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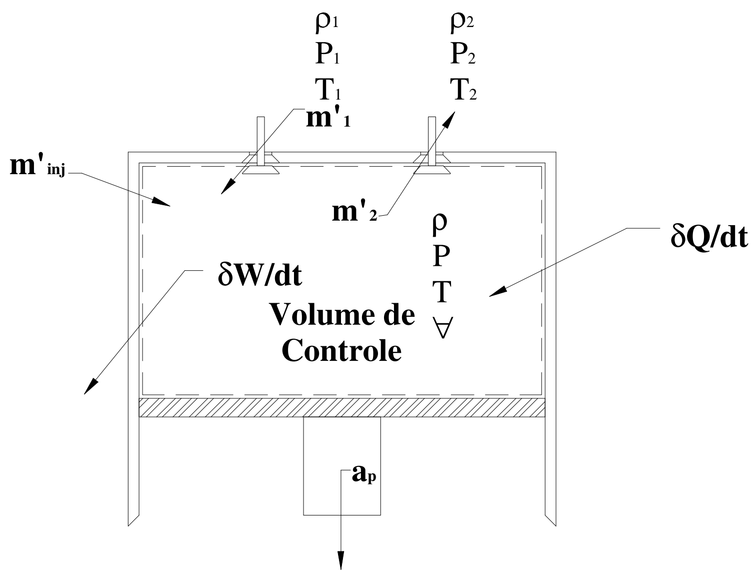
ANEXO D

DEDUÇÕES ALGÉBRICAS:

D.1 MODELAGEM MATEMÁTICA ZERO-DIMENSIONAL DE UMA ZONA, PARA UM CILINDRO DE MOTOR A PISTÃO



Equação da Conservação da Massa:

$$\iint_{S.C.} \rho\ \vec{V} \cdot d\vec{A} = -\frac{\partial}{\partial t} \iiint_{V.C.} \rho\, d\forall$$

$$\iint_{A_1} \rho_1\ \vec{V}_1 \cdot d\vec{A} + \iint_{A_2} \rho_2\ \vec{V}_2 \cdot d\vec{A} = -\frac{\partial}{\partial t} \iiint_{V.C.} \rho\, d\forall$$

Integrando ao longo do volume de controle e considerando uma distribuição homogênea no interior do mesmo, tem-se,

$$-\dot{m}_{inj} - \dot{m}_1 + \dot{m}_2 = -\frac{d}{dt}(\rho \forall)$$

onde, o sub-índice 1 é para o fluxo que entra no volume de controle e o sub-índice 2 é para o fluxo que deixa o volume de controle. A vazão mássica de injeção direta de combustível no cilindro é representada por

$$\dot{m}_{inj}$$

Mas a massa específica do gás que sai é igual a do gás no interior do cilindro, supondo uma distribuição homogênea dentro do mesmo, assim,

.

$$\rho_2 = \rho$$

$$\rho \frac{d\forall}{dt} + \forall \frac{d\rho}{dt} = \dot{m}_1 + \dot{m}_{inj} - \dot{m}_2$$

$$\forall \frac{d\rho}{dt} = \dot{m}_1 + \dot{m}_{inj} - \dot{m}_2 - \rho \frac{d\forall}{dt}$$

$$\frac{d\rho}{dt} = \frac{1}{\forall}\left(\dot{m}_1 + \dot{m}_{inj} - \dot{m}_2 - \rho \frac{d\forall}{dt} \right)$$

Mudando da variável independente tempo para posição angular do virabrequim (υ), tem-se,

$$\frac{d\theta}{dt} = \omega \Rightarrow dt = \frac{d\theta}{\omega}$$

onde, ϖ = velocidade de rotação (rad/s)

Portanto,

$$\frac{d\rho}{d\theta} = \frac{1}{\omega \forall}\left(\dot{m}_1 + \dot{m}_{inj} - \dot{m}_2 - \rho\,\omega \frac{d\forall}{d\theta} \right)$$

Equação da Conservação de Energia:

$$\frac{\delta Q}{dt} - \frac{\delta W_s}{dt} = \iint_{S.C.} \left(e + \frac{P}{\rho} \right) \rho \left(\vec{V} \cdot \vec{n} \right) dA + \frac{\partial}{\partial t} \iiint_{V.C} e \rho \, d\forall + \frac{\delta W_\mu}{dt}$$

onde, $e = u + \frac{V^2}{2} + g z$

para o motor alternativo,

$$\frac{\delta W_s}{dt} = P \frac{d\forall}{dt}$$

$$\frac{\delta Q}{dt} = \frac{\delta Q_f}{dt} + \frac{\delta Q_l}{dt}$$

.

portanto,

$$\frac{\delta Q_f}{dt}+\frac{\delta Q_1}{dt}-P\frac{d\mathcal{V}}{dt}=-\left(e_1+\frac{P_1}{\rho_1}\right)\dot{m}_1+\left(e_2+\frac{P_2}{\rho_2}\right)\dot{m}_2+\frac{d}{dt}(e\rho\mathcal{V})+\frac{\delta W_\mu}{dt}$$

mas, $e+\dfrac{P}{\rho}=h+\dfrac{V^2}{2}+gz$

$$\mathcal{V}\frac{d(\rho e)}{dt}+\rho e\frac{d\mathcal{V}}{dt}=\frac{\delta Q_f}{dt}+\frac{\delta Q_1}{dt}-P\frac{d\mathcal{V}}{dt}+\frac{d}{dt}(e\rho\mathcal{V})-\frac{\delta W_\mu}{dt}+$$
$$+\dot{m}_1\left(h_1+\frac{V_1^2}{2}+gz_1\right)-\dot{m}_2\left(h_2+\frac{V_2^2}{2}+gz_2\right)$$

$$\rho\mathcal{V}\frac{de}{dt}+e\mathcal{V}\frac{d\rho}{dt}+\rho e\frac{d\mathcal{V}}{dt}=\frac{\delta Q_f}{dt}+\frac{\delta Q_1}{dt}-P\frac{d\mathcal{V}}{dt}-\frac{\delta W_\mu}{dt}+$$
$$+\dot{m}_1\left(h_1+\frac{V_1^2}{2}+gz_1\right)-\dot{m}_2\left(h_2+\frac{V_2^2}{2}+gz_2\right)$$

$$\rho\mathcal{V}\frac{d}{dt}\left(u+\frac{V^2}{2}+gz\right)=-\left(u+\frac{V^2}{2}+gz\right)\left(\mathcal{V}\frac{d\rho}{dt}+\rho\frac{d\mathcal{V}}{dt}\right)+\frac{\delta Q_f}{dt}+\frac{\delta Q_1}{dt}-P\frac{d\mathcal{V}}{dt}-\frac{\delta W_\mu}{dt}+$$
$$+\dot{m}_1\left(h_1+\frac{V_1^2}{2}+gz_1\right)-\dot{m}_2\left(h_2+\frac{V_2^2}{2}+gz_2\right)$$

$$\rho\mathcal{V}\frac{du}{dt}+\rho\mathcal{V}V\frac{dV}{dt}+\rho\mathcal{V}\frac{d(gz)}{dt}=-\left(h+\frac{V^2}{2}+gz-\frac{P}{\rho}\right)\left(\mathcal{V}\frac{d\rho}{dt}+\rho\frac{d\mathcal{V}}{dt}\right)+\frac{\delta Q_f}{dt}+\frac{\delta Q_1}{dt}-P\frac{d\mathcal{V}}{dt}-$$
$$-\frac{\delta W_\mu}{dt}+\dot{m}_1\left(h_1+\frac{V_1^2}{2}+gz_1\right)-\dot{m}_2\left(h_2+\frac{V_2^2}{2}+gz_2\right)$$

$$\rho\mathcal{V}c_v\frac{dT}{dt}=-\rho\mathcal{V}\left(V\frac{dV}{dt}+\frac{d(gz)}{dt}\right)-\left(h+\frac{V^2}{2}+gz-\frac{P}{\rho}\right)\left(\mathcal{V}\frac{d\rho}{dt}+\rho\frac{d\mathcal{V}}{dt}\right)+\frac{\delta Q_f}{dt}+\frac{\delta Q_1}{dt}-P\frac{d\mathcal{V}}{dt}-$$
$$-\frac{\delta W_\mu}{dt}+\dot{m}_1\left(h_1+\frac{V_1^2}{2}+gz_1\right)-\dot{m}_2\left(h+\frac{V_2^2}{2}+gz_2\right)$$

mas, $c_v=\dfrac{R}{(\gamma-1)}$ (para gás ideal) e desprezando a energia potencial,

.

$$\frac{dT}{dt}=\frac{(\gamma-1)}{\rho\forall R}\left[-\rho\forall\left(V\frac{dV}{dt}\right)-\left(h+\frac{V^2}{2}-\frac{P}{\rho}\right)\left(\forall\frac{d\rho}{dt}+\rho\frac{d\forall}{dt}\right)+\frac{\delta Q_f}{dt}+\frac{\delta Q_l}{dt}-P\frac{d\forall}{dt}-\right.$$
$$\left.-\frac{\delta W_\mu}{dt}+\dot{m}_1\left(h_1+\frac{V_1^2}{2}\right)-\dot{m}_2\left(h+\frac{V_2^2}{2}\right)\right]$$

trocando a variável independente,

$$\frac{dT}{d\theta}=\frac{(\gamma-1)}{\rho\forall\omega R}\left[-\rho\forall\omega\left(V\frac{dV}{d\theta}\right)-\omega\left(h+\frac{V^2}{2}-\frac{P}{\rho}\right)\left(\forall\frac{d\rho}{d\theta}+\rho\frac{d\forall}{d\theta}\right)+\omega\frac{\delta Q_f}{d\theta}+\omega\frac{\delta Q_l}{d\theta}-\right.$$
$$\left.-\omega P\frac{d\forall}{d\theta}-\omega\frac{\delta W_\mu}{d\theta}+\dot{m}_1\left(h_1+\frac{V_1^2}{2}\right)+\dot{m}_{inj}h_f-\dot{m}_2\left(h+\frac{V_2^2}{2}\right)\right]$$

Equação da Conservação da Quantidade de Movimento:

$$\sum\vec{F}_{ext}=\iint_{S.C.}\vec{V}\rho\left(\vec{V}\cdot\vec{n}\right)dA+\frac{\partial}{\partial t}\iiint_{V.C.}\rho V\,d\forall$$

integrando ao longo do volume de controle,

$$\sum\vec{F}_{ext}=-\rho_1 V_1^2 A_1+\rho V_2^2 A_2+\frac{d}{dt}\left(\rho V\forall\right)$$

$$\sum\vec{F}_{ext}=-\rho_1 V_1^2 A_1+\rho V_2^2 A_2+\rho V\frac{d\forall}{dt}+\forall\left(V\frac{d\rho}{dt}+\rho\frac{dV}{dt}\right)$$

mas, o balanço de forças que atuam no gás confinado no cilindro,

$$\sum\vec{F}_{ext}=-M_p a_p-M_g g+F_\mu$$

onde,

$M_p =$ massa do pistão, dos anéis e de parte oscilante da biela
$a_p =$ aceleração do pistão.

.

portanto,

$$\frac{dV}{dt}=\frac{1}{\rho \forall}\left[-M_p\, a_p - M_g\, g + F_\mu + \rho_1 V_1^2 A_1 - \rho V_2^2 A_2 - \rho V \frac{d\forall}{dt} - \forall V \frac{d\rho}{dt}\right]$$

trocando a variável independente,

$$\frac{dV}{d\theta}=\frac{1}{\rho \forall \omega}\left[-M_p\, a_p - M_g\, g + F_\mu + \dot{m}_1 V_1 - \dot{m}_2 V_2 - \rho V \omega \frac{d\forall}{d\theta} - \forall V \omega \frac{d\rho}{d\theta}\right]$$

Equação de Estado: (Gás Ideal)

$$P=\rho R T$$

tirando o logarítmo e derivando em relação ao ângulo do virabrequim,

$$\frac{1}{P}\frac{dP}{d\theta}=\frac{1}{\rho}\frac{d\rho}{d\theta}+\frac{1}{R}\frac{dR}{d\theta}+\frac{1}{T}\frac{dT}{d\theta}$$

$$\frac{dP}{d\theta}=\frac{P}{\rho}\frac{d\rho}{d\theta}+\frac{P}{R}\frac{dR}{d\theta}+\frac{P}{T}\frac{dT}{d\theta}$$

onde,

$$R=\sum_{i=1}^{N} C_i\, R_i$$

$$\frac{dR}{d\theta}=\sum_{i=1}^{N} R_i \frac{d\, C_i}{d\theta}$$

D.2 DEPURAÇÃO QUEIMA NO CILINDRO UMA ZONA

Conservação da massa:

$$\frac{d\rho}{d\theta}=\frac{1}{\omega \forall}\left(\dot{m}_1 + \dot{m}_{inj} - \dot{m}_2 - \rho \omega \frac{d\forall}{d\theta}\right)$$

Eliminando o fluxo de massa, temos,

$$\frac{d\rho}{d\theta}=\frac{1}{\forall}\left(-\rho \frac{d\forall}{d\theta}\right)$$

.

Conservação da energia:

$$\frac{dT}{d\theta}=\frac{(\gamma-1)}{\rho\mathcal{V}\omega R}\left[-\rho\mathcal{V}\omega\left(V\frac{dV}{d\theta}\right)-\omega\left(h+\frac{V^2}{2}-\frac{P}{\rho}\right)\left(\mathcal{V}\frac{d\rho}{d\theta}+\rho\frac{d\mathcal{V}}{d\theta}\right)+\omega\frac{\delta Q_f}{d\theta}+\omega\frac{\delta Q_l}{d\theta}-\right.$$

$$\left.-\omega P\frac{d\mathcal{V}}{d\theta}-\omega\frac{\delta W_\mu}{d\theta}+\dot{m}_1\left(h_1+\frac{V_1^2}{2}\right)+\dot{m}_{inj}h_f-\dot{m}_2\left(h+\frac{V_2^2}{2}\right)\right]$$

Desprezando alguns termos e eliminando o fluxo de massa através do VC, temos,

$$\frac{dT}{d\theta}=\frac{(\gamma-1)}{\rho\mathcal{V}R}\left[-\left(c_pT-\frac{P}{\rho}\right)\left(\mathcal{V}\frac{d\rho}{d\theta}+\rho\frac{d\mathcal{V}}{d\theta}\right)+\frac{\delta Q_f}{d\theta}-P\frac{d\mathcal{V}}{d\theta}\right]$$

Equação de estado:

$$\frac{1}{P}\frac{dP}{d\theta}=\frac{1}{\rho}\frac{d\rho}{d\theta}+\frac{1}{T}\frac{dT}{d\theta}$$

$$\gamma=\frac{c_p}{c_v}=\frac{c_p}{c_p-R}$$

$$c_p=\frac{\gamma}{(\gamma-1)}R$$

Portanto,

$$\frac{dT}{d\theta}=\frac{(\gamma-1)}{\rho\mathcal{V}R}\left[-\left(\frac{\gamma}{(\gamma-1)}RT-\frac{P}{\rho}\right)\left(\mathcal{V}\frac{d\rho}{d\theta}+\rho\frac{d\mathcal{V}}{d\theta}\right)+\frac{\delta Q_f}{d\theta}-P\frac{d\mathcal{V}}{d\theta}\right]$$

Mas, da equação da conservação da massa,

$$\mathcal{V}\frac{d\rho}{d\theta}+\rho\frac{d\mathcal{V}}{d\theta}=0$$

Assim,

$$\frac{dT}{d\theta}=\frac{(\gamma-1)}{\rho\mathcal{V}R}\left[\frac{\delta Q_f}{d\theta}-P\frac{d\mathcal{V}}{d\theta}\right]$$

Usando a equação do gás ideal na forma diferencial, temos,

.

$$\frac{1}{T}\frac{dT}{d\theta} = \frac{1}{P}\frac{dP}{d\theta} - \frac{1}{\rho}\frac{d\rho}{d\theta}$$

$$\frac{1}{T}\frac{dT}{d\theta} = \frac{1}{P}\frac{dP}{d\theta} - \frac{1}{\rho}\frac{d\rho}{d\theta}$$

Combinando com a conservação de massa,

$$\frac{dT}{d\theta} = \frac{T}{P}\frac{dP}{d\theta} - \frac{T}{\rho}\frac{d\rho}{d\theta} = \frac{T}{P}\frac{dP}{d\theta} + \frac{T}{V}\frac{dV}{d\theta}$$

Assim,

$$\frac{T}{P}\frac{dP}{d\theta} + \frac{T}{V}\frac{dV}{d\theta} = \frac{(\gamma-1)}{\rho V R}\left[\frac{\delta Q_f}{d\theta} - P\frac{dV}{d\theta}\right]$$

Mas, da equação do gás ideal,

$$\rho R = \frac{P}{T}$$

Assim,

$$\frac{T}{P}\frac{dP}{d\theta} + \frac{T}{V}\frac{dV}{d\theta} = \frac{T(\gamma-1)}{PV}\left[\frac{\delta Q_f}{d\theta} - P\frac{dV}{d\theta}\right]$$

$$\frac{\delta Q_f}{d\theta} - P\frac{dV}{d\theta} = \frac{1}{(\gamma-1)}\left[V\frac{dP}{d\theta} + P\frac{dV}{d\theta}\right]$$

$$\frac{\delta Q_f}{d\theta} = \frac{1}{(\gamma-1)}\left[V\frac{dP}{d\theta} + P\frac{dV}{d\theta}\right] + P\frac{dV}{d\theta}$$

$$\frac{dT}{d\theta} = T\left[\frac{1}{P}\frac{dP}{d\theta} + \frac{1}{V}\frac{dV}{d\theta}\right]$$

$$\frac{d\rho}{d\theta} = \frac{1}{V}\left(-\rho\frac{dV}{d\theta}\right) = -\frac{m}{V^2}\frac{dV}{d\theta}$$

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D.3 DEPURAÇÃO DE RAZÃO DE SANGRIA NO TURBOCOMPRESSOR

Balanço de energia no turbocompressor:

$$\dot{m}_c\, c_{pc}\,(T_{2c}-T_{1c})=\dot{m}_t\, c_{pt}\,(T_{2t}-T_{1t})+(\dot{m}_c-\dot{m}_t)c_{pt}T_{1t}$$

Razão de temperaturas: (x = c ou t) $\tau_x=\dfrac{T_{2x}}{T_{1x}}$

$$\dot{m}_c\, c_{pc}\, T_{1c}(\tau_c-1)=\dot{m}_t\, c_{pt}\, T_{1t}(\tau_t-1)+(\dot{m}_c-\dot{m}_t)c_{pt}T_{1t}$$

Razão de bypass pela válvula de sangria da turbina: $\beta=\dfrac{(\dot{m}_c-\dot{m}_t)}{\dot{m}_c}=1-\dfrac{\dot{m}_t}{\dot{m}_c}$

$$c_{pc}\, T_{1c}(\tau_c-1)=(1-\beta)c_{pt}\, T_{1t}(\tau_t-1)+\beta\, c_{pt}T_{1t}$$

$$c_{pc}\, T_{1c}(\tau_c-1)=c_{pt}\, T_{1t}(\tau_t-1)\;-\beta\, c_{pt}\, T_{1t}(\tau_t-1)+\beta\, c_{pt}T_{1t}$$

$$\beta=\frac{c_{pc}\, T_{1c}(\tau_c-1)-c_{pt}\, T_{1t}(\tau_t-1)}{-c_{pt}\, T_{1t}(\tau_t-1)+c_{pt}T_{1t}}$$

$$\beta=\frac{c_{pc}\, T_{1c}(\tau_c-1)-c_{pt}\, T_{1t}(\tau_t-1)}{c_{pt}\, T_{1t}(2-\tau_t)}$$