574). Cambridge (UK): Oxford University Press.

Dehaene, S., Izard, V., & Piazza, M. (2005). *Control over Non-Numerical Parameters in Numerosity Experiments*. Retrieved from: http://www.unicog.org/docs/DocumentationDotsGeneration.doc

De Smedt, B. & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of experimental child psychology*, 108(2), 278-92. doi: 10.1016/j.jecp.2010.09.003

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, 2(2), 48-55. doi: 10.1016/j.tine.2013.06.001

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement.

Journal of Experimental Child Psychology, 103, 469-479. doi:10.1016/j.jecp.2009.01.010

Desoete, A. Ceulemans, A., De Weerdt, F., & Pieters, S. (2012). Can we predict mathematical learning disabilities from symbolic and non-symbolic comparison tasks in kindergarten? Findings from a longitudinal study. *British Journal of Educational Psychology*, 82, 64-81. doi: 10.1348/2044-8279.002002

Gallistel, C. R. & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in cognitive sciences*, 4(2), 59-65. doi: 10.1016/S1364-6613(99)01424-2

Garland, A. & Low, J. (2014). Addition and subtraction in wild New Zealand robins. *Behavioral processes*, 109, 103-110. doi: 10.1016/j.beproc.2014.08.022

Geary D. C. (1993). Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychology Bulletin*, 114(2), 345-62. doi: 10.1037//0033-2909.114.2.345

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of experimental child psychology*, 88(2), 121-151. doi:10.1016/j.jecp.2004.03.002

Gebuis, T., & Reynvoet, B. (2011). Generating nonsymbolic number stimuli. *Behavior research methods*, 43(4), 981-986. doi:10.3758/s13428-011-0097-5

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., Simms, V., & Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PloS ONE*, 8(6), e67374. doi: 10.1371/journal.pone.0067374

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition,* 115(3), 394-406. doi: 10.1016/j.cognition.2010.02.002

Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science*, 306(5695), 496-499. doi: 10.1126/science.1094492

Gross, H. J., Pahl, M., Si, A., Zhu, H., Tautz, J., & Zhang, S. (2009). Number-based visual generalization in the honeybee. *PloS ONE,* 4(1), e4263. doi: 10.1371/journal.pone.0004263

Halberda, J. & Feigenson, L. (2008). Developmental change in the acuity of the "Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental psychology*, 44(5), 1457-1465. doi: 10.1037/a0012682

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, 455, 665-668. doi:10.1038/nature07246

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, 93(3), 615-626. doi: 10.1037/0022-0663.93.3.615

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience,* 6(6), 435-448. doi:10.1038/nrn1684

Inglis, M., Attridge, N., Batchelor, S., & Gilmore, C. (2011). Non-verbal number acuity correlates with symbolic mathematics achievement: but only in children. *Psychonomic Bulletin & Review,* 18(6), 1222-1229. doi: 10.3758/s13423-011-0154-1

Iuculano, T., Tang, J., Hall, C. W. B., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science*, 11(5), 669-680. doi: 10.1111/j.1467-7687.2008.00716.x

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences*, 16(2), 10382-10385. doi: 10.1073/pnas.0812142106

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child development*, 74(3), 834-850. doi:10.1111/1467-8624.00571

Jordan, K. E., MacLean, E. L., & Brannon, E. M. (2008). Monkeys match and tally quantities across senses. *Cognition*, 108, 617-625. doi: 10.1016/j.cognition.2008.05.006

Júlio-Costa, A., Starling-Alves, I., Lopes-Silva, J. B., Wood, G., & Haase, V. G. (2015). Stable measures of number sense accuracy in math learning disability: Is it time to proceed from basic science to clinical application?. *PsyCh journal*, 4(4), 218-225. doi: 10.1002/pchj.114

Kovas, Y., Giampietro, V., Viding, E., Ng, V., Brammer, M., Barker, G. J., Happe, F. G. E., & Plomin, R. (2009). Brain correlates of non-symbolic numerosity estimation in low and high mathematical ability children. *PLoS ONE*, 4(2), e4587. doi:10.1371/journal.pone.0004587

Krusche, P., Uller, C., & Dicke, U. (2010). Quantity discrimination in salamanders. *Journal of Experimental Biology*, 213(11), 1822-1828. doi: 10.1242/jeb.039297

Kucian, K., Loenneker, T., Dietrich, T., Dosch, M., Martin, E., & von Aster, M. (2006). Impaired neural networks for approximate calculation in dyscalculic children: a functional MRI study. *Behavioral and Brain Functions*, 2(31). doi: 10.1186/1744-9081-2-31

Kucian, K., Loennekera, T., Martin, E., & von Aster, M. (2011). Non-symbolic numerical distance effect in children with and without developmental dyscalculia: a parametric MRI study. *Developmental Neuropsychology*, 36(6), 741-765. doi: 10.1080/87565641.2010.549867

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: a study of 8-9-year-old students. *Cognition*, 93(2), 99-125. doi:10.1016/j.cognition.2003.11.004

Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *Journal of experimental child psychology*, 103(3), 309-24. doi:10.1016/j.jecp.2009.03.006

Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of experimental child psychology*, 103(4), doi:546-65. 10.1016/j.jecp.2008.12.006

Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2016). From ‘sense of number’to ‘sense of magnitude’–The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, 1-62. doi: 10.1017/S0140525X16000960

Lewis, C., Hitch, G. J., & Walker, P. (1994). The prevalence of specific arithmetic difficulties and specific reading difficulties in 9-to 10-year-old boys and girls. *Journal of Child Psychology and Psychiatry*, 35(2), 283-292. doi: 10.1111/j.1469-7610.1994.tb01162.x

Mazzocco, M. M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch, & M. M. M. Mazzocco (Eds), *Why Is Math So Hard for Some Children? The Nature and Origins of Mathematical Learning Difficulties and Disabilities* (pp. 29-47). Baltimore: Brookes.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia). *Child Development*, 82(4), 1224-1237. doi: 10.1111/j.1467-8624.2011.01608.x

McComb, K., Packer, C., & Pusey, A. (1994). Roaring and numerical assessment in contests between groups of female lions, Panthera leo. *Animal Behaviour*, 47, 379-387. doi: 10.1006/anbe.1994.1052

Mejias, S., Mussolin, C., Rousselle, L., Gregoire, J., & Noel, M. P. (2011). Numerical and nonnumerical estimation in children with and without mathematical learning disabilities. *Child Neuropsychology,* 18(6), 550-575. doi: 10.1080/09297049.2011.625355

Moeller, K., Pixner, S., Zuber, J. Kaufmann, L., & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance—A longitudinal study on numerical development. *Research in Developmental Disabilities*, 32, 1837-1851. doi: 10.1016/j.ridd.2011.03.012

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature*, 215, 1519-1520. doi:10.1038/2151519a0

Mundy, E. & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology*, 103, 490-502. doi:10.1016/j.jecp.2009.02.003

Mussolin, C., Mejias, S., & Noël, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition*, 115(1), 10-25. doi:10.1016/j.jecp.2009.02.003

Noël, M. P. & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: an explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience*. doi: 10.3389/fnhum.2011.00165

Oliveira-Ferreira, F., Costa, D. S., Micheli, L. R., Oliveira, L. D. F. S., Pinheiro-Chagas, P., & Haase, V. G. (2012b). School Achievement Test: Normative data for a representative sample of elementary school children. *Psychology & Neuroscience,* 5(2), 157-164. doi: 10.3922/j.psns.2012.2.05

Oliveira-Ferreira, F., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2012a). Explaining school mathematics performance from symbolic and nonsymbolic magnitude processing: similarities and differences between typical and low-achieving children. *Psychology & Neuroscience*, 5(1), 37-46. doi: 10.3922/j.psns.2012.1.06

Olsson, L., Östergren, R., & Träff, U. (2016). Developmental dyscalculia: A deficit in the approximate number system or an access deficit?. *Cognitive Development*, 39, 154-167. doi: 10.1016/j.cogdev.2016.04.006

Ostad, S. A. (1997). Developmental differences in addition strategies: A comparison of mathematically disabled and mathematically normal children. *British Journal of Educational Psychology*, 67(3), 345-357. doi: 10.1111/j.2044-8279.1997.tb01249.x

Ostad, S. A. (1999). Developmental progression of subtraction strategies: A comparison of mathematically normal and mathematically disabled children. *European Journal of Special Needs Education*, 14(1), 21-36. doi: 10.1080/0885625990140103

Parsons, S. & Bynner, J. (2005). *Does numeracy matter more?.* London: National Research and Development Centre for adult literacy and numeracy.

Paulsen, D. J., Woldorff, M. G., & Brannon, E. M. (2010). Individual differences in nonverbal number discrimination correlate with event-related potentials and measures of probabilistic reasoning. *Neuropsychologia,* 48(13), 3687-3695. doi: 10.1016/j.neuropsychologia.2010.08.014

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences,* 14(12), 542-551. doi:10.1016/j.tics.2010.09.008

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41. doi:10.1016/j.cognition.2010.03.012

Piazza, M., Izard, V., Pinel, P., LeBihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human parietal cortex. *Neuron*, 44(3), 547-555. doi:10.1016/j.neuron.2004.10.014

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science,* 306(5695), 499-503. doi: 10.1126/science.1102085

Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., Willmes, K., & Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation?. *PloS ONE*, 9(11), e111155. doi: 10.1371/journal.pone.0111155

Price, G. R., Holloway, I. D., Rasanen, P., Vesterinen, M., & Ansari, D. (2007). Impaired parietal magnitude processing in developmental dyscalculia. *Current Biology*, 17(24), R1042–R1043. doi: 10.1016/j.cub.2007.10.013

Reynvoet, B., De Smedt, B., & Van den Bussche, E. (2009). Children’s representation of symbolic magnitude: The development of the priming distance effect. *Journal of Experimental Child Psychology*, 103(4), 480-489. doi:10.1016/j.jecp.2009.01.007

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, 102(3), 361-95. doi:10.1016/j.cognition.2006.01.005

Schleger F., Landerl, K., Muenssinger, J., Draganova, R., Reinl, M., Kiefer-Schmidt, I., Weiss, M., Wacker-Gußmann, A., Huotilainen, M., & Preissl, H. (2014). Magnetoencephalographic signatures of numerosity discrimination in fetuses and neonates. *Developmental neuropsychology*, 39(4), 316-329. doi: 10.1080/87565641.2014.914212

Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2016). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: a meta-analysis. *Developmental science*. doi:10.1111/desc.12372

Shalev, R. S., Auerbach, J., Manor, O., & Gross-Tsur, V. (2000). Developmental dyscalculia: prevalence and prognosis. *European child & adolescent psychiatry*, 9, S58-S64. doi:10.1007/s007870070009

Siegler, R. S., & Robinson, M. (1982). The development of numerical understandings. *Advances in child development and behavior*, 16, 241-312. doi: 10.1016/S0065-2407(08)60072-5

Skagerlund, K., & Träff, U. (2014). Development of magnitude processing in children with developmental dyscalculia: space, time, and number. *Frontiers in psychology*. doi: 10.3389/fpsyg.2014.00675

Stein, L. M. (1994). *TDE – Teste do Desempenho Escolar. Manual para aplicação e interpretação*. São Paulo: Casa do Psicólogo.

Szücs, D., Nobes, A., Devine, A., Gabriel, F. C., & Gebuis, T. (2013). Visual stimulus parameters seriously compromise the measurement of approximate number system acuity and comparative effects between adults and children. *Frontiers in Psychology*, *4*, 444. doi: 10.3389/fpsyg.2013.00444

Verguts, T. & Van Opstal, F. (2005). Dissociation of the distance effect and size effect in one-digit numbers. *Psychonomic Bulletin & Review*, 12(5), 925-930. doi:10.3758/BF03196787

Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in cognitive sciences*, 7(11), 483-488. doi: 10.1016/j.tics.2003.09.002

Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, 358(6389), 749-750. doi:10.1038/358749a0

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental science*, 8(1), 88-101. doi: 10.1111/j.1467-7687.2005.00395.x

Symbolic and nonsymbolic number processing influences on math achievement

Isabella Starling-Alves[1], Ricardo J. Moura[2], Annelise Júlio-Costa[1], Vitor Geraldi Haase[1,3,4,5], and Renato Bortoloti[1,3,4,5]

1.Programa de Pós-Graduação em Neurociências, Universidade Federal de Minas Gerais, Brasil

2. Programa de Pós-Graduação em Ciências do Comportamento, Universidade de Brasília, Brasil

3. Departamento de Psicologia, Universidade Federal de Minas Gerais, Brasil

4. Programa de Pós-Graduação em Psicologia: Cognição e Comportamento, Universidade Federal de Minas Gerais, Brasil

5. Instituto Nacional de Ciência e Tecnologia: Comportamento, Cognição e Ensino

Abstract

The nonsymbolic and symbolic processing in children with mathematical difficulties (MD) has received much attention in the last years. Usually, these abilities are assessed by numerical magnitude comparison tasks. However, while large numerosities are typically used in the nonsymbolic comparison tasks, few studies have investigated symbolic comparisons with two-digit numbers. This study investigated the nonsymbolic and the two-digit number processing in children with MD, and the relationship of these abilities with basic arithmetic calculation and numerical transcoding skills. Children with MD presented impairments in the nonsymbolic processing. They also presented difficulties in a two-digit number comparison task, although no compatibility effect was observed. There was a significant correlation between nonsymbolic processing and multiplication and numerical transcoding scores. The two-digit number processing, in turn, was correlated to addition, subtraction, and multiplication, as well as numerical transcoding scores and syntactic errors frequency. Impairments in the analog number representation in children with MD were endorsed, as well as the importance of basic numerical processing to math achievement.

Keywords: mathematical difficulties, ANS, nonsymbolic comparison, two-digit symbolic comparison, compatibility effect

Symbolic and nonsymbolic number processing influences on math achievement

The numerical reasoning skills are crucial for our daily lives. Besides its relevance for school achievement (Halberda, Mazzocco, & Feigenson, 2008; Schneider et al., 2016), numerical skills are related to career (Parsons & Bynner, 2005) and health outcomes (Reyna & Brainerd, 2007), and play an important role in a nation's economic success (Hanushek, Woessmann, Jamison, & Jamison, 2008). In this sense, difficulties with numerical reasoning are attracting widespread research interest in order to find their underlying mechanisms.

The persistent difficulty to learn math with a neurobiological etiology is known as developmental dyscalculia (DD), a disorder that affects approximately 6% of age-school children (Shalev, Auerbach, Manor, & Gross-Tsur, 2000). Children with DD often struggle to solve addition, subtraction, and multiplication (Landerl, Bevan, & Butterworth, 2004), write numbers (Moura et al., 2013), and represent and manipulate numerosities (Pinheiro-Chagas et al., 2014; Rousselle & Noël, 2007).

Different criteria have been applied to identify DD. While some studies use a criterion of discrepancy between intelligence and math achievement scores, others use an arbitrary cut-off point in standardized math achievement tests, varying from the 5^{th} to the 30^{th} percentile (Mazzocco, 2007). In addition, different terms have been used to DD, as math learning disabilities (e.g. Mazzocco, Feigenson, & Halberda, 2011), mathematical disabilities (e.g. Geary, 1993), and mathematical difficulties (e.g. Jordan, Hanich, & Kaplan, 2003). In this paper, the term MD will be used to refer to low math achievement in general (Mazzocco, 2007).

The triple-code model (Dehaene, 1992) assumes that there are three main numerical representation codes: analog, auditory-verbal, and visual-Arabic. Number related tasks, independently of their complexity, are tied to a specific input and output code, so it is possible to transcode one representation into another (e.g. an analog representation into an Arabic one). The auditory-verbal and the visual-Arabic codes are symbolic and result from cultural artifacts. On the other hand, the analog code consists in a primitive non-symbolic representation of number, and is the foundation for symbolic representations.

The abstract representation of numerical magnitudes in an analog fashion relies on the approximate number system (ANS; Halberda & Feigenson, 2008), which allows nonsymbolic comparisons (Piazza et al., 2010), estimations (Mazzocco et al., 2011) and approximate calculations (Pinheiro-Chagas et al., 2014). An extensive literature demonstrates that the ANS is spatially organized along a mental number line that is logarithmically compressed, and oriented from left (smaller numbers) to right (greater magnitudes) (Galton, 1881; Dehaene, 2003; Hubbard, Piazza, Pinel, & Dehaene, 2005). One evidence of the mental number line is the distance effect (Moyer & Landauer, 1967): the closer two numbers are from each other, the harder it is to perform comparisons between them. For example, one may present higher reaction time (RT) and lower accuracy to compare 4 and 6 (distance 2) than 4 and 9 (distance 5).

The ability to distinguish different magnitudes also depends on the ratio between them. The ANS responds to the Weber's law (Dehaene, 2003), which states that the minimum noticeable change in a stimulus is a constant ratio of the original stimulus (Laming, 1986), and is indexed by the internal Weber fraction (Halberda et al., 2008). The internal Weber fraction assesses the variability in the representation of a specific magnitude, and a high Weber fraction indicates a poor ANS (see Piazza, Izard, Pinel, LeBihan, & Dehaene, 2004, and Halberda et al., 2008).

Piazza et al. (2010) investigated the ANS acuity of 10 years-old children with MD and typical achievement (TA), pre-school children (5 year-olds), and adults using a nonsymbolic magnitude comparison task, indexed by the Weber fraction. Results showed that the Weber fraction decreased significantly throughout development, from kindergarten to adulthood. The Weber fractions of children with MD were higher when compared to TA peers paired by age, but corresponded to the Weber fraction of 5 years-old children. In sum, children with MD presented a developmental delay in the ANS acuity.

Several studies corroborated the ANS impairments in children with MD, as assessed by nonsymbolic magnitude comparison (Costa et al., 2011; Mazzocco et al., 2011; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007), estimation (Mazzocco et al., 2011; Mejias, Mussolin, Rousselle, Gregoire, & Noël., 2011; Pinheiro-Chagas et al., 2014) and approximate calculation (Pinheiro-Chagas et al., 2014) tasks. Accordingly, some studies have suggested that the main cause of MD is a deficit in the ANS, which is referred as the core deficit hypothesis (Piazza et al., 2010; Pinheiro-Chagas et al., 2014). The core deficit hypothesis sustains that children with MD present a deficit in the analog representations of number, resulting in poor performance in measures of the ANS, and leading to deficits in the symbolic representations (Piazza et al., 2010). However, this hypothesis has been queried by the access deficit hypothesis (Noël & Rousselle, 2011; Rousselle and Noël, 2007), which proposes that children with MD present an intact ANS, but a poor symbolic magnitude processing resulting from a deficit in attaching symbolic magnitudes to their correspondent analog representations.

The symbolic magnitude comparison is the most used task to assess symbolic number processing. Several studies have corroborated that children with MD present impairments in single-digits comparison tasks (De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007), but these observations are inconclusive (Landerl, Fussenegger, Moll, & Willburguer, 2009; Starling-Alves et al., in preparation). Landerl et al.

(2009) failed to find differences between children with MD and TA in single-digit number comparisons, but a different outcome was observed in a two-digit number comparison task. Ashkenazi, Mark-Zigdon, and Henik (2009) also showed that the comparison between numbers composed by two digits is sensitive to deficits in MD. In the study of Ashkenazi et al. (2009), children with MD were presented with single and two-digit number comparison tasks. In the single-digit number comparison, MD and TA differed only in accuracy, but not in RT. Otherwise, in the two-digit number comparison, the distance effect was larger in MD in contrast to TA. The unit-decade compatibility effect was also investigated in this study, but there was no difference between groups.

The unit-decade compatibility effect (hereinafter compatibility effect) is a measure that underlies the automatic magnitude representation of two-digit numbers (Nuerk & Willmes, 2005; Nuerk, Weger, and Willmes, 2004), and occurs when the performance in a comparison task depends on the magnitude of units and decades. Nuerk, Weger, and Willmes (2001) showed that when two-digit numbers present greater decade and unit (compatible trial, e.g. 47 vs. 23, 4 is greater than 2, 7 is greater than 3) in a symbolic comparison task, lower RT and higher accuracy are observed. In contrast, numbers that present greater decade and smaller unit (incompatible trial, e.g. 43 vs. 27, 4 is greater than 2, but 3 is smaller than 7) lead to higher RT and lower accuracy. The processing of two-digit numbers is related to the knowledge of place-value rules (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011), since it is important to notice that the greater decade indicates the greater number.

The place-value of each component of a two-digit number is automatically processed in typical achievement (Kallai & Tzelgov, 2012), but children with MD present difficulties in this domain (Cirino et al., 2007; Hanich, Jordan, Kaplan, & Dick, 2001). The place-value knowledge is distinguished as important for writing Arabic numbers (Barrouillet, Bernardin, & Camos, 2004, Camos, 2008). Barrouillet et al. (2004) proposed a developmental, asemantic and procedural (ADAPT) model of writing Arabic numbers. This model assumes that transcoding a verbal number into an Arabic code is independent of a semantic representation, and relies on the application of procedural rules. For familiar numbers, the spoken language indicates the units, teens, decades, and the separators hundred and thousand, and this information is stored in the long-term memory. Thus, the procedures to write an Arabic number require the identification of the place-value of each component digit of a number. For example, for transcoding “twenty-four” into “24”, it is necessary to identify that “-four” indicates a unit associated to “4” and “twenty” indicates a decade associated to “2”, and then write the numbers in the right sequence. While a familiar number is directly retrieved from lexicon, unfamiliar numbers require more procedural rules and higher working memory capacity. Throughout development, two main factors contribute to better performance in transcoding: the familiarity of the number and the improvement of procedural abilities.

Moura et al. (2013) showed that children with MD presented lower achievement in number transcoding when compared to TA peers in early and middle elementary school. The type of transcoding errors, lexical or syntactic, committed by MD also differed from TA. Lexical errors occur when an element of a number is replaced (e.g. “thirty-two” -> “35”), whereas syntactic errors occur when the magnitude of the number is modified (e.g. “two hundred fifty-nine” -> “200509”) or the position of the digits is switched (e.g. “twenty-five” -> “52”). While children with TA presented syntactic errors only in the early elementary school, and achieved proficiency in writing numbers in middle elementary school, children with MD presented lexical and syntactic errors in early elementary school, and syntactic errors in the middle elementary school.

A growing body of literature has shown an association between numerical processing and math achievement (Bonny & Lourenco, 2013; Bugden, Price, McLean, & Ansari, 2012; Halberda et al., 2008; Libertus, Feigenson, & Halberda, 2011; Pinheiro-Chagas et al., 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012). In a recent meta-analysis, Schneider et al. (2016) showed that both nonsymbolic and symbolic magnitude processing are important for mathematical competence, throughout the lifespan.

The ANS is recognized as a start-up tool for the acquisition of symbolic number knowledge and mathematical proficiency (Piazza, 2010). Halberda et al. (2008), in a seminal study, showed that the Weber fraction of 14-years-old children was significantly correlated to past math achievement, back to kindergarten. The relationship between the ANS, assessed by a nonsymbolic comparison task, and scores in standardized mathematic tests (Inglis, Attridge, Batchelor, & Gilmore, 2011; Landerl & Kölle, 2009; Libertus, et al., 2011) and basic arithmetic operations (Lyons & Beilock, 2011; Mundy & Gilmore, 2009; Pinheiro-Chagas et al., 2014) has been corroborated.

The symbolic number processing is also distinguished as important for math achievement. Mundy and Gilmore (2009) showed a moderate correlation between accuracy and RT in a symbolic magnitude comparison task, with single-digit numbers, and calculation skills in the beginning of elementary school. This association has been supported by other studies (Bugden et al., 2012; Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Holloway & Ansari, 2009), even when a symbolic magnitude comparison task with two-digit number was used (Landerl & Köle, 2009; Moeller et al., 2011; Sasanguie et al., 2012). In a longitudinal study, Moeller et al. (2011) showed that performance in a two-digit number comparison task in first grade predicted math achievement in third grade. However, the relationship between two-digit processing and math achievement is still less explored than single-digit processing, especially in number writing skills.

The present study

There is still a considerable controversy surrounding the core deficit hypothesis and the access deficit hypothesis. The miscellaneous methods used in the literature makes it difficult to compare studies. Different outcomes are observed regarding the used index, which vary from accuracy and RT (Landerl et al., 2004; De Smedt & Gilmore, 2011; Kucian et al., 2006) to distance effect (Ashkenazi et al., 2009; Kovas et al., 2009; Rousselle & Noël, 2007) and also Weber fraction (Mazzocco et al., 2011; Piazza et al. 2010; Pinhero-Chagas et al., 2014; Starling-Alves et al., in preparation). Additionally, stimuli in nonsymbolic and symbolic magnitude comparison tasks cover different magnitude ranges. Usually, larger magnitudes are used in nonsymbolic comparisons (e.g. De Smedt & Gilmore; 2011; Piazza et al. 2010; Pinheiro-Chagas et al., 2014; Rousselle & Noël, 2007), in contrast to magnitudes from one to nine in the symbolic comparisons (e.g. De Smedt & Gilmore, 2011; Mussolin, Mejias, & Noël, 2010; Rousselle & Noël, 2007). Few studies have investigated the compatibility effect, which allows the investigation on the automatic processing of two-digit numbers, in MD. (Ashkenazi et al., 2009; Landerl, 2013; Landerl & Kovas, 2009; Landerl et al., 2009).

Regardless of the magnitude or index used, an association between both nonsymbolic and single-digit number processing and children's math achievement has been addressed by several studies (Bugden et al., 2012; Lyons & Beilock, 2011; Moeller et al., 2011; Pinheiro-Chagas et al., 2014; Sasanguie et al., 2012). However, the relationship between two-digit numbers and math achievement, especially numerical transcoding skills, has not been investigated in depth yet.

In this study, the nonsymbolic and the symbolic number processing in children with MD, and the association between these abilities with math achievement were investigated. Children with MD and TA were presented with nonsymbolic magnitude comparison and two-digit number comparison tasks, both with similar magnitude stimuli. Then, the relationship between these tasks and addition, subtraction, multiplication and numerical transcoding skills was examined.

Method

Participants

One hundred and twenty-eight children voluntarily participated in this experiment. Children were students from third to fifth grade of public schools of Belo Horizonte, Brazil. Intelligence, scholar achievement, and numerical transcoding abilities were assessed in groups. Children with spelling difficulties (percentile < 25^{th} in a spelling task) or low intelligence (percentile < 10^{th} in the Raven's Coloured Progressive Matrices) did not take part in the posterior phases of the study (n = 41). Thus, 87 children with TA and MD were selected to complete an individual assessment, comprising tasks of math achievement and magnitude comparison tasks. Twelve children were excluded from analysis because they presented either a poor R^2 (< .20) in the fitting procedure to calculate the Weber fraction, or presented a value of Weber fraction that exceeded the discriminability of the task (Weber fraction > .60).

The final sample was composed by 55 children in the TA group (30 male, 25 female, age range: 8-11 years , M = 9.07, SD = .57), and 20 in the MD group (11 female, 9 male, age range: 8-10 years, M = 9.05, SD = .51). Groups were matched in age, F (1,73) = .02, p = .88, η^2 = .001, sex, X^2 = .53, p = .32, φ = .08, and intelligence, F (1,73) = 2.01, p =.16, η^2 = .027. The descriptive data is presented in table 3.1.

Table 3.1: Descriptive data of the individual assessment sample.

	TA (n=55)		MD (n=20)		χ^2	df	p	φ
Sex (% female)	45.45		55.00		.53	1	.32	.08
	Mean	**SD**	**Mean**	**SD**	F	df	p	η^2
Age (years)	9.07	.57	9.05	.51	.02	1;73	.88	.001
Raven (Z score)	1.15	.08	.92	.14	2.01	1;73	.16	.027

Note. TA = typically achieving children; MD = children with mathematics difficulties; Z score (mean = 0, SD = 1).

Materials

The materials used in this study were the same of Starling-Alves et al. (in preparation; see page 5 of this thesis). However, different versions of Nonsymbolic Magnitude Comparison task and Symbolic Magnitude Comparison task were applied. Moreover, an Arabic Number Transcoding Task was included.

Arabic Number Transcoding Task. To evaluate number transcoding skills, children were instructed to write the Arabic forms of dictated numbers. This task consists of 81 items, up to four digits (2 one-digit numbers, 6 two-digit numbers, 19 three-digit numbers, and 54 four-digit numbers). The numbers were selected based on their transcoding complexity, defined by the number of transcoding rules (algorithms) necessary to convert between notations, according to the ADAPT model (Barrouillet et al., 2004; Camos, 2008). The items covered different complexity levels, from more simple numbers (2 rules) to more complex ones (7 rules). As shown in previous investigations, this task presents high internal consistency (*KR20* = 0.96; Moura et al., 2013, 2015; Lopes-Silva et al., 2014). The total score in this task was given by a z-score calculated for each child based on scores distribution of the total sample. In addition, an error frequency score was calculated by dividing the total syntactic and lexical errors of children by the sample size of their groups.

Nonsymbolic Magnitude Comparison. This version of the task (Figure 3.1) was similar to the one used in Starling-Alves et al. (in preparation; see page 5 of this thesis). However, there were two reference numerosities (see Piazza et al., 2010): 32 dots contrasted to 20, 24, 28, 30, 34, 36, 38, 40, or 44 dots, and 16 dots contrasted to 10, 12, 13, 14, 15, 17, 18, 19, 20, or 22 dots. For analysis of accuracy and RT, distances were grouped in short, comprising distances 1, 2, and 3, and large, distances 4, 6, 8, and 12. Each numerosity was presented 8 times, each time in a different configuration, in a total of 80 trials. Data were trimmed in two interactive steps for each child to exclude responses ± 3 *SD* from the mean RT. As a measure of the ANS acuity, the internal Weber fraction calculation was based on the Log-Gaussian model of number representation (Dehaene, 2007), with the methods described by Piazza et al. (2004), combining the error rates for ratios with reference 16 and 32.

Two-digit Number Comparison. In this symbolic magnitude comparison task (Figure 3.2), children were instructed to compare two-digit Arabic numbers presented on the computer screen and decide which was the greater (see Nuerk et al., 2001). The numbers were presented on the screen printed in white over a black background. The numerosities ranged from 21 to 89, and the distance between decades, the distance between units, and the decade-unit compatibility were controlled. Large conditions were defined when the distance between decades or between units ranged from 4 to 8 (e.g. 21 vs. 67, 34 vs. 81, 38 vs. 91, 81 vs. 39), whereas short decade conditions occurred when the distance between decades or between units ranged from 1 to 3 (e.g. 43 vs. 59, 29 vs. 47, 54 vs. 37). In the congruent conditions, the greater number presented greater decade and unit (e.g. 59 vs. 43), and in the incongruent conditions, the greater number presented greater decade but smaller unit (e.g. 81 vs. 34).

The task comprised two blocks of 56 trials each. Before the testing trial, a fixation point – a cross, printed in white – was presented on the screen for 1000ms. The task trial consisted of the presentation of two numbers, shown on the screen for 5000ms, and followed by a mask, presented for 500ms. The intertrial interval was also 500ms. Children were instructed to press the “H” key on the keyboard if the greater number was presented at the top of the computer screen, and “B” if the greater number was presented at the bottom. Decade distance, unit distance, and compatibility conditions presentation in the top or the bottom of the screen was counterbalanced. Data were trimmed in two interactive steps for each child to exclude responses ± 3 *SD* from the mean reaction time.

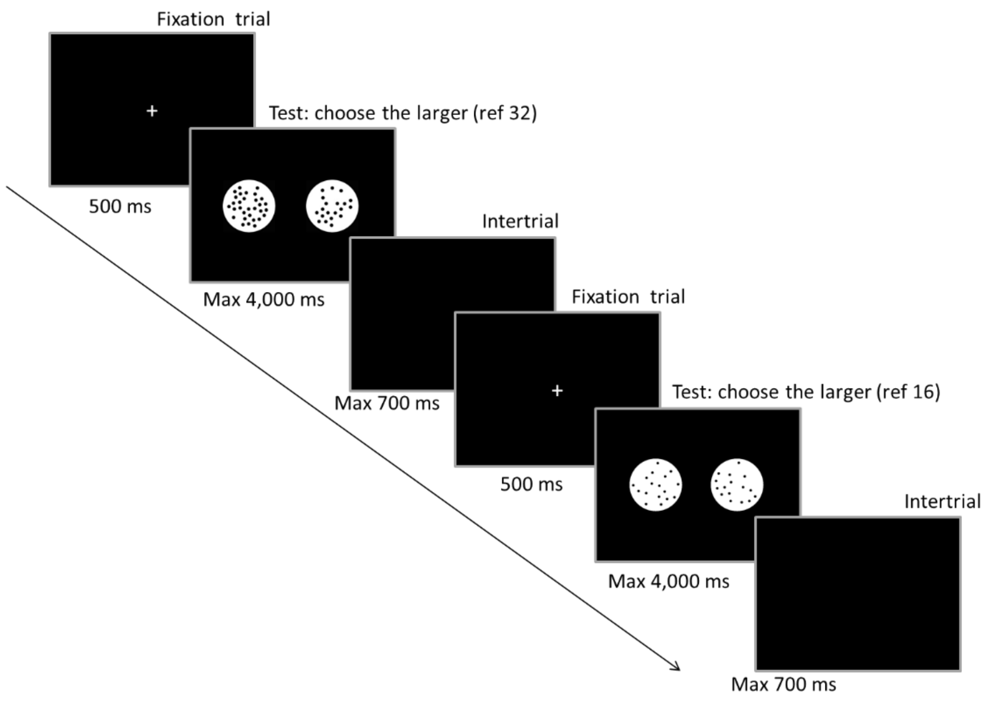

Figure 3.1. Illustration of the nonsymbolic comparison task, with arrows indicating the time curse of the task: fixation trial, experimental trial, and intertrial interval.

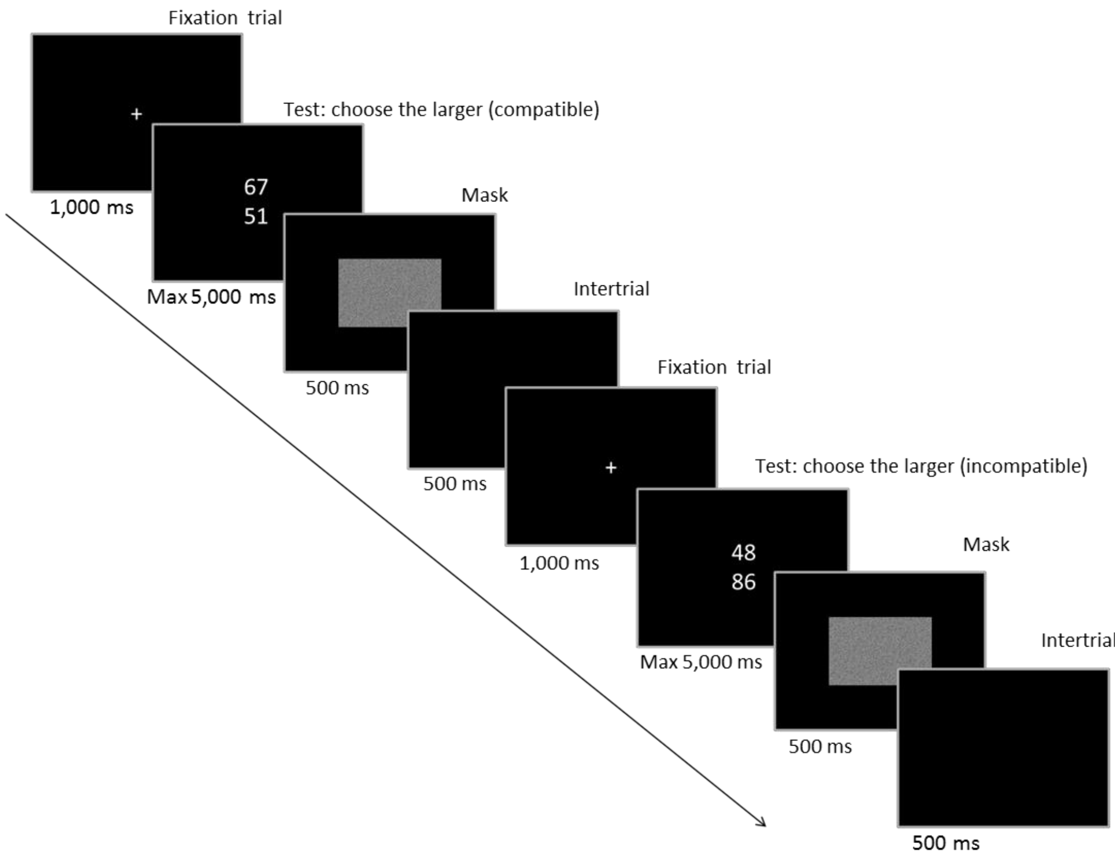

Figure 3.2. Illustration of the symbolic comparison task, with arrows indicating the time curse of the task: fixation trial, experimental trial, mask, and intertrial interval.

Procedures

The procedures were the same as in Starling-Alves et al. (in preparation; see page 5 of this thesis).

Results

Nonsymbolic magnitude comparison

First, 2x2 mixed-ANOVAs, with distance (2 levels: large and short) as within-subjects factor, and groups (2 levels: TA and MD) as between-subjects were run for accuracy and RT. For RT (Figure 3.3 - A), there was a main effect of distance, F (1,73) = 74.03, p <.001, η^2 = .507, with higher RT for short distances (M = .83, SD = .01) when compared to large ones (M = .68, *SD* = .01). There was no main effect of groups, *F* (1,73) = .37, *p* = .54, η^2 = .005, and no interaction between factors, *F*(1,73) = 3.20, *p* = .08, η^2 = .043.

For accuracy (Figure 3.3 - B), there was a main effect of distance, *F* (1,73) = 213.09, *p* <.001, η^2 = .745, with higher accuracy for large (*M* = .83, *SD* = .01) distances than short (*M* = .68, *SD* = .01) ones. There was also a main effect of groups, *F* (1,73) = 13.15, *p* <.01, η^2 = .153, with higher accuracy for TA (*M* = .80, *SD* = .01) than MD (*M* = .75, *SD* = .01). There was no significant interaction between factors, *F* (1,73) = .01, *p* = .98, η^2 = .00.

The Weber fraction was also indexed. Children with MD presented higher values of Weber fraction (*M* = 0.25, *SD* = 0.06) when compared to TA (*M* = 0.20, *SD* = 0.06), *F* (1,73) = 8.67, *p*<.01, η^2 = .106 (Figure 3.4).

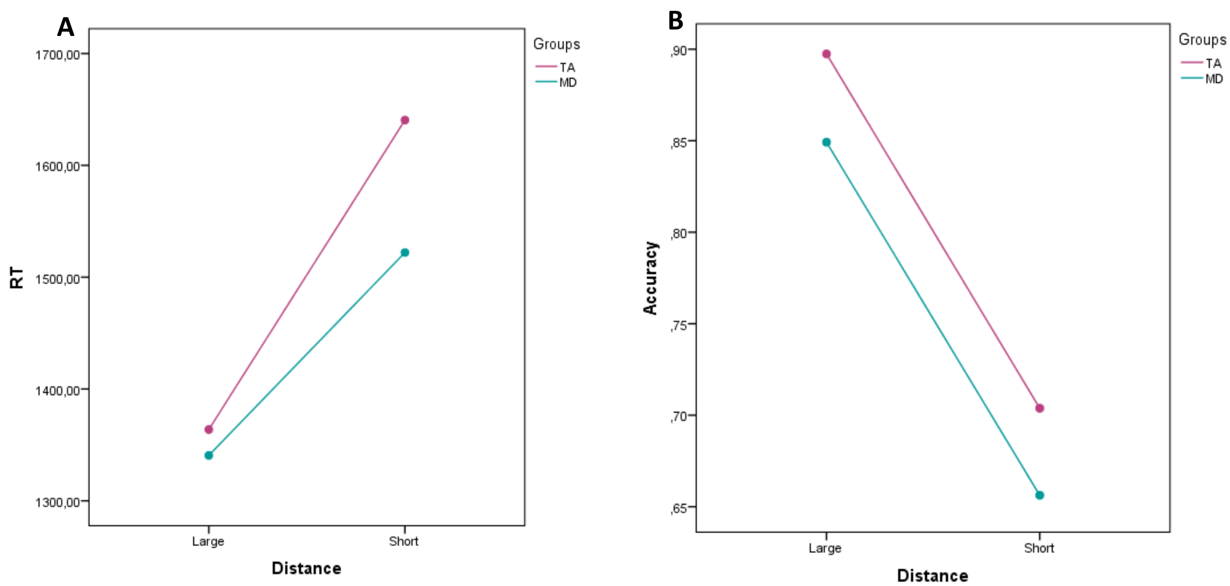

Figure 3.3. Distance effect in nonsymbolic comparison task. *a*. Reaction time (RT) – there was a significant Distance Effect, with significant difference between large and short distances. There was no difference between MD (mathematical difficulties) and TA (typical achievement), and no interaction between factors. *b*. Accuracy – As in RT, the distance effect was significant, with higher accuracy for large than short distance. Children with MD presented lower accuracy when compared to TA. There was no interaction between factors.

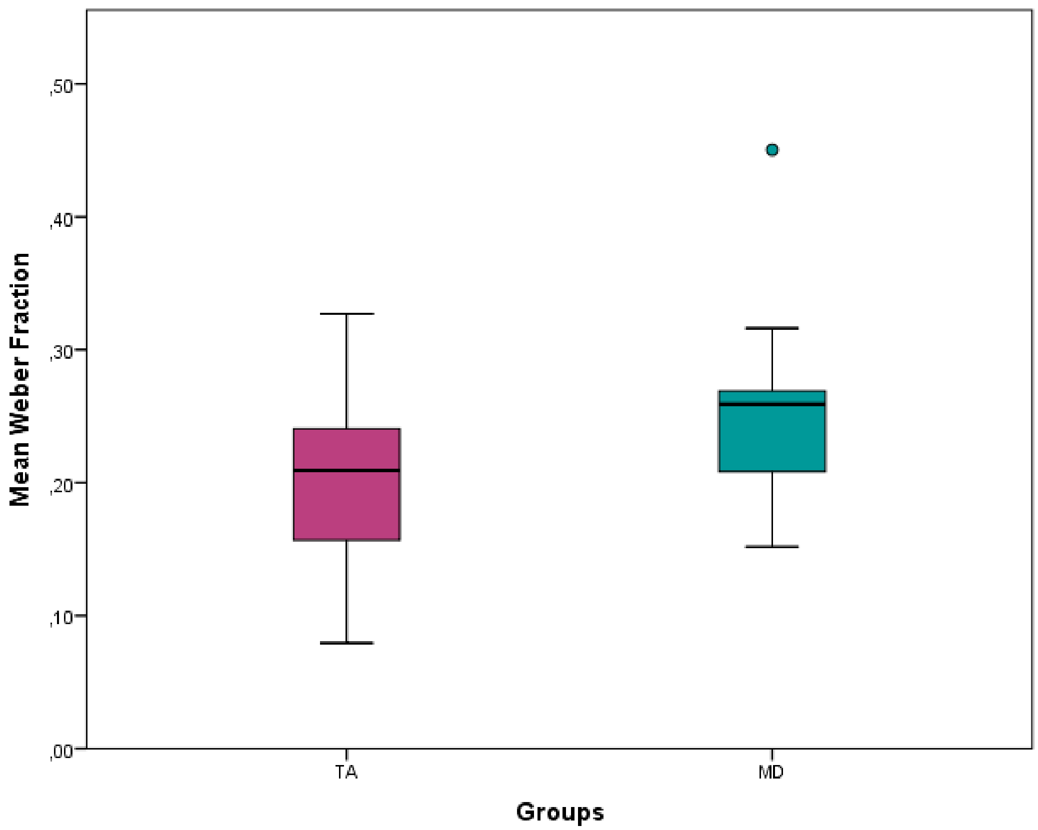

Figure 3.4. Weber fraction indexed by the nonsymbolic comparison task. Children with MD (mathematical difficulties) presented a higher nonsymbolic Weber fraction than TA (typical achievement) peers.

Two-digit number comparison

In the two-digit number comparison task, 2 x 2 mixed-ANOVAs were conducted for RT and accuracy, with compatibility (2 levels: compatible and incompatible) as within-subjects, and group (2 levels: MD and TA) as between-subjects factor. For RT (Figure 3.5 - A), there was a significant main effect of group, F (1,73) = 4.55, p <.05, η^2 = .059, with higher RT for MD group (M = 1403.66, SD = 72.53) when compared to TA (M = 1223.06, SD = 43.74). There was no main effect of compatibility, F (1,43) = 3.61, p = 0.62, η^2 = .047, and no significant interaction between factors, F (1,73) = 2.12, p = .15, η^2 = .028.

Similar results were found for accuracy (Figure 3.5 - B). There was a significant main effect for group, F(1,73) = 19.72, p <.001, η^2 = .213, with higher accuracy for TA children (M =

53.41, *SD* = .26) when compared to MD (*M* = 51.15, *SD* = .44). There was no significant main effect of compatibility, $F(1,73) = 1.12$, $p = .29$, $\eta^2 = .015$, or interaction, $F(1,73) = .47$, $p = .49$, $\eta^2 = .006$.

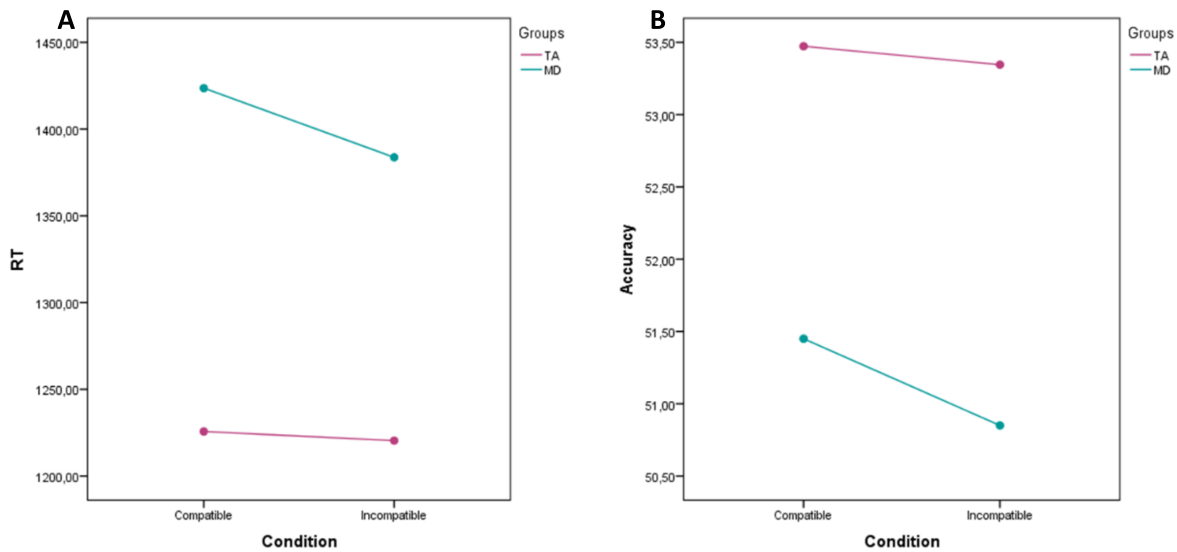

Figure 3.5. Compatibility effect. *a.* Reaction time (RT) – there was a significant difference between MD (mathematical difficulties) and TA (typical achievement), but no difference between compatible and incompatible trials and no interaction between factors. *b.* Accuracy – As in RT, there was a significant difference between groups, but no compatibility effect or interaction between factors.

Number processing and math achievement

Groups were compared regarding their performance in Basic Arithmetic Operations and Numerical transcoding. Then, Pearson correlation between math achievement and nonsymbolic and symbolic processing were run. The Weber fraction was used as an index of nonsymbolic processing, and the total accuracy and mean RT were used for symbolic processing, since there was no difference between compatible and incompatible conditions.

A significant difference between groups was found in addition, $F(1,73) = 4.97$, $p < .05$, $\eta^2 = .064$, with higher accuracy for TA (*M* = 20.41, *SD* = 5.62) than MD (*M* = 17.05, *SD* = 6.20), and subtraction, $F(1,73) = 5.83$, $p < .05$, $\eta^2 = .074$, with higher accuracy for TA group (*M* =

14.58, *SD* = 6.28) than MD (*M* = 10.75, *SD* = 5.43) as well. Finally, children with MD also presented difficulties in multiplication (*M* = 6.20, *SD* = 5.62), when compared to TA peers (*M* = 10.85, *SD* = 7.09), F (1, 73) = 6.99, $p < .05$, $\eta^2 = .087$.

The Weber fraction presented significant correlation only with multiplication, $r = -.27$, $p <.05$. A significant correlation was also observed between mean accuracy, in symbolic comparison task, and addition, $r = .27$, $p <.05$, and subtraction scores, $r = .24$, $p <.05$. The mean RT in symbolic comparison task correlated with scores in addition, $r = -.37$, $p <.001$, subtraction, $r = -.40$, $p <.001$, and multiplication, $r = -.27$, $p <.05$.

For numerical transcoding, MD group presented lower *z*-scores (*M* = -.60, *SD* = .20) when compared to TA (*M* = .26, *SD* = .12), $F(1,73) = 13.56$, $p < .001$, $\eta^2 = .157$. There was a significant correlation between transcoding *z*-scores and mean accuracy, $r = .38$, $p <.01$, and RT in symbolic comparison, $r = -.54$, $p <.001$, and Weber fraction, $r = -.25$, $p <.05$.The frequency of syntactic and lexical errors was also investigated. A score for errors type was calculated by dividing the total frequency of syntactic and lexical errors for each child by the correspondent group size. This analysis minimize cofounds originated from unequal sample sizes.

A 2x2 mixed ANOVA (Figure 3.6) was calculated with error type as within-subjects (2 levels: syntactic and lexical) and groups as between-subjects factor (2 levels: MD and TA). There was a main-effect of error type, $F(1,73) = 14.31$, $p < .001$, $\eta^2 = .164$, with higher syntactic errors (*M* = .56, *SD* = .12) than lexical ones (*M* = .10, *SD* = .08), and a main-effect of group, $F(1,73) = 19.43$, $p < .001$, $\eta^2 = .210$, with higher errors frequency for MD (*M* = .59, *SD* = .10) than TA group (*M* = .06, *SD* = .06). There was a significant interaction between factors, $F(1,73) = 14.31$, $p < .001$, $\eta^2 = .164$, indicating that children with MD presented more syntactic errors (*M*

= 1.01, *SD* = .20) than lexical ones (*M* = .17, *SD* = .03), and children with TA presented similar frequency of syntactic (*M* = .08, *SD* = .12) and lexical errors (*M* = .03, *SD* = .02).

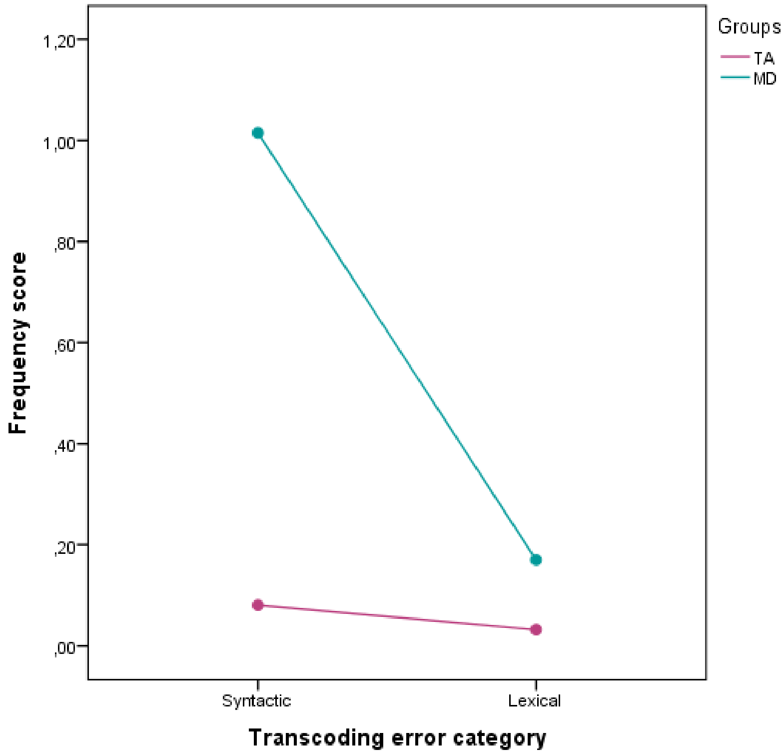

Figure 3.6. Error frequency in the numerical transcoding task analysis. There was significant difference between error categories, and between children with MD (mathematical difficulties) and TA (typical achievement). A significant interaction was observed between factors.

There was a significant correlation between syntactic errors and accuracy, $r = -.38$, $p < .01$, and RT, $r = .58$, $p < .001$, in the two-digit comparison task. No significant correlation was observed between syntactic errors and Weber fraction, $r = .21$, $p = .07$, and between lexical errors and accuracy, $r = -.20$, $p = .08$, and RT, $r = .12$, $p < .30$ in two-digit number comparison task, and Weber fraction, $r = .22$, $p = .07$.

Discussion

In the present study, the numerical processing skills of children with MD were investigated. Children with MD presented impairments in the ANS acuity, as assessed by a nonsymbolic comparison task, and in the symbolic number processing, as assessed by a two-digit number comparison task. In this task, the compatibility effect was not observed either in MD or TA groups. The relationship between numerical processing skills and math achievement was also examined. There was a significant association between the nonsymbolic processing and numerical transcoding and multiplication scores, and between symbolic processing and numerical transcoding, addition, subtraction, and multiplication.

In the nonsymbolic comparison task, RT, accuracy, distance effect and Weber fraction were calculated. A significant distance effect was observed for both RT and accuracy. The distance effect occurs when it is more difficult to distinguish between numerically close numbers (i.g. 32 and 36, distance 4) than between numbers that are far away from each other in a mental number line (i.g. 32 and 44, distance 12) (Dehaene, 2003; Moyer & Landauer, 1967), and has been replicated by different studies (Castronovo & Göbel, 2012; Holloway & Ansari, 2009; Landerl & Köle, 2009). Although no difference between groups was observed in RT, children with MD presented lower accuracy and higher Weber fraction when compared to TA peers. The Weber fraction is distinguished as a sensible measure of the ANS acuity (Anobile et al., 2016; Piazza et al., 2010; Schneider et al., 2016), which has been assumed as impaired in children with MD (Costa et al., 2011; Mazzocco et al., 2011; Pinheiro-Chagas et al., 2014; Olsson et al., 2016).

Children with MD also presented impairments in the symbolic processing. The symbolic processing is usually assessed by single-digit number comparison tasks (De Smedt & Gilmore, 2011; Holloway & Ansari, 2009; Iuculano et al., 2008; Landerl et al., 2004; Mundy & Gilmore, 2009; Sasanguie et al., 2012; Vanbinst, Ansari, Ceulemans, Ghesquière, & De Smedt, 2015), but some studies show that a two-digit number comparison task might be more sensitive to deficits in MD (Landerl & Kölle, 2009; Landerl et al., 2009; Ashkenazi et al., 2009). According to

Skagerlund and Träff (2014), it is possible that a greater exposure of children to single-digit numbers in school results in an efficient processing of small symbolic magnitudes. In this study, children with MD presented higher RT and lower accuracy in a two-digit number comparison task when compared to TA children.

These results are in line with the core deficit hypothesis. The core deficit hypothesis proposes that a deficit in analog abstract representations of number - the ANS - is the main cause of MD (Piazza et al., 2010; Mazzocco et al., 2011; Pinheiro-Chagas et al. 2014). As the symbolic number representations are built upon nonsymbolic ones (Dehaene, 1992; Feigenson, Dehaene, & Spelke, 2004), a poor ANS might lead to deficits in symbolic magnitudes processing (Piazza et al., 2010; Skagerlund & Träff, 2014). In opposition to the core deficit hypothesis, the access deficit hypothesis suggests that children with MD present an intact ANS, but an impairment in linking symbolic numbers to their analog representations, which explain deficits in symbolic number processing (Rousselle & Noël, 2007; Noël & Rousselle, 2011). The core deficit hypothesis was corroborated since children with MD presented low ANS acuity and impairments in two-digit number processing.

The two-digit number processing can be investigated by the compatibility effect, which accounts for the automatic activation of numerical magnitudes, and occurs when higher RT and lower accuracy are observed in incompatible two-digit number comparisons, in contrast to lower RT and higher accuracy in compatible items (Nuerk & Willmes, 2005). In this study, no compatibility effect was observed in TA and MD groups. The compatibility effect has been underexplored in children with MD, and the literature present inconclusive findings (Ashkenazi, et al., 2009; Landerl, 2013; Landerl & Köle, 2009; Landerl et al., 2009). Landerl and Köle (2009) investigated the compatibility effect in children with MD cross-sectionally. They were the first to show the compatibility effect in children with MD, and evinced that this group process two-digit numbers in a decomposed fashion. Differently, Ashkenazi, et al. (2009) found no significant compatibility effect in children with MD. These authors proposed that the compatibility effect in children might be influenced by the distance between decades and between units, such that sets with small decade distance and large unit distance rise stronger compatibility effect. According to Nuerk and Willmes (2005), when distance effect in decades interacts with the distance effect in units, the compatibility effect might be null or even reverse.

The absence of the compatibility effect in children is indicative of poor ability to process two-digit numbers automatically (Ashkenazi, et al., 2009; Mussolin & Noël, 2008). Difficulties in the automatic processing of numbers is one of the cognitive deficits associated with MD. In a case-study, our research group showed that a child with impaired ANS presented difficulties in automatic processing of numbers, e.g. in comparing nonsymbolic magnitudes and performing multiplication tasks, but not in domains that required controlled processing (Haase et al., 2014), e.g. performing subtraction tasks.

In the present study, the basic arithmetic operations and numerical transcoding skills of children with MD were also investigated. Children with MD presented lower scores in addition, subtraction, multiplication and numerical transcoding tasks when compared to TA peers, in line with previous studies (Landerl et al., 2004; Moura et al., 2013). An association between numerical processing and math achievement has been widely shown in the literature (Chen & Li, 2014; Schneider et al., 2016), and there is evidence that the nonsymbolic magnitude processing is associated with basic addition, subtraction, and multiplication (Lyons and Beilock, 2011; Mundy & Gilmore, 2009; Pinheiro-Chagas et al., 2014). Surprisingly, the Weber fraction was only correlated to scores in numerical transcoding and multiplication. Lopes-Silva et al. (2014)

showed a significant association between transcoding scores and the Weber fraction. However, this association was weak and did not remain after general cognitive domains, as intelligence and working memory, were controlled.

In Brazil, children in 3rd and 4th grades are learning simple and complex multiplication. One possibility is that nonsymbolic representations of number are more recruited during the acquisition of new mathematical knowledge, in comparison to consolidated math skills, solved by retrieval (Ashcraft, 1992; Bonny & Lourenco, 2013). McCrink and Spelke (2010) showed that 5 to 7 years-old children, not formally presented to multiplication, could understand nonsymbolic multiplicative patterns relying on the ANS. However, studies with adults suggest that multiplication is related to retrieval and to the symbolic processing of number (Lemer, Dehaene, Spelke, & Cohen, 2003; Park & Brannon, 2013; Vanbinst et al., 2015). A developmental shift in the strategies to solve multiplication and the role of the ANS in this process are still to be clarified by longitudinal studies.

Additionally, the symbolic processing was significantly associated with addition, subtraction, multiplication and numerical transcoding. The RT in the two-digit number comparison task was moderately correlated to addition, subtraction, multiplication and numerical transcoding scores, and the accuracy was correlated to addition, subtraction and numerical transcoding. These results corroborate previous findings (Landerl & Köle, 2009; Moeller et al., 2011; Sasanguie et al., 2012; Schneider et al., 2016) that show that the symbolic number processing is relevant for math achievement. In a meta-analysis, Schneider et al. (2016) punctuated that the symbolic number processing present stronger association to math achievement than the nonsymbolic number processing. However, Price and Fuchs (2016) showed that the symbolic number processing mediates the relationship between the abstract analog representation of number and math achievement. Accordingly, the ANS may influence math skills by allowing the development of symbolic numerical representations (Dehaene, 1992).

Finally, the error types in numerical transcoding were taken into account. In accordance with previous findings (Moura et al., 2013), children with MD presented higher frequencies of syntactic errors than lexical ones, whereas children with TA presented similar low frequency of syntactic and lexical errors. In addition, poor representation of two-digit number – lower accuracy and higher RT – was associated to higher frequency of syntactic errors. The two-digit number comparison task indicates the automatic processing of numbers and the knowledge of the place-value rules (Moeller et al., 2011), which are important for transcoding procedures. To properly transcode a number, the place-value of each component of the number needs to be identified, associated to a digit and written in the correct sequence (Barrouillet et al., 2004). Syntactic errors occur when the magnitude of the number is modified or the position of the digits within a number is switched, and are indicative of poor acquisition of transcoding procedures rules (Barrouillet et al., 2004, Camos, 2008; Moura et al., 2013), which might be the case of children with MD.

In sum, children with MD presented poor ANS acuity and a poor performance in a two-digit number comparison task, corroborating the core deficit hypothesis. The compatibility effect was not observed, suggesting that elementary school children cannot fully process numbers automatically. Children with MD also presented difficulties in addition, subtraction, multiplication, and numerical transcoding tasks. Both the symbolic and the nonsymbolic comparison tasks were associated with math achievement. Finally, difficulties in two-digit number processing were associated with high frequencies of syntactic errors in numerical transcoding, suggesting that the automatic processing of number and place-value skills are important to the knowledge of transcoding rules.

To the best of our knowledge, this was the first study to investigate the nonsymbolic and symbolic magnitude processing in children with MD, presenting both the Weber fraction as an index of ANS acuity, and the unit-digit compatibility effect. In addition, no previous studies had investigated the relationship between two-digit number processing and numerical transcoding errors frequency. Further studies should clarify the two-digit number processing in MD and its relationship with arithmetic and numerical transcoding, especially with longitudinal designs.

References

Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental psychology*, 52(8), 1206. doi: 10.1037/dev0000155

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, *44*(1), 75-106. doi: 10.1016/0010-0277(92)90051-I

Ashkenazi, S., Mark-Zigdon, N., & Henik, A. (2009). Numerical distance effect in developmental dyscalculia. *Cognitive Development*, 24(4), 387-400. doi:10.1016/j.cogdev.2009.09.006

Barrouillet, P., Bernardin, S., & Camos, V. (2004). Time constraints and resource sharing in adults' working memory spans. *Journal of Experimental Psychology: General*, 133(1), 83. doi:10.1037/0096-3445.133.1.83

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of experimental child psychology*, 114(3), 375-388. doi: 10.1016/j.jecp.2012.09.015

Bugden, S., Price, G. R., McLean, D. A., & Ansari, D. (2012). The role of the left intraparietal sulcus in the relationship between symbolic number processing and children's arithmetic competence. *Developmental Cognitive Neuroscience*, 2(4), 448-457. doi: 10.1016/j.dcn.2012.04.001

Camos, V. (2008). Low working memory capacity impedes both efficiency and learning of number transcoding in children. *Journal of experimental child psychology*, 99(1), 37-57. doi: 10.1016/j.jecp.2007.06.006

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS ONE*, 7(4), e33832. doi: 10.1371/journal.pone.0033832

Chen, Q. & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, 148, 163-172. doi:10.1016/j.actpsy.2014.01.016

Costa, A. J., Silva, J. B. L., Chagas, P. P., Krinzinger, H., Lonneman, J., Willmes,K., Wood, G., & Haase, V.G. (2011). A hand full of numbers: a role for offloading in arithmetics learning? *Frontiers in Psychology*. doi: 10.3389/fpsyg.2011.00368

Cirino, P. T., Fletcher, J. M., Ewing-Cobbs, L., Barnes, M. A., & Fuchs, L. S. (2007). Cognitive arithmetic differences in learning difficulty groups and the role of behavioral inattention. *Learning Disabilities Research & Practice*, 22(1), 25-35. doi: 10.1111/j.1540-5826.2007.00228.x

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44(1), 1-42. doi: 10.1016/0010-0277(92)90049-N

Dehaene, S. (2003). The neural basis of the Weber–Fechner law: a logarithmic mental number line. *Trends in cognitive sciences*, 7(4), 145-147. doi:10.1016/S1364-6613(03)00055-X

Dehaene, S. (2007). Symbols and quantities in parietal cortex: elements of a mathematical theory of number representation and manipulation. In P. Haggard, Y. Rossetti, & M. Kawato (Eds.), *Sensorimotor Foundations of Higher Cognition: Attention and Performance* (pp. 527-574). Cambridge (UK): Oxford University Press.

De Smedt, B. & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of experimental child psychology*, 108(2), 278-92. doi: 10.1016/j.jecp.2010.09.003

Desoete, A., Ceulemans, A., De Weerdt, F., & Pieters, S. (2012). Can we predict mathematical learning disabilities from symbolic and non-symbolic comparison tasks in kindergarten? Findings from a longitudinal study. *British Journal of Educational Psychology*, 82(1), 64-81. doi:10.1348/2044-8279.002002

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8, 307-314. doi:10.1016/j.tics.2004.05.002

Galton, F. (1881). Visualised numerals. *The Journal of the Anthropological Institute of Great Britain and Ireland*, 10, 85-102.

Geary D. C. (1993). Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychology Bulletin*, 114(2), 345-62. doi: 10.1037//0033-2909.114.2.345

Haase, V. G., Júlio-Costa, A., Lopes-Silva, J. B., Starling-Alves, I., Antunes, A. M., Pinheiro-Chagas, P., & Wood, G. (2014). Contributions from specific and general factors to unique deficits: two cases of mathematics learning difficulties. *Frontiers in Psychology*. doi:10.3389/fpsyg.2014.00102

Halberda, J. & Feigenson, L. (2008). Developmental change in the acuity of the "Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental psychology*, 44(5), 1457-1465. doi: 10.1037/a0012682

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, 455, 665-668. doi:10.1038/nature07246

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, 93(3), 615. doi: 10.1037/0022-0663.93.3.615

Hanushek, E. A., Woessmann, L., Jamison, E. A., & Jamison, D. T. (2008). Education and economic growth. *Education Next*, 8(2).

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of experimental child psychology*, 103(1), 17-29. doi:10.1016/j.jecp.2008.04.001

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience*, 6(6), 435-448. doi:10.1038/nrn1684

Inglis, M., Attridge, N., Batchelor, S., & Gilmore, C. (2011). Non-verbal number acuity correlates with symbolic mathematics achievement: but only in children. *Psychonomic Bulletin & Review*, 18(6), 1222-1229. doi: 10.3758/s13423-011-0154-1

Iuculano, T., Tang, J., Hall, C. W. B., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science*, 11(5), 669-680. doi: 10.1111/j.1467-7687.2008.00716.x

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child development*, 74(3), 834-850. doi:10.1111/1467-8624.00571

Kallai, A. Y., & Tzelgov, J. (2012). The place-value of a digit in multi-digit numbers is processed automatically. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 38(5), 1221. doi: 10.1037/a0027635

Kovas, Y., Giampietro, V., Viding, E., Ng, V., Brammer, M., Barker, G. J., Happe, F. G. E., & Plomin, R. (2009). Brain correlates of non-symbolic numerosity estimation in low and high mathematical ability children. *PLoS ONE*, 4(2), e4587. doi:10.1371/journal.pone.0004587

Kucian, K., Loenneker, T., Dietrich, T., Dosch, M., Martin, E., & von Aster, M. (2006). Impaired neural networks for approximate calculation in dyscalculic children: a functional MRI study. *Behavioral and Brain Functions*, 2(31). doi: 10.1186/1744-9081-2-31

Landerl, K. (2013). Development of numerical processing in children with typical and dyscalculic arithmetic skills—a longitudinal study. *Frontiers in Psychology*. doi:10.3389/fpsyg.2013.00459

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: a study of 8-9-year-old students. *Cognition*, 93(2), 99-125. doi:10.1016/j.cognition.2003.11.004

Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *Journal of experimental child psychology*, 103(3), 309-24. doi:10.1016/j.jecp.2009.03.006

Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of experimental child psychology*, 103(4), doi:546-65. 10.1016/j.jecp.2008.12.006

Laming, D. (1986). Weber's law. *Sensory analysis*. *Academic Press, London*, 1-17.

Lemer, C., Dehaene, S., Spelke, E., & Cohen, L. (2003). Approximate quantities and exact number words: Dissociable systems. *Neuropsychologia*, *41*(14), 1942-1958. doi:10.1016/S0028-3932(03)00123-4

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental science*, 14(6), 1292-1300. doi: 10.1111/j.1467-7687.2011.01080.x

Lopes-Silva, J. B., Moura, R., Júlio-Costa, A., Haase, V. G., & Wood, G. (2014). Phonemic awareness as a pathway to number transcoding. doi: 10.3389/fpsyg.2014.00013

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, 121(2), 256-261. doi: 10.1016/j.cognition.2011.07.009

Mazzocco, M. M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch, & M. M. M. Mazzocco (Eds), Why Is Math So Hard for Some Children? The Nature and Origins of Mathematical Learning Difficulties and Disabilities (pp. 29-47). Baltimore: Brookes.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia). *Child Development*, 82(4), 1224-1237. doi: 10.1111/j.1467-8624.2011.01608.x

McCrink, K., & Spelke, E. S. (2010). Core multiplication in childhood. *Cognition*, 116(2), 204-216. doi:10.1016/j.cognition.2010.05.003

Mejias, S., Mussolin, C., Rousselle, L., Gregoire, J., & Noel, M. P. (2011). Numerical and nonnumerical estimation in children with and without mathematical learning disabilities. *Child Neuropsychology*, 18(6), 550-575. doi: 10.1080/09297049.2011.625355

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance—A longitudinal study on numerical development. *Research in developmental disabilities*, 32(5), 1837-1851. doi:10.1016/j.ridd.2011.03.012

Moura, R., Lopes-Silva, J. B., Vieira, L. R., Paiva, G. M., de Almeida Prado, A. C., Wood, G., & Haase, V. G. (2015). From "Five" to 5 for 5 minutes: arabic number transcoding as a short, specific, and sensitive screening tool for mathematics learning difficulties. *Archives of Clinical Neuropsychology*, 30(1), 88-98. doi:10.1093/arclin/acu071

Moura, R., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: the role of working memory and procedural and lexical competencies. *Journal of experimental child psychology*, 116(3), 707-727. doi: 10.1016/j.jecp.2013.07.008

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature*, 215, 1519-1520. doi:10.1038/2151519a0

Mundy, E. & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology*, 103, 490-502. doi:10.1016/j.jecp.2009.02.003

Mussolin, C., Mejias, S., & Noël, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition*, 115(1), 10-25. doi:10.1016/j.jecp.2009.02.003

Mussolin, C., & Noël, M. P. (2008). Automaticity for numerical magnitude of two-digit Arabic numbers in children. *Acta psychologica*, 129(2), 264-272. doi: 10.1016/j.actpsy.2008.08.001

Noël, M. P. & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: an explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience*. doi: 10.3389/fnhum.2011.00165

Nuerk, H. C., Weger, U., & Willmes, K. (2001). Decade breaks in the mental number line? Putting the tens and units back in different bins. *Cognition*, 82(1), B25-B33. doi: 10.1016/S0010-0277(01)00142-1

Nuerk, H. C., Weger, U., & Willmes, K. (2004). On the perceptual generality of the unit-decade compatibility effect. *Experimental Psychology*, 51(1), 72-79. doi:10.1027/1618-3169.51.1.72

Nuerk, H. C., & Willmes, K. (2005). On the magnitude representations of two-digit numbers. *Psychology Science*, 47(1), 52-72.

Olsson, L., Östergren, R., & Träff, U. (2016). Developmental dyscalculia: A deficit in the approximate number system or an access deficit?. *Cognitive Development*, 39, 154-167. doi: 10.1016/j.cogdev.2016.04.006

Park, J., & Brannon, E. M. (2013). Training the approximate number system improves math proficiency. *Psychological science*, 24 (10), 2013 – 2019. doi: 10.1177/0956797613482944

Parsons, S. & Bynner, J. (2005). *Does numeracy matter more?*. London: National Research and Development Centre for adult literacy and numeracy.

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences*, 14(12), 542-551. doi:10.1016/j.tics.2010.09.008

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41. doi:10.1016/j.cognition.2010.03.012

Piazza, M., Izard, V., Pinel, P., LeBihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human parietal cortex. *Neuron*, 44(3), 547-555. doi:10.1016/j.neuron.2004.10.014

Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., Willmes, K., & Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation?. *PloS ONE*, 9(11), e111155. doi: 10.1371/journal.pone.0111155

Price, G. R., & Fuchs, L. S. (2016). The mediating relation between symbolic and nonsymbolic foundations of math competence. *PloS one*, *11*(2), e0148981. doi:10.1371/journal.pone.0148981

Price, G. R., Holloway, I. D., Rasanen, P., Vesterinen, M., & Ansari, D. (2007). Impaired parietal magnitude processing in developmental dyscalculia. *Current Biology*, 17(24), R1042–R1043. doi: 10.1016/j.cub.2007.10.013

Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, 17(2), 147-159. doi:10.1016/j.lindif.2007.03.010

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, 102(3), 361-95. doi:10.1016/j.cognition.2006.01.005

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology*, 30(2), 344-357. doi: 10.1111/j.2044-835X.2011.02048.x

Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2016). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: a meta-analysis. *Developmental science*. doi:10.1111/desc.12372

Shalev, R. S., Auerbach, J., Manor, O., & Gross-Tsur, V. (2000). Developmental dyscalculia: prevalence and prognosis. *European child & adolescent psychiatry*, 9, S58-S64. doi:10.1007/s007870070009

Skagerlund, K., & Träff, U. (2014). Development of magnitude processing in children with developmental dyscalculia: space, time, and number. *Frontiers in psychology*, 5, 675. doi: 10.3389/fpsyg.2014.00675

Starling-Alves, I., Moura, R. J., Júlio-Costa, A., Haase, V.G., & Bortoloti, R. (in prep). Core deficit or access hypothesis: which one rules in mathematical difficulties?

Vanbinst, K., Ceulemans, E., Ghesquière, P., & De Smedt, B. (2015). Profiles of children's arithmetic fact development: A model-based clustering approach. *Journal of experimental child psychology*, *133*, 29-46. doi: 10.1016/j.jecp.2015.01.00

General Discussion

In the current thesis, the nonsymbolic and the symbolic magnitude processing in children with Mathematical Difficulties (MD) and the relationship of these abilities with math achievement were investigated in two studies. Here, a general overview of studies is presented and discussed.

In the first study, numerical magnitude processing of children with MD and typical achievement (TA) were assessed by a nonsymbolic magnitude and a single-digit number comparison task. In order to assure that indexes used in both tasks were comparable, accuracy, reaction time (RT), distance effect, and a nonsymbolic and a symbolic Weber fraction were presented. The Weber fraction takes into account the errors committed in each ratio in a comparison task, and accounts for the variability in the representation of magnitudes (Halberda, Mazzocco, & Feigenson, 2008; Piazza et al., 2010). As the symbolic magnitude representations are attached to the ANS (Dehaene, 1992), and respond to the distance effect (Moyer & Landauer, 1967), they could possibly be indexed by the Weber fraction.

Children with MD presented difficulties in the nonsymbolic magnitude comparison task, presenting lower accuracy and higher Weber fractions when compared to TA. However, in the symbolic comparison task with single-digit numbers, no differences between groups were observed. The distance effect was observed– higher RT and lower accuracy for close magnitudes – for both the nonsymbolic and the symbolic comparison tasks, but there was no difference between groups in this domain.

Children with MD presented difficulties in addition, subtraction, and multiplication skills. The relationship between number processing and math achievement was investigated through linear regression analysis separately for MD and TA groups, with the nonsymbolic and the symbolic Weber fraction as factors and scores in addition, subtraction, and multiplication as dependent variables. For TA children, the nonsymbolic and the symbolic Weber fractions were significant predictors of addition, subtraction, and multiplication. However, for the MD group, no regression reached significance. Therefore, it is possible that children with MD do not engage the number processing to solve arithmetic problems, and use different cognitive resources (Pinheiro-Chagas et al., 2014).

Since no differences between MD and TA groups were found in the single-digit number processing, in the second study, the two-digit number processing and its association with math achievement were investigated. Children with MD and TA responded to a nonsymbolic magnitude comparison task, and a two-digit number comparison task, such that oth tasks had similar magnitude ranges.

In the second study, children with MD presented difficulties in the nonsymbolic magnitude processing and in the two-digit number processing. In the nonsymbolic number comparison task, children with MD presented lower accuracy and higher Weber fraction than TA. In the two-digit number task, in turn, children with MD presented higher RT and lower accuracy than the TA, but no compatibility effect was observed for both groups. The compatibility effect occurs when higher RT and lower accuracy are observed in incompatible two-digit number comparisons, in contrast to lower RT and higher accuracy in compatible items, and allows the investigation of automatic processing of number (Nuerk & Willmes, 2005). Unfortunately, as the design of the two-digit task was based on the distance between decades and units, and the compatibility between them, rather than on the ratio between numbers, it was not possible to calculate a symbolic Weber fraction.

Regarding math achievement, children with MD presented difficulties in addition, subtraction, multiplication and numerical transcoding skills. The association between number processing and math achievement was investigated through a correlation analysis. The nonsymbolic Weber fraction was significantly correlated with multiplication and numerical transcoding scores. On the other hand, in the two-digit number comparison task, accuracy was correlated with addition, subtraction, and numerical transcoding scores, and RT was correlated with addition, subtraction, multiplication and numerical transcoding scores. An error analysis was conducted in the numerical transcoding task, and children with MD presented more syntactic and lexical errors than children with TA. There was a significant correlation between syntactic errors frequency and accuracy and RT in the two-digit number comparison task. No correlation was significant for lexical errors.

Overall, children with MD presented impairments in the ANS, supporting the core deficit hypothesis. The core deficit hypothesis proposes that a deficit in the ANS is the main cause of MD (Mazzocco, Feigensn, & Halberda, 2011; Piazza et al. 2010; Pinheiro-Chagas et al., 2014). The ANS is dedicated to the representation of abstract magnitudes in an analog fashion (Feigenson, Dehaene, & Spelke, 2004), and allows nonsymbolic magnitude comparison, estimation and calculation (Dehaene, 2001). The validity of the ANS has been taken into question (Gebuis, & Reynvoet, 2011; Gilmore et al., 2013; Leibovich, Katzin, Harel, Henik, 2016), but there is evidence that this system is part of a generalized magnitude system (Skagerlund & Träff, 2014; Walsh, 2003), and is independent of visual properties (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016).

According the triple-code model (Dehaene, 1992), the ANS is the foundation for the symbolic representations of number. Therefore, it was expected that children with low ANS acuity would also present impairments in the symbolic processing. When a single-digit number comparison task was used, no differences between children with MD and TA were found. However, a two-digit number comparison task was sensible to symbolic processing difficulties in MD, which is in line with other studies (Ashkenazi, Mark-Zigdon, & Henik, 2009; Landerl, Fussenegger, Moll, & Willburguer, 2009). Skagerlund and Träff (2014) suggested that, in schools, there might be a greater exposure of children to single-digit numbers than to two-digit numbers. Accordingly, elementary school children present an efficient processing of small symbolic magnitudes, and greater symbolic magnitudes tasks are more sensible to difficulties. In the two-digit number comparison task, no compatibility effect was observed for MD and TA. One possibility is that elementary school children are still unable to process larg numbers automatically, as proposed by Ashkenazi et al. (2009).

In the present thesis, the association between numerical magnitudes processing and math achievement was observed, in line with several studies (Bugden, Price, McLean, & Ansari, 2012; Castronovo & Göbel, 2012; De Smedt, Verschaffel, & Ghesquière, 2009; Halberda et al., 2008; Inglis, Attridge, Batchelor, & Gilmore, 2011; Paulsen, Woldorff, & Brannon, 2010). The symbolic number processing presented a stronger correlation with math achievement than the nonsymbolic processing. Price and Fuchs (2016) showed that the symbolic number processing mediates the relationship between the ANS and math achievement. In this sense, the ANS may influence math skills indirectly, by allowing the development of symbolic numerical representations (Dehaene, 1992).

As mathematical abilities are important for our daily lives, it is necessary to identify underlying mechanisms of MD. Further studies might still clarify the importance of nonsymbolic and symbolic magnitude processing to math achievement, especially through longitudinal designs, as well as confirm the validity of a symbolic Weber fraction and its relationship with MD.

References

Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental psychology*, 52(8), 1206. doi: 10.1037/dev0000155

Ashkenazi, S., Mark-Zigdon, N., & Henik, A. (2009). Numerical distance effect in developmental dyscalculia. *Cognitive Development*, 24(4), 387-400. doi:10.1016/j.cogdev.2009.09.006

Bugden, S., Price, G. R., McLean, D. A., & Ansari, D. (2012). The role of the left intraparietal sulcus in the relationship between symbolic number processing and children's arithmetic competence. *Developmental Cognitive Neuroscience*, *2*(4), 448-457. doi:10.1016/j.dcn.2012.04.001

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS ONE*, 7(4), e33832. doi: 10.1371/journal.pone.0033832

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44(1), 1-42. doi: 10.1016/0010-0277(92)90049-N

Dehaene, S. (2001). Précis of the number sense. *Mind & Language*, 16(1), 16-36. doi: 10.1111/1468-0017.00154

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, 103, 469-479. doi:10.1016/j.jecp.2009.01.010

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8, 307-314. doi:10.1016/j.tics.2004.05.002

Gebuis, T., & Reynvoet, B. (2011). Generating nonsymbolic number stimuli. *Behavior research methods*, 43(4), 981-986. doi:10.3758/s13428-011-0097-5

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., Simms, V., & Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PloS ONE*, 8(6), e67374. doi: 10.1371/journal.pone.0067374

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, 455, 665-668. doi:10.1038/nature07246

Inglis, M., Attridge, N., Batchelor, S., & Gilmore, C. (2011). Non-verbal number acuity correlates with symbolic mathematics achievement: but only in children. *Psychonomic Bulletin & Review*, 18(6), 1222-1229. doi: 10.3758/s13423-011-0154-1

Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *Journal of experimental child psychology*, 103(3), 309-24. doi:10.1016/j.jecp.2009.03.006

Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2016). From 'sense of number'to 'sense of magnitude'–The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, 1-62. doi: 10.1017/S0140525X16000960

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia). *Child Development*, 82(4), 1224-1237. doi: 10.1111/j.1467-8624.2011.01608.x

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature*, 215, 1519-1520. doi:10.1038/2151519a0

Nuerk, H. C., & Willmes, K. (2005). On the magnitude representations of two-digit numbers. *Psychology Science*, 47(1), 52-72.

Paulsen, D. J., Woldorff, M. G., & Brannon, E. M. (2010). Individual differences in nonverbal number discrimination correlate with event-related potentials and measures of probabilistic reasoning. *Neuropsychologia,* 48(13), 3687-3695. doi: 10.1016/j.neuropsychologia.2010.08.014

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41. doi:10.1016/j.cognition.2010.03.012

Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., Willmes, K., & Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation?. *PloS ONE*, 9(11), e111155. doi: 10.1371/journal.pone.0111155

Price, G. R., & Fuchs, L. S. (2016). The mediating relation between symbolic and nonsymbolic foundations of math competence. *PloS one*, *11*(2), e0148981. doi:10.1371/journal.pone.0148981

Skagerlund, K., & Träff, U. (2014). Development of magnitude processing in children with developmental dyscalculia: space, time, and number. *Frontiers in psychology*. doi: 10.3389/fpsyg.2014.00675

Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in cognitive sciences*, 7(11), 483-488. doi: 10.1016/j.tics.2003.09.002