UNIVERSIDADE FEDERAL DE MINAS GERAIS

ESCOLA DE ENGENHARIA

DEPARTAMENTO DE ENGENHARIA DE ESTRUTURAS

CURSO DE ESPECIALIZAÇÃO EM ESTRUTURAS

LAGE COGUMELO – DIMENSIONAMENTO Á FLEXÃO E PUNÇÃO

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2015

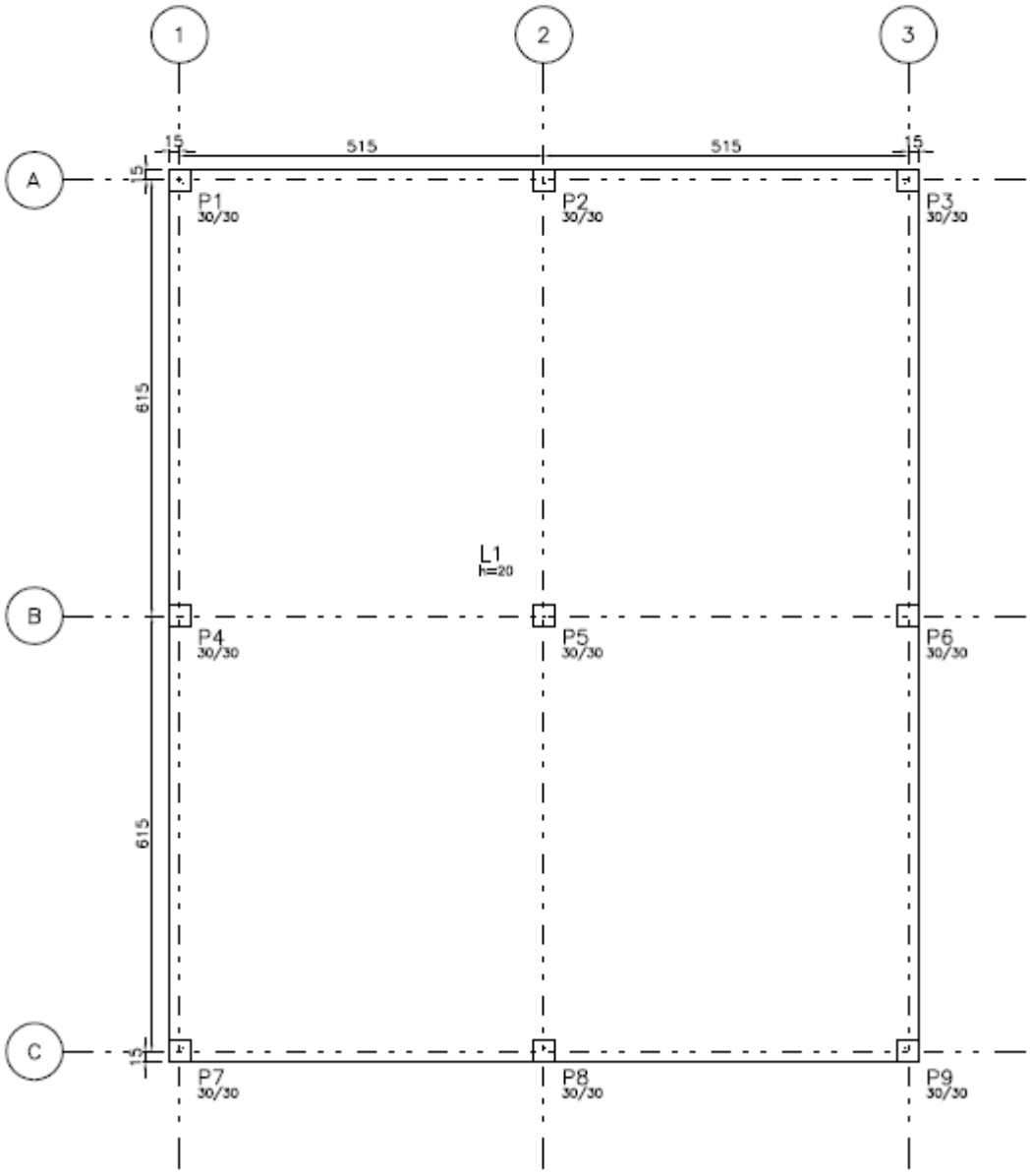
Ser engenheiro civil é motivo de grande satisfação e de muita responsabilidade. A qualidade de um serviço de engenharia está diretamente relacionada com a capacidade técnica e prática dos profissionais envolvidos. Cursar a Especialização em Estruturas tem como propósito aumentar o embasamento teórico no referido assunto, crescer a rede de contatos com colegas de profissão e com os mestres da UFMG que ensinam com tanta sabedoria e diligência.

Após ser graduado pela FUNEDI / UEMG no fim de 2012, ingressei como aluno regular no curso de Especialização em Estruturas. As disciplinas cursadas foram: Projeto de Estruturas de Concreto I, Projeto de Estruturas de Aço I, Projeto de Estruturas de Concreto II, Projeto de Estruturas de Aço II, Projeto de Estruturas de Aço com Perfis Estruturais Formados a Frio, Projeto de Estruturas Mistas de Aço e Concreto, Projeto de Estruturas de Fundação e Concepção de Estruturas de Edifícios.

Para conclusão do Curso de Especialização, além da obrigatoriedade de cursar oito disciplinas afins, é necessária a entrega de um Trabalho Prático Final. A proposta desenvolvida neste trabalho foi dimensionar e detalhar, à flexão e à punção, uma laje lisa de concreto armado. Esse tema foi escolhido visando aperfeiçoamento no procedimento de cálculo desse tipo de laje. Todo processo de cálculo utilizado está de acordo com a NBR 6118:2003. As dimensões da laje são de 10,60m x 12,60m com 9 pilares de apoio. A primeira etapa do Trabalho prático é dimensionar e detalhar a laje lisa à flexão. A segunda etapa é verificar a punção todas as regiões da laje que estão em contado direto com os pilares.

O curso de Especialização em Estruturas da UFMG superou as minhas expectativas, me dando mais segurança e aumentando a qualidade dos meus serviços de engenharia, o que me fez um engenheiro mais satisfeito com a profissão. Agradeço a todos os mestres do ensino e a todos os funcionários desta instituição. Em especial, agradeço ao grande amigo Ney Amorim Silva, meu professor e orientador neste trabalho final, por toda a amizade construída, por toda a dedicação e disponibilidade dedicada a mim.

Fôrma da laje lisa



1 – Espessura da laje:

$$Lx = 515cm$$

$$Ly = 615cm$$

$$L \geq \begin{cases} Lx = 515cm \\ Ly = 615cm \end{cases} \therefore L = 615cm$$

$$\frac{L}{36} \leq h \leq \frac{L}{40} \therefore \frac{615cm}{36} \leq h \leq \frac{615cm}{40} \therefore 15cm \leq h \leq 17cm$$

$$h = 16cm \ \therefore h_{min} = 16cm \therefore ok!$$

2 – Dimensões dos pilares:

Obs: A dimensão “a” do pilar é paralela ao lado “Lx” da laje e a dimensão “b” do pilar é paralela ao lado “Ly” da laje.

$$a \geq \begin{cases} 30cm \\ \frac{Lx}{20} = \frac{515cm}{20} = 25{,}75cm \\ \frac{h_{pilar}}{15} = \frac{(296cm - 16cm)}{15} = 18{,}67cm \end{cases} \therefore a = 30cm$$

$$b \geq \begin{cases} 30cm \\ \frac{Ly}{20} = \frac{615cm}{20} = 30{,}75cm \cong 30cm \\ \frac{h_{pilar}}{15} = \frac{(296cm - 16cm)}{15} = 18{,}67cm \end{cases} \therefore b = 30cm$$

3 – Carregamentos:

3.1 – Alvenaria nas bordas:

$$0{,}20m \times 2{,}80m \times 13\frac{KN}{m^3} = 7{,}28\frac{KN}{m}$$

3.2 – Cargas distribuídas sobre a área da laje:

3.2.1 – Peso próprio da laje:

$$2500\frac{kgf}{m^3} \times 0{,}16m = 400\ \frac{Kgf}{m^2}$$

3.2.2 – Revestimento:

$100\ \frac{kgf}{m^2}$

3.2.3 – Sobrecarga:

$300\frac{kgf}{m^2}$

3.2.4 – Carga característica total:

$$400\frac{\text{kgf}}{\text{m}^2} + 100\frac{\text{kgf}}{\text{m}^2} + 300\frac{\text{kgf}}{\text{m}^2} = 800\frac{\text{kgf}}{\text{m}^2} = 8{,}0\frac{\text{KN}}{\text{m}^2}$$

4 – Carregamento nos pórticos:

Obs: Considera-se “q” como carga acidental e “g” como carga permanente.

$$q = 300\frac{kgf}{m^2}$$

$$g = 400\frac{kgf}{m^2} + 100\frac{kgf}{m^2} = 500\frac{kgf}{m^2}$$

$$q = 300\frac{kgf}{m^2} \leq 0{,}75 \times g = 0{,}75 \times 500\frac{kgf}{m^2} = 375\frac{kgf}{m^2}$$

Portanto, $q \leq 0{,}75g$. Esta situação define qual o modelo de carregamento a ser adotado.

4.1 – Direção x:

4.1.1 – Inércia das “vigas” dos eixos A e C:

$$Iv = \frac{3{,}225m \times (0{,}16m)^3}{12} = 1{,}10 \times 10^{-3}m^4$$

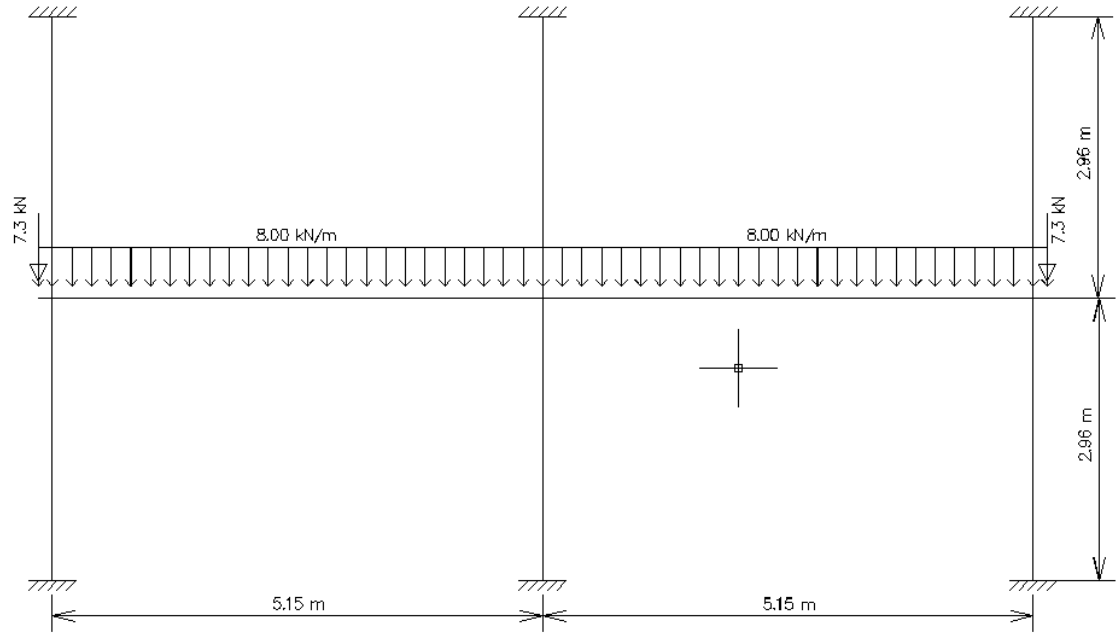
4.1.2 – Inércia das “vigas” do eixo B:

$$Iv = \frac{6{,}15m \times (0{,}16m)^3}{12} = 2{,}10 \times 10^{-3}m^4$$

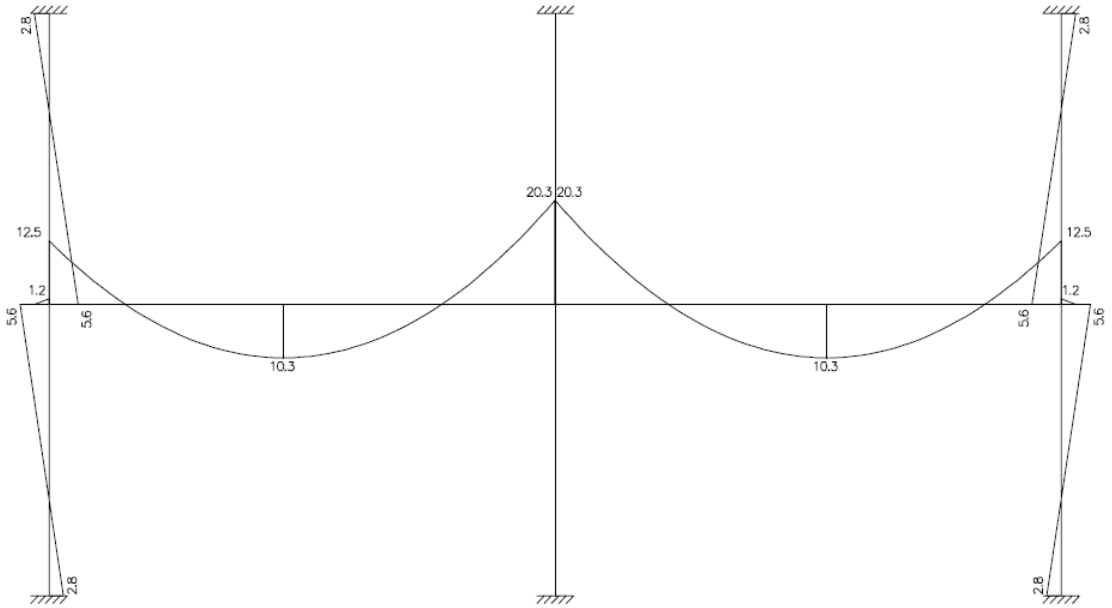
4.1.3 – Inércia dos “pilares”:

$$Ip = \frac{0{,}30m \times (0{,}30m)^3}{12} = 6{,}75 \times 10^{-4} m^4$$

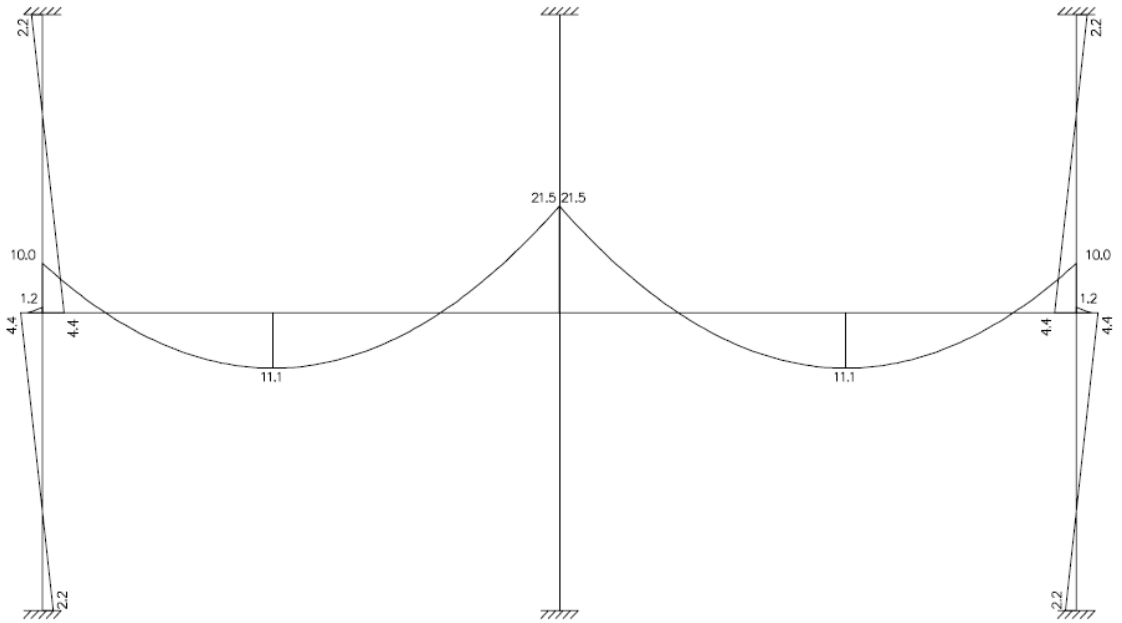
4.1.4 – Modelo estrutural considerando o carregamento por faixa de 1m:



4.1.5 – Diagrama de momento fletor referente aos Eixos A e C (KN.m):



4.1.6 – Diagrama de momento fletor referente ao Eixo B (KN.m):



4.2 – Direção y:

4.2.1 – Inércia das “vigas” dos eixos 1 e 3:

$$Iv = \frac{2{,}725m \times (0{,}16m)^3}{12} = 9{,}30 \times 10^{-4} m^4$$

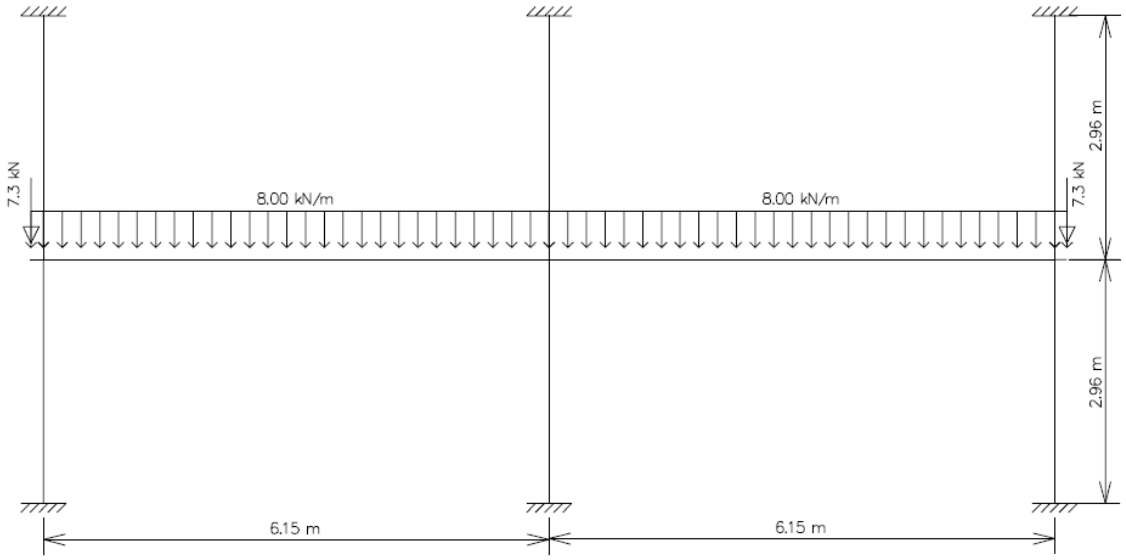
4.2.2 – Inércia das “vigas” do eixo 2:

$$Iv = \frac{5{,}15m \times (0{,}16m)^3}{12} = 1{,}76 \times 10^{-3} m^4$$

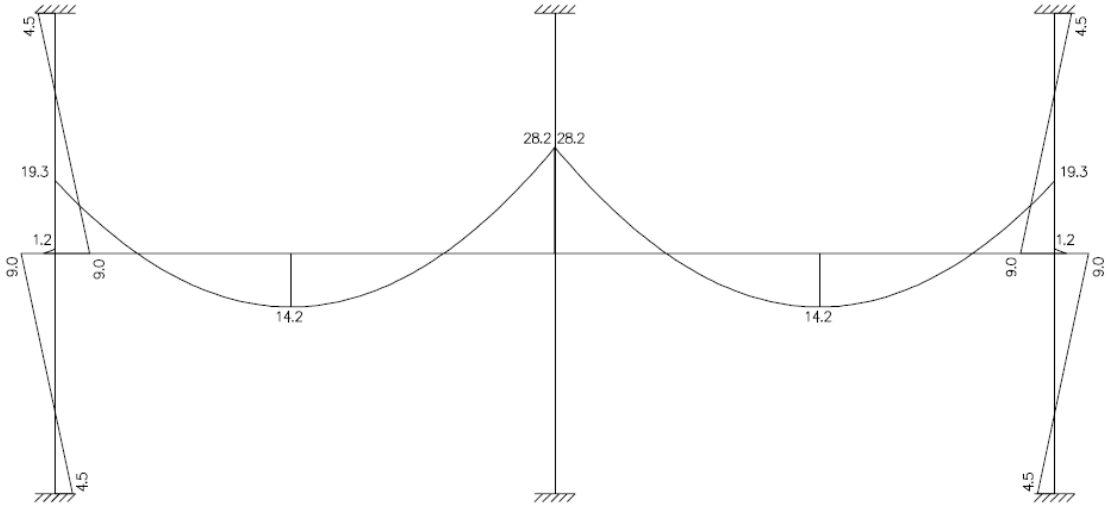
4.2.3 – Inércia dos “pilares”:

$$Ip = \frac{0{,}30m \times (0{,}30m)^3}{12} = 6{,}75 \times 10^{-4} m^4$$

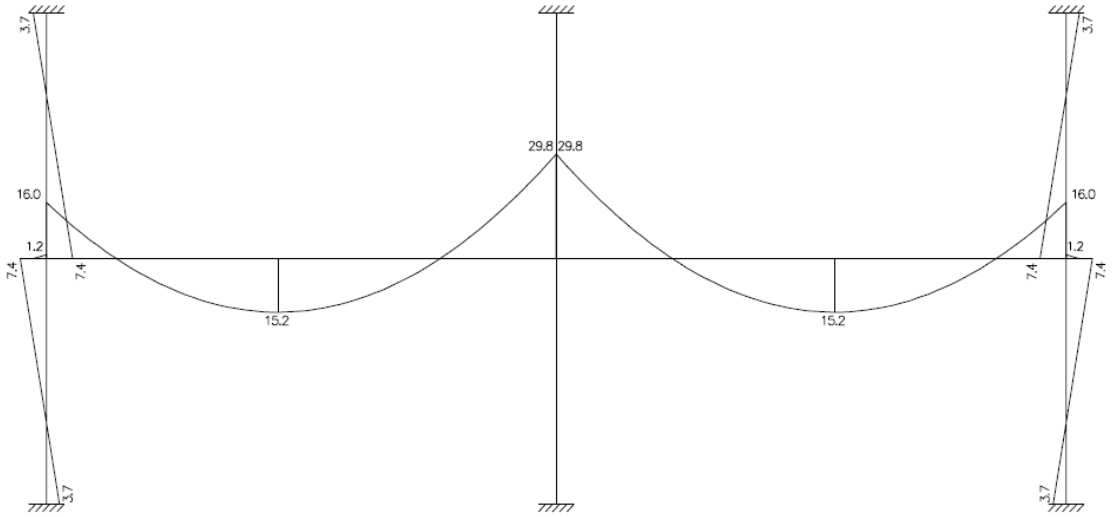
4.2.4 – Modelo estrutural considerando o carregamento por faixa de 1m:



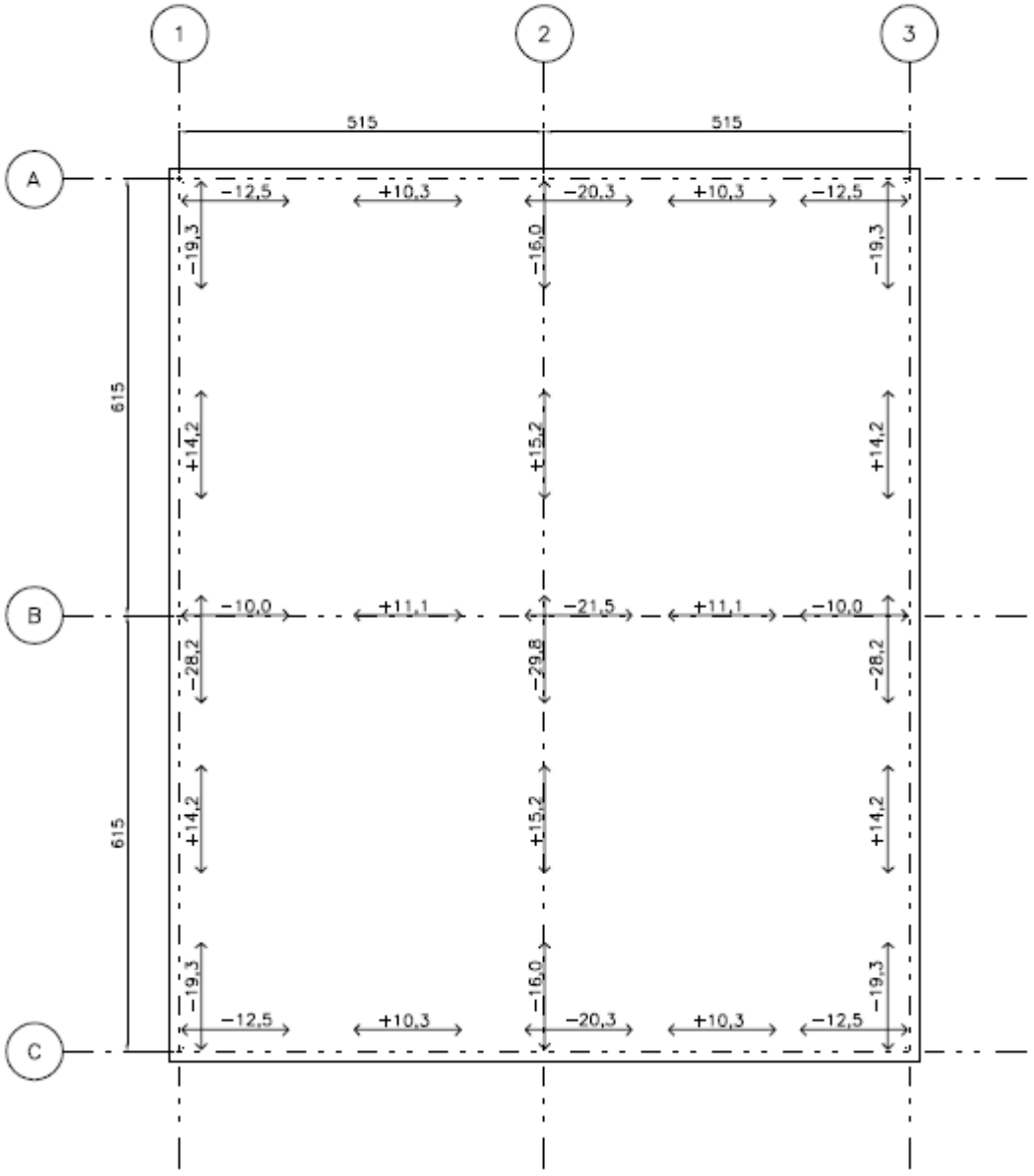
4.2.5 – Diagrama de momento fletor referente aos Eixos 1 e 3 (KN.m):



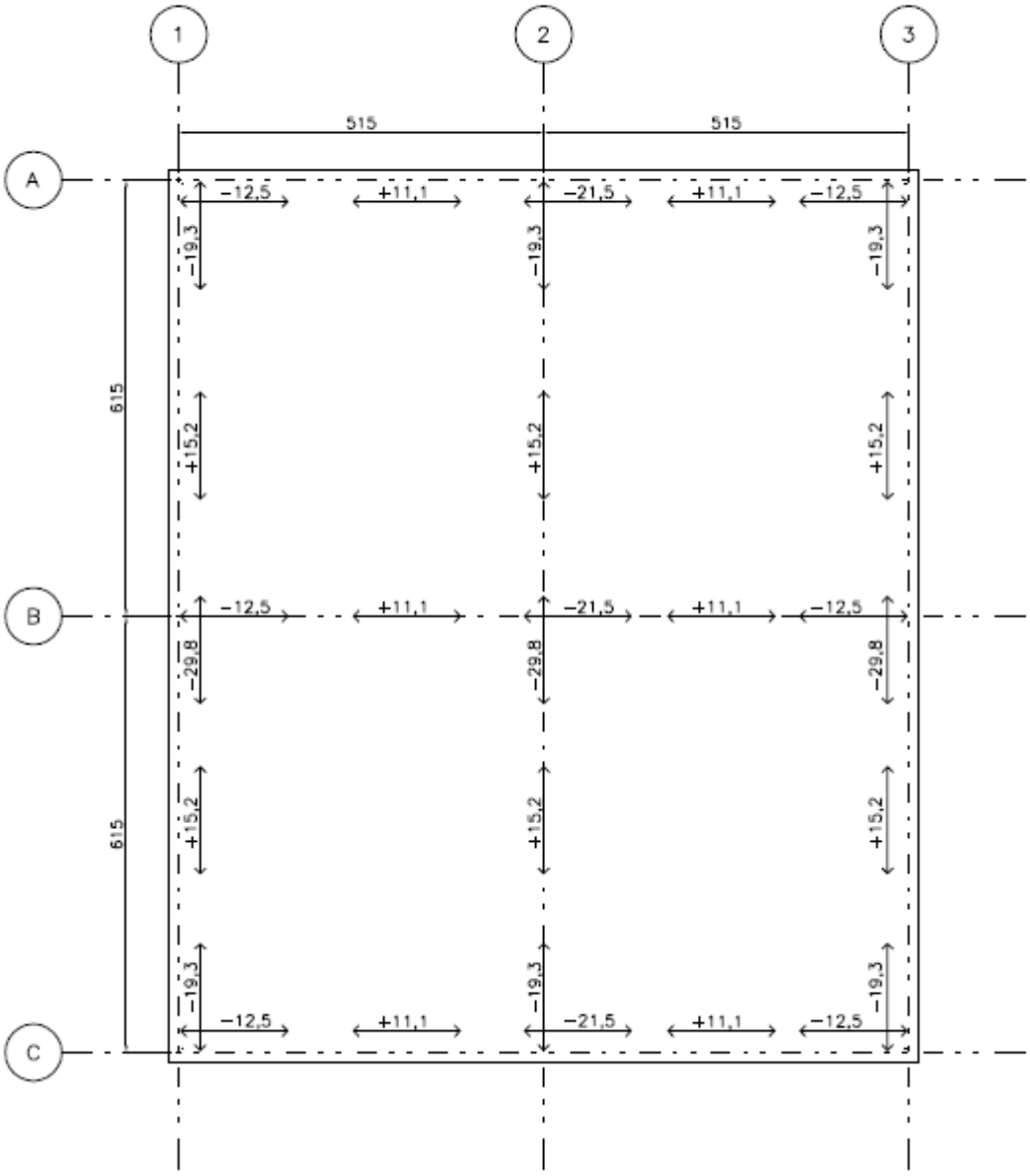
4.2.6 – Diagrama de momento fletor referente ao Eixo 2 (KN.m):



Momentos calculados (KN.m)



Momentos adotados (KN.m)



5 – Dimensionamento:

5.1 – Largura das faixas na direção x:

$$\frac{615cm}{4} = 153{,}75cm$$

5.2 – Largura das faixas na direção y:

$$\frac{515cm}{4} = 128{,}75cm$$

5.3 – Altura útil da laje:

$$H = 16cm \therefore d' = 4{,}5cm$$

$$d = H - d' = 16cm - 4{,}5cm = 11{,}5cm$$

5.4 – Área de aço mínima:

$$As_{min} = 0{,}15\%.b.h = 0{,}15\% \times 100cm \times 16cm = 2{,}4\frac{cm^2}{m\ de\ laje}$$

5.5 – Área de aço na direção x:

5.5.1 – Faixas externas:

MOMENTO NO PAINEL (kN.m)	% POR FAIXA	MOMENTOS POR FAIXA (kN.m)	d (cm)	fc (KN/cm²)	b (cm)	k	As total (cm²)	As por metro (cm²)	DIÂMETRO
-12,5 x 6,15 = -76,9	37,5%	-29,5	11,5	1,52	153,75	0,13	8,64	5,62	Ø10 c/ 14cm
-21,5 x 6,15 = -132,2	37,5%	-49,6	11,5	1,52	153,75	0,22	15,56	10,12	Ø12,5 c/ 12cm
11,1 x 6,15 = 68,3	27,5%	18,8	11,5	1,52	153,75	0,09	5,84	3,8	Ø8 c/ 13cm

5.5.2 Faixas internas:

MOMENTO NO PAINEL (kN.m)	% POR FAIXA	MOMENTOS POR FAIXA (kN.m)	d (cm)	fc (KN/cm²)	b (cm)	k	As total (cm²)	As por metro (cm²)	DIÂMETRO
-12,5 x 6,15 = -76,9	12,5%	-9,6	11,5	1,52	153,75	0,04	2,52	*2,40	Ø8 c/ 20cm
-21,5 x 6,15 = -132,2	12,5%	-16,5	11,5	1,52	153,75	0,07	4,49	2,92	Ø8 c/ 17cm
11,1 x 6,15 = 68,3	22,5%	15,4	11,5	1,52	153,75	0,07	4,49	2,92	Ø8 c/ 17cm

* Área de aço mínima.

5.6 – Área de aço na direção y:

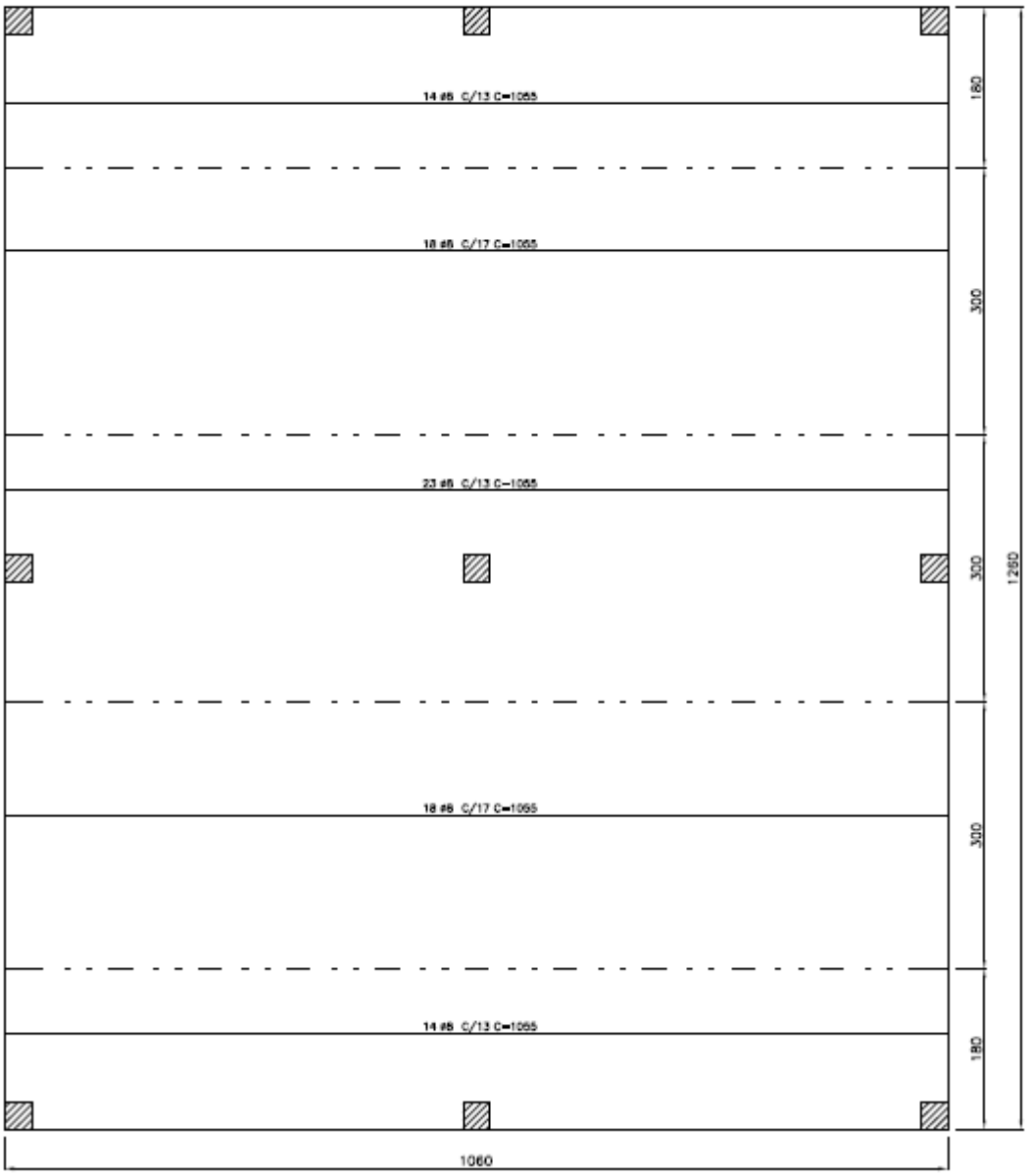
5.6.1 – Faixas externas:

MOMENTO NO PAINEL (kN.m)	% POR FAIXA	MOMENTOS POR FAIXA (kN.m)	d (cm)	fc (KN/cm²)	b (cm)	k	As total (cm²)	As por metro (cm²)	DIÂMETRO
-19,3 x 5,15 = -99,4	37,5%	37,3	11,5	1,52	128,75	0,2	11,67	9,06	Ø12,5 c/ 13cm
-29,8 x 5,15 = -153,5	37,5%	57,6	11,5	1,52	128,75	0,31	19,85	15,42	Ø16 c/ 13cm
15,2 x 5,15 = 78,3	27,5%	21,5	11,5	1,52	128,75	0,12	6,64	5,16	Ø10 c/ 15cm

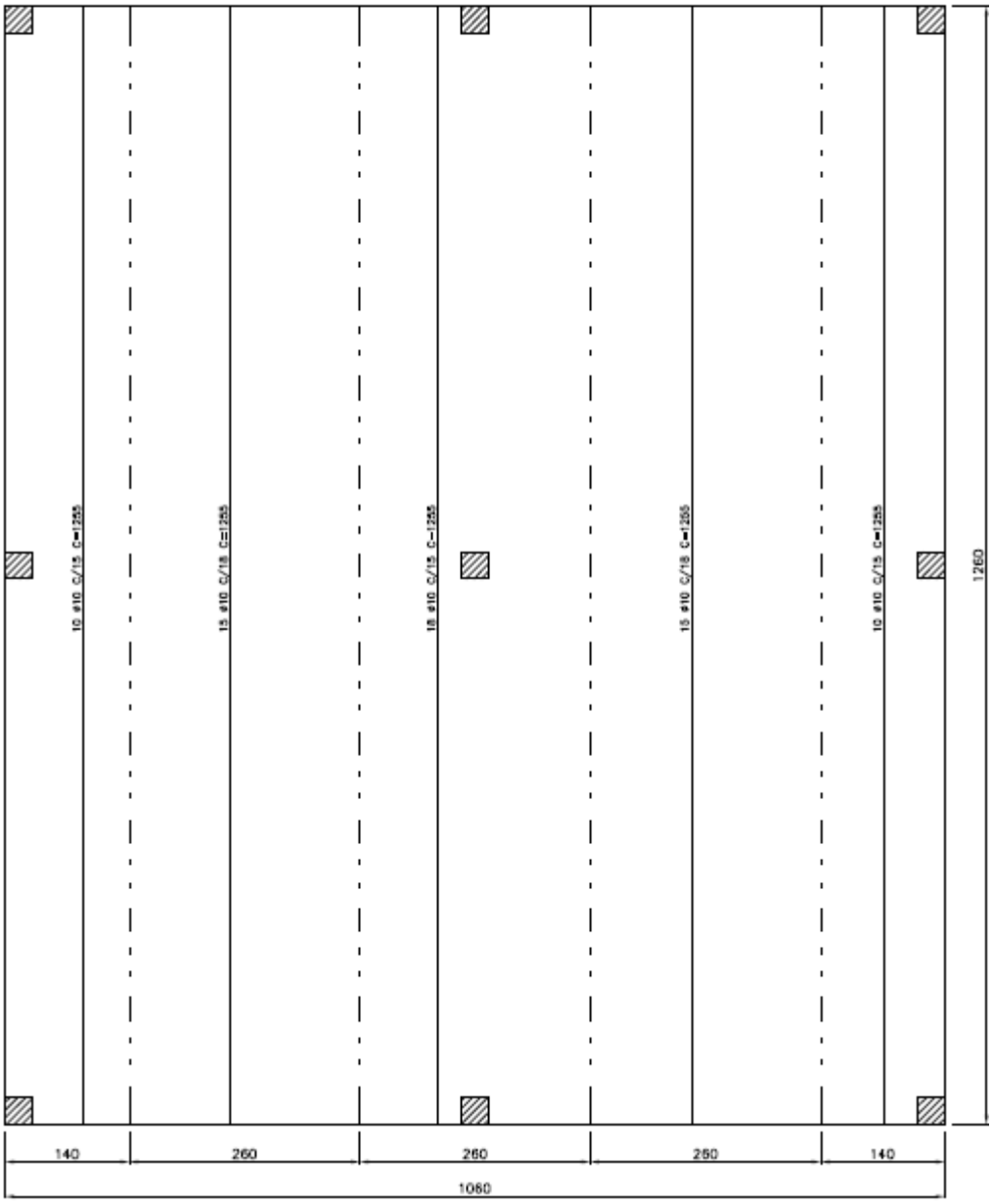
5.6.2 – Faixas internas:

MOMENTO NO PAINEL (kN.m)	% POR FAIXA	MOMENTOS POR FAIXA (kN.m)	d (cm)	fc (KN/cm²)	b (cm)	k	As total (cm²)	As por metro (cm²)	DIÂMETRO
-19,3 x 5,15 = -99,4	12,5%	12,4	11,5	1,52	128,75	0,07	3,76	2,92	Ø8 c/ 17cm
-29,8 x 5,15 = -153,5	12,5%	19,2	11,5	1,52	128,75	0,10	5,46	4,24	Ø10 c/ 18cm
15,2 x 5,15 = 78,3	22,5%	17,6	11,5	1,52	128,75	0,10	5,46	4,24	Ø10 c/ 18cm

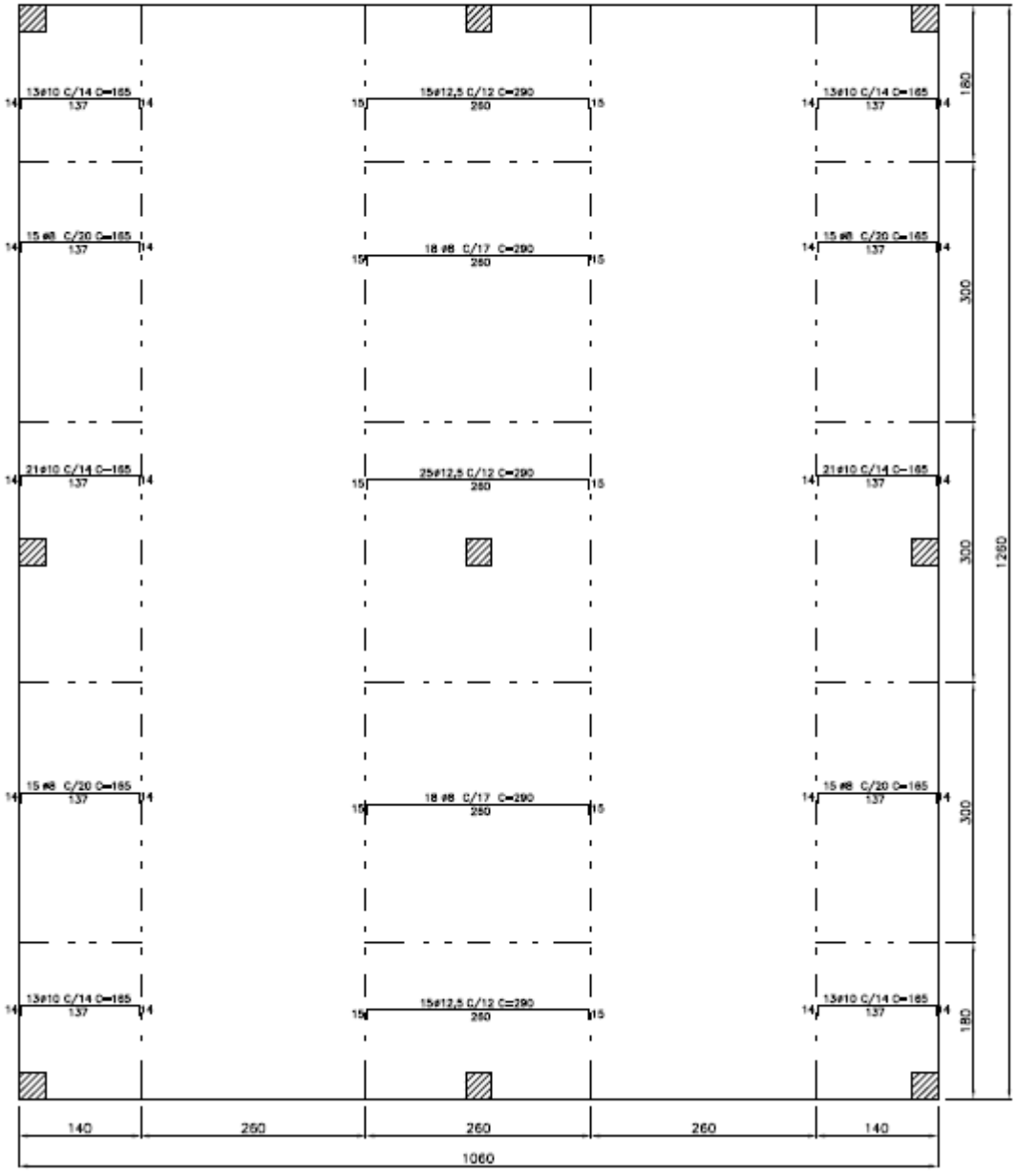
Armadura positiva, direção x



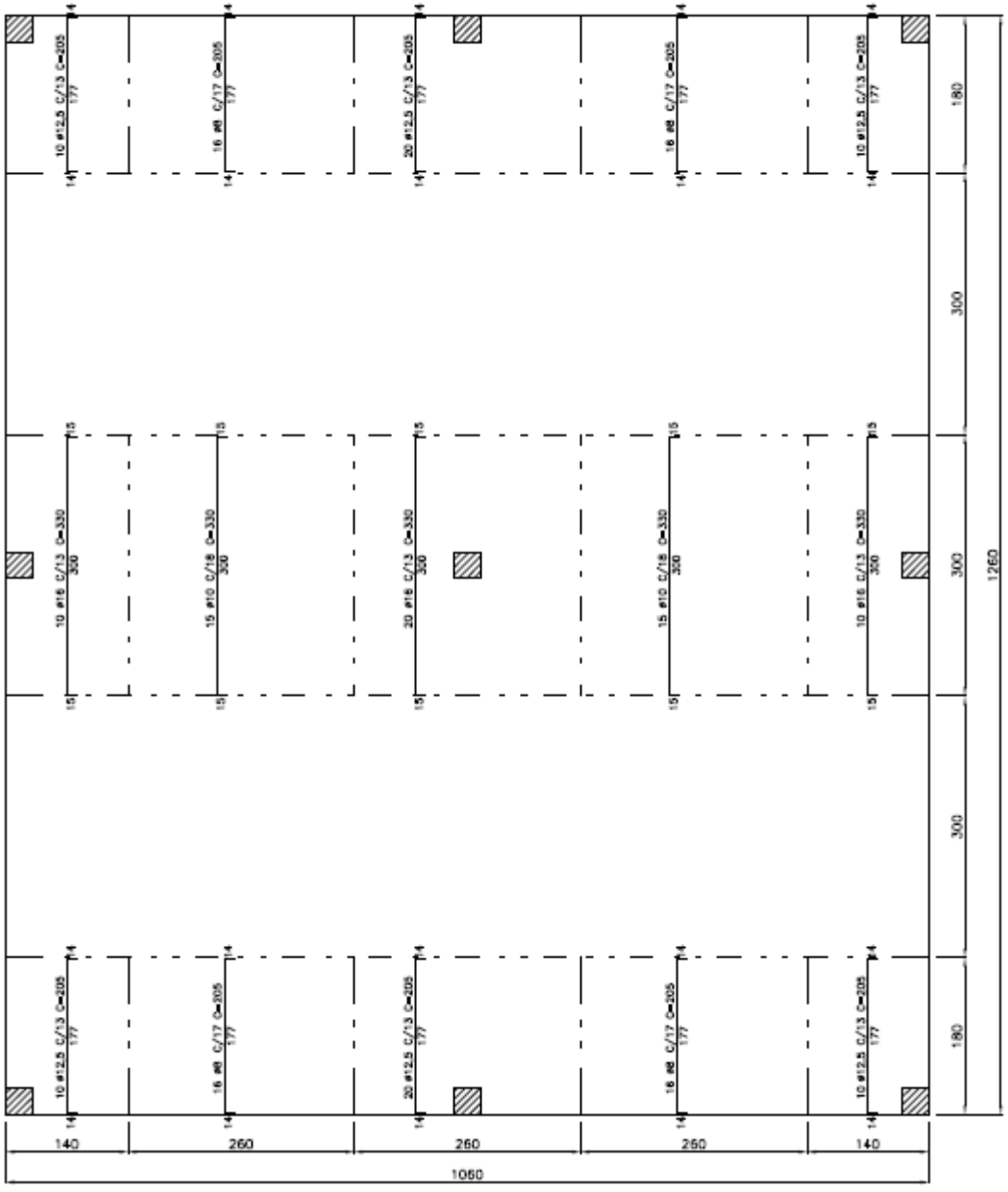
Armadura positiva, direção y



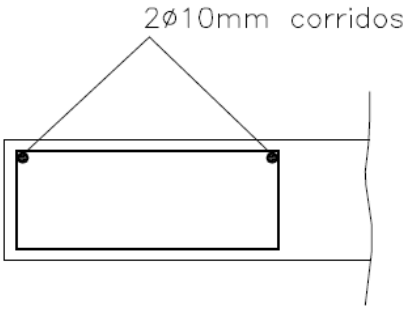
Armadura negativa, direção x

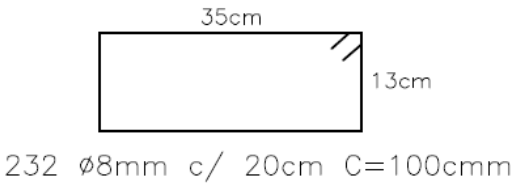


Armadura negativa, direção y



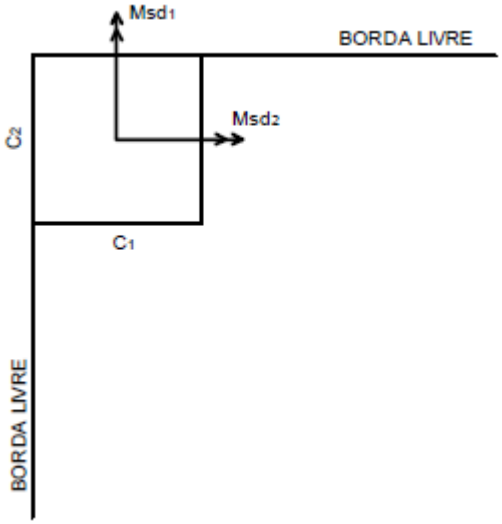
5.7 – Reforços de borda:





PUNÇÃO

Verificação dos pilares P1, P3, P7 e P9, usando como referência o pilar P1:



1 – Dados:

$C_1 = 40cm$

$C_2 = 40cm$

$d = h - d' = 16cm - 4{,}5cm = 11{,}5cm$

$Fsd = Fz.1{,}4 = 113{,}63KN \times 1{,}4 = 159{,}08KN\ (Compressão)$

$Msd_1 = M_y.1{,}4 = 1806KN.cm \times 1{,}4 = 2528{,}4KN.cm$

$Msd_2 = M_x.1{,}4 = 2453KN.cm \times 1{,}4 = 3434{,}2KN.cm$

$$a_{01} \leq \begin{cases} 1{,}5.d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ C_1 = 40cm \end{cases} \therefore a_{01} = 17{,}25cm$$

$$a_{02} \leq \begin{cases} 1{,}5.d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ C_2 = 40cm \end{cases} \therefore a_{02} = 17{,}25cm$$

$$a_1 \leq \begin{cases} 1{,}5.d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ 0{,}5.C_1 = 0{,}5 \times 40cm = 20cm \end{cases} \therefore a_1 = 17{,}25cm$$

$$a_2 \leq \begin{cases} 1{,}5.d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ 0{,}5.C_2 = 0{,}5 \times 40cm = 20cm \end{cases} \therefore a_2 = 17{,}25cm$$

2 – Verificação do Contorno C:

$u^* = a_{01} + a_{02} = 17{,}25cm + 17{,}25cm = 34{,}5cm$

2.1 – Análise de Msd_1:

$$wp_1 = \frac{{C_1}^2}{4} + \frac{C_1.C_2}{2} = \frac{(40cm)^2}{4} + \frac{40cm \times 40cm}{2} = 1200cm^2$$

$${e_1}^* = \frac{C_1.a_1 - {a_1}^2 + a_2.C_1}{2.(a_1 + a_2)} = \frac{40cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 40cm}{2 \times (17{,}25cm + 17{,}25cm)}$$

$$e_1{}^* = 15{,}69cm$$

$$Msd_1{}^* = Fsd.e_1{}^* = 159{,}08KN \times 15{,}69cm = 2495{,}97KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 2528{,}4KN.cm - 2495{,}97KN.cm = 32{,}43KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 32{,}43KN.cm$$

$$\frac{C_1}{C_2} = \frac{40cm}{40cm} = 1 \therefore K_1 = 0{,}60$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} = \frac{159{,}08KN}{34{,}5cm \times 11{,}5cm} + 0{,}6 \times \frac{32{,}43KN.cm}{1200cm^2 \times 11{,}5cm}$$

$$Tsd_1 = 0{,}402\frac{KN}{cm^2} = 4{,}02MPa$$

$$Trd_2 = 0{,}27.\propto_v.fcd = 0{,}27 \times \left(1 - \frac{fck}{250}\right) \times \left(\frac{fck}{1{,}4}\right) = 0{,}27 \times \left(1 - \frac{25MPa}{250}\right) \times \left(\frac{25MPa}{1{,}4}\right)$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd_1 = 4{,}02MPa < Trd_2 = 4{,}34MPa \therefore ok!$$

2.2 – Análise de Msd_2:

$$wp_2 = \frac{C_2{}^2}{4} + \frac{C_2.C_1}{2} = \frac{(40cm)^2}{4} + \frac{40cm \times 40cm}{2} = 1200cm^2$$

$$e_2{}^* = \frac{C_2.a_2 - a_2{}^2 + a_1.C_2}{2.(a_2 + a_1)} = \frac{40cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 40cm}{2 \times (17{,}25cm + 17{,}25cm)}$$

$$e_2{}^* = 15{,}69cm$$

$$Msd_2{}^* = Fsd.e_2{}^* = 159{,}08KN \times 15{,}69cm = 2495{,}97KN.cm$$

$$Msd_{2_{fim}} \geq \begin{cases} 0 \\ Msd_2 - Msd_2{}^* = 3434{,}2KN.cm - 2495{,}97KN.cm = 938{,}23KN.cm \end{cases}$$

$$Msd_{2_{fim}} = 932{,}23KN.cm$$

$$\frac{C_2}{C_1} = \frac{40cm}{40cm} = 1 \therefore K_2 = 0{,}60$$

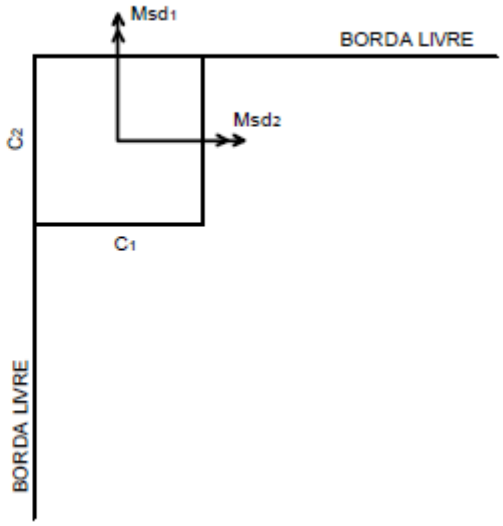
$$Tsd_2 = \frac{Fsd}{u^*.d} + K_2.\frac{Msd_{2_{fim}}}{wp_2.d} = \frac{159{,}08KN}{34{,}5cm \times 11{,}5cm} + 0{,}6 \times \frac{938{,}23KN.cm}{1200cm^2 \times 11{,}5cm}$$

$$Tsd_2 = 0{,}442\frac{KN}{cm^2} = 4{,}42MPa$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd_2 = 4{,}42MPa > Trd_2 = 4{,}34MPa \therefore não\ ok!$$

Obs: Ao verificar o contorno C, chega-se à conclusão que será necessário mudar algo na estrutura para atender à condição proposta. Vou aumentar a seção dos pilares da estrutura para 50cm x 50cm.



3 – Dados:

$$C_1 = 50cm$$

$$C_2 = 50cm$$

$$d = h - d' = 16cm - 4{,}5cm = 11{,}5cm$$

$$Fsd = Fz \times 1{,}4 = 113{,}63KN \times 1{,}4 = 159{,}08KN\ (Compressão)$$

$$Msd_1 = M_y \times 1{,}4 = 1806KN.cm \times 1{,}4 = 2528{,}4KN.cm$$

$$Msd_2 = M_x \times 1{,}4 = 2453KN.cm \times 1{,}4 = 3434{,}2KN.cm$$

$$a_{01} \leq \begin{cases} 1{,}5 \times d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ C_1 = 50cm \end{cases} \therefore a_{01} = 17{,}25cm$$

$$a_{02} \leq \begin{cases} 1{,}5 \times d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ C_2 = 50cm \end{cases} \therefore a_{02} = 17{,}25cm$$

$$a_1 \leq \begin{cases} 1{,}5 \times d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ 0{,}5 \times C_1 = 0{,}5 \times 50cm = 25cm \end{cases} \therefore a_1 = 17{,}25cm$$

$$a_2 \leq \begin{cases} 1{,}5 \times d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ 0{,}5 \times C_2 = 0{,}5 \times 50cm = 25cm \end{cases} \therefore a_2 = 17{,}25cm$$

4 – Verificação do Contorno C:

$$u^* = a_{01} + a_{02} = 17{,}25cm + 17{,}25cm = 34{,}5cm$$

4.1 – Análise de Msd_1:

$$wp_1 = \frac{{C_1}^2}{4} + \frac{C_1.C_2}{2} = \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} = 1875cm^2$$

$${e_1}^* = \frac{C_1.a_1 - {a_1}^2 + a_2.C_1}{2.(a_1 + a_2)} = \frac{50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm}{2 \times (17{,}25cm + 17{,}25cm)}$$

$${e_1}^* = 20{,}69cm$$

$${Msd_1}^* = Fsd.{e_1}^* = 159{,}08KN \times 20{,}69cm = 3291{,}37KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - {Msd_1}^* = 2528{,}4KN.cm - 3291{,}37KN.cm = -762{,}97KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} = \frac{159{,}08KN}{34{,}5cm \times 11{,}5cm} + 0{,}60 \times \frac{0}{1875cm^2 \times 11{,}5cm}$$

$$Tsd_1 = 0{,}402\frac{KN}{cm^2} = 4{,}02MPa$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd_1 = 4{,}02MPa < Trd_2 = 4{,}34MPa \therefore ok!$$

4.2 – Análise de Msd_2:

$$wp_2 = \frac{{C_2}^2}{4} + \frac{C_2.C_1}{2} = \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} = 1875cm^2$$

$${e_2}^* = \frac{C_2.a_2 - {a_2}^2 + a_1.C_2}{2.(a_2 + a_1)} = \frac{50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm}{2 \times (17{,}25cm + 17{,}25cm)}$$

$${e_2}^* = 20{,}69cm$$

$${Msd_2}^* = Fsd.{e_2}^* = 159{,}08KN \times 20{,}69cm = 3291{,}37KN.cm$$

$$Msd_{2_{fim}} \geq \begin{cases} 0 \\ Msd_2 - {Msd_2}^* = 3434{,}2KN.cm - 3291{,}37KN.cm = 142{,}83KN.cm \end{cases}$$

$$Msd_{2_{fim}} = 142{,}83KN.cm$$

$$\frac{C_2}{C_1} = \frac{50cm}{50cm} = 1 \therefore K_2 = 0{,}60$$

$$Tsd_2 = \frac{Fsd}{u^*.d} + K_2.\frac{Msd_{2_{fim}}}{wp_2.d} = \frac{159{,}08KN}{34{,}5cm \times 11{,}5cm} + 0{,}6 \times \frac{142{,}83KN.cm}{1875cm^2 \times 11{,}5cm}$$

$$Tsd_2 = 0{,}405\frac{KN}{cm^2} = 4{,}05MPa$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd_2 = 4{,}05MPa < Trd_2 = 4{,}34MPa \therefore \ ok!$$

5 – Verificação do Contorno C':

$$u^* = a_1 + a_2 + \pi.d = 17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm = 70{,}63cm$$

5.1 – Análise de Msd_1:

$$wp_1 = \frac{{C_1}^2}{4} + \frac{C_1.C_2}{2} + 2.C_2.d + 4.d^2 + \frac{\pi.d.C_1}{2}$$

$$= \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} + 2 \times 50cm \times 11{,}5cm + 4 \times (11{,}5cm)^2 = 3554cm^2$$

$$= \frac{\pi \times 11{,}5cm \times 50cm}{2} = 903{,}21cm^2$$

$$wp_1 = 3554cm^2 + 903{,}21cm^2 = 4457{,}21cm^2$$

$$e_1{}^* = \frac{C_1.a_1 - a_1{}^2 + a_2.C_1 + 4.a_2.d + 8.d^2 + \pi.d.C_1}{2.(a_1 + a_2 + \pi.d)}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm = 1427{,}44\text{cm}^2$$

$$= 4 \times 17{,}25cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 3657{,}92cm^2$$

$$= 2 \times (17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm) = 141{,}26cm$$

$$e_1{}^* = \frac{1427{,}44cm^2 + 3657{,}92cm^2}{141{,}26cm} = 36\text{cm}$$

$$Msd_1{}^* = Fsd.e_1{}^* = 159{,}08KN \times 36cm = 5726{,}88KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 2528{,}4KN.cm - 5726{,}88KN.cm = -3198{,}48KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} = \frac{159{,}08KN}{70{,}63cm \times 11{,}5cm} + 0{,}60 \times \frac{0}{4457{,}21cm^2 \times 11{,}5cm}$$

$$Tsd_1 = 0{,}20\,KN/cm^2 = 2{,}00MPa$$

$$\text{Ø}10mm \text{ à } cada\ 14cm \therefore \rho_x = \frac{As_x}{Ac} = \frac{0{,}8cm^2}{14cm \times 16cm} = 0{,}0035$$

$$\text{Ø}12.5mm \text{ à } cada\ 13cm \therefore \rho_y = \frac{As_y}{Ac} = \frac{1{,}25cm^2}{14cm \times 16cm} = 0{,}0059$$

$$\rho = \sqrt{\rho_x.\rho_y} = \sqrt{0{,}0035 \times 0{,}0059} = 0{,}0045$$

$$Trd_1 = 0{,}13.\left(1 + \sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}}$$

$$Trd_1 = 0{,}13 \times \left(1 + \sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0045 \times 25MPa)^{\frac{1}{3}} = 0{,}68MPa$$

$$Tsd_1 = 2{,}00MPa > Trd_1 = 0{,}68MPa \therefore Armar \text{ à } punção!$$

5.1.1 – Cálculo da armadura de punção:

$\alpha = 90° \therefore sen90° = 1{,}0$

$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}0cm$

$$fywd = 250 + 185.\frac{(h-15)}{20} = 250 + 185 \times \frac{(16cm-15)}{20} = 259{,}20MPa \therefore 26\frac{KN}{cm^2}$$

$$Trd_3 = 0{,}10.\left(1 + \sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}} + 1{,}5.\frac{d}{s_r}.\frac{Asw.fywd.sen\propto}{u.d}$$

$$= 0{,}10 \times \left(1 + \sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0045 \times 25MPa)^{\frac{1}{3}} = 0{,}52\text{MPa}$$

$$= 1{,}5 \times \frac{11{,}5cm}{8cm} \times \frac{Asw \times 259{,}2MPa \times 1}{70{,}63cm \times 11{,}5cm} = 0{,}69 \times Asw \times \frac{MPa}{cm^2}$$

$$Trd_3 = 0{,}52MPa + 0{,}69 \times Asw \times \frac{MPa}{cm^2}$$

$Tsd_1 \leq Trd_3$

$$2MPa \leq 0{,}52MPa + 0{,}69 \times Asw \times \frac{MPa}{cm^2}$$

$Asw_1 \geq 2{,}14cm^2$

5.2 – Análise de Msd_2:

$$wp_2 = \frac{{C_2}^2}{4} + \frac{C_2.C_1}{2} + 2.C_1.d + 4.d^2 + \frac{\pi.d.C_2}{2}$$

$$= \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} + 2 \times 50cm \times 11{,}5cm + 4 \times (11{,}5cm)^2 = 3554cm^2$$

$$= \frac{\pi \times 11{,}5cm \times 50cm}{2} = 903{,}21cm^2$$

$$wp_2 = 3554cm^2 + 903{,}21cm^2 = 4457{,}21cm^2$$

$${e_2}^* = \frac{C_2.a_2 - {a_2}^2 + a_1.C_2 + 4.a_1.d + 8.d^2 + \pi.d.C_2}{2.(a_2 + a_1 + \pi.d)}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm = 1427{,}44\text{cm}^2$$

$$= 4 \times 17{,}25cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 3657{,}92cm^2$$

$$= 2 \times (17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm) = 141{,}26cm$$

$$e_2{}^* = \frac{1427{,}44cm^2 + 3657{,}92cm^2}{141{,}26cm} = 36\text{cm}$$

$$Msd_2{}^* = Fsd.e_2{}^* = 159{,}08KN \times 36cm = 5726{,}88KN.cm$$

$$Msd_{2_{fim}} \geq \begin{cases} 0 \\ Msd_2 - Msd_2{}^* = 3434{,}2KN.cm - 5726{,}88KN.cm = -2292{,}68KN.cm \end{cases}$$

$$Msd_{2_{fim}} = 0$$

$$\frac{C_2}{C_1} = \frac{50cm}{50cm} = 1 \therefore K_2 = 0{,}60$$

$$Tsd_2 = \frac{Fsd}{u^*.d} + K_2.\frac{Msd_{2_{fim}}}{wp_2.d} = \frac{159{,}08KN}{70{,}63cm \times 11{,}5cm} + 0{,}60 \times \frac{0}{4457{,}21cm^2 \times 11{,}5cm}$$

$$Tsd_2 = 0{,}20\,KN/cm^2 = 2{,}00MPa$$

$$Trd_1 = 0{,}68MPa$$

$$Tsd_2 = 2{,}00MPa > Trd_1 = 0{,}68MPa \therefore Armar \text{ à } punção!$$

5.2.1 – Cálculo da armadura de punção:

$$\alpha = 90° \therefore sen90° = 1{,}0$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}0cm$$

$$fywd = 250 + 185.\frac{(h-15)}{20} = 250 + 185 \times \frac{(16cm-15)}{20} = 259{,}20MPa \therefore 26\frac{KN}{cm^2}$$

$$Trd_3 = 0{,}10.\left(1 + \sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}} + 1{,}5.\frac{d}{s_r}.\frac{Asw.fywd.sen\propto}{u.d}$$

$$= 0{,}10 \times \left(1 + \sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0045 \times 25MPa)^{\frac{1}{3}} = 0{,}52\text{MPa}$$

$$= 1{,}5 \times \frac{11{,}5cm}{8cm} \times \frac{Asw \times 259{,}2MPa \times 1}{70{,}63cm \times 11{,}5cm} = 0{,}69 \times Asw \times \frac{MPa}{cm^2}$$

$$Trd_3 = 0{,}52MPa + 0{,}69 \times Asw \times \frac{MPa}{cm^2}$$

$$Tsd_2 \leq Trd_3$$

$$2MPa \leq 0{,}52MPa + 0{,}69 \times Asw \times \frac{MPa}{cm^2}$$

$$Asw_2 \geq 2{,}14cm^2$$

5.3 – Armadura de punção total calculada:

$$Asw \geq Asw_1 + Asw_2 \therefore Asw \geq 2{,}14cm^2 + 2{,}14cm^2 \therefore Asw \geq 4{,}28cm^2$$

5.4 – Cálculo da armadura de punção obrigatória:

$$Asw \geq \frac{1}{2}.\frac{Fsd}{fywd} \therefore Asw \geq \frac{1}{2} \times \frac{159{,}08KN}{26\frac{KN}{cm^2}} \therefore Asw \geq 3{,}06cm^2$$

Serão no mínimo 3 contornos de armação à punção, então:

$$Asw \geq \frac{3{,}06cm^2}{3\ contornos} = 1{,}02cm^2$$

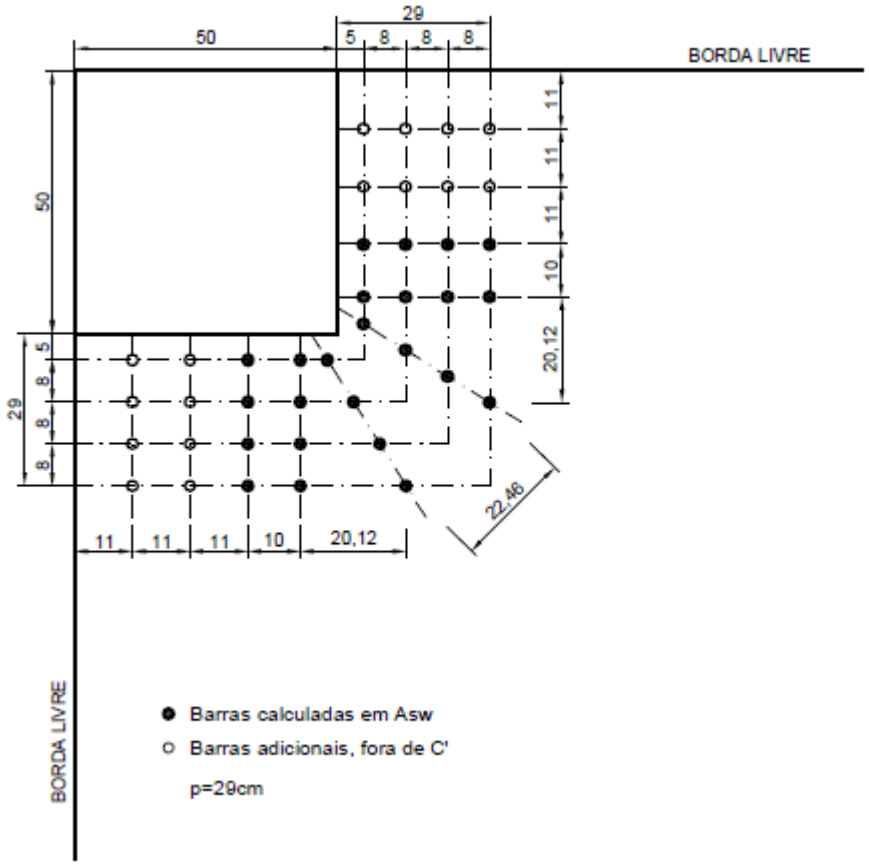
$$Asw \therefore 4{,}28cm^2 > 1{,}02cm^2 \therefore ok!$$

5.5 – Disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}00cm$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}00cm$$

$$s_e \leq 2.d \therefore s_e \leq 2 \times 11{,}5cm \therefore s_e \leq 23cm \therefore s_e = 23{,}00cm$$



$6Ø10.0mm = 4{,}8cm^2 > 4{,}28cm^2 \therefore ok! \therefore Por\ contorno\ serão\ 10Ø10.0mm$

6 – Verificação do Contorno C":

$p = 29cm$

$$u^* = a_1 + a_2 + \pi.d + \frac{\pi.p}{2} = 17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm + \frac{\pi \times 29cm}{2}$$

$$u^* = 116{,}18cm$$

6.1 – Análise de Msd_1:

$$wp_1 = \frac{{C_1}^2}{4} + \frac{C_1.C_2}{2} + 2.C_2.d + 4.d^2 - \frac{\pi.d.C_1}{2} + C_2.p + 4.d.p + \frac{\pi.p.C_1}{4} + p^2$$

$$= \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} + 2 \times 50cm \times 11{,}5cm + 4 \times (11{,}5cm)^2 = 3554cm^2$$

$$= \frac{\pi \times 11{,}5cm \times 50cm}{2} = 903{,}21cm^2$$

$$= 50cm \times 29cm + 4 \times 11{,}5cm \times 29cm + \frac{\pi \times 29cm \times 50cm}{4} + (29cm)^2 = 4763{,}83cm^2$$

$$wp_1 = 3554cm^2 - 903{,}21cm^2 + 4763{,}83cm^2 = 7414{,}62cm^2$$

$$e_1{}^* = \frac{C_1.a_1 - a_1{}^2 + a_2.C_1 + 4.a_2.d + 8.d^2 + \pi.d.C_1 + 2.a_2.p + 8.d.p + \frac{\pi.p.C_1}{2} + 2.p^2}{2.\left(a_1 + a_2 + \pi.d + \frac{\pi.p}{2}\right)}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm = 1427{,}44\text{cm}^2$$

$$= 4 \times 17{,}25cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 3657{,}92cm^2$$

$$= 2 \times 17{,}25cm \times 29cm + 8 \times 11{,}5cm \times 29cm = 3668{,}5\text{cm}^2$$

$$= \frac{\pi \times 29\text{cm} \times 50\text{cm}}{2} + 2 \times (29cm)^2 = 3959{,}65cm^2$$

$$= 2 \times \left(17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm + \frac{\pi \times 29cm}{2}\right) = 232{,}36cm$$

$$e_1{}^* = \frac{1427{,}44cm^2 + 3657{,}92cm^2 + 3668{,}5cm^2 + 3959{,}65cm^2}{232{,}36cm} = 54{,}71\text{cm}$$

$$Msd_1{}^* = Fsd.e_1{}^* = 159{,}08KN \times 54{,}71cm = 8703{,}27KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 2528{,}4KN.cm - 8703{,}27KN.cm = -6174{,}87KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} = \frac{159{,}08KN}{116{,}18cm \times 11{,}5cm} + 0{,}60 \times \frac{0}{7414{,}62cm^2 \times 11{,}5cm}$$

$$Tsd_1 = 0{,}119\frac{KN}{cm^2} = 1{,}19MPa$$

$$Trd_1 = 0{,}68MPa$$

$$Tsd_1 = 1{,}19MPa > Trd_1 = 0{,}68MPa \therefore \textit{Colocar mais um contorno de armação!}$$

7 – Inserir mais um contorno de armação:

Obs: Como o momento final é nulo e continuará sendo nulo com o aumento da excentricidade, consegue-se estimar o valor de “p” para que a solicitação seja menor que a resistência.

$$Tsd_1 \leq Trd_1 \therefore Tsd_1 \leq 0{,}68MPa \therefore Tsd_1 \leq 0{,}068\frac{KN}{cm^2}$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + 0 = \frac{159{,}08KN}{u^*.11{,}5cm} = \frac{13{,}83}{u^*}\frac{KN}{cm}$$

$$\frac{13{,}83}{u^*}\frac{KN}{cm} \leq 0{,}068\frac{KN}{cm^2} \therefore u^* \geq 204cm$$

$$u^* = a_1 + a_2 + \pi.d + \frac{\pi.p}{2} = 17{,}25cm \times 2 + \pi \times 11{,}5cm + \frac{\pi \times p}{2}$$

$$u^* = 70{,}63cm + 1{,}57 \times p$$

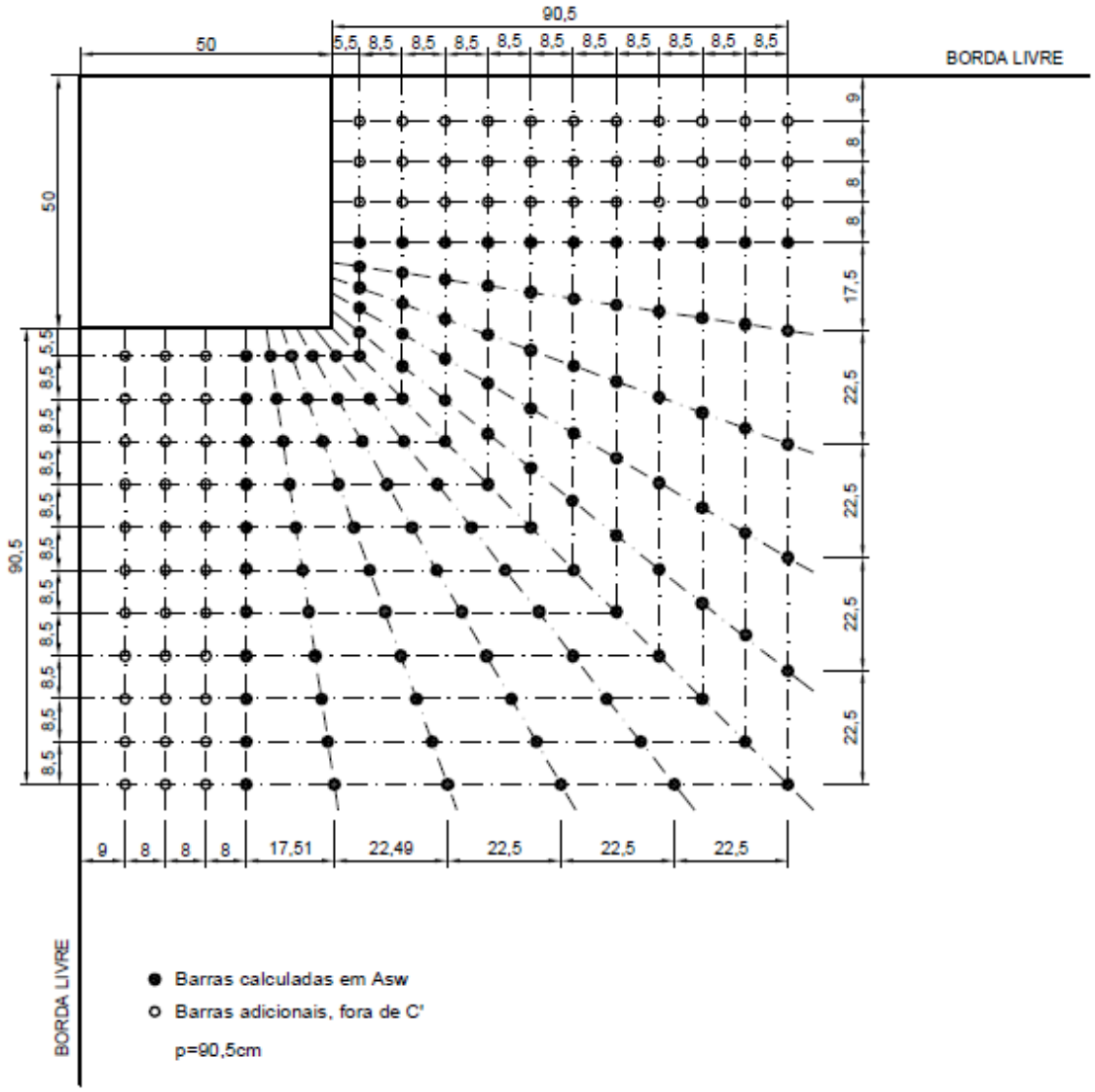
$$70{,}63cm + 1{,}57 \times p \geq 204cm \therefore p \geq 84{,}95cm$$

8 – Nova disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}50cm$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}50cm$$

$$s_e \leq 2.d \therefore s_e \leq 2 \times 11{,}5cm \therefore s_e \leq 23cm \therefore s_e = 23{,}00cm$$



$11Ø8.0mm = 5{,}5cm^2 > 4{,}28cm^2 \therefore ok! \therefore Por\ contorno\ serão\ 17Ø8.0mm$

9 – Verificação do novo contorno C":

$$p = 90{,}5cm$$

$$u^* = a_1 + a_2 + \pi.d + \frac{\pi.p}{2} = 17{,}25cm + 17{,}25cm + \pi.11{,}5cm + \frac{\pi.90{,}5cm}{2}$$

$$u^* = 212{,}79cm$$

9.1 – Análise de Msd_1:

$$wp_1 = \frac{{C_1}^2}{4} + \frac{C_1.C_2}{2} + 2.C_2.d + 4.d^2 - \frac{\pi.d.C_1}{2} + C_2.p + 4.d.p + \frac{\pi.p.C_1}{4} + p^2$$

$$= \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} + 2 \times 50cm \times 11{,}5cm + 4 \times (11{,}5cm)^2 = 3554cm^2$$

$$= \frac{\pi \times 11{,}5cm \times 50cm}{2} = 903{,}21cm^2$$

$$= 50cm \times 90{,}5cm + 4 \times 11{,}5cm \times 90{,}5cm + \frac{\pi \times 90{,}5cm \times 50cm}{4} + (90{,}5cm)^2 = 20432{,}18cm^2$$

$$wp_1 = 3554cm^2 - 903{,}21cm^2 + 20432{,}18cm^2 = 23082{,}97cm^2$$

$$e_1{}^* = \frac{C_1.a_1 - a_1{}^2 + a_2.C_1 + 4.a_2.d + 8.d^2 + \pi.d.C_1 + 2.a_2.p + 8.d.p + \frac{\pi.p.C_1}{2} + 2.p^2}{2.\left(a_1 + a_2 + \pi.d + \frac{\pi.p}{2}\right)}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm = 1427{,}44\text{cm}^2$$

$$= 4 \times 17{,}25cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 3657{,}92cm^2$$

$$= 2 \times 17{,}25cm \times 90{,}5cm + 8 \times 11{,}5cm \times 90{,}5cm = 11448{,}25\text{cm}^2$$

$$= \frac{\pi \times 90{,}5\text{cm} \times 50\text{cm}}{2} + 2 \times (90{,}5cm)^2 = 23488{,}35cm^2$$

$$= 2 \times \left(17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm + \frac{\pi \times 90{,}5cm}{2}\right) = 425{,}57cm$$

$$e_1{}^* = \frac{1427{,}44cm^2 + 3657{,}92cm^2 + 11448{,}25cm^2 + 23488{,}35cm^2}{425{,}57cm} = 94{,}04\text{cm}$$

$$Msd_1{}^* = Fsd.e_1{}^* = 159{,}08KN \times 94{,}04cm = 14959{,}88KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 2528{,}4KN.cm - 14959{,}88KN.cm = -12431{,}48KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} = \frac{159{,}08KN}{212{,}79cm \times 11{,}5cm} + 0{,}60 \times \frac{0}{23082{,}97cm^2 \times 11{,}5cm}$$

$$Tsd_1 = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} = \frac{159{,}08KN}{212{,}79cm \times 11{,}5cm} + 0 = 0{,}065\frac{KN}{cm^2} = 0{,}65MPa$$

$$Trd_1 = 0{,}68MPa$$

$$Tsd_1 = 0{,}65MPa < Trd_1 = 0{,}68MPa \therefore ok!$$

9.2 – Análise de Msd_2:

$$wp_2 = \frac{{C_1}^2}{4} + \frac{C_2.C_1}{2} + 2.C_1.d + 4.d^2 - \frac{\pi.d.C_2}{2} + C_1.p + 4.d.p + \frac{\pi.p.C_2}{4} + p^2$$

$$= \frac{(50cm)^2}{4} + \frac{50cm \times 50cm}{2} + 2 \times 50cm \times 11{,}5cm + 4 \times (11{,}5cm)^2 = 3554cm^2$$

$$= \frac{\pi \times 11{,}5cm \times 50cm}{2} = 903{,}21cm^2$$

$$= 50cm \times 90{,}5cm + 4 \times 11{,}5cm \times 90{,}5cm + \frac{\pi \times 90{,}5cm \times 50cm}{4} + (90{,}5cm)^2 = 20432{,}18cm^2$$

$$wp_2 = 3554cm^2 - 903{,}21cm^2 + 20432{,}18cm^2 = 23082{,}97cm^2$$

$${e_2}^* = \frac{C_2.a_2 - {a_2}^2 + a_1.C_2 + 4.a_1.d + 8.d^2 + \pi.d.C_2 + 2.a_1.p + 8.d.p + \frac{\pi.p.C_2}{2} + 2.p^2}{2.\left(a_2 + a_1 + \pi.d + \frac{\pi.p}{2}\right)}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + 17{,}25cm \times 50cm = 1427{,}44\text{cm}^2$$

$$= 4 \times 17{,}25cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 3657{,}92cm^2$$

$$= 2 \times 17{,}25cm \times 90{,}5cm + 8 \times 11{,}5cm \times 90{,}5cm = 11448{,}25\text{cm}^2$$

$$= \frac{\pi \times 90{,}5\text{cm} \times 50\text{cm}}{2} + 2 \times (90{,}5cm)^2 = 23488{,}35cm^2$$

$$= 2 \times \left(17{,}25cm + 17{,}25cm + \pi \times 11{,}5cm + \frac{\pi \times 90{,}5cm}{2}\right) = 425{,}57cm$$

$${e_2}^* = \frac{1427{,}44cm^2 + 3657{,}92cm^2 + 11448{,}25cm^2 + 23488{,}35cm^2}{425{,}57cm} = 94{,}04\text{cm}$$

$${Msd_2}^* = Fsd.{e_2}^* = 159{,}08KN \times 94{,}04cm = 14959{,}88KN.cm$$

$$Msd_{2_{fim}} \geq \begin{cases} 0 \\ Msd_2 - {Msd_2}^* = 3434{,}2KN.cm - 14959{,}88KN.cm = -11525{,}68KN.cm \end{cases}$$

$$Msd_{2_{fim}} = 0$$

$$\frac{C_2}{C_1} = \frac{50cm}{50cm} = 1 \therefore K_2 = 0{,}60$$

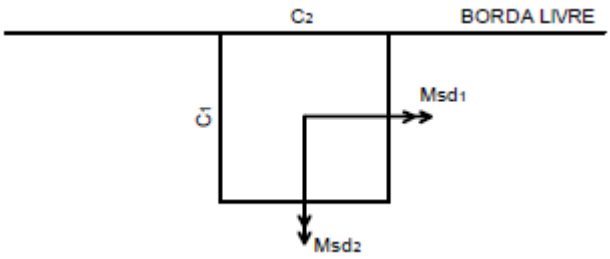
$$Tsd_2 = \frac{Fsd}{u^*.d} + K_2.\frac{Msd_{2_{fim}}}{wp_2.d} = \frac{159{,}08KN}{212{,}79cm \times 11{,}5cm} + 0{,}60 \times \frac{0}{23082{,}97cm^2 \times 11{,}5cm}$$

$$Tsd_2 = 0{,}065\frac{KN}{cm^2} = 0{,}65MPa$$

$$Trd_1 = 0{,}68MPa$$

$$Tsd_1 = 0{,}65MPa < Trd_1 = 0{,}68MPa \therefore ok!$$

Verificação dos pilares P2, P8, usando como referência o pilar P2:



1 – Dados:

$$C_1 = 50cm$$

$$C_2 = 50cm$$

$$d = h - d' = 16cm - 4{,}5cm = 11{,}5cm$$

$$Fsd = Fz.1{,}4 = 170{,}36KN \times 1{,}4 = 238{,}50KN\ (Compressão)$$

$$Msd_1 = M_x.1{,}4 = 3811KN.cm \times 1{,}4 = 5335{,}4KN.cm$$

$$Msd_2 = M_y.1{,}4 = 0\ KN.cm \times 1{,}4 = 0\ KN.cm$$

$$a_0 \leq 1{,}5.d \therefore a_0 \leq 1{,}5 \times 11{,}5cm \therefore a_0 \leq 17{,}25cm \therefore a_0 = 17{,}25cm$$

$$a \leq \begin{cases} 1{,}5.d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ 0{,}5.C_2 = 0{,}5 \times 50cm = 25cm \end{cases} \therefore a = 17{,}25cm$$

2 – Verificação do Contorno C:

$$u^* = 2.a_0 + C_2 = 2 \times 17{,}25cm + 50cm = 84{,}5cm$$

$$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} = \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} = 2500cm^2$$

$$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 = \frac{(50cm)^2}{4} + 50cm \times 50cm = 3125cm^2$$

$$e^* = \frac{C_1.a_0 - {a_0}^2 + \frac{C_1.C_2}{2}}{2.a_0 + C_2} = \frac{50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm \times 50cm}{2}}{2 \times 17{,}25cm + 50cm}$$

$$e^* = 21{,}48cm$$

$${Msd_1}^* = Fsd.e^* = 238{,}5KN \times 21{,}48cm = 5122{,}98KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 5335{,}4KN.cm - 5122{,}98KN.cm = 212{,}42KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 212{,}42KN.cm$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{238{,}50KN}{84{,}5cm \times 11{,}5cm} + 0{,}6 \times \frac{212{,}42KN.cm}{2500cm^2 \times 11{,}5cm} + 0{,}45 \times \frac{0}{3125cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}25\frac{KN}{cm^2} = 2{,}5MPa$$

$$Trd_2 = 0{,}27.\propto_v.fcd = 0{,}27 \times \left(1 - \frac{fck}{250}\right) \times \left(\frac{fck}{1{,}4}\right) = 0{,}27 \times \left(1 - \frac{25MPa}{250}\right) \times \left(\frac{25MPa}{1{,}4}\right)$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd = 2{,}5MPa < Trd_2 = 4{,}34MPa \therefore ok!$$

2 – Verificação do Contorno C':

$$u^* = 2.a + C_2 + 2.\pi.d = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm = 156{,}76cm$$

$$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} + 2.C_2.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_1$$

$$= \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} + 2 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} = 3650\mathrm{cm}^2$$

$$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 2864{,}42cm^2$$

$$wp_1 = 3650cm^2 + 2864{,}42cm^2 = 6514{,}42cm^2$$

$$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 + 4.C_1.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_2$$

$$= \frac{(50cm)^2}{4} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} = 5425\mathrm{cm}^2$$

$$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 2864{,}42cm^2$$

$$wp_2 = 5425cm^2 + 2864{,}42cm^2 = 8289{,}42cm^2$$

$$e^* = \frac{C_1.a - a^2 + \frac{C_1 + C_2}{2} + 2.C_2.d + 8.d^2 + \pi.d.C_1}{2.a + C_2 + 2.\pi.d} =$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm + 50cm}{2} = 614{,}94cm^2$$

$$= 2 \times 50cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 4014{,}42cm^2$$

$$= 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm = 156{,}76cm$$

$$e^* = \frac{614{,}94cm^2 + 4014{,}42cm^2}{156{,}76cm} = 29{,}53cm$$

$$Msd_1{}^* = Fsd.e^* = 238{,}5KN \times 29{,}53cm = 7042{,}91KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 5335{,}4KN.cm - 7042{,}91KN.cm = -1707{,}51KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{238{,}50KN}{156{,}76cm \times 11{,}5cm} + 0{,}6 \times \frac{0}{2500cm^2 \times 11{,}5cm} + 0{,}45 \times \frac{0}{3125cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}132\frac{KN}{cm^2} = 1{,}32MPa$$

$$Ø12{,}5mm \text{ à } cada\ 12cm \therefore \rho_x = \frac{As_x}{Ac} = \frac{1{,}25cm^2}{12cm \times 16cm} = 0{,}0065$$

$$Ø12.5mm \text{ à } cada\ 13cm \therefore \rho_y = \frac{As_y}{Ac} = \frac{1{,}25cm^2}{13cm \times 16cm} = 0{,}006$$

$$\rho = \sqrt{\rho_x.\rho_y} = \sqrt{0{,}0065 \times 0{,}006} = 0{,}0062$$

$$Trd_1 = 0{,}13.\left(1+\sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}}$$

$$Trd_1 = 0{,}13 \times \left(1+\sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0062 \times 25MPa)^{\frac{1}{3}} = 0{,}75MPa$$

$$Tsd = 1{,}32MPa > Trd_1 = 0{,}75MPa \therefore Armar \text{ à } punção!$$

2.1 – Cálculo da armadura de punção:

$$\alpha = 90° \therefore sen90° = 1{,}0$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}0cm$$

$$fywd = 250 + 185.\frac{(h-15)}{20} = 250 + 185 \times \frac{(16cm-15)}{20} = 259{,}20MPa \therefore 26\frac{KN}{cm^2}$$

$$Trd_3 = 0{,}10.\left(1+\sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}} + 1{,}5.\frac{d}{s_r}.\frac{Asw.fywd.sen\propto}{u.d}$$

$$= 0{,}10 \times \left(1+\sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0062 \times 25MPa)^{\frac{1}{3}} = 0{,}58MPa$$

$$= 1{,}5 \times \frac{11{,}5cm}{8cm} \times \frac{Asw \times 259{,}2MPa \times 1}{156{,}76cm \times 11{,}5cm} = 0{,}31 \times Asw \times \frac{MPa}{cm^2}$$

$$Trd_3 = 0{,}58MPa + 0{,}31 \times Asw \times \frac{MPa}{cm^2}$$

$$Tsd \leq Trd_3$$

$$1{,}32MPa \leq 0{,}58MPa + 0{,}31 \times Asw \times \frac{MPa}{cm^2}$$

$$Asw \geq 2{,}39cm^2$$

2.2 – Cálculo da armadura de punção obrigatória:

$$Asw \geq \frac{1}{2}.\frac{Fsd}{fywd} \therefore Asw \geq \frac{1}{2} \times \frac{238{,}50KN}{26\frac{KN}{cm^2}} \therefore Asw \geq 4{,}59cm^2$$

Serão no mínimo 3 contornos de armação à punção, então:

$$Asw \geq \frac{4{,}59cm^2}{3\ contornos} = 1{,}53cm^2$$

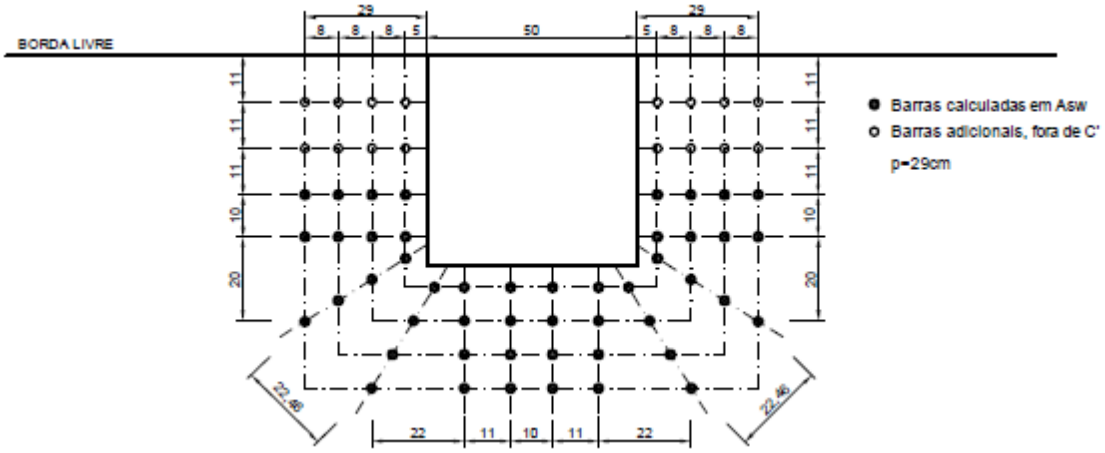
$$Asw \therefore 2{,}39cm^2 > 1{,}53cm^2 \therefore ok!$$

2.3 – Disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}00cm$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}00cm$$

$$s_e \leq 2.d \therefore s_e \leq 2 \times 11{,}5cm \therefore s_e \leq 23cm \therefore s_e = 23{,}00cm$$



$$12\text{Ø}5.0mm = 2{,}40cm^2 > 2{,}39cm^2 \therefore ok! \therefore Por\ contorno\ serão\ 16\text{Ø}5.0mm$$

3 – Verificação do Contorno C”:

$$p = 29cm$$

$$u^* = 2.a + C_2 + 2.\pi.d + \pi.p$$

$$u^* = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 29cm$$

$$u^* = 247{,}86cm$$

$$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} + 2.C_2.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_1 + C_2.\mathrm{p} + 8.\mathrm{d}.\mathrm{p} + \frac{\pi.p.C_1}{2} + 2.p^2$$

$$= \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} + 2 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 4708\mathrm{cm}^2$$

$$= \pi \times 11{,}5cm \times 50cm + 50cm \times 29cm + 8 \times 11{,}5cm \times 29cm = 5924{,}42cm^2$$

$$= \frac{\pi \times 29cm \times 50cm}{2} + 2 \times (29cm)^2 = 3959{,}65cm^2$$

$$wp_1 = 4708cm^2 + 5924{,}42cm^2 + 3959{,}65cm^2 = 14592{,}07cm^2$$

$$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 + 4.C_1.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_2 + 2.C_1.p + 8.d.p + \frac{\pi.p.C_2}{2} + 2.p^2$$

$$= \frac{(50cm)^2}{4} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 6483\mathrm{cm}^2$$

$$= \pi \times 11{,}5cm \times 50cm + 2 \times 50cm \times 29cm + 8 \times 11{,}5cm \times 29cm = 7374{,}42cm^2$$

$$= \frac{\pi \times 29cm \times 50cm}{2} + 2 \times (29cm)^2 = 3959{,}65cm^2$$

$$wp_2 = 6483cm^2 + 7374{,}42cm^2 + 3959{,}65cm^2 = 17817{,}07cm^2$$

$$e^* = \frac{C_1.a - a^2 + \frac{C_1 + C_2}{2} + 2.C_2.d + 8.d^2 + \pi.d.C_1 + C_2.p + 8.d.p + \frac{\pi.p.C_1}{2} + 2.p^2}{2.a + C_2 + 2.\pi.d + \pi.p}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm + 50cm}{2} + 2 \times 50cm \times 11{,}5cm = 1764{,}94cm^2$$

$$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm + 50cm \times 29cm = 4314{,}42cm^2$$

$$= 8 \times 11{,}5cm \times 29cm + \frac{\pi \times 29cm \times 50cm}{2} + 2 \times (29cm)^2 = 6627{,}65cm^2$$

$$= 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 29cm = 247{,}86cm$$

$$e^* = \frac{1764{,}94cm^2 + 4314{,}42cm^2 + 6627{,}65cm^2}{247{,}86cm} = 51{,}27m$$

$${Msd_1}^* = Fsd.e^* = 238{,}5KN \times 51{,}27cm = 12227{,}90KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - {Msd_1}^* = 5335{,}4KN.cm - 12227{,}9KN.cm = -6892{,}5KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{238{,}50KN}{247{,}86cm \times 11{,}5cm} + 0{,}6.\frac{0}{14592{,}07cm^2 \times 11{,}5cm} + 0{,}45.\frac{0}{17817{,}07cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}084\frac{KN}{cm^2} = 0{,}84MPa$$

$$Trd_1 = 0{,}75MPa$$

$$Tsd = 0{,}84MPa > Trd_1 = 0{,}75MPa \therefore Colocar\ mais\ um\ contorno\ de\ armação!$$

4 – Inserir mais um contorno de armação:

Obs: Como o momento final é nulo e continuará sendo nulo com o aumento da excentricidade, consegue-se estimar o valor de "p" para que a solicitação seja menor que a resistência.

$$Tsd \leq Trd_1 \therefore Tsd \leq 0{,}75MPa \therefore Tsd \leq 0{,}075\frac{KN}{cm^2}$$

$$Tsd = \frac{Fsd}{u^*.d} + 0 = \frac{238{,}5KN}{u^*.11{,}5cm} = \frac{20{,}74}{u^*}\frac{KN}{cm}$$

$$\frac{20{,}74}{u^*}\frac{KN}{cm} \leq 0{,}075\frac{KN}{cm^2} \therefore u^* \geq 276{,}53cm$$

$$u^* = 2.a + C_2 + 2.\pi.d + \pi.p$$

$$u^* = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times p$$

$$u^* = 156{,}76cm + \pi \times p$$

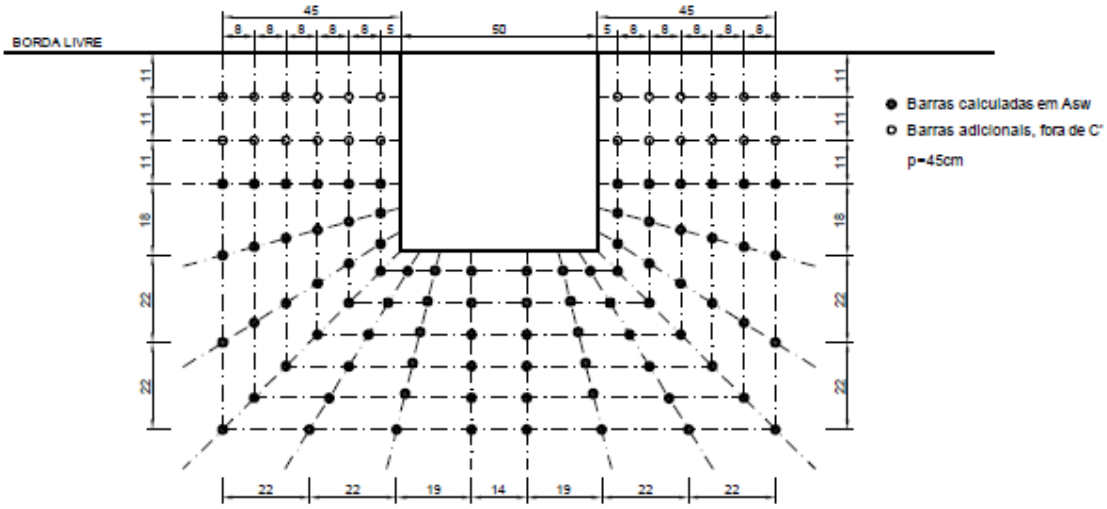
$$156{,}76cm + \pi \times p \geq 276{,}53cm \therefore p \geq 38{,}12cm$$

5 – Nova disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}00cm$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}00cm$$

$$s_s \leq 2.d \therefore s_s \leq 2 \times 11{,}5cm \therefore s_s \leq 23cm \therefore s_s = 23{,}00cm$$



$14Ø5.0mm = 2{,}74cm^2 > 2{,}39cm^2 \therefore ok! \therefore$ *Por contorno serão* $18Ø5.0mm$

9 – Verificação do novo contorno C":

$p = 45cm$

$u^* = 2.a + C_2 + 2.\pi.d + \pi.p$

$u^* = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 45cm$

$u^* = 298{,}13cm$

$$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} + 2.C_2.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_1 + C_2.\mathrm{p} + 8.\mathrm{d}.\mathrm{p} + \frac{\pi.p.C_1}{2} + 2.p^2$$

$$= \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} + 2 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 4708\mathrm{cm}^2$$

$$= \pi \times 11{,}5cm \times 50cm + 50cm \times 45cm + 8 \times 11{,}5cm \times 45cm = 8196{,}42cm^2$$

$$= \frac{\pi \times 45cm \times 50cm}{2} + 2 \times (45cm)^2 = 7584{,}29cm^2$$

$$wp_1 = 4708cm^2 + 8196{,}42cm^2 + 7584{,}29cm^2 = 20488{,}71cm^2$$

$$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 + 4.C_1.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_2 + 2.C_1.p + 8.d.p + \frac{\pi.p.C_2}{2} + 2.p^2$$

$$= \frac{(50cm)^2}{4} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 6483\mathrm{cm}^2$$

$$= \pi \times 11{,}5cm \times 50cm + 2 \times 50cm \times 45cm + 8 \times 11{,}5cm \times 45cm = 10446{,}42cm^2$$

$$= \frac{\pi \times 45cm \times 50cm}{2} + 2 \times (45cm)^2 = 7584{,}29cm^2$$

$$wp_2 = 6483cm^2 + 10446{,}42cm^2 + 7584{,}29cm^2 = 24513{,}71cm^2$$

$$e^* = \frac{C_1.a - a^2 + \frac{C_1 + C_2}{2} + 2.C_2.d + 8.d^2 + \pi.d.C_1 + C_2.p + 8.d.p + \frac{\pi.p.C_1}{2} + 2.p^2}{2.a + C_2 + 2.\pi.d + \pi.p}$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm + 50cm}{2} + 2 \times 50cm \times 11{,}5cm = 1764{,}94cm^2$$

$$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm + 50cm \times 45cm = 5114{,}42cm^2$$

$$= 8 \times 11{,}5cm \times 45cm + \frac{\pi \times 45cm \times 50cm}{2} + 2 \times (45cm)^2 = 11724{,}29cm^2$$

$$= 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 45cm = 298{,}13cm$$

$$e^* = \frac{1764{,}94cm^2 + 5114{,}42cm^2 + 11724{,}29cm^2}{298{,}13cm} = 62{,}4cm$$

$${Msd_1}^* = Fsd.e^* = 238{,}5KN \times 62{,}4cm = 14882{,}4KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - {Msd_1}^* = 5335{,}4KN.cm - 14882{,}4KN.cm = -9547KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

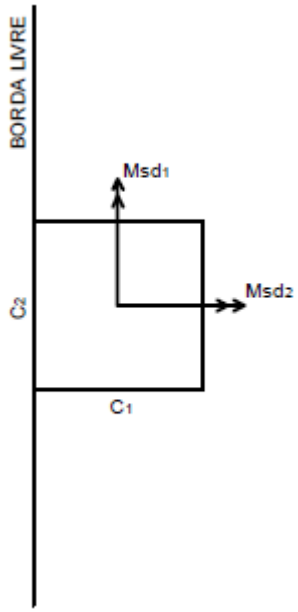
$$Tsd = \frac{238{,}50KN}{298{,}13cm \times 11{,}5cm} + 0{,}6.\frac{0}{20488{,}71cm^2 \times 11{,}5cm} + 0{,}45.\frac{0}{24513{,}71cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}070\frac{KN}{cm^2} = 0{,}70MPa$$

$$Trd_1 = 0{,}75MPa$$

$$Tsd = 0{,}70MPa < Trd_1 = 0{,}75MPa \therefore ok!$$

Verificação dos pilares P4 e P6, usando como referência o pilar P4:



1 – Dados:

$C_1 = 50cm$

$C_2 = 50cm$

$d = h - d' = 16cm - 4{,}5cm = 11{,}5cm$

$Fsd = Fz.1{,}4 = 178{,}84KN \times 1{,}4 = 250{,}38KN\ (Compressão)$

$Msd_1 = M_y.1{,}4 = 2706KN.cm \times 1{,}4 = 3788{,}4KN.cm$

$Msd_2 = M_x.1{,}4 = 0\ KN.cm \times 1{,}4 = 0\ KN.cm$

$a_0 \leq 1{,}5.d \therefore a_0 \leq 1{,}5 \times 11{,}5cm \therefore a_0 \leq 17{,}25cm \therefore a_0 = 17{,}25cm$

$$a \leq \begin{cases} 1{,}5.d = 1{,}5 \times 11{,}5cm = 17{,}25cm \\ 0{,}5.C_2 = 0{,}5 \times 50cm = 25cm \end{cases} \therefore a = 17{,}25cm$$

2 – Verificação do Contorno C:

$u^* = 2.a_0 + C_2 = 2 \times 17{,}25cm + 50cm = 84{,}5cm$

$$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} = \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} = 2500cm^2$$

$$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 = \frac{(50cm)^2}{4} + 50cm \times 50cm = 3125cm^2$$

$$e^* = \frac{C_1.a_0 - {a_0}^2 + \frac{C_1.C_2}{2}}{2.a_0 + C_2} = \frac{50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm \times 50cm}{2}}{2 \times 17{,}25cm + 50cm}$$

$e^* = 21{,}48cm$

${Msd_1}^* = Fsd.e^* = 250{,}38KN \times 21{,}48cm = 5378{,}16KN.cm$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - {Msd_1}^* = 3788{,}4KN.cm - 5378{,}16KN.cm = -1589{,}76KN.cm \end{cases}$$

$Msd_{1_{fim}} = 0$

$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{250{,}38KN}{84{,}5cm \times 11{,}5cm} + 0{,}6 \times \frac{0}{2500cm^2 \times 11{,}5cm} + 0{,}45 \times \frac{0}{3125cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}258\frac{KN}{cm^2} = 2{,}58MPa$$

$$Trd_2 = 0{,}27.\propto_v.fcd = 0{,}27 \times \left(1 - \frac{fck}{250}\right) \times \left(\frac{fck}{1{,}4}\right) = 0{,}27 \times \left(1 - \frac{25MPa}{250}\right) \times \left(\frac{25MPa}{1{,}4}\right)$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd = 2{,}58MPa < Trd_2 = 4{,}34MPa \therefore ok!$$

3 – Verificação do Contorno C':

$$u^* = 2.a + C_2 + 2.\pi.d = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm = 156{,}76cm$$

$$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} + 2.C_2.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_1$$

$$= \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} + 2 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} = 3650\mathrm{cm}^2$$

$$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 2864{,}42cm^2$$

$$wp_1 = 3650cm^2 + 2864{,}42cm^2 = 6514{,}42cm^2$$

$$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 + 4.C_1.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_2$$

$$= \frac{(50cm)^2}{4} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} = 5425\mathrm{cm}^2$$

$$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 2864{,}42cm^2$$

$$wp_2 = 5425cm^2 + 2864{,}42cm^2 = 8289{,}42cm^2$$

$$e^* = \frac{C_1.a - a^2 + \frac{C_1 + C_2}{2} + 2.C_2.d + 8.d^2 + \pi.d.C_1}{2.a + C_2 + 2.\pi.d} =$$

$$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm + 50cm}{2} = 614{,}94cm^2$$

$$= 2 \times 50cm \times 11{,}5cm + 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm = 4014{,}42cm^2$$

$$= 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm = 156{,}76cm$$

$$e^* = \frac{614{,}94cm^2 + 4014{,}42cm^2}{156{,}76cm} = 29{,}53cm$$

$$Msd_1{}^* = Fsd.e^* = 250{,}38KN \times 29{,}53cm = 7393{,}72KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 3788{,}4KN.cm - 7393{,}72KN.cm = -3605{,}32KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{250{,}38KN}{156{,}76cm \times 11{,}5cm} + 0{,}6 \times \frac{0}{6514{,}42cm^2 \times 11{,}5cm} + 0{,}45 \times \frac{0}{8289{,}42cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}139\frac{KN}{cm^2} = 1{,}39MPa$$

$$Ø10mm \text{ à } cada\ 14cm \therefore \rho_x = \frac{As_x}{Ac} = \frac{0{,}8cm^2}{14cm \times 16cm} = 0{,}0036$$

$$Ø16mm \text{ à } cada\ 13cm \therefore \rho_y = \frac{As_y}{Ac} = \frac{2cm^2}{13cm \times 16cm} = 0{,}0096$$

$$\rho = \sqrt{\rho_x.\rho_y} = \sqrt{0{,}0036 \times 0{,}0096} = 0{,}0059$$

$$Trd_1 = 0{,}13.\left(1 + \sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}}$$

$$Trd_1 = 0{,}13 \times \left(1 + \sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0062 \times 25MPa)^{\frac{1}{3}} = 0{,}75MPa$$

$$Tsd = 1{,}39MPa > Trd_1 = 0{,}75MPa \therefore Armar \text{ à } punção!$$

3.1 – Cálculo da armadura de punção:

$$\alpha = 90° \therefore sen90° = 1{,}0$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}0cm$$

$$fywd = 250 + 185.\frac{(h-15)}{20} = 250 + 185 \times \frac{(16cm-15)}{20} = 259{,}20MPa \therefore 26\frac{KN}{cm^2}$$

$$Trd_3 = 0{,}10.\left(1 + \sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}} + 1{,}5.\frac{d}{s_r}.\frac{Asw.fywd.sen\propto}{u.d}$$

$$= 0{,}10 \times \left(1 + \sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0059 \times 25MPa)^{\frac{1}{3}} = 0{,}57MPa$$

$$= 1{,}5 \times \frac{11{,}5cm}{8cm} \times \frac{Asw \times 259{,}2MPa \times 1}{156{,}76cm \times 11{,}5cm} = 0{,}31 \times Asw \times \frac{MPa}{cm^2}$$

$$Trd_3 = 0{,}57MPa + 0{,}31 \times Asw \times \frac{MPa}{cm^2}$$

$$Tsd \leq Trd_3$$

$$1{,}4MPa \leq 0{,}57MPa + 0{,}31 \times Asw \times \frac{MPa}{cm^2}$$

$$Asw \geq 2{,}68cm^2$$

3.2 – Cálculo da armadura de punção obrigatória:

$$Asw \geq \frac{1}{2}.\frac{Fsd}{fywd} \therefore Asw \geq \frac{1}{2} \times \frac{250{,}38KN}{26\frac{KN}{cm^2}} \therefore Asw \geq 4{,}82cm^2$$

Serão no mínimo 3 contornos de armação à punção, então:

$$Asw \geq \frac{4{,}82cm^2}{3\ contornos} = 1{,}61cm^2$$

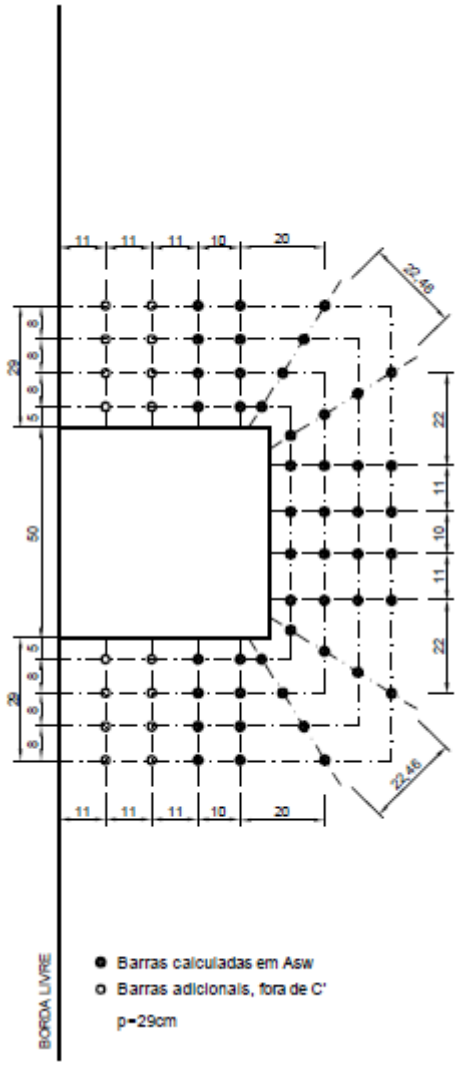
$$Asw \therefore 2{,}68cm^2 > 1{,}61cm^2 \therefore ok!$$

3.3 – Disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}00cm$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}00cm$$

$$s_e \leq 2.d \therefore s_e \leq 2 \times 11{,}5cm \therefore s_e \leq 23cm \therefore s_e = 23{,}00cm$$



$$12\text{Ø}6.3mm = 3{,}78cm^2 > 2{,}68cm^2 \therefore ok! \therefore Por\ contorno\ serão\ 16\text{Ø}6.3mm$$

4 – Verificação do Contorno C":

$p = 29cm$

$u^* = 2.a + C_2 + 2.\pi.d + \pi.p$

$u^* = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 29cm$

$u^* = 247{,}86cm$

$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} + 2.C_2.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_1 + C_2.\mathrm{p} + 8.\mathrm{d}.\mathrm{p} + \frac{\pi.p.C_1}{2} + 2.p^2$

$= \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} + 2 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 4708\mathrm{cm}^2$

$= \pi \times 11{,}5cm \times 50cm + 50cm \times 29cm + 8 \times 11{,}5cm \times 29cm = 5924{,}42cm^2$

$= \frac{\pi \times 29cm \times 50cm}{2} + 2 \times (29cm)^2 = 3959{,}65cm^2$

$wp_1 = 4708cm^2 + 5924{,}42cm^2 + 3959{,}65cm^2 = 14592{,}07cm^2$

$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 + 4.C_1.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_2 + 2.C_1.p + 8.d.p + \frac{\pi.p.C_2}{2} + 2.p^2$

$= \frac{(50cm)^2}{4} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 6483\mathrm{cm}^2$

$= \pi \times 11{,}5cm \times 50cm + 2 \times 50cm \times 29cm + 8 \times 11{,}5cm \times 29cm = 7374{,}42cm^2$

$= \frac{\pi \times 29cm \times 50cm}{2} + 2 \times (29cm)^2 = 3959{,}65cm^2$

$wp_2 = 6483cm^2 + 7374{,}42cm^2 + 3959{,}65cm^2 = 17817{,}07cm^2$

$e^* = \frac{C_1.a - a^2 + \frac{C_1 + C_2}{2} + 2.C_2.d + 8.d^2 + \pi.d.C_1 + C_2.p + 8.d.p + \frac{\pi.p.C_1}{2} + 2.p^2}{2.a + C_2 + 2.\pi.d + \pi.p}$

$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm + 50cm}{2} + 2 \times 50cm \times 11{,}5cm = 1764{,}94cm^2$

$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm + 50cm \times 29cm = 4314{,}42cm^2$

$= 8 \times 11{,}5cm \times 29cm + \frac{\pi \times 29cm \times 50cm}{2} + 2 \times (29cm)^2 = 6627{,}65cm^2$

$= 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 29cm = 247{,}86cm$

$$e^* = \frac{1764{,}94cm^2 + 4314{,}42cm^2 + 6627{,}65cm^2}{247{,}86cm} = 51{,}27m$$

$$Msd_1{}^* = Fsd.e^* = 250{,}38KN \times 51{,}27cm = 12836{,}98KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 3788{,}4KN.cm - 12836{,}98KN.cm = -9048{,}58KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{250{,}38KN}{247{,}86cm \times 11{,}5cm} + 0{,}6.\frac{0}{14592{,}07cm^2 \times 11{,}5cm} + 0{,}45.\frac{0}{17817{,}07cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}088\frac{KN}{cm^2} = 0{,}88MPa$$

$$Trd_1 = 0{,}75MPa$$

$$Tsd = 0{,}88MPa > Trd_1 = 0{,}75MPa \therefore Colocar\ mais\ um\ contorno\ de\ armação!$$

5 – Inserir mais um contorno de armação:

Obs: Como o momento final é nulo e continuará sendo nulo com o aumento da excentricidade, consegue-se estimar o valor de “p” para que a solicitação seja menor que a resistência.

$$Tsd \leq Trd_1 \therefore Tsd \leq 0{,}75MPa \therefore Tsd \leq 0{,}075\frac{KN}{cm^2}$$

$$Tsd = \frac{Fsd}{u^*.d} + 0 = \frac{250{,}38KN}{u^*.11{,}5cm} = \frac{21{,}77}{u^*}\frac{KN}{cm}$$

$$\frac{21{,}77}{u^*}\frac{KN}{cm} \leq 0{,}075\frac{KN}{cm^2} \therefore u^* \geq 290{,}27cm$$

$$u^* = 2.a + C_2 + 2.\pi.d + \pi.p$$

$$u^* = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times p$$

$$u^* = 156{,}76cm + \pi \times p$$

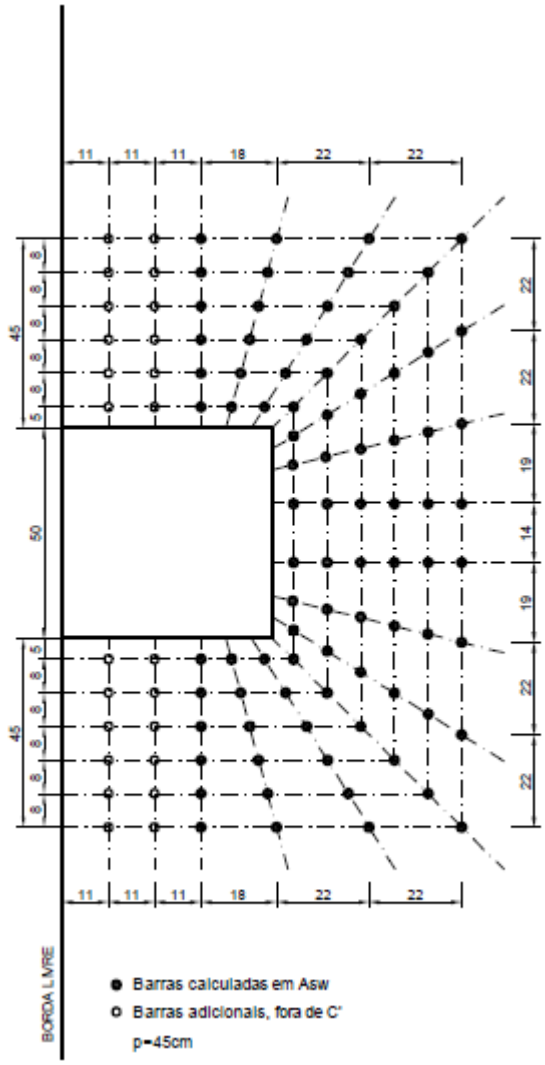
$$156{,}76cm + \pi \times p \geq 290{,}27cm \therefore p \geq 42{,}51cm$$

6 – Nova disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}00cm$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}00cm$$

$$s_e \leq 2.d \therefore s_e \leq 2 \times 11{,}5cm \therefore s_e \leq 23cm \therefore s_e = 23{,}00cm$$



$$14\text{Ø}5.0mm = 2{,}74cm^2 > 2{,}68cm^2 \therefore ok! \therefore Por\ contorno\ serão\ 18\text{Ø}5.0mm$$

7 – Verificação do novo contorno C":

$p = 45cm$

$u^* = 2.a + C_2 + 2.\pi.d + \pi.p$

$u^* = 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 45cm$

$u^* = 298{,}13cm$

$wp_1 = \frac{{C_1}^2}{2} + \frac{C_1.C_2}{2} + 2.C_2.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_1 + C_2.\mathrm{p} + 8.\mathrm{d}.\mathrm{p} + \frac{\pi.p.C_1}{2} + 2.p^2$

$= \frac{(50cm)^2}{2} + \frac{50cm \times 50cm}{2} + 2 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 4708\mathrm{cm}^2$

$= \pi \times 11{,}5cm \times 50cm + 50cm \times 45cm + 8 \times 11{,}5cm \times 45cm = 8196{,}42cm^2$

$= \frac{\pi \times 45cm \times 50cm}{2} + 2 \times (45cm)^2 = 7584{,}29cm^2$

$wp_1 = 4708cm^2 + 8196{,}42cm^2 + 7584{,}29cm^2 = 20488{,}71cm^2$

$wp_2 = \frac{{C_2}^2}{4} + C_1.C_2 + 4.C_1.\mathrm{d} + 8.\mathrm{d}^2 + \pi.\mathrm{d}.C_2 + 2.C_1.p + 8.d.p + \frac{\pi.p.C_2}{2} + 2.p^2$

$= \frac{(50cm)^2}{4} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 8 \times (11{,}5\mathrm{cm})^2 = 6483\mathrm{cm}^2$

$= \pi \times 11{,}5cm \times 50cm + 2 \times 50cm \times 45cm + 8 \times 11{,}5cm \times 45cm = 10446{,}42cm^2$

$= \frac{\pi \times 45cm \times 50cm}{2} + 2 \times (45cm)^2 = 7584{,}29cm^2$

$wp_2 = 6483cm^2 + 10446{,}42cm^2 + 7584{,}29cm^2 = 24513{,}71cm^2$

$e^* = \frac{C_1.a - a^2 + \frac{C_1 + C_2}{2} + 2.C_2.d + 8.d^2 + \pi.d.C_1 + C_2.p + 8.d.p + \frac{\pi.p.C_1}{2} + 2.p^2}{2.a + C_2 + 2.\pi.d + \pi.p}$

$= 50cm \times 17{,}25cm - (17{,}25cm)^2 + \frac{50cm + 50cm}{2} + 2 \times 50cm \times 11{,}5cm = 1764{,}94cm^2$

$= 8 \times (11{,}5cm)^2 + \pi \times 11{,}5cm \times 50cm + 50cm \times 45cm = 5114{,}42cm^2$

$= 8 \times 11{,}5cm \times 45cm + \frac{\pi \times 45cm \times 50cm}{2} + 2 \times (45cm)^2 = 11724{,}29cm^2$

$= 2 \times 17{,}25cm + 50cm + 2 \times \pi \times 11{,}5cm + \pi \times 45cm = 298{,}13cm$

$$e^* = \frac{1764{,}94cm^2 + 5114{,}42cm^2 + 11724{,}29cm^2}{298{,}13cm} = 62{,}4cm$$

$$Msd_1{}^* = Fsd.e^* = 250{,}38KN \times 62{,}4cm = 15623{,}71KN.cm$$

$$Msd_{1_{fim}} \geq \begin{cases} 0 \\ Msd_1 - Msd_1{}^* = 3788{,}4KN.cm - 15623{,}71KN.cm = -11835{,}31KN.cm \end{cases}$$

$$Msd_{1_{fim}} = 0$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{2.C_1} = \frac{50cm}{2 \times 50cm} = 0{,}5 \therefore K_2 = 0{,}45$$

$$Tsd = \frac{Fsd}{u^*.d} + K_1.\frac{Msd_{1_{fim}}}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

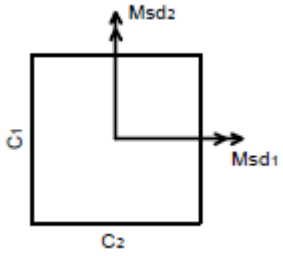
$$Tsd = \frac{250{,}38KN}{298{,}13cm \times 11{,}5cm} + 0{,}6.\frac{0}{20488{,}71cm^2 \times 11{,}5cm} + 0{,}45.\frac{0}{24513{,}71cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}073\frac{KN}{cm^2} = 0{,}73MPa$$

$$Trd_1 = 0{,}75MPa$$

$$Tsd = 0{,}73MPa < Trd_1 = 0{,}75MPa \therefore ok!$$

Verificação do pilar P5:



1 – Dados:

$C_1 = 50cm$

$C_2 = 50cm$

$d = h - d' = 16cm - 4{,}5cm = 11{,}5cm$

$Fsd = Fz.1{,}4 = 253{,}38KN \times 1{,}4 = 354{,}73KN\ (Compressão)$

$Msd_1 = M_x.1{,}4 = 0\ KN.cm \times 1{,}4 = 0\ KN.cm$

$Msd_2 = M_y.1{,}4 = 0\ KN.cm \times 1{,}4 = 0\ KN.cm$

2 – Verificação do Contorno C:

$$u = 2.(C_1 + C_2) = 2 \times (50cm + 50cm) = 200cm$$

$$wp_1 = \frac{{C_1}^2}{2} + C_1.C_2 = \frac{(50cm)^2}{2} + 50cm \times 50cm = 3750cm^2$$

$$wp_2 = \frac{{C_2}^2}{2} + C_2.C_1 = \frac{(50cm)^2}{2} + 50cm \times 50cm = 3750m^2$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{C_1} = \frac{50cm}{50cm} = 1 \therefore K_2 = 0{,}60$$

$$Tsd = \frac{Fsd}{u.d} + K_1.\frac{Msd_1}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{354{,}73KN}{200cm \times 11{,}5cm} + 0{,}6 \times \frac{0}{3750cm^2 \times 11{,}5cm} + 0{,}6 \times \frac{0}{3750cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}154\frac{KN}{cm^2} = 1{,}54MPa$$

$$Trd_2 = 0{,}27.\propto_v.fcd = 0{,}27 \times \left(1 - \frac{fck}{250}\right) \times \left(\frac{fck}{1{,}4}\right) = 0{,}27 \times \left(1 - \frac{25MPa}{250}\right) \times \left(\frac{25MPa}{1{,}4}\right)$$

$$Trd_2 = 4{,}34MPa$$

$$Tsd = 1{,}54MPa < Trd_2 = 4{,}34MPa \therefore ok!$$

3 – Verificação do Contorno C':

$$u = 2.(C_1 + C_2) + 4.\pi.d = 2 \times (50cm + 50cm) + 4 \times \pi \times 11{,}5cm = 344{,}51cm$$

$$wp_1 = \frac{{C_1}^2}{2} + C_1.C_2 + 4.C_2.\mathrm{d} + 16.\mathrm{d}^2 + 2.\pi.\mathrm{d}.C_1$$

$$= \frac{(50cm)^2}{2} + 50\mathrm{cm} \times 50\mathrm{cm} + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} = 6050\mathrm{cm}^2$$

$$= 16 \times (11{,}5\mathrm{cm})^2 + 2 \times \pi \times 11{,}5\mathrm{cm} \times 50\mathrm{cm} = 5728{,}83\mathrm{cm}^2$$

$$wp_1 = 6050cm^2 + 5728{,}83cm^2 = 11778{,}83cm^2$$

$$wp_2 = \frac{{C_2}^2}{2} + C_2.C_1 + 4.C_1.\mathrm{d} + 16.\mathrm{d}^2 + 2.\pi.\mathrm{d}.C_2$$

$$= \frac{(50cm)^2}{2} + 50\mathrm{cm} \times 50\mathrm{cm} + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} = 6050\mathrm{cm}^2$$

$$= 16 \times (11{,}5\mathrm{cm})^2 + 2 \times \pi \times 11{,}5\mathrm{cm} \times 50\mathrm{cm} = 5728{,}83\mathrm{cm}^2$$

$$wp_2 = 6050cm^2 + 5728{,}83cm^2 = 11778{,}83cm^2$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{C_1} = \frac{50cm}{50cm} = 1 \therefore K_2 = 0{,}60$$

$$Tsd = \frac{Fsd}{u.d} + K_1.\frac{Msd_1}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{354{,}73KN}{344{,}51cm \times 11{,}5cm} + 0{,}6 \times \frac{0}{11778{,}83m^2 \times 11{,}5cm} + 0{,}6 \times \frac{0}{11778{,}83cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}090\frac{KN}{cm^2} = 0{,}90MPa$$

$$Ø12{,}5mm \text{ à } cada\ 12cm \therefore \rho_x = \frac{As_x}{Ac} = \frac{1{,}25cm^2}{12cm \times 16cm} = 0{,}0065$$

$$Ø16mm \text{ à } cada\ 13cm \therefore \rho_y = \frac{As_y}{Ac} = \frac{2cm^2}{13cm \times 16cm} = 0{,}0096$$

$$\rho = \sqrt{\rho_x.\rho_y} = \sqrt{0{,}0065 \times 0{,}0096} = 0{,}0079$$

$$Trd_1 = 0{,}13.\left(1 + \sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}}$$

$$Trd_1 = 0{,}13 \times \left(1 + \sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0079 \times 25MPa)^{\frac{1}{3}} = 0{,}81MPa$$

$$Tsd = 0{,}90MPa > Trd_1 = 0{,}81MPa \therefore Armar \text{ à } punção!$$

3.1 – Cálculo da armadura de punção:

$$\alpha = 90° \therefore sen90° = 1{,}0$$

$$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}0cm$$

$$fywd = 250 + 185.\frac{(h-15)}{20} = 250 + 185 \times \frac{(16cm-15)}{20} = 259{,}20MPa \therefore 26\frac{KN}{cm^2}$$

$$Trd_3 = 0{,}10.\left(1+\sqrt{\frac{20}{d}}\right).(100.\rho.fck)^{\frac{1}{3}} + 1{,}5.\frac{d}{s_r}.\frac{Asw.fywd.sen\propto}{u.d}$$

$$= 0{,}10 \times \left(1+\sqrt{\frac{20}{11{,}5cm}}\right) \times (100 \times 0{,}0079 \times 25MPa)^{\frac{1}{3}} = 0{,}63MPa$$

$$= 1{,}5 \times \frac{11{,}5cm}{8cm} \times \frac{Asw \times 259{,}2MPa \times 1}{344{,}51cm \times 11{,}5cm} = 0{,}14 \times Asw \times \frac{MPa}{cm^2}$$

$$Trd_3 = 0{,}63MPa + 0{,}14 \times Asw \times \frac{MPa}{cm^2}$$

$$Tsd \leq Trd_3$$

$$0{,}90MPa \leq 0{,}63MPa + 0{,}14 \times Asw \times \frac{MPa}{cm^2}$$

$$Asw \geq 1{,}93cm^2$$

3.2 – Cálculo da armadura de punção obrigatória:

$$Asw \geq \frac{1}{2}.\frac{Fsd}{fywd} \therefore Asw \geq \frac{1}{2} \times \frac{354{,}73KN}{26\frac{KN}{cm^2}} \therefore Asw \geq 6{,}84cm^2$$

Serão no mínimo 3 contornos de armação à punção, então:

$$Asw \geq \frac{6{,}84cm^2}{3\ contornos} = 2{,}28cm^2$$

$$Asw \therefore 1{,}93cm^2 < 2{,}28cm^2 \therefore Não\ ok!$$

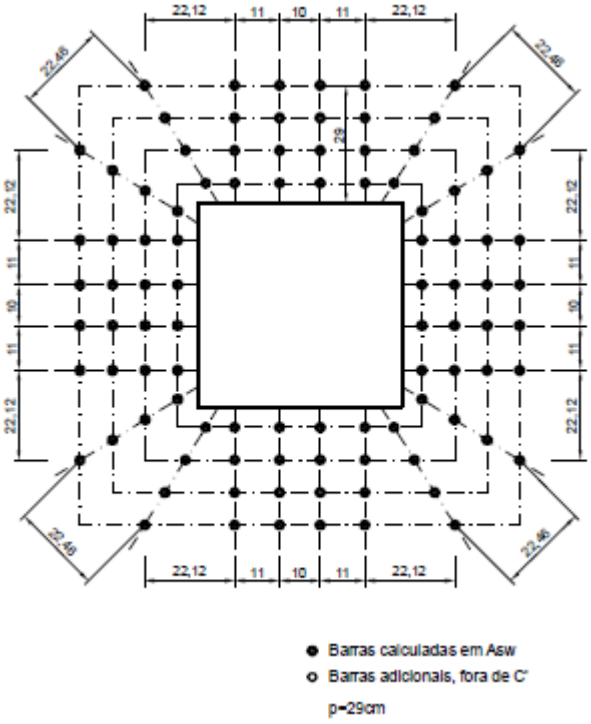
Obs: Portanto, se for detalhado apenas 3 contornos de armação, é necessário armar considerando 2,28cm^2 por contorno ao invés de 1,93cm^2 por contorno.

3.3 – Disposição construtiva:

$$s_0 \leq 0{,}5.d \therefore s_0 \leq 0{,}5 \times 11{,}5cm \therefore s_0 \leq 5{,}75cm \therefore s_0 = 5{,}00cm$$

$s_r \leq 0{,}75.d \therefore s_r \leq 0{,}75 \times 11{,}5cm \therefore s_r \leq 8{,}63cm \therefore s_r = 8{,}00cm$

$s_s \leq 2.d \therefore s_s \leq 2 \times 11{,}5cm \therefore s_s \leq 23cm \therefore s_s = 23{,}00cm$



$24\text{Ø}5.0mm = 4{,}80cm^2 > 1{,}93cm^2 \therefore ok! \therefore Por\ contorno\ serão\ 24\text{Ø}5.0mm$

4 – Verificação do contorno C":

$p = 29cm$

$u = 2.(C_1 + C_2) + 4.\pi.d + 2.\pi.p$

$u = 2 \times (50cm + 50cm) + 4 \times \pi \times 11{,}5cm + 2 \times \pi \times 29cm = 526{,}73cm$

$wp_1 = \frac{{C_1}^2}{2} + C_1.C_2 + 4.C_2.\mathrm{d} + 16.\mathrm{d}^2 + 2.\pi.\mathrm{d}.C_1 + 2.C_2.\mathrm{p} + 16.\mathrm{d}.\mathrm{p} + 4.p^2 + \pi.C_1.\mathrm{p}$

$= \frac{(50cm)^2}{2} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 16 \times (11{,}5\mathrm{cm})^2 = 8166\mathrm{cm}^2$

$= 2 \times \pi \times 11{,}5cm \times 50cm + 2 \times 50cm \times 29cm = 6512{,}83cm^2$

$= 16 \times 11{,}5cm \times 29cm + 4 \times (29cm)^2 + \pi \times 50cm \times 29cm = 13255{,}31cm^2$

$wp_1 = 8166cm^2 + 6512{,}83cm^2 + 13255{,}31cm^2 = 27934{,}14cm^2$

$$wp_2 = \frac{{C_2}^2}{2} + C_2.C_1 + 4.C_1.\mathrm{d} + 16.\mathrm{d}^2 + 2.\pi.\mathrm{d}.C_2 + 2.C_1.\mathrm{p} + 16.\mathrm{d}.\mathrm{p} + 4.p^2 + \pi.C_2.\mathrm{p}$$

$$= \frac{(50cm)^2}{2} + 50cm \times 50cm + 4 \times 50\mathrm{cm} \times 11{,}5\mathrm{cm} + 16 \times (11{,}5\mathrm{cm})^2 = 8166\mathrm{cm}^2$$

$$= 2 \times \pi \times 11{,}5cm \times 50cm + 2 \times 50cm \times 29cm = 6512{,}83cm^2$$

$$= 16 \times 11{,}5cm \times 29cm + 4 \times (29cm)^2 + \pi \times 50cm \times 29cm = 13255{,}31cm^2$$

$$wp_2 = 8166cm^2 + 6512{,}83cm^2 + 13255{,}31cm^2 = 27934{,}14cm^2$$

$$\frac{C_1}{C_2} = \frac{50cm}{50cm} = 1 \therefore K_1 = 0{,}60$$

$$\frac{C_2}{C_1} = \frac{50cm}{50cm} = 1 \therefore K_2 = 0{,}60$$

$$Tsd = \frac{Fsd}{u.d} + K_1.\frac{Msd_1}{wp_1.d} + K_2.\frac{Msd_2}{wp_2.d}$$

$$Tsd = \frac{354{,}73KN}{526{,}73cm \times 11{,}5cm} + 0{,}6.\frac{0}{27934{,}14cm^2 \times 11{,}5cm} + 0{,}6.\frac{0}{27934{,}14cm^2 \times 11{,}5cm}$$

$$Tsd = 0{,}059\frac{KN}{cm^2} = 0{,}59MPa$$

$$Trd_1 = 0{,}81MPa$$

$$Tsd = 0{,}59MPa < Trd_1 = 0{,}81MPa \therefore ok!$$